Quantum Algorithm for Scattering Amplitudes with Reduction Formula

based on T. Li, WKL, E. Wang, H. Xing, arXiv:2301.04179 [hep-ph]

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Outline

- Motivation

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- LSZ reduction formula

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- The quantum algorithm

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- Polology in Gross-Neveu model

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- Conclusion

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- How can quantum advantage be manifested in the field of particle physics?

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  - Monte-Carlo lattice calculations cannot handle real time dynamics.
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- **Quantum computing offers a possibly viable way to compute scattering amplitudes for general quantum field theories, with complexity scaling polynomially with energies and number of particles.** [Jordan, Lee, Preskill (2014)]

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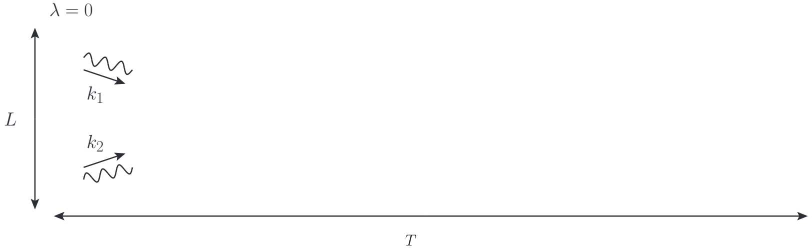
  1. Incoming particles are prepared as spatially widely separated wave packets, with coupling constants turned off.



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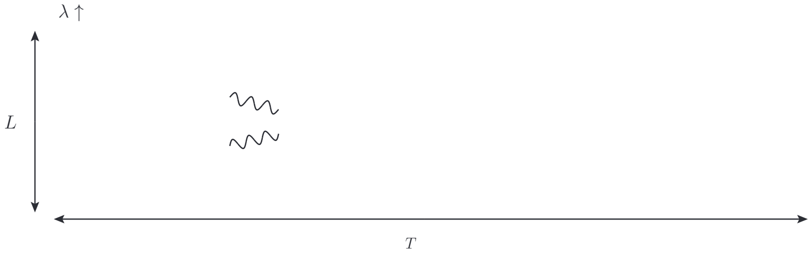

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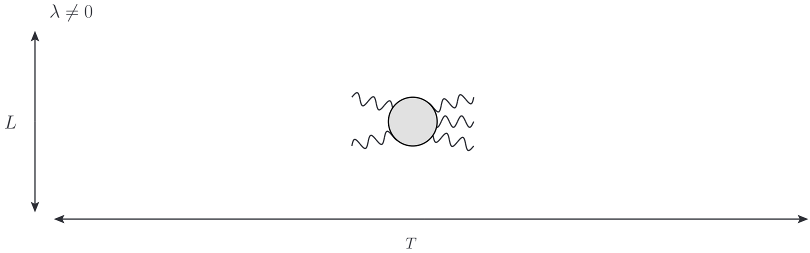

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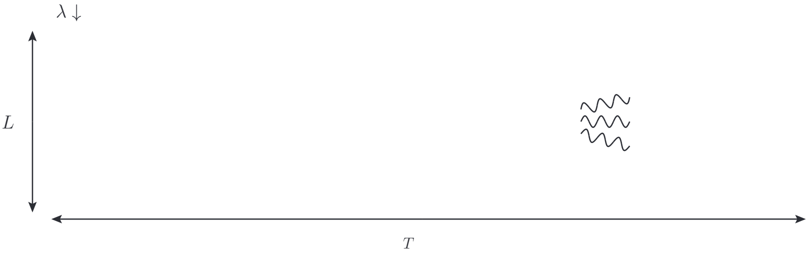

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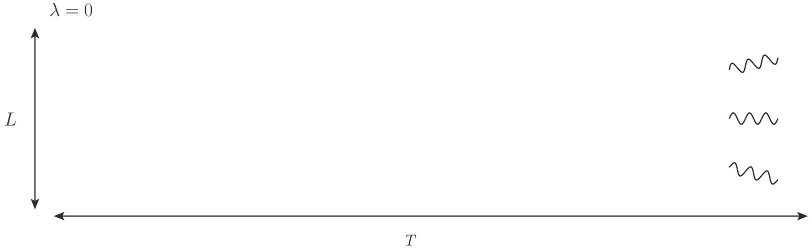

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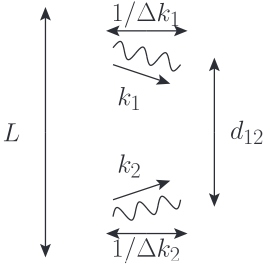

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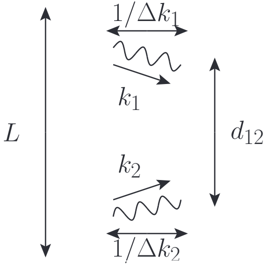

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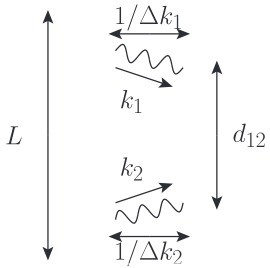

  2. Adiabatic turn on and turn off of coupling constants require a long time span of evolution $\implies$ unwanted broadening of wave packets $\implies$ induce most theoretical uncertainties in the strong-coupling regime.
  3. Coupling constants are turned off at the beginning $\implies$ bound states cannot be incorporated as incoming particles.

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LSZ reduction formula

Consider $h(\boldsymbol{k}_1) + \cdots + h(\boldsymbol{k}_{n_{\rm in}}) \to h(\boldsymbol{p}_1) + \cdots + h(\boldsymbol{p}_{n_{\rm out}})$
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Lehmann-Symanzik-Zimmermann (LSZ) reduction formula:

$$i\mathcal{M} = R^{n/2} \lim_{\substack{p_i^2 \to m^2 \\ k_j^2 \to m^2}} G(\{p_i\},\{k_j\}) \left(\prod_{r=1}^{n_{\text{out}}} K^{-1}(p_r) \right) \left(\prod_{s=1}^{n_{\text{in}}} K^{-1}(k_s) \right)$$

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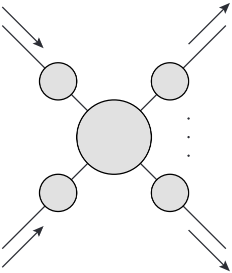

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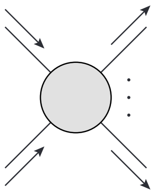

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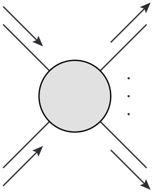

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  Propagator $K(p)$ also has a simple pole at $p^2 = m^2$:

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  - In practice, when the continuum theory is approximated by a theory on the lattice, these singularities are tamed and the pole structure $\frac{1}{p^2-m^2+i\epsilon}$ is approximated by some bounded function of p^2 which approaches it in the continuum and infinite-volume limits.

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  - The large cancellation in the continuum limit among the components in the LSZ reduction formula could potentially cause problems on numerical stability in practical calculations. Detailed study of the approach to the continuum limit of the LSZ reduction formula is left for the future.

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  - Since **only states with zero spatial momentum are involved in our formalism**, QAOA can be applied easily: one simply uses input reference states and alternating operators which are constructed to be translation-invariant.

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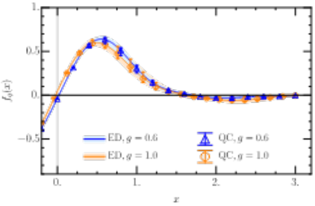

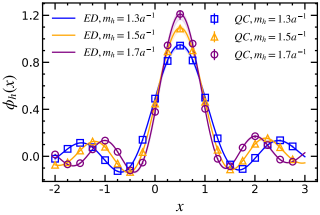

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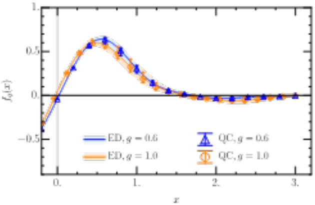

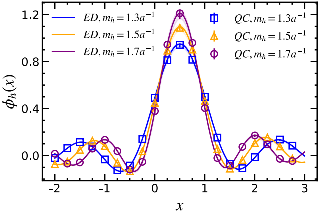

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- **No adiabatic turn on or turn off of coupling constants** $\Longrightarrow$ no associated extra time evolution $\Longrightarrow$ less computational depth for time evolution.
- **Bound states are allowed as incoming or outgoing particles**, since coupling constants are never turned off. The field operator ϕ is not necessarily a fundamental field of the theory. Any operator which has the same quantum numbers as the external particle h can be used.

Complexity of LSZ formalism

- Suppose we have N lattice sites and T temporal sites, and we need n_q qubits at each lattice site. Overall complexity in our formalism is estimated to be $\mathcal{O}(2^n n n_q N^{n+1} T^n)$.

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- Let $\Lambda_{\max}$ be the largest energy scale in the scattering process. Overall complexity in our formalism is estimated to be $\mathcal{O}(\Lambda_{\max}^{2n+1} \log \Lambda_{\max})$, which is polynomial in $\Lambda_{\max}$. In JLP, complexity also scales polynomially in $\Lambda_{\max}$.

Polology in Gross-Neveu model

Consider Gross-Neveu model ($1+1$-d Nambu-Jona-Lasinio (NJL) model):

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_q)\psi + g(\bar{\psi}\psi)^2$$

We aim to simulate the fermion propagator

$$K_\psi(p) = \int d^2x\, e^{ip\cdot x} \langle \Omega | T\{\psi(x)\bar{\psi}(0)\} | \Omega \rangle$$

and the connected fermion 4-point function corresponding to $2 \to 2$ scattering of a quark and an antiquark, $q(\boldsymbol{k_1})\bar{q}(\boldsymbol{k_2}) \to q(\boldsymbol{p_1})\bar{q}(\boldsymbol{p_2})$,

$$\begin{aligned} & G_\psi^{\alpha\beta\gamma\delta}(p_1, p_2, k_1) \\ &= \int d^2x_1 d^2x_2 d^2y_1\, e^{i(p_1\cdot x_1 + p_2 \cdot x_2 - k_1 \cdot y_1)} \\ &\quad \times \langle \Omega | \psi^\alpha(x_1)\bar{\psi}^\beta(x_2)\bar{\psi}^\gamma(y_1)\psi^\delta(0)|\Omega\rangle_{\text{con}} \end{aligned}$$

Jordan-Wigner transformation

How to represent a QFT with qubits?

$$\psi(0,\mathbf{z}) = \begin{pmatrix} \psi_1(0,\mathbf{z}) \\ \psi_2(0,\mathbf{z}) \end{pmatrix} \equiv \begin{pmatrix} \varphi_{2n} \\ \varphi_{2n+1} \end{pmatrix} \quad , 0 \leq n \leq \frac{N}{2} - 1$$

Express φ_n in terms of Pauli operators on qubits:

$$\varphi_n \equiv \Xi^3_n \sigma^+_n$$

where $\Xi^3_n \equiv \prod_{n'<n} \sigma^3_{n'}$.
Hamiltonian is split into 4 (symmetry-preserving) parts: $H = H_1 + H_2 + H_3 + H_4$

$$H_1 = \sum_{n=\text{even}}^{\frac{N}{2}-1} \frac{1}{4} \left(\sigma^1_n \sigma^2_{n+1} - \sigma^2_n \sigma^1_{n+1} \right)$$

$$H_2 = \sum_{n=\text{even}}^{\frac{N}{2}-1} \frac{g}{2} \sigma^3_n \sigma^3_{n+1}$$

$$H_3 = H_1(n = \text{even} \to n = \text{odd}) + \frac{1}{4} \Xi^3_{N-1} \left(\sigma^2_{N-1} \sigma^1_0 - \sigma^1_{N-1} \sigma^2_0 \right)$$

$$H_4 = \sum_{n=0}^{\frac{N}{2}-1} \frac{m_q}{2} (-1)^n (I - \sigma^3_n) - \frac{g}{2} (I - \sigma^3_n)$$

QAOA and VQE

Energy eigenstates are found by a variational method: quantum alternating operator ansatz (QAOA) & quantum-number-resolving variational quantum eigensolver (VQE)

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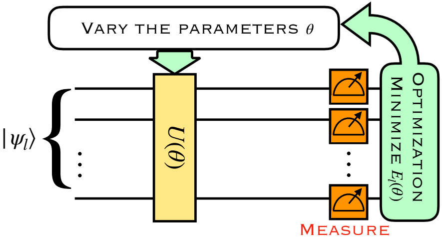

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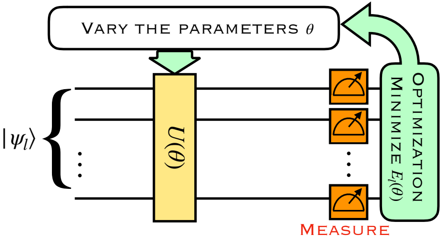

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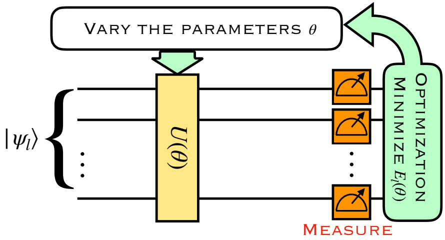

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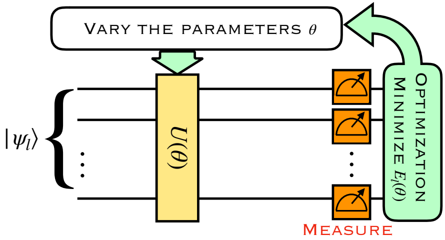
4. Eigenstates are given by $|h_{li}\rangle = U(\theta^*)\,|\psi_{li}\rangle_{\mathrm{ref}}$, with θ^* optimized values of θ.



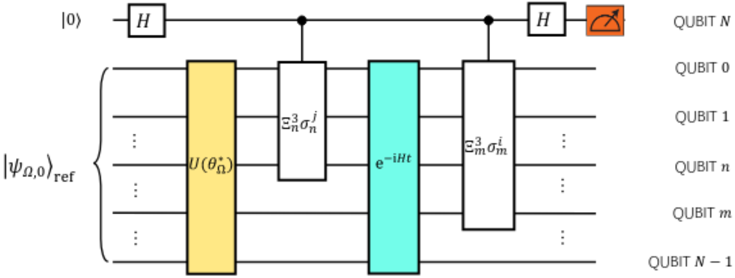
QAOA and VQE

For example, for $l =$quantum numbers of the vacuum $|\Omega\rangle$, we can take

$$|\psi_{\Omega,1}\rangle_{\text{ref}} = |010101\ldots01\rangle$$
$$|\psi_{\Omega,2}\rangle_{\text{ref}} = \frac{1}{\sqrt{N/2}}\left(|1001\ldots01\rangle + |0110\ldots01\rangle + \cdots + |0101\ldots10\rangle\right)$$

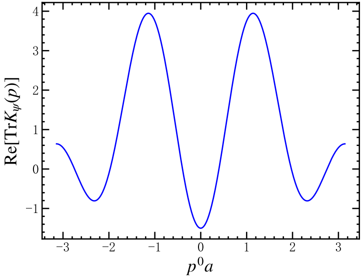
Correlation functions

Two-point function $\langle\Omega|T\{\psi(x)\bar{\psi}(0)\}|\Omega\rangle$:



Circuits for n-point functions can be similarly constructed.

The classical simulation is performed on a desktop workstation with 16 cores, using opensource packages QuSpin [Weinberg, Bukov (2017)] and projectQ [Steiger, Häner, Troyer (2018)], with 14 qubits (7 lattice sites).



Real part of $\mathrm{Tr}K_\psi(p)$ as a function of $p^0 a$ with $p^1 = 0$

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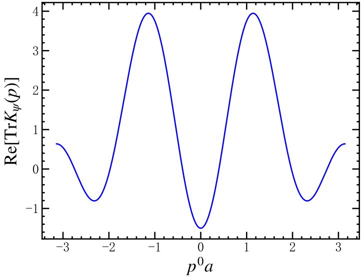

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- The peaks at $p^0 a = \pm 1.14$ and ± 3.14 correspond to the poles from the two lowest-lying states with the same quantum numbers as the quark field, as is verified by solving for the mass spectrum with direct numerical diagonalization of the discretized Hamiltonian ($m_h a = 1.19, 3.25$).

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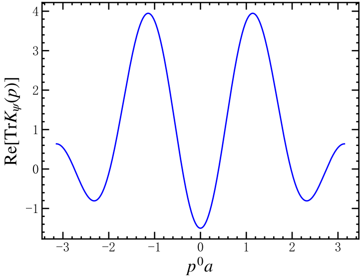

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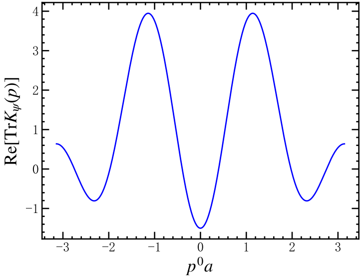

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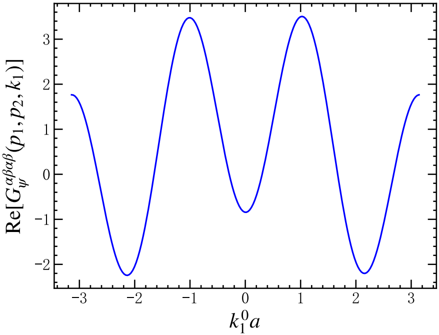
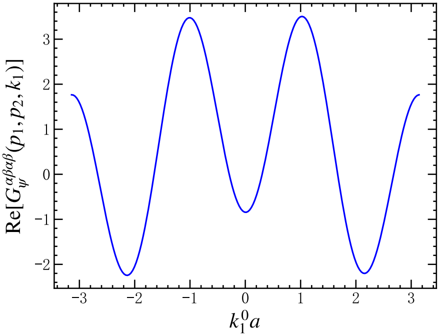
- The peaks at $p^0 a = \pm 1.14$ can be interpreted as a quark, and the peaks at $p^0 a = \pm 3.14$ can be interpreted as a bound state made up of two quarks and one antiquark.
- In the continuum limit, a pole corresponds to a peak of infinite height, while in the discretized model we consider here the peaks have finite height.



Real part of $G_\psi^{\alpha\beta\alpha\beta}(p_1, p_2, k_1)$ as a function of $k_1^0 a$, with $k_1 = (k_1^0, 0)$, $p_1 = (0, 0)$, $p_2 = (k_1^0, \pi/a)$



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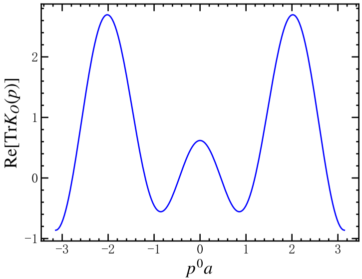
- Similar to the case of the propagator, the peaks at $k_1^0 a = \pm 1.01$ and ± 3.13 correspond to the poles from the two lowest-lying states with the same quantum numbers as the quark field.

Propagator for $q\bar{q}$ bound state

In order to demonstrate the power of the LSZ reduction formula in handling scatterings of bound-state particles, we also simulate the propagator of the composite operator $O(x) = \bar{\psi}(x)\psi(x)$

$$K_O(p) = \int d^2x\, e^{ip\cdot x} \langle \Omega | T\{O(x)O(0)\} | \Omega \rangle_{\text{con}}$$

PROPAGATOR FOR $q\bar{q}$ BOUND STATE



Real part of $\mathrm{Tr}K_O(p)$ as a function of p^0a with $p^1 = 0$

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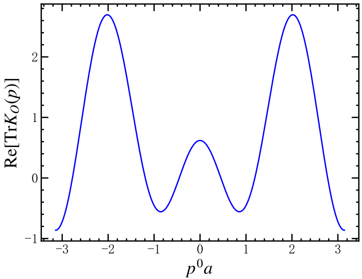

Real part of $\mathrm{Tr} K_O(p)$ as a function of $p^0 a$ with $p^1 = 0$

- The peaks at $p^0 a = \pm 2.02$ correspond to the poles from the second lowest-lying state h_O with the same quantum numbers as the vacuum, as is verified by solving for the mass spectrum with direct numerical diagonalization ($m_{h_O} a = 1.98$).

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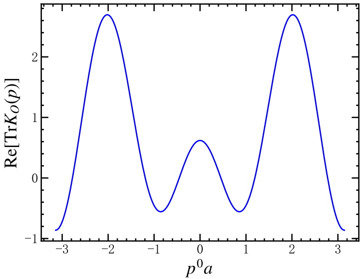

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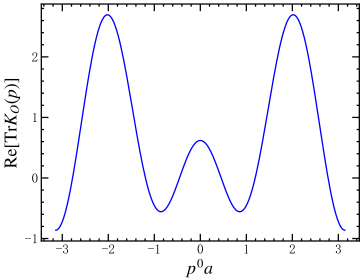

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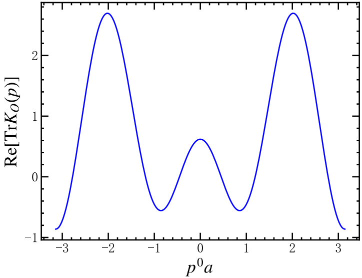

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This simple example shows that the quantum algorithm succeeds in recovering the expected pole structure of both the propagator and the connected n-point function, which is crucial to the implementation of the LSZ reduction formula.

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- As a proof of concept, in a simple model, the Gross-Neveu model, we demonstrated by simulations on classical hardware that the propagator and the connected 4-point function obtained from the quantum algorithm has the desired pole structure crucial to the implementation of the LSZ reduction formula.

Thank you.