Drell–Yan production in QCD: q_T resummation at N^{3}LL accuracy and fiducial cross sections at N^{3}LO

Giancarlo Ferrera

Milan University & INFN, Milan

DESY – Zeuthen – December 16th 2021

Outline

- **Fixed-order calculations within the q_T subtraction formalism**
- **Transverse-momentum resummation**
- **Drell–Yan production at N^3LL+N^3LO**
- **Fiducial power corrections within the q_T subtraction formalism**

Precise QCD predictions and the LHC

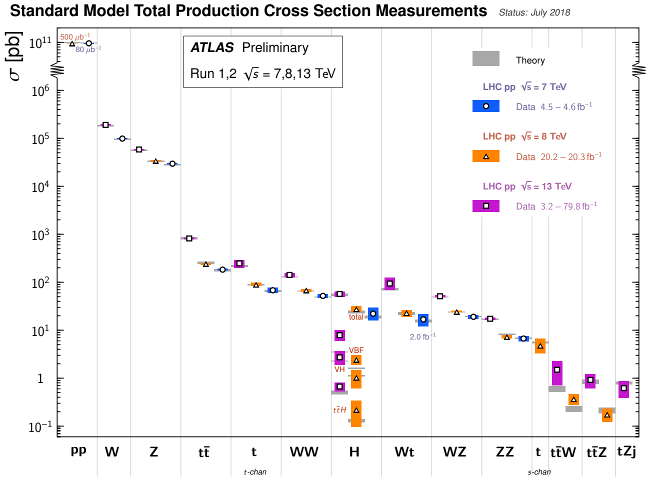
The success of the LHC will be crucial for High Energy Physics.

How to **increase the discovery power of the LHC?**

The LHC is a (large) hadron collider machine: all the interesting high-p_T reactions initiate by QCD hard scattering of partons.

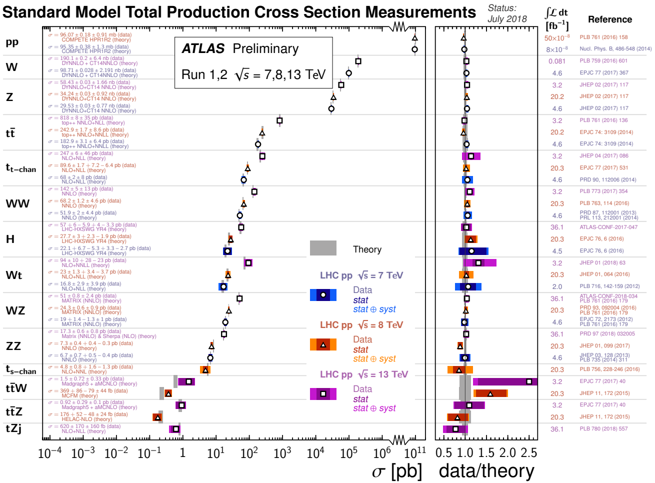
To fully exploit the information contained in the LHC experimental data (and aventually claiming for new-physics signals) precise theoretical predictions of QCD dynamics is necessary.

LHC key results



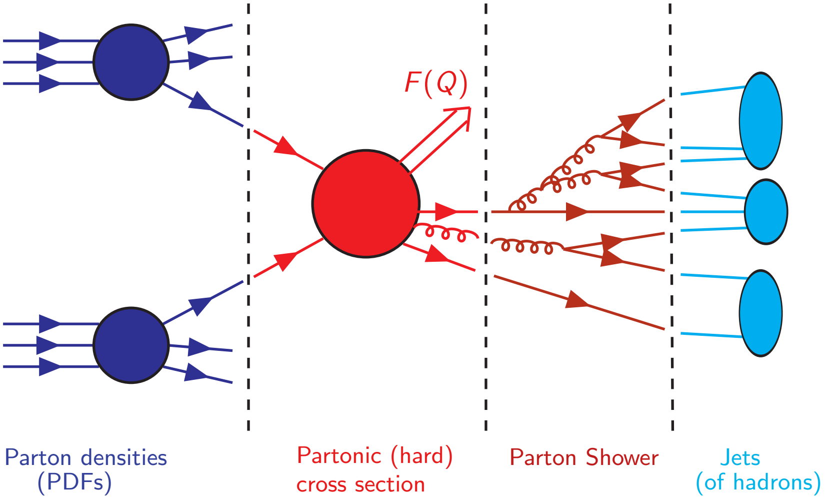
- Very good agreement between experimental results and SM theoretical predictions of the high-Q^2 processes

LHC key results

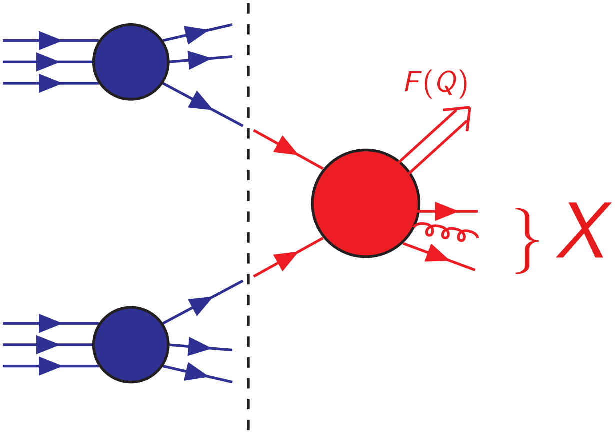


- Very good agreement between experimental results and SM theoretical predictions of the high-Q^2 processes

Theoretical predictions at the LHC



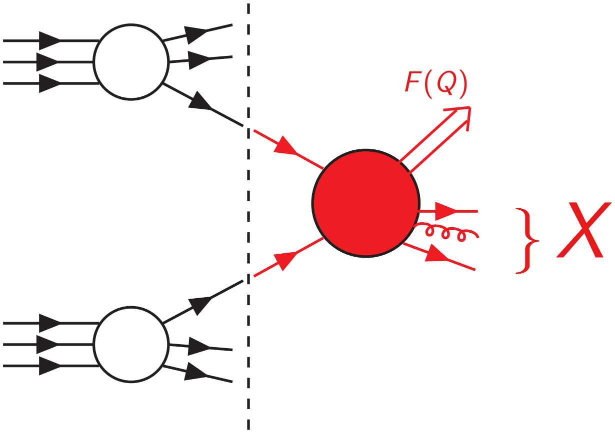
Theoretical predictions at the LHC



Parton densities
(PDFs)

Partonic (hard)
cross section

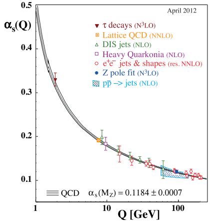
Theoretical predictions at the LHC



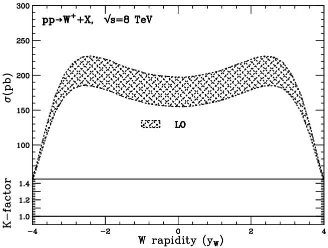
Partonic (hard)
cross section

Fixed-order calculations

Fixed-order calculations



The QCD coupling
$\alpha_S(Q) \sim 1/(\beta_0 \ln Q^2/\Lambda^2_{QCD}) \sim 0.1$
(for $Q \sim m_H$).



- Factorization theorem

$$\sigma = \sum_{a,b} f_a(M^2) \otimes f_b(M^2) \otimes \hat{\sigma}_{ab}(\alpha_S) + \mathcal{O}\left(\frac{\Lambda}{M}\right)$$

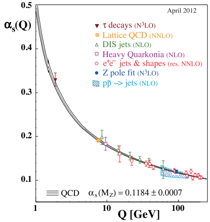
- Perturbation theory at **leading order (LO)**:

$$\hat{\sigma}(\alpha_S) = \hat{\sigma}^{(0)}$$

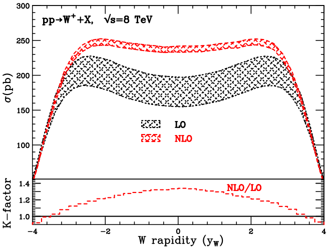
- **LO result:** only **order of magnitude** estimate.
  NLO: first reliable estimate.
  NNLO & beyond: precise prediction & **robust uncertainty**.

- Higher-order calculations **not an easy task** due to **infrared (IR) singularities** (impossible direct use of numerical techniques).

Fixed-order calculations



The QCD coupling
$\alpha_S(Q) \sim 1/(\beta_0 \ln Q^2/\Lambda^2_{QCD}) \sim 0.1$
(for $Q \sim m_H$).



- Factorization theorem

$$\sigma = \sum_{a,b} f_a(M^2) \otimes f_b(M^2) \otimes \hat{\sigma}_{ab}(\alpha_S) + \mathcal{O}\left(\frac{\Lambda}{M}\right)$$

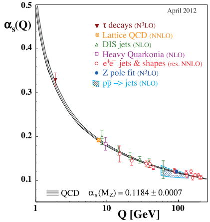
- Perturbation theory at **next order (NLO)**:

$$\hat{\sigma}(\alpha_S) = \hat{\sigma}^{(0)} + \alpha_S\, \hat{\sigma}^{(1)}$$

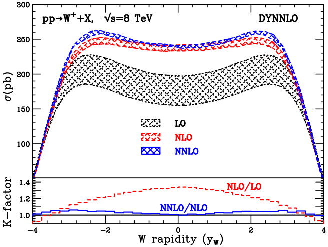
- **LO result:** only **order of magnitude** estimate.
  NLO: first reliable estimate.
  NNLO & beyond: precise prediction & **robust uncertainty**.

- Higher-order calculations **not an easy task** due to **infrared (IR) singularities** (impossible direct use of numerical techniques).

Fixed-order calculations



The QCD coupling
$\alpha_S(Q) \sim 1/(\beta_0 \ln Q^2/\Lambda^2_{QCD}) \sim 0.1$
(for $Q \sim m_H$).



- Factorization theorem

$$\sigma = \sum_{a,b} f_a(M^2) \otimes f_b(M^2) \otimes \hat{\sigma}_{ab}(\alpha_S) + \mathcal{O}\left(\frac{\Lambda}{M}\right)$$

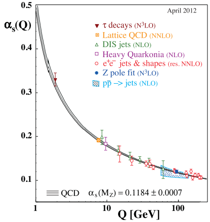
- Perturbation theory at **NNLO & beyond**:

$$\hat{\sigma}(\alpha_S) = \hat{\sigma}^{(0)} + \alpha_S\,\hat{\sigma}^{(1)} + \alpha_S^2\,\hat{\sigma}^{(2)} + \cdots$$

- **LO result:** only **order of magnitude** estimate.
  NLO: first reliable estimate.
  NNLO & beyond: precise prediction & **robust uncertainty**.

- Higher-order calculations **not an easy task** due to **infrared (IR) singularities** (impossible direct use of numerical techniques).

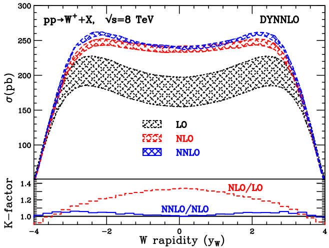
Fixed-order calculations



The QCD coupling

$\alpha_S(Q) \sim 1/(\beta_0 \ln Q^2/\Lambda^2_{QCD}) \sim 0.1$

(for $Q \sim m_H$).



- Factorization theorem

$$\sigma = \sum_{a,b} f_a(M^2) \otimes f_b(M^2) \otimes \hat{\sigma}_{ab}(\alpha_S) + \mathcal{O}\left(\frac{\Lambda}{M}\right)$$

- Perturbation theory at **NNLO & beyond**:

$$\hat{\sigma}(\alpha_S) = \hat{\sigma}^{(0)} + \alpha_S\,\hat{\sigma}^{(1)} + \alpha_S^2\,\hat{\sigma}^{(2)} + \cdots$$

- **LO result:** only **order of magnitude** estimate.
  NLO: first reliable estimate.
  NNLO & beyond: precise prediction & robust uncertainty.

- Higher-order calculations **not an easy task** due to **infrared (IR) singularities** (impossible direct use of numerical techniques).

Drell-Yan process

LO
Drell, Yan (1974)

NLO
Altarelli, Ellis, Greco
Martinelli, (1980-84)

NNLO
Hamberg, van Neerven, Matsuura (1991)
Melnikov, Petriello (2006)
Catani, Cieri, de Florian, G.F., Grazzini (2009)

N^{3}LO
Duhr, Mistlberger (2021)
Camarda, GF, Cieri (2021)

Fiducial cross sections at higher order

- Experiments have finite acceptance important to provide exclusive theoretical predictions.
- Beyond LO infrared singularities in *real* and *virtual* corrections prevent the straightforward implementation of Monte Carlo numerical techniques.
- Subtraction method: introduction of auxiliary QCD cross section *in a general way* exploiting the universality of the soft and collinear emission. Fully formalized at NLO [Frixione,Kunszt,Signer('96) (FKS), Catani,Seymour('97) (CS)]. It allows (relatively) straightforward calculations (once the QCD amplitudes are available). Fully general formalism beyond NLO still lacking.

$$\sigma^{NLO} = \int_{m+1} d\sigma^R(\epsilon) \;+\; \int_m d\sigma^V(\epsilon)$$

$$= \int_{m+1} \left[d\sigma^R(\epsilon) - d\sigma^A(\epsilon) \right]_{\epsilon=0} \;+\; \int_m \left[d\sigma^V(\epsilon) + \int_1 d\sigma^A(\epsilon) \right]_{\epsilon=0}$$

Fiducial cross sections at higher order

- Experiments have finite acceptance important to provide exclusive theoretical predictions.
- Beyond LO infrared singularities in *real* and *virtual* corrections prevent the straightforward implementation of Monte Carlo numerical techniques.
- Subtraction method: introduction of auxiliary QCD cross section *in a general way* exploiting the universality of the soft and collinear emission. Fully formalized at NLO [Frixione,Kunszt,Signer('96) (FKS), Catani,Seymour('97) (CS)]. It allows (relatively) straightforward calculations (once the QCD amplitudes are available). Fully general formalism beyond NLO still lacking.

$$\sigma^{NLO} = \int_{m+1} d\sigma^R(\epsilon) \; + \; \int_m d\sigma^V(\epsilon)$$

$$= \int_{m+1} \left[d\sigma^R(\epsilon) - d\sigma^A(\epsilon) \right]_{\epsilon=0} \; + \; \int_m \left[d\sigma^V(\epsilon) + \int_1 d\sigma^A(\epsilon) \right]_{\epsilon=0}$$

Fiducial cross sections at higher order

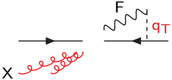
- Experiments have finite acceptance important to provide exclusive theoretical predictions.
- Beyond LO infrared singularities in *real* and *virtual* corrections prevent the straightforward implementation of Monte Carlo numerical techniques.
- Subtraction method: introduction of auxiliary QCD cross section *in a general way* exploiting the universality of the soft and collinear emission. Fully formalized at NLO [Frixione,Kunszt,Signer('96) (FKS), Catani,Seymour('97) (CS)]. It allows (relatively) straightforward calculations (once the QCD amplitudes are available). Fully general formalism beyond NLO still lacking.

$$\sigma^{NLO} = \int_{m+1} d\sigma^R(\epsilon) \; + \; \int_m d\sigma^V(\epsilon)$$

$$= \int_{m+1} \left[d\sigma^R(\epsilon) - d\sigma^A(\epsilon) \right]_{\epsilon=0} \; + \; \int_m \left[d\sigma^V(\epsilon) + \int_1 d\sigma^A(\epsilon) \right]_{\epsilon=0}$$

The q_T-subtraction method at NNLO (and beyond)

$$h_1(p_1) + h_2(p_2) \;\rightarrow\; F(M, q_T) + X$$



F is one or more colourless particles (vector bosons, photons, Higgs bosons,...) [Catani,Grazzini('07)].

- **Observation:** at LO the q_T of the F is exactly zero.

$$d\sigma^{F}_{N^nLO}|_{q_T\neq 0} = d\sigma^{F+\text{jets}}_{N^{n-1}LO} \;,$$

for $q_T \neq 0$ the N^nLO IR divergences cancelled with the $N^{n-1}LO$ subtraction method.

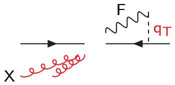
- The only remaining N^nLO singularities are associated with the $q_T \rightarrow 0$ limit.
- **Key point**: treat the N^nLO singularities at $q_T = 0$ by an additional subtraction using the universality of logarithmically-enhanced contributions from q_T resummation formalism [Catani,de Florian,Grazzini('00)].

$$d\sigma^{F}_{N^nLO} \xrightarrow{q_T\rightarrow 0} d\sigma^{F}_{LO} \otimes \Sigma(q_T/M)dq_T^2$$

$$= d\sigma^{F}_{LO} \otimes \sum_{n=1}^{\infty}\sum_{k=1}^{2n}\left(\frac{\alpha_S}{\pi}\right)^n \Sigma^{(n,k)}\frac{M^2}{q_T^2}\ln^{k-1}\frac{M^2}{q_T^2}\,d^2q_T$$

The q_T-subtraction method at NNLO (and beyond)

$$h_1(p_1) + h_2(p_2) \;\rightarrow\; F(M, q_T) + X$$



F is one or more colourless particles (vector bosons, photons, Higgs bosons,...) [Catani,Grazzini('07)].

- **Observation:** at LO the q_T of the F is exactly zero.

$$d\sigma^{F}_{N^nLO}|_{q_T \neq 0} = d\sigma^{F+\text{jets}}_{N^{n-1}LO} \quad ,$$

for $q_T \neq 0$ the N^nLO IR divergences cancelled with the $N^{n-1}LO$ subtraction method.

- The only remaining N^nLO singularities are associated with the $q_T \rightarrow 0$ limit.

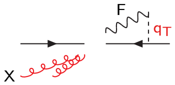
- **Key point**: treat the N^nLO singularities at $q_T = 0$ by an additional subtraction using the universality of logarithmically-enhanced contributions from q_T resummation formalism [Catani,de Florian,Grazzini('00)].

$$d\sigma^{F}_{N^nLO} \xrightarrow{q_T \to 0} d\sigma^{F}_{LO} \otimes \Sigma(q_T/M) dq_T^2$$

$$= d\sigma^{F}_{LO} \otimes \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \left(\frac{\alpha_S}{\pi}\right)^n \Sigma^{(n,k)} \frac{M^2}{q_T^2} \ln^{k-1} \frac{M^2}{q_T^2} \, d^2 q_T$$

The q_T-subtraction method at NNLO (and beyond)

$$h_1(p_1) + h_2(p_2) \ \rightarrow \ F(M, q_T) + X$$



F is one or more colourless particles (vector bosons, photons, Higgs bosons,...) [Catani,Grazzini('07)].

- **Observation:** at LO the q_T of the F is exactly zero.

$$d\sigma^{F}_{N^nLO}|_{q_T \neq 0} = d\sigma^{F+\mathrm{jets}}_{N^{n-1}LO} \ ,$$

for $q_T \neq 0$ the N^nLO IR divergences cancelled with the $N^{n-1}LO$ subtraction method.

- The only remaining N^nLO singularities are associated with the $q_T \rightarrow 0$ limit.
- **Key point**: treat the N^nLO singularities at $q_T = 0$ by an additional subtraction using the universality of logarithmically-enhanced contributions from q_T resummation formalism [Catani,de Florian,Grazzini('00)].

$$d\sigma^{F}_{N^nLO} \xrightarrow{q_T \rightarrow 0} d\sigma^{F}_{LO} \otimes \Sigma(q_T/M) dq_T^2$$

$$= d\sigma^{F}_{LO} \otimes \sum_{n=1}^{\infty} \sum_{k=1}^{2n} \left(\frac{\alpha_S}{\pi}\right)^n \Sigma^{(n,k)} \frac{M^2}{q_T^2} \ln^{k-1} \frac{M^2}{q_T^2} \, d^2 q_T$$

The final result valid also for $q_T = 0$ is:

$$d\sigma^F_{N^nLO} = \mathcal{H}^F_{N^nLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+jets}_{N^{n-1}LO} - d\sigma^{CT}_{N^{n-1}LO} \right] \quad ,$$

where $$\mathcal{H}^F_{N^nLO} = \left[1 + \frac{\alpha_S}{\pi} \mathcal{H}^{F(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{F(2)} + \cdots + \left(\frac{\alpha_S}{\pi}\right)^n \mathcal{H}^{F(n)} \right]$$

- The choice of the counter-term has some arbitrariness but it must behave $d\sigma^{CT} \xrightarrow{q_T \to 0} d\sigma^F_{LO} \otimes \Sigma(q_T/M) dq_T^2$ where $\Sigma(q_T/M)$ is universal.
- $d\sigma^{CT}$ regularizes the $q_T = 0$ singularity of $d\sigma^{F+jets}$: *real* and *real-virtual* N^nLO contributions.
- The finite part of *multi-loop virtual* corrections is contained in the hard-collinear function $\mathcal{H}^F_{N^nLO}$. Its process dependent part can be directly related to the all-order virtual amplitude by an universal (process independent) factorization formula [Catani,Cieri,de Florian,G.F.,Grazzini('14)].
- Final state partons only appear in $d\sigma^{F+jets}$ so that N^nLO IR-safe cuts are included in the N^{n-1}LO computation: observable-independent N^nLO extension of a N^{n-1}LO subtraction formalism.

The final result valid also for $q_T = 0$ is:

$$d\sigma^F_{N^nLO} = \mathcal{H}^F_{N^nLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+jets}_{N^{n-1}LO} - d\sigma^{CT}_{N^{n-1}LO} \right] \quad ,$$

where $$\mathcal{H}^F_{N^nLO} = \left[1 + \frac{\alpha_S}{\pi} \mathcal{H}^{F(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{F(2)} + \cdots + \left(\frac{\alpha_S}{\pi}\right)^n \mathcal{H}^{F(n)} \right]$$

- The choice of the counter-term has some arbitrariness but it must behave $d\sigma^{CT} \xrightarrow{q_T \to 0} d\sigma^F_{LO} \otimes \Sigma(q_T/M) dq_T^2$ where $\Sigma(q_T/M)$ is universal.
- $d\sigma^{CT}$ regularizes the $q_T = 0$ singularity of $d\sigma^{F+jets}$: *real* and *real-virtual* N^nLO contributions.
- The finite part of *multi-loop virtual* corrections is contained in the hard-collinear function $\mathcal{H}^F_{N^nLO}$. Its process dependent part can be directly related to the all-order virtual amplitude by an universal (process independent) factorization formula [Catani,Cieri,de Florian,G.F.,Grazzini('14)].
- Final state partons only appear in $d\sigma^{F+jets}$ so that N^nLO IR-safe cuts are included in the N^{n-1}LO computation: observable-independent N^nLO extension of a N^{n-1}LO subtraction formalism.

The final result valid also for $q_T = 0$ is:

$$d\boldsymbol{\sigma}^{F}_{N^nLO} = \boldsymbol{\mathcal{H}}^{F}_{N^nLO} \otimes d\boldsymbol{\sigma}^{F}_{LO} + \left[d\boldsymbol{\sigma}^{F+jets}_{N^{n-1}LO} - d\boldsymbol{\sigma}^{CT}_{N^{n-1}LO} \right] ,$$

where $$\boldsymbol{\mathcal{H}}^{F}_{N^nLO} = \left[1 + \frac{\boldsymbol{\alpha}_S}{\boldsymbol{\pi}} \boldsymbol{\mathcal{H}}^{F(1)} + \left(\frac{\boldsymbol{\alpha}_S}{\boldsymbol{\pi}} \right)^2 \boldsymbol{\mathcal{H}}^{F(2)} + \cdots + \left(\frac{\boldsymbol{\alpha}_S}{\boldsymbol{\pi}} \right)^n \boldsymbol{\mathcal{H}}^{F(n)} \right]$$

- The choice of the counter-term has some arbitrariness but it must behave $d\boldsymbol{\sigma}^{CT} \xrightarrow{q_T \to 0} d\boldsymbol{\sigma}^{F}_{LO} \otimes \Sigma(q_T/M) dq_T^2$ where $\Sigma(q_T/M)$ is universal.
- $d\boldsymbol{\sigma}^{CT}$ regularizes the $q_T = 0$ singularity of $d\boldsymbol{\sigma}^{F+jets}$: *real* and *real-virtual* N^nLO contributions.
- The finite part of *multi-loop virtual* corrections is contained in the hard-collinear function $\boldsymbol{\mathcal{H}}^{F}_{N^nLO}$. Its process dependent part can be directly related to the all-order virtual amplitude by an universal (process independent) factorization formula [Catani,Cieri,de Florian,G.F.,Grazzini('14)].
- Final state partons only appear in $d\boldsymbol{\sigma}^{F+jets}$ so that N^nLO IR-safe cuts are included in the $N^{n-1}LO$ computation: observable-independent N^nLO extension of a $N^{n-1}LO$ subtraction formalism.

The final result valid also for $q_T = 0$ is:

$$d\sigma^F_{N^nLO} = \mathcal{H}^F_{N^nLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+jets}_{N^{n-1}LO} - d\sigma^{CT}_{N^{n-1}LO} \right] ,$$

where $$\mathcal{H}^F_{N^nLO} = \left[1 + \frac{\alpha_S}{\pi} \mathcal{H}^{F(1)} + \left(\frac{\alpha_S}{\pi} \right)^2 \mathcal{H}^{F(2)} + \cdots + \left(\frac{\alpha_S}{\pi} \right)^n \mathcal{H}^{F(n)} \right]$$

- The choice of the counter-term has some arbitrariness but it must behave $d\sigma^{CT} \xrightarrow{q_T \to 0} d\sigma^F_{LO} \otimes \Sigma(q_T/M) dq_T^2$ where $\Sigma(q_T/M)$ is universal.
- $d\sigma^{CT}$ regularizes the $q_T = 0$ singularity of $d\sigma^{F+jets}$: *real* and *real-virtual* N^nLO contributions.
- The finite part of *multi-loop virtual* corrections is contained in the hard-collinear function $\mathcal{H}^F_{N^nLO}$. Its process dependent part can be directly related to the all-order virtual amplitude by an universal (process independent) factorization formula [Catani,Cieri,de Florian,G.F.,Grazzini('14)].
- Final state partons only appear in $d\sigma^{F+jets}$ so that N^nLO IR-safe cuts are included in the N^{n-1}LO computation: observable-independent N^nLO extension of a N^{n-1}LO subtraction formalism.

Universality of hard factors at all orders

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- *All-order universal* factorization formula relates $H_c^F(\alpha_S)$ to the virtual amplitude

$$\mathcal{M}_{ab\to F} = \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \mathcal{M}_{ab\to F}^{(n)}\,,$$ renormalized virtual amplitude (UV finite but IR divergent).

$$\tilde{I}(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \tilde{I}^{(n)}(\epsilon)\,,$$ IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

$$\widetilde{\mathcal{M}}_{ab\to F} = \left[1 - \tilde{I}(\epsilon, M^2)\right] \mathcal{M}_{ab\to F}\,,$$ hard-virtual subtracted amplitude (IR finite).

$$H_q^F(\alpha_S) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}|^2}{|\mathcal{M}_{q\bar{q}\to F}^{(0)}|^2}$$

- Process independent part of $\mathcal{H}^F$ coefficients calculated at NNLO in [Catani,Grazzini ('11)],[Catani,Cieri,de Florian, G.F.,Grazzini ('12)] at N^{3}LO in [Luo et al. ('19,'20)],[Erbert et al. ('20)]

Universality of hard factors at all orders

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- *All-order universal* factorization formula relates $H_c^F(\alpha_S)$ to the virtual amplitude

$$\mathcal{M}_{ab\to F} = \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \mathcal{M}_{ab\to F}^{(n)} ,$$

renormalized virtual amplitude (UV finite but IR divergent).

$$\tilde{I}(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \tilde{I}^{(n)}(\epsilon) ,$$

IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

$$\widetilde{\mathcal{M}}_{ab\to F} = \left[1 - \tilde{I}(\epsilon, M^2)\right] \mathcal{M}_{ab\to F} \quad ,$$

hard-virtual subtracted amplitude (IR finite).

$$H_q^F(\alpha_S) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}|^2}{|\mathcal{M}_{q\bar{q}\to F}^{(0)}|^2}$$

- Process independent part of $\mathcal{H}^F$ coefficients calculated at NNLO in [Catani,Grazzini ('11)],[Catani,Cieri,de Florian, G.F.,Grazzini ('12)] at N^{3}LO in [Luo et al. ('19,'20)],[Erbert et al. ('20)]

Universality of hard factors at all orders

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- *All-order universal* factorization formula relates $H_c^F(\alpha_S)$ to the virtual amplitude

$$\mathcal{M}_{ab\to F} = \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \mathcal{M}^{(n)}_{ab\to F} \,,$$

renormalized virtual amplitude (UV finite but IR divergent).

$$\tilde{I}(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \tilde{I}^{(n)}(\epsilon) \,,$$

IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

$$\widetilde{\mathcal{M}}_{ab\to F} = \left[1 - \tilde{I}(\epsilon, M^2)\right] \mathcal{M}_{ab\to F} \quad ,$$

hard-virtual subtracted amplitude (IR finite).

$$H_q^F(\alpha_S) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}|^2}{|\mathcal{M}^{(0)}_{q\bar{q}\to F}|^2}$$

- Process independent part of $\mathcal{H}^F$ coefficients calculated at NNLO in [Catani,Grazzini ('11)],[Catani,Cieri,de Florian, G.F.,Grazzini ('12)] at N^{3}LO in [Luo et al. ('19,'20)],[Erbert et al. ('20)]

Universality of hard factors at all orders

- *Process-dependence* is fully encoded in the hard-virtual factor $H_c^F(\alpha_S)$.
- *All-order universal* factorization formula relates $H_c^F(\alpha_S)$ to the virtual amplitude

$$\mathcal{M}_{ab\to F} = \sum_{n=0}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \mathcal{M}_{ab\to F}^{(n)}\,,$$

renormalized virtual amplitude (UV finite but IR divergent).

$$\tilde{I}(\epsilon, M^2) = \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \tilde{I}^{(n)}(\epsilon)\,,$$

IR subtraction *universal* operators (contain IR ϵ-poles and IR finite terms)

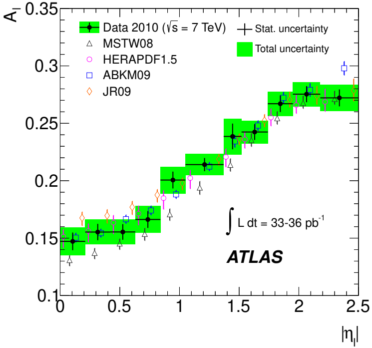
$$\widetilde{\mathcal{M}}_{ab\to F} = \left[1 - \tilde{I}(\epsilon, M^2)\right] \mathcal{M}_{ab\to F}\quad ,$$

hard-virtual subtracted amplitude (IR finite).

$$H_q^F(\alpha_S) = \frac{|\widetilde{\mathcal{M}}_{q\bar{q}\to F}|^2}{|\mathcal{M}_{q\bar{q}\to F}^{(0)}|^2}$$

- Process independent part of $\mathcal{H}^F$ coefficients calculated at NNLO in [Catani,Grazzini ('11)],[Catani,Cieri,de Florian, G.F.,Grazzini ('12)] at N^3LO in [Luo et al. ('19,'20)],[Erbert et al. ('20)]

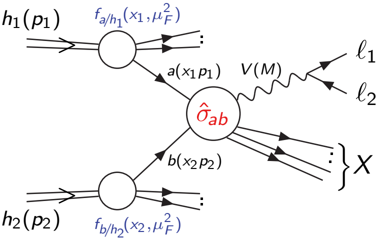
NNLO QCD predictions compared with LHC data



Lepton charge asymmetry from $W \to l\nu_l$ decay. Comparison between experimental data and NNLO predictions (DYNNLO [Catani,Cieri,de Florian, G.F.,Grazzini ('09),('10)]) using various PDFs (from [ATLAS Coll.('12)]).

Transverse-momentum resummation

q_T resummation



$$h_1(p_1) + h_2(p_2) \rightarrow F(M, q_T) + X$$

where $F = \gamma^*, Z^0, W^{\pm}, H, HH, \ldots$

pQCD *collinear* factorization formula ($M \gg \Lambda_{QCD}$):

$$\frac{d\sigma}{dq_T^2}(q_T,M,s) = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)\, \frac{d\hat\sigma_{ab}}{dq_T^2}(q_T,M,\hat s;\alpha_S,\mu_R^2,\mu_F^2).$$

Fixed-order perturbative expansion not reliable for $q_T \ll M$:

$$\int_0^{q_T^2} d\bar q_T^2 \frac{d\hat\sigma}{d\bar q_T^2} \overset{q_T \ll M}{\sim} \sigma_0 + \alpha_S\left[c_{12}\ln^2\frac{M^2}{q_T^2} + c_{11}\ln\frac{M^2}{q_T^2} + c_{10}\right] + \cdots$$

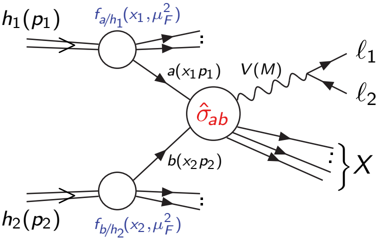
$\alpha_S \ln(M^2/q_T^2) \gg 1$: need for resummation of large logs.

$$\frac{d\sigma}{dq_T^2} = \frac{d\sigma^{(res)}}{dq_T^2} + \frac{d\sigma^{(fin)}}{dq_T^2};$$

$$\int_0^{q_T^2} d\bar q_T^2 \frac{d\hat\sigma^{(fin)}}{d\bar q_T^2} \overset{q_T\to 0}{=} 0$$

$$\int_0^{q_T^2} d\bar q_T^2 \frac{d\hat\sigma^{(res)}}{d\bar q_T^2} \overset{q_T\to 0}{\sim} \sigma_0 + \sum_n \sum_{m=0}^{2n} c_{nm}\,\alpha_S^n \ln^m \frac{M^2}{q_T^2}$$

q_T resummation



$$h_1(p_1) + h_2(p_2) \;\rightarrow\; F(M, q_T) + X$$

where $\quad F = \gamma^*, Z^0, W^\pm, H, HH, \ldots$

pQCD *collinear* factorization formula ($M \gg \Lambda_{QCD}$):

$$\frac{d\sigma}{dq_T^2}(q_T,M,s) = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2\; f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)\, \frac{d\hat\sigma_{ab}}{dq_T^2}(q_T,M,\hat s;\alpha_S,\mu_R^2,\mu_F^2).$$

Fixed-order perturbative expansion not reliable for $q_T \ll M$:

$$\int_0^{q_T^2} d\bar q_T^2\, \frac{d\hat\sigma}{d\bar q_T^2} \overset{q_T \ll M}{\sim} \sigma_0 + \alpha_S \left[c_{12} \ln^2 \frac{M^2}{q_T^2} + c_{11} \ln \frac{M^2}{q_T^2} + c_{10} \right] + \cdots$$

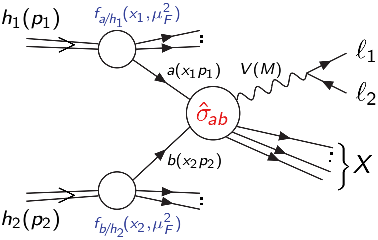
$\alpha_S \ln(M^2/q_T^2) \gg 1$: need for resummation of large logs.

$$\frac{d\sigma}{dq_T^2} = \frac{d\sigma^{(res)}}{dq_T^2} + \frac{d\sigma^{(fin)}}{dq_T^2};$$

$$\int_0^{q_T^2} d\bar q_T^2\, \frac{d\hat\sigma^{(fin)}}{d\bar q_T^2} \overset{q_T\to 0}{=} 0$$

$$\int_0^{q_T^2} d\bar q_T^2\, \frac{d\hat\sigma^{(res)}}{d\bar q_T^2} \overset{q_T\to 0}{\sim} \sigma_0 + \sum_n \sum_{m=0}^{2n} c_{nm}\, \alpha_S^n \ln^m \frac{M^2}{q_T^2}$$

q_T resummation



$$h_1(p_1) + h_2(p_2) \;\rightarrow\; F(M, q_T) + X$$

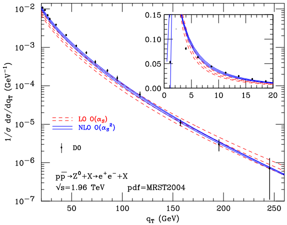
where $\quad F = \gamma^*, Z^0, W^\pm, H, HH, \ldots$

pQCD *collinear* factorization formula ($M \gg \Lambda_{QCD}$):

$$\frac{d\sigma}{dq_T^2}(q_T,M,s) = \sum_{a,b}\int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1,\mu_F^2)\, f_{b/h_2}(x_2,\mu_F^2)\, \frac{d\hat{\sigma}_{ab}}{dq_T^2}(q_T,M,\hat{s};\alpha_S,\mu_R^2,\mu_F^2).$$

Fixed-order perturbative expansion not reliable for $q_T \ll M$:

$$\int_0^{q_T^2} d\bar{q}_T^2\, \frac{d\hat{\sigma}}{d\bar{q}_T^2} \overset{q_T \ll M}{\sim} \sigma_0 + \alpha_S\left[c_{12}\ln^2\frac{M^2}{q_T^2} + c_{11}\ln\frac{M^2}{q_T^2} + c_{10}\right] + \cdots$$



$\alpha_S \ln(M^2/q_T^2) \gg 1$: need for resummation of large logs.

$$\frac{d\sigma}{dq_T^2} = \frac{d\sigma^{(res)}}{dq_T^2} + \frac{d\sigma^{(fin)}}{dq_T^2};$$

$$\int_0^{q_T^2} d\bar{q}_T^2\, \frac{d\hat{\sigma}^{(fin)}}{d\bar{q}_T^2} \overset{q_T\to 0}{=} 0$$

$$\int_0^{q_T^2} d\bar{q}_T^2\, \frac{d\hat{\sigma}^{(res)}}{d\bar{q}_T^2} \overset{q_T\to 0}{\sim} \sigma_0 + \sum_n \sum_{m=0}^{2n} c_{nm}\, \alpha_S^n \ln^m \frac{M^2}{q_T^2}$$

Soft gluon exponentiation

Sudakov resummation feasible when:
dynamics AND kinematics factorize
$\Rightarrow$ exponentiation.

- Dynamics factorization: general propriety of QCD matrix element for soft emissions based on colour coherence. It is the analogous of the independent multiple soft-photon emission is QED:

$$dw_n(q_1,\ldots,q_n) \simeq \frac{1}{n!}\prod_{i=1}^{n} dw_1(q_i)$$

- Kinematics factorization: not valid in general. For q_T distribution it holds in the impact parameter space (Fourier transform)

$$\int d^2\mathrm{q_T}\ \exp(-i\mathrm{b}\cdot\mathrm{q_T})\,\delta\left(\mathrm{q_T}-\sum_{j=1}^{n}\mathrm{q_{T_j}}\right) = \exp(-i\mathrm{b}\cdot\sum_{j=1}^{n}\mathrm{q_{T_j}}) = \prod_{j=1}^{n}\exp(-i\mathrm{b}\cdot\mathrm{q_{T_j}})\,.$$

- Exponentiation holds in the impact parameter space. Results have then to be transformed back to the physical space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$.

q_T resummation in QCD

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(res)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \sum_{n=0}\sum_{m=0}^{2n} c_{nm}\,\alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(fin)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \mathcal{O}\left(\frac{Q_T^2}{M^2}\right)$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathrm{b}}{4\pi} e^{i\mathrm{b}\cdot\mathrm{q_T}}\, \mathcal{W}(b, M),$$

In the Mellin moments space we have:

$$\mathcal{W}_N(b,M) = \mathcal{H}_N(\alpha_S) \times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\} \qquad \text{where} \qquad L \equiv \log(M^2 b^2)$$

$$\mathcal{G}_N(\alpha_S, L) = -\int_{1/b^2}^{M^2} \frac{dq^2}{q^2}\left[A(\alpha_S(q^2)) + \widetilde{B}_N(\alpha_S(q^2))\right] = L\, g^{(1)}(\alpha_S L) + g_N^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g_N^{(3)}(\alpha_S L) + \cdots$$

$$A(\alpha_S) = \frac{\alpha_S}{\pi}A^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 A^{(2)} + \left(\frac{\alpha_S}{\pi}\right)^3 A^{(3)} + \cdots; \quad \widetilde{B}_N(\alpha_S) = \frac{\alpha_S}{\pi}\widetilde{B}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \widetilde{B}_N^{(2)} + \cdots; \quad \mathcal{H}_N(\alpha_S) = \sigma^{(0)}\left[1 + \frac{\alpha_S}{\pi}\mathcal{H}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}_N^{(2)} + \cdots\right]$$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$, $(\sigma^{(0)})$; NLL ($\sim \alpha_S^n L^n$): $g_N^{(2)}$, $\mathcal{H}_N^{(1)}$; NNLL ($\sim \alpha_S^n L^{n-1}$): $g_N^{(3)}$, $\mathcal{H}_N^{(2)}$;

Resummed result at small q_T *matched* with fixed "finite" part at large q_T:
uniform accuracy for $q_T \ll M$ and $q_T \sim M$.

q_T resummation in QCD

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(res)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \sum_{n=0} \sum_{m=0}^{2n} c_{nm}\, \alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(fin)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \mathcal{O}\left(\frac{Q_T^2}{M^2}\right)$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathrm{b}}{4\pi} e^{i\mathrm{b}\cdot\mathrm{q_T}}\, \mathcal{W}(b, M),$$

In the Mellin moments space we have:

$$\mathcal{W}_N(b,M) = \mathcal{H}_N(\alpha_S) \times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\} \qquad \text{where} \qquad L \equiv \log(M^2 b^2)$$

$$\mathcal{G}_N(\alpha_S, L) = -\int_{1/b^2}^{M^2} \frac{dq^2}{q^2}\left[A(\alpha_S(q^2)) + \widetilde{B}_N(\alpha_S(q^2))\right] = L\, g^{(1)}(\alpha_S L) + g_N^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g_N^{(3)}(\alpha_S L) + \cdots$$

$$A(\alpha_S) = \frac{\alpha_S}{\pi}A^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 A^{(2)} + \left(\frac{\alpha_S}{\pi}\right)^3 A^{(3)} + \cdots; \quad \widetilde{B}_N(\alpha_S) = \frac{\alpha_S}{\pi}\widetilde{B}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \widetilde{B}_N^{(2)} + \cdots; \quad \mathcal{H}_N(\alpha_S) = \sigma^{(0)}\left[1 + \frac{\alpha_S}{\pi}\mathcal{H}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}_N^{(2)} + \cdots\right]$$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$, $(\sigma^{(0)})$; NLL ($\sim \alpha_S^n L^n$): $g_N^{(2)}$, $\mathcal{H}_N^{(1)}$; NNLL ($\sim \alpha_S^n L^{n-1}$): $g_N^{(3)}$, $\mathcal{H}_N^{(2)}$;

Resummed result at small q_T *matched* with fixed "finite" part at large q_T:
uniform accuracy for $q_T \ll M$ and $q_T \sim M$.

$\mathbf{q_T}$ resummation in QCD

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(res)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \sum_{n=0}\sum_{m=0}^{2n} c_{nm}\,\alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(fin)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \mathcal{O}(\frac{Q_T^2}{M^2})$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1, \;\; \log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathrm{b}}{4\pi} e^{i\mathrm{b}\cdot\mathrm{q_T}}\, \mathcal{W}(b, M),$$

In the Mellin moments space we have:

$$\mathcal{W}_N(b,M) = \mathcal{H}_N(\alpha_S) \times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\} \qquad \text{where} \qquad L \equiv \log(M^2 b^2)$$

$$\mathcal{G}_N(\alpha_S, L) = -\int_{1/b^2}^{M^2} \frac{dq^2}{q^2}\left[A(\alpha_S(q^2)) + \widetilde{B}_N(\alpha_S(q^2))\right] = L\, g^{(1)}(\alpha_S L) + g_N^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g_N^{(3)}(\alpha_S L) + \cdots$$

$$A(\alpha_S) = \frac{\alpha_S}{\pi} A^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 A^{(2)} + \left(\frac{\alpha_S}{\pi}\right)^3 A^{(3)} + \cdots; \;\; \widetilde{B}_N(\alpha_S) = \frac{\alpha_S}{\pi}\widetilde{B}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \widetilde{B}_N^{(2)} + \cdots; \;\; \mathcal{H}_N(\alpha_S) = \sigma^{(0)}\left[1 + \frac{\alpha_S}{\pi}\mathcal{H}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}_N^{(2)} + \cdots\right]$$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$, ($\sigma^{(0)}$); NLL ($\sim \alpha_S^n L^n$): $g_N^{(2)}$, $\mathcal{H}_N^{(1)}$; NNLL ($\sim \alpha_S^n L^{n-1}$): $g_N^{(3)}$, $\mathcal{H}_N^{(2)}$;

Resummed result at small q_T *matched* with fixed "finite" part at large q_T:
uniform accuracy for $q_T \ll M$ and $q_T \sim M$.

$\mathbf{q_T}$ resummation in QCD

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(res)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \sum_{n=0}\sum_{m=0}^{2n} c_{nm}\, \alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(fin)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \mathcal{O}\left(\frac{Q_T^2}{M^2}\right)$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathrm{b}}{4\pi} e^{i\mathrm{b}\cdot\mathrm{q_T}}\, \mathcal{W}(b, M),$$

In the Mellin moments space we have:

$$\mathcal{W}_N(b,M) = \mathcal{H}_N(\alpha_S) \times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\} \qquad \text{where} \qquad L \equiv \log(M^2 b^2)$$

$$\mathcal{G}_N(\alpha_S, L) = -\int_{1/b^2}^{M^2} \frac{dq^2}{q^2}\left[A(\alpha_S(q^2)) + \widetilde{B}_N(\alpha_S(q^2))\right] = L\, g^{(1)}(\alpha_S L) + g_N^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g_N^{(3)}(\alpha_S L) + \cdots$$

$$A(\alpha_S) = \frac{\alpha_S}{\pi} A^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 A^{(2)} + \left(\frac{\alpha_S}{\pi}\right)^3 A^{(3)} + \cdots; \quad \widetilde{B}_N(\alpha_S) = \frac{\alpha_S}{\pi}\widetilde{B}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \widetilde{B}_N^{(2)} + \cdots; \quad \mathcal{H}_N(\alpha_S) = \sigma^{(0)}\left[1 + \frac{\alpha_S}{\pi}\mathcal{H}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}_N^{(2)} + \cdots\right]$$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$, ($\sigma^{(0)}$); NLL ($\sim \alpha_S^n L^n$): $g_N^{(2)}$, $\mathcal{H}_N^{(1)}$; NNLL ($\sim \alpha_S^n L^{n-1}$): $g_N^{(3)}$, $\mathcal{H}_N^{(2)}$;

Resummed result at small q_T *matched* with fixed "finite" part at large q_T:
uniform accuracy for $q_T \ll M$ and $q_T \sim M$.

q_T resummation in QCD

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(res)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \sum_{n=0}\sum_{m=0}^{2n} c_{nm}\, \alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(fin)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \mathcal{O}\left(\frac{Q_T^2}{M^2}\right)$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathrm{b}}{4\pi} e^{i\mathrm{b}\cdot\mathrm{q_T}}\, \mathcal{W}(b, M),$$

In the Mellin moments space we have:

$$\mathcal{W}_N(b,M) = \mathcal{H}_N(\alpha_S) \times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\} \qquad \text{where} \qquad L \equiv \log(M^2 b^2)$$

$$\mathcal{G}_N(\alpha_S, L) = -\int_{1/b^2}^{M^2} \frac{dq^2}{q^2}\left[A(\alpha_S(q^2)) + \widetilde{B}_N(\alpha_S(q^2))\right] = L\, g^{(1)}(\alpha_S L) + g_N^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g_N^{(3)}(\alpha_S L) + \cdots$$

$$A(\alpha_S) = \frac{\alpha_S}{\pi} A^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 A^{(2)} + \left(\frac{\alpha_S}{\pi}\right)^3 A^{(3)} + \cdots; \quad \widetilde{B}_N(\alpha_S) = \frac{\alpha_S}{\pi} \widetilde{B}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \widetilde{B}_N^{(2)} + \cdots; \quad \mathcal{H}_N(\alpha_S) = \sigma^{(0)}\left[1 + \frac{\alpha_S}{\pi}\mathcal{H}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}_N^{(2)} + \cdots\right]$$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$, ($\sigma^{(0)}$); NLL ($\sim \alpha_S^n L^n$): $g_N^{(2)}$, $\mathcal{H}_N^{(1)}$; NNLL ($\sim \alpha_S^n L^{n-1}$): $g_N^{(3)}$, $\mathcal{H}_N^{(2)}$;

Resummed result at small q_T *matched* with fixed "finite" part at large q_T:
uniform accuracy for $q_T \ll M$ and $q_T \sim M$.

q_T resummation in QCD

$$\frac{d\hat\sigma}{dq_T^2} = \frac{d\hat\sigma^{(res)}}{dq_T^2} + \frac{d\hat\sigma^{(fin)}}{dq_T^2};$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat\sigma^{(res)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \sum_{n=0}\sum_{m=0}^{2n} c_{nm}\,\alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat\sigma^{(fin)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \mathcal{O}(\frac{Q_T^2}{M^2})$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1, \;\; \log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat\sigma^{(res)}}{dq_T^2} = \frac{M^2}{\hat s}\int \frac{d^2\mathrm{b}}{4\pi} e^{i\mathrm{b}\cdot\mathrm{q_T}}\,\mathcal{W}(b,M),$$

In the Mellin moments space we have:

$$\mathcal{W}_N(b,M) = \mathcal{H}_N(\alpha_S)\times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\} \qquad \text{where} \qquad L\equiv \log(M^2b^2)$$

$$\mathcal{G}_N(\alpha_S,L) = -\int_{1/b^2}^{M^2}\frac{dq^2}{q^2}\left[A(\alpha_S(q^2)) + \widetilde{B}_N(\alpha_S(q^2))\right] = L\,g^{(1)}(\alpha_S L) + g_N^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi}g_N^{(3)}(\alpha_S L) + \cdots$$

$$A(\alpha_S) = \frac{\alpha_S}{\pi}A^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 A^{(2)} + \left(\frac{\alpha_S}{\pi}\right)^3 A^{(3)} + \cdots; \;\; \widetilde{B}_N(\alpha_S) = \frac{\alpha_S}{\pi}\widetilde{B}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2\widetilde{B}_N^{(2)} + \cdots; \;\; \mathcal{H}_N(\alpha_S) = \sigma^{(0)}\left[1 + \frac{\alpha_S}{\pi}\mathcal{H}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2\mathcal{H}_N^{(2)} + \cdots\right]$$

LL ($\sim\alpha_S^n L^{n+1}$): $g^{(1)}$, $(\sigma^{(0)})$; NLL ($\sim\alpha_S^n L^n$): $g_N^{(2)}$, $\mathcal{H}_N^{(1)}$; NNLL ($\sim\alpha_S^n L^{n-1}$): $g_N^{(3)}$, $\mathcal{H}_N^{(2)}$;

Resummed result at small q_T *matched* with fixed "finite" part at large q_T:
uniform accuracy for $q_T \ll M$ and $q_T \sim M$.

$\mathbf{q_T}$ resummation in QCD

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(res)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \sum_{n=0}\sum_{m=0}^{2n} c_{nm}\,\alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(fin)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \mathcal{O}\left(\frac{Q_T^2}{M^2}\right)$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1$, $\log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathrm{b}}{4\pi} e^{i\mathrm{b}\cdot\mathrm{q_T}}\, \mathcal{W}(b, M),$$

In the Mellin moments space we have:

$$\mathcal{W}_N(b,M) = \mathcal{H}_N(\alpha_S) \times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\} \qquad \text{where} \qquad L \equiv \log(M^2 b^2)$$

$$\mathcal{G}_N(\alpha_S, L) = -\int_{1/b^2}^{M^2} \frac{dq^2}{q^2}\left[A(\alpha_S(q^2)) + \widetilde{B}_N(\alpha_S(q^2))\right] = L\, g^{(1)}(\alpha_S L) + g_N^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g_N^{(3)}(\alpha_S L) + \cdots$$

$$A(\alpha_S) = \frac{\alpha_S}{\pi} A^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 A^{(2)} + \left(\frac{\alpha_S}{\pi}\right)^3 A^{(3)} + \cdots; \quad \widetilde{B}_N(\alpha_S) = \frac{\alpha_S}{\pi}\widetilde{B}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \widetilde{B}_N^{(2)} + \cdots; \quad \mathcal{H}_N(\alpha_S) = \sigma^{(0)}\left[1 + \frac{\alpha_S}{\pi}\mathcal{H}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}_N^{(2)} + \cdots\right]$$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$, $(\sigma^{(0)})$; NLL ($\sim \alpha_S^n L^n$): $g_N^{(2)}$, $\mathcal{H}_N^{(1)}$; NNLL ($\sim \alpha_S^n L^{n-1}$): $g_N^{(3)}$, $\mathcal{H}_N^{(2)}$;

Resummed result at small q_T *matched* with fixed "finite" part at large q_T:
uniform accuracy for $q_T \ll M$ and $q_T \sim M$.

$\mathbf{q_T}$ resummation in QCD

$$\frac{d\hat{\sigma}}{dq_T^2} = \frac{d\hat{\sigma}^{(res)}}{dq_T^2} + \frac{d\hat{\sigma}^{(fin)}}{dq_T^2};$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(res)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \sum_{n=0}\sum_{m=0}^{2n} c_{nm}\,\alpha_S^n \log^m \frac{M^2}{Q_T^2}$$

$$\int_0^{Q_T^2} dq_T^2 \left[\frac{d\hat{\sigma}^{(fin)}}{dq_T^2}\right]_{f.o.} \overset{Q_T\to 0}{\sim} \mathcal{O}\left(\frac{Q_T^2}{M^2}\right)$$

Resummation holds in impact parameter space: $q_T \ll M \Leftrightarrow Mb \gg 1, \;\; \log M/q_T \gg 1 \Leftrightarrow \log Mb \gg 1$

$$\frac{d\hat{\sigma}^{(res)}}{dq_T^2} = \frac{M^2}{\hat{s}} \int \frac{d^2\mathrm{b}}{4\pi} e^{i\mathrm{b}\cdot\mathrm{q_T}}\, \mathcal{W}(b,M),$$

In the Mellin moments space we have:

$$\mathcal{W}_N(b,M) = \mathcal{H}_N(\alpha_S) \times \exp\left\{\mathcal{G}_N(\alpha_S, L)\right\} \qquad \text{where} \qquad L \equiv \log(M^2 b^2)$$

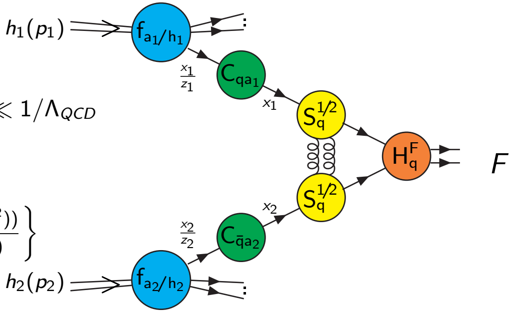
$$\mathcal{G}_N(\alpha_S, L) = -\int_{1/b^2}^{M^2} \frac{dq^2}{q^2}\left[A(\alpha_S(q^2)) + \widetilde{B}_N(\alpha_S(q^2))\right] = L\, g^{(1)}(\alpha_S L) + g_N^{(2)}(\alpha_S L) + \frac{\alpha_S}{\pi} g_N^{(3)}(\alpha_S L) + \cdots$$

$$A(\alpha_S) = \frac{\alpha_S}{\pi} A^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 A^{(2)} + \left(\frac{\alpha_S}{\pi}\right)^3 A^{(3)} + \cdots; \;\; \widetilde{B}_N(\alpha_S) = \frac{\alpha_S}{\pi}\widetilde{B}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \widetilde{B}_N^{(2)} + \cdots; \;\; \mathcal{H}_N(\alpha_S) = \sigma^{(0)}\left[1 + \frac{\alpha_S}{\pi}\mathcal{H}_N^{(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}_N^{(2)} + \cdots\right]$$

LL ($\sim \alpha_S^n L^{n+1}$): $g^{(1)}$, ($\sigma^{(0)}$); NLL ($\sim \alpha_S^n L^n$): $g_N^{(2)}$, $\mathcal{H}_N^{(1)}$; NNLL ($\sim \alpha_S^n L^{n-1}$): $g_N^{(3)}$, $\mathcal{H}_N^{(2)}$;

Resummed result at small q_T *matched* with fixed "finite" part at large q_T:
uniform accuracy for $q_T \ll M$ and $q_T \sim M$.

Transverse-momentum resummation formula



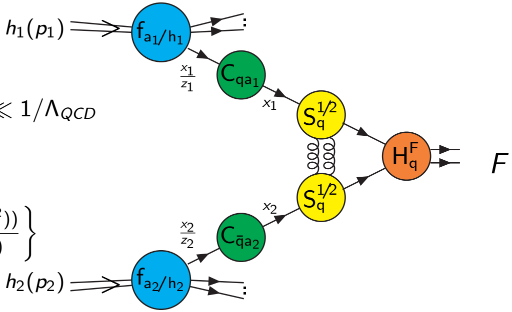
$$M \gg \Lambda_{QCD}\ , \quad b \gg 1/M\ , \quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2))$$
$$\times\ \exp\left\{ -\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\ \beta(\alpha_S(q^2))\ \frac{d\ln C(\alpha_S(q^2))}{d\ln\alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{d^2\mathbf{q_T}\ dM^2\ dy\ d\Omega} = \frac{M^2}{s}\left[d\sigma^{(0)}_{q\bar{q},F}\right] H_q^F(x_1p_1, x_2p_2; \Omega; \alpha_S(M^2)) \sum_{a_1,a_2}\int \frac{d^2\mathrm{b}}{(2\pi)^2}\ e^{i\mathrm{b}\cdot\mathbf{q_T}}\ S_q(M,b)$$
$$\times \int_{x_1}^1 \frac{dz_1}{z_1}\ C_{qa_1}(z_1;\alpha_S(b_0^2/b^2))\ f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^1 \frac{dz_2}{z_2}\ C_{\bar{q}\,a_2}(z_2;\alpha_S(b_0^2/b^2))\ f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$\widetilde{F}_{q_f/h}(x,b,M) = \sum_a \int_x^1 \frac{dz}{z}\ \sqrt{S_q(M,b)}\,C_{q_f\,a}(z;\alpha_S(b_0^2/b^2))\ f_{a/h}(x/z, b_0^2/b^2)$$

Transverse-momentum resummation formula



$$M \gg \Lambda_{QCD} \;, \quad b \gg 1/M \;, \quad b \ll 1/\Lambda_{QCD}$$

$$C(\alpha_S(b_0^2/b^2)) = C(\alpha_S(M^2))$$
$$\times \;\exp\left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \; \beta(\alpha_S(q^2)) \; \frac{d \ln C(\alpha_S(q^2))}{d \ln \alpha_S(q^2)} \right\}$$

$$\frac{d\sigma_F^{(res)}}{d^2\mathbf{q_T}\, dM^2\, dy\, d\Omega} = \frac{M^2}{s} \left[d\sigma^{(0)}_{q\bar{q},F} \right] H_q^F(x_1 p_1, x_2 p_2; \Omega; \alpha_S(M^2)) \sum_{a_1, a_2} \int \frac{d^2\mathrm{b}}{(2\pi)^2} \; e^{i\mathrm{b}\cdot\mathbf{q_T}} \; S_q(M, b)$$
$$\times \int_{x_1}^1 \frac{dz_1}{z_1} \; C_{qa_1}(z_1; \alpha_S(b_0^2/b^2)) \; f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) \int_{x_2}^1 \frac{dz_2}{z_2} \; C_{\bar{q}\,a_2}(z_2; \alpha_S(b_0^2/b^2)) \; f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$\widetilde{F}_{q_f/h}(x, b, M) = \sum_a \int_x^1 \frac{dz}{z} \; \sqrt{S_q(M, b)}\, C_{q_f\,a}(z; \alpha_S(b_0^2/b^2)) \; f_{a/h}(x/z, b_0^2/b^2)$$

$\mathbf{q_T}$ recoil and lepton angular distribution

- The dependence of the resummed cross section on the leptonic variable Ω is

$$\frac{d\hat{\sigma}^{(0)}}{d\Omega} = \hat{\sigma}^{(0)}(M^2)\, F(\mathbf{q_T}/M; M^2, \Omega) \;\; , \quad \text{with} \quad \int d\Omega\, F(\mathbf{q_T}/M; \Omega) = 1 \, .$$

  the q_T dependence arise as a *dynamical* q_T-recoil of the vector boson due to *soft* and *collinear* multiparton emissions.

- This dependence cannot be *unambiguously* calculated through resummation (it is not singular)

$$F(\mathbf{q_T}/M; M^2, \Omega) = F(0/M; M^2, \Omega) + \mathcal{O}(\mathbf{q_T^2}/M^2) \;\; ,$$

  - After the matching between *resummed* and *finite* component the $\mathcal{O}(\mathbf{q_T^2}/M^2)$ ambiguity starts at $\mathcal{O}(\alpha_S^3)$ ($\mathcal{O}(\alpha_S^2)$) at NNLL+NNLO (NLL+NLO).
  - After integration over leptonic variable Ω the ambiguity *completely cancel*.

- A general procedure to treat the q_T recoil in q_T resummed calculations introduced in [Catani,de Florian,G.F.,Grazzini('15)].

- This procedure is directly related to the choice of a particular (among the infinite ones) vector boson rest frame to generate the lepton momenta: e.g. the Collins–Soper rest frame.

$\mathbf{q_T}$ recoil and lepton angular distribution

- The dependence of the resummed cross section on the leptonic variable Ω is

$$\frac{d\hat{\sigma}^{(0)}}{d\Omega} = \hat{\sigma}^{(0)}(M^2)\, F(\mathbf{q_T}/M; M^2, \Omega) \;\; , \;\; \text{with} \;\; \int d\Omega\, F(\mathbf{q_T}/M; \Omega) = 1 \; .$$

  the q_T dependence arise as a *dynamical* q_T-recoil of the vector boson due to *soft* and *collinear* multiparton emissions.

- This dependence cannot be *unambiguously* calculated through resummation (it is not singular)

$$F(\mathbf{q_T}/M; M^2, \Omega) = F(0/M; M^2, \Omega) + \mathcal{O}(\mathbf{q_T^2}/M^2) \;\; ,$$

  - After the matching between *resummed* and *finite* component the $\mathcal{O}(\mathbf{q_T^2}/M^2)$ ambiguity starts at $\mathcal{O}(\alpha_S^3)$ $(\mathcal{O}(\alpha_S^2))$ at NNLL+NNLO (NLL+NLO).
  - After integration over leptonic variable Ω the ambiguity *completely cancel*.

- A general procedure to treat the q_T recoil in q_T resummed calculations introduced in [Catani,de Florian,G.F.,Grazzini('15)].

- This procedure is directly related to the choice of a particular (among the infinite ones) vector boson rest frame to generate the lepton momenta: e.g. the Collins–Soper rest frame.

$\mathbf{q_T}$ recoil and lepton angular distribution

- The dependence of the resummed cross section on the leptonic variable Ω is

$$\frac{d\hat{\sigma}^{(0)}}{d\Omega} = \hat{\sigma}^{(0)}(M^2)\, F(\mathbf{q_T}/M; M^2, \Omega) \;\; , \quad \text{with} \quad \int d\Omega\, F(\mathbf{q_T}/M; \Omega) = 1 \, .$$

  the q_T dependence arise as a *dynamical* q_T-recoil of the vector boson due to *soft* and *collinear* multiparton emissions.

- This dependence cannot be *unambiguously* calculated through resummation (it is not singular)

$$F(\mathbf{q_T}/M; M^2, \Omega) = F(0/M; M^2, \Omega) + \mathcal{O}(\mathbf{q_T^2}/M^2) \;\; ,$$

  - After the matching between *resummed* and *finite* component the $\mathcal{O}(\mathbf{q_T^2}/M^2)$ ambiguity starts at $\mathcal{O}(\alpha_S^3)$ $(\mathcal{O}(\alpha_S^2))$ at NNLL+NNLO (NLL+NLO).
  - After integration over leptonic variable Ω the ambiguity *completely cancel*.

- A general procedure to treat the q_T recoil in q_T resummed calculations introduced in [Catani,de Florian,G.F.,Grazzini('15)].

- This procedure is directly related to the choice of a particular (among the infinite ones) vector boson rest frame to generate the lepton momenta: e.g. the Collins–Soper rest frame.

$\mathbf{q_T}$ recoil and lepton angular distribution

- The dependence of the resummed cross section on the leptonic variable Ω is

$$\frac{d\hat{\sigma}^{(0)}}{d\Omega} = \hat{\sigma}^{(0)}(M^2)\, F(\mathbf{q_T}/M; M^2, \Omega) \;\;, \quad \text{with} \quad \int d\Omega\, F(\mathbf{q_T}/M; \Omega) = 1 \,.$$

  the q_T dependence arise as a *dynamical* q_T-recoil of the vector boson due to *soft* and *collinear* multiparton emissions.

- This dependence cannot be *unambiguously* calculated through resummation (it is not singular)

$$F(\mathbf{q_T}/M; M^2, \Omega) = F(0/M; M^2, \Omega) + \mathcal{O}(\mathbf{q_T^2}/M^2) \;\;,$$

  - After the matching between *resummed* and *finite* component the $\mathcal{O}(\mathbf{q_T^2}/M^2)$ ambiguity starts at $\mathcal{O}(\alpha_S^3)$ $\left(\mathcal{O}(\alpha_S^2)\right)$ at NNLL+NNLO (NLL+NLO).
  - After integration over leptonic variable Ω the ambiguity *completely cancel*.

- A general procedure to treat the q_T recoil in q_T resummed calculations introduced in [Catani,de Florian,G.F.,Grazzini('15)].

- This procedure is directly related to the choice of a particular (among the infinite ones) vector boson rest frame to generate the lepton momenta: e.g. the Collins–Soper rest frame.

q_T resummation: numerical implementations

- q_T resummation performed for Drell–Yan process up to NNLL+NNLO by using the formalism developed in [Catani,de Florian,Grazzini('01)], [Bozzi,Catani,de Florian,Grazzini('06,'08)]. We have included
  - NNLL logarithmic contributions to all orders (i.e. up to $\exp(\sim \alpha_S^n L^{n-1})$);
  - NNLO corrections (i.e. up to $\mathcal{O}(\alpha_S^2)$) at small q_T;
  - NLO corrections (i.e. up to $\mathcal{O}(\alpha_S^2)$) at large q_T;
  - NNLO result (i.e. up to $\mathcal{O}(\alpha_S^2)$) for the total cross section.
- We have implemented the calculation in the publicly available codes:

  DYqT: computes resummed q_T spectrum, inclusive over other kinematical variables [Bozzi,Catani,de Florian,G.F.,Grazzini('09,'11)]

  DYRes: computes resummed q_T spectrum and related distributions, it retains full kinematics of the vector boson and of its leptonic decay products (possible to apply arbitrary cuts on these variables, and to plot the corresponding distributions) [Catani,de Florian,G.F.,Grazzini('15)]

  http://pcteserver.mi.infn.it/~ferrera/research.html.

q_T resummation: numerical implementations

- q_T resummation performed for Drell–Yan process up to NNLL+NNLO by using the formalism developed in [Catani,de Florian,Grazzini('01)], [Bozzi,Catani,de Florian,Grazzini('06,'08)]. We have included
  - NNLL logarithmic contributions to all orders (i.e. up to $\exp(\sim\alpha_S^n L^{n-1})$);
  - NNLO corrections (i.e. up to $\mathcal{O}(\alpha_S^2)$) at small q_T;
  - NLO corrections (i.e. up to $\mathcal{O}(\alpha_S^2)$) at large q_T;
  - NNLO result (i.e. up to $\mathcal{O}(\alpha_S^2)$) for the total cross section.
- We have implemented the calculation in the publicly available codes:

  DYqT: computes resummed q_T spectrum, inclusive over other kinematical variables [Bozzi,Catani,de Florian,G.F.,Grazzini('09,'11)]

  DYRes: computes resummed q_T spectrum and related distributions, it retains full kinematics of the vector boson and of its leptonic decay products (possible to apply arbitrary cuts on these variables, and to plot the corresponding distributions) [Catani,de Florian,G.F.,Grazzini('15)]

  http://pcteserver.mi.infn.it/~ferrera/research.html.

q_T resummation: numerical implementations

- q_T resummation performed for Drell–Yan process up to NNLL+NNLO by using the formalism developed in [Catani,de Florian,Grazzini('01)], [Bozzi,Catani,de Florian,Grazzini('06,'08)]. We have included
  - NNLL logarithmic contributions to all orders (i.e. up to $\exp(\sim\alpha_S^n L^{n-1})$);
  - NNLO corrections (i.e. up to $\mathcal{O}(\alpha_S^2)$) at small q_T;
  - NLO corrections (i.e. up to $\mathcal{O}(\alpha_S^2)$) at large q_T;
  - NNLO result (i.e. up to $\mathcal{O}(\alpha_S^2)$) for the total cross section.
- We have implemented the calculation in the publicly available codes:

  DYqT: computes resummed q_T spectrum, inclusive over other kinematical variables [Bozzi,Catani,de Florian,G.F.,Grazzini('09,'11)]

  DYRes: computes resummed q_T spectrum and related distributions, it retains full kinematics of the vector boson and of its leptonic decay products (possible to apply arbitrary cuts on these variables, and to plot the corresponding distributions) [Catani,de Florian,G.F.,Grazzini('15)]

  http://pcteserver.mi.infn.it/~ferrera/research.html.

Fast predictions for Drell-Yan processes: DYTurbo

[Camarda, Boonekamp, Bozzi, Catani, Cieri, Cuth, G.F., de Florian, Glazov, Grazzini, Vincter, Schott ('20)]

DYTurbo project

- Optimised version of DYNNLO, DYqT, DYRes with improvements in

Software	Numerical integration
Code profiling	Quadrature with interpolating functions
Loop vectorisation	Factorisation of integrals
Hoisting	Analytic integration
Loop unrolling	
Multi-threading	

- Achieved significant enhancement in time performance for a given numerical precision
- The main application is the measurement of the W mass at the LHC
- Other applications: PDF fits including qt-resummation for cross-section predictions, $\sin^2\theta_W$, $\alpha_s(m_Z)$
- Two main modes of operation: Vegas integration and Quadrature rules based on interpolating functions

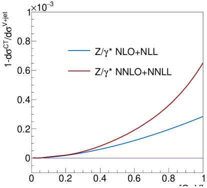
Stefano Camarda 5

Fast predictions for Drell-Yan processes: DYTurbo

[Camarda, Boonekamp, Bozzi, Catani, Cieri, Cuth, G.F., de Florian, Glazov, Grazzini, Vincter, Schott ('20)]

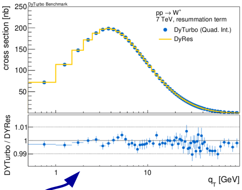
Closure tests and benchmark

- Matching conditions implies relation between the terms which can be used to test their numerical precision



$$\lim_{q_T \to 0} 1 - d\sigma^{\rm CT(res)}/d\sigma^{\rm V+jet} = 0$$

→ tested at 10^{-5}



- DYTurbo predictions fare benchmarked with DYRes at NNLL, and with other programs at NNLO

	SHERPA	DYNNLO	FEWZ	DYTurbo (Quad.)
$\sigma(pp \to W^+ \to l^+\nu)$ [pb]	3204 ± 4	3191 ± 7	3207 ± 2	3196 ± 7
$\sigma(pp \to W^- \to l^-\nu)$ [pb]	2252 ± 3	2243 ± 6	2238 ± 1	2248 ± 4
$\sigma(pp \to Z/\gamma \to l^+l^-)$ [pb]	502.0 ± 0.6	502.4 ± 0.4	504.6 ± 0.1	502.8 ± 1.0

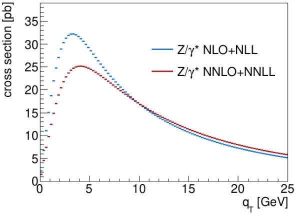
Small differences between FEWZ and the other predictions are expected due to phase space with p_T^l symmetric cuts, and different subtraction scheme

Stefano Camarda 9

Fast predictions for Drell-Yan processes: DYTurbo

[Camarda, Boonekamp, Bozzi, Catani, Cieri, Cuth, G.F., de Florian, Glazov, Grazzini, Vincter, Schott ('20)]

Example calculation



- Example calculation for Z p_T spectrum at 13 TeV
  - No cuts on the leptons
  - Full rapidity range
  - 100 p_T bins
  - 20 parallel threads

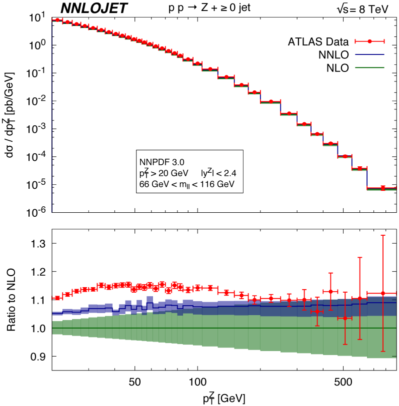
Time required	RES	CT	V+jet
NLO+NLL	6 s	0.2 s	4 min
NNLO+NNLL	10 s	0.7 s	3.4 h

- The most demanding calculation is V+jet

  → can use APPLgrid/FASTnlo for this term

Stefano Camarda 11

NNLO QCD predictions at large q_T



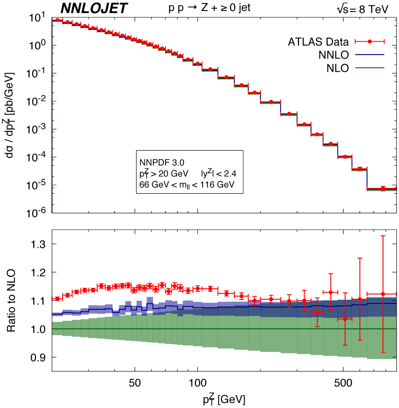
- ATLAS data ($\sqrt{s} = 8\ TeV$) [1512.02192] (2.8% luminosity uncertainty not shown).
- NNLO (i.e.$\mathcal{O}(\alpha_S^3)$) QCD predictions [G.-De Ridder, Gehrmann, Glover, Huss, Morgan('16)]. NNLO correction positive (∼6-8%) and reduce scale dependence (factor 2 around $\mu = \sqrt{M^2 + q_T^2}$).

$Z\ q_T$ spectrum ($q_T > 20\ GeV$).

In the small q_T region effects of soft-gluon resummation are essential

At the LHC 90% of the $W^\pm$ and Z^0 are produced with $q_T \lesssim 20\ GeV$

NNLO QCD predictions at large q_T



- ATLAS data ($\sqrt{s} = 8\ TeV$) [1512.02192] (2.8% luminosity uncertainty not shown).
- NNLO (i.e.$\mathcal{O}(\alpha_S^3)$) QCD predictions [G.-De Ridder, Gehrmann, Glover, Huss, Morgan('16)]. NNLO correction positive ($\sim$6-8%) and reduce scale dependence (factor 2 around $\mu = \sqrt{M^2 + q_T^2}$).

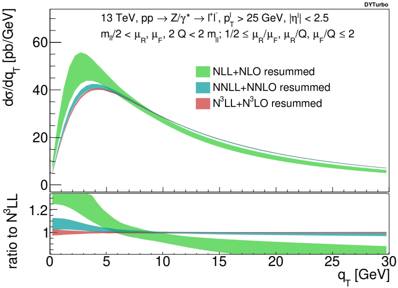
Z q_T spectrum ($q_T > 20\ GeV$).

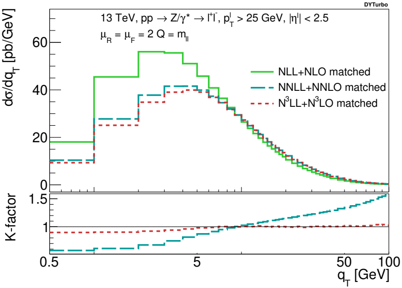
In the small q_T region effects of soft-gluon resummation are essential

At the LHC 90% of the $W^{\pm}$ and Z^0 are produced with $q_T \lesssim 20\,GeV$

Z/γ^* production at N^{3}LL+N^{3}LO

[Camarda,Cieri,G.F.('21)]





DYTurbo results. Resummed (left) and matched (right) NLL, NNLL and N^{3}LL bands for Z/γ^* q_T spectrum.

Lower panel: ratio with respect to the N^{3}LL central value.

Fiducial power corrections

Fiducial power corrections within the q_T subtraction

[Camarda,Cieri,G.F.('21)]

$$\sigma^F_{fid} = \int_{cuts} \mathcal{H}^F \otimes d\sigma^F_{LO} + \int_{cuts} \left[d\sigma^{F+\text{jets}}_{q_T > q_T^{cut}} - d\sigma^{CT}_{q_T > q_T^{cut}} \right] + \mathcal{O}\left((q_T^{cut}/M)^p \right)$$

- $d\sigma^{F+\text{jets}}$ and $d\sigma^{CT}$ are *separately* divergent, their sum is finite. A lower limit $q_T > q_T^{cut}$ is necessary with a power correction ambiguity $\mathcal{O}\left((q_T^{cut}/M)^p\right)$.
- Typical cuts on the p_T and rapidities of the final state particles leads to linear ($p = 1$) "fiducial" power corrections (FPC) $\left((q_T^{cut}/M)^p\right)$ [Alekhin et al.('21)].
- The limit $q_T^{cut} \to 0$ leads to large cancellations and large numerical integration uncertainties.
- **Key point**: FPC are absent in resummed calculations when q_T recoil is correctly taken into account.

$$\int_{cuts} d\sigma^{F+\text{jets}} \xrightarrow{q_T \to 0} \int_{cuts} d\widetilde{\sigma}^{CT} + \mathcal{O}\left((q_T/M)^2 \right)$$

where $d\widetilde{\sigma}^{CT}$ is the q_T subtraction counteterm with the Born amplitude $\hat{\sigma}^{(0)}$ evaluated with the recoil kinematics [Catani,de Florian,G.F.,Grazzini('15)].

Fiducial power corrections within the q_T subtraction

[Camarda,Cieri,G.F.('21)]

$$\sigma^F_{fid} = \int_{cuts} \mathcal{H}^F \otimes d\sigma^F_{LO} + \int_{cuts} \left[d\sigma^{F+\text{jets}}_{q_T > q_T^{cut}} - d\sigma^{CT}_{q_T > q_T^{cut}} \right] + \mathcal{O}\left((q_T^{cut}/M)^p \right)$$

- $d\sigma^{F+\text{jets}}$ and $d\sigma^{CT}$ are *separately* divergent, their sum is finite. A lower limit $q_T > q_T^{cut}$ is necessary with a power correction ambiguity $\mathcal{O}\left((q_T^{cut}/M)^p \right)$.
- Typical cuts on the p_T and rapidities of the final state particles leads to linear $(p = 1)$ "fiducial" power corrections (FPC) $\left((q_T^{cut}/M)^p \right)$ [Alekhin et al.('21)].
- The limit $q_T^{cut} \to 0$ leads to large cancellations and large numerical integration uncertainties.
- **Key point**: FPC are absent in resummed calculations when q_T recoil is correctly taken into account.

$$\int_{cuts} d\sigma^{F+\text{jets}} \xrightarrow{q_T \to 0} \int_{cuts} d\widetilde{\sigma}^{CT} + \mathcal{O}\left((q_T/M)^2 \right)$$

where $d\widetilde{\sigma}^{CT}$ is the q_T subtraction counteterm with the Born amplitude $\hat{\sigma}^{(0)}$ evaluated with the recoil kinematics [Catani,de Florian,G.F.,Grazzini('15)].

Fiducial power corrections within the q_T subtraction

[Camarda,Cieri,G.F.('21)]

$$\sigma^F_{fid} = \int_{cuts} \mathcal{H}^F \otimes d\sigma^F_{LO} + \int_{cuts} \left[d\sigma^{F+\text{jets}}_{q_T > q_T^{cut}} - d\sigma^{CT}_{q_T > q_T^{cut}} \right] + \mathcal{O}\left((q_T^{cut}/M)^p \right)$$

- $d\sigma^{F+\text{jets}}$ and $d\sigma^{CT}$ are *separately* divergent, their sum is finite. A lower limit $q_T > q_T^{cut}$ is necessary with a power correction ambiguity $\mathcal{O}\left((q_T^{cut}/M)^p \right)$.
- Typical cuts on the p_T and rapidities of the final state particles leads to linear $(p = 1)$ "fiducial" power corrections (FPC) $\left((q_T^{cut}/M)^p \right)$ [Alekhin et al.('21)].
- The limit $q_T^{cut} \to 0$ leads to large cancellations and large numerical integration uncertainties.
- **Key point**: FPC are absent in resummed calculations when q_T recoil is correctly taken into account.

$$\int_{cuts} d\sigma^{F+\text{jets}} \xrightarrow{q_T \to 0} \int_{cuts} d\widetilde{\sigma}^{CT} + \mathcal{O}\left((q_T/M)^2 \right)$$

where $d\widetilde{\sigma}^{CT}$ is the q_T subtraction counteterm with the Born amplitude $\hat{\sigma}^{(0)}$ evaluated with the recoil kinematics [Catani,de Florian,G.F.,Grazzini('15)].

Fiducial power corrections within the q_T subtraction

[Camarda,Cieri,G.F.('21)]

q_T subtraction formula without FPC

$$\sigma^F_{fid} = \int_{cuts} \mathcal{H}^F \otimes d\sigma^F_{LO} + \int_{cuts} \left[d\sigma^{F+\text{jets}}_{q_T > q_T^{cut}} - d\sigma^{CT}_{q_T > q_T^{cut}} \right] + \int_{cuts} d\sigma^{FPC} + \mathcal{O}\left((q_T^{cut}/M)^2\right)$$

with

$$d\sigma^{FPC} = \left[d\widetilde{\sigma}^{CT}_{q_T < q_T^{cut}} - d\sigma^{CT}_{q_T < q_T^{cut}} \right]$$

- $d\sigma^{FPC}$ is *universal* and IR finite.
- can be treated as a *local* subtraction: integration for $q_T < q_T^{cut}$ extended at arbitrary small q_T (e.g. $q_T/M \sim 10^{-6}$ GeV)
- Equivalent methods have been proposed by [Ebert et al.('20), Buonocore et al.('21)]

Fiducial power corrections within the q_T subtraction

[Camarda,Cieri,G.F.('21)]

q_T subtraction formula without FPC

$$\sigma^F_{fid} = \int_{cuts} \mathcal{H}^F \otimes d\sigma^F_{LO} + \int_{cuts} \left[d\sigma^{F+\text{jets}}_{q_T > q_T^{cut}} - d\sigma^{CT}_{q_T > q_T^{cut}} \right] + \int_{cuts} d\sigma^{FPC} + \mathcal{O}\left((q_T^{cut}/M)^2 \right)$$

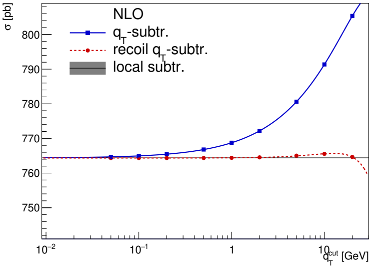
with

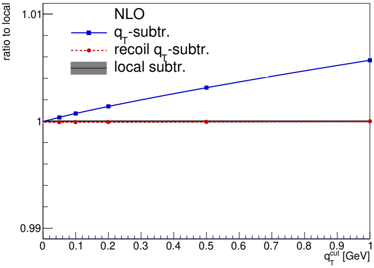
$$d\sigma^{FPC} = \left[d\widetilde{\sigma}^{CT}_{q_T < q_T^{cut}} - d\sigma^{CT}_{q_T < q_T^{cut}} \right]$$

- $d\sigma^{FPC}$ is *universal* and IR finite.
- can be treated as a *local* subtraction: integration for $q_T < q_T^{cut}$ extended at arbitrary small q_T (e.g. $q_T/M \sim 10^{-6}$ GeV)
- Equivalent methods have been proposed by [Ebert et al.('20), Buonocore et al.('21)]

Fiducial power corrections at NLO

Z/γ^* production and decay at the LHC (13 TeV).
CUTS on leptons: $p_T > 25$ GeV, $|\eta| < 2.5$, $66 < M_{ll} < 116$ GeV,
$q_T < 100$ GeV.

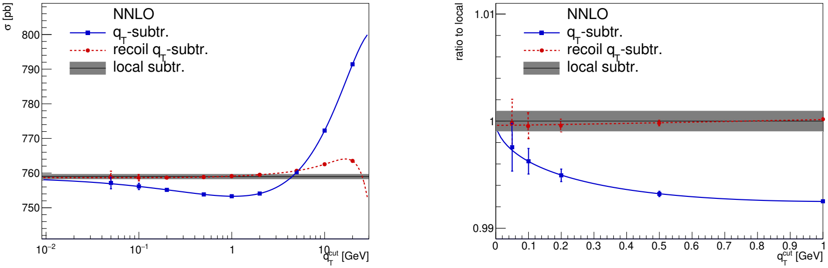




NLO results with the q_T subtraction method (blue squared points) and q_T subtraction method without FPC (red circled points) at various values of q_T^{cut}, and with a local subtraction method (black line).

Fiducial power corrections at NNLO

Z/γ^* production and decay at the LHC (13 TeV).
CUTS on leptons: $p_T > 25$ GeV, $|\eta| < 2.5$, $66 < M_{ll} < 116$ GeV,
$q_T < 100\,$GeV.



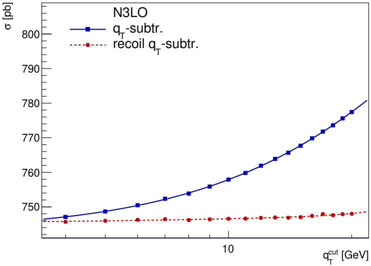
NNLO results with the q_T subtraction method (blue squared points) and q_T subtraction method without FPC (red circled points) at various values of q_T^{cut}, and with a local subtraction method (black line).

Fiducial power corrections at NLO and NNLO

Order	NLO	NNLO
q_T subtr. ($q_T^{cut} = 1\,\text{GeV}$)	$768.8 \pm 0.1\,\text{pb}$	$753.3 \pm 0.3\,\text{pb}$
q_T subtr. ($q_T^{cut} = 0.5\,\text{GeV}$)	$766.8 \pm 0.1\,\text{pb}$	$753.8 \pm 0.2\,\text{pb}$
recoil q_T subtr.	$764.4 \pm 0.1\,\text{pb}$	$759.1 \pm 0.3\,\text{pb}$
local subtraction	$764.4 \pm 0.1\,\text{pb}$	$759.0 \pm 0.7\,\text{pb}$

Fiducial cross sections at the LHC ($\sqrt{s} = 13$ TeV): fixed-order results at NLO and NNLO. The uncertainties refer to an estimate of the numerical uncertainties.

Fiducial power corrections at N^{3}LO



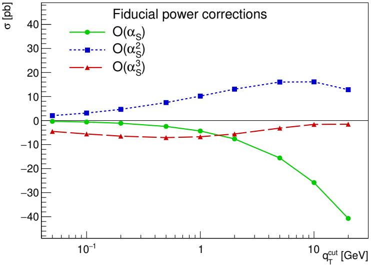
Z/γ^* production and decay at the LHC (13 TeV).

CUTS on leptons: $p_T > 25$ GeV, $|\eta| < 2.5$, $66 < M_{ll} < 116$ GeV, $q_T < 100\,\text{GeV}$.

Order	N^{3}LO
q_T subtr. ($q_T^{\rm cut} = 4\,\text{GeV}$)	$747.1 \pm 0.7\,\text{pb}$
recoil q_T subtr.	$745.7 \pm 0.7\,\text{pb}$

Fiducial cross sections at the LHC ($\sqrt{s} = 13$ TeV): fixed-order results at N^{3}LO. The uncertainties refer to an estimate of the numerical uncertainties.

Fiducial power corrections up to N^{3}LO



- Flip sign of the FPC with order. Alternating-sign "unphysical" factorial growth of the FO expansion due to symmetric cuts [Salam,Slade('21)].
- Unphysical behaviour can be removed within resummed perturbative predictions. *However* the goal of having precise FO calculations is very relevant.
- No reduction of FPC with higher orders. At N^{3}LO with $q_T^{cut} = 0.05$ GeV -0.4% (+0.3% from α_S^2 and a -0.7% α_S^3.
- Our method is crucial when *local* calculations are not available or when large numerical uncertainties are associated to the $q_T \to 0$ limit (e.g. at N^3LO).

Conclusions

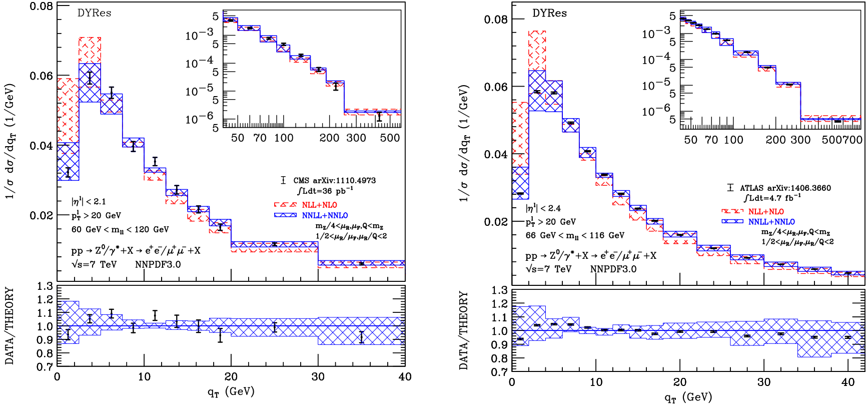
- To fully exploit the information contained in the experimental data, and **to increase the LHC discovery power**, precise theoretical predictions are necessary $\Rightarrow$ computation of higher-order pQCD corrections.
- Discussed formalisms necessary to perform fixed-order and q_T resummed predictions up to N^{3}LL+N^{3}LO and presented results for Drell–Yan production at the LHC.
- Presented a method to remove linear fiducial power corrections within the q_T-subtraction formalism.
- Computations encoded in the fast and numerically precise publicly available code `DYTurbo`: https://dyturbo.hepforge.org

Conclusions

- To fully exploit the information contained in the experimental data, and **to increase the LHC discovery power**, precise theoretical predictions are necessary ⇒ computation of higher-order pQCD corrections.
- Discussed formalisms necessary to perform fixed-order and q_T resummed predictions up to N^{3}LL+N^{3}LO and presented results for Drell–Yan production at the LHC.
- Presented a method to remove linear fiducial power corrections within the q_T-subtraction formalism.
- Computations encoded in the fast and numerically precise publicly available code DYTurbo: https://dyturbo.hepforge.org

Back up slides

DYRes **results:** q_T spectrum of Z boson at the LHC



NLL+NLO and NNLL+NNLO bands for Z/γ^* q_T spectrum compared with CMS (left) and ATLAS (right) data.
Lower panel: ratio with respect to the NNLL+NNLO central value.

Program performances: for high statistic runs (i.e. few per mille accuracy on cross sections) *on a single CPU*: ~ 1 day at full NLL, ~ 3 days at full NNLL.