Frontiers in Numerical Relativity: Binary black holes with high mass-ratio and eccentricity

Image: *Nils Vu*

Harald Pfeiffer
MPI for Gravitational Physics

DESY Theory Seminar
Zeuthen
Dec 14, 2023

LIGO-Virgo-KAGRA Gravitational Wave Observations

LIGO-Virgo-KAGRA | Aaron Geller | Northwestern


-0.26s

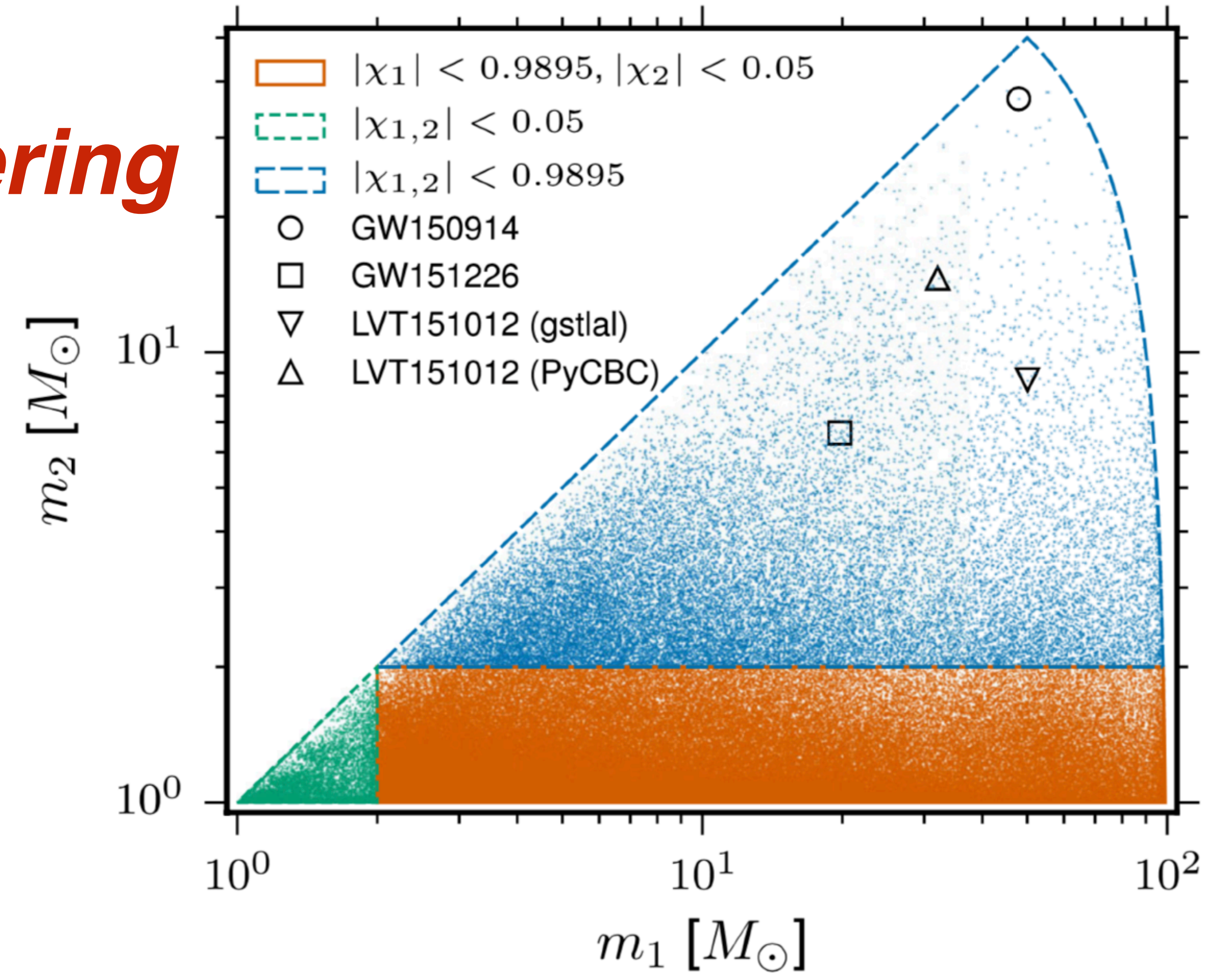
$$L_{\text{GW peak}} \sim 10^{23} L_\odot \sim 50 L_{\text{universe}}$$

$$P = 1/20s$$
$$v = 50{,}000km/s$$
$$T = 0.1s$$

$$E_{\text{GW}} = 3M_\odot$$
$$L_{\text{GW}} \sim \frac{30M_\odot c^2}{sec}$$
$$L_\odot = \frac{0.007M_\odot c^2}{10^{10}\text{yr}}$$

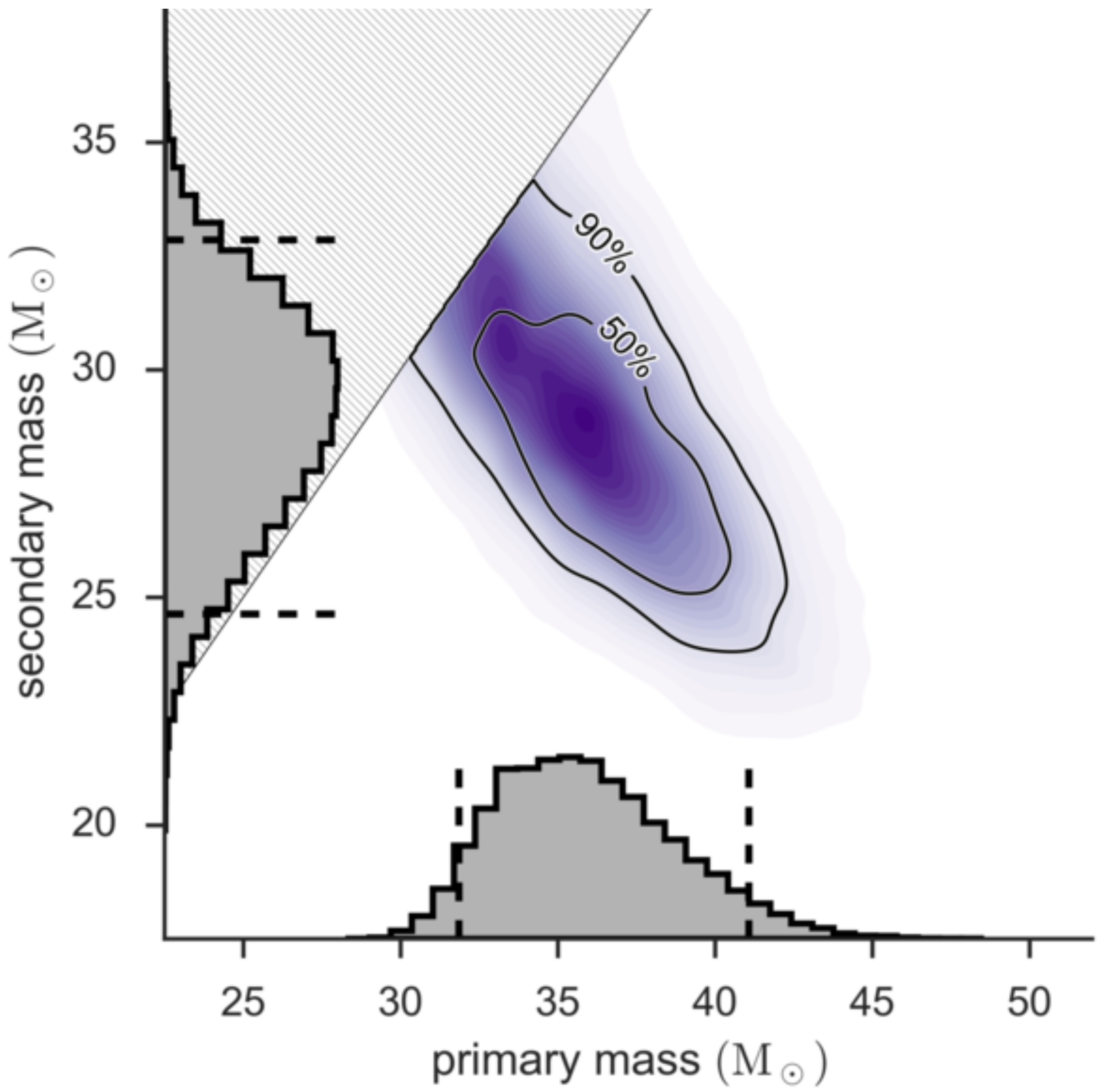
Waveform knowledge ***essential for GW astronomy***

Detection by matched filtering



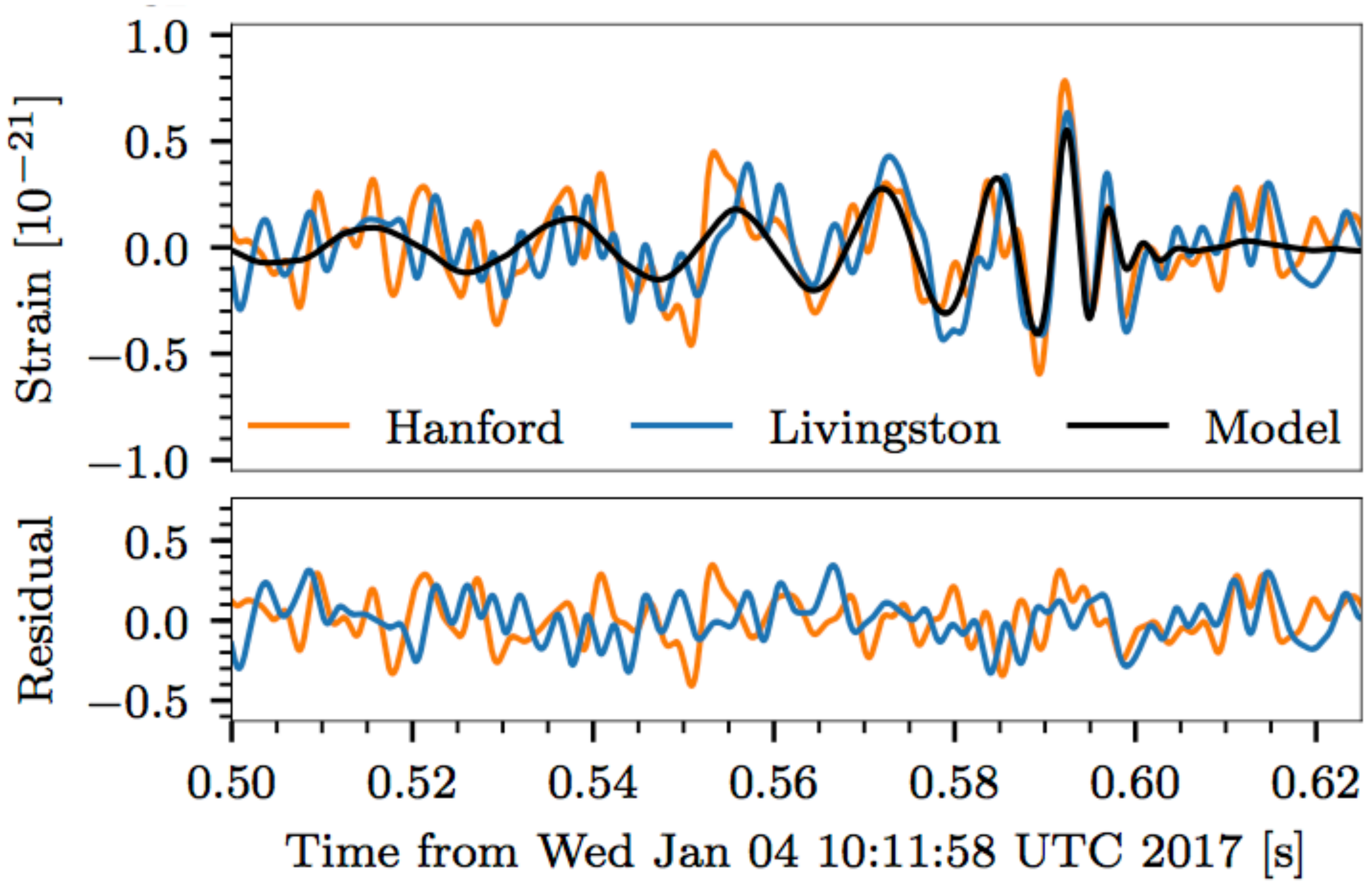
LIGO+Virgo, PRX 2016 (1606.04856)

Parameter estimation



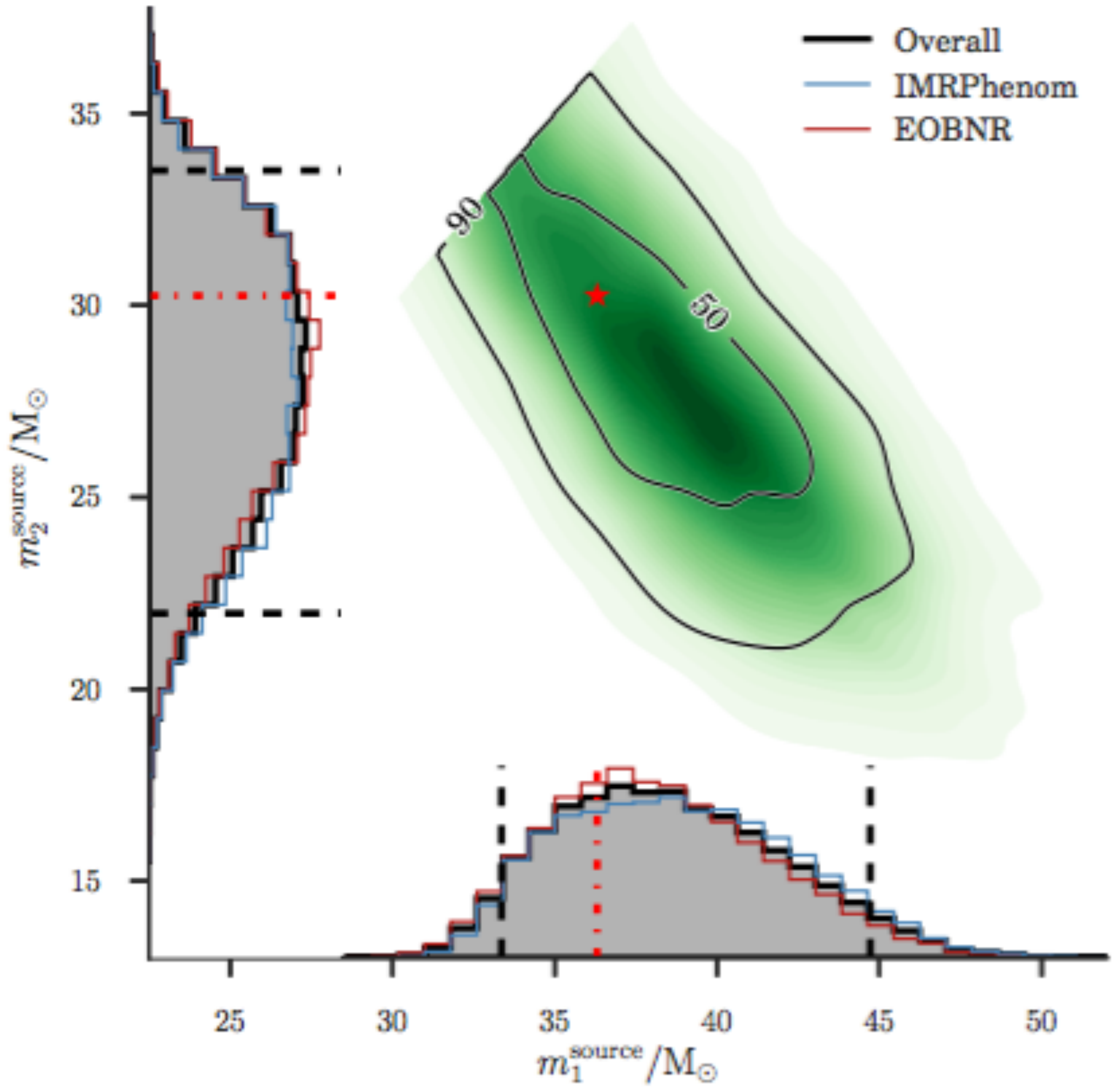
"GW150914" Abbott+ PRL 12016

Testing GR



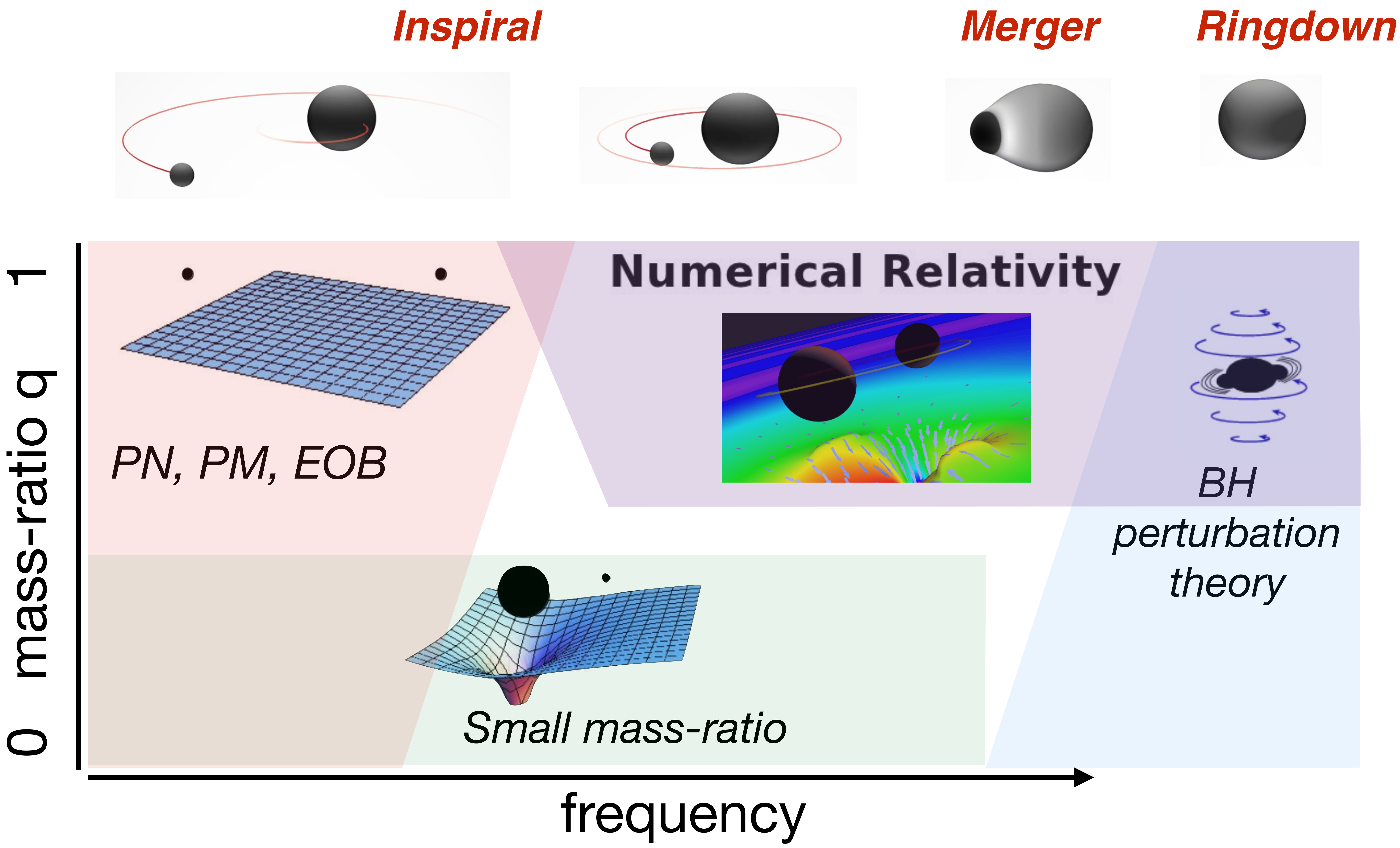
LIGO+Virgo, PRL 2017 (1706.01812)

Validation



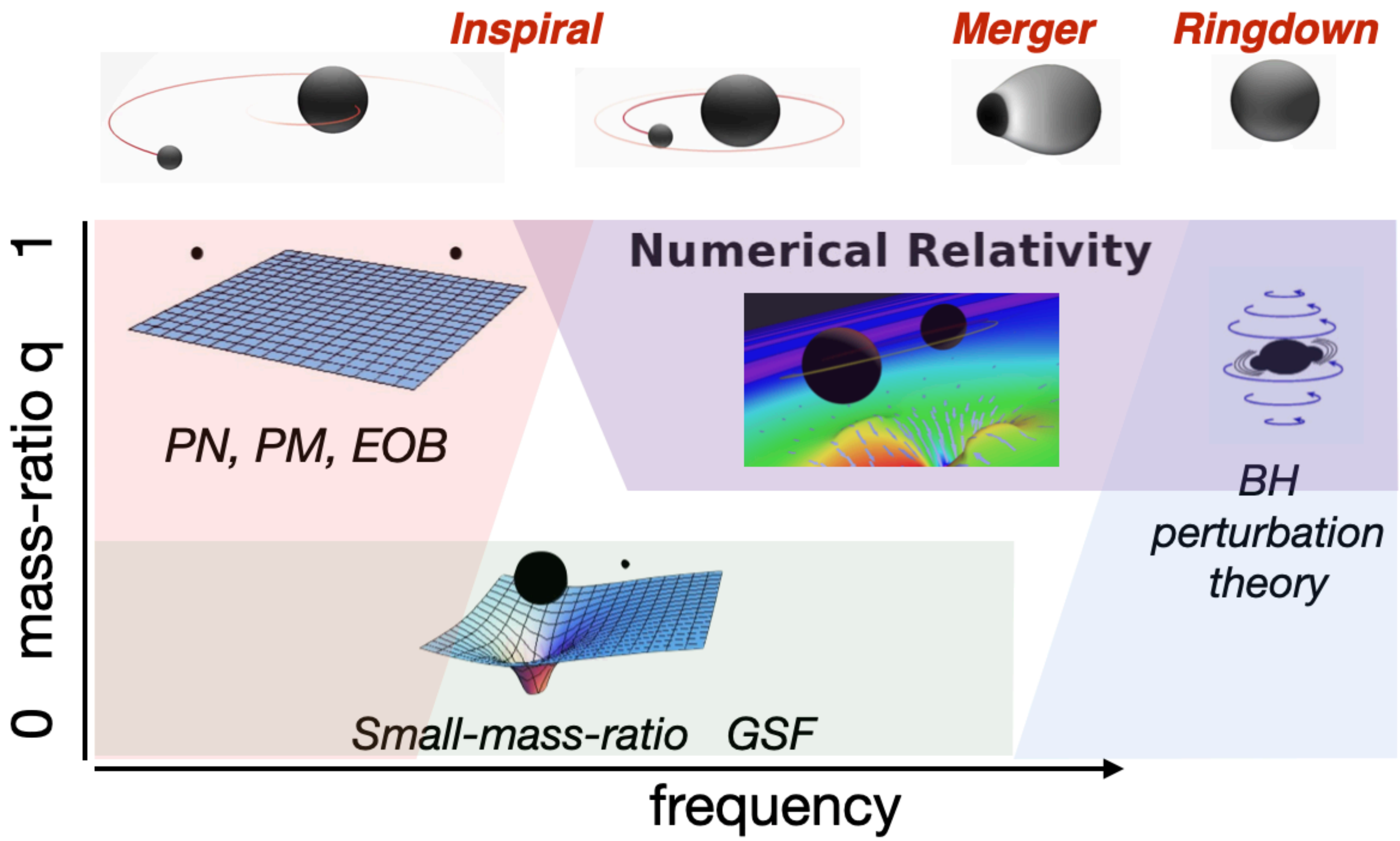
LIGO & Virgo: CQG 2017 (1611.07531)

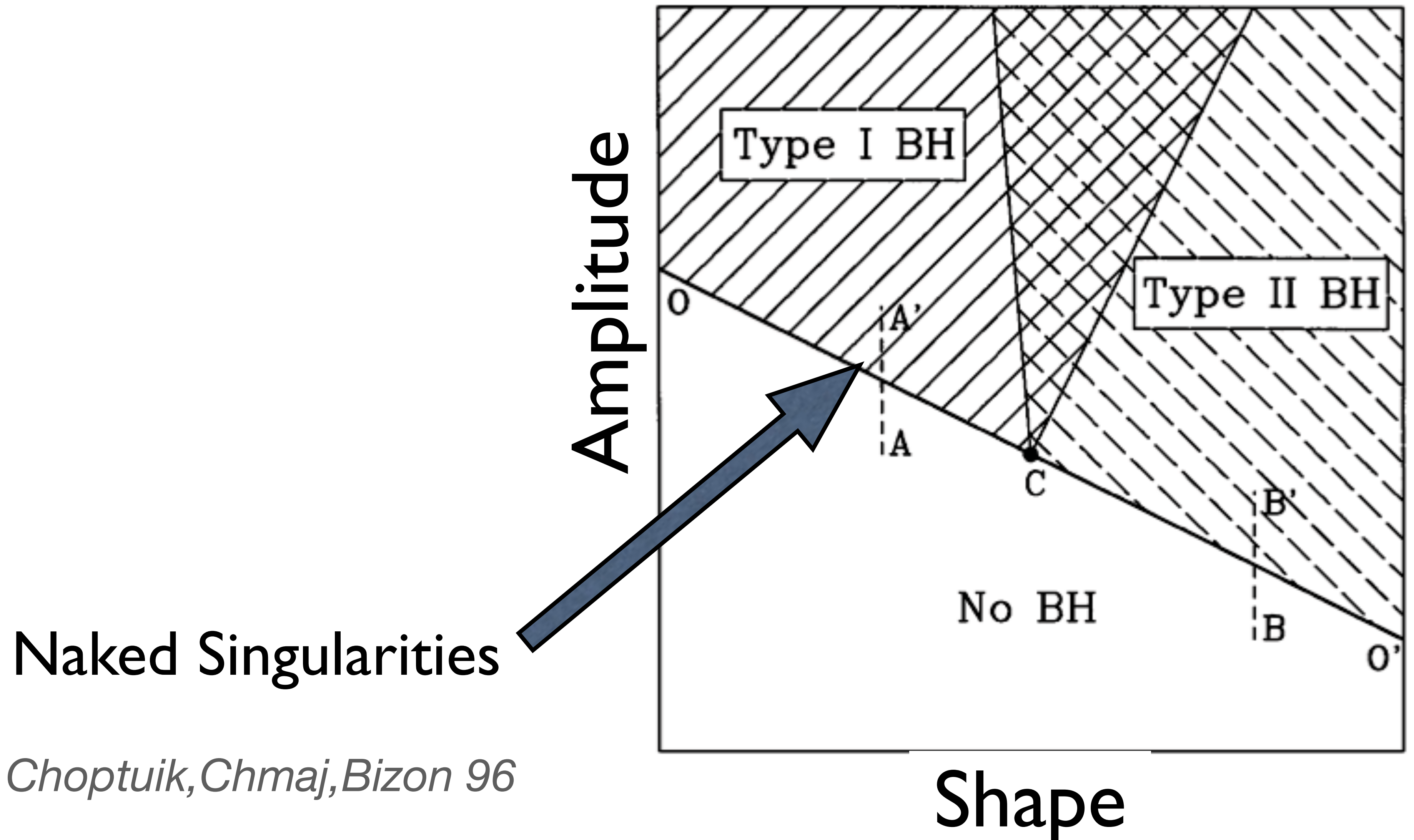
Methods for modeling BBH



Role of NR

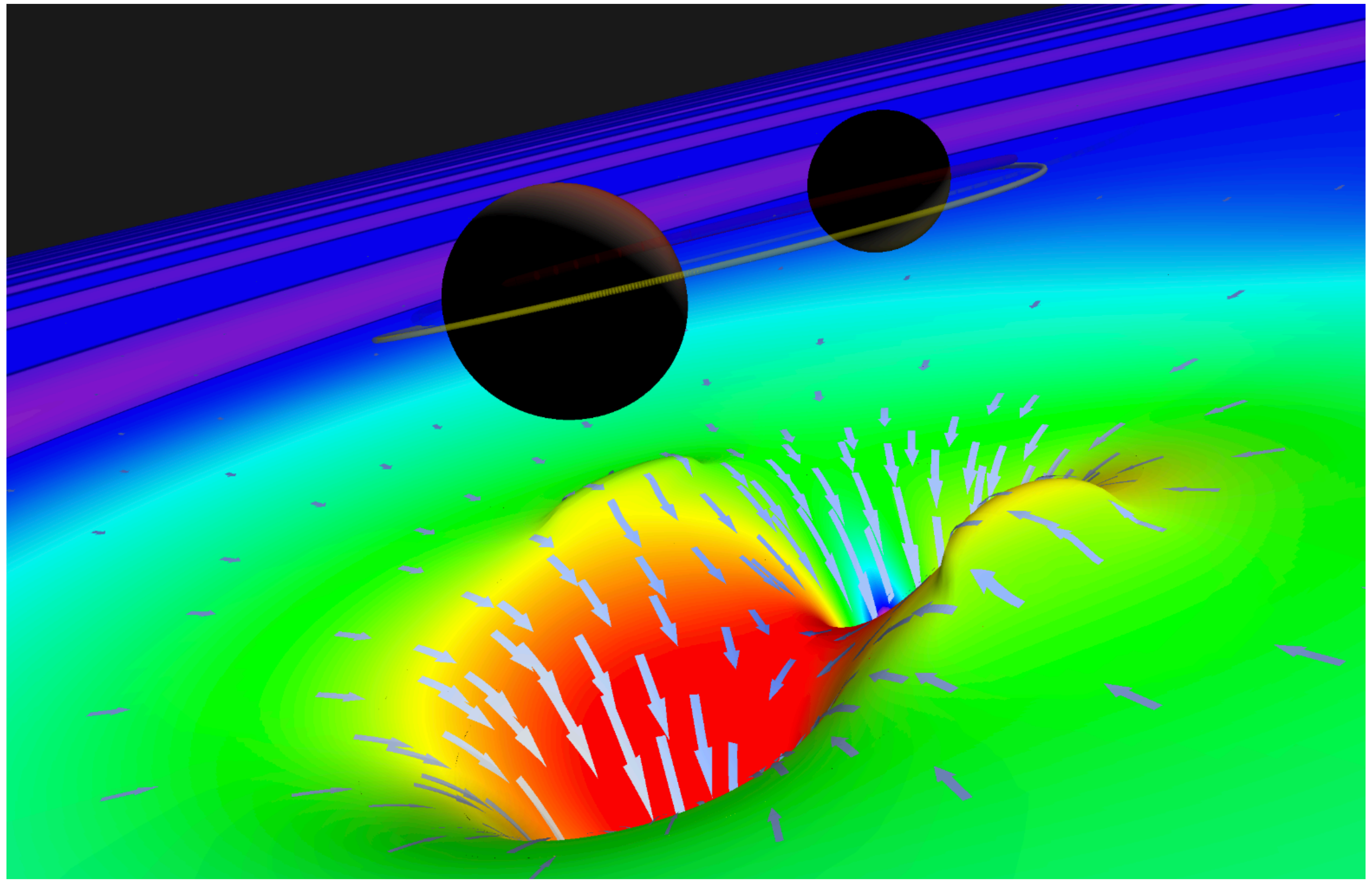
- **Solutions of GR** for GW-Astro for late inspiral + merger
  - cover parameter space
  - **error estimates**
- **Regions of validity** of perturbative methods
  - all available perturbation orders needed for science
- **Properties of GR** in nonlinear & dynamic regime





Choptuik,Chmaj,Bizon 96

Numerical Relativity



Solving Einstein Equations - Basic idea

- Goal: Space-time metric $g_{ab}(x^i, t)$ satisfying

$$R_{ab}[g_{ab}] = 0$$

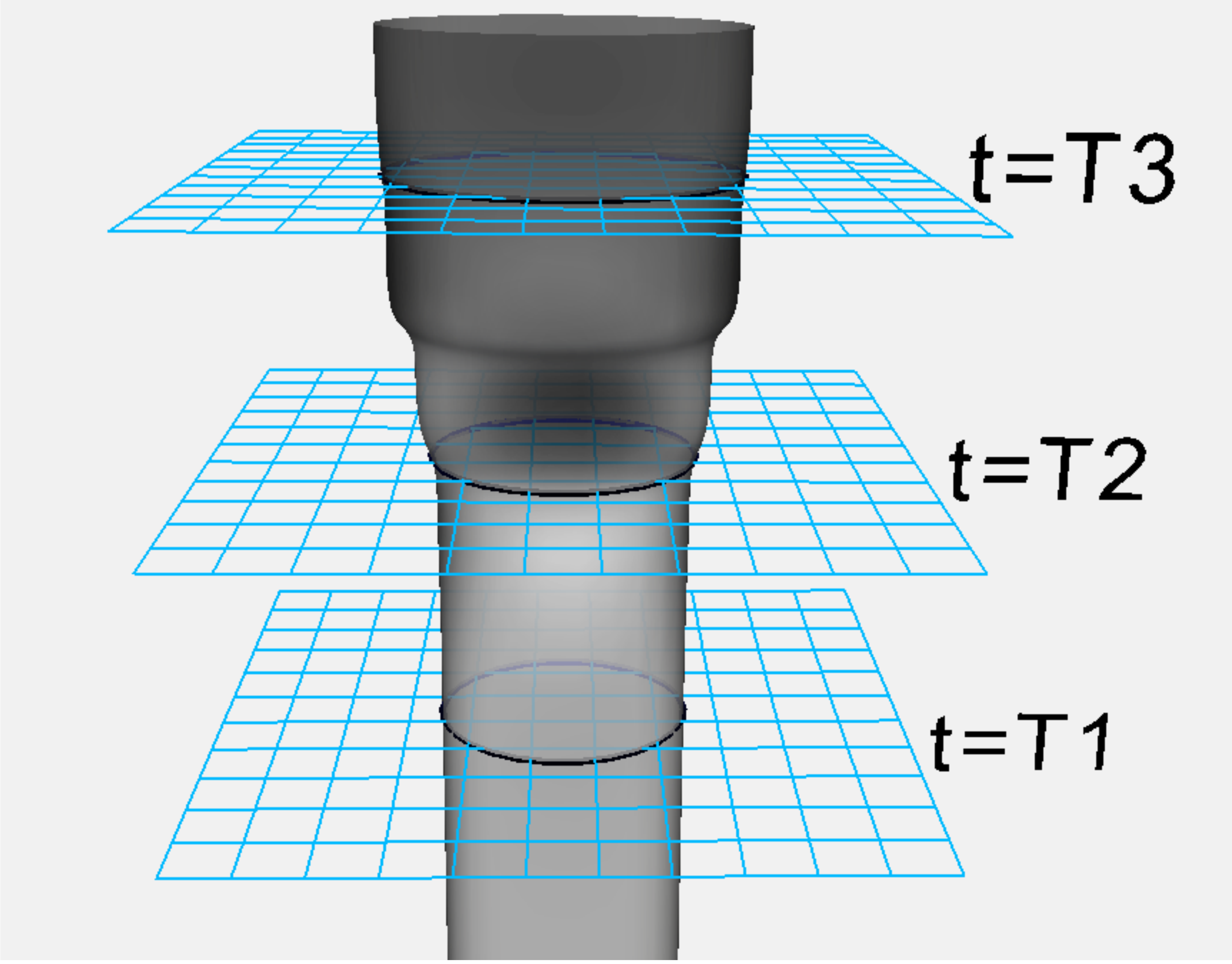
- Split spacetime into space and time

- Evolution equations

$$\partial_t g_{ij} = \ldots$$
$$\partial_t K_{ij} = \ldots$$

- Constraints

$$R[g_{ij}] + K^2 - K_{ij}K^{ij} = 0$$
$$\nabla_j \left(K^{ij} - g^{ij}K\right) = 0$$

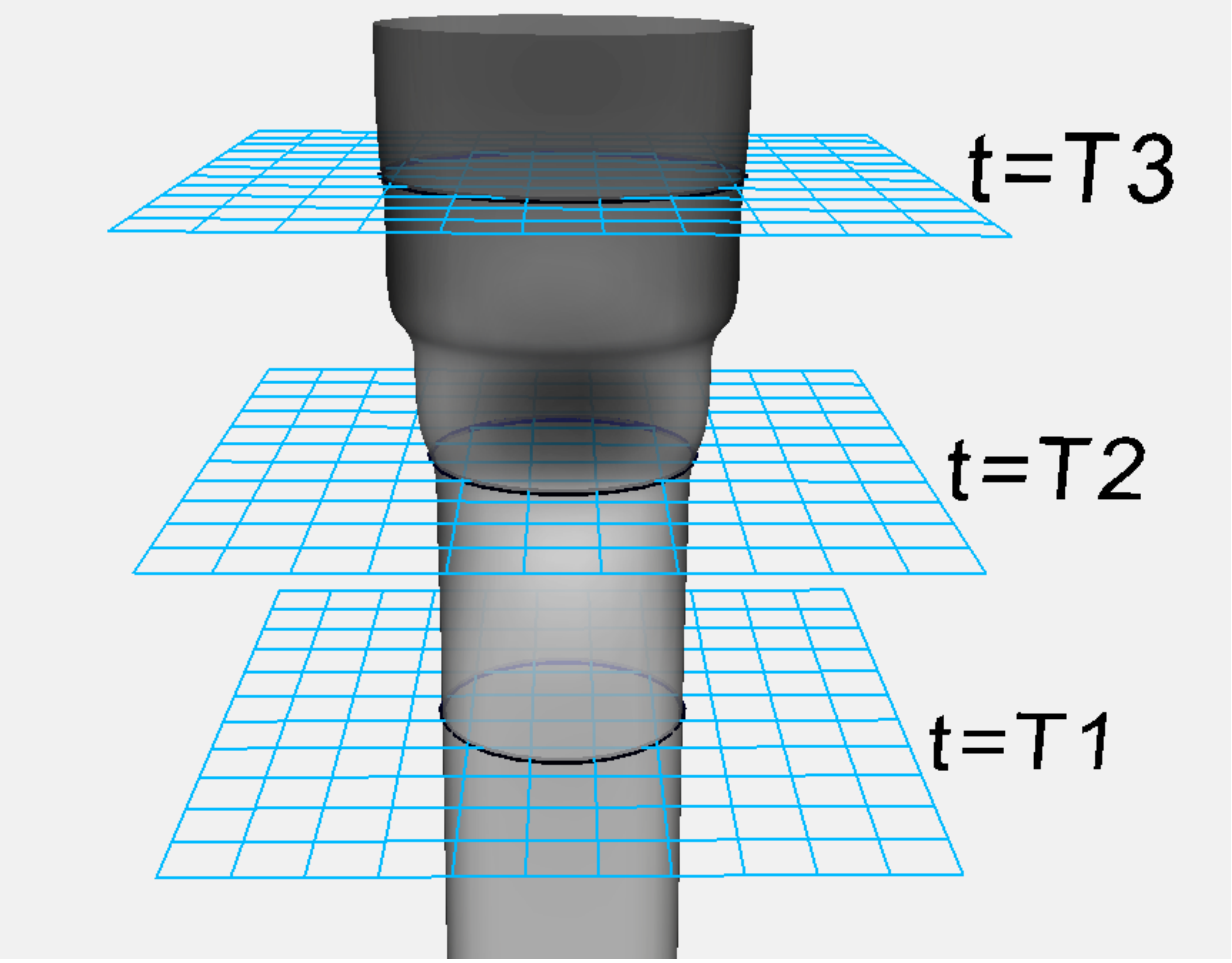


cf. Maxwell's equations

$$\partial_t \vec{E} = \nabla \times \vec{B}$$
$$\partial_t \vec{B} = -\nabla \times \vec{E}$$
$$\nabla \cdot \vec{E} = 0$$
$$\nabla \cdot \vec{B} = 0$$

Why is this hard?

- ADM equations ill-posed;
  rewrite as hyperbolic system
- **Singularities** inside black holes
- Constraints difficult to preserve
- **Coordinate freedom**
  - How to choose coordinates for a space-time one does not know yet?
- Many common numerical challenges
  - 20-50 variables
  - 10,000 FLOP / grid-point / time-step
  - Different length scales
  - **High accuracy requirements**



The first 50 Years of numerical relativity for BBH

1964 — **~1999** — **~2005** — **2015**

1962 *ADM*
3+1 formulation

1964
Hahn-Lindquist
2 wormholes

1984
Unruh
excision

1992,3
Choptuik; Abrahams+Evans
critical phenomena

1997
Brandt-Brügmann
puncture data

1994-98
BBH Grand Challenge

1999-00
AEI/PSU
grazing collisions

2000-04
AEI/UTB-NASA
revive crashing codes (Lazarus)

~2000 *Choptuik; Schnetter;Brügmann*
mesh refinement

2005
Gundlach ea
constraint damping

2005 *Pretorius*
inspiral-merger-ringdown (IMR) w/ harmonic

2005-06
Campanelli+; Baker+
IMR w/ BSSN & moving punctures

2006-08
Scheel..HP+ *SXS*
IMR w/ spectral

2007-
Ajith, AEI, Jena
phenom GW models

2009-
UMD, SXS
EOB GW models

2011
Schmidt ea; Boyle ea
Radiation aligned frame

2011
Lousto ea
q=100

2014-
precessing GW models

2015
Szilagyi *ea*
175 orbits

1975-77
Smarr-Eppley
head-on collision

1979 *York*
kinematics and dynamics of GR

1989-95
Bona-Masso
modified ADM, (hyperbolicity)

1994 *Cook*
Bowen-York initial data

1994-95
NCSA-WashU
improved head-on collision

1999
BSSN
evolution system

1999 *York*
conformal thin sandwich ID

1999-2005 *York, Cornell, Caltech, LSU*
hyperbolic formulations

2000-02
Alcubierre
gauge conditions

2004
Brügmann ea
one orbit

2003-08
Cook, Pfeiffer ea
improved ID

2000 *Ashtekar*
isolated horizons

2006,07
Baker ea; Gonzalez ea
non-spinning BBH kicks

2007 *SXS*
PN-NR comparison

2007-11
RIT; Jena; AEI;…
BBH superkicks

2008
all of NR
NINJA

2009-11
Bishop, ...
Cauchy characteristic extraction

2010
Bernuzzi ea
C4z

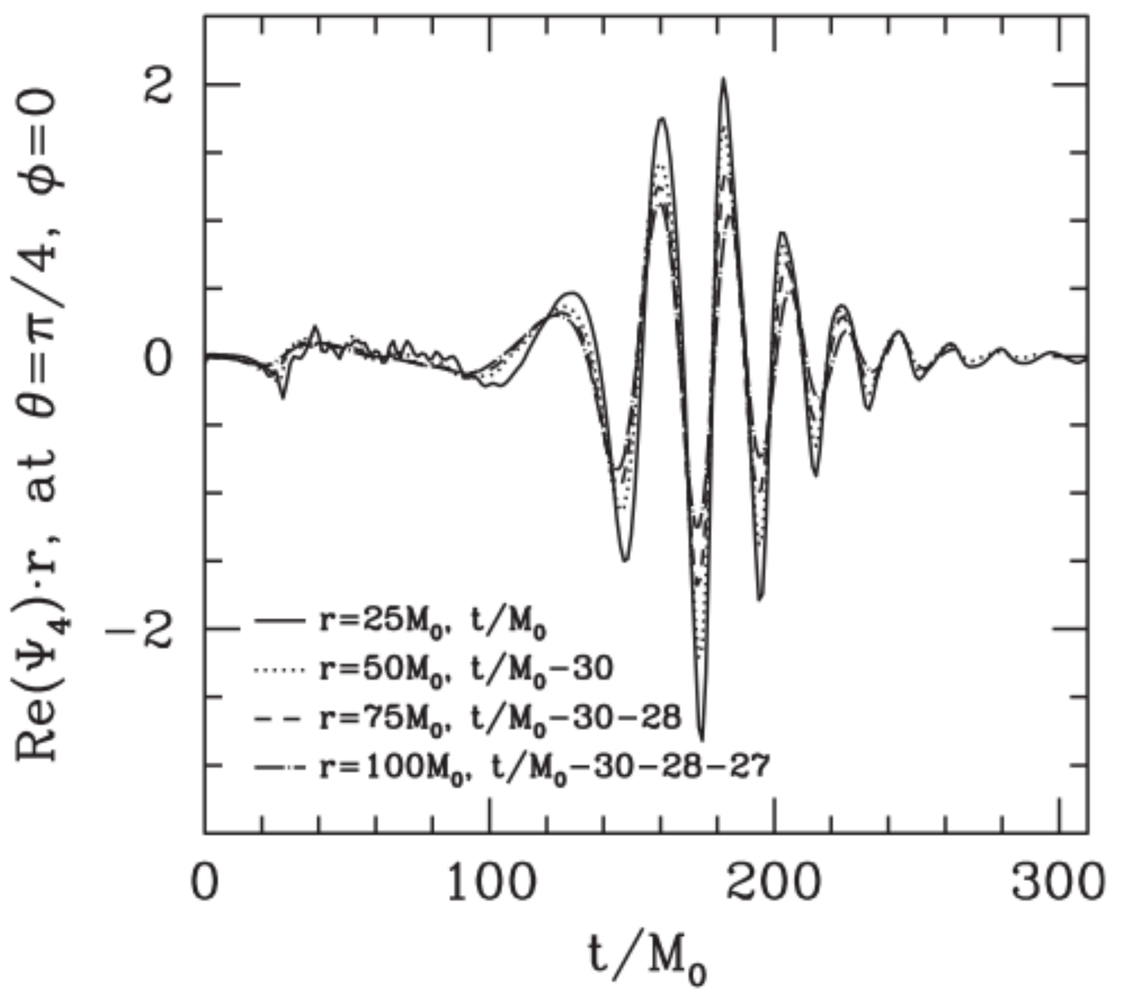
2011
Lovelace ea
$S/M^2=0.97$

2011-
Le Tiec ea
self-force studies

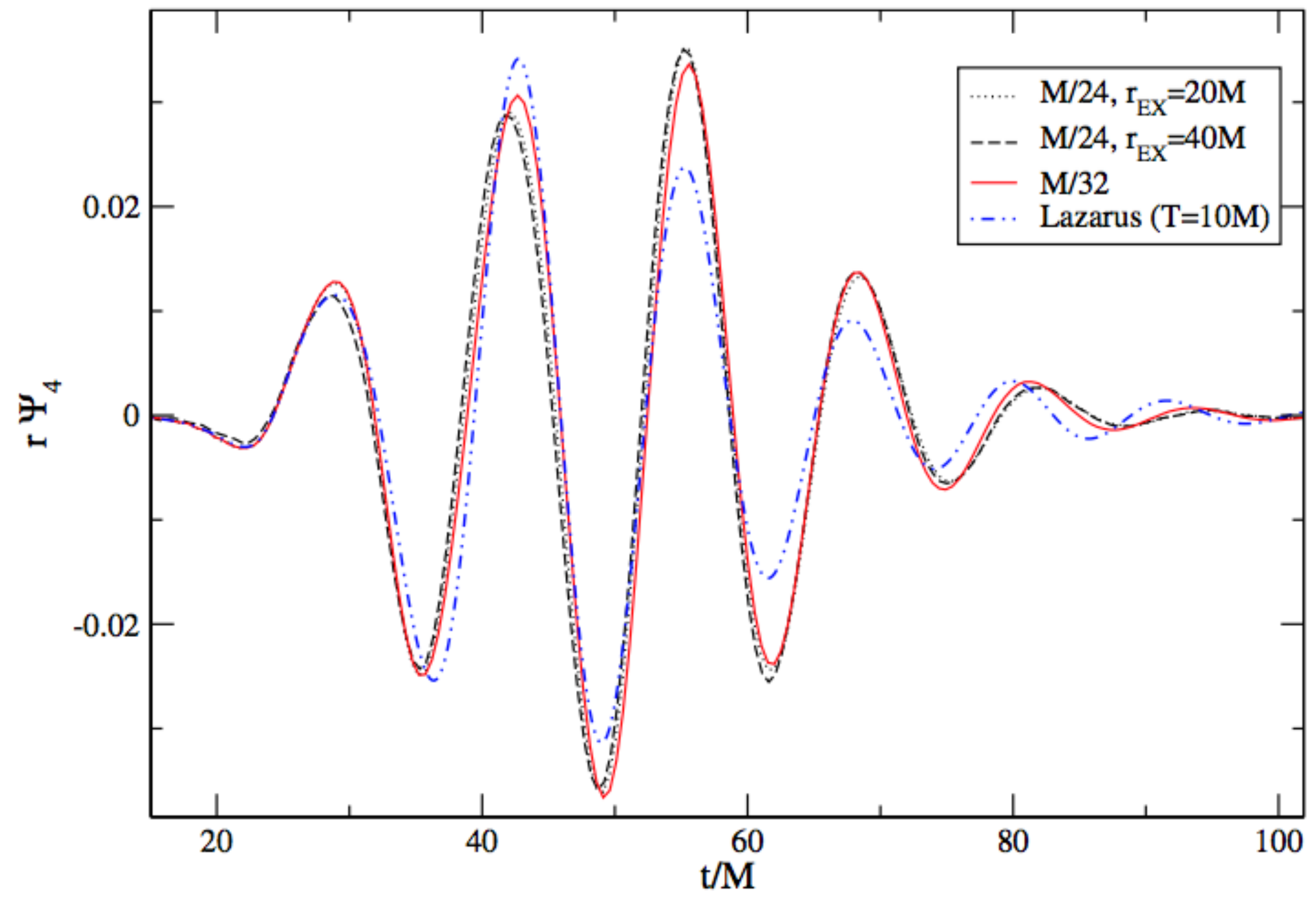
2013
GaTech; SXS
Precessing parameter studies

Courtesy Carlos Lousto, updated by HP

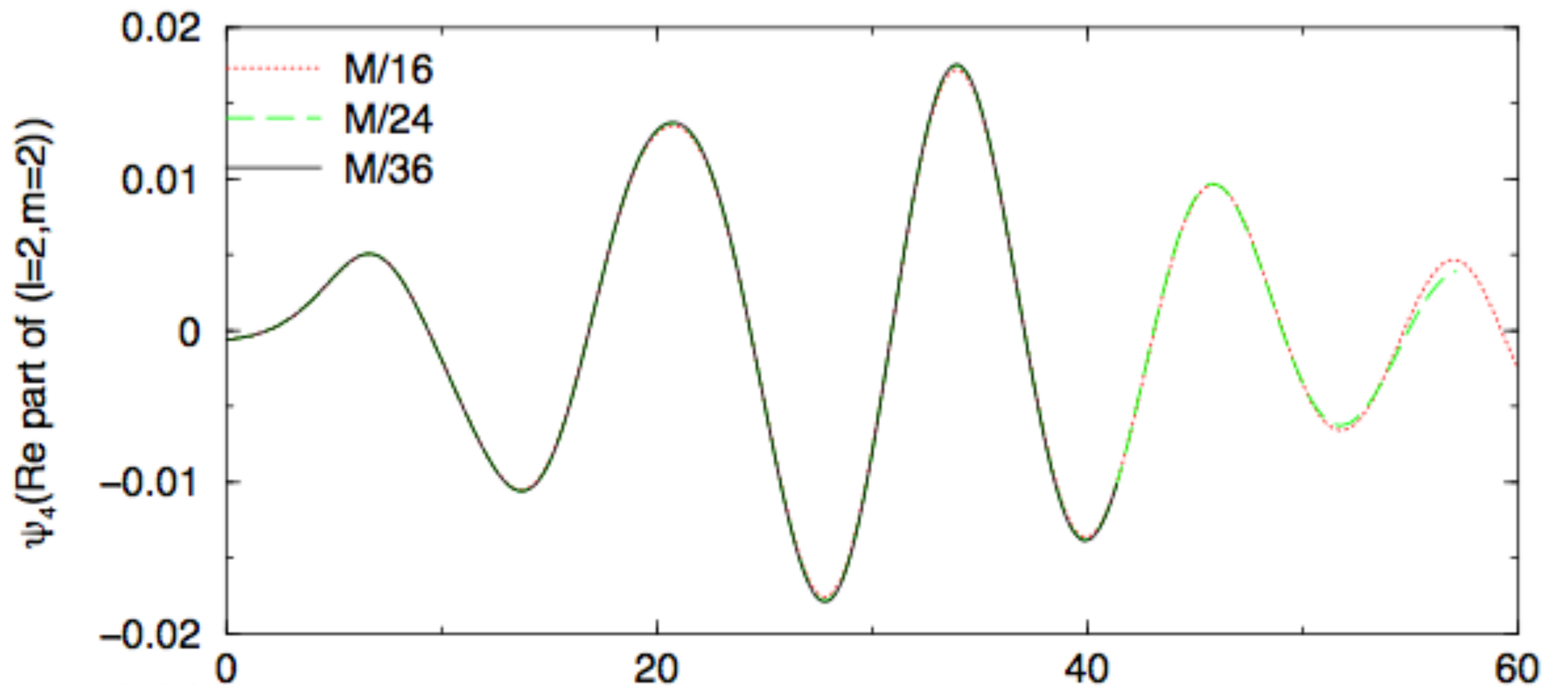
2005: First working BBH inspirals



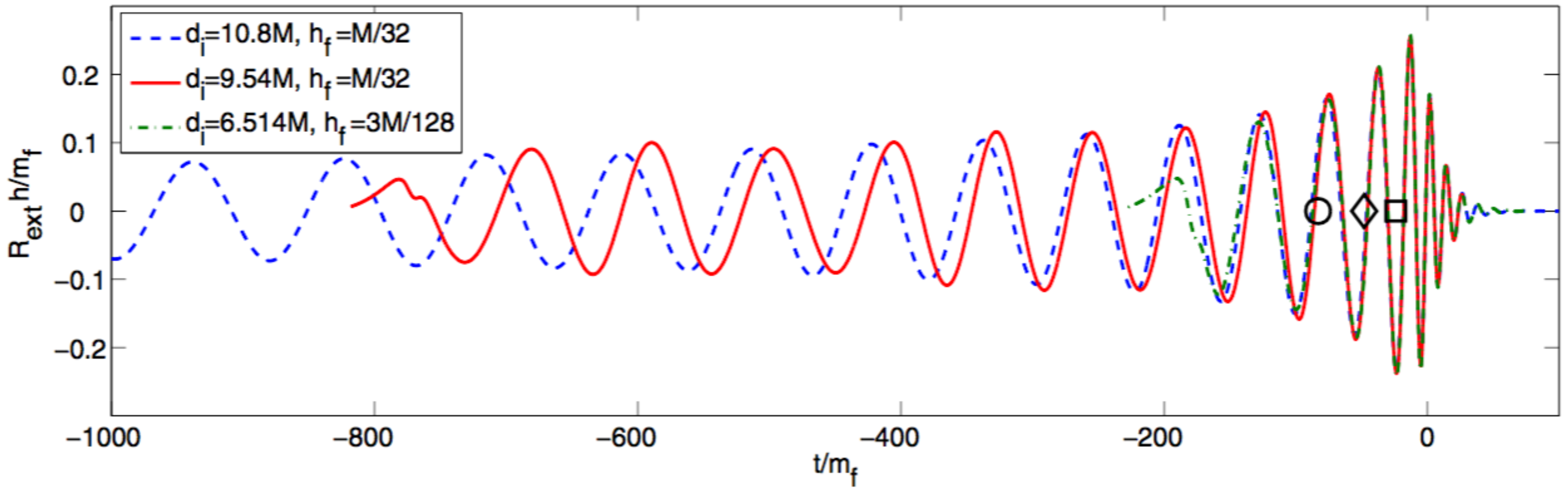
Pretorius 05



Baker+06



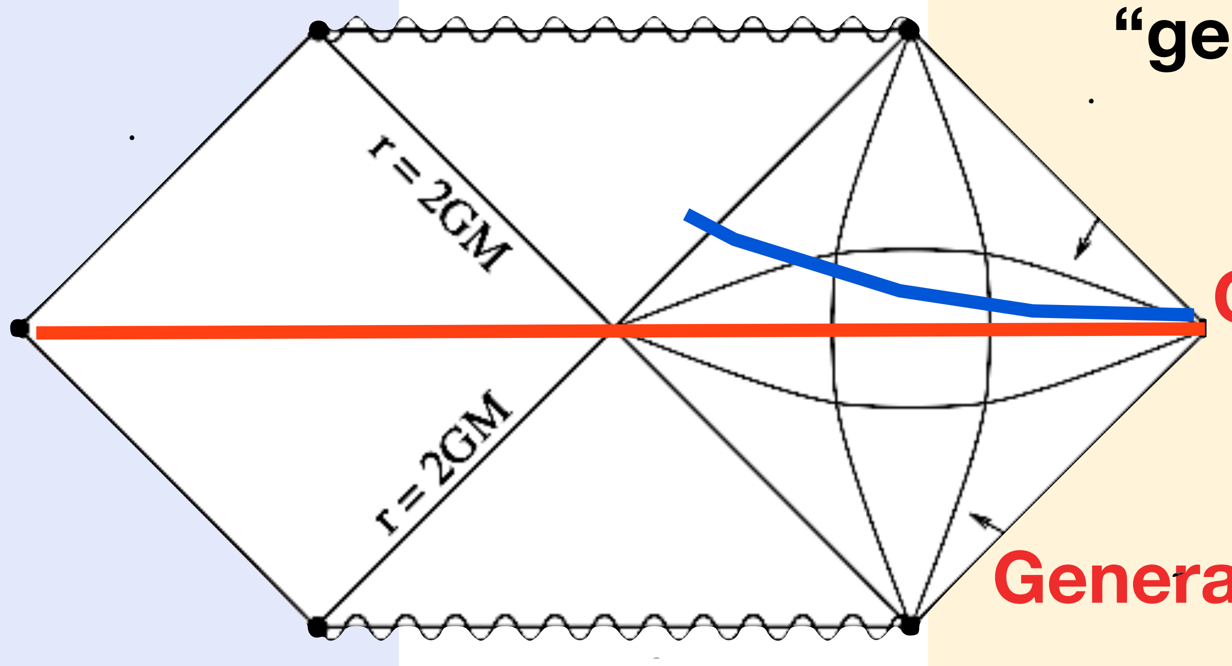
Campanelli+06



Baker+07

Important early result:
Simplicity of merger
Continuous transition
inspiral → ringdown

Major approaches towards BBH simulations



“BSSN & Moving punctures”

Puncture initial-data
$\chi \lesssim 0.9$ (but see Zlochower+ 17)

BSSN or CC4z

Moving puncture
mergers “easy”

Sommerfeld outer BC

4th to 8th order finite-difference

BHs advect through static grids

GW extrapolation
(Healy,Lousto ’20 for LazEv COM correction)

LazEv, Maya, ETK, BAM,
Goddard, GRchombo, …

“generalized harmonic & spectral”

Quasi-equilibrium excision data
$\chi \lesssim 0.999$

Generalized-Harmonic Evolution System

BH excision
mergers difficult

Constraint preserving,
minimally reflective outer BC

Spectral methods

Moving grid
long, phase-accurate inspirals

GW extrapolation & center-of-mass correction

Cauchy-characteristic extraction
accurate m=0 modes, GW memory

SpEC (SXS collaboration)

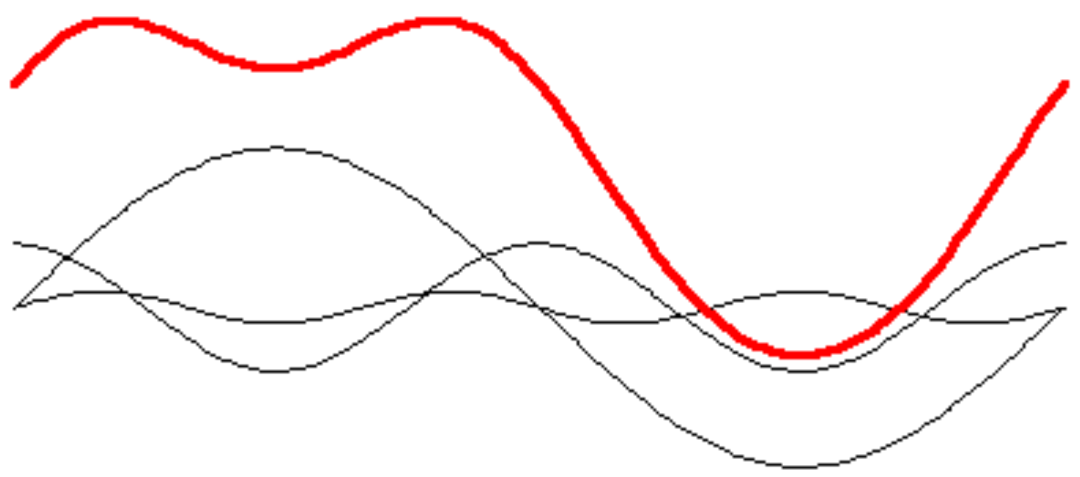
Spectral Einstein Code (SpEC)

Simulating eXtreme Spacetimes collaboration

http://www.black-holes.org/SpEC.html

- **Expand in basis-functions**

$$u(x,t) = \sum_{k=1}^{N} \tilde{u}_k(t)\Phi_k(x)$$

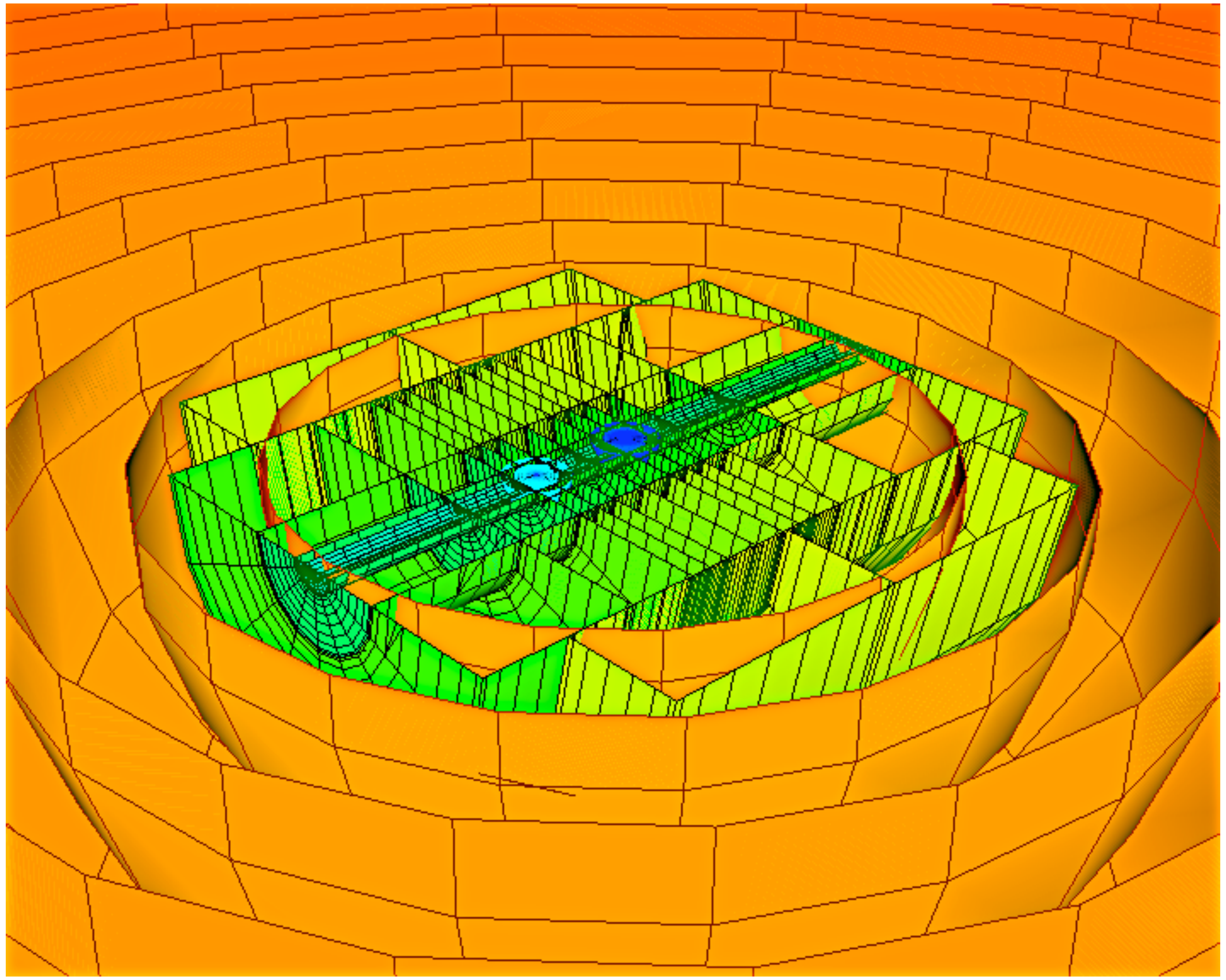


- **Compute derivatives analytically**

$$u'(x,t) = \sum_{k=1}^{N} \tilde{u}_k(t)\Phi'_k(x)$$

- ***Exponentially fast convergence***
  - for smooth problems

Combine w/ domain-decomposition



Einstein constraints: Formalism

$$R + (\mathrm{tr}K)^2 - K^2 = 0$$
$$\nabla \cdot (K - g\,\mathrm{tr}K) = 0$$

$g = \psi^4 \tilde{g}$

Lichnerowicz 44

$K = \frac{1}{3}\mathrm{tr}K\, g + A$

$$\tilde{\nabla}^2 \psi \quad = \dots$$
$$\tilde{\nabla}\cdot\left(\frac{1}{\tilde{\sigma}}\tilde{\mathbb{L}}V\right) = \dots$$

coupled nonlinear elliptic PDEs in 3D

A → conformal scaling, $A = \psi^{-10}\tilde{A}$ → $\tilde{A}$

A ↓ TT decomp.

$\tilde{A}$ ↓ conformal TT decomp.

$A = A_{\mathrm{TT}} + \frac{1}{\sigma}(\mathbb{L}V)$ → conformal scaling, $A_{\mathrm{TT}} = \psi^{-10}\tilde{A}_{\mathrm{TT}}$, $\sigma = \psi^6\tilde{\sigma}$ → $\tilde{A} = \tilde{A}_{\mathrm{TT}} + \frac{1}{\tilde{\sigma}}(\tilde{\mathbb{L}}V)$

Hamiltonian picture ≡ Lagrangian picture

York(+) 72; 74; 99, HP, York 03

Applied to binary black holes

- **Asymptotics/boundary conditions**

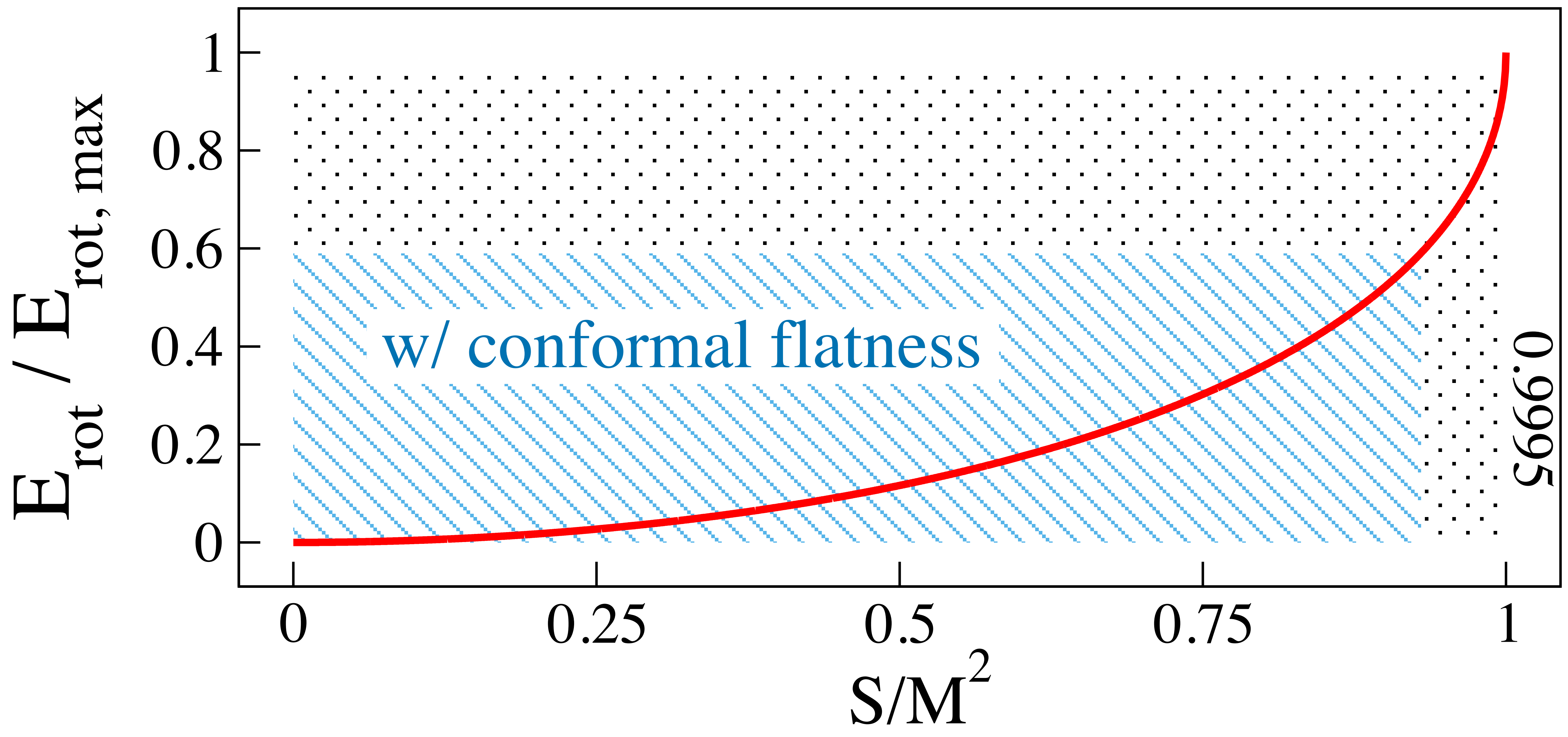
  Brandt, Brügmann 97; Cook,HP 04

- **Elliptic solver**

  HP+ 02; Ansorg 04; Vu,HP+ 21a,b

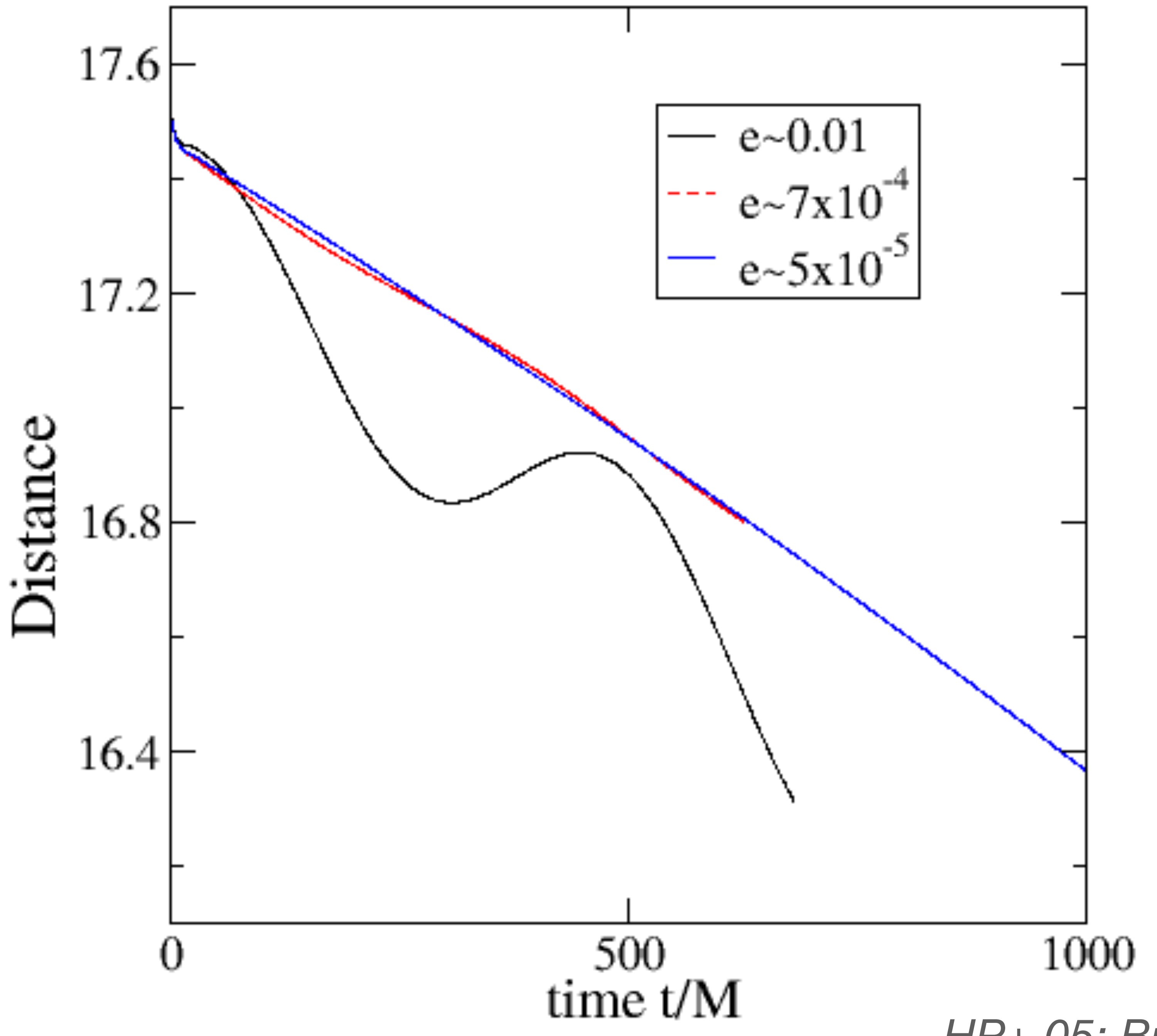
- **Spins > 0.9**

  Lovelace..HP+ 08



$$\tilde{\nabla}^2\psi \;= \dots$$
$$\tilde{\nabla}\cdot\left(\frac{1}{\tilde{N}}\tilde{\mathbb{L}}\beta\right) = \dots$$
$$\tilde{\nabla}^2\tilde{N} \;= \dots$$

- **Control eccentricity**



HP+ 05; Buonanno..HP+ 08
Nee, HP+ (in prep)

Generalized Harmonic Evolution System

- Einstein's equations

$$0 = R_{ab}[g_{ab}] = -\frac{1}{2}\Box g_{ab} + \nabla_{(a}\Gamma_{b)} + \text{lower order terms}, \qquad \Gamma_a = -g_{ab}\Box x^b.$$

- Generalized harmonic coordinates $g_{ab}\Box x^b \equiv H_a(x^a, g_{ab})$
  (Friedrich 1985, Pretorius 2005; $H = 0$ used since 1920's)

$$\Box g_{ab} = \text{lower order terms.}$$

Excellent GH exposition: Lindblom et al 2006

$\Rightarrow$ Constraint $C_a \equiv H_a - g_{ab}\Box x^b = 0$

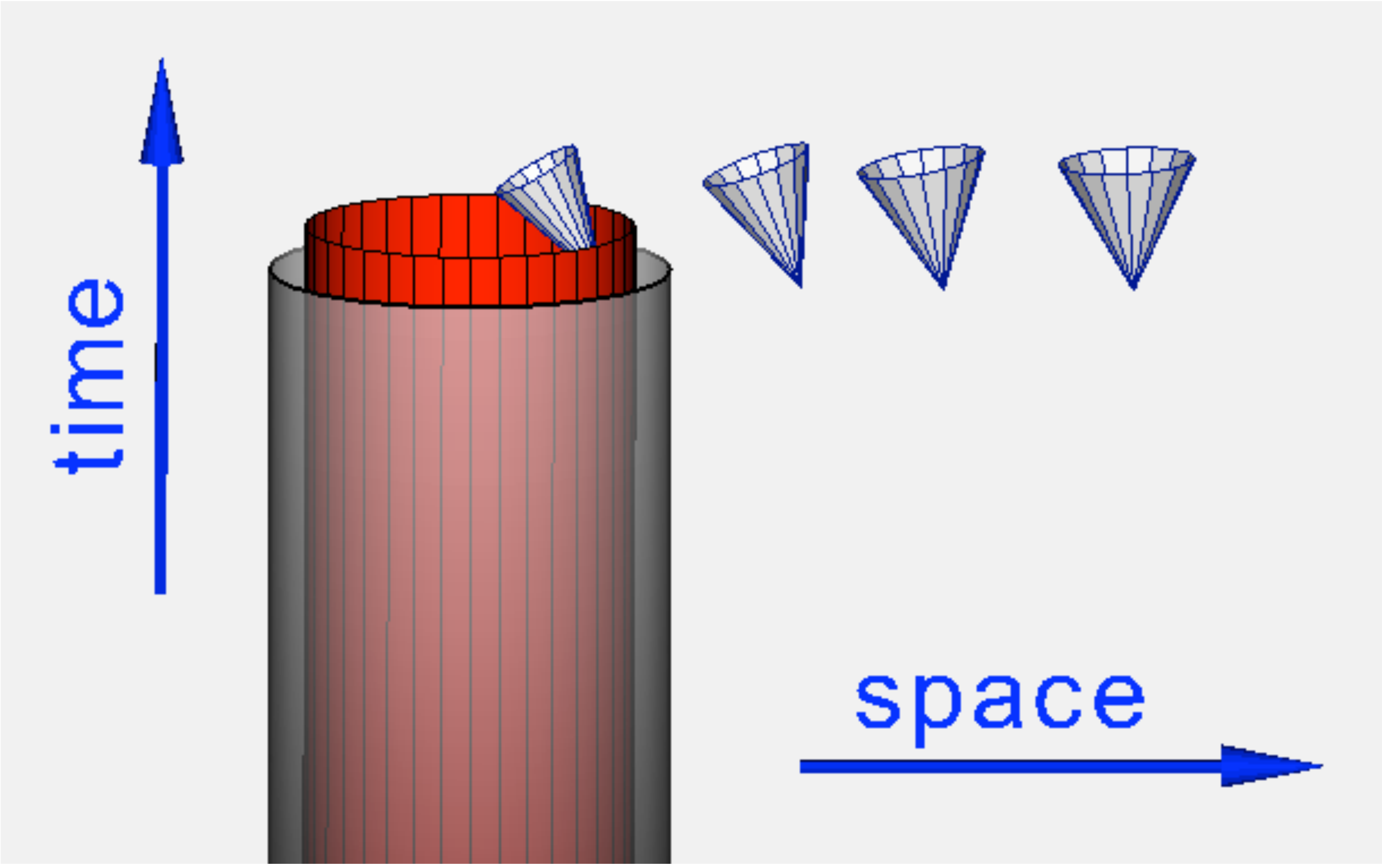
- Constraint damping (Gundlach, et *al.*, Pretorius, 2005)

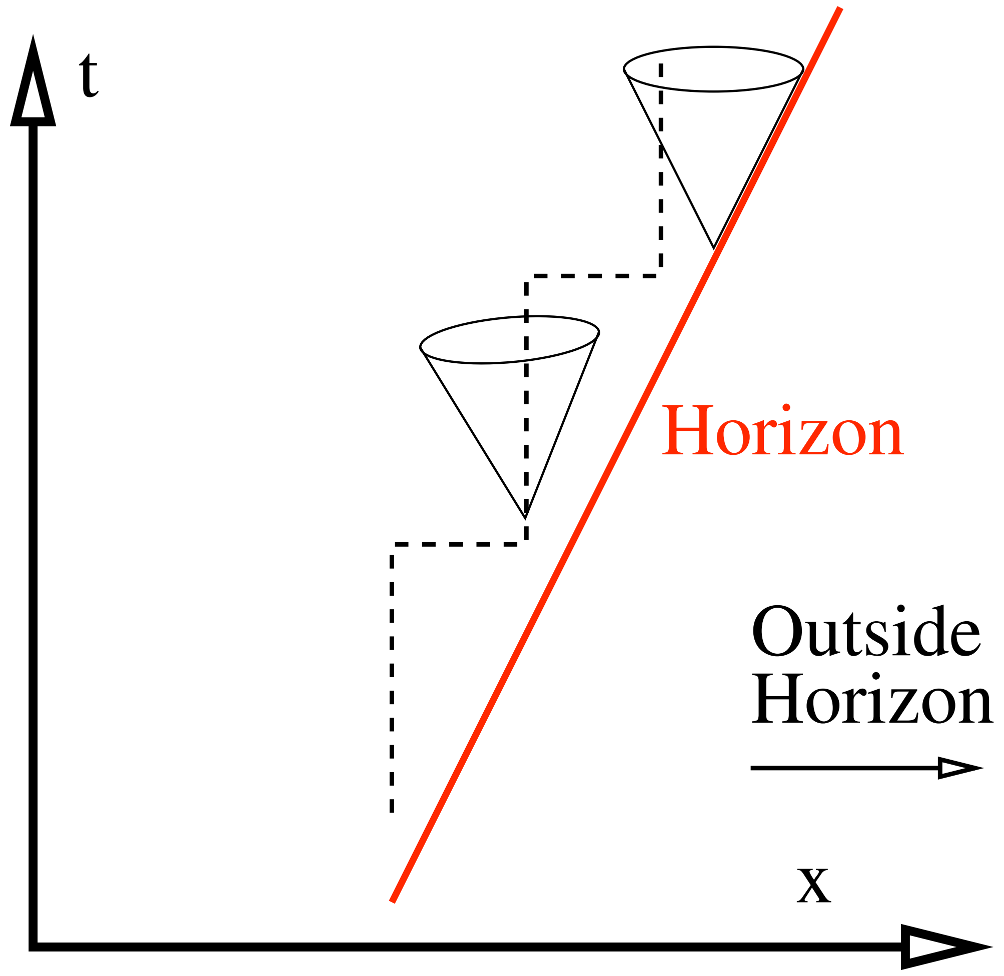
$$\Box g_{ab} = \gamma \left[t_{(a}C_{b)} - \frac{1}{2} g_{ab} t^c C_c \right] + \text{lower order terms}$$

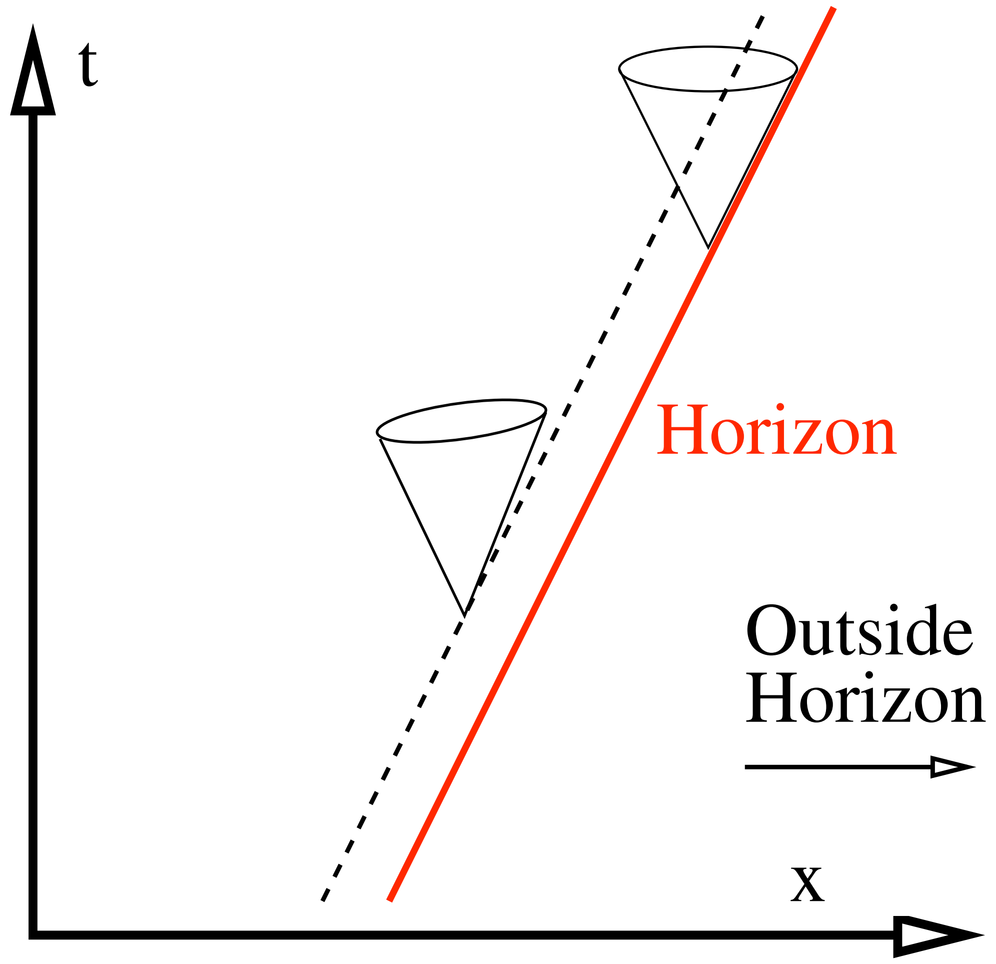
$$\partial_t C_a \sim -\gamma C_a.$$

BH Excision

- Excise inside BH horizons
- Excision boundaries:
  - **follow BHs continuously**
  - conform to shape of AH







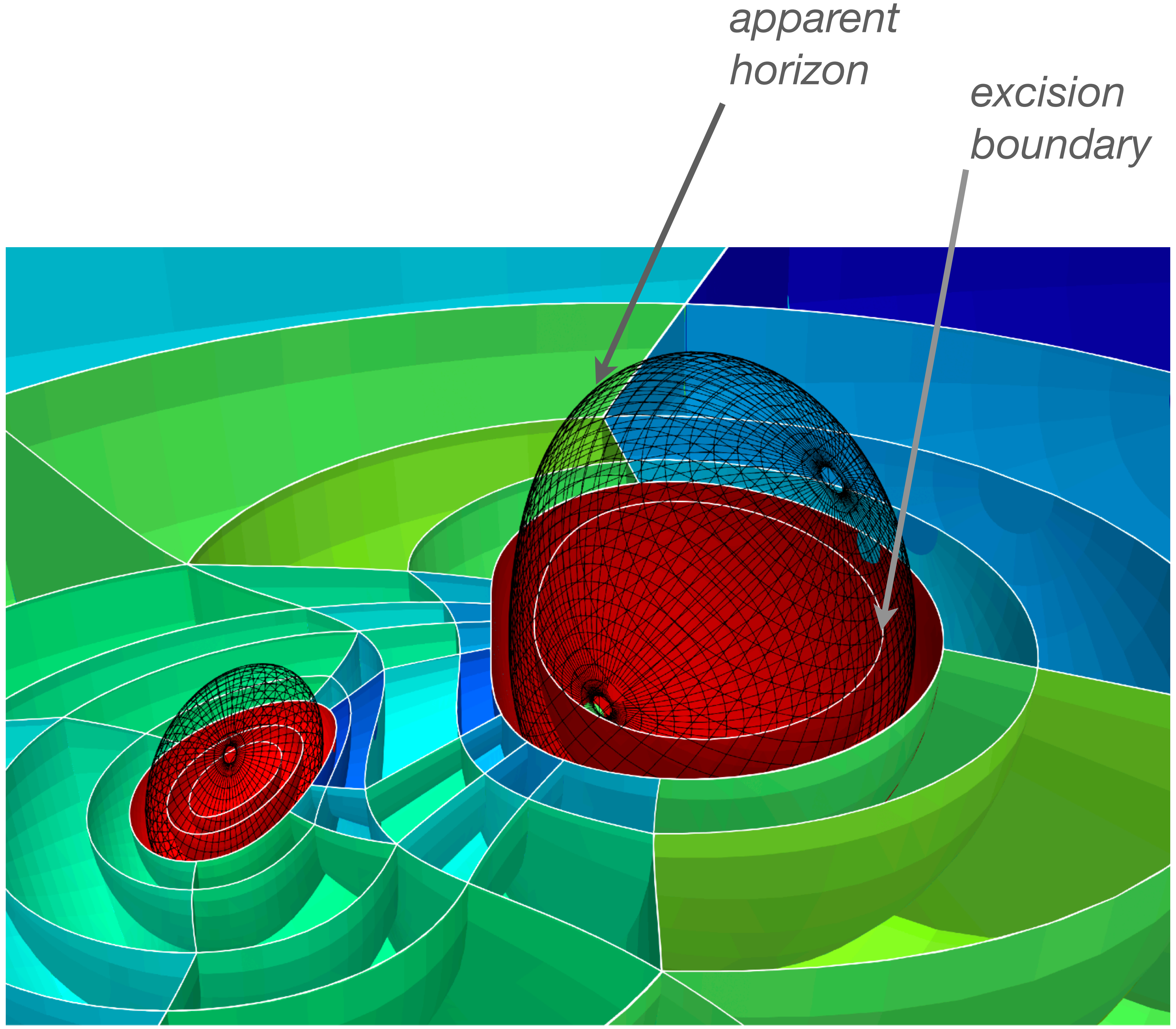
Scheel, HP+ 08, Szilagyi+ 08, Hemberger+ 13

BH Excision

- Excise inside BH horizons
- Excision boundaries:
  - **follow BHs continuously**
  - conform to shape of AH
- **Horizon tracking & shape-control**

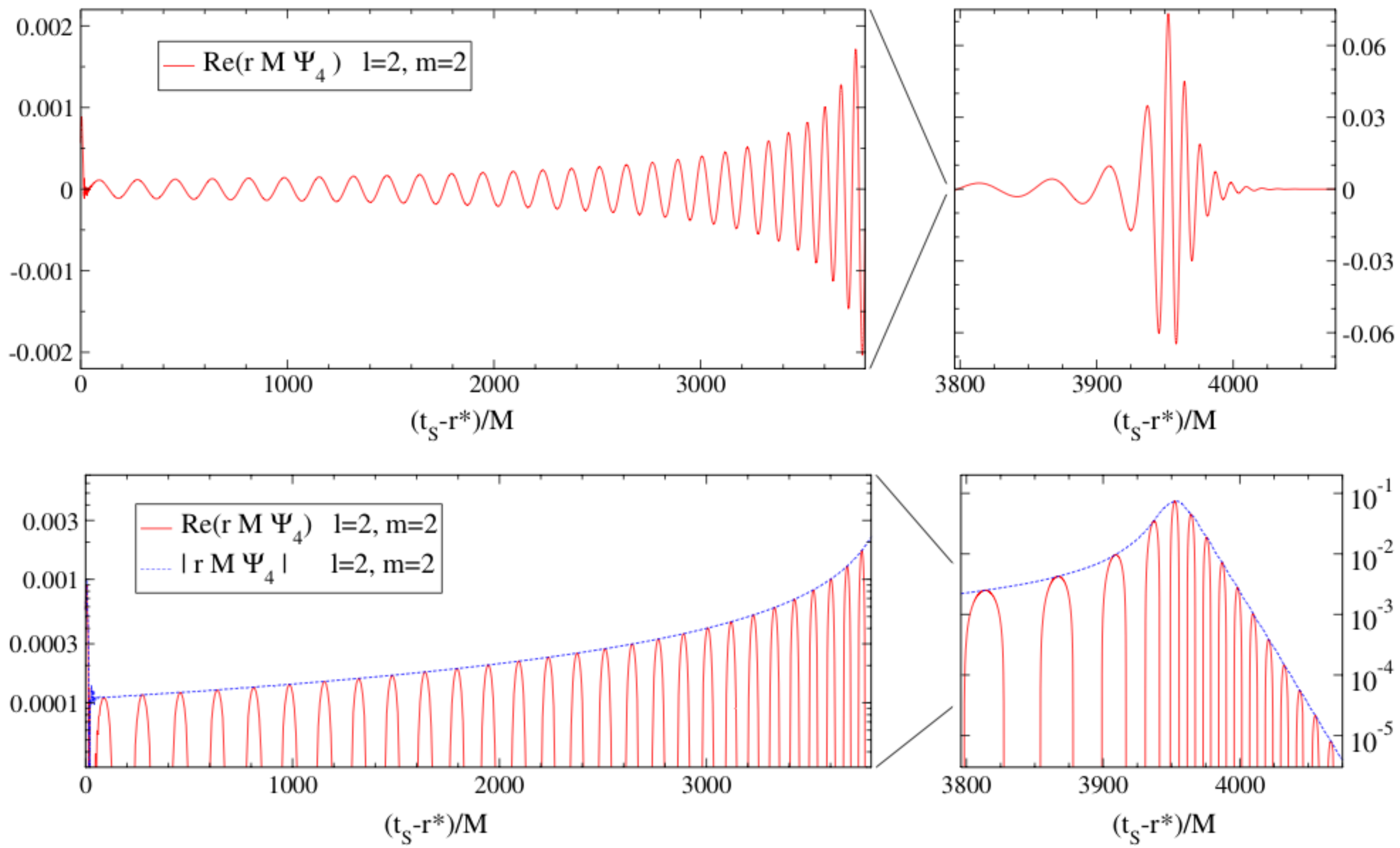
$$\vec{x}_{\text{inertial}} = a(t)\mathbf{R}(t)\vec{\xi}_{\text{grid}} + \text{deformations}$$

  - {a(t), R(t),...} determined by feedback-loop
    - find AH(t) in $\vec{x}_{\text{inertial}}$
    - adjust {a(t), R(t),...} to keep excision boundaries inside AH

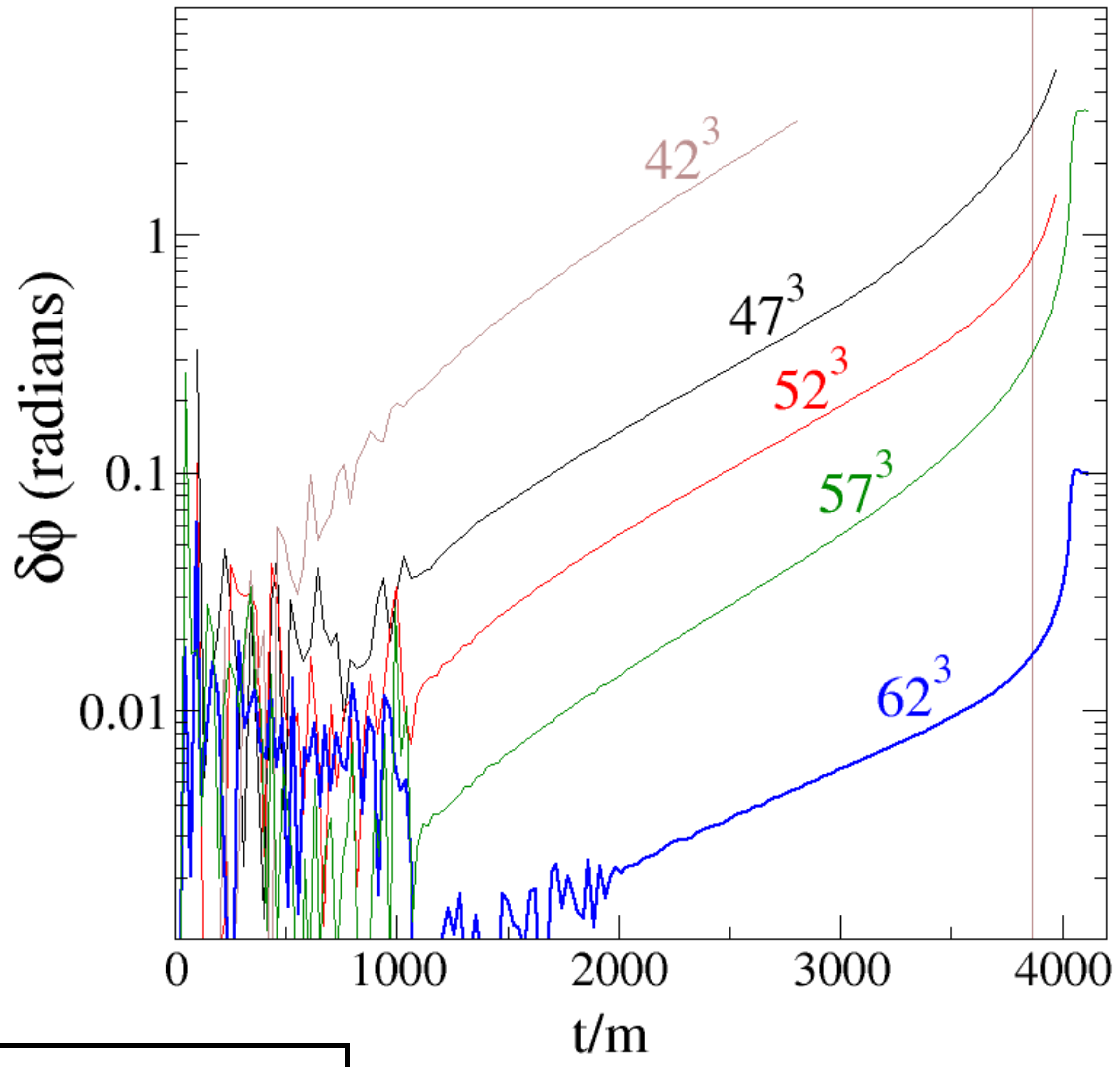


Scheel, HP+ 08, Szilagyi+ 08, Hemberger+ 13

Accuracy of SpEC (circular inspiral)



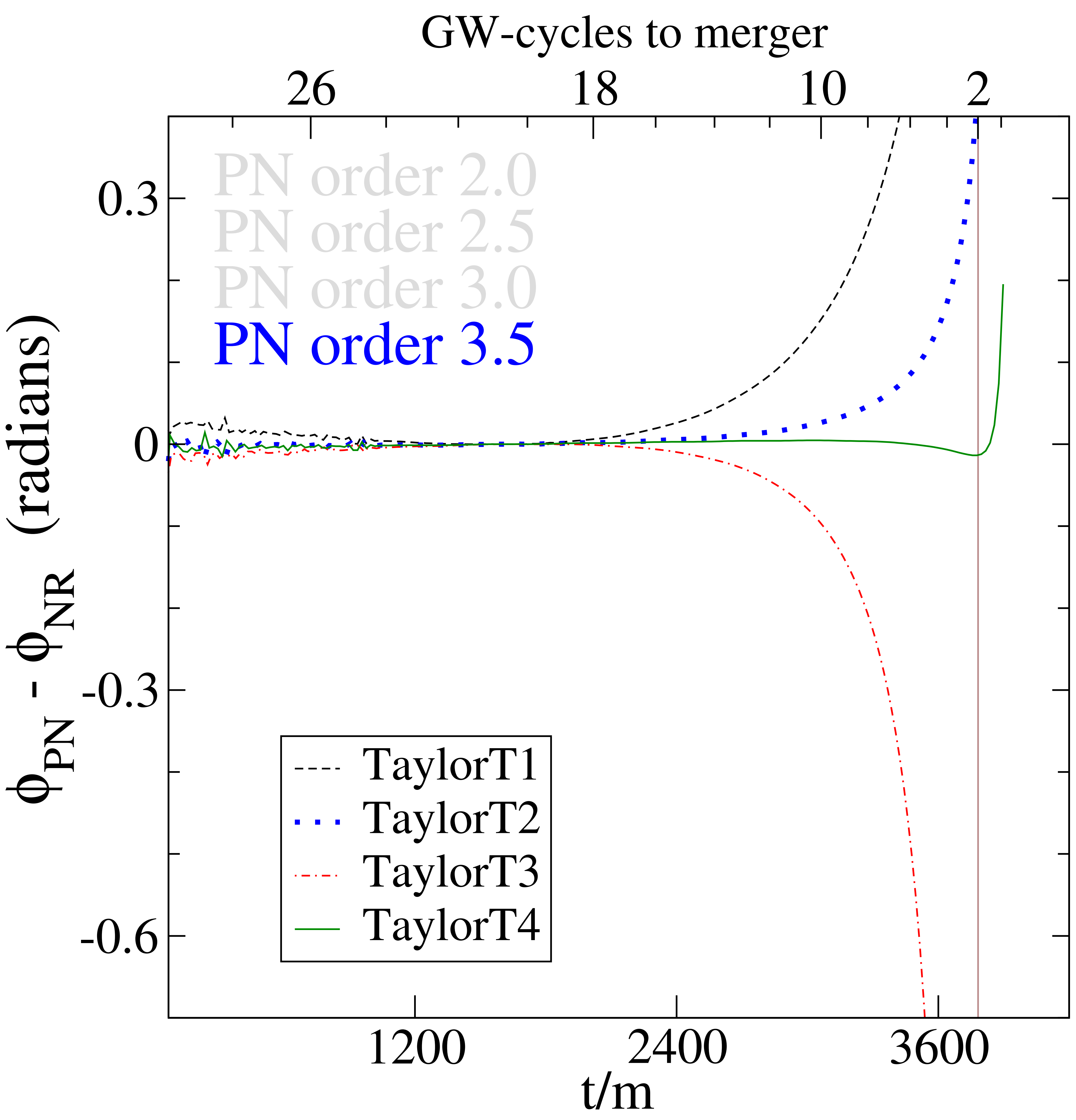
Phase error at different resolutions



- ***Rapid convergence*** *due to spectral methods*
- *Small errors due to moving grid*
- *Best code for long inspirals (but mergers hard)*

Scheel,HP+ 09

post-Newtonian vs. NR



TaylorT1...T4
Different choices to truncate energy balance equation

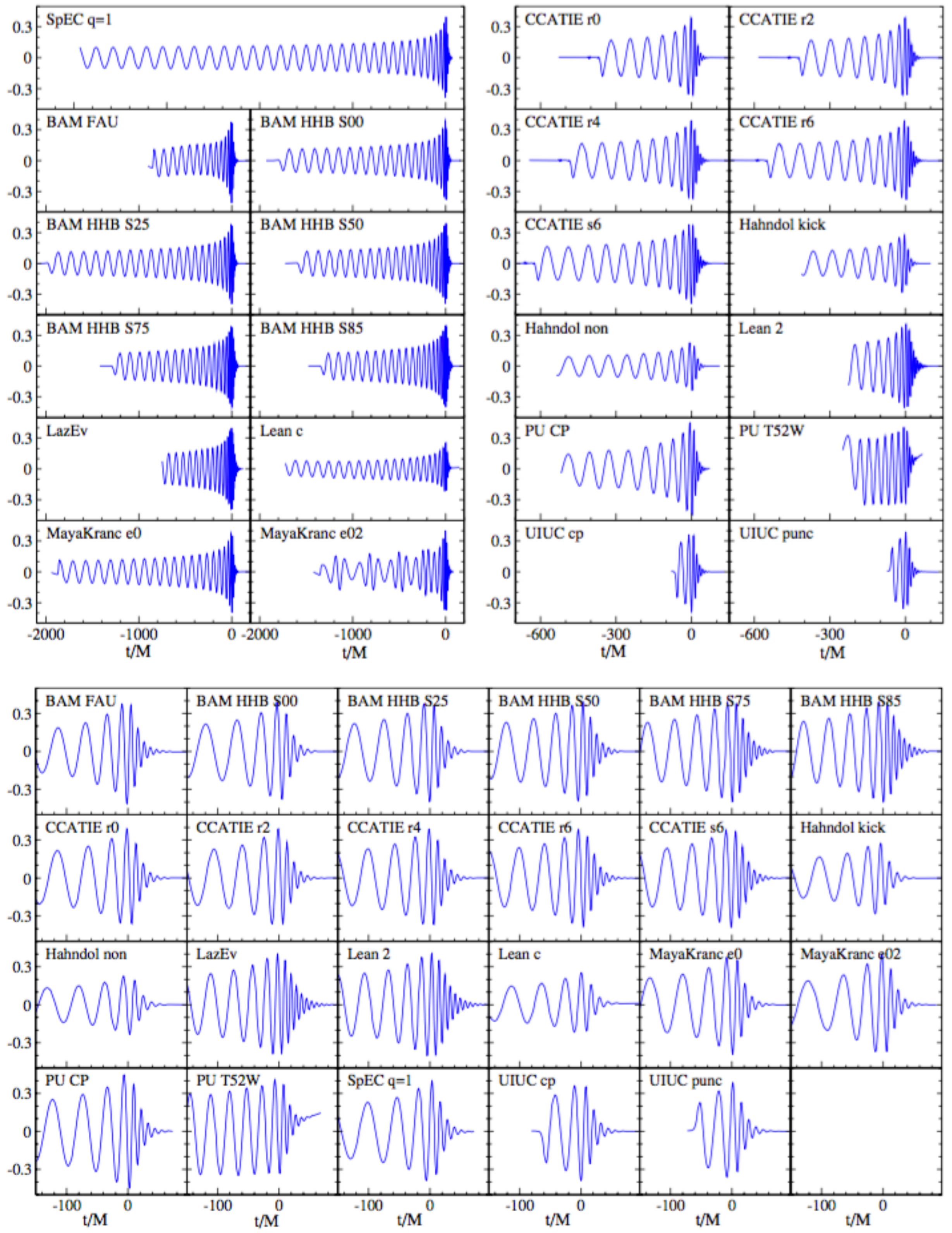
$$\frac{dE}{dt} = -F_{\mathrm{GW}}$$

Boyle..HP+ 07

NR Parameter space exploration

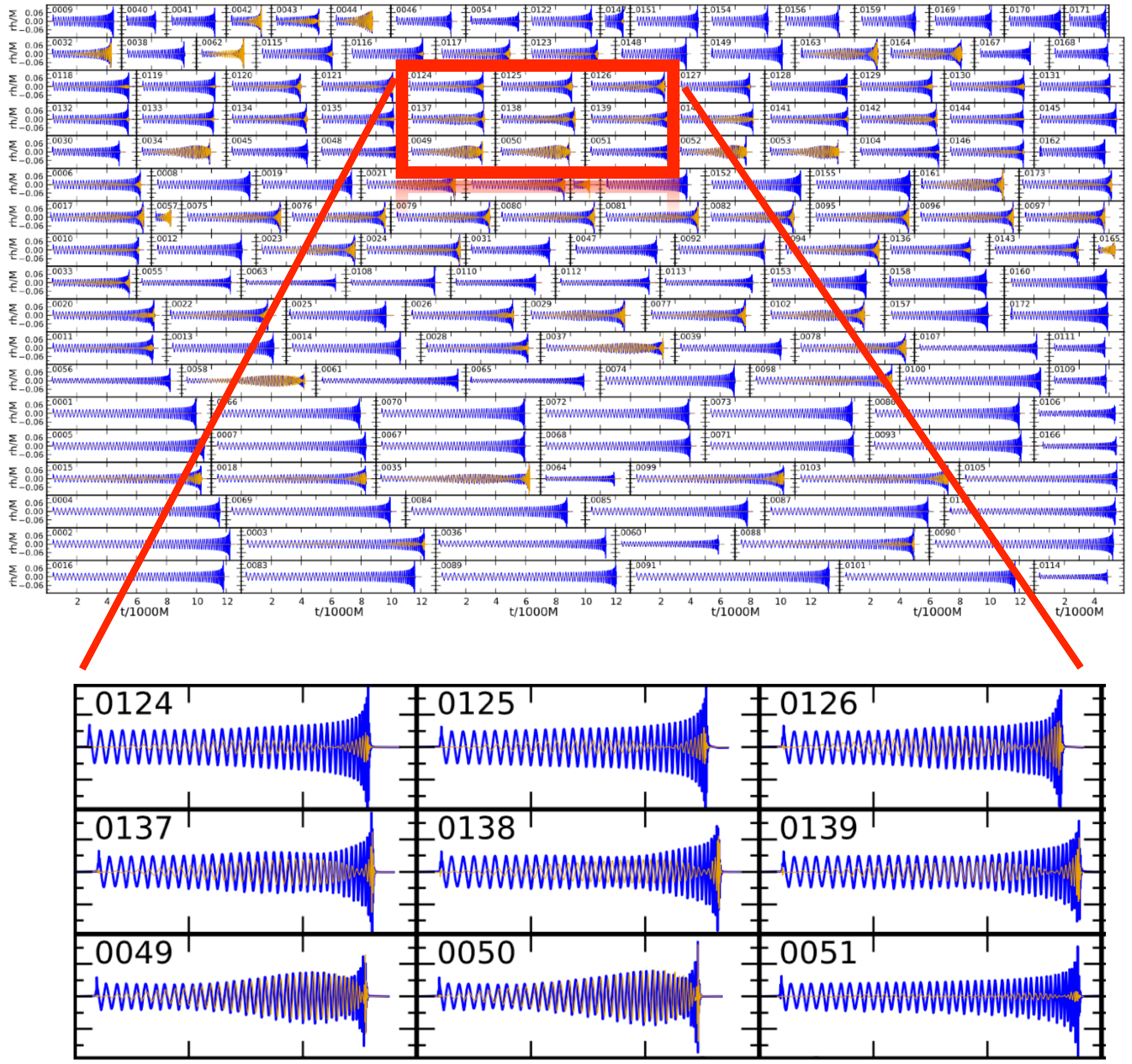
NINJA

Aylott .. HP+ 09



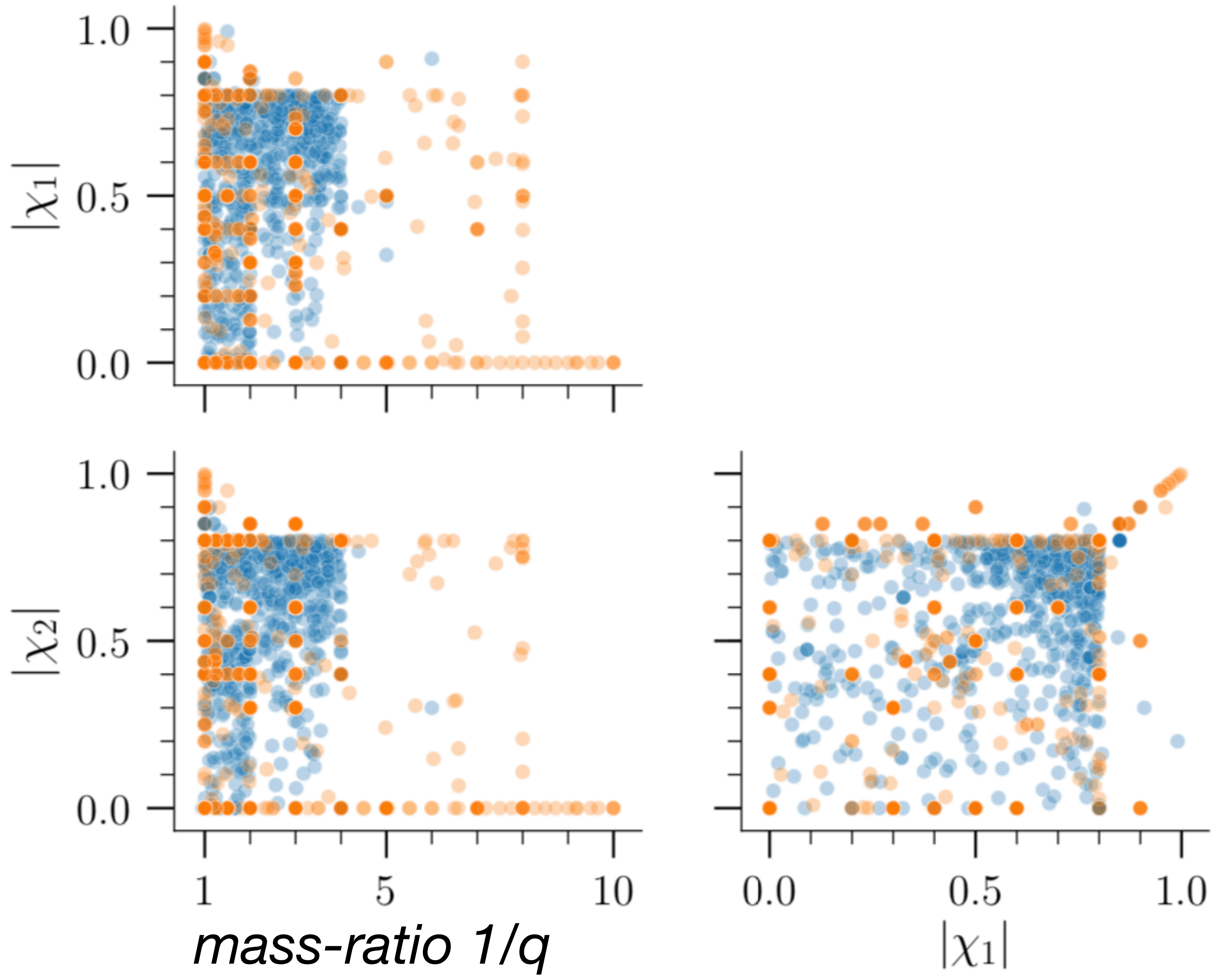
1st SXS Catalog

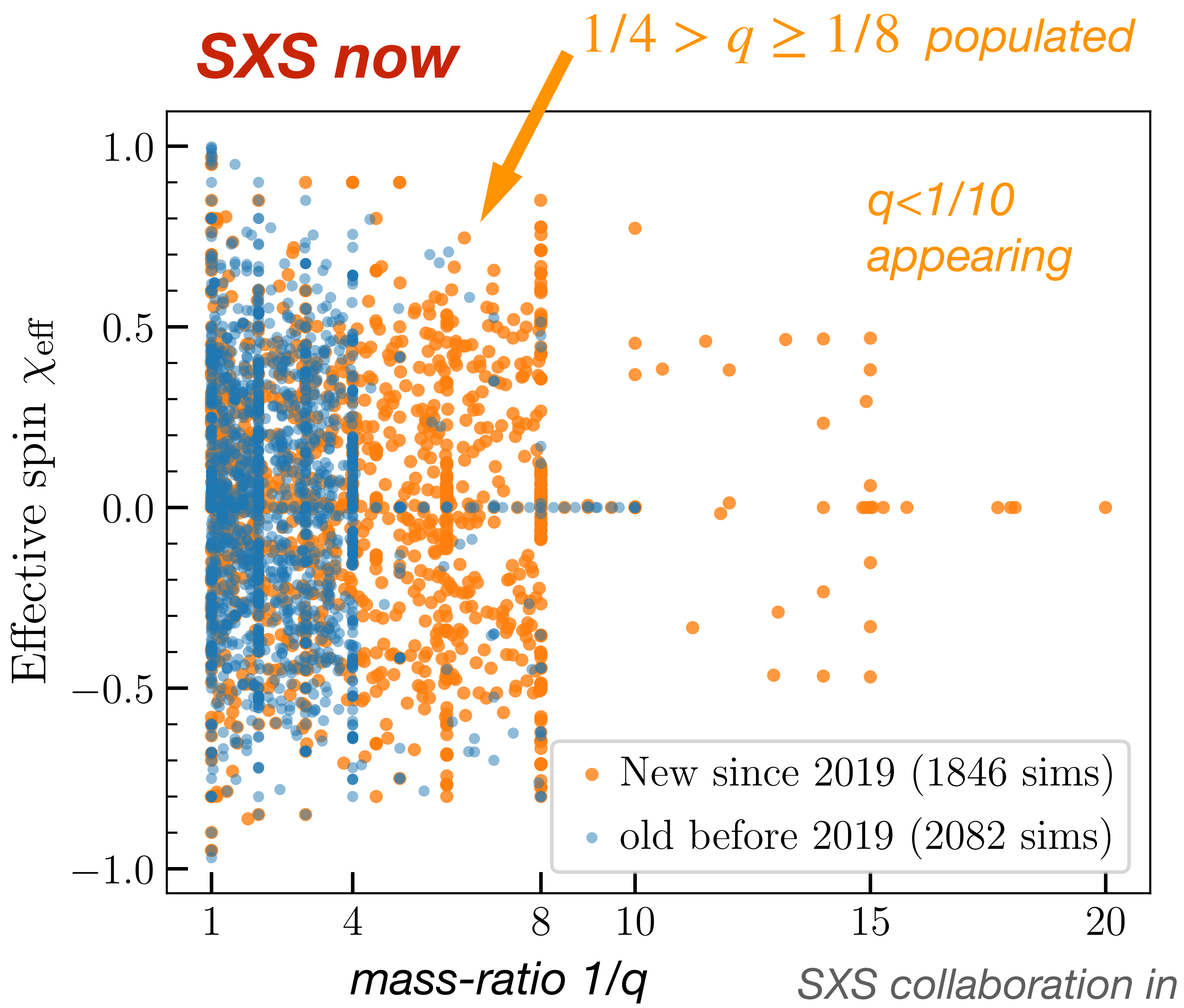
Mroue .. HP+ 13



2nd SXS Catalog

Boyle..HP+ 19





SXS collaboration in prep

More parameter space exploration efforts

$q \geq 1/15$

Catalog	Started	Updating?	Simulations	m_1/m_2 range	$\lvert\chi_1\rvert$ range	$\lvert\chi_2\rvert$ range	Precessing?	Median $N_{\rm cyc}$	Public?
NINJA [98, 115]	2008	✗	63	1–10	0–0.95	0–0.95	✗	15	✗
NRAR [120]	2013	✗	25	1–10	0–0.8	0–0.6	✓	24	✗
Georgia Tech [122]	2016	✓	452	1–15	0–0.8	0–0.8	✓	4	✓
RIT (2017) [123]	2017	✓	126	1–6	0–0.85	0–0.85	✓	16	✓
RIT (2020) [124]	2017	✓	777	1-15	0–0.95	0–0.95	✓	19	✓
NCSA (2019) [125]	2019	✗	89	1–10	0	0	✗	20	✗
SXS (2018)	2013	✓	337	1–10	0–0.995	0–0.995	✓	23	✓
SXS (2019)	2013	✓	2018	1–10	0–0.998	0–0.998	✓	39	✓

eccentric

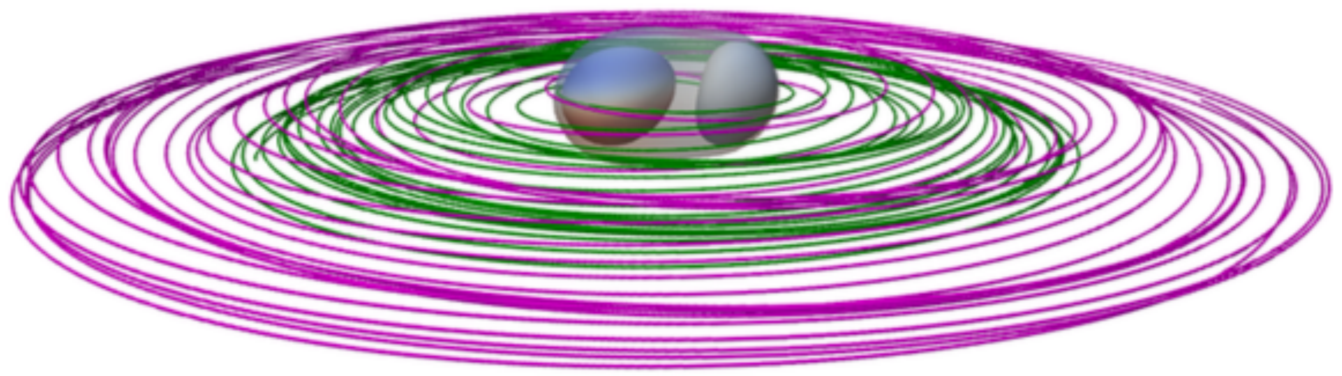
longest sims

highest spins

Table from Boyle et al 2019 (1904.04831)

AND...
Palma group (Husa+), Cardiff group (Hannam+)
Eccentric catalog (Healy+Lousto (PRD 2022), 2202.00018)
Maya 2nd catalog (Ferguson+ 2309.00262)

Parameter space: NR records

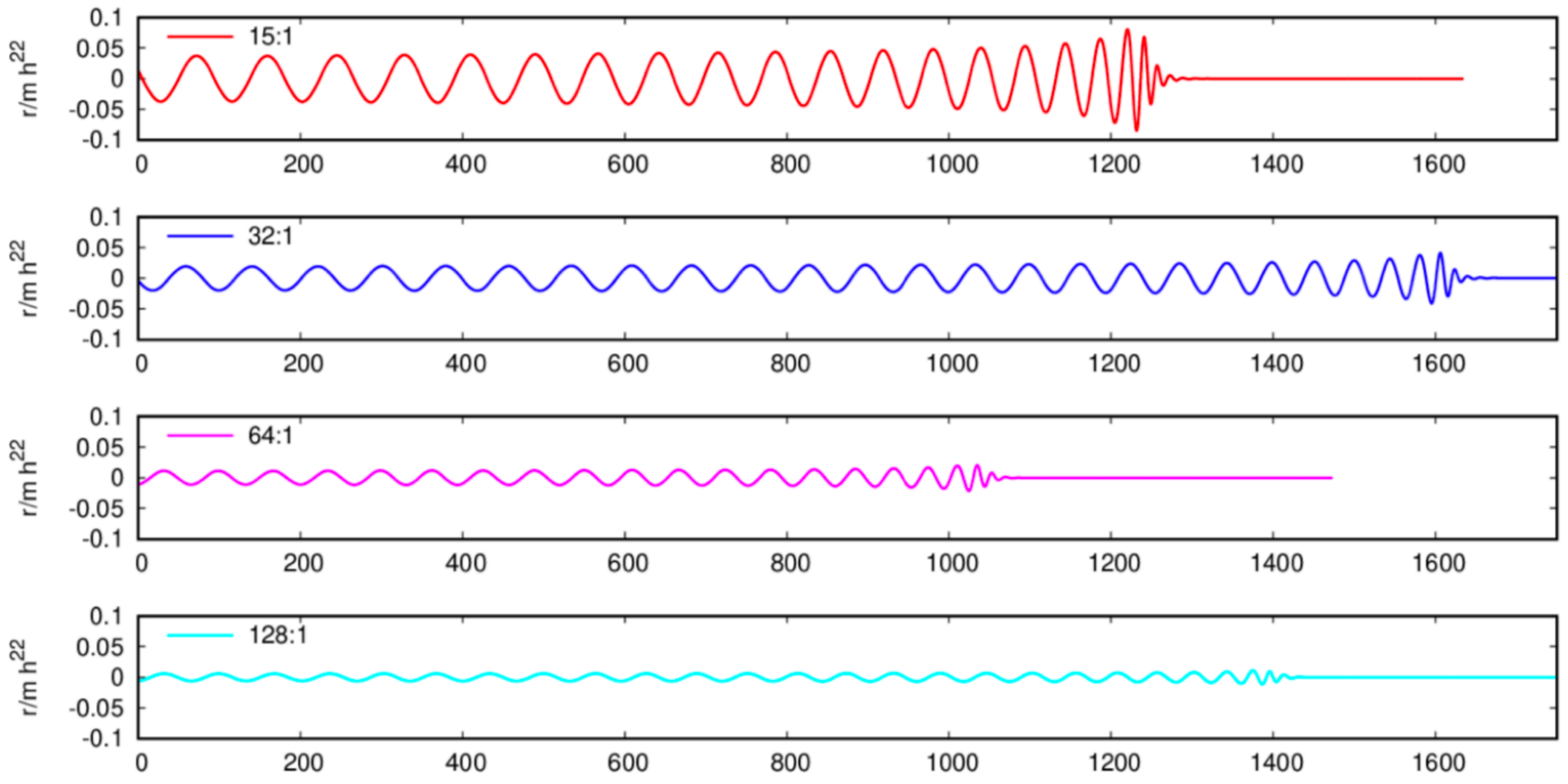


q=1: $S/M^2 = 0.998$
q<1: $S/M^2 = 0.95$

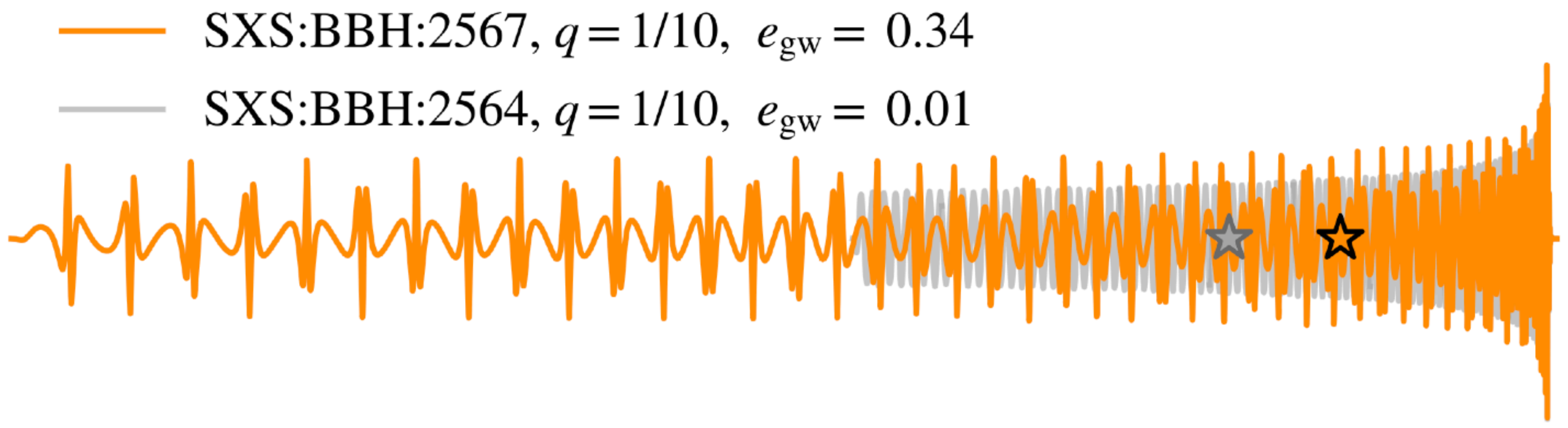
Scheel+ 14
Lovelace..HP+15
Boyle..HP+ 19

q = 1/128

Lousto, Healy 20

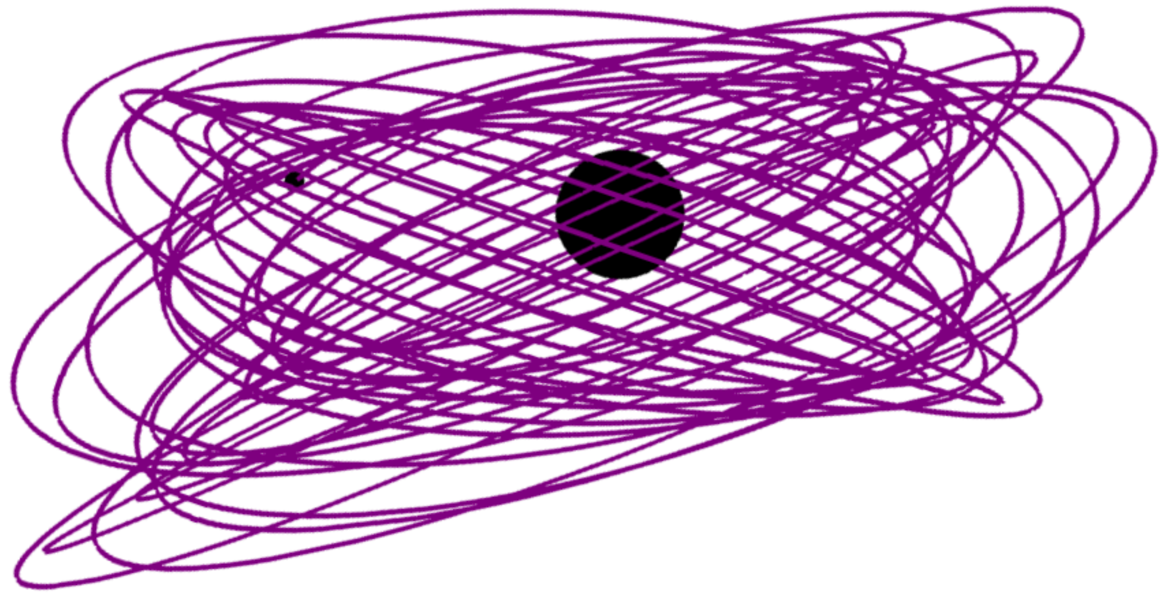


eccentric, high q



Ramos-Buades, .. HP+ 22

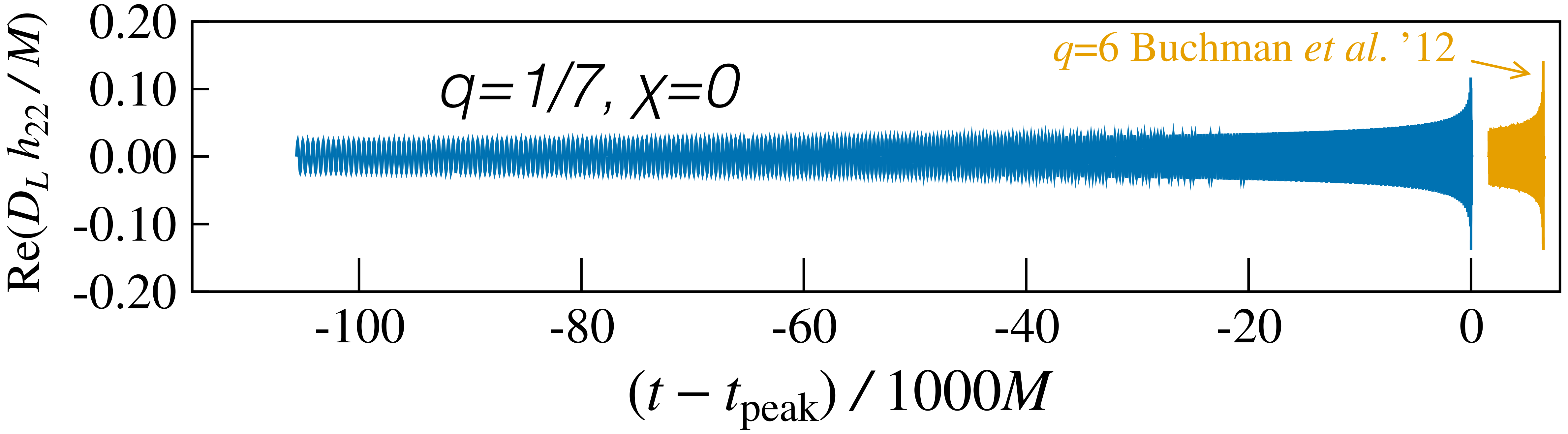
***eccentric, precessing* q = 1/7**



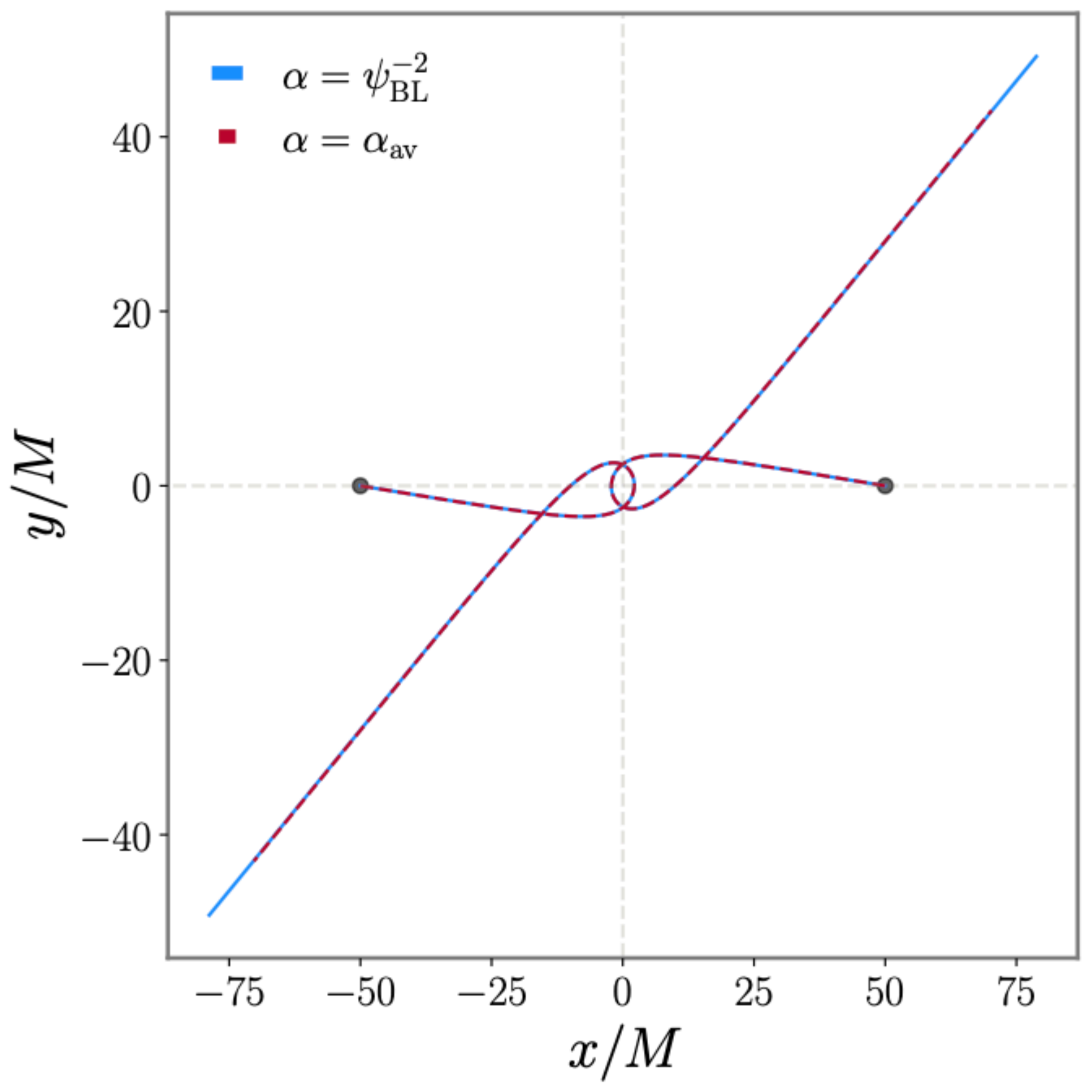
Lewis, Zimmerman, HP 17

350 *GW cycles*

Szilagyi..HP+15



hyperbolic scatter w/ spin

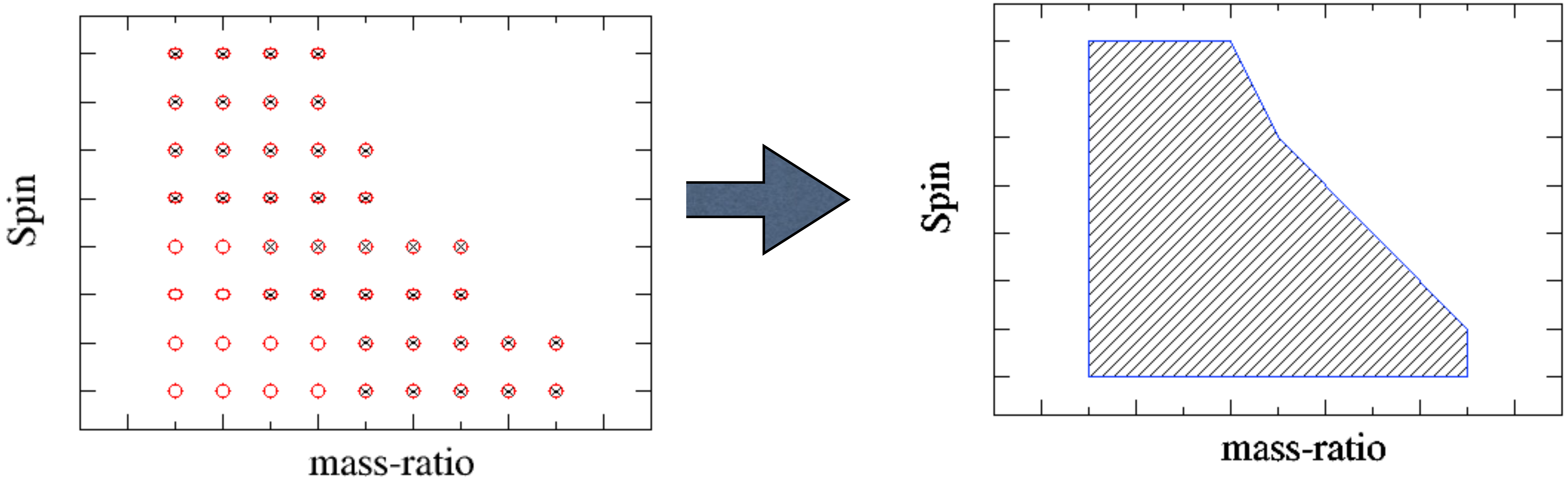


Rettegno, Pratten+ 23

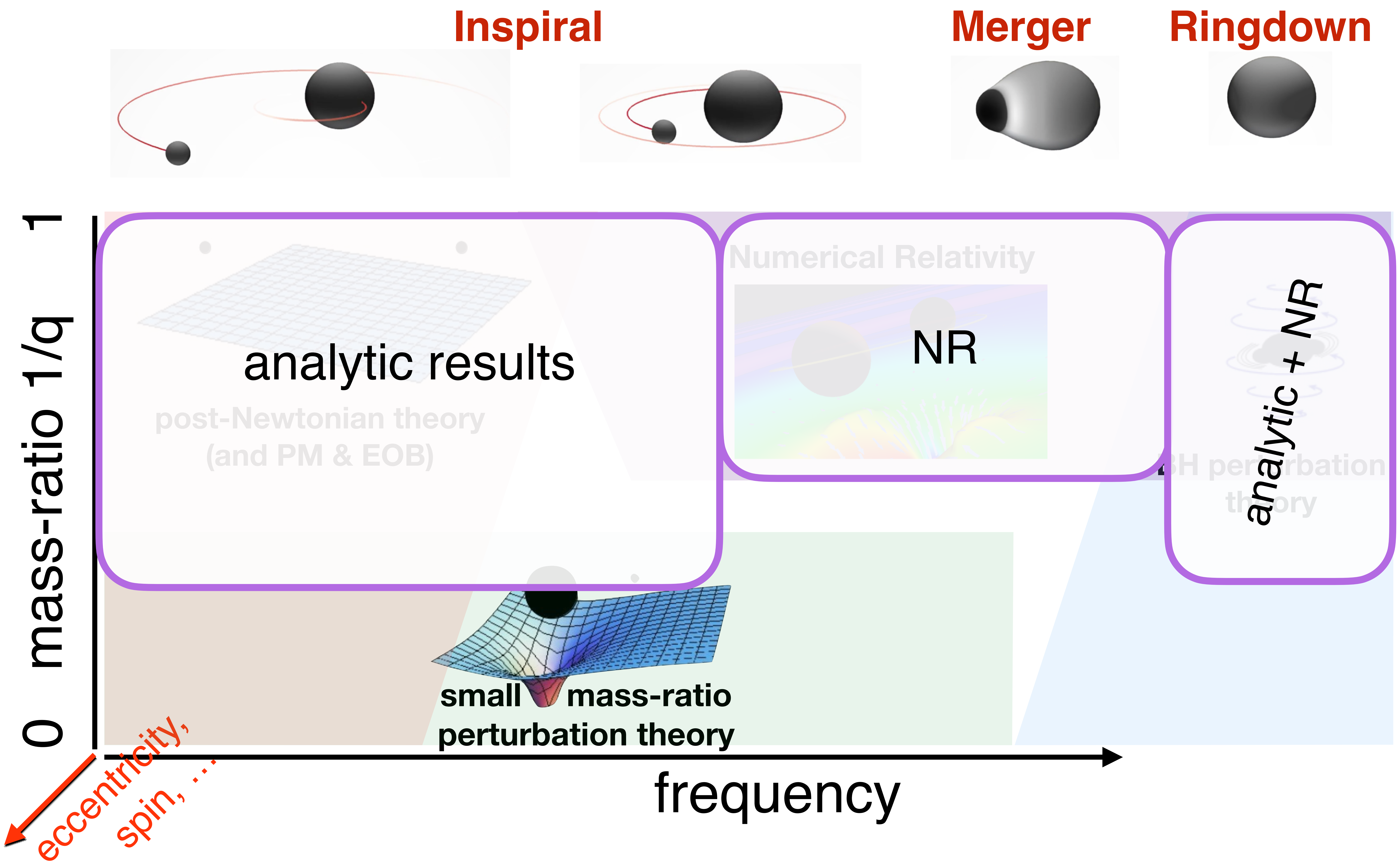

$M = 50\ M_\odot$
$q = 1 : 6$
$\vec{\chi}_1 = (0.17,\ -0.38,\ 0.68)$
$\vec{\chi}_2 = (0.00,\ 0.00,\ 0.00)$
$e = 0.40$
$F\ [L_\odot\ sr^{-1}]$
10^{22}
10^{21}
10^{20}
10^{19}
10^{18}
10^{17}
10^{16}
-1340 ms
$h_\times$
h_+

Waveform models

- **Continuous** in parameters $\theta = \{m_1, m_2, \vec{S}_1, \vec{S}_2; \quad e, l; \quad \iota, \phi, \psi; \quad \mathrm{RA}, \mathrm{dec}, D_L, T_c\}$
- **Fast** evaluation
- Cover parameter-space
- Accurate



LIGO/Virgo IMR waveform models



Inspiral-merger-ringdown BH-BH waveform models

Effective one body (EOB)

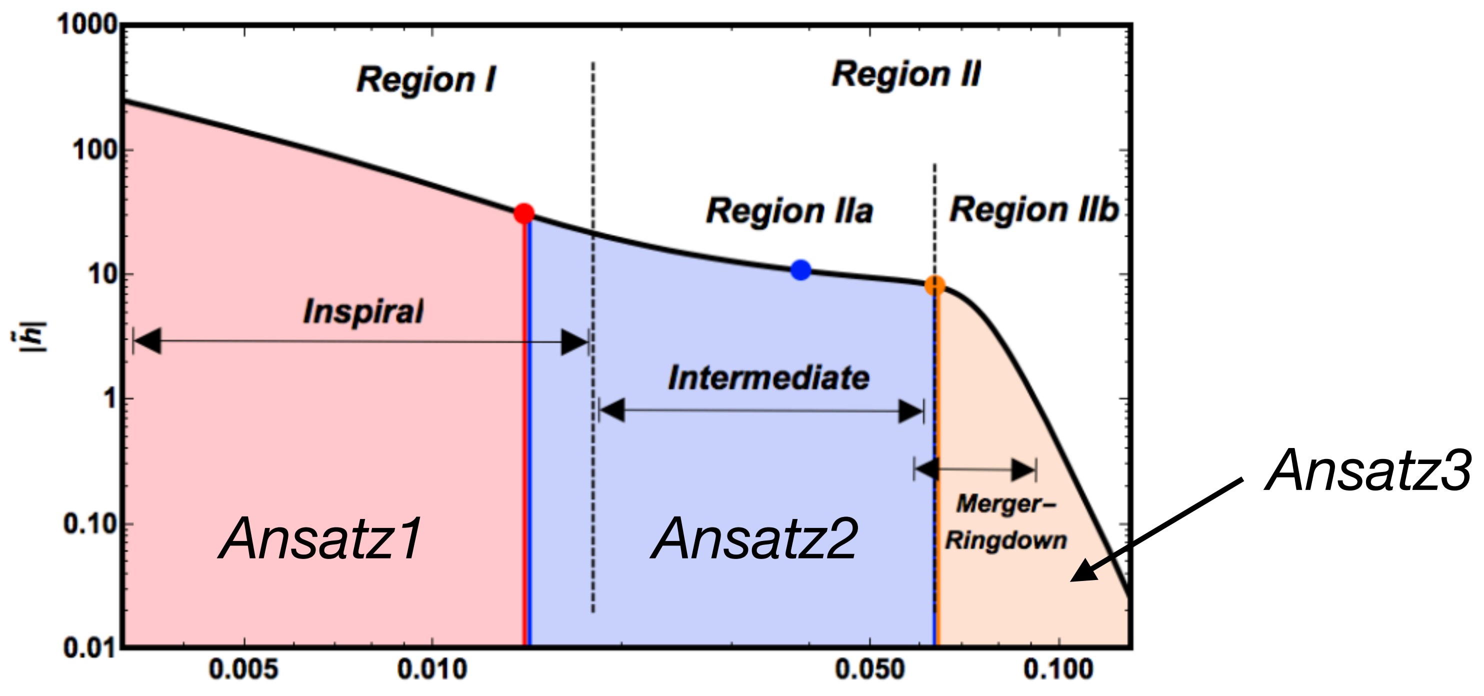
$$H = \mu\sqrt{p_r^2 + A(r)\left[1 + \frac{p_\varphi^2}{r^2} + 2(4-3\nu)\nu\frac{p_r^4}{r^2}\right]}$$

Hamiltonian dynamics dynamics $\Rightarrow h_{lm}(\theta;\ t)$

Buonanno, Damour 99

EOBNR Buonanno+ 09, Pan+11
SEOBNR (aligned spins) Taracchini+ 12
SEOBNv3 (precessing) Pan+ 14, Taracchini+14,
SEOBNRv4 {HM, P, PHM, HM_ROM} Bohe+ 17, Cotesta+ 18, Cotesta+ 20, Ossokine+ 20
TEOBResumS, TEOBResumSM Nagar+ 18, 20
SEOBNRv5{HM,P,PHM,E,PEHM}

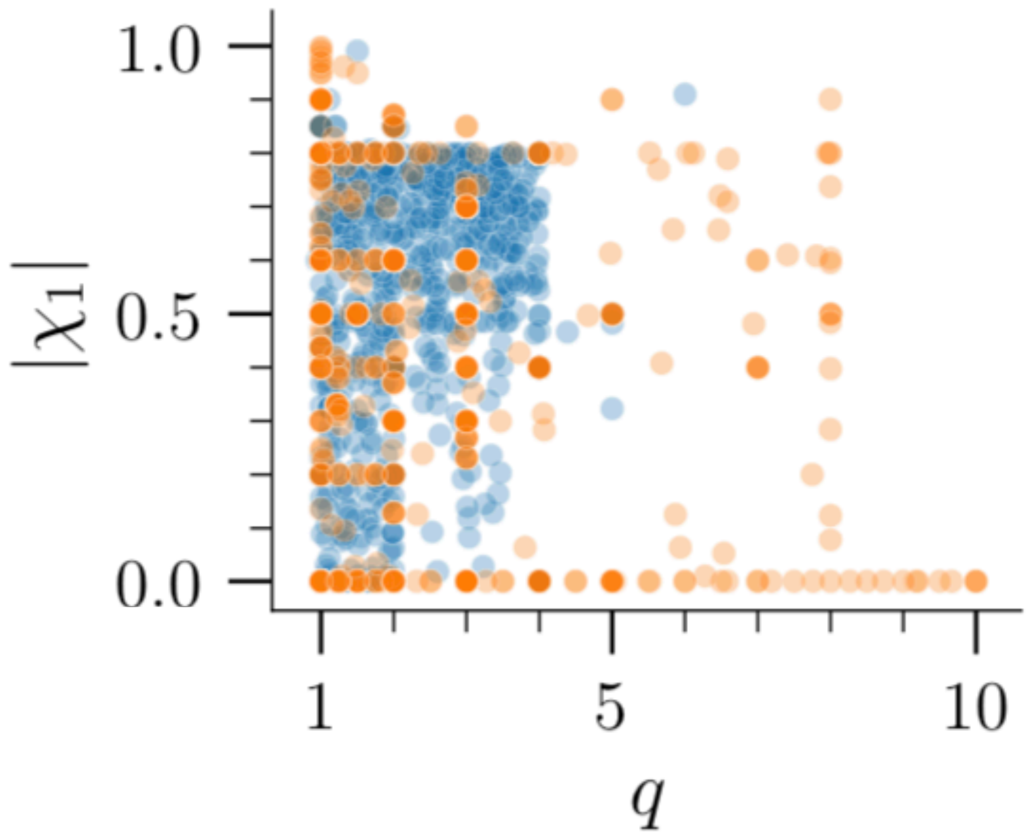
Phenomenological (Phenom)



Power-series for $\tilde{h}_{lm}(\theta; f)$

Phenom Ajith+ 08
Phenom{B,C} (aligned spins) Ajith+ 11, Santamaria+ 10
PhenomD Husa+ 15, Khan+ 15
PhenomHM London+ 18
Phenom{P, Pv2} (precessing) Hannam+ 13
Phenom{Pv3, Pv3HM} Khan+ 19, Khan+ 20
PhenomX{AS, HM, P, PHM} Pratten+ 20, Garcia-Quiros+ 20ab, Pratten+ 21
PhenomPNR Hamilton+ 21
PhenomX_{O4a, Taylor}

NR surrogate Models



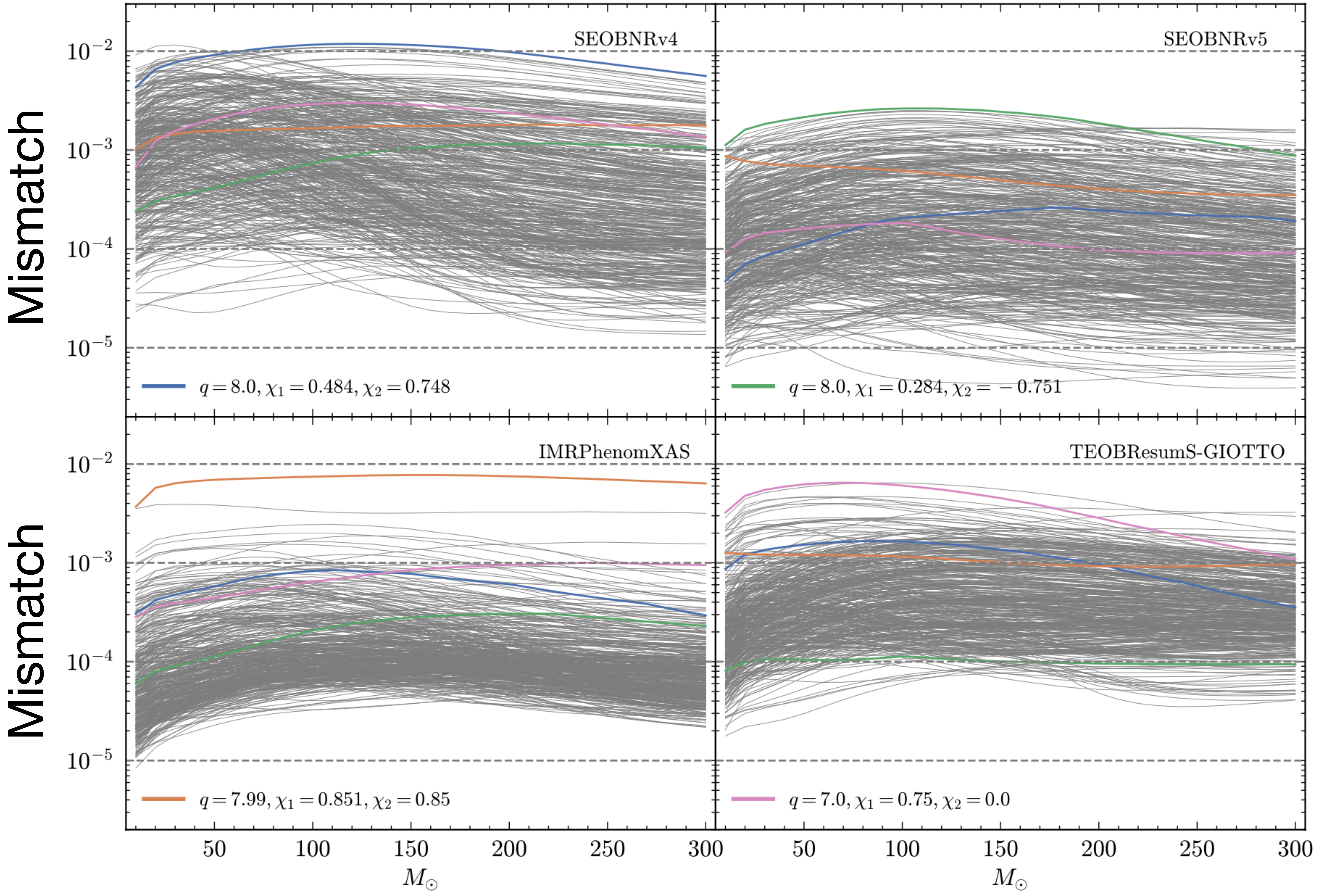
Direct interpolation in parameters θ

Blackman+ 15,17,18
Varma+ 18,19
Islam+ 21

Major application of NR: waveform modeling

- **Assess accuracy** of waveform models
- Determine **importance of improvements**
  - higher order
  - additional physics
    - higher modes, precession, memory
- **Calibrate parameters** in model to improve agreement
    - GW modes & fluxes
    - merger dynamics
    - ringdown attachment
    - ...

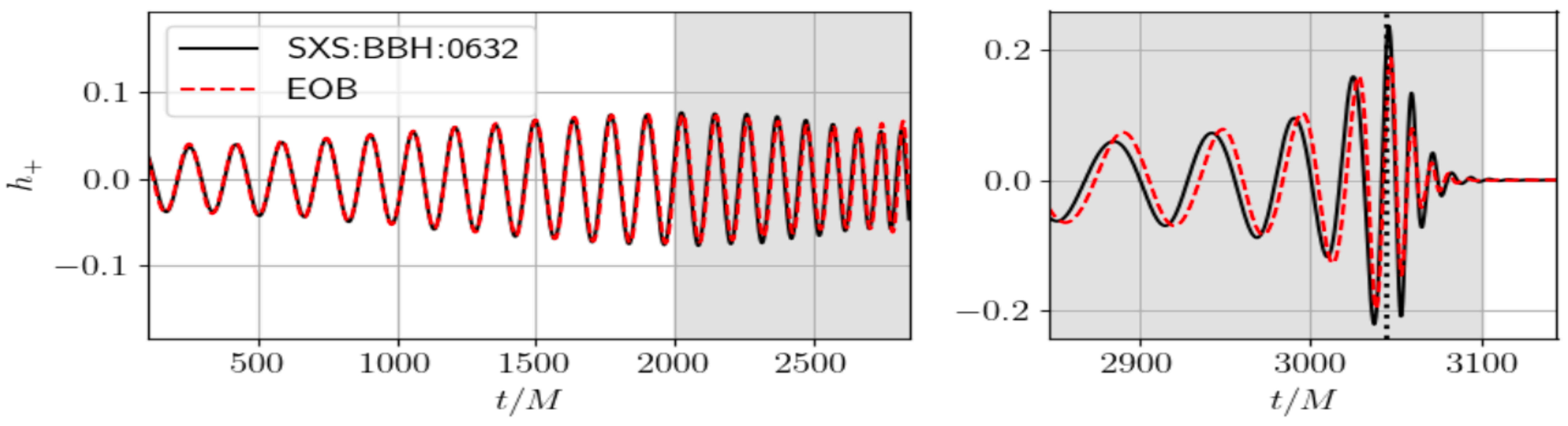
Errors in 4 waveform models as compared to 442 NR simulations



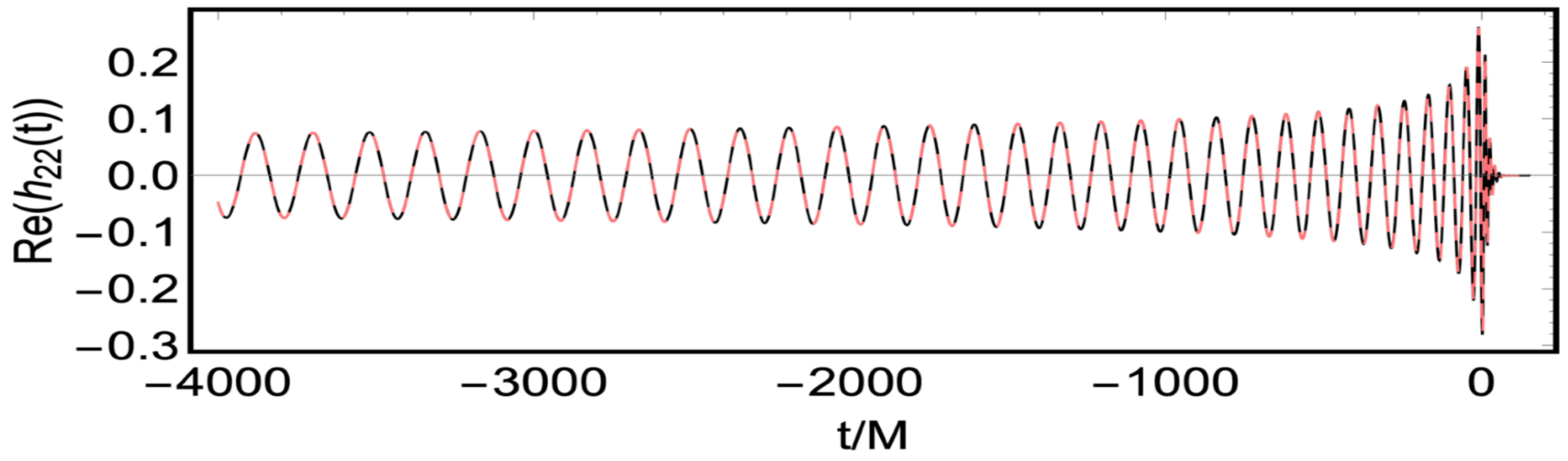
As example: Pompili+ 2303.18039

Example plots of waveform models

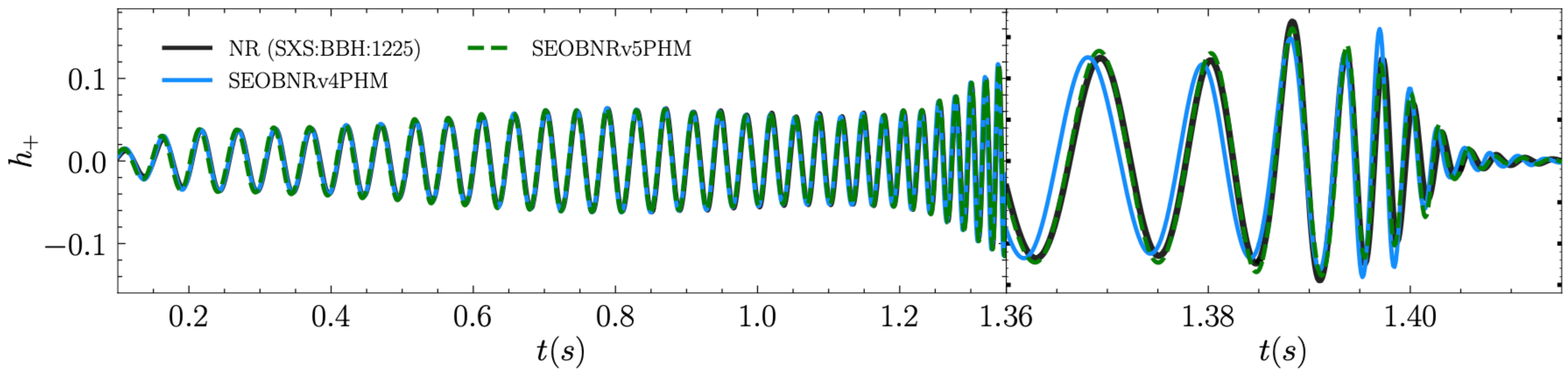
TEOBResumS
Plot from Gamba+ 22



IMRPhenomT
Plot from Estelles+ 20

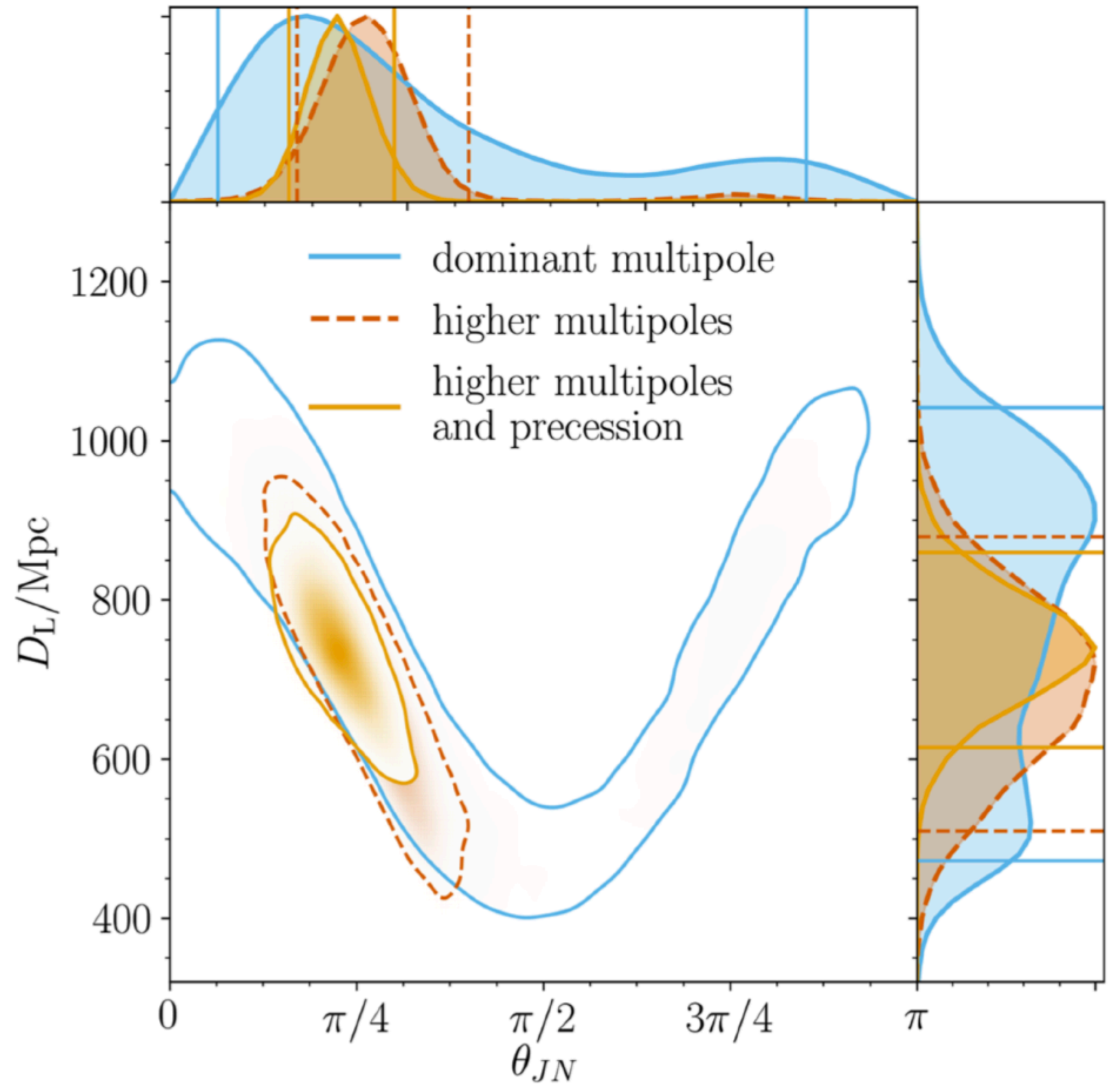


SEOBNR
Plot from Ramos-Buades+ 23

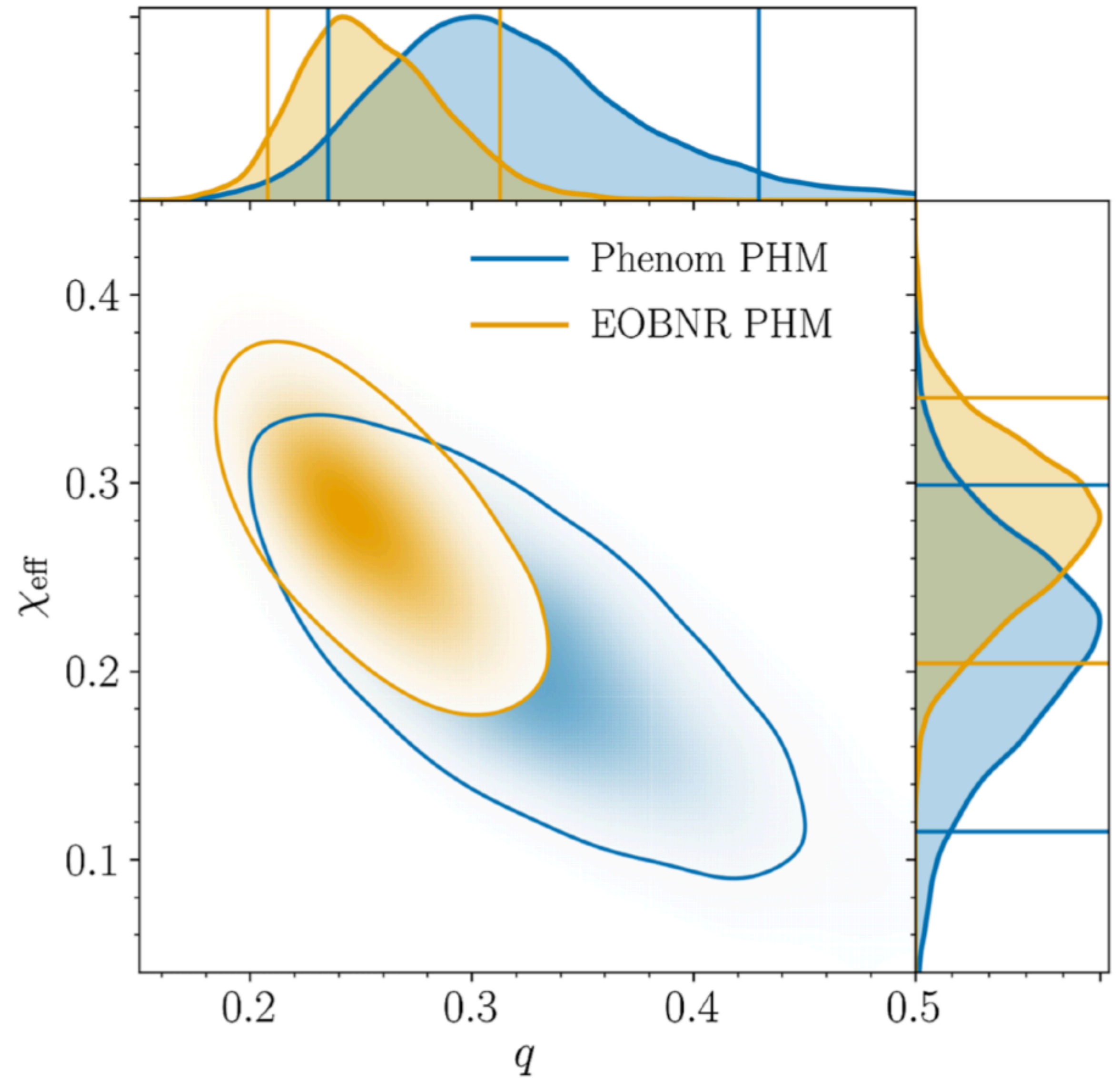


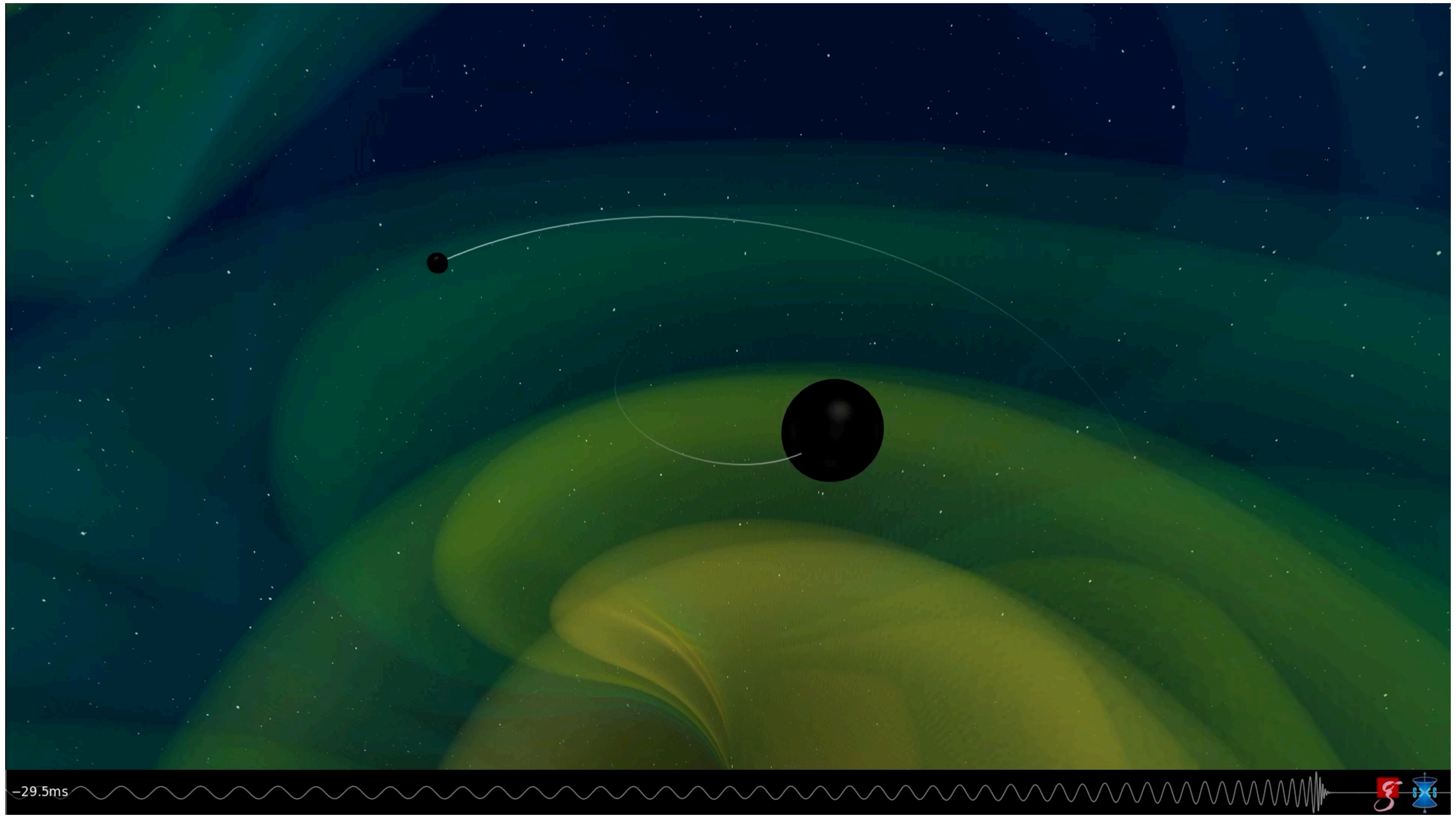
case-study: GW190412 $(30+8)M_\odot$ at SNR=19

higher-modes break degeneracies



Abbott et al,
PRD 102 043015 (2020)





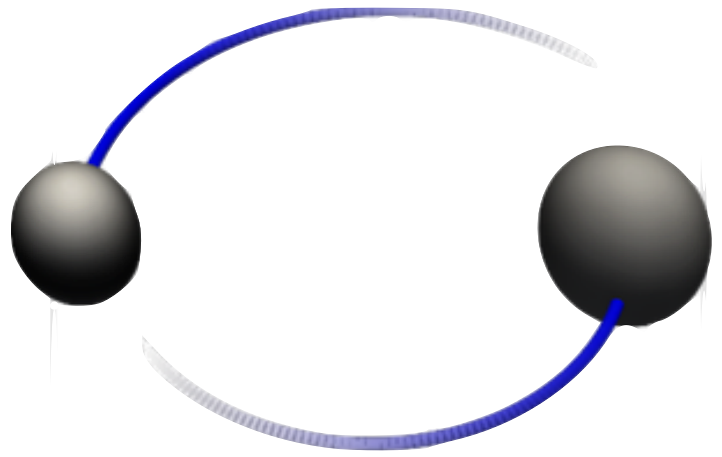
N. Vu, HP

some differences between waveform models remain even at current SNR

IMRPhenomPv3HM Khan et al 19; 20
SEOBNRv4PHM Ossokine+ 20

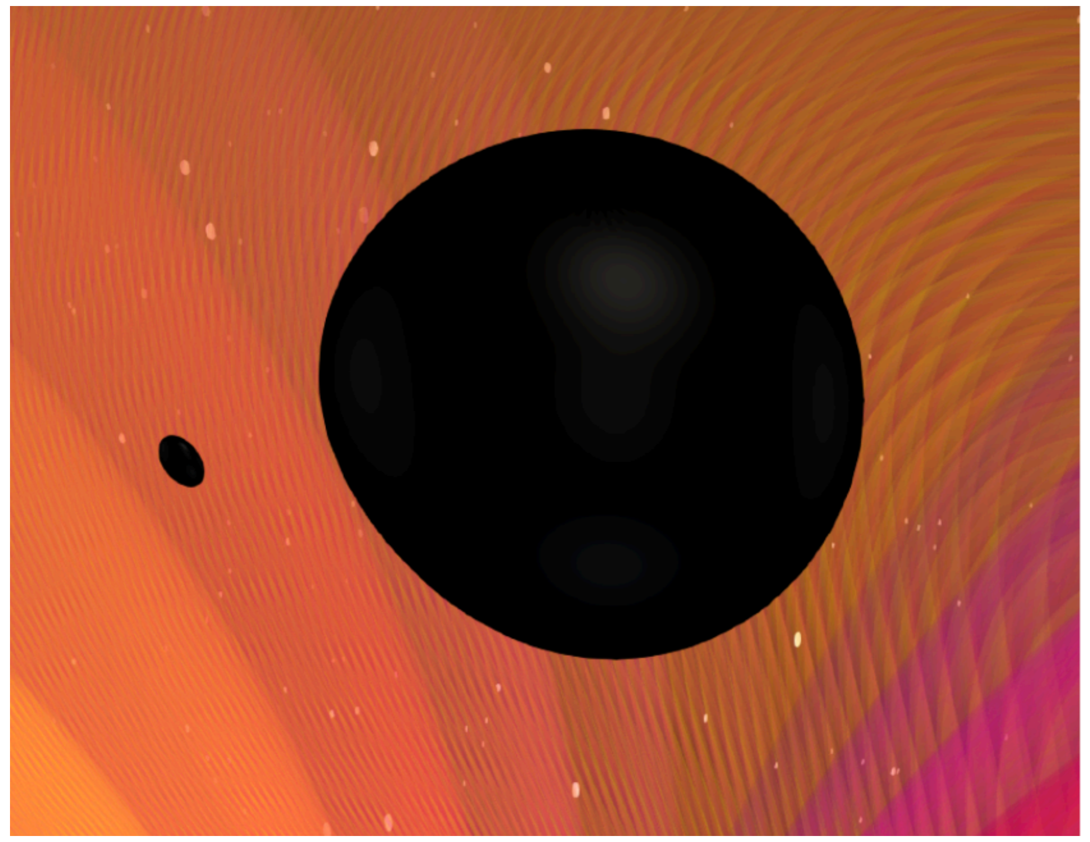
(some)
recent NR developments

Binaries at all mass-ratios



LVC

GW150914
q~1



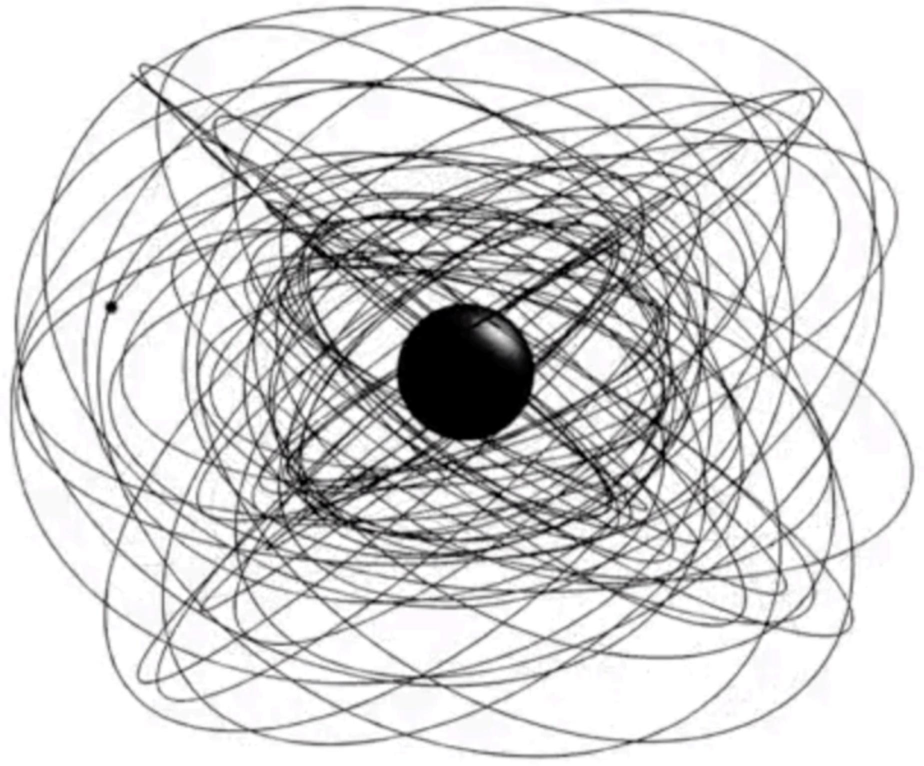
GW190814

q~0.1

Vu, HP

Intermediate mass BH
$(10+1000)M_\odot$
$(10^3+10^6)M_\odot$

NASA



S. Drasco

EMRI
$(10+10^6)M_\odot$

?

NR
$q \gtrsim 1/20$

Small-mass-ratio approximation (GSF)

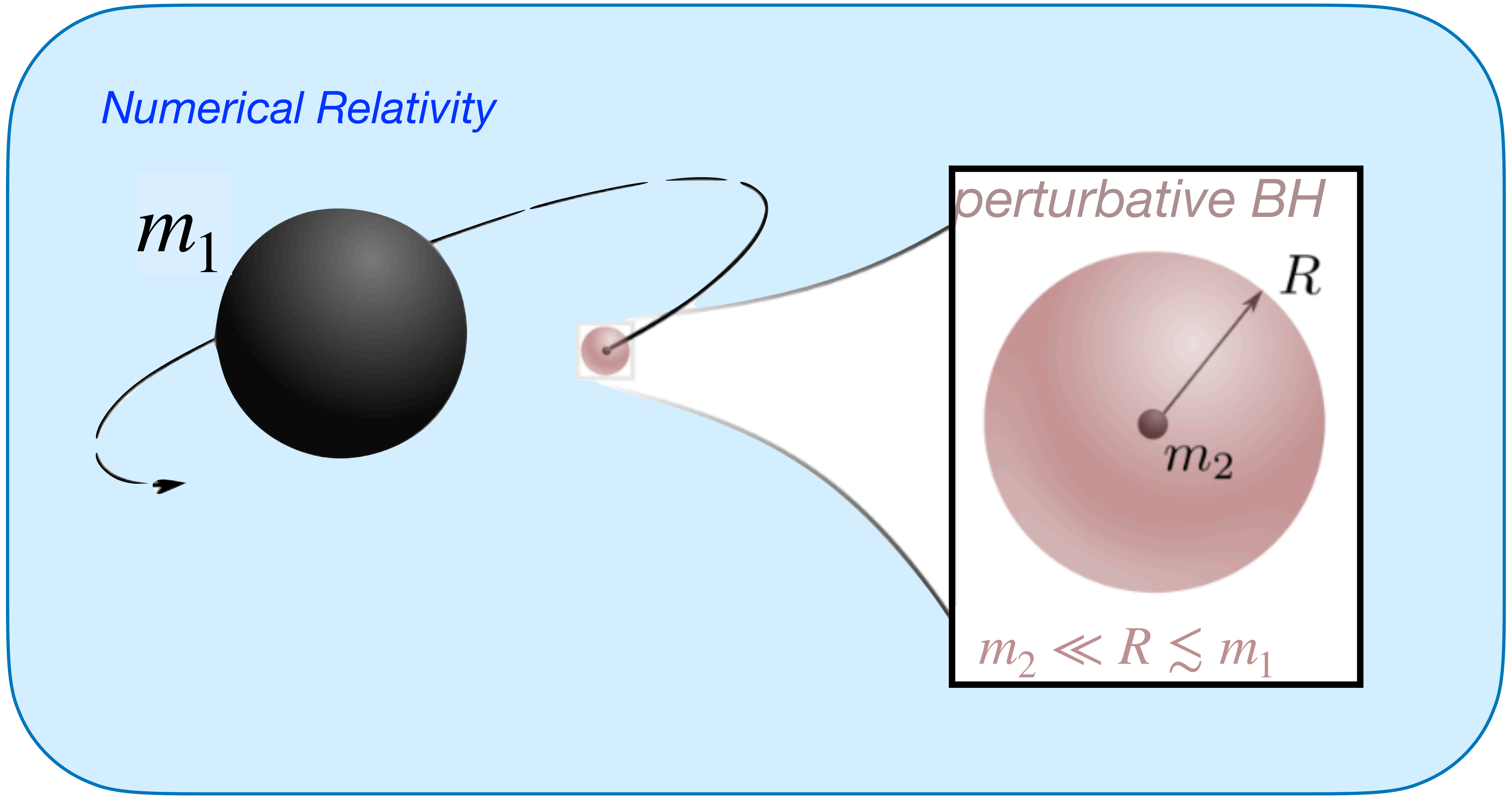
expansion in symmetric mass-ratio $\nu = q/(1+q)^2$

$$\text{Cost} \sim \frac{T}{\Delta t} \sim \frac{q^{-1}(M\Omega_i)^{-8/3}}{q} \sim \mathbf{q^{-2}\,(M\Omega_i)^{-8/3}}$$

New methods for small-q Numerical Relativity

- **Worldtube excision**
  - **Excise region** with radius R around m_2
  - In excised region, use **tidally perturbed BH** metric
  - Determine internal BH-perturbations by matching to NR
  - Set NR boundary conditions from internal solution

- Courant limit **increased by factor** $R/m_2 \sim 1/q \gg 1$



Dhesi, Rüter, *Pound, Barack, HP PRD 104 (2021) 124002*
Wittek, Dhesi, *Barack, HP, Pound+ PRD 108 (2023) 024041*
Wittek, Pound, Barack,HP+ in prep

Idea from Bernard Schutz

Warm up: Scalar charge

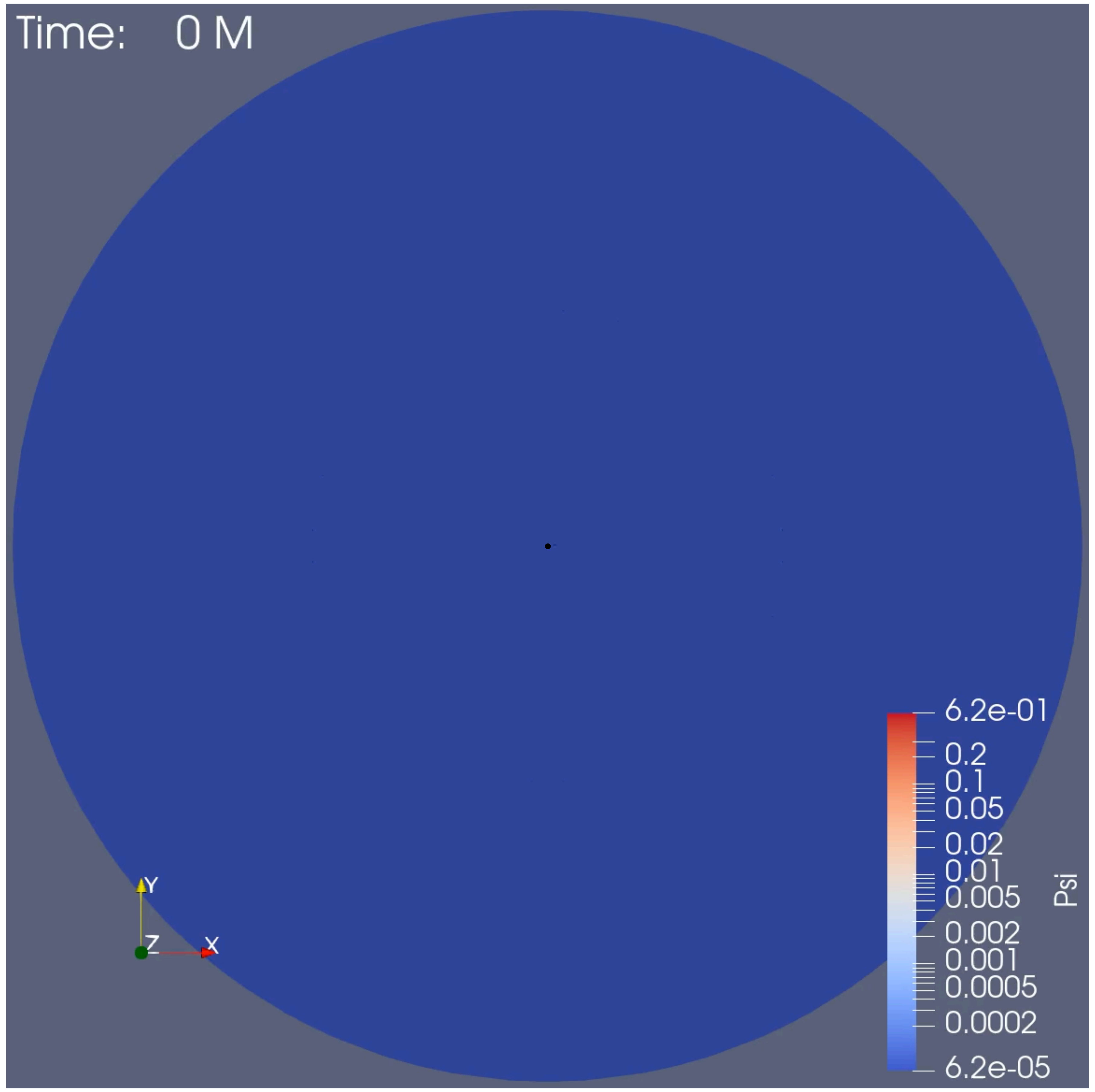
- Scalar charge orbiting Schwarzschild BH

$$g^{ab}\nabla_a\nabla_b\Phi(x^c) = -4\pi\rho(x^c)$$

- Goal: handle **point-charge**

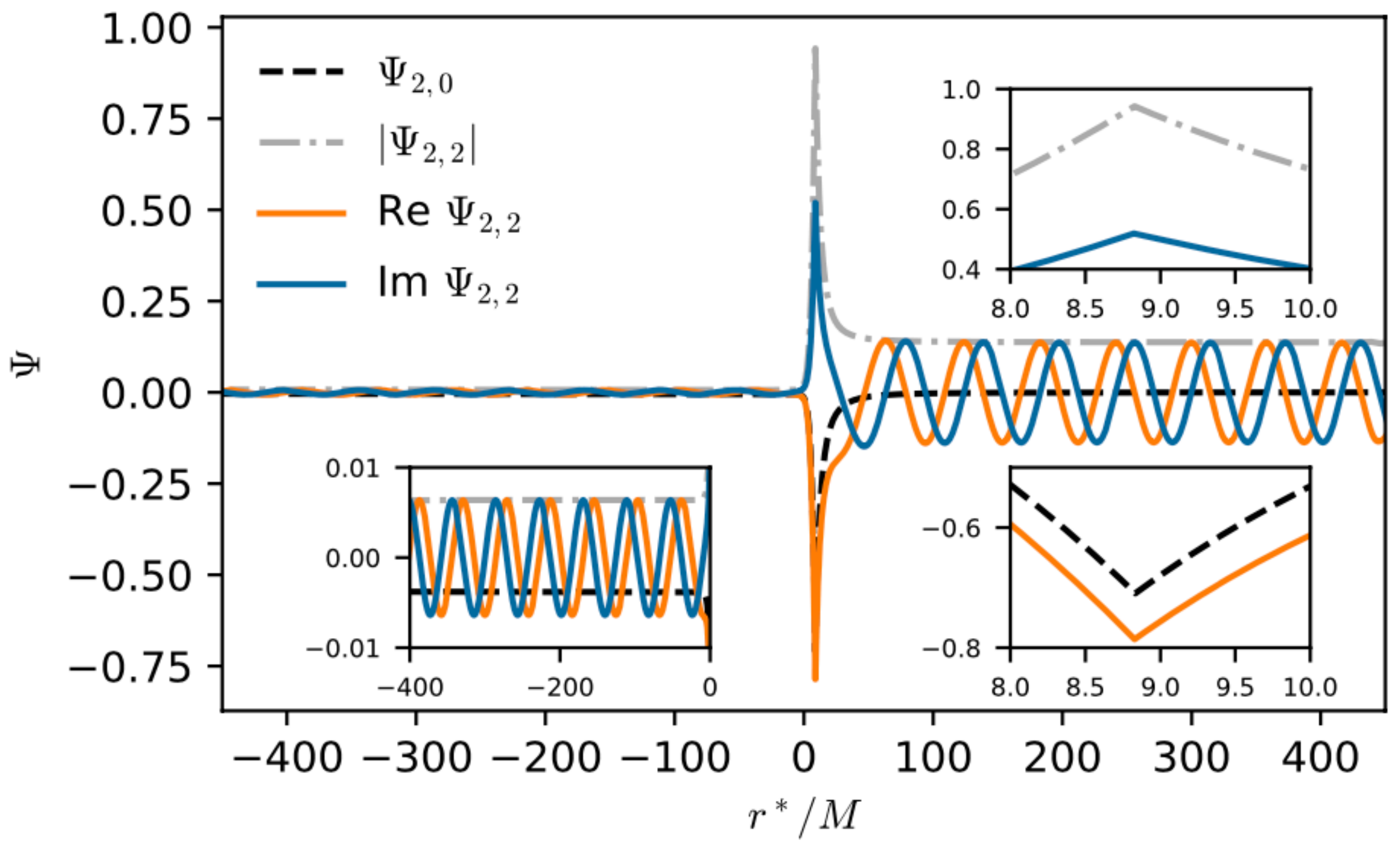
$$\rho(x^c) = q\int \frac{\delta^4[x^c - x_p^c(\tau)]}{\sqrt{-g}}\,d\tau$$

- Perturbative solution inside worldtube
  - $\Phi^A = \Phi^S + \Phi^R$
  - $\Phi^S(x^i,t)$ (singular part — known analytically)
  - $\Phi^R(x^i,t) = \xi_0(t) + \xi_1^i(t)(x^i - c^i) + \xi_2^{ij}(t)(x^i - c^i)(x^j - c^j) + \ldots$
    - coefficients ξ specify perturbation & need to be determined from NR



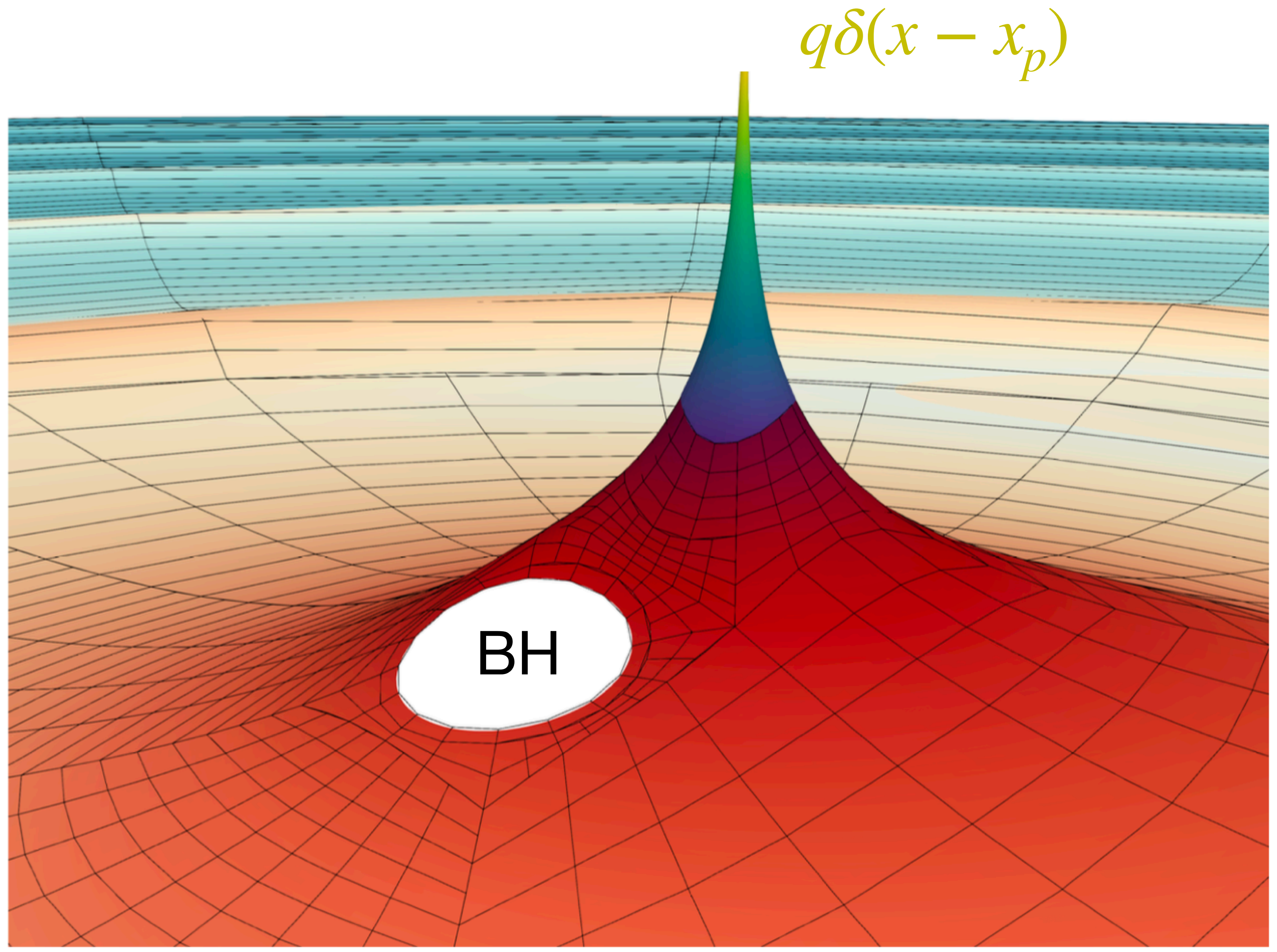
So far: Scalar point charge

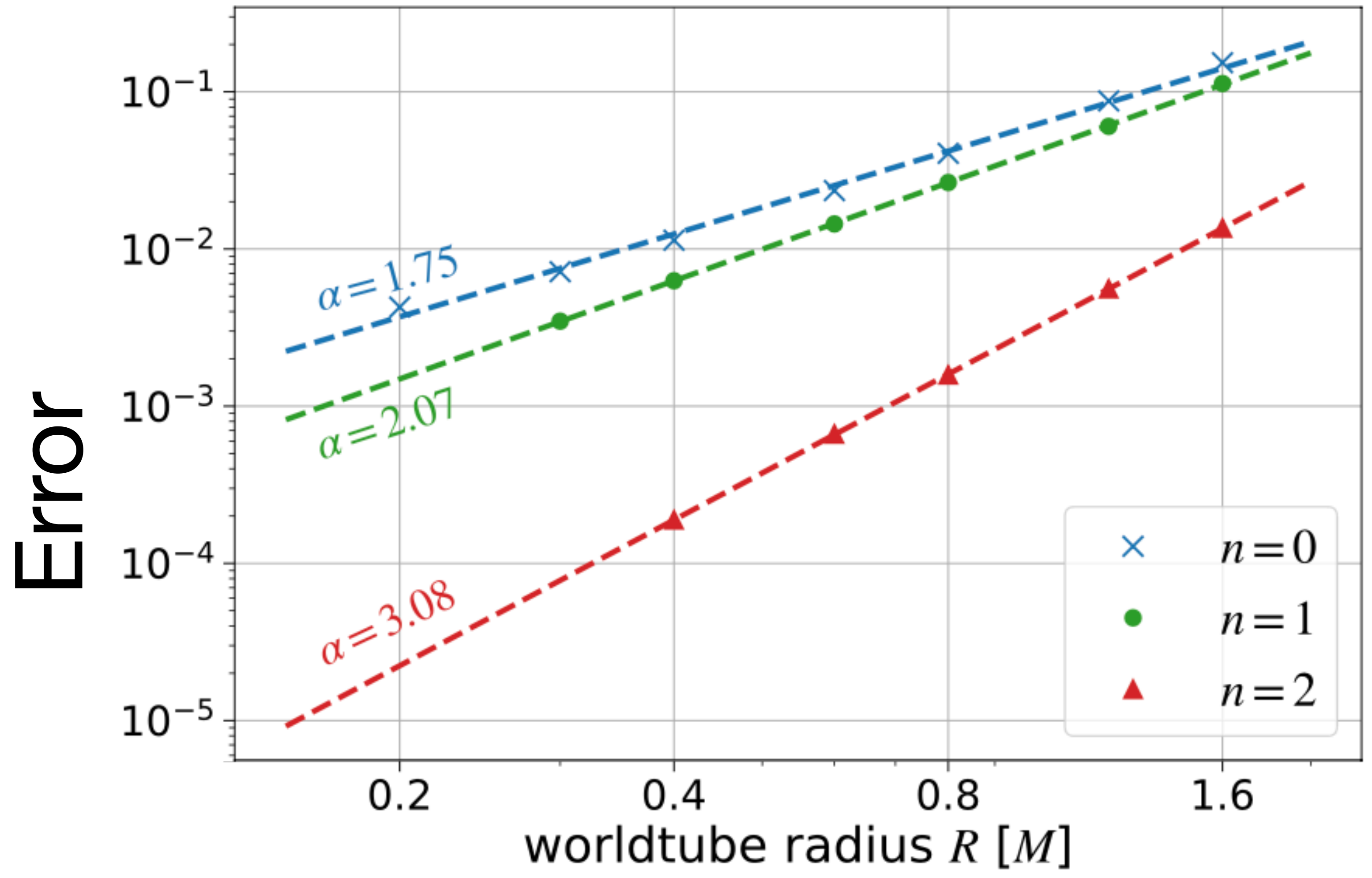
1+1D (circular motion) $\Phi_{lm}(r, t)$



__Dhesi, Rüter__, Pound, Barack, HP
PRD 104 (2021) 124002

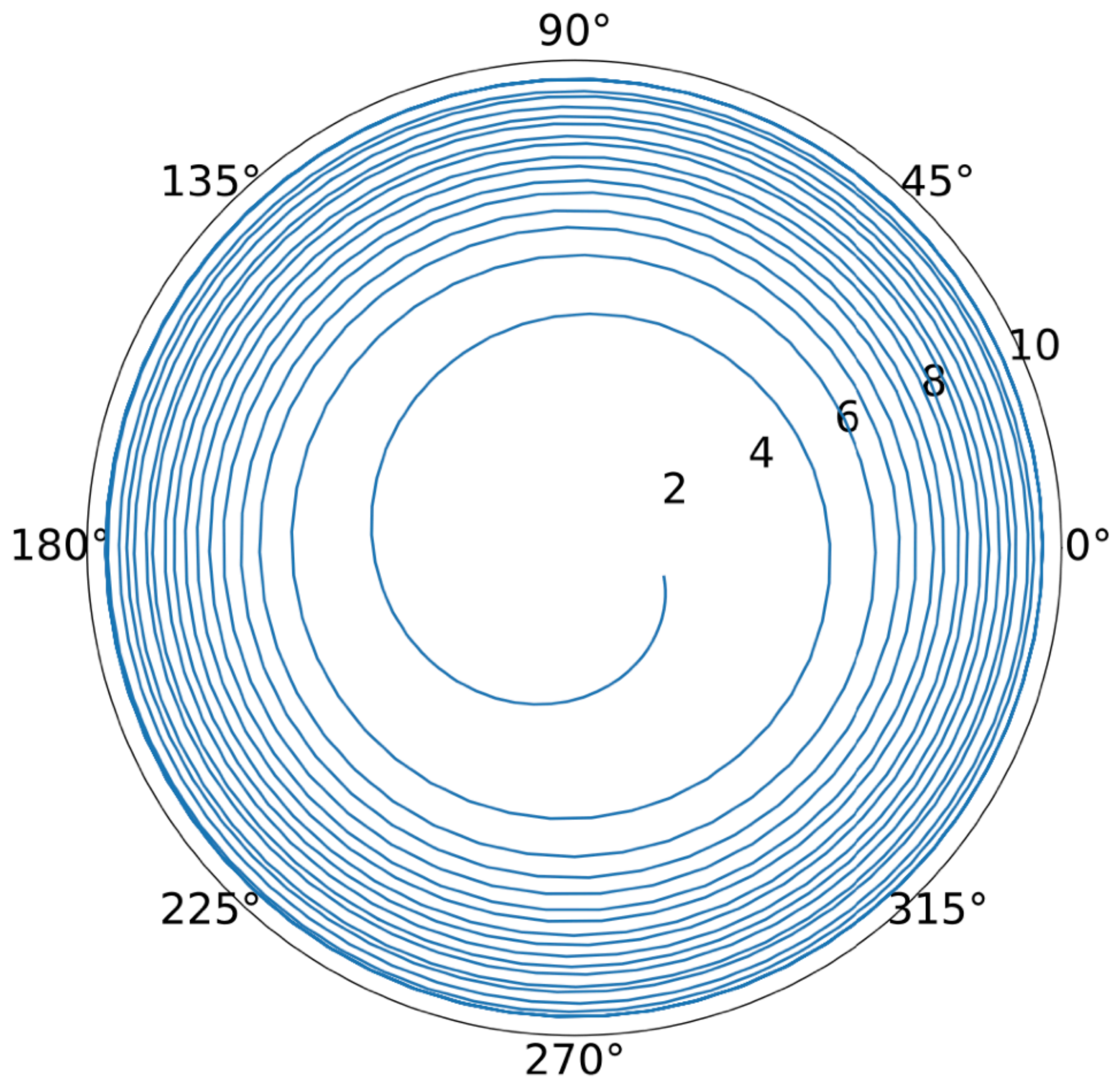
3+1D (circular motion) $\Phi(\vec{x}, t)$





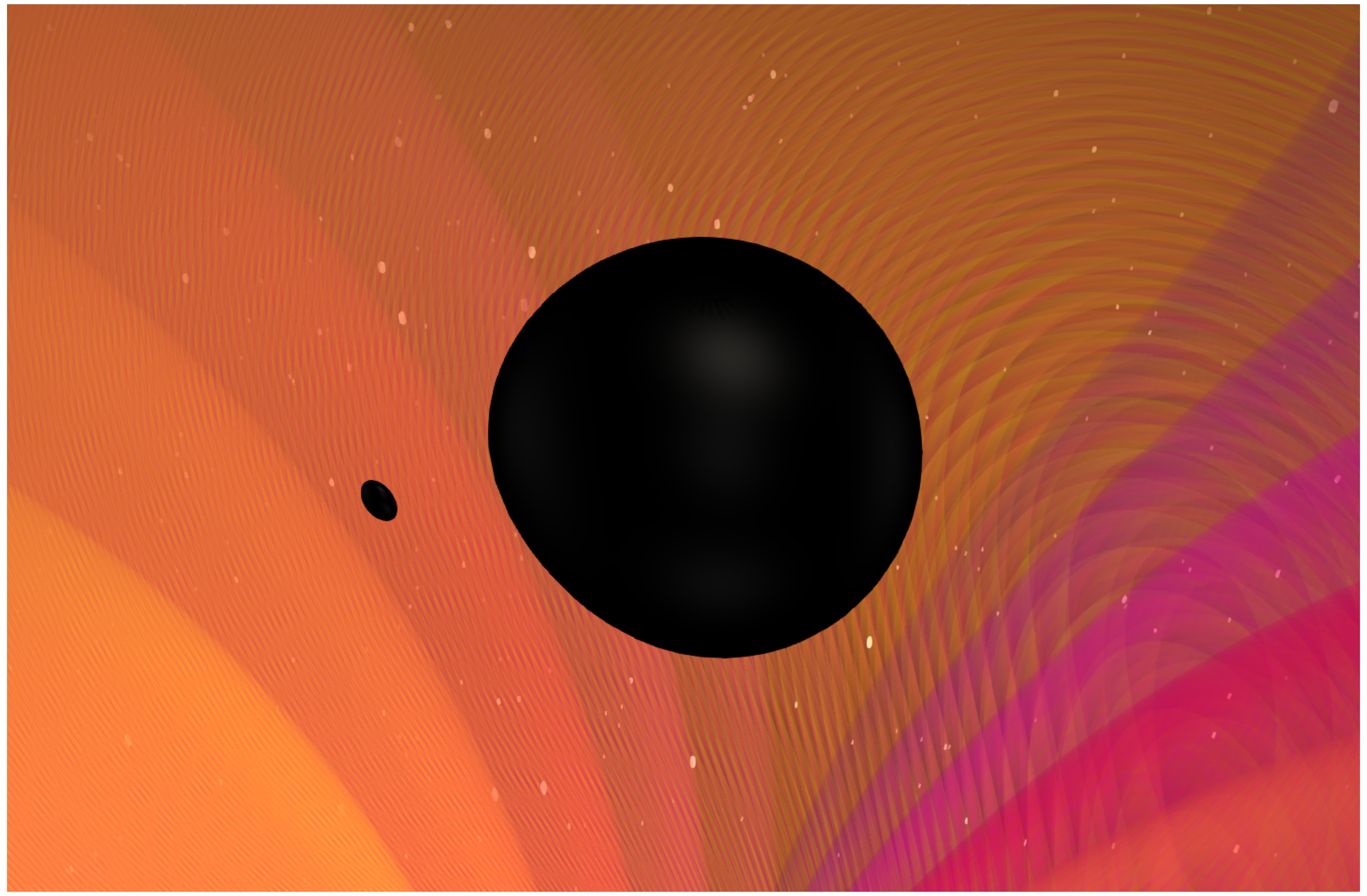
__Wittek, Dhesi__, Barack, HP, Pound+
PRD 108 (2023) 024041

3+1 D (self-force driven inspiral)
$\vec{x}_p(t)$ **and** $\Phi(\vec{x}, t)$



__Wittek__+ in prep

Interplay NR & small mass-ratio perturbation theory



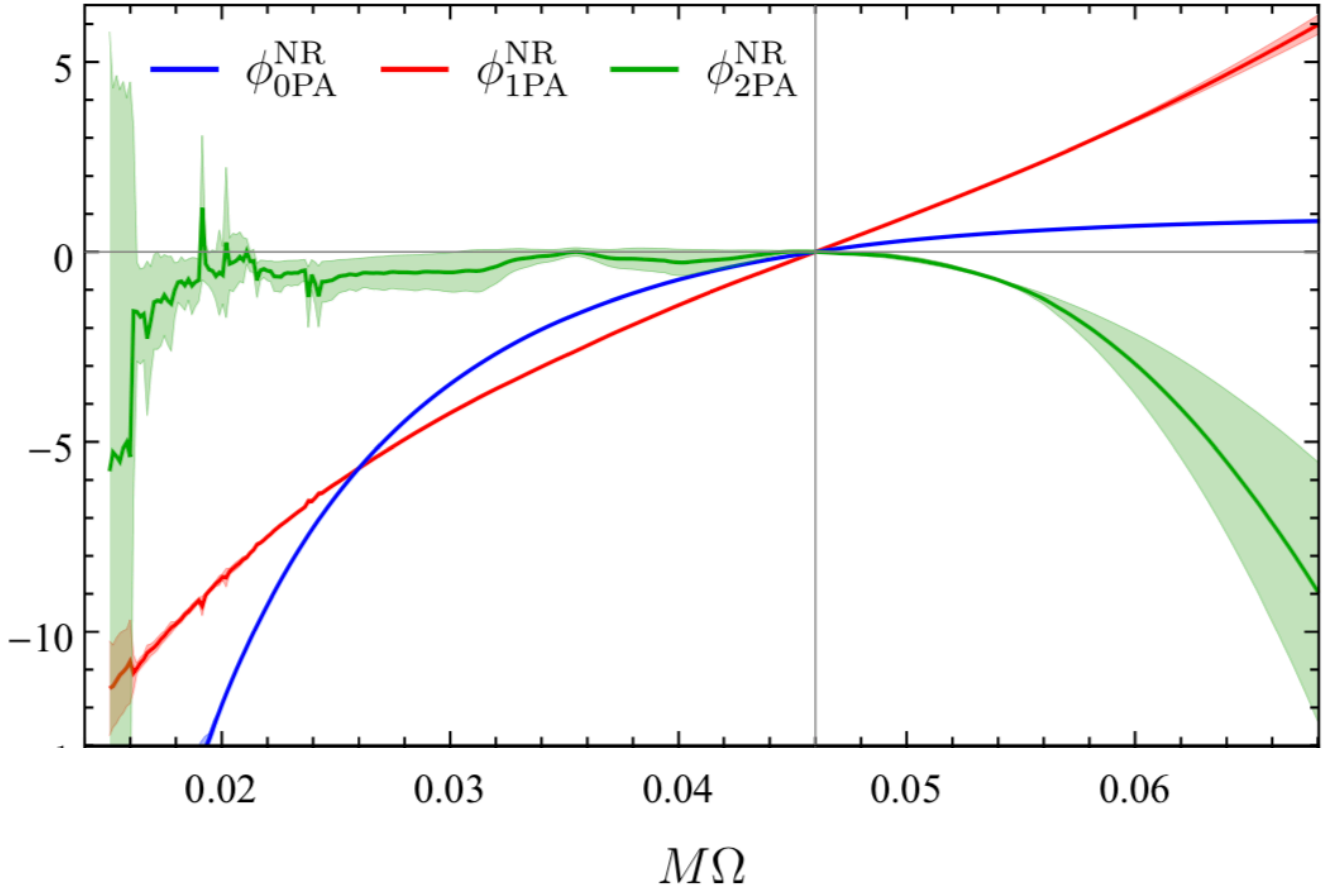
van de Meent & HP, PRL 125 181101 (2020) — non-spinning, non-eccentric
***Ramos-Buades**, vdMeent, HP+. PRD 106 124040 (2022) — non-spinning, eccentric*

Extract GSF information directly from NR

$$\Phi(M\Omega) = \frac{1}{\nu}\Phi_0(M\Omega) + \Phi_1(M\Omega) + \nu\,\Phi_2(M\Omega) + \ldots + \frac{1}{\nu^{1/2}}\Phi_{\text{resonances}} + \frac{1}{\nu^{1/5}}\Phi_{\text{plunge}}$$

- **Fit $\Phi_{NR}(\Omega_{NR})$ to SMR expansion**
  - 55 NR sims at different ν
  - non-spinning

- Different orders in ν
  - $\Phi_0(M\Omega)$ - agrees with 0PA
  - $\Phi_1(M\Omega)$ - hereby computed
  - $\Phi_2(M\Omega)$ - remarkably small

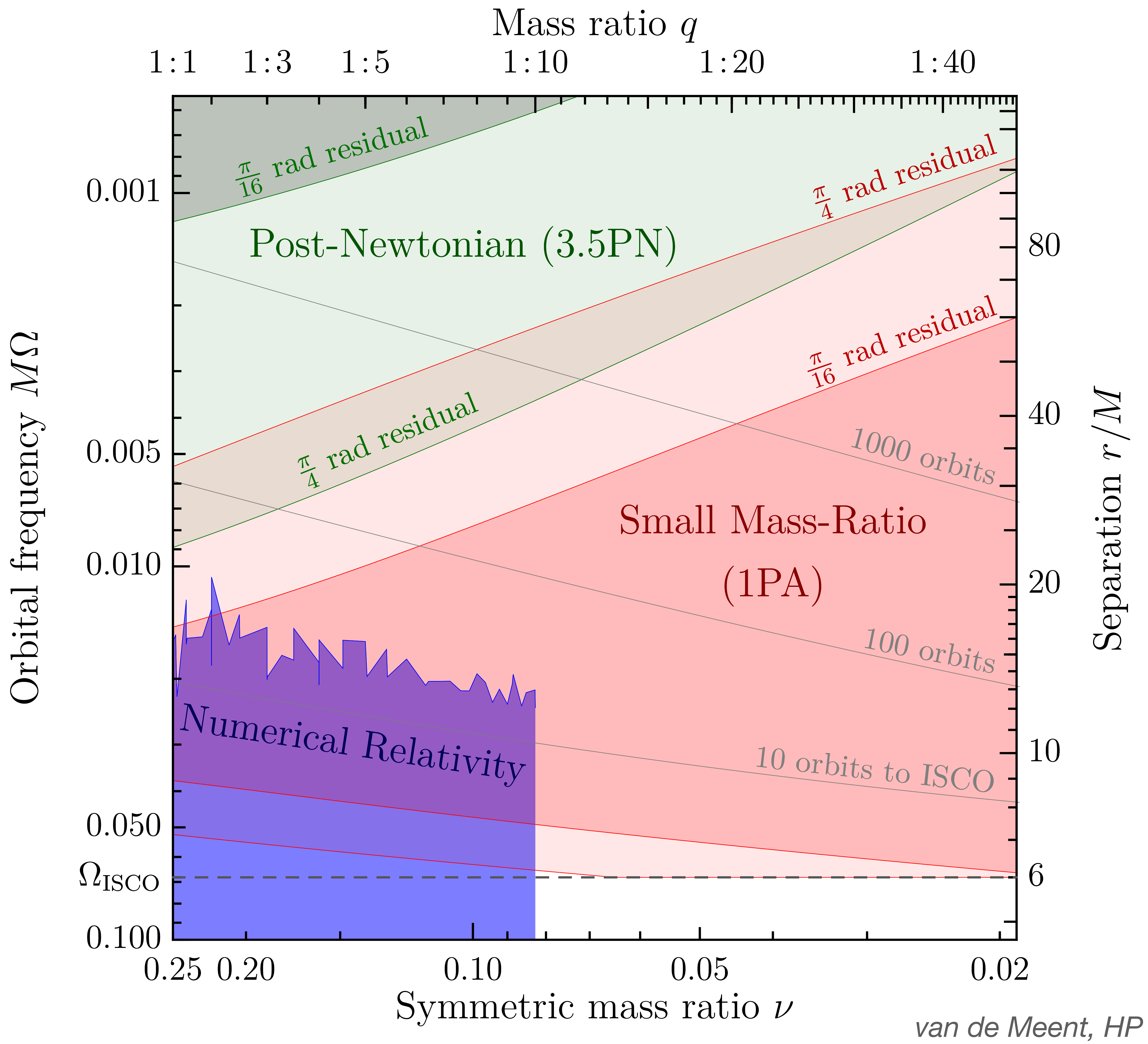


van de Meent, HP 2020

Extract GSF information directly from NR

$$\Phi(M\Omega) = \frac{1}{\nu}\Phi_0(M\Omega) + \Phi_1(M\Omega) + \nu\,\Phi_2(M\Omega) + \ldots + \frac{1}{\nu^{1/2}}\Phi_{\text{resonances}} + \frac{1}{\nu^{1/5}}\Phi_{\text{plunge}}$$

- **Fit $\Phi_{\mathrm{NR}}(\Omega_{\mathrm{NR}})$ to SMR expansion**
  - 55 NR sims at different ν
  - non-spinning
- Different orders in ν
  - $\Phi_0(M\Omega)$ - agrees with 0PA
  - $\Phi_1(M\Omega)$ - hereby computed
  - $\Phi_2(M\Omega)$ - remarkably small
- **Predict region of validity of SMR**
  - quasi-circular & non-spinning

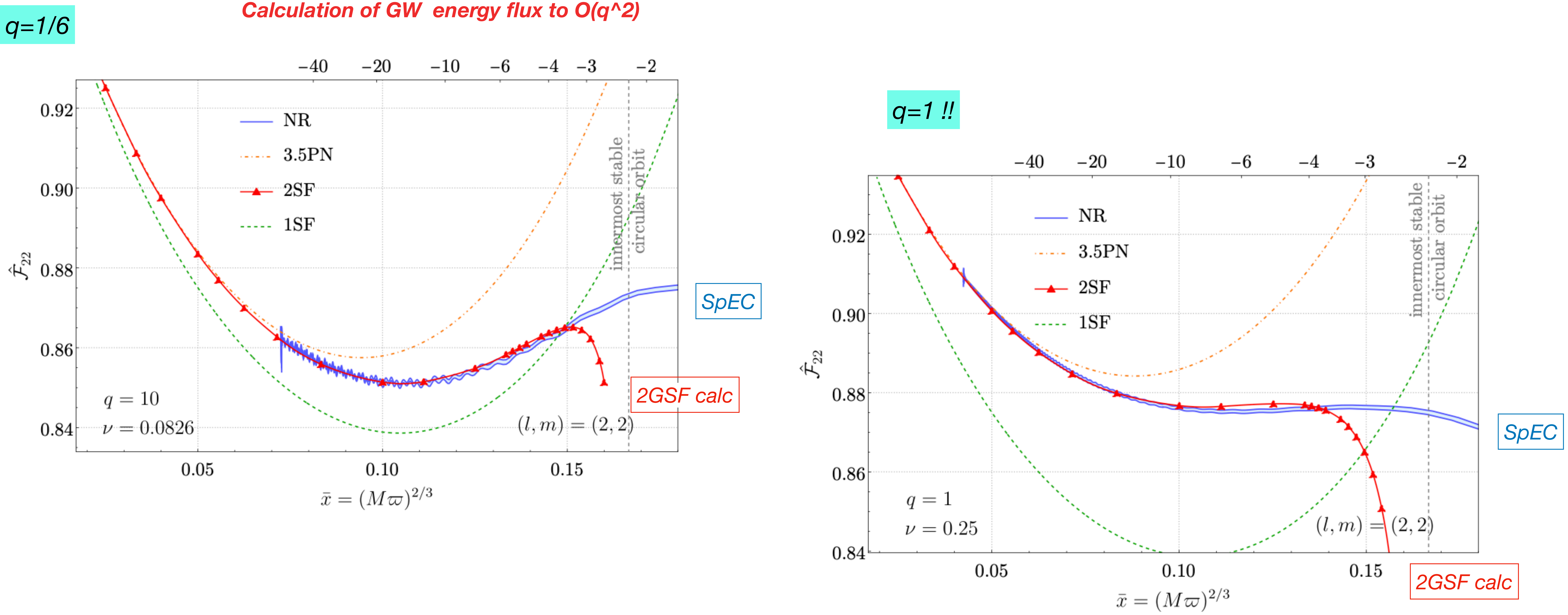


van de Meent, HP 2020

Confirmed by actual 2-SMR calculations

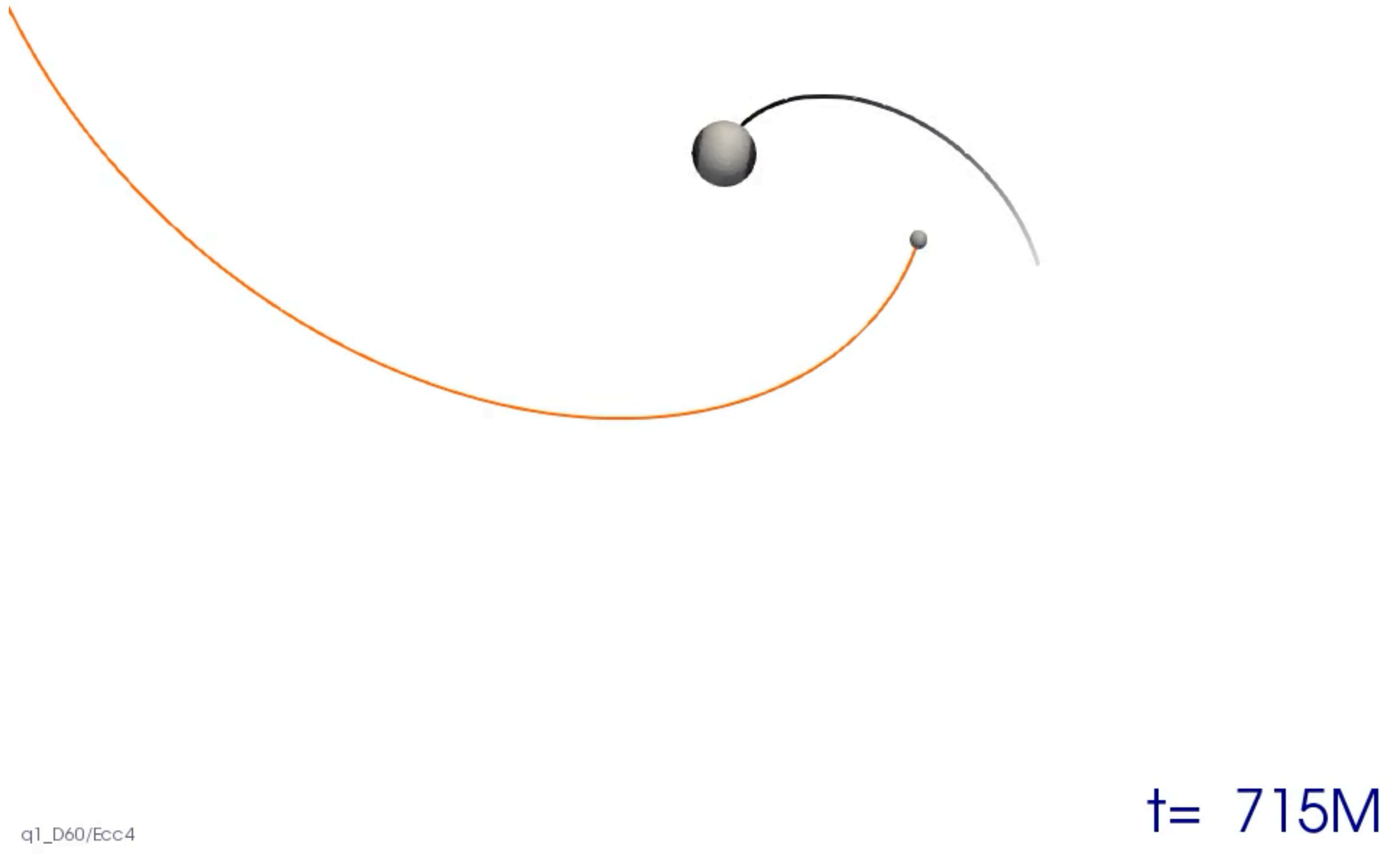
Warburton, Pound, Wardell, Miller, Durkan 21

Calculation of GW energy flux to O(q^2)



Eccentric BBH simulations

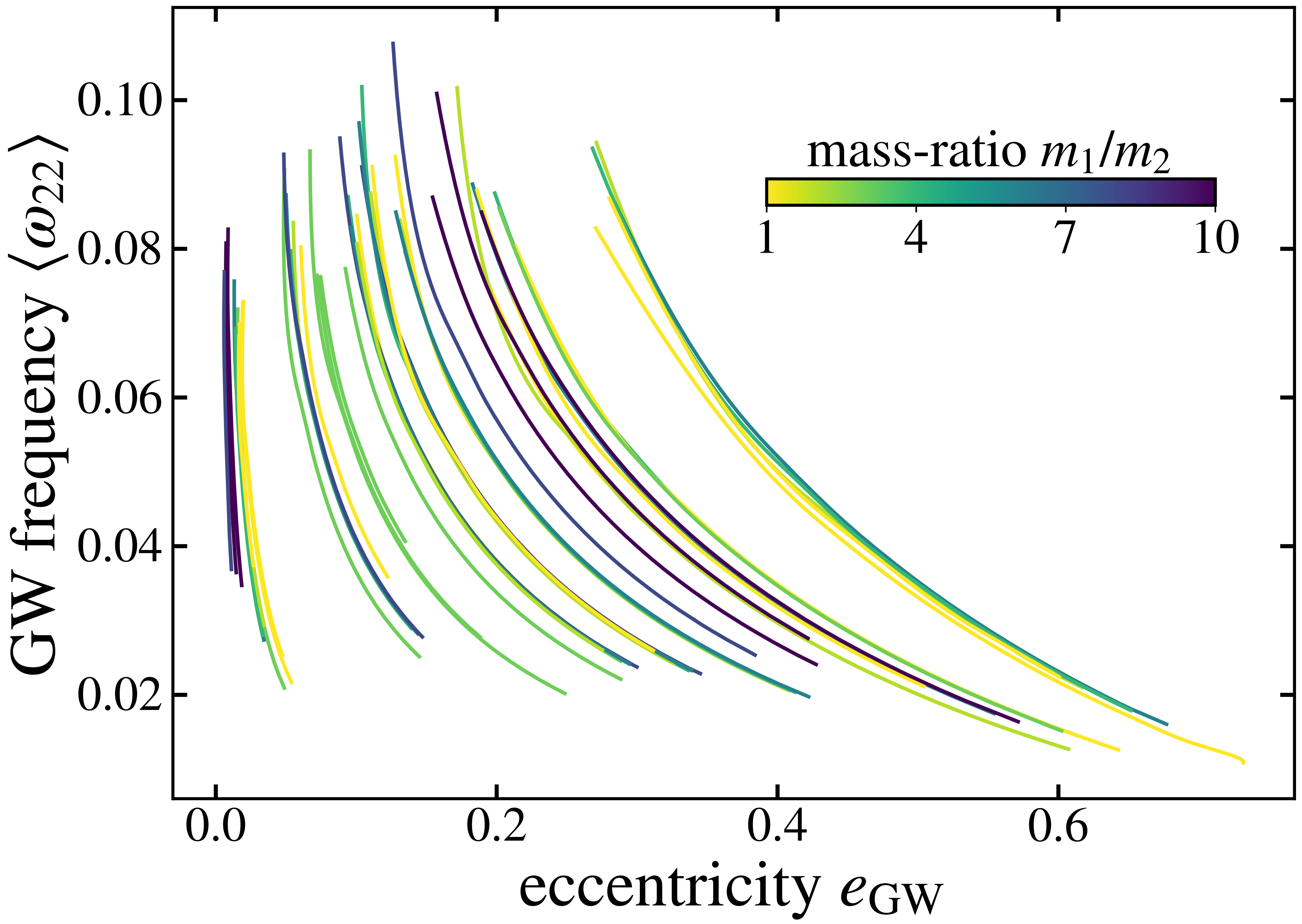
- **Why** eccentric NR?
  - **measure** eccentricity of GW signals
  - **distinguish “deviation from GR”** from “eccentric GR”
  - **EMRIs will be eccentric** in LISA

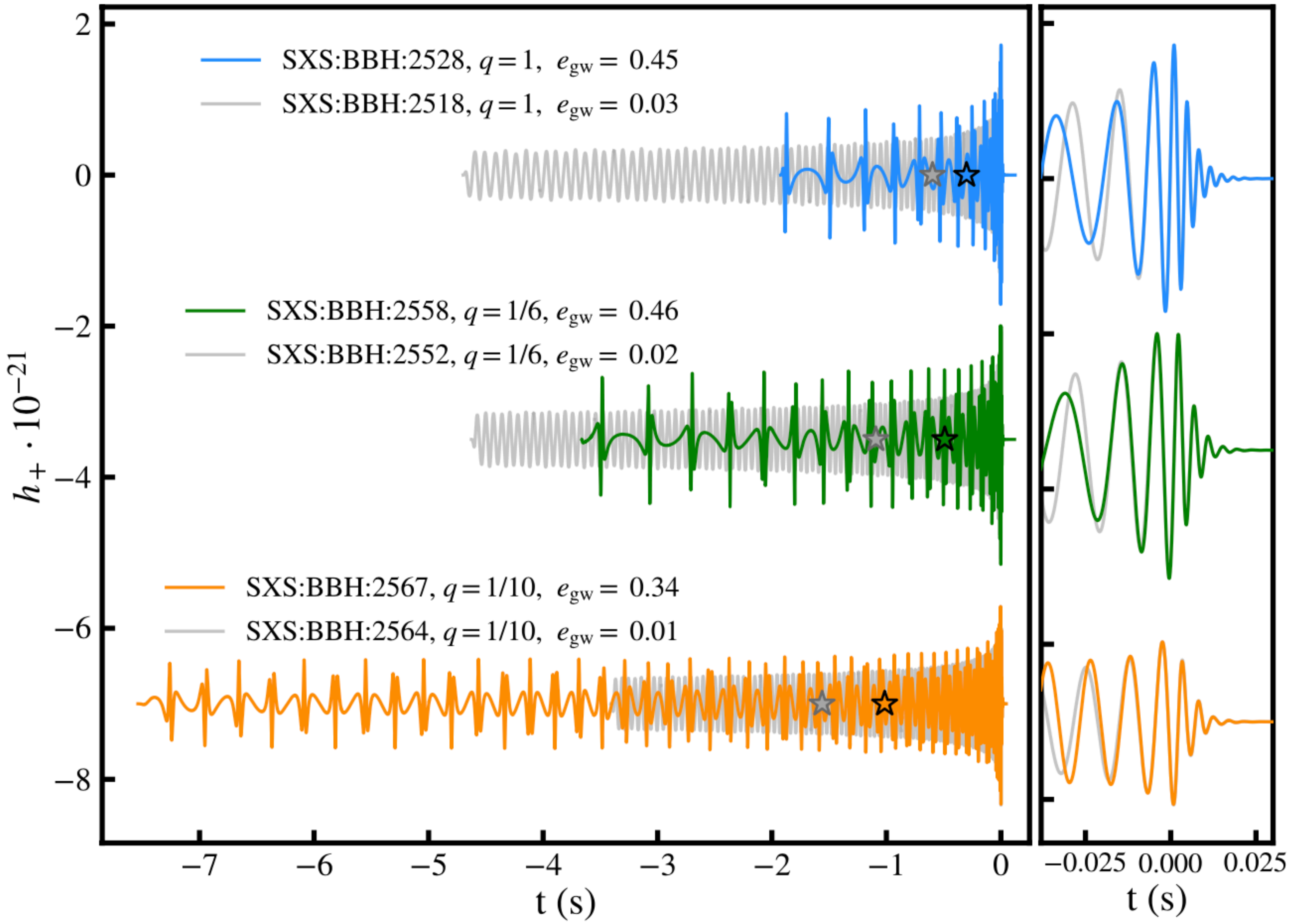


NB:
- All animations based on full 3-D PDE solutions, not just ODEs for centers of BHs.
- BH Horizons shaded by their curvature, showing tidal deformations at periastron passages

Eccentric NR, q=1—0.1

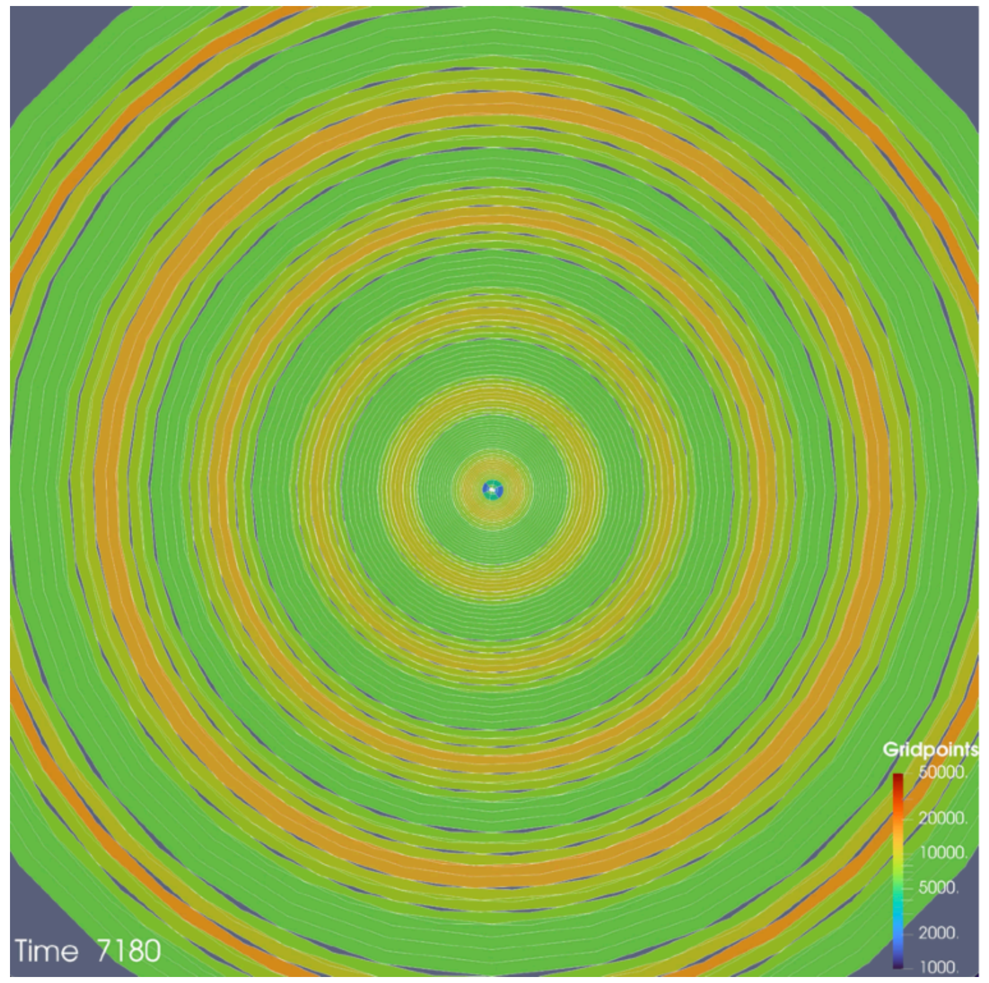
- new NR sims
  - q=1–0.1, e=0–0.7
  - three resolutions each
  - NR errors analysed

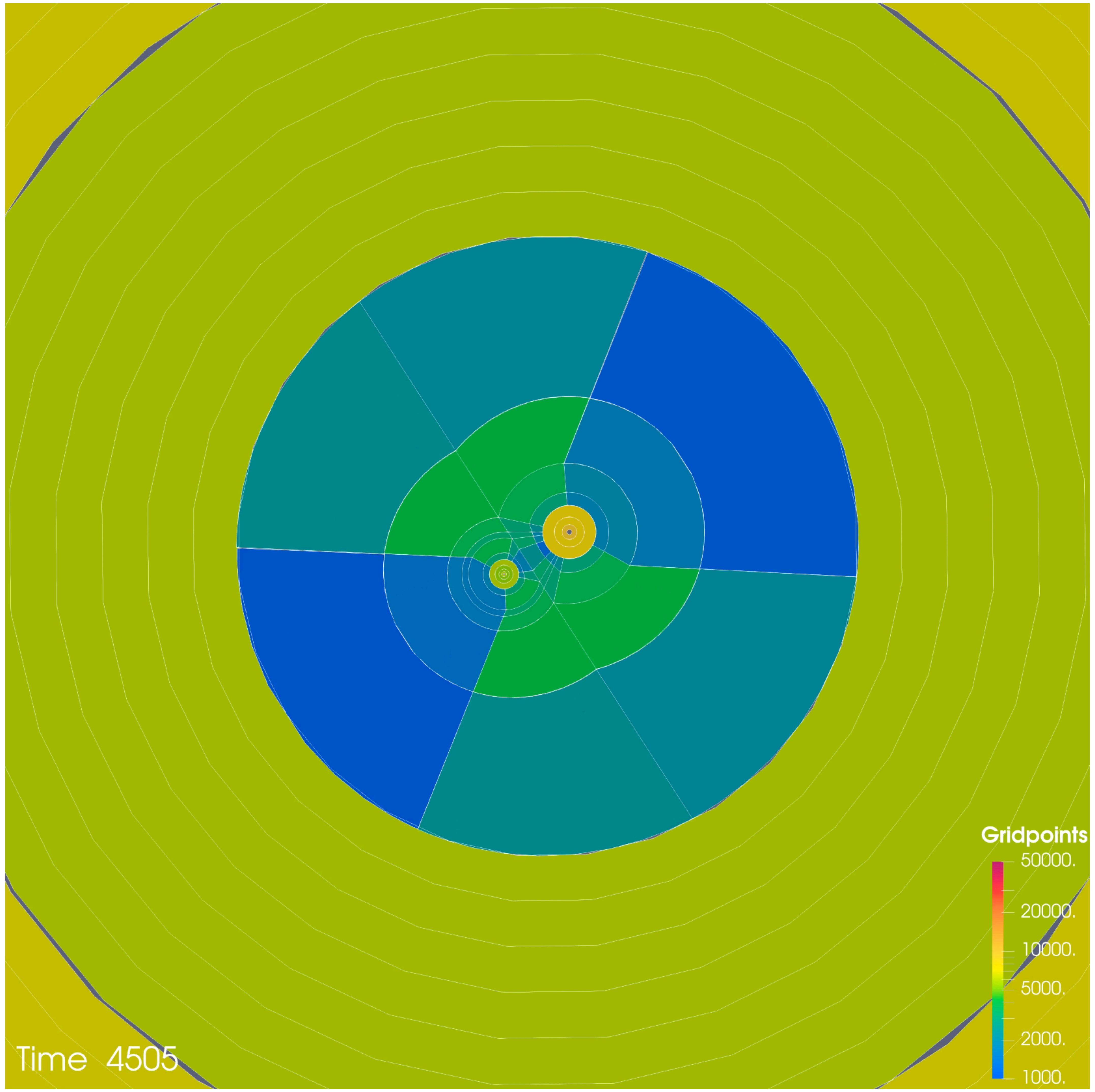




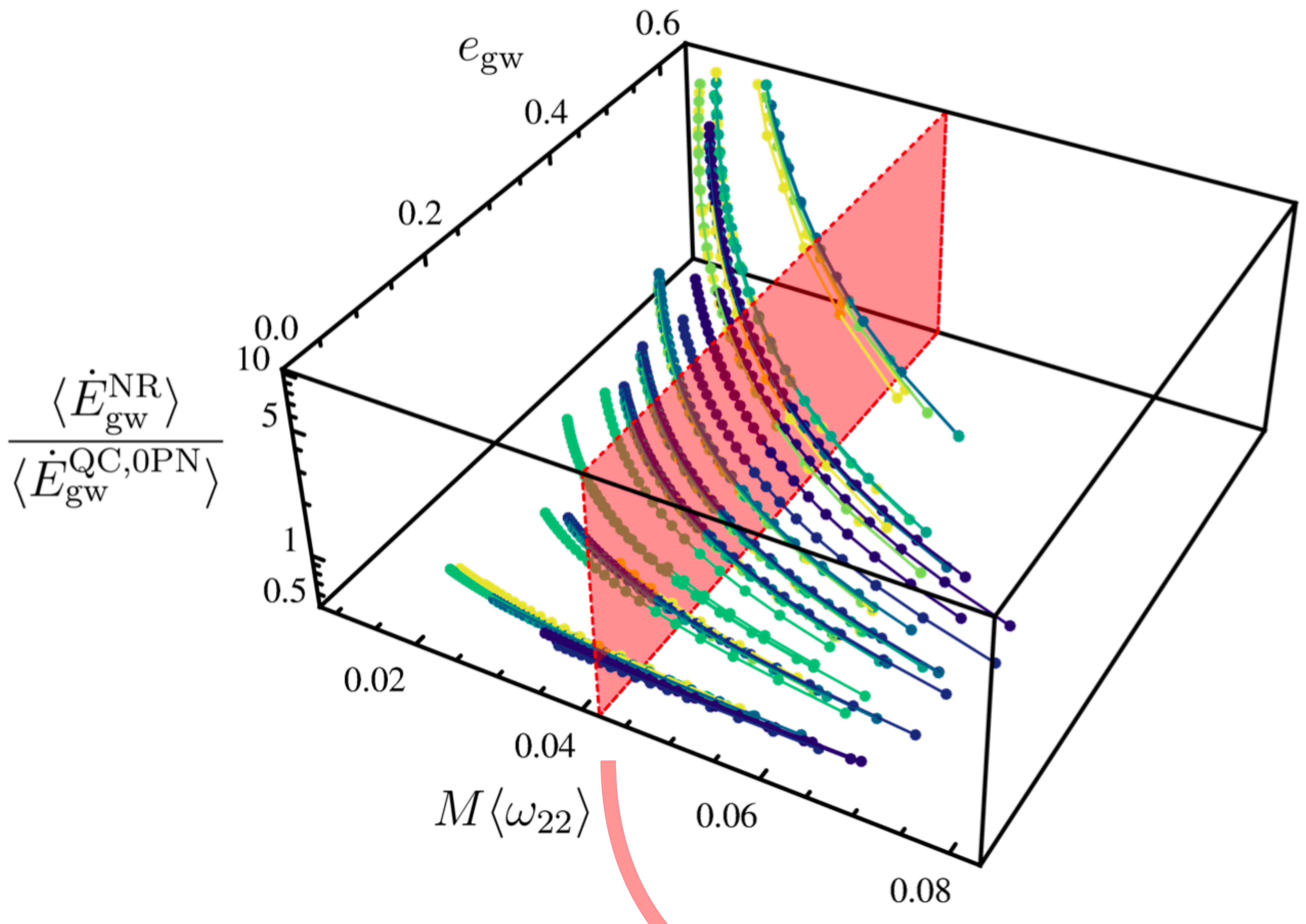
Eccentric NR, q=1—0.1

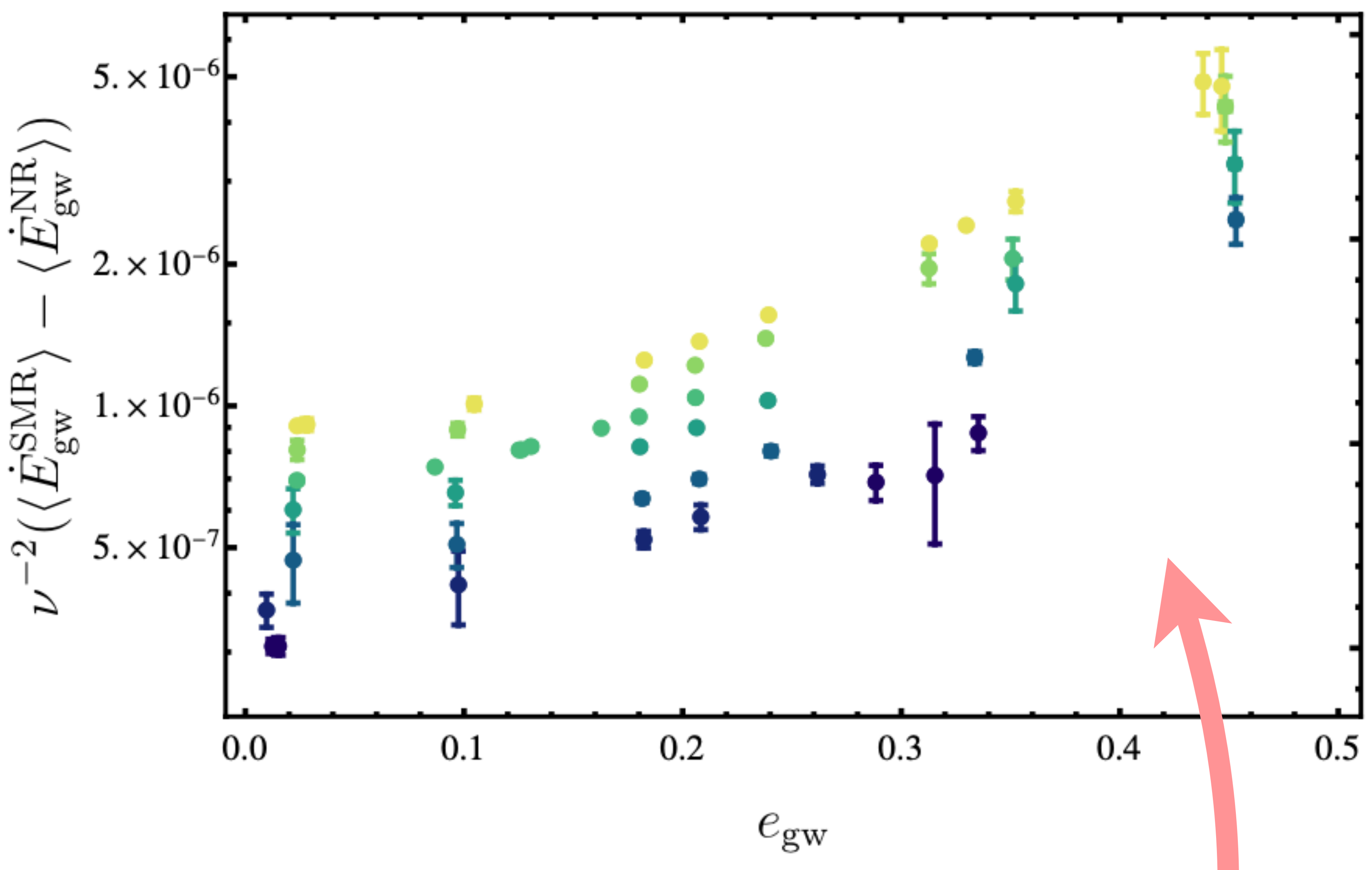
- new NR sims
  - q=1–0.1, e=0–0.7
  - three resolutions each
  - NR errors analysed
- **GW pulses visible in AMR**



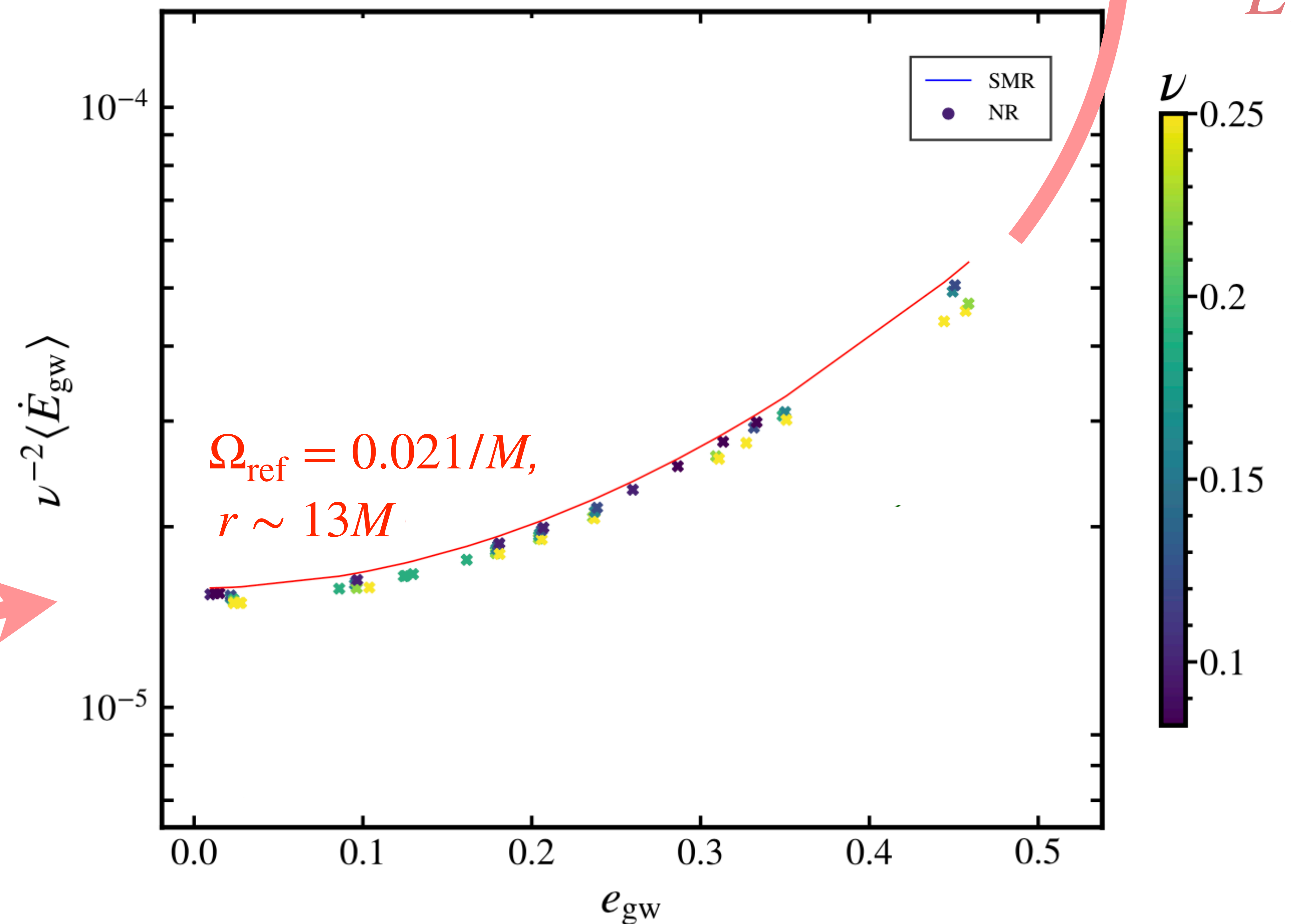


$\langle \dot{E}_{\mathrm{GW,NR}} \rangle$ for 52 simulations



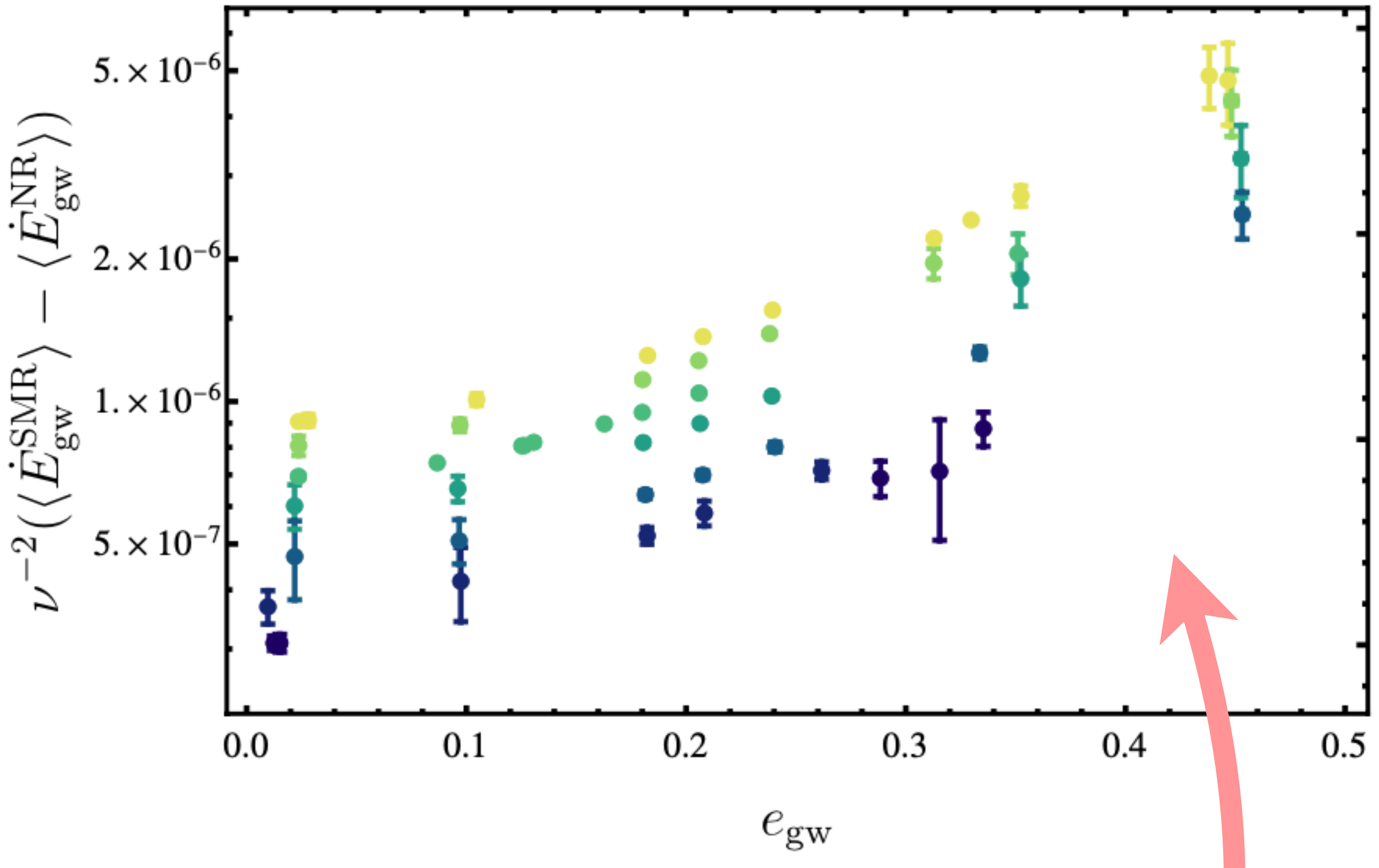


Difference
$\dot{E}_{\mathrm{SMR}} - \dot{E}_{\mathrm{NR}}$

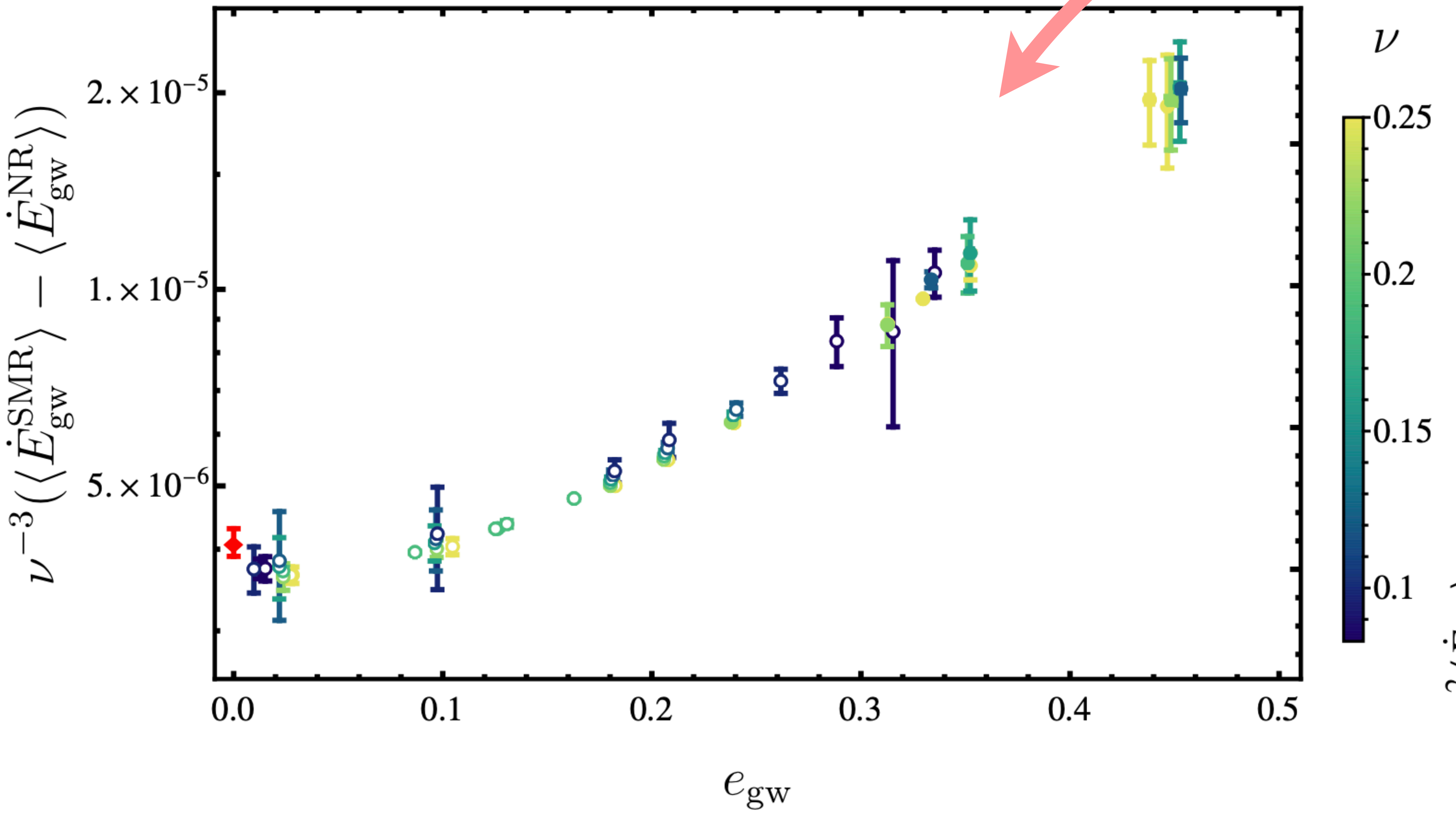


At fixed Ω_{ref}
accounting for $\dot{E} \propto \nu^2$

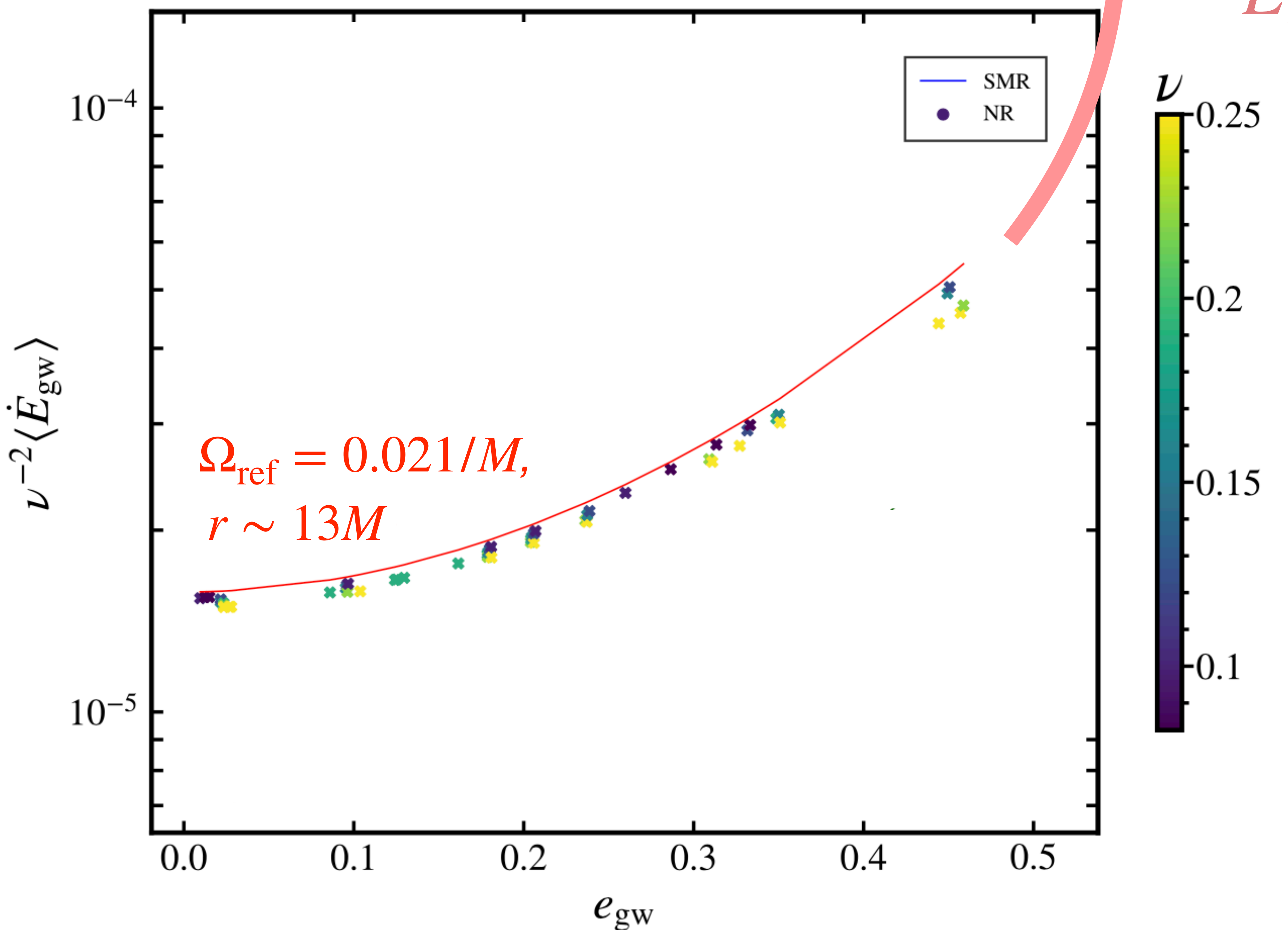
$\langle \dot{E}_{\rm GW,NR} \rangle$ for 52 simulations



Rescale by ν



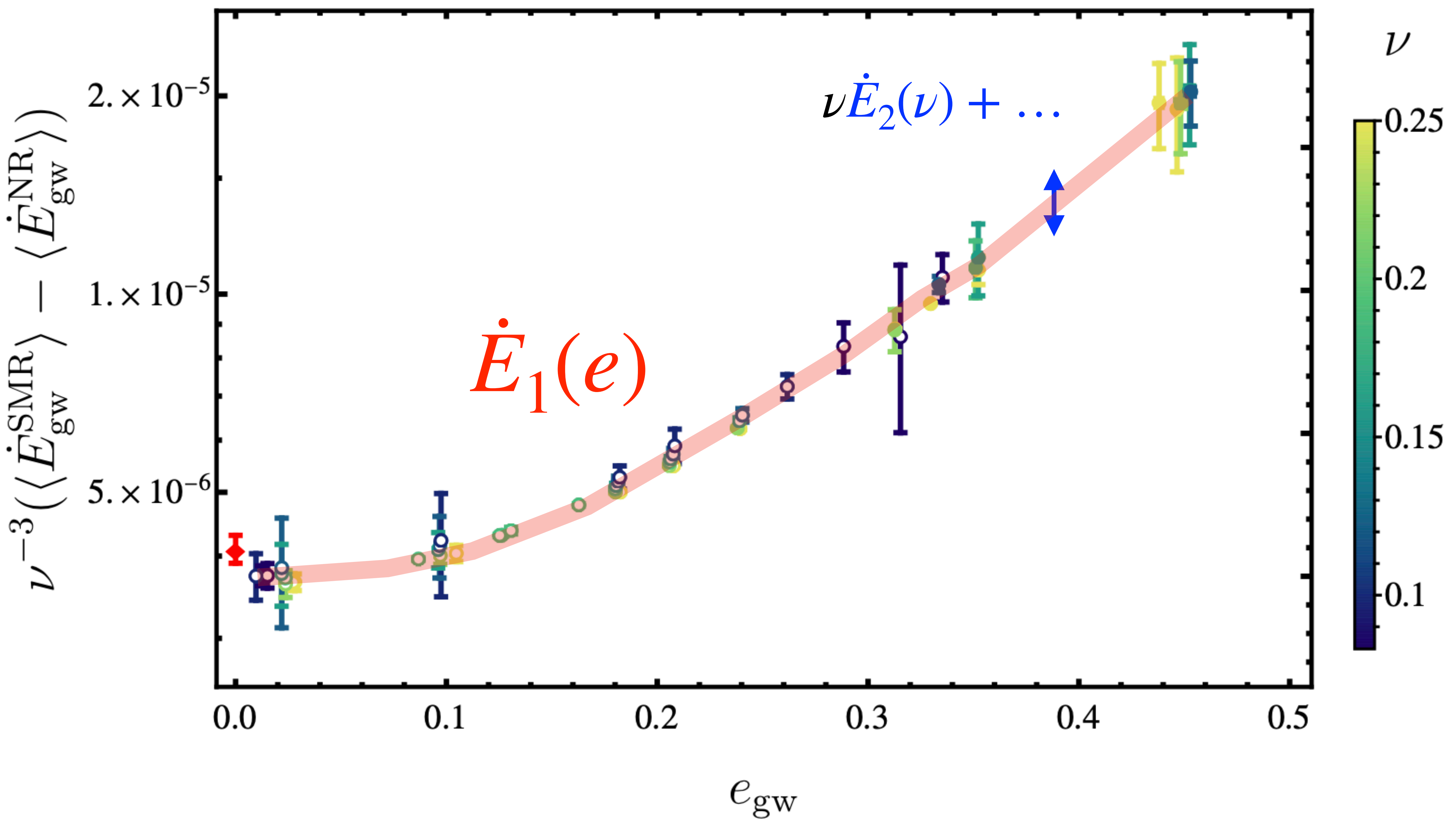
Difference $\dot{E}_{\rm SMR} - \dot{E}_{\rm NR}$



Interpretation

GW fluxes at fixed frequency

$$\dot{E}_{\rm GW}(\nu, e) = \nu^2\dot{E}_0(e)+\nu^3\dot{E}_1(e)+\nu^4\dot{E}_2(e) + \dots$$



- ***Data collapses to one line:***
  (SMR − NR) $\propto \nu^3$

  ⇒NR recovers $\dot{E}_0$ from SMR

  ⇒ collapsed curve gives next-to-leading order contribution

  $\dot{E}_1(e) = \mathcal{O}(\nu^3)$ *— unknown before*

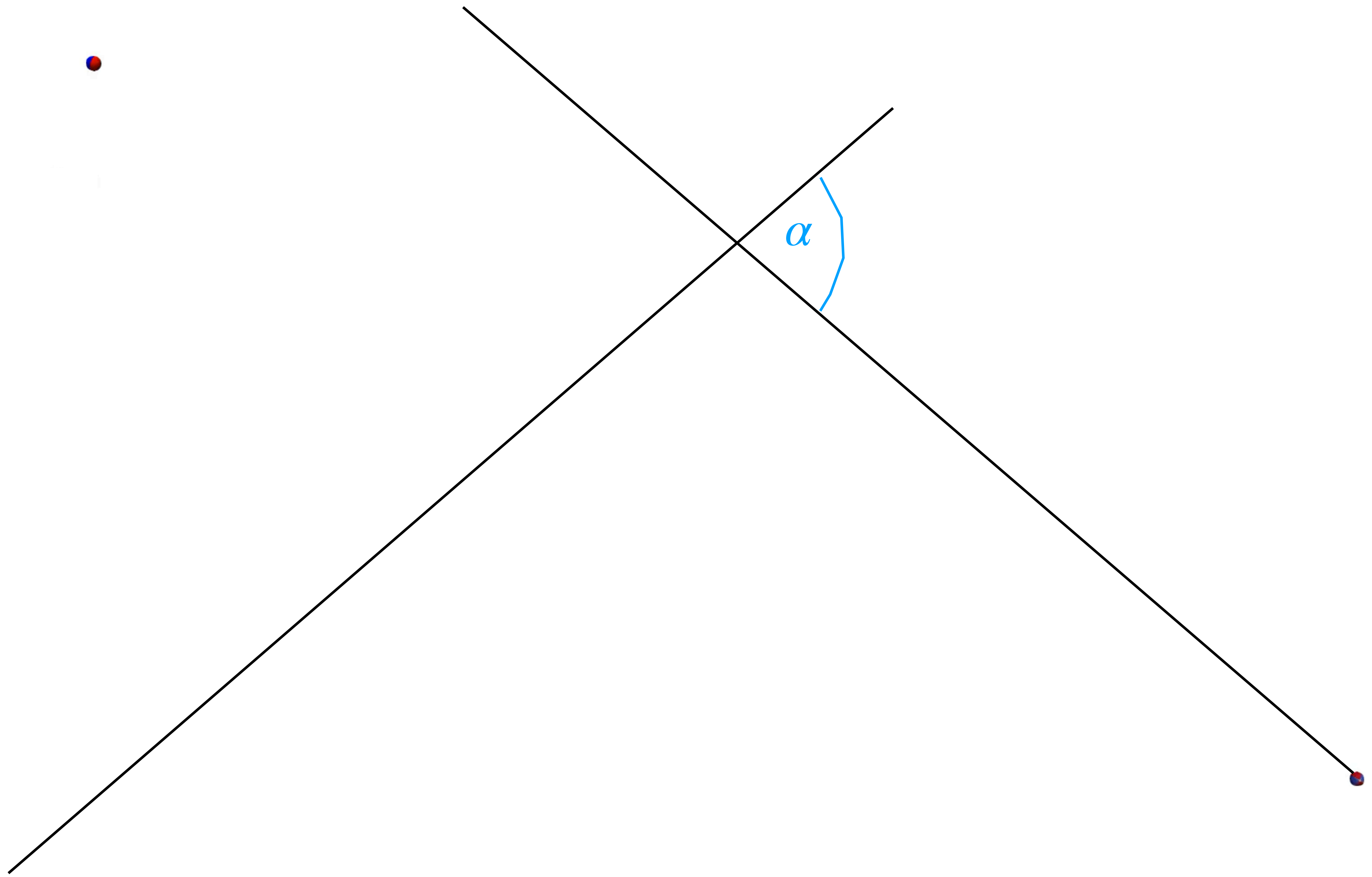
  ⇒ nearly vanishing spread: next-to-next-to-leading-order contribution is small

  $\dot{E}_2(e) = \mathcal{O}(\nu^4)$ *— insignificant*

BH scattering

BH Scattering

- **Clean probe** of strong field dynamics
- **Gauge invariant** quantities
  - scattering angle(s) α
  - $E_{\rm GW}$, $L_{\rm GW}$, $\Delta v_{\infty,i}$
  - Δm_i, $\Delta \vec{S}_i$

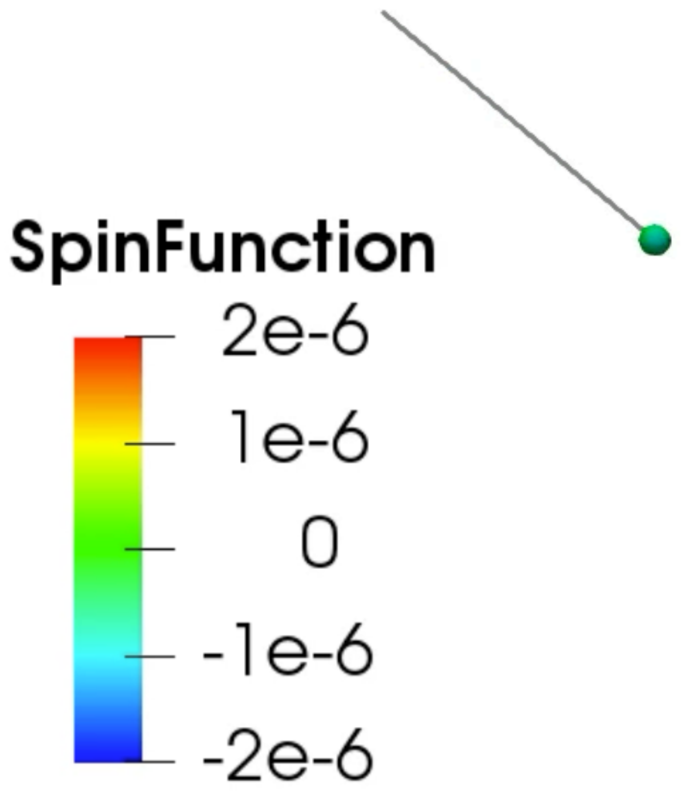


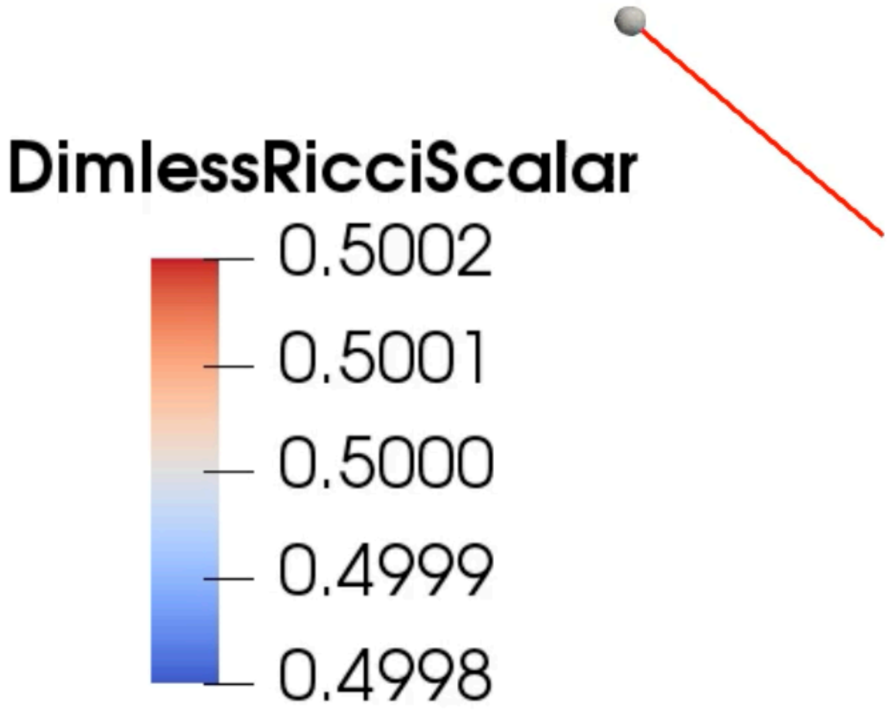
BH Scattering

- **Clean probe** of strong field dynamics

- **Gauge invariant** quantities
  - scattering angle(s) α
  - E_{GW}, L_{GW}, $\Delta v_{\infty,i}$
  - Δm_i, $\Delta \vec{S}_i$

- **Time-dependent data**
  - GWs
  - Horizons

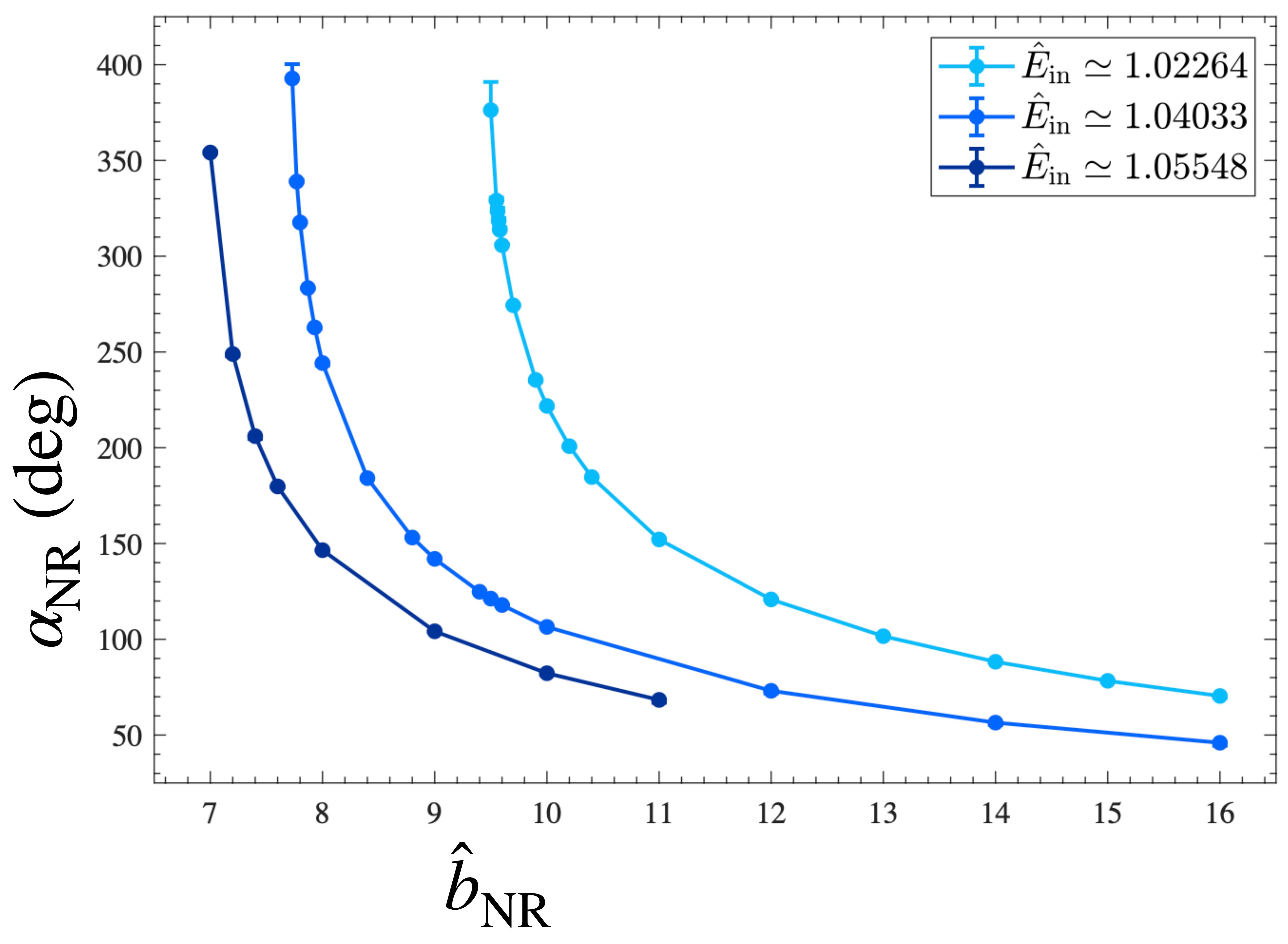




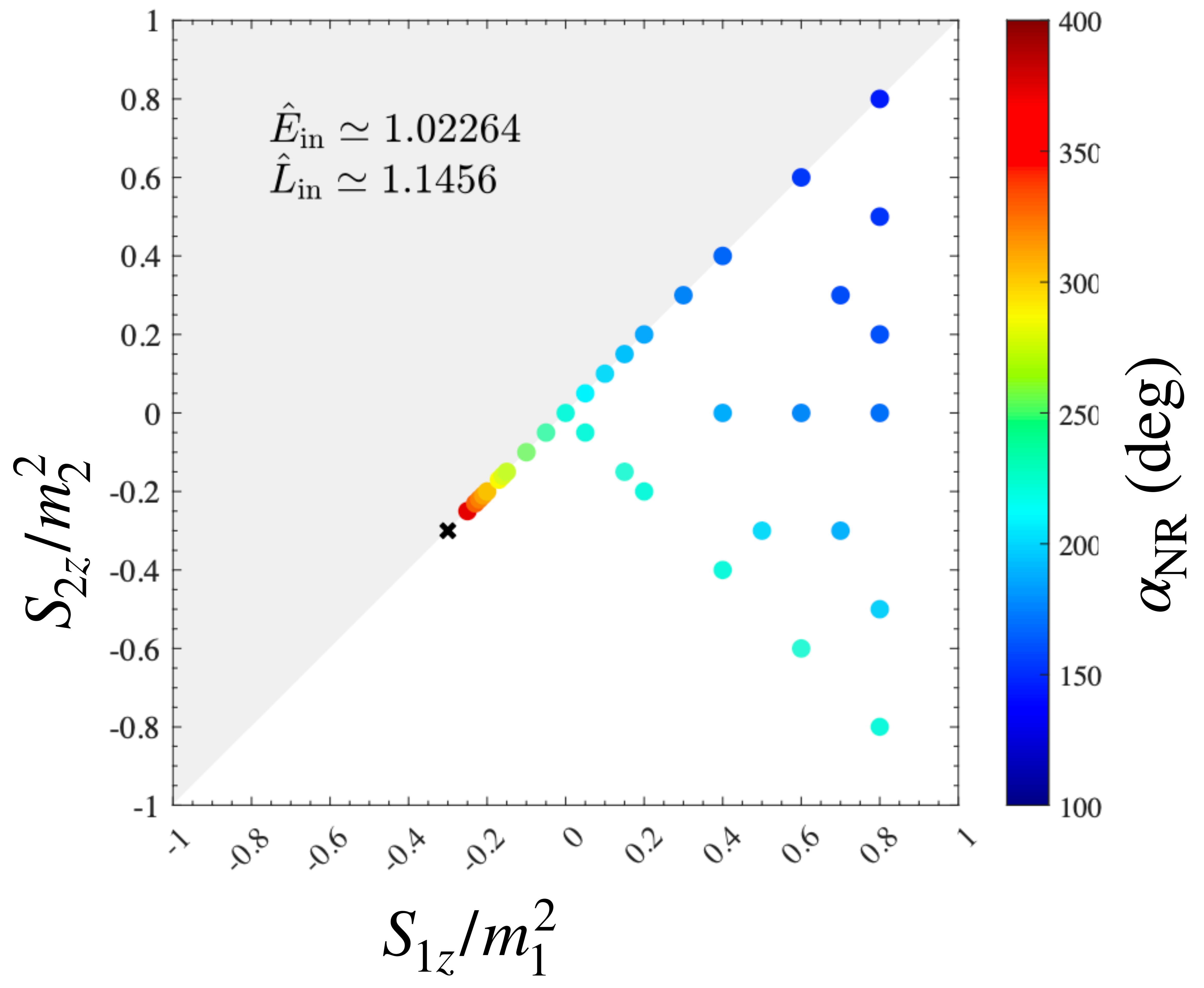
Scattering angles from NR

Most impressive results: Rettegno, Pratten, Thomas, Schmidt, Damour 2307.06999

non-spinning



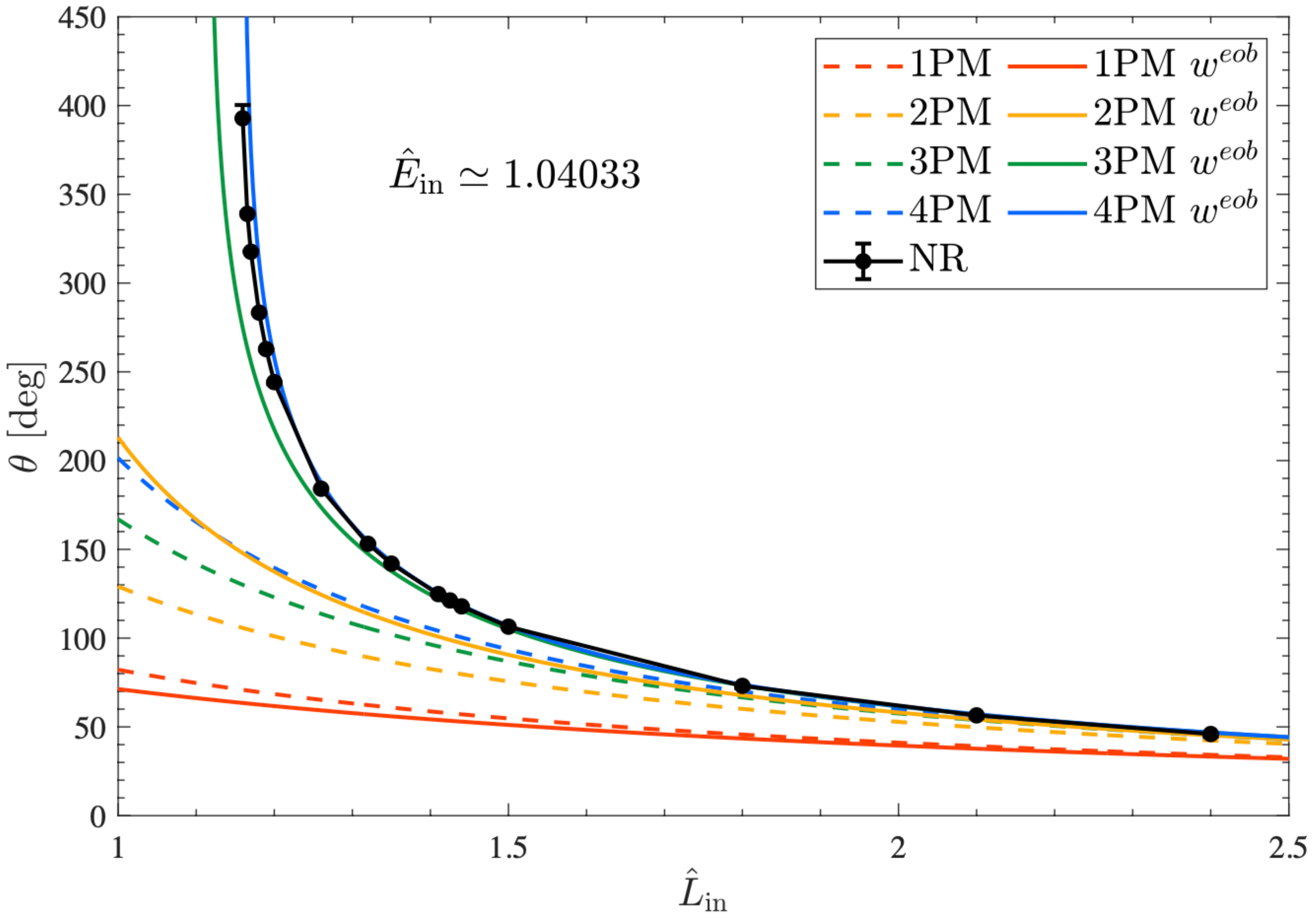
spinning BHs



Scattering angles from NR

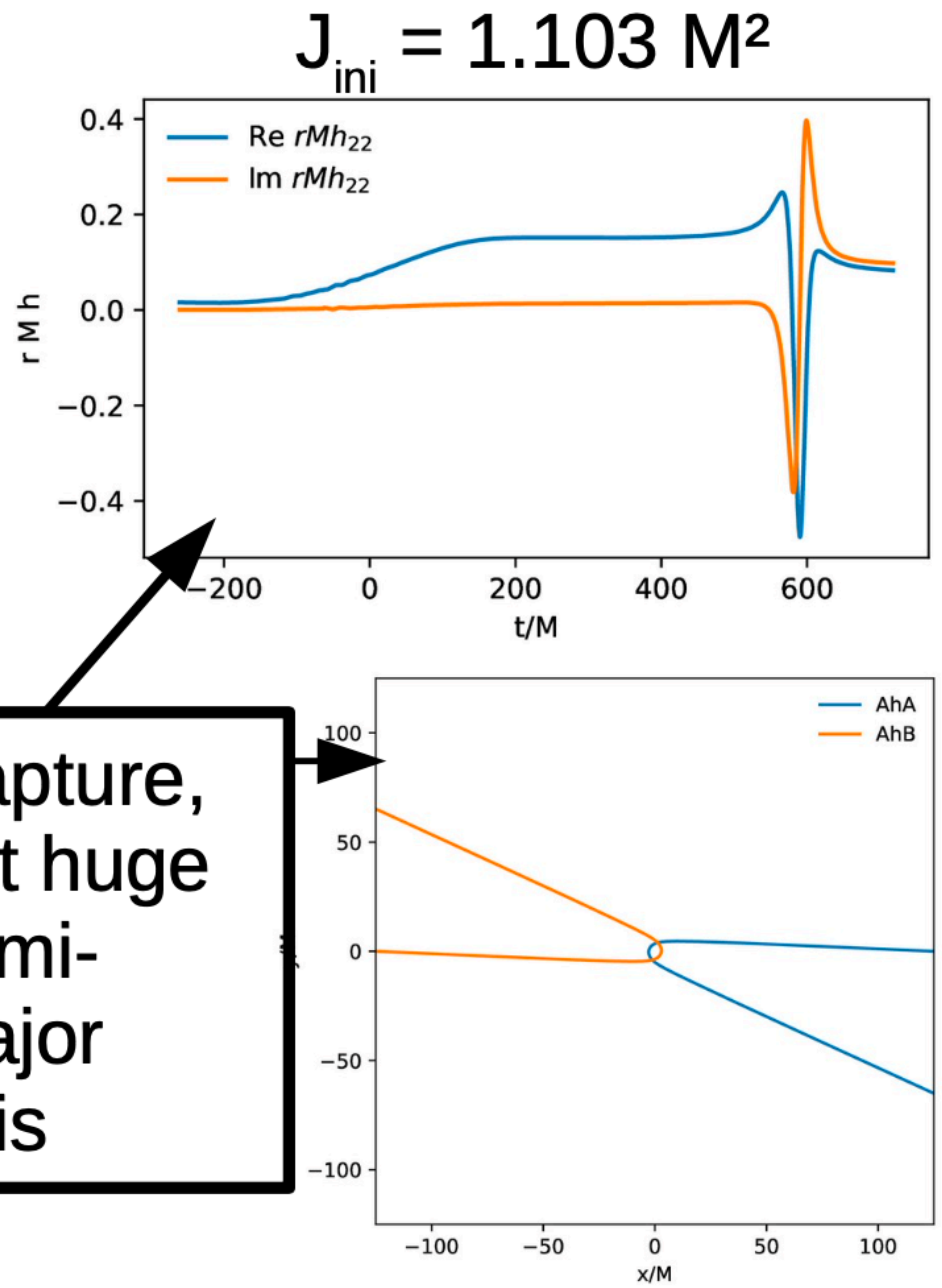
Most impressive results: Rettegno, Pratten, Thomas, Schmidt, Damour 2307.06999

non-spinning, equal-mass

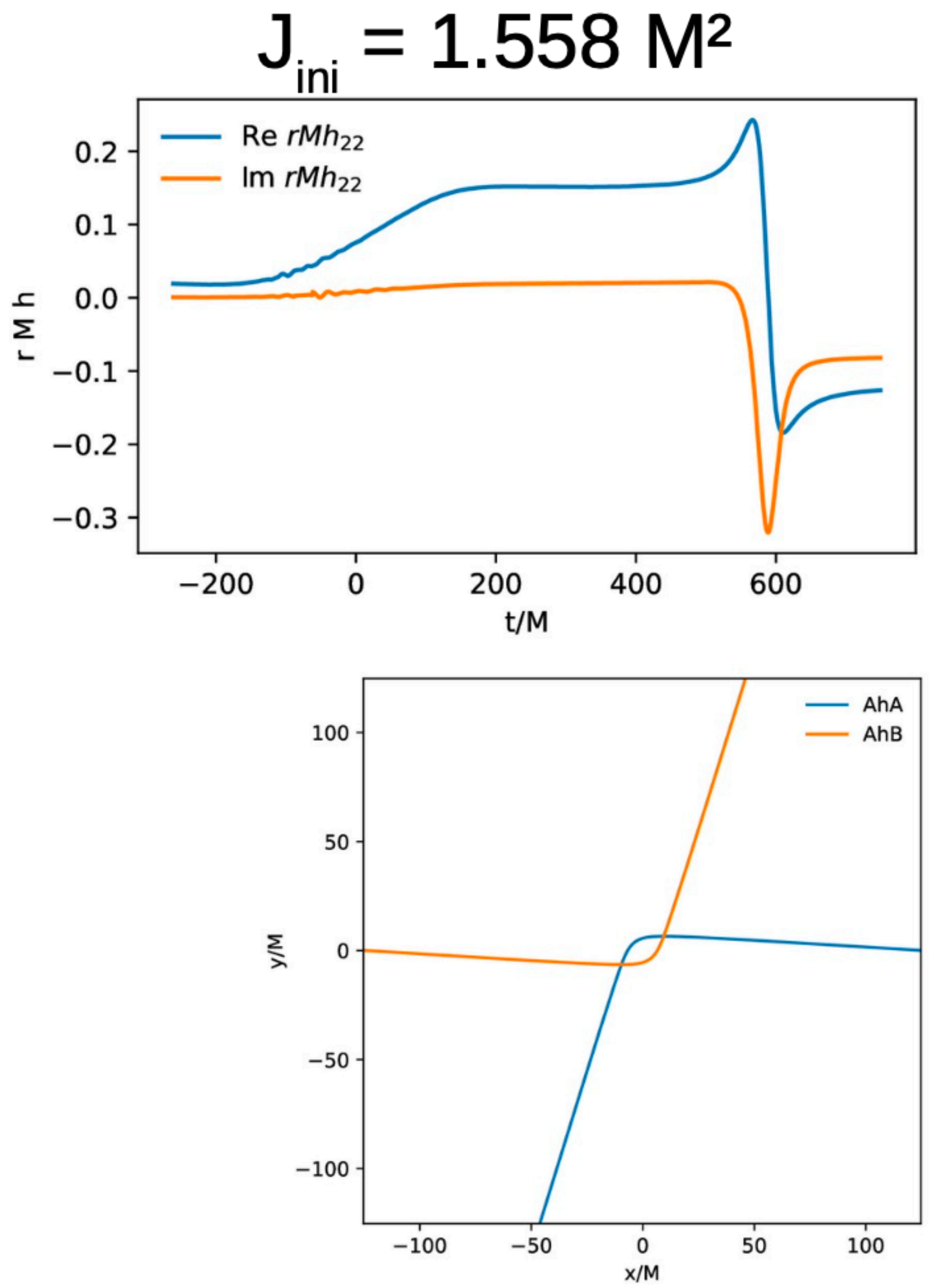


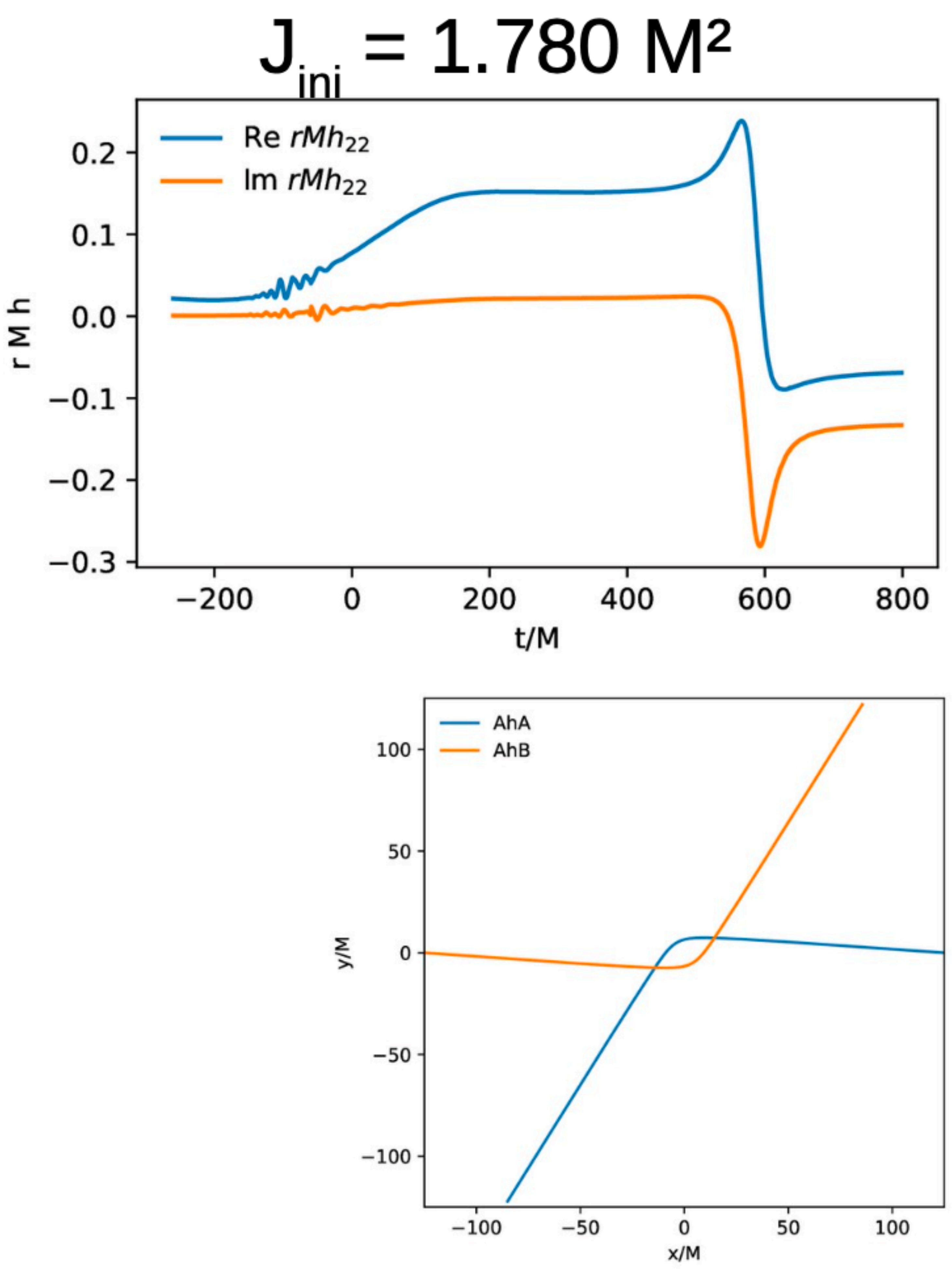
Some first waveforms

E=0.0223 M



Capture, but huge semi-major axis





Rüter, HP, SXS in prep

Summary

- **NR for quasi-circular binaries mature pillar of GW astronomy**
  - must keep up with improving detectors!
- **High mass-ratio challenging**
  - new methods promising
  - growing evidence that 2nd order SMR may be surprisingly accurate
- **Eccentric and hyperbolic systems** gain in attention
  - super-large parameter spaces

Rüter, HP