# Precision theory for high energy collider physics 

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## Precision physics at hadron colliders

- Precision tests of the Standard Model
- Measurements of masses and couplings
- Interplay of calculations and measurements
- Accuracy on many cross sections now $\approx(1 . .5) \%$
- Ultimate precision frontier at hadron colliders: 1\%
- Require theory predictions accurate at this level

Standard Model Production Cross Section Measurements
Status: February 2022


## State-of-the-art

- Precise predictions: perturbation theory expansion of observables
- Experimental measurements: fiducial cross sections
- theory predictions account for experimental cuts and definition of final state
- Automated tools for LO and NLO QCD and electroweak (2010's)
- infrastructure from event generator programs
- HERWIG, PYTHIA, SHERPA, aMC@NLO
- standard interface to one-loop amplitude providers
- BlackHat, GoSam, Recola, OpenLoops, NJet, MadLoop, CutTools
- Combined with parton shower

- full event properties with NLO accuracy on differential cross sections


## State-of-the-art

- NNLO QCD predictions for $2 \rightarrow 2$ processes (NNLO revolution, $2015 \rightarrow$ )
- accomplished during past 10 years on case-by-case basis
- as parton-level event generators (full final state information)
- computationally expensive
- current frontier at NNLO: $2 \rightarrow 3$
- Typical size of corrections and uncertainty
- NLO corrections: 10..100\%, uncertainty: 10..30\%
- NNLO corrections: $2 . .15 \%$, uncertainty: $3 . .8 \%$
- expect N3LO to yield uncertainty at level of $1 \%$.



## Fixed-order perturbation theory

- One extra parton per order in perturbation series
- Partons are combined into jets using same algorithm as in experiment


LO


NLO


NNLO


N3LO

- No algorithm dependence at leading order
- Theoretical description more accurate with increasing order
- Parton shower: multiple emissions, approximate description


## Ingredients to fixed order calculations

- Matrix elements with extra real (R) or virtual (V) partons

|  | Matrix elements | Parton evolution |
| :---: | :---: | :---: |
| LO | Born | 1-loop |
| NLO | R, V | 2-loop |
| NNLO | RR, RV, VV | 3-loop |
| N3LO | RRR, RRV, RVV, VVV | 4-loop |

- Infrared singularities in all R-type and V-type subprocesses
- sum of all subprocesses finite
- require procedure to arrange IR cancellations between subprocesses
- Incoming hadrons: parton distributions
- mass factorization of initial-state radiation and parton evolution


## Ingredients to fixed order calculations

- Different final state multiplicity for real and virtual corrections
- R: n+1 particles; V: n particles
- application of event selection, fiducial cuts: evaluate separately
- Upcycling of lower-order calculations
- only purely virtual correction (V, VV, VVV, ....) genuinely new
- real radiation corrections from higher-multiplicity calculations at lower order
- e.g. Higgs boson production: NNLO RV contribution = NLO V contribution to $\mathrm{H}+$ jet
- stability: use analytic one-loop amplitudes if available
- Cancellation of infrared singularities between subprocesses
- must evaluate integrals of type [Z.Kunszt, D.Soper]

$$
\mathcal{I}=\lim _{\epsilon \rightarrow 0}\left[\int_{0}^{1} \frac{d x}{x} x^{\epsilon} F(x)-\frac{1}{\epsilon} F(0)\right]
$$

## Methods

$$
\mathcal{I}=\lim _{\epsilon \rightarrow 0}\left[\int_{0}^{1} \frac{d x}{x} x^{\epsilon} F(x)-\frac{1}{\epsilon} F(0)\right]
$$

- Subtraction
- subtract singular (soft and/or collinear behavior) from R, integrate and add back

$$
\mathcal{I}=\lim _{\epsilon \rightarrow 0}\left[\int_{0}^{1} \frac{d x}{x} x^{\epsilon}(F(x)-F(0))+F(0) \int_{0}^{1} \frac{d x}{x} x^{\epsilon}-\frac{1}{\epsilon} F(0)\right]
$$

- many variants at NLO and NNLO: dipole, FKS, antenna, residue, sector-improved,.....
[S.Catani, M.Seymour; S.Frixione, Z.Kunszt, A.Signer; A.Gehrmann-De Ridder, N.Glover, TG; M.Czakon; F.Caola, K.Melnikov, R.Röntsch; V.del Duca, C.Duhr, A.Kardos, Z.Trocsanyi, G.Somogyi; G.Bertolotti, L.Magnea, G.Pelliccioli, A.Ratti, C.Signorile-Signorile, P.Torrielli, S.Uccirati]
- Slicing
- cut off singular region from phase space integral, add integrated below-cut contribution

$$
\mathcal{I} \approx \lim _{\epsilon \rightarrow 0}\left[\int_{\delta}^{1} \frac{d x}{x} x^{\epsilon} F(x)+F(0) \int_{0}^{\delta} \frac{d x}{x} x^{\epsilon}-\frac{1}{\epsilon} F(0)\right]=\int_{\delta}^{1} \frac{d x}{x} x^{\epsilon} F(x)+F(0) \ln \delta
$$

- variants up to N3LO, depending on slicing variable: $\mathrm{q}_{\mathrm{T}}, \mathrm{N}$-jettiness
[S.Catani, M.Grazzini; R.Boughezal, X.Liu, F.Petriello; J.Gaunt, M.Stahlhofen, F.Tackmann, J.Walsh]


## NNLO subtraction

- Structure of NNLO cross section

$$
\begin{aligned}
\mathrm{d} \sigma_{N N L O}= & \int_{\mathrm{d} \Phi_{m+2}}\left(\mathrm{~d} \sigma_{N N L O}^{R}-\mathrm{d} \sigma_{N N L O}^{S}\right) \\
& +\int_{\mathrm{d} \Phi_{m+1}}\left(\mathrm{~d} \sigma_{N N L O}^{V, 1}-\mathrm{d} \sigma_{N N L O}^{V S, 1}\right)+\int_{\mathrm{d} \Phi_{m+1}} \mathrm{~d} \sigma_{N N L O}^{M F, 1} \\
& +\int_{\mathrm{d} \Phi_{m}} \mathrm{~d} \sigma_{N N L O}^{V, 2}+\int_{\mathrm{d} \Phi_{m+2}} \mathrm{~d} \sigma_{N N L O}^{S}+\int_{\mathrm{d} \Phi_{m+1}} \mathrm{~d} \sigma_{N N L O}^{V S, 1}+\int_{\mathrm{d} \Phi_{m}} \mathrm{~d} \sigma_{N N L O}^{M F, 2}
\end{aligned}
$$

- Real and virtual contributions: $\mathrm{d} \sigma_{N N L O}^{R}, \mathrm{~d} \sigma_{N N L O}^{V, 1}, \mathrm{~d} \sigma_{N N L O}^{V, 2}$
- Subtraction term for double real radiation: $\mathrm{d} \sigma_{N N L O}^{S}$
- Subtraction term for one-loop single real radiation: $\mathrm{d} \sigma_{N N L O}^{V S, 1}$
- Mass factorization terms: $\mathrm{d} \sigma_{N N L O}^{M F, 1}, \mathrm{~d} \sigma_{N N L O}^{M F, 2}$
- Each line finite and free of poles $\rightarrow$ numerical implementation


## Antenna subtraction

- Subtraction terms constructed from antenna functions
- Antenna function contains all emission between two partons

- Phase space factorization

$$
d \Phi_{m+1}\left(p_{1}, \ldots, p_{m+1} ; q\right)=d \Phi_{m}\left(p_{1}, \ldots, \tilde{p}_{I}, \tilde{p}_{K}, \ldots, p_{m+1} ; q\right) \cdot d \Phi_{X_{i j k}}\left(p_{i}, p_{j}, p_{k} ; \tilde{p}_{I}+\tilde{p}_{K}\right)
$$

- Integrated subtraction term

$$
\mathcal{X}_{i j k}=\int d \Phi_{X_{i j k}} X_{i j k}
$$

## Antenna subtraction

- Colour-ordered pair of hard partons (radiators)
- Hard quark-antiquark pair
- Hard quark-gluon pair
- Hard gluon-gluon pair
- NLO [D. Kosower; J. Campbell, M. Cullen, E.W.N. Glover]
- Three-parton antenna: one unresolved parton
- NNLO [A. Gehrmann-De Ridder, E.W.N. Glover, TG]

- Four-parton antenna: two unresolved partons
- Three-parton antenna at one loop
- Products of NLO antenna functions
- Soft antenna function



## Antenna subtraction: incoming hadrons

- Three antenna types [A. Daleo, D. Maitre, TG]
- Final-final antenna

- Initial-final antenna
- Initial-intial antenna





## NNLOJET code

- NNLO parton level event generator
- Based on antenna subtraction
- Provides infrastructure
- Process management
- Phase space, histogram routines
- Validation and testing
- Parallel computing (MPI) support for warm-up and production
- ApplGrid/fastNLO interfaces in development
- Processes implemented at NNLO
- Z+(0,1) jet, $\gamma+1$ jet, $\mathrm{H}+(0,1) \mathrm{jet}, \mathrm{W}+(0,1) \mathrm{jet}, \mathrm{H}+2 \mathrm{jet}(\mathrm{VBF})$
- DIS-2j, LHC-2j
- Typical runtimes: 60’000-250'000 core-hours


## Triple-differential Drell-Yan cross section

- Lepton pair production: EW precision observable

$$
\frac{\mathrm{d}^{3} \sigma}{\mathrm{~d} m_{l l} \mathrm{~d} y_{l l} \mathrm{~d} \cos \theta^{*}}=\frac{\pi \alpha^{2}}{3 m_{l l} s} \sum_{q} P_{q}\left(\cos \theta^{*}\right)\left[f_{q}\left(x_{1}, Q^{2}\right) f_{\bar{q}}\left(x_{2}, Q^{2}\right)+(q \leftrightarrow \bar{q})\right]
$$



- ATLAS 8 TeV measurement [1710.05167]

| Observable | Central-Central | Central-Forward |
| :---: | :---: | :---: |
| $m_{l l}[\mathrm{GeV}]$ | $[46,66,80,91,102,116,150,200]$ | $[66,80,91,102,116,150]$ |
| $\left\|y_{l l}\right\|$ | $[0,0.2,0.4,0.6,0.8,1,1.2$, | $[1.2,1.6,2,2.4,2.8,3.6]$ |
| $\cos \theta^{*}$ | $1.4,1.6,1.8,2,2.2,2.4]$ |  |
| Total Bin Count: | $[-1,-0.7,-0.4,0,0.4,0.7,1]$ | $[-1,-0.7,-0.4,0,0.4,0.7,1]$ |



## Triple-differential Drell-Yan cross section

- Measured with fiducial event selection cuts (on single leptons)

| Central-Central | Central-Forward |
| :---: | :---: |
| $p_{T}^{l}>20 \mathrm{GeV}$ | $p_{T, F}^{l}>20 \mathrm{GeV} \quad p_{T, C}^{l}>25 \mathrm{GeV}$ |
| $\left\|y^{l}\right\|<2.4$ | $2.5<\left\|y_{F}^{l}\right\|<4.9 \quad\left\|y_{C}^{l}\right\|<2.4$ |
| $46 \mathrm{GeV}<m_{l l}<200 \mathrm{GeV}$ | $66 \mathrm{GeV}<m_{l l}<150 \mathrm{GeV}$ |

- Fiducial cuts influence acceptances in triple-differential bins


## Triple-differential Drell-Yan cross section

- Leading order: fiducial cuts intersect bin definitions
[A.Gehrmann-De Ridder, E.W.N.Glover, A.Huss, C.Preuss, D.Walker, TG]




## Triple-differential Drell-Yan cross section

- Leading-order forbidden bins
- require finite $Q_{T}$ of lepton pair
- shown here: symmetric lepton pair
$\rightarrow$ prediction starts only at NLO
- lower accuracy
- potential perturbative instabilities


Minimum $Q_{T}$ Values in the $\left(\left|y_{11}\right|, \cos \theta^{*}\right)$ plane [CC]


Minimum $Q_{T}$ Values in the ( $\left.\left|y_{11}\right|, \cos \theta^{*}\right)$ plane [CF]


## Triple-differential Drell-Yan cross section

## Forbidden bins at leading order

- large theory uncertainty, poor agreement with data
- $\mathrm{O}\left(\alpha_{s}{ }^{3}\right)$ corrections (Drell-Yan $\mathrm{N}^{3}$ LO) obtained from V+jet at NNLO
[R.Boughezal, J.Campbell, K.Ellis, C.Focke, W.Giele, X.Liu, F.Petriello; MCFM: T.Neumann, J.Campbell;
NNLOJET: A.Gehrmann-De Ridder, N.Glover, A.Huss, T.Morgan, D.Walker, TG]
- use NNLOJET implementation
- replace jet requirement by (small) $Q_{T}$ cut
- numerical convergence at small $Q_{T}$ challenging


## State-of-the-art theory prediction



- QCD NNLO ( $\alpha_{s}{ }^{2}$ ) plus N3LO $\left(\alpha_{s}{ }^{3}\right)$ in LO-forbidden bins
- combined with (NLO+HO) EW corrections [c.Carloni Calame, G.Motagna, A.Nicrosini, A.Vicini]


## Triple-differential Drell-Yan cross section




Future applications

- measurement of $\sin ^{2} \Theta w$
- determination of parton distributions


## Photon+jet production at NNLO

- Photon+jet production
- multi-differential measurements
- probe of gluon distribution
- several production modes: direct, fragmentation, secondary
- Photon isolation
- required for photon identification
- sensitive on photon fragmentation function
- extension of NNLO antenna subtraction: identified particles [R. Schürmann, TG]

fixed cone: $E_{\text {had }}<\varepsilon E_{\gamma}+E_{0}$


## Photon+jet production at NNLO

- NNLO corrections [X. Chen, E.W.N. Glover, A. Huss, M. Höfer, R. Schürmann, TG]
- reduce theory uncertainty to $\sim 5 \%$ level
- considerably improve description of kinematical shapes



## Identified hadrons at NNLO

- Fragmentation antenna functions
- antenna functions (final-final or initial-final) differential in the momentum fraction $z$ of one hard final-state radiator
- Computation of integrated fragmentation antennae [L.Bonino, R.Schürmann, G.Stagnitto, TG]
- NLO and NNLO real-virtual: no integration needed, expansion in distributions
- NNLO double-real: phase space integration ( $2 \rightarrow 3$ phase space with constraints)
- reduction to phase space master integrals
- computation from differential equations
- boundary conditions from integration over z

| family | master | deepest pole | at $x=1$ | at $z=1$ |
| :---: | :---: | :---: | :---: | :---: |
|  | I[0] | $\epsilon^{0}$ | $(1-x)^{1-2 \epsilon}$ | $(1-z)^{1-2 \epsilon}$ |
| A | $I[5]$ | $\epsilon^{-1}$ | $(1-x)^{-2 \epsilon}$ | $(1-z)^{1-2 \epsilon}$ |
|  | $I[2,3,5]$ | $\epsilon^{-2}$ | $(1-x)^{-1-2 \epsilon}$ | $(1-z)^{-1-2}$ |
| B | $I[7]$ | $\epsilon^{0}$ | $(1-x)^{1-2 \epsilon}$ | $(1-z)^{1-2 \epsilon}$ |
|  | $I[-2,7]$ | $\epsilon^{0}$ | $(1-x)^{1-2 \epsilon}$ | $(1-z)^{1-2 \epsilon}$ |
|  | $I[-3,7]$ | $\epsilon^{0}$ | $(1-x)^{1-2 \epsilon}$ | $(1-z)^{1-2 \epsilon}$ |
|  | $I[2,3,7]$ | $\epsilon^{-2}$ | $(1-x)^{-2 \epsilon}$ | $(1-z)^{-1-2 \epsilon}$ |
| C | $I[5,7]$ | $\epsilon^{-1}$ | $(1-x)^{-2 \epsilon}$ | $(1-z)^{1-2 \epsilon}$ |
|  | $I[3,5,7]$ | $\epsilon^{-2}$ | $(1-x)^{-2 \epsilon}$ | $(1-z)^{-2 \epsilon}$ |
| D | $I[1]$ | $\epsilon^{0}$ | $(1-x)^{-2 \epsilon}$ | $(1-z)^{-2 \epsilon}$ |
|  | $I[1,4]$ | $\epsilon^{0}$ | $(1-x)^{-2 \epsilon}$ | $(1-z)^{-2 \epsilon}$ |
|  | $I[1,3,4]$ | $\epsilon^{-1}$ | $(1-x)^{-2 \epsilon}$ | $(1-z)^{-1-2 \epsilon}$ |
| E | $I[1,3,5]$ | $\epsilon^{-2}$ | $(1-x)^{-2 \epsilon}$ | $(1-z)^{-1-2 \epsilon}$ |
| G | $I[1,3,8]$ | $\epsilon^{-2}$ | $(1-x)^{-2 \epsilon}$ | $(1-z)^{-1-2 \epsilon}$ |
| H | $I[1,4,5]$ | $\epsilon^{-1}$ | $(1-x)^{-1-2 \epsilon}$ | $(1-z)^{-2 \epsilon}$ |
| I | $I[2,4,5]$ | $\epsilon^{-2}$ | $(1-x)^{-1-2 \epsilon}$ | $(1-z)^{-2 \epsilon}$ |
| J | [ [4, 7] | $\epsilon^{0}$ | $(1-x)^{-2 \epsilon}$ | $(1-z)^{-2 \epsilon}$ |
|  | $I[3,4,7]$ | $\epsilon^{-1}$ | $(1-x)^{-2 \epsilon}$ | $(1-z)^{-2 \epsilon}$ |
| K | $I[3,5,8]$ | $\epsilon^{-2}$ | $(1-x)^{-1-2 \epsilon}$ | $(1-z)^{-2 \epsilon}$ |
| L | $I[4,5,7]$ | $\epsilon^{-1}$ | $(1-x)^{-1-2 \epsilon}$ | $(1-z)^{-2 \epsilon}$ |
| M | $I[4,5,8]$ | $\epsilon^{-1}$ | $(1-x)^{-1-2 \epsilon}$ | $(1-z)^{-2 \epsilon}$ |

## Identified hadrons at NNLO

- Semi-inclusive DIS (SIDIS)
- resolve flavour structure of light quark sea ( $\pi, \mathrm{K}$ production)
- tag heavy flavours (D production)
- important process in polarized DIS (spin structure of the proton)
- studied at EMC, SMC, HERMES, COMPASS
- will be probed extensively at BNL Electron-Ion Collider (EIC)

- NNLO SIDIS coefficient functions [L.Bonino, R.Schürmann, G.Stagnitto, TG]
- computation very similar to initial-final fragmentation antenna functions
- confirm earlier partial results and approximations [D.Anderle, D.de Florian, w. Vogelsang]
- on non-singlet leading colour: agree with independent results [S.Goyal, S.Moch, V.Pathak, N.Rana, V.Ravindran]


## Identified hadrons at NNLO: SIDIS



## Towards N3LO for fiducial cross sections

Inclusive coefficient functions (total cross section) at N3LO

- computed analytically
- three-loop form factors (VVV)
- inclusive phase space up to triple emission (RRR,RRV,RVV)
- 100s of loop and phase-space master integrals


## Results

- Deep inelastic structure functions
[S.Moch, J.Vermaseren, A.Vogt; J.Blümlein, P.Marquard, C.Schneider, K.Schönwald]
- Higgs boson production [C.Anastasiou, C.Duhr, F.Dulat, F.Herzog, B.Mistlberger]



## Towards N3LO for fiducial cross sections

## Three-loop amplitudes for $2 \rightarrow 2$ processes (VVV)

- algebraic complexity of integral reduction, computation of master integrals
- recent innovations
- finite-field methods [A.von Manteuffel, R.Schabinger; T.Peraro]
- canonical integral basis [J.Henn]
- minimal tensor decomposition [T.Peraro, L.Tancredi]
- first results
- four-parton amplitudes [F.Caola, A.Chakraborty, G.Gambuti, A.von Manteuffel, L.Tancredi]

- parton-photon amplitudes
[P.Bargiela, F.Caola, A.Chakraborty, G.Gambuti, A.von Manteuffel, L.Tancredi]
- V+3-parton amplitudes (planar)
[P.Jakubcik, C.Mella, N.Syrrakos, L.Tancredi, TG]


## Towards N3LO for fiducial cross sections

Infrared singularity structure of real radiation understood

- RRR: four-parton collinear factors [v.del Duca, C.Duhr, R.Haindl, A.Lazopoulos, M.Michel]
- RRR: triple-soft current [S.Catani, L.Cieri, D.Colferai, F.Coradeschi, A.Torrini; V.del Duca, C.Duhr, R.Haindl, z.Liu]
- RRV: three-parton collinear factors at one loop [s.Catani, D.de Florian, G.Rodrigo; M.Czakon, S.Sapeta]
- RRV: one-loop double-soft current [s.Catani, L.Cieri; Y.zhu; M.Czakon, F.Eschment, T.Schellenberger]
- RVV: simple collinear factors at two loops [c.Duhr, M.Jaquier, TG]
- RVV: two-loop soft current [y.Li, H.X.Zhu; C.Duhr, TG; L.Dixon, E.Herrmann, K.Yan, H.X.Z.Zhu]

Require scheme for infrared cancellations

## Towards N3LO for fiducial cross sections

Infrared cancellations: challenges

- subtraction $\quad \mathcal{I}=\lim _{\epsilon \rightarrow 0}\left[\int_{0}^{1} \frac{d x}{x} x^{\epsilon}(F(x)-F(0))+F(0) \int_{0}^{1} \frac{d x}{x} x^{\epsilon}-\frac{1}{\epsilon} F(0)\right]$
- construction of subtraction term (completeness, overcompensation)
- integration of building blocks (analytical or numerical)
- slicing

$$
\mathcal{I} \approx \lim _{\epsilon \rightarrow 0}\left[\int_{\delta}^{1} \frac{d x}{x} x^{\epsilon} F(x)+F(0) \int_{0}^{\delta} \frac{d x}{x} x^{\epsilon}-\frac{1}{\epsilon} F(0)\right]
$$

- analytic computation of below-cut contribution
- numerical importance of power-suppressed terms, value of slicing parameter


## N3LO for Drell-Yan observables

Slicing parameter: transverse momentum ( $\mathrm{q}_{T}$ slicing) [s.Catani, M.Grazzin]

$$
\frac{d \sigma_{X}^{N 3 L O}}{d O}=\mathcal{H}_{N 3 L O} \otimes \frac{d \sigma_{X}^{L O}}{d O}+\left[\int_{q_{T, X}} \frac{d \sigma_{X+j}^{N N L O}}{d O}-\frac{d \sigma_{X, C T}^{N N L O}}{d O}\left(q_{T}\right)\right]
$$

- below-cut contribution from expansion of N3LL $\mathrm{q}_{\mathrm{T}}$ resummation to $\mathrm{O}\left(\alpha_{s}{ }^{3}\right)$ [W.Bizon, P.Monni, E.Re, P.Torrielli; S.Camrada, L.Cieri, G.Ferrera;T.Becher, T.Neumann; W.L.Ju, M.Schönherr]
- ingredients: three-loop soft and beam functions [Y.Li, H.X.Zhu; M.Ebert, B.Mistlberger, G.Vita; M.X.Luo, T.Z.Yang, Y.J.Zhu]
- check: independence on $\mathrm{q}_{\mathrm{T}, \text { cut }}$ slicing parameter
- check: reproduce inclusive coefficient functions (no ingredients or methodology in common!) [X.Chen, E.W.N.Glover, A.Huss, T.Z.Yang, H.X.Zhu, TG]



## N3LO for Drell-Yan observables

## Results: fiducial distributions


single lepton distribution in NC Drell-Yan, matched to N3LL resummation (RadISH) [X.Chen, E.W.N.Glover, A.Huss, P.F.Monni, E.Re, L.Rottoli, P.Torrielli, TG]

transverse mass distribution in W boson production (CDF II cuts) [X.Chen, E.W.N.Glover, A.Huss, T.Z.Yang, H.X.Zhu, TG]

charged lepton distribution in W boson production (ATLAS 5.02 TeV)
[J.Campbell, T.Neumann]

## Towards N3LO for fiducial cross sections

Subtraction methods at N3LO: work in progress

- integrating N3LO antenna functions
- final-final kinematics [X.Chen, M.Marcoli, P.Jakubcik, G.Stagnitto]
- initial-final kinematics [G.Fontana, K.Schönwald, TG]

Shortcut for simple processes: Projection to Born
[M.Cacciari, F.Dreyer, A.Karlberg, G.Salam, G.Zanderighi]

$$
\frac{d \sigma_{X}^{N 3 L O}}{d O}=\frac{d \sigma_{X+j}^{N N L O}}{d O}-\frac{d \sigma_{X+j}^{N N L O}}{d O_{B}}+\frac{d \sigma_{X}^{N 3 L O, \text { incl }}}{d O_{B}}
$$

- Higgs production in vector boson fusion [F.Dreyer, A. Karlberg]
- Higgs production in gluon fusion, including $\mathrm{H} \rightarrow \gamma \gamma$
[X.Chen, N.Glover, A.Huss, B.Mistlberger, A.Pelloni]



## Parton distributions at N3LO

Caveat: current N3LO predictions use NNLO parton distributions

- inherent inconsistency, difficult to quantify


## N3LO parton distributions require

- four-loop Altarelli-Parisi splitting functions
- use four-loop OPE, haunted by ghosts
- ongoing: lower Mellin moments, specific color and flavor combinations [G.Falcioni, F.Herzog, S.Moch, A.Vogt; A.von Manteuffel, V.Sotnikov, T.Z.Yang, TG]
- N3LO coefficient functions for relevant observables
- DIS and inclusive DY known [S.Moch, J.Vermaseren, A.Vogt;
J.Blümlein, P.Marquard, C.Schneider, K.Schönwald; C.Duhr, B.Mistlberger]
- fiducial cross sections next frontier

First approximate N3LO parton distribution fits



## Summary

- LHC embarks on a decade-long program of precision physics
- Ultimate precision challenge for QCD
- predictions for complex final states at per-cent level accuracy
- Theory ready to face this challenge
- NNLO predictions becoming the new standard
- N3LO concepts, techniques and tools developing rapidly
- Stay tuned

