Multiple soft radiation at one-loop order and the emission of a soft quark-antiquark pair

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Outline

- Motivation
- Introduction
- General features of multiple soft QCD radiation at one-loop level
- Explicit form of IR and UV divergent (ε-pole) terms of one-loop soft current
- Explicit form of qqbar soft current by including finite terms ($\mathcal{O}(\varepsilon^0)$)
- Soft-qqbar radiation at the squared amplitude level
- Outlook

Motivation

- The soft and collinear singularities have a process-independent structure, and they are controlled by universal factorization formulae and corresponding soft/collinear factors

These factorization properties are relevant for both fixed-order and resummed QCD calculations

- Soft/collinear factorization formulae can be used to organize and greatly simplify the cancellation mechanism of the infrared (IR) divergences in fixed-order calculations between phase space soft/collinear singularities and virtual IR divergences

- Real and virtual radiative corrections in scattering amplitudes are kinematically strongly unbalanced (close to the exclusive boundary of the phase space). The cancellation of IR divergences among them produces large logs

Soft/collinear factorization formulae and the corresponding **singular factors** are the basic ingredients for the explicit computation and resummation of these large logarithms

Motivation

- The singular factors at $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\alpha_s^2)$ for soft and collinear factorization of scattering amplitudes are known since long time
- At $\mathcal{O}(\alpha_s)$ they have been essential to devise fully general methods to carry out next-to-leading order (NLO) QCD calculations

Frixione, Kunszt, Signer (1995); Frixione (1997)
Catani, Seymour (1996)

- At $\mathcal{O}(\alpha_s^2)$ similar considerations to NNLO QCD methods

Campbell, Glover (1997)
Catani, Grazzini (1998,1999,2000)
Bern, Del Duca, Schmidt (1998)
Kosower, Uwer (1999)
Bern, Del Duca, Kilgore, Schmidt (1999)
Czakon (2011)
Bierenbaum, Czakon, Mitov (2011); Czakon, Mitov (2018)
Catani, de Florian, Rodrigo (2011)
Sborlini, de Florian, Rodrigo (2013)

Soft/collinear factorization contributes to resummed calculations up to next-to-next-to-leading logarithmic accuracy (NNLL)

Becher, Broggio, Ferroglia (2014)
Luisoni, Marzani (2015)

- At $\mathcal{O}(\alpha_s^3)$ soft/collinear factorization can be used in the context of N3LO calculations and resummed computations at N3LL accuracy

Del Duca, Frizzo, Maltoni (1999)
Birthwright, Glover, Khoze, Marquard (2005)
Del Duca, Duhr, Haindl, Lazopoulos, Michel (2019) (2007)
Catani, de Florian, Rodrigo (2003)

Process-independent singular factors for the various collinear limits

Sborlini, de Florian, Rodrigo (2014) Badger, Buciuni, Peraro (2015) Bern, Dixon, Kosower (2004) Badger, Glover (2004)
Duhr, Gehrmann, Jaquier (2014) Catani, de Florian, Rodrigo (2011)

Soft currents

- Triple soft gluon radiation at the tree level
- Double soft emission at one loop level (has been consider recently)
- Single soft-gluon radiation at two loop order

Catani, Colferai, Torrini (2019)
Zhu (2020) Catani, LC (2021)
Badger, Glover (2004) Li, Zhu (2013)
Duhr, Gehrmann (2013) Dixon, Herrmann, Yan, Zhu (2019)

Introduction

Soft Factorization

- We consider the amplitude $\mathcal{M}$ of a generic scattering process whose external particles are QCD partons and possibly, additional non-QCD particles

$$\mathcal{M}^{c_1,c_2,\ldots}_{s_1,s_2,\ldots}(p_1,p_2,\ldots) \equiv \left(\langle c_1,c_2,\ldots| \otimes \langle s_1,s_2,\ldots|\right) \, |\mathcal{M}(p_1,p_2,\ldots)\rangle$$

Colour indices

Spin (e. g. helicity) indices

Outgoing momenta

- The amplitude $\mathcal{M}$ can be evaluated in QCD perturbation theory as a power series expansion in the QCD coupling gs

$$\mathcal{M} = \mathcal{M}^{(0)} + \mathcal{M}^{(1)} + \mathcal{M}^{(2)} + \ldots \quad ,$$

$\mathcal{M}(1)$ includes an extra factor of gs^2 → (i.e., $\mathcal{M}^{(1)}/\mathcal{M}^{(0)} \propto g_S^2$)

- We regularize ultraviolet (UV) and infrared (IR) divergences by performing the analytic continuation of the loop momenta and phase -space in d=4-2ε space-time dimensions

Introduction

Soft Factorization

- **Let us assume that $\mathcal{M}$ is in the kinematical configuration where one or more the momenta of the external massless partons (gluons or massless quarks or antiquarks) become soft**

$$\mathcal{M}(\underbrace{\lambda q_1, \dots, \lambda q_m}_{\text{Soft rescaled momenta by } \lambda}, p_1 \dots, p_n) \sim \underbrace{\frac{1}{(\lambda)^m} \bmod (\ln^r \lambda)}_{\text{Dominant singular terms}} + \underbrace{\dots}_{\text{Subdominant terms } \mathcal{O}(\sqrt{\lambda})} \quad , \qquad (\lambda \to 0)$$

- **In the soft multiparton limit, the dominant singular behaviour of $\mathcal{M}$ can be expressed by the following process-independent (universal) factorization formula**

Bern, Chalmers (1995)
Bern, Del Duca, Kilgore, Schmidt (1995)
Catani, Grazzini (1999, 2000)
Feige, Schwartz (2014)

$$|\mathcal{M}(\underbrace{q_1, \dots, q_m}_{\text{Soft momenta}}, p_1, \dots, p_n)\rangle = \underbrace{\boldsymbol{J}(q_1, \dots, q_m)}_{\text{Soft multiparton current}} \, |\mathcal{M}(p_1, \dots, p_n)\rangle + \underbrace{\dots}_{\text{Subdominant terms}}$$

Introduction

Soft Factorization

- In the soft multiparton limit, the dominant singular behaviour of $\mathscr{M}$ can be expressed by the following process-independent (universal) factorization formula

$$|\mathcal{M}(\underbrace{q_1, \ldots, q_m}_{\text{Soft momenta}}, p_1, \ldots, p_n)\rangle = \underbrace{\boldsymbol{J}}_{\text{Soft multiparton current}}(q_1, \ldots, q_m)\, |\mathcal{M}(p_1, \ldots, p_n)\rangle + \underbrace{\ldots}_{\text{Subdominant terms}}$$

- In the case of tree-level scattering amplitudes the factorization formula can be simply derived by considering soft-parton radiation from the hard-parton external legs of the amplitude and by directly applying the eikonal approximation for emission vertices and propagators.

Bassetto, Ciafaloni, Marchesini (1983) Berends, Giele (1989) Catani, Grazzini (1999)

- At one-loop level the soft current can still be computed by using the eikonal approximation for soft-parton radiation from the external hard partons, and this discussion generalizes to two-loop and higher-loop orders.

Bern, Chalmers (1995) Bern, Del Duca, Kilgore, Schmidt (1995) Catani, Grazzini (2000)

Introduction

Soft Factorization

- As for the amplitude $\mathscr{M}$, the soft current can be evaluated in QCD perturbation theory

$$\boldsymbol{J} = \boldsymbol{J}^{(0)} + \boldsymbol{J}^{(1)} + \boldsymbol{J}^{(2)} + \dots$$

- Therefore

$$|\mathcal{M}^{(0)}(q_1, \dots, q_m, p_1, \dots, p_n)\rangle \simeq \boldsymbol{J}^{(0)}(q_1, \dots, q_m)\, |\mathcal{M}^{(0)}(p_1, \dots, p_n)\rangle$$

The symbol $\simeq$ means we are neglecting subdominant terms

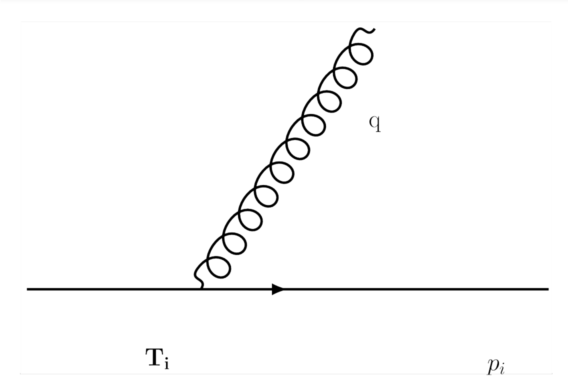
The loop label refers to the unrenormalized current

$$\begin{aligned} |\mathcal{M}^{(1)}(q_1, \dots, q_m, p_1, \dots, p_n)\rangle &\simeq \boldsymbol{J}^{(1)}(q_1, \dots, q_m)\, |\mathcal{M}^{(0)}(p_1, \dots, p_n)\rangle \\ &+ \boldsymbol{J}^{(0)}(q_1, \dots, q_m)\, |\mathcal{M}^{(1)}(p_1, \dots, p_n)\rangle \end{aligned}$$

Introduction

Tree-level soft currents

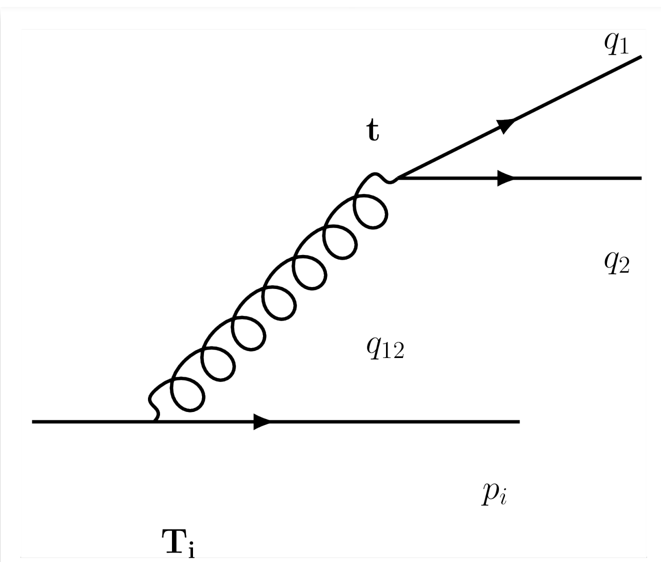
- For the emission of a single soft gluon



$$\boldsymbol{J}^{(0)}(q) = g_{\mathrm{S}}\,\mu^{\epsilon} \sum_{i\in H} \boldsymbol{T}_i \,\frac{p_i\cdot\varepsilon(q)}{p_i\cdot q} \equiv \boldsymbol{J}^{(0)}_{\nu}(q)\varepsilon^{\nu}(q)$$

Bassetto, Ciafaloni, Marchesini (1983)

- For the emission of a soft qqbar pair



$$\boldsymbol{J}^{(0)}(q_1,q_2) = -\,(g_{\mathrm{S}}\mu^{\epsilon})^2 \sum_{i\in H} \boldsymbol{t}^c\, T_i^c\, \frac{p_i\cdot j(1,2)}{p_i\cdot q_{12}}$$

Catani, Grazzini (1999)

$$j^{\nu}(1,2) \equiv \frac{\overline{u}(q_1)\,\gamma^{\nu}\,v(q_2)}{q_{12}^2}\;, \qquad q_{12} = q_1 + q_2$$

Introduction

Tree-level soft currents

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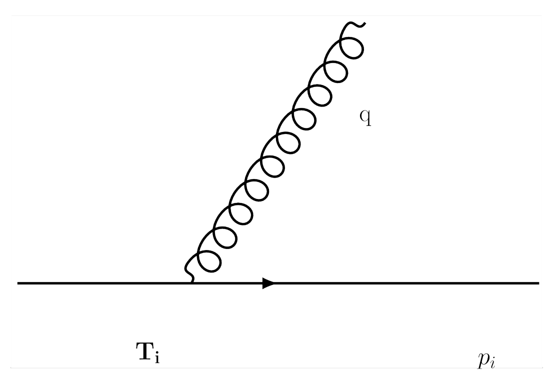


Ti is the colour charge of the hard parton I

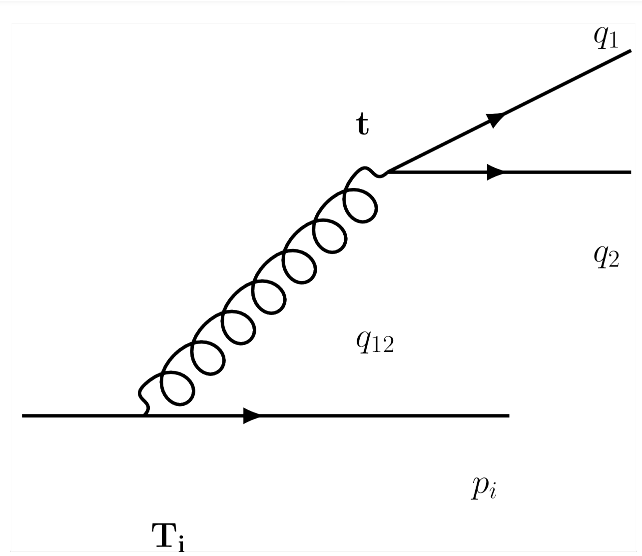
$$\boldsymbol{J}^{(0)}(q) = g_{\rm S}\,\mu^{\epsilon} \sum_{i \in H} \boldsymbol{T}_i \frac{p_i \cdot \varepsilon(q)}{p_i \cdot q} \equiv \boldsymbol{J}^{(0)}_{\nu}(q)\varepsilon^{\nu}(q)$$

The spin polarisation vector of the gluon

Bassetto, Ciafaloni, Marchesini (1983)

The sum extends over all hard partons

- **For the emission of a soft qqbar pair**



$$\boldsymbol{J}^{(0)}(q_1, q_2) = -\,(g_{\rm S}\mu^{\epsilon})^2 \sum_{i \in H} \boldsymbol{t}^c\, T_i^c \frac{p_i \cdot j(1,2)}{p_i \cdot q_{12}}$$

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Note 1: The soft factor radiation for two gluons from tree level colour-ordered sub amplitudes with external gluons and with an additional quark-antiquark pair was computed by Berends and Giele (1989).

Note 2: The soft current for the emission of two soft gluons in a generic scattering amplitude was given by Catani and Grazzini (1999). The tree-level current for the emission of three soft gluons was computed by Catani, Colferai, Torrini (2019).

One-loop current for multiple soft emission

The one-loop soft current for single gluon emission

Bern, Del Duca, Schmidt (1999)
Catani, Grazzini (2000)

From massless hard partons

Bierenbaum, Czakon, Mitov (2011); Czakon, Mitov (2018)

From massive hard partons (heavy quarks)

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Catani, Grazzini (2000)

$$
\begin{aligned}
\boldsymbol{J}^{(1)\,a} \quad &= \quad -\,(g_{\rm S}\,\mu^{\epsilon})^{3}\; c_{\Gamma}\,\frac{1}{\epsilon^{2}}\,\Gamma(1-\epsilon)\Gamma(1+\epsilon)\;\; i f_{abc} \\
&\times \quad \sum_{\substack{i,j\,\in H\\ i\neq j}} T_i^b\,T_j^c \left(\frac{p_i^{\nu}}{p_i\cdot q}-\frac{p_j^{\nu}}{p_j\cdot q}\right)\varepsilon_{\nu}(q)\;\frac{(-2p_i\cdot q-i0)^{-\epsilon}\,(-2p_j\cdot q-i0)^{-\epsilon}}{(-2p_i\cdot p_j-i0)^{-\epsilon}}
\end{aligned}
$$

One-loop current for multiple soft emission

The one-loop soft current for single gluon emission

Bern, Del Duca, Schmidt (1999)

Catani, Grazzini (2000)

a is the colour index of the soft gluon

$$c_\Gamma \equiv \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{(4\pi)^{2-\epsilon}\,\Gamma(1-2\epsilon)}$$

Purely non-abelian -> in accord with absence of one-loop corrections to the soft current for single soft photon emissions in massless QED

Yennie, Frauschi, Suura (1961); Grammer, Yennie (1973)

$$\begin{aligned}\boldsymbol{J}^{(1)\,a} &= -\left(g_S\,\mu^\epsilon\right)^3 \, c_\Gamma \, \frac{1}{\epsilon^2}\,\Gamma(1-\epsilon)\Gamma(1+\epsilon)\;\, i f_{abc} \\ &\times \sum_{\substack{i,j\,\in H \\ i\neq j}} T_i^b\,T_j^c\left(\frac{p_i^\nu}{p_i\cdot q}-\frac{p_j^\nu}{p_j\cdot q}\right)\varepsilon_\nu(q)\;\frac{\left(-2p_i\cdot q-i0\right)^{-\epsilon}\left(-2p_j\cdot q-i0\right)^{-\epsilon}}{\left(-2p_i\cdot p_j-i0\right)^{-\epsilon}}\end{aligned}$$

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It involves non-abelian colour correlations with two hard partons, at most

“x - i 0” denotes the Feynman prescription for analytic continuation in different kinematical regions

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Its kinematics structure has a rational dependence analogous to the tree-level case

$$\boldsymbol{J}^{(0)}(q) = g_{\rm S}\,\mu^\epsilon \sum_{i\in H} \boldsymbol{T}_i\, \frac{p_i\cdot\varepsilon(q)}{p_i\cdot q} \equiv \boldsymbol{J}^{(0)}_\nu(q)\varepsilon^\nu(q)$$

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→ Which is modified by through logarithmic corrections by the loop interactions

→ $$q^2_{\perp\,ij} = \frac{2(p_i\cdot q)(p_j\cdot q)}{p_i\cdot p_j}$$

Outline

Where we are?

- Motivation
- Introduction
- General features of multiple soft QCD radiation at one-loop level
- Explicit form of IR and UV divergent (ε-pole) terms of one-loop soft current
- Explicit form of qqbar soft current by including finite terms ($\mathcal{O}(\varepsilon^0)$)
- Soft-qqbar radiation at the squared amplitude level
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One-loop current for multiple soft emission

The structure of one-loop current for multiple soft radiation Catani, LC (2021)

- In general at L-loop order the current $J^{(L)}$ has poles of the type $1/\varepsilon^k$ with $2L \geq k \geq 1$.

- The ε-pole contributions to the one-loop soft current have a general structure, whose explicit form can be directly derived from the known universal structure of the IR and UV divergences of one-loop scattering amplitudes Giele, Glover (1992): Kunszt, Signer, Trocsanyi (1994) Catani, Seymour (1996) Catani (1998)

- The procedure to derive the ε-pole contributions is completely analogous to that used for the study of the multiparton collinear limit of scattering amplitudes Catani, de Florian, Rodrigo (2003) Catani, de Florian, Rodrigo (2011)

- The general all-order representation of the ε-pole contributions to J is

$$\boldsymbol{J}(q_1,\dots,q_m) = \mathbf{V}(q_1,\dots,q_m,p_1,\dots,p_n)\,\boldsymbol{J}^{[\text{no }\epsilon-\text{poles}]}(q_1,\dots,q_m)\,\mathbf{V}^{-1}(p_1,\dots,p_n)$$

One-loop current for multiple soft emission

The structure of one-loop current for multiple soft radiation Catani, LC (2021)

- Using the known expression of the one-loop term $V^{(1)}$ of the operator V we obtain

Giele, Glover (1992): Kunszt, Signer, Trocsanyi (1994)
Catani, Seymour (1996)
Catani (1998)

Valid for m ≥ 1 soft partons

$$
\begin{aligned}
\boldsymbol{J}^{(1)}(q_1,\dots,q_m) = -g_S^2\, c_\Gamma \Bigg\{ &\sum_{k\in S}\left[\frac{1}{\epsilon^2}C_k + \frac{1}{\epsilon}(\gamma_k - b_0)\right]\boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
&+ \frac{1}{\epsilon}\Bigg[\sum_{\substack{k,l\in S\\ k\neq l}} \ln\left(\frac{-2q_k\cdot q_l - i0}{\mu^2}\right)\boldsymbol{T}_k\cdot\boldsymbol{T}_l + \sum_{\substack{i\in H\\ k\in S}} \ln\left(\frac{-2p_i\cdot q_k - i0}{\mu^2}\right)\, 2\,\boldsymbol{T}_i\cdot\boldsymbol{T}_k\Bigg]\boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
&- \frac{1}{\epsilon}\sum_{\substack{i,j\in H\\ i\neq j}} \ln\left(\frac{-2p_i\cdot p_j - i0}{\mu^2}\right)\left[\boldsymbol{J}^{(0)}(q_1,\dots,q_m)\,,\ \boldsymbol{T}_i\cdot\boldsymbol{T}_j\right]\Bigg\} + \mathcal{O}(\epsilon^0)\ ,
\end{aligned}
$$

C_k is the Casimir of the parton k ($T^2_k = C_k$)

$$
\gamma_q = \gamma_{\bar q} = \frac{3}{2}\,C_F\ , \qquad \gamma_g = \frac{1}{6}\,(11\,C_A - 4T_R\,N_f)
$$

$$
b_0 = \frac{1}{6}\,(11\,C_A - 4T_R\,N_f)
$$

One-loop current for multiple soft emission

The structure of one-loop current for multiple soft radiation Catani, LC (2021)

Prop. / Pole	NEARLY ON-SHELL	VERY SOFT	// TO ONE OF SOFT PARTONS
C_k/ε^2			
γ_k/ε			
$\mathcal{R}e$ ln(arg)/ε			
$\mathcal{I}m$ ln(arg)/ε			

$$
\begin{aligned}
\boldsymbol{J}^{(1)}(q_1,\dots,q_m) = -g_S^2\, c_\Gamma \Big\{ & \sum_{k\in S}\Big[\frac{1}{\epsilon^2}C_k + \frac{1}{\epsilon}(\gamma_k - b_0)\Big]\boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
& + \frac{1}{\epsilon}\Big[\sum_{\substack{k,l\in S\\ k\neq l}} \ln\Big(\frac{-2q_k\cdot q_l - i0}{\mu^2}\Big)\boldsymbol{T}_k\cdot\boldsymbol{T}_l + \sum_{\substack{i\in H\\ k\in S}} \ln\Big(\frac{-2p_i\cdot q_k - i0}{\mu^2}\Big)\, 2\,\boldsymbol{T}_i\cdot\boldsymbol{T}_k\Big]\boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
& - \frac{1}{\epsilon}\sum_{\substack{i,j\in H\\ i\neq j}} \ln\Big(\frac{-2p_i\cdot p_j - i0}{\mu^2}\Big)\Big[\boldsymbol{J}^{(0)}(q_1,\dots,q_m)\,,\ \boldsymbol{T}_i\cdot\boldsymbol{T}_j\Big]\Big\} + \mathcal{O}(\epsilon^0)\ ,
\end{aligned}
$$

m ≥ 1 soft partons

One-loop current for multiple soft emission

The structure of one-loop current for multiple soft radiation
Catani, LC (2021)

Prop. / Pole	NEARLY ON-SHELL	VERY SOFT	// TO ONE OF SOFT PARTONS
C_k/ε^2	✓	✓	✓
γ_k/ε			
$\mathscr{Re}$ ln(arg)$/\varepsilon$ $\mathscr{Im}$			

$$
\begin{aligned}
\boldsymbol{J}^{(1)}(q_1,\dots,q_m) = -g_S^2\, c_\Gamma \Big\{ &\sum_{k\in S} \left[\frac{1}{\epsilon^2} C_k + \frac{1}{\epsilon}(\gamma_k - b_0) \right] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
&+ \frac{1}{\epsilon} \Big[\sum_{\substack{k,l\,\in S\\ k\neq l}} \ln\left(\frac{-2q_k\cdot q_l - i0}{\mu^2}\right) \boldsymbol{T}_k\cdot\boldsymbol{T}_l + \sum_{\substack{i\in H\\ k\in S}} \ln\left(\frac{-2p_i\cdot q_k - i0}{\mu^2}\right)\, 2\,\boldsymbol{T}_i\cdot\boldsymbol{T}_k \Big] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
&- \frac{1}{\epsilon} \sum_{\substack{i,j\,\in H\\ i\neq j}} \ln\left(\frac{-2p_i\cdot p_j - i0}{\mu^2}\right) \left[\boldsymbol{J}^{(0)}(q_1,\dots,q_m)\,,\ \boldsymbol{T}_i\cdot\boldsymbol{T}_j \right] \Big\} + \mathcal{O}(\epsilon^0) \;,
\end{aligned}
$$

m ≥ 1 soft partons

One-loop current for multiple soft emission

The structure of one-loop current for multiple soft radiation Catani, LC (2021)

Prop. / Pole	NEARLY ON-SHELL	VERY SOFT	// TO ONE OF SOFT PARTONS
C_k/ε^2	✓	✓	✓
γ_k/ε			
$\mathcal{R}e$ ln(arg)/ε			
$\mathcal{I}m$ ln(arg)/ε			

$$
\begin{aligned}
\boldsymbol{J}^{(1)}(q_1,\dots,q_m) = &-g_S^2\, c_\Gamma \Bigg\{ \sum_{k\in S} \left[\frac{1}{\epsilon^2} C_k + \frac{1}{\epsilon}(\gamma_k - b_0)\right] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
&+ \frac{1}{\epsilon}\Bigg[\sum_{\substack{k,l\in S\\ k\neq l}} \ln\left(\frac{-2q_k\cdot q_l - i0}{\mu^2}\right) \boldsymbol{T}_k\cdot\boldsymbol{T}_l + \sum_{\substack{i\in H\\ k\in S}} \ln\left(\frac{-2p_i\cdot q_k - i0}{\mu^2}\right) 2\,\boldsymbol{T}_i\cdot\boldsymbol{T}_k \Bigg] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
&- \frac{1}{\epsilon} \sum_{\substack{i,j\in H\\ i\neq j}} \ln\left(\frac{-2p_i\cdot p_j - i0}{\mu^2}\right) \left[\boldsymbol{J}^{(0)}(q_1,\dots,q_m)\,,\ \boldsymbol{T}_i\cdot\boldsymbol{T}_j\right] \Bigg\} + \mathcal{O}(\epsilon^0)\ ,
\end{aligned}
$$

m ≥ 1 soft partons

One-loop current for multiple soft emission

The structure of one-loop current for multiple soft radiation Catani, LC (2021)

Prop. / Pole	NEARLY ON-SHELL	VERY SOFT	// TO ONE OF SOFT PARTONS
C_k/ε^2	✓	✓	✓
γ_k/ε	✓	✗	✓
$\mathcal{R}e$ ln(arg)/ε $\mathcal{I}m$			

$$\begin{aligned}
\boldsymbol{J}^{(1)}(q_1,\dots,q_m) = -g_S^2\, c_\Gamma \Bigg\{ & \sum_{k\in S} \left[\frac{1}{\epsilon^2} C_k + \frac{1}{\epsilon} (\gamma_k - b_0) \right] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
& + \frac{1}{\epsilon} \Bigg[\sum_{\substack{k,l\in S \\ k\neq l}} \ln\left(\frac{-2q_k\cdot q_l - i0}{\mu^2}\right) \boldsymbol{T}_k\cdot\boldsymbol{T}_l + \sum_{\substack{i\in H \\ k\in S}} \ln\left(\frac{-2p_i\cdot q_k - i0}{\mu^2}\right) 2\,\boldsymbol{T}_i\cdot\boldsymbol{T}_k \Bigg] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
& - \frac{1}{\epsilon} \sum_{\substack{i,j\in H \\ i\neq j}} \ln\left(\frac{-2p_i\cdot p_j - i0}{\mu^2}\right) \left[\boldsymbol{J}^{(0)}(q_1,\dots,q_m)\,,\ \boldsymbol{T}_i\cdot\boldsymbol{T}_j \right] \Bigg\} + \mathcal{O}(\epsilon^0) \ ,
\end{aligned}$$

m ≥ 1 soft partons

One-loop current for multiple soft emission

The structure of one-loop current for multiple soft radiation Catani, LC (2021)

Pole \ Prop.	NEARLY ON-SHELL	VERY SOFT	// TO ONE OF SOFT PARTONS
C_k/ε^2	✓	✓	✓
γ_k/ε	✓	✗	✓
$\ln(\text{arg})/\varepsilon$ $\mathcal{R}e$			
$\ln(\text{arg})/\varepsilon$ $\mathcal{I}m$			

$$
\begin{aligned}
\boldsymbol{J}^{(1)}(q_1,\dots,q_m) = & -g_S^2\, c_\Gamma \Bigg\{ \sum_{k\in S} \left[\frac{1}{\epsilon^2} C_k + \frac{1}{\epsilon}(\gamma_k - b_0) \right] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
& + \frac{1}{\epsilon} \Bigg[\sum_{\substack{k,l\in S\\ k\neq l}} \ln\left(\frac{-2q_k\cdot q_l - i0}{\mu^2}\right) \boldsymbol{T}_k\cdot\boldsymbol{T}_l + \sum_{\substack{i\in H\\ k\in S}} \ln\left(\frac{-2p_i\cdot q_k - i0}{\mu^2}\right) 2\,\boldsymbol{T}_i\cdot\boldsymbol{T}_k \Bigg] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
& - \frac{1}{\epsilon} \sum_{\substack{i,j\in H\\ i\neq j}} \ln\left(\frac{-2p_i\cdot p_j - i0}{\mu^2}\right) \left[\boldsymbol{J}^{(0)}(q_1,\dots,q_m)\,,\, \boldsymbol{T}_i\cdot\boldsymbol{T}_j \right] \Bigg\} + \mathcal{O}(\epsilon^0)\ ,
\end{aligned}
$$

$m \geq 1$ soft partons

One-loop current for multiple soft emission

The structure of one-loop current for multiple soft radiation Catani, LC (2021)

Pole \ Prop.	NEARLY ON-SHELL	VERY SOFT	// TO ONE OF SOFT PARTONS
C_k/ε^2	✓	✓	✓
γ_k/ε	✓	✗	✓
$\ln(\text{arg})/\varepsilon$ ($\mathcal{R}e$ / $\mathcal{I}m$)	$\mathcal{R}e$: ✓ $\mathcal{I}m$: ✗	✓	✗

$$
\begin{aligned}
\boldsymbol{J}^{(1)}(q_1,\dots,q_m) = & -g_S^2\, c_\Gamma \Bigg\{ \sum_{k\in S}\left[\frac{1}{\epsilon^2}C_k + \frac{1}{\epsilon}(\gamma_k - b_0)\right] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
& + \frac{1}{\epsilon}\left[\sum_{\substack{k,l\in S\\ k\neq l}} \ln\left(\frac{-2q_k\cdot q_l - i0}{\mu^2}\right) \boldsymbol{T}_k\cdot \boldsymbol{T}_l + \sum_{\substack{i\in H\\ k\in S}} \ln\left(\frac{-2p_i\cdot q_k - i0}{\mu^2}\right) 2\,\boldsymbol{T}_i\cdot \boldsymbol{T}_k\right] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
& - \frac{1}{\epsilon}\sum_{\substack{i,j\in H\\ i\neq j}} \ln\left(\frac{-2p_i\cdot p_j - i0}{\mu^2}\right) \left[\boldsymbol{J}^{(0)}(q_1,\dots,q_m)\,,\, \boldsymbol{T}_i\cdot \boldsymbol{T}_j\right] \Bigg\} + \mathcal{O}(\epsilon^0)\ ,
\end{aligned}
$$

$m \geq 1$ soft partons

One-loop current for multiple soft emission

The structure of one-loop current for multiple soft radiation Catani, LC (2021)

Prop. / Pole	NEARLY ON-SHELL	VERY SOFT	// TO ONE OF SOFT PARTONS
C_k/ε^2	✓	✓	✓
γ_k/ε	✓	✗	✓
$\ln(\mathrm{arg})/\varepsilon$ — $\mathcal{Re}$	✓	✓	✗
$\ln(\mathrm{arg})/\varepsilon$ — $\mathcal{Im}$	✗		

$$
\begin{aligned}
\boldsymbol{J}^{(1)}(q_1,\dots,q_m) = -g_S^2\, c_\Gamma \Bigg\{ & \sum_{k\in S} \left[\frac{1}{\epsilon^2} C_k + \frac{1}{\epsilon}(\gamma_k - b_0) \right] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
& + \frac{1}{\epsilon} \Bigg[\sum_{\substack{k,l\,\in S\\ k\neq l}} \ln\!\left(\frac{-2q_k\cdot q_l - i0}{\mu^2} \right) \boldsymbol{T}_k\cdot\boldsymbol{T}_l + \sum_{\substack{i\in H\\ k\in S}} \ln\!\left(\frac{-2p_i\cdot q_k - i0}{\mu^2} \right) 2\,\boldsymbol{T}_i\cdot\boldsymbol{T}_k \Bigg] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
& - \frac{1}{\epsilon} \sum_{\substack{i,j\,\in H\\ i\neq j}} \ln\!\left(\frac{-2p_i\cdot p_j - i0}{\mu^2} \right) \left[\boldsymbol{J}^{(0)}(q_1,\dots,q_m)\,,\ \boldsymbol{T}_i\cdot\boldsymbol{T}_j \right] \Bigg\} + \mathcal{O}(\epsilon^0)\ ,
\end{aligned}
$$

$m \geq 1$ soft partons

One-loop current for multiple soft emission

The structure of one-loop current for multiple soft radiation Catani, LC (2021)

- We can apply our formula to specific cases

$$
\begin{aligned}
\boldsymbol{J}^{(1)}(q_1,\dots,q_m) = -g_S^2\, c_\Gamma \Bigg\{ \sum_{k\in S} \left[\frac{1}{\epsilon^2} C_k + \frac{1}{\epsilon}(\gamma_k - b_0)\right] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
+ \frac{1}{\epsilon} \Bigg[\sum_{\substack{k,l\,\in S\\ k\neq l}} \ln\left(\frac{-2q_k\cdot q_l - i0}{\mu^2}\right) \boldsymbol{T}_k\cdot\boldsymbol{T}_l + \sum_{\substack{i\in H\\ k\in S}} \ln\left(\frac{-2p_i\cdot q_k - i0}{\mu^2}\right)\; 2\,\boldsymbol{T}_i\cdot\boldsymbol{T}_k \Bigg] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
- \frac{1}{\epsilon} \sum_{\substack{i,j\,\in H\\ i\neq j}} \ln\left(\frac{-2p_i\cdot p_j - i0}{\mu^2}\right) \left[\boldsymbol{J}^{(0)}(q_1,\dots,q_m)\,,\ \boldsymbol{T}_i\cdot\boldsymbol{T}_j\right] \Bigg\} + \mathcal{O}(\epsilon^0)\ ,
\end{aligned}
$$

Valid for m ≥ 1 soft partons

- For m = 1 it reproduces the poles in

$$
\begin{aligned}
\boldsymbol{J}^{(1)\,a} \;&=\; -\,(g_S\,\mu^\epsilon)^3\; c_\Gamma\, \frac{1}{\epsilon^2}\,\Gamma(1-\epsilon)\Gamma(1+\epsilon)\;\; i f_{abc} \\
&\times \sum_{\substack{i,j\,\in H\\ i\neq j}} T_i^b\, T_j^c \left(\frac{p_i^\nu}{p_i\cdot q} - \frac{p_j^\nu}{p_j\cdot q}\right) \varepsilon_\nu(q)\, \frac{(-2p_i\cdot q - i0)^{-\epsilon}\,(-2p_j\cdot q - i0)^{-\epsilon}}{(-2p_i\cdot p_j - i0)^{-\epsilon}}
\end{aligned}
$$

Soft current single gluon emission

One-loop current for multiple soft emission

The structure of one-loop current for multiple soft radiation Catani, LC (2021)

- We can apply our formula to specific cases

$$\boldsymbol{J}^{(1)}(q_1,\dots,q_m) = -g_S^2\, c_\Gamma \Big\{ \sum_{k\in S} \left[\frac{1}{\epsilon^2} C_k + \frac{1}{\epsilon}(\gamma_k - b_0)\right] \boldsymbol{J}^{(0)}(q_1,\dots,q_m)$$

$$+ \frac{1}{\epsilon}\Big[\sum_{\substack{k,l\,\in S\\ k\neq l}} \ln\left(\frac{-2q_k\cdot q_l - i0}{\mu^2}\right) \boldsymbol{T}_k\cdot\boldsymbol{T}_l + \sum_{\substack{i\in H\\ k\in S}} \ln\left(\frac{-2p_i\cdot q_k - i0}{\mu^2}\right)\; 2\,\boldsymbol{T}_i\cdot\boldsymbol{T}_k\Big] \boldsymbol{J}^{(0)}(q_1,\dots,q_m)$$

$$- \frac{1}{\epsilon} \sum_{\substack{i,j\,\in H\\ i\neq j}} \ln\left(\frac{-2p_i\cdot p_j - i0}{\mu^2}\right) \left[\boldsymbol{J}^{(0)}(q_1,\dots,q_m)\,,\; \boldsymbol{T}_i\cdot\boldsymbol{T}_j\right] \Big\} + \mathcal{O}(\epsilon^0)\;,$$

Valid for m ≥ 1 soft partons

- For m = 2, emission of two soft partons (either two gluons or a qqbar pair)
    - For two gluons the ε-pole structure is known Zhu (2020)
    - For a qqbar pair the ε-pole structure is known Zhu (2020) Catani, LC (2021)

One-loop current for multiple soft emission

The structure of one-loop current for multiple soft radiation Catani, LC (2021)

- We can apply our formula to specific cases

$$\boldsymbol{J}^{(1)}(q_1,\dots,q_m) = -g_{\rm S}^2\, c_\Gamma \left\{ \sum_{k\in S} \left[\frac{1}{\epsilon^2} C_k + \frac{1}{\epsilon}(\gamma_k - b_0) \right] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \right.$$

$$+ \frac{1}{\epsilon} \left[\sum_{\substack{k,l\,\in S\\ k\neq l}} \ln\left(\frac{-2q_k\cdot q_l - i0}{\mu^2} \right) \boldsymbol{T}_k \cdot \boldsymbol{T}_l + \sum_{\substack{i\in H\\ k\in S}} \ln\left(\frac{-2p_i\cdot q_k - i0}{\mu^2} \right)\; 2\,\boldsymbol{T}_i \cdot \boldsymbol{T}_k \right] \boldsymbol{J}^{(0)}(q_1,\dots,q_m)$$

$$\left. - \frac{1}{\epsilon} \sum_{\substack{i,j\,\in H\\ i\neq j}} \ln\left(\frac{-2p_i\cdot p_j - i0}{\mu^2} \right)\; \left[\boldsymbol{J}^{(0)}(q_1,\dots,q_m)\,,\; \boldsymbol{T}_i \cdot \boldsymbol{T}_j \right] \right\} + \mathcal{O}(\epsilon^0)\;\;,$$

Valid for m ≥ 1 soft partons

- For m = 2, emission of two soft partons (either two gluons or a qqbar pair)
  - For two gluons the ε-pole structure is known Zhu (2020)
  - For a qqbar pair the ε-pole structure is known Zhu (2020) Catani, LC (2021)

To be precise extra exp{-$\varepsilon\gamma_E$}

One-loop current for multiple soft emission

The structure of one-loop current for multiple soft radiation Catani, LC (2021)

- An alternative expression is

$$
\begin{aligned}
\boldsymbol{J}^{(1)}(q_1,\dots,q_m) \underset{\mathrm{CS}}{=} & - g_{\mathrm{S}}^2 \left(\frac{-q_{1\dots m}^2 - i0}{\mu^2} \right)^{-\epsilon} c_\Gamma \left\{ \sum_{k\in S} \left[\frac{1}{\epsilon^2} C_k + \frac{1}{\epsilon} (\gamma_k - b_0) \right] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \right. \\
& + \frac{1}{\epsilon} \left[\sum_{\substack{k,l \in S \\ k \neq l}} \ln \left(\frac{-2q_k \cdot q_l - i0}{-q_{1\dots m}^2 - i0} \right) \boldsymbol{T}_k \cdot \boldsymbol{T}_l + \sum_{\substack{i\in H \\ k\in S}} \ell_{ik}(q_{1\dots m})\, 2\, \boldsymbol{T}_i \cdot \boldsymbol{T}_k \right] \boldsymbol{J}^{(0)}(q_1,\dots,q_m) \\
& \left. + \frac{1}{\epsilon} \sum_{\substack{i,j \in H \\ i \neq j}} L_{ij}(q_{1\dots m}) \left[\boldsymbol{J}^{(0)}(q_1,\dots,q_m)\, ,\ \boldsymbol{T}_i \cdot \boldsymbol{T}_j \right] \right\} + \mathcal{O}(\epsilon^0) \ , \qquad (m \geq 2)
\end{aligned}
$$

$$
q_{1\dots m} \equiv \sum_{k\in S} q_k = q_1 + \dots + q_m
$$

$$
\ell_{ik}(q_{1\dots m}) \equiv \ln \left(\frac{-p_i \cdot q_k - i0}{-p_i \cdot q_{1\dots m} - i0} \right)
$$

$$
L_{ij}(q_{1\dots m}) = L_{ji}(q_{1\dots m}) \equiv \ln \left(\frac{-p_i \cdot q_{1\dots m} - i0}{-p_i \cdot p_j - i0} \right) + \ln \left(\frac{-2p_j \cdot q_{1\dots m} - i0}{-q_{1\dots m}^2 - i0} \right)
$$

Outline

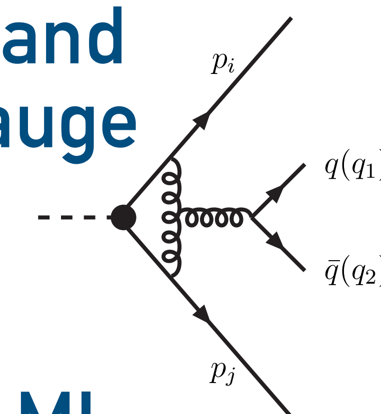
Where we are?

- Motivation
- Introduction
- General features of multiple soft QCD radiation at one-loop level
- Explicit form of IR and UV divergent (ε-pole) terms of one-loop soft current
- Explicit form of qqbar soft current by including finite terms ($\mathcal{O}(\varepsilon^0)$)
- Soft-qqbar radiation at the squared amplitude level
- Outlook

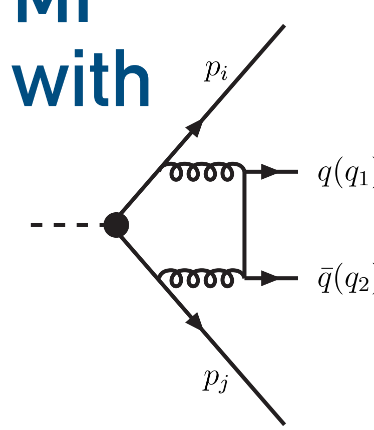
Soft qqbar emission: the one-loop current
Catani, LC (2021)

Introduction

- In order to derive the soft current at one-loop we use the general (process-independent) method that was introduced for the single gluon soft current at one loop Catani, Grazzini (2000)
- We have evaluated a set of one-loop Feynman diagrams in which the external-leg hard partons are coupled to virtual gluons by using the eikonal approximation (for both vertices and propagators). Other vertices and propagators are computed using customary QCD Feynman rules
- We perform the calculation by using both the Feynman gauge and the axial gauge $n.A = 0$, with and auxiliary light-like ($n^2 = 0$) gauge vector n. This provide us with an explicit check of the gauge invariance of the procedure and the calculation



- We have performed and independent calculation of all the soft MI integrals that enter in our calculation, which are in agreement with those encountered in literature Catani, Grazzini (2000)
Anastasiou, Duhr, Dulat, Furlan, Herzog, Mistlberger (2015)



Soft qqbar emission: the one-loop current

Catani, LC (2021)

Results

- We define tree-level and one-loop rescaled current as follows

$$\boldsymbol{J}^{(0)}(q_1,q_2) = (g_S\,\mu^\epsilon)^2\ \hat{\boldsymbol{J}}^{(0)}(q_1,q_2)\ ,$$

$$\boldsymbol{J}^{(1)}(q_1,q_2) = (g_S\,\mu^\epsilon)^4\left(-q_{12}^2 - i0\right)^{-\epsilon} c_\Gamma\,\hat{\boldsymbol{J}}^{(1)}(q_1,q_2)\ ,$$

$$\hat{\boldsymbol{J}}^{(1)}(q_1,q_2) = \hat{\boldsymbol{J}}^{(1,\,\mathrm{div})}(q_1,q_2) + \hat{\boldsymbol{J}}^{(1,\,\mathrm{fin})}(q_1,q_2)\ ,$$

- The explicit expressions of the components are

$$\hat{\boldsymbol{J}}^{(1,\,\mathrm{div})}(q_1,q_2) = -2\left[\frac{1}{\epsilon^2}C_F + \frac{1}{\epsilon}\left(\frac{3}{2}C_F - \frac{1}{6}(11\,C_A - 4\,T_R\,N_f)\right)\right]\hat{\boldsymbol{J}}^{(0)}(q_1,q_2)$$
$$-\ \frac{2}{\epsilon}\,j_\nu(1,2)\,\boldsymbol{t}^a\boldsymbol{t}^b \sum_{\substack{i,j\,\in H\\ i\neq j}} T_i^a\,T_j^b\left(\frac{p_i^\nu}{p_i\cdot q_{12}} - \frac{p_j^\nu}{p_j\cdot q_{12}}\right)(L_{ij} + \ell_{i1} + \ell_{j2})$$

$$q^2_{12\perp ij} = \frac{2(p_i\cdot q_{12})(p_j\cdot q_{12})}{p_i\cdot p_j} - q^2_{12}$$

$$\hat{\boldsymbol{J}}^{(1,\,\mathrm{fin})}(q_1,q_2) = \left[\left(-8-(\delta_R-1)\right)C_F + \left(\frac{76}{9} - \frac{\pi^2}{3} + \frac{1}{3}(\delta_R-1)\right)C_A - \frac{20}{9}\,T_R N_f\right]\hat{\boldsymbol{J}}^{(0)}(q_1,q_2)$$
$$+\ j_\nu(1,2)\,\boldsymbol{t}^a\boldsymbol{t}^b \sum_{\substack{i,j\,\in H\\ i\neq j}} T_i^a\,T_j^b\left[\left(\frac{p_i^\nu}{p_i\cdot q_{12}} - \frac{p_j^\nu}{p_j\cdot q_{12}}\right)\left(L_{ij}^2 + (\ell_{i1}-\ell_{j2})^2\right)\right.$$
$$\left. +\ \frac{q_{12}^2}{q^2_{12\perp ij}}\left(\frac{p_i^\nu}{p_i\cdot q_{12}} + \frac{p_j^\nu}{p_j\cdot q_{12}}\right) 2\,L_{ij}\left(\ell_{i1}-\ell_{j2}\right)\right] + \mathcal{O}(\epsilon)\ ,$$

Soft qqbar emission: the one-loop current

Catani, LC (2021)

Results

- The explicit expressions of the components are

$$\hat{\boldsymbol{J}}^{(1)}(q_1,q_2) = \hat{\boldsymbol{J}}^{(1,\,\mathrm{div})}(q_1,q_2) + \hat{\boldsymbol{J}}^{(1,\,\mathrm{fin})}(q_1,q_2)$$

$$\begin{aligned}\hat{\boldsymbol{J}}^{(1,\,\mathrm{div})}(q_1,q_2) \;=&\; -2\left[\frac{1}{\epsilon^2}\,C_F + \frac{1}{\epsilon}\left(\frac{3}{2}\,C_F - \frac{1}{6}(11\,C_A - 4\,T_R\,N_f)\right)\right]\hat{\boldsymbol{J}}^{(0)}(q_1,q_2)\\ &- \frac{2}{\epsilon}\,j_\nu(1,2)\,\boldsymbol{t}^a\boldsymbol{t}^b \sum_{\substack{i,j\,\in H\\ i\neq j}} T_i^a\,T_j^b\left(\frac{p_i^\nu}{p_i\cdot q_{12}} - \frac{p_j^\nu}{p_j\cdot q_{12}}\right)(L_{ij} + \ell_{i1} + \ell_{j2})\end{aligned}$$

Not purely non-abelian colour correlations

The soft current for lepton-antilepton in massless QED is non-vanishing

The ε-pole structure agrees with our general result applied to a qqbar pair

$$\begin{aligned}\hat{\boldsymbol{J}}^{(1,\,\mathrm{fin})}(q_1,q_2) =& \left[(-8-(\delta_R-1))C_F + \left(\frac{76}{9} - \frac{\pi^2}{3} + \frac{1}{3}(\delta_R-1)\right)C_A - \frac{20}{9}\,T_R N_f\right]\hat{\boldsymbol{J}}^{(0)}(q_1,q_2)\\ &+ j_\nu(1,2)\,\boldsymbol{t}^a\boldsymbol{t}^b \sum_{\substack{i,j\,\in H\\ i\neq j}} T_i^a\,T_j^b\left[\left(\frac{p_i^\nu}{p_i\cdot q_{12}} - \frac{p_j^\nu}{p_j\cdot q_{12}}\right)\left(L_{ij}^2 + (\ell_{i1}-\ell_{j2})^2\right)\right.\\ &\left.+ \frac{q_{12}^2}{q_{12\perp ij}^2}\left(\frac{p_i^\nu}{p_i\cdot q_{12}} + \frac{p_j^\nu}{p_j\cdot q_{12}}\right) 2\,L_{ij}\,(\ell_{i1}-\ell_{j2})\right] + \mathcal{O}(\epsilon)\quad ,\end{aligned}$$

These results are valid in arbitrary kinematical regions

Soft qqbar emission: the one-loop current

Catani, LC (2021)

Results

- The explicit expressions of the components are

$$\hat{\boldsymbol{J}}^{(1)}(q_1,q_2) = \hat{\boldsymbol{J}}^{(1,\,\mathrm{div})}(q_1,q_2) + \hat{\boldsymbol{J}}^{(1,\,\mathrm{fin})}(q_1,q_2)$$

$$\begin{aligned}\hat{\boldsymbol{J}}^{(1,\,\mathrm{div})}(q_1,q_2) \;&=\; -2\left[\frac{1}{\epsilon^2}C_F + \frac{1}{\epsilon}\left(\frac{3}{2}C_F - \frac{1}{6}(11\,C_A - 4\,T_R \boxed{N_f)}\right)\right]\hat{\boldsymbol{J}}^{(0)}(q_1,q_2)\\ &\quad - \frac{2}{\epsilon}\,j_\nu(1,2)\,\boldsymbol{t}^a\boldsymbol{t}^b \sum_{\substack{i,j\,\in H\\ i\neq j}} T_i^a\,T_j^b\left(\frac{p_i^\nu}{p_i\cdot q_{12}} - \frac{p_j^\nu}{p_j\cdot q_{12}}\right)(L_{ij}+\ell_{i1}+\ell_{j2})\end{aligned}$$

$$\begin{aligned}\hat{\boldsymbol{J}}^{(1,\,\mathrm{fin})}(q_1,q_2) = \Bigg[&\left(-8-(\delta_R-1)\right)C_F + \left(\frac{76}{9} - \frac{\pi^2}{3} + \frac{1}{3}(\delta_R-1)\right)C_A - \frac{20}{9}\boxed{T_R N_f}\;\hat{\boldsymbol{J}}^{(0)}(q_1,q_2)\\ &+\; j_\nu(1,2)\,\boldsymbol{t}^a\boldsymbol{t}^b \sum_{\substack{i,j\,\in H\\ i\neq j}} T_i^a\,T_j^b\left[\left(\frac{p_i^\nu}{p_i\cdot q_{12}} - \frac{p_j^\nu}{p_j\cdot q_{12}}\right)\left(L_{ij}^2 + (\ell_{i1}-\ell_{j2})^2\right)\right.\\ &+\left.\frac{q_{12}^2}{q_{12\perp ij}^2}\left(\frac{p_i^\nu}{p_i\cdot q_{12}} + \frac{p_j^\nu}{p_j\cdot q_{12}}\right)2\,L_{ij}\,(\ell_{i1}-\ell_{j2})\right] + \mathcal{O}(\epsilon)\ ,\end{aligned}$$

Not purely non-abelian colour correlations

The soft current for lepton-antilepton in massless QED is non-vanishing

The ε-pole structure agrees with our general result applied to a qqbar pair

These terms are due to vacuum polarization of Nf massless quarks

The trascendentality is = 2. The dilogarithms which appear in the MI cancel in the complete result

There are no additional colour-correlations structures at any higher order in the ε expansion

Soft qqbar emission: the one-loop current

Catani, LC (2021)

Results

- The explicit expressions of the components are

$$\hat{\boldsymbol{J}}^{(1)}(q_1,q_2) = \hat{\boldsymbol{J}}^{(1,\,\mathrm{div})}(q_1,q_2) + \hat{\boldsymbol{J}}^{(1,\,\mathrm{fin})}(q_1,q_2)$$

$$\hat{\boldsymbol{J}}^{(1,\,\mathrm{div})}(q_1,q_2) \;=\; -\,2\left[\frac{1}{\epsilon^2}\,C_F + \frac{1}{\epsilon}\left(\frac{3}{2}\,C_F - \frac{1}{6}(11\,C_A - 4\,T_R\,\boxed{N_f})\right)\right]\hat{\boldsymbol{J}}^{(0)}(q_1,q_2)$$
$$-\;\frac{2}{\epsilon}\,j_\nu(1,2)\;\boldsymbol{t}^a\boldsymbol{t}^b\sum_{\substack{i,j\,\in H\\ i\neq j}} T_i^a\,T_j^b\left(\frac{p_i^\nu}{p_i\cdot q_{12}} - \frac{p_j^\nu}{p_j\cdot q_{12}}\right)(L_{ij}+\ell_{i1}+\ell_{j2})$$

Not purely non-abelian colour correlations

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$$\hat{\boldsymbol{J}}^{(1,\,\mathrm{fin})}(q_1,q_2) = \Bigg[\left(-8-(\delta_R-1)\right)C_F + \left(\frac{76}{9} - \frac{\pi^2}{3} + \frac{1}{3}(\delta_R-1)\right)C_A - \frac{20}{9}\,\boxed{T_R N_f}\;\hat{\boldsymbol{J}}^{(0)}(q_1,q_2)$$
$$+\;j_\nu(1,2)\;\boldsymbol{t}^a\boldsymbol{t}^b\sum_{\substack{i,j\,\in H\\ i\neq j}} T_i^a\,T_j^b\left[\left(\frac{p_i^\nu}{p_i\cdot q_{12}} - \frac{p_j^\nu}{p_j\cdot q_{12}}\right)\left(L_{ij}^2+(\ell_{i1}-\ell_{j2})^2\right)\right.$$
$$\left.+\;\frac{q_{12}^2}{q_{12\perp ij}^2}\left(\frac{p_i^\nu}{p_i\cdot q_{12}} + \frac{p_j^\nu}{p_j\cdot q_{12}}\right)2\,L_{ij}\,(\ell_{i1}-\ell_{j2})\right] + \mathcal{O}(\epsilon)\;\;,$$

These terms are due to vacuum polarization of Nf massless quarks

Regularization scheme parameter

The trascendentality is = 2. The dilogarithms which appear in the MI cancel in the complete result

There are no additional colour-correlations structures at any higher order in the ε expansion

Regularization scheme parameter

$$\delta_R = 1 \quad (\mathrm{CDR},\ \mathrm{HV})\;, \qquad\qquad \delta_R = 0 \quad (\mathrm{4DH},\ \mathrm{DR})$$

Soft qqbar emission: the one-loop current

Catani, LC (2021)

Results

- The explicit expression of the finite component is

$$
\begin{aligned}
\hat{\boldsymbol{J}}^{(1,\,\mathrm{fin})}(q_1,q_2) = &\Bigg[\Big(-8-(\delta_R-1)\Big)C_F + \left(\frac{76}{9}-\frac{\pi^2}{3}+\frac{1}{3}(\delta_R-1)\right)C_A - \frac{20}{9}\,T_R N_f\Bigg]\hat{\boldsymbol{J}}^{(0)}(q_1,q_2) \\
&+\ j_\nu(1,2)\,\boldsymbol{t}^a\boldsymbol{t}^b \sum_{\substack{i,j\,\in H \\ i\neq j}} T_i^a\,T_j^b\ \boxed{\left(\frac{p_i^\nu}{p_i\cdot q_{12}}-\frac{p_j^\nu}{p_j\cdot q_{12}}\right)}\Big(L_{ij}^2+(\ell_{i1}-\ell_{j2})^2\Big) \\
&+\ \frac{q_{12}^2}{q_{12\perp ij}^2}\left(\frac{p_i^\nu}{p_i\cdot q_{12}}+\frac{p_j^\nu}{p_j\cdot q_{12}}\right)2\,L_{ij}\,\big(\ell_{i1}-\ell_{j2}\big)\Bigg] + \mathcal{O}(\epsilon)\ ,
\end{aligned}
$$

Collinear singularity if q^2_{12} -> 0 (if the momenta of the soft quark and anti quark are parallel)

Behaviour which is present in

$\hat{\boldsymbol{J}}^{(1,\,\mathrm{div})}$

$\boldsymbol{J}^{(0)}$

Soft qqbar emission: the one-loop current

Catani, LC (2021)

Results

- The explicit expression of the finite component is

$$\hat{\boldsymbol{J}}^{(1,\,\mathrm{fin})}(q_1,q_2) = \Bigg[\Big(-8-(\delta_R-1)\Big)C_F + \left(\frac{76}{9} - \frac{\pi^2}{3} + \frac{1}{3}(\delta_R-1)\right)C_A - \frac{20}{9}\,T_R N_f \Bigg] \hat{\boldsymbol{J}}^{(0)}(q_1,q_2)$$
$$+ \; j_\nu(1,2)\,\boldsymbol{t}^a \boldsymbol{t}^b \sum_{\substack{i,j\,\in H \\ i\neq j}} T_i^a\,T_j^b \Bigg[\boxed{\left(\frac{p_i^\nu}{p_i\cdot q_{12}} - \frac{p_j^\nu}{p_j\cdot q_{12}} \right)} \left(L_{ij}^2 + (\ell_{i1}-\ell_{j2})^2\right)$$
$$+ \; \boxed{\frac{q_{12}^2}{q_{12\perp ij}^2} \left(\frac{p_i^\nu}{p_i\cdot q_{12}} + \frac{p_j^\nu}{p_j\cdot q_{12}} \right)} 2\,L_{ij}\,(\ell_{i1}-\ell_{j2}) \Bigg] + \mathcal{O}(\epsilon) \;\;,$$

Collinear singularity if q^2_{12} -> 0 (if the momenta of the soft quark and anti quark are parallel)

Behaviour which is present in

$$\hat{\boldsymbol{J}}^{(1,\,\mathrm{div})}$$

$$\boldsymbol{J}^{(0)}$$

This rational factor has no collinear singularity at q^2_{12} -> 0, but potentially leads to a singularity in the limit $q^2_{12\perp ij} \to 0$

$$q^2_{12\perp\, ij} = \frac{2(p_i\cdot q_{12})(p_j\cdot q_{12})}{p_i\cdot p_j} - q_{12}^2$$

It is the transverse component of the momentum q_{12} of the soft qqbar pair with respect to the momenta pi and pj of the colour-correlated hard partons in a reference frame in which pi and pj are back-to-back

$$\frac{1}{q^2_{12\perp ij}} L_{ij} \underset{q_{12\perp ij}\to 0}{\simeq} \frac{1}{q^2_{12\perp ij}} \left[2\pi i\ \mathrm{sign}(q_{12}^2)\ \Theta\!\left(\frac{-p_i\cdot q_{12}}{p_i\cdot p_j}\right) \Theta\!\left(\frac{-p_j\cdot q_{12}}{p_i\cdot p_j}\right) + \mathcal{O}\!\left(\frac{q^2_{12\perp ij}}{q^2_{12}}\right)\right]$$

The current has a one-loop singularity of absorptive origin

Soft qqbar emission: the one-loop current

Catani, LC (2021)

Results

- Few comments about the singularity

$$\frac{1}{q^2_{12\perp ij}} L_{ij} \underset{q_{12\perp ij}\to 0}{\simeq} \frac{1}{q^2_{12\perp ij}}\left[2\pi i\,\mathrm{sign}(q^2_{12})\,\Theta\!\left(\frac{-p_i\cdot q_{12}}{p_i\cdot p_j}\right)\Theta\!\left(\frac{-p_j\cdot q_{12}}{p_i\cdot p_j}\right) + \mathcal{O}\!\left(\frac{q^2_{12\perp ij}}{q^2_{12}}\right)\right]$$

The singularity is partly screened by the logarithmic function Lij

The current has a one-loop singularity of absorptive origin

$$\frac{1}{q^2_{12\perp ij}} L_{ij} \underset{q_{12\perp ij}\to 0}{\simeq} \frac{1}{q^2_{12\perp ij}}\, 2\pi i\,\Theta(-p_i^0)\,\Theta(-p_j^0)\ , \qquad (q_1^0>0,\ q_2^0>0)$$

Considering the physically most relevant region

$$\boldsymbol{t}^a\boldsymbol{t}^b\,T_i^a\,T_j^b\,\frac{L_{ij}}{q^2_{12\perp ij}}(\ell_{i1}-\ell_{j2}) \underset{q_{12\perp ij}\to 0}{\longrightarrow} -f^{abc}\boldsymbol{t}^c\,T_i^a\,T_j^b\,\frac{\pi\,\Theta(-p_i^0)\,\Theta(-p_j^0)}{q^2_{12\perp ij}}\,(\ell_{i1}-\ell_{j2})\ , \quad (q_1^0, q_2^0>0)$$

The singularity has a purely non-abelian character

As we have just discussed, the singularity of the soft-qqbar current in the limit $q_{12\perp ij} \to 0$ originates from one-loop interactions of the two soft partons. Therefore, we expect the presence of the transverse-momentum singularity also in the case of double soft-gluon emission at one-loop level.

We have checked → Zhu (2020)

Outline

Where we are?

- Motivation
- Introduction
- General features of multiple soft QCD radiation at one-loop level
- Explicit form of IR and UV divergent (ε-pole) terms of one-loop soft current
- Explicit form of qqbar soft current by including finite terms ($\mathcal{O}(\varepsilon^0)$)
- Soft-qqbar radiation at the squared amplitude level
- Outlook

Soft qqbar radiation: squared amplitude and current

Catani, LC (2021)

Introduction

Summed over colours and spins over its external legs

$$|\mathcal{M}|^2 = \langle \mathcal{M} | \, \mathcal{M} \rangle$$

The square of the soft-emission factorization formula gives

$$|\mathcal{M}(q_1, \ldots, q_m, p_1, \ldots, p_n)|^2 \simeq \langle \mathcal{M}(p_1, \ldots, p_n) | \; | \boldsymbol{J}(q_1, \ldots, q_m) \, |^2 \; | \mathcal{M}(p_1, \ldots, p_n) \rangle$$

The all-loop squared current summed over the colours and spins of the soft partons

$$\begin{aligned} |\boldsymbol{J}(q_1, \ldots, q_m)|^2 &= \left[J^{\,c_1, \ldots, c_m}_{\,s_1, \ldots, s_m}(q_1, \ldots, q_m) \right]^\dagger J^{\,c_1, \ldots, c_m}_{\,s_1, \ldots, s_m}(q_1, \ldots, q_m) \\ &\equiv \left[\boldsymbol{J}(q_1, \ldots, q_m) \right]^\dagger \boldsymbol{J}(q_1, \ldots, q_m) \ . \end{aligned}$$

Soft qqbar radiation: squared amplitude and current

Catani, LC (2021)

Introduction

Summed over colours and spins over its external legs

$$|\mathcal{M}|^2 = \langle \mathcal{M} | \, \mathcal{M} \rangle$$

The square of the soft-emission factorization formula gives

$$|\mathcal{M}(q_1, \ldots, q_m, p_1, \ldots, p_n)|^2 \simeq \langle \mathcal{M}(p_1, \ldots, p_n)| \; |\boldsymbol{J}(q_1, \ldots, q_m)\,|^2 \; |\mathcal{M}(p_1, \ldots, p_n)\rangle$$

The all-loop squared current summed over the colours and spins of the soft partons

$$\begin{aligned} |\boldsymbol{J}(q_1, \ldots, q_m)|^2 \;&=\; \left[J^{\,c_1,\ldots,c_m}_{\,s_1,\ldots,s_m}(q_1, \ldots, q_m)\right]^\dagger J^{\,c_1,\ldots,c_m}_{\,s_1,\ldots,s_m}(q_1, \ldots, q_m) \\ &\equiv\; [\boldsymbol{J}(q_1, \ldots, q_m)]^\dagger \; \boldsymbol{J}(q_1, \ldots, q_m) \;\; . \end{aligned}$$

The squared current is a colour operator that depends on the colour charges (and momenta) of the hard partons in $\mathcal{M}$

The colour charges produce colour correlations and, therefore, this part is not proportional to $\mathcal{M}^2$ in the general case

We define

$$|\boldsymbol{J}(q_1, q_2)|^2 \equiv (g_\mathrm{S}\, \mu^\epsilon)^4 \; |\hat{\boldsymbol{J}}(q_1, q_2)|^2_{(0\ell)} + (g_\mathrm{S}\, \mu^\epsilon)^6 \left(|q^2_{12}|\right)^{-\epsilon} \, c_\Gamma \; |\hat{\boldsymbol{J}}(q_1, q_2)|^2_{(1\ell)} + \mathcal{O}(g^8_\mathrm{S})$$

Tree-level

One-loop

Soft qqbar radiation: squared amplitude and current

Catani, LC (2021)

Introduction

- The tree-level squared current is

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(0\ell)} = \left[\hat{\boldsymbol{J}}^{(0)}(q_1,q_2)\right]^\dagger \hat{\boldsymbol{J}}^{(0)}(q_1,q_2)$$

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(0\ell)} = T_R \sum_{i,j\in H} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \;\; \mathcal{I}_{ij}(q_1,q_2)$$

Catani, Grazzini (1999)

$$\mathcal{I}_{ij}(q_1,q_2) = \frac{(p_i\cdot q_1)\,(p_j\cdot q_2) + (p_j\cdot q_1)\,(p_i\cdot q_2) - (p_i\cdot p_j)\,(q_1\cdot q_2)}{(q_1\cdot q_2)^2\,(p_i\cdot q_{12})\,(p_j\cdot q_{12})}$$

Symmetric with respect to the interchange q1 <-> q2 and i <-> j

Soft qqbar radiation: squared amplitude and current

Catani, LC (2021)

Introduction

- The tree-level squared current is

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(0\ell)} = \left[\hat{\boldsymbol{J}}^{(0)}(q_1,q_2)\right]^\dagger \, \hat{\boldsymbol{J}}^{(0)}(q_1,q_2)$$

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(0\ell)} = T_R \sum_{i,j\in H} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \;\; \mathcal{I}_{ij}(q_1,q_2)$$

Catani, Grazzini (1999)

$$\mathcal{I}_{ij}(q_1,q_2) = \frac{(p_i\cdot q_1)\,(p_j\cdot q_2) + (p_j\cdot q_1)\,(p_i\cdot q_2) - (p_i\cdot p_j)\,(q_1\cdot q_2)}{(q_1\cdot q_2)^2\,(p_i\cdot q_{12})\,(p_j\cdot q_{12})}$$

Symmetric with respect to the interchange q1 <-> q2 and i <-> j

Using colour charge conservation, the tree-level current can be recast in the following form

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(0\ell)} \underset{\rm CS}{=} -\frac{1}{2}\,T_R \sum_{\substack{i,j\,\in H\\ i\neq j}} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \;\; w_{ij}(q_1,q_2)$$

$$w_{ij}(q_1,q_2) = \mathcal{I}_{ii}(q_1,q_2) + \mathcal{I}_{jj}(q_1,q_2) - 2\,\mathcal{I}_{ij}(q_1,q_2)$$

Soft qqbar radiation: squared amplitude and current

Catani, LC (2021)

Introduction

- The tree-level squared current is

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(0\ell)} = \left[\hat{\boldsymbol{J}}^{(0)}(q_1,q_2)\right]^\dagger \hat{\boldsymbol{J}}^{(0)}(q_1,q_2)$$

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(0\ell)} = T_R \sum_{i,j\in H} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \;\; \mathcal{I}_{ij}(q_1,q_2)$$

Catani, Grazzini (1999)

$$\mathcal{I}_{ij}(q_1,q_2) = \frac{(p_i\cdot q_1)\,(p_j\cdot q_2) + (p_j\cdot q_1)\,(p_i\cdot q_2) - (p_i\cdot p_j)\,(q_1\cdot q_2)}{(q_1\cdot q_2)^2\,(p_i\cdot q_{12})\,(p_j\cdot q_{12})}$$

Symmetric with respect to the interchange q1 <-> q2 and i <-> j

These two expressions are not identical at the algebraic level, but they are fully equivalent by acting onto scattering amplitudes (or, generically, colour-single states)

Using colour charge conservation, the tree-level current can be recast in the following form

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(0\ell)} \underset{\text{CS}}{=} -\frac{1}{2}\,T_R \sum_{\substack{i,j\,\in H\\ i\neq j}} \boldsymbol{T}_i \cdot \boldsymbol{T}_j \;\; w_{ij}(q_1,q_2)$$

Two-particle correlations between the hard partons. Their colour structure has the form of dipole contributions **T**i . **T**j.

$$w_{ij}(q_1,q_2) = \mathcal{I}_{ii}(q_1,q_2) + \mathcal{I}_{jj}(q_1,q_2) - 2\,\mathcal{I}_{ij}(q_1,q_2)$$

Symmetric with respect to the interchange q1 <-> q2 and i <-> j

Soft qqbar radiation: squared amplitude and current

Catani, LC (2021)

Results

- The one-loop squared current

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(1\ell)} = \left(\frac{-q_{12}^2 - i0}{|q_{12}^2|}\right)^{-\epsilon} \left[\hat{\boldsymbol{J}}^{(0)}(q_1,q_2)\right]^\dagger \hat{\boldsymbol{J}}^{(1)}(q_1,q_2) + \text{ h.c.}$$

$$d^{abc} = \frac{1}{T_R}\,\mathrm{Tr}\left(\{t^a,t^b\}\,t^c\right)$$

We define the d-conjugated (quadratic) charge operator **D**i of the parton i

$$\widetilde{D}_i^a \equiv d^{abc}\,T_i^b\,T_i^c$$

With indices {a,b,c} in the adjoint representation of SU(Nc). It is odd under charge conjugation

Soft qqbar radiation: squared amplitude and current

Catani, LC (2021)

Results

- The one-loop squared current

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(1\ell)} = \left(\frac{-q_{12}^2 - i0}{|q_{12}^2|}\right)^{-\epsilon} \left[\hat{\boldsymbol{J}}^{(0)}(q_1,q_2)\right]^\dagger \hat{\boldsymbol{J}}^{(1)}(q_1,q_2) + \text{ h.c.}$$

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$$\widetilde{D}_i^a \equiv d^{abc}\,T_i^b\,T_i^c$$

With indices {a,b,c} in the adjoint representation of SU(Nc). It is odd under charge conjugation

$$\begin{aligned}|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(1\ell)} &= -\frac{1}{2}\,T_R \sum_{\substack{i,j\,\in H\\ i\neq j}} \left[\,\boldsymbol{T}_i\cdot\boldsymbol{T}_j\;\; w_{ij}^{[S]}(q_1,q_2) + \widetilde{\boldsymbol{D}}_i\cdot\boldsymbol{T}_j\;\; w_{ij}^{[A]}(q_1,q_2)\right]\\
&- T_R \sum_{\substack{i,j,k\,\in H\\ \mathrm{dist.}\{i,j,k\}}} T_i^a\,T_j^b\,T_k^c \left[f^{abc}\,F_{ijk}^{[S]}(q_1,q_2) + d^{abc}\left(F_{ijk}^{[A]}(q_1,q_2) - \frac{1}{2}F_{iji}^{[A]}(q_1,q_2) - \frac{1}{2}F_{ijj}^{[A]}(q_1,q_2)\right)\right]\end{aligned}$$

Valid to arbitrary orders in the ε expansion

Soft qqbar radiation: squared amplitude and current

Catani, LC (2021)

Results

- The one-loop squared current

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(1\ell)} = \left(\frac{-q_{12}^2 - i0}{|q_{12}^2|}\right)^{-\epsilon} \left[\hat{\boldsymbol{J}}^{(0)}(q_1,q_2)\right]^\dagger \hat{\boldsymbol{J}}^{(1)}(q_1,q_2) + \text{ h.c.}$$

$$d^{abc} = \frac{1}{T_R}\,\text{Tr}\left(\{t^a,t^b\}\,t^c\right)$$

With indices {a,b,c} in the adjoint representation of SU(Nc). It is odd under charge conjugation

We define the d-conjugated (quadratic) charge operator **D**i of the parton i

$$\widetilde{D}_i^a \equiv d^{abc}\,T_i^b\,T_i^c$$

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(1\ell)} = -\frac{1}{2}\,T_R \sum_{\substack{i,j\in H\\ i\neq j}} \left[\boldsymbol{T}_i\cdot\boldsymbol{T}_j\;\; w_{ij}^{[S]}(q_1,q_2) + \widetilde{\boldsymbol{D}}_i\cdot\boldsymbol{T}_j\;\; w_{ij}^{[A]}(q_1,q_2)\right]$$

Two hard-parton correlations

$$-\,T_R \sum_{\substack{i,j,k\in H\\ \text{dist.}\{i,j,k\}}} T_i^a\,T_j^b\,T_k^c \left[f^{abc}\,F_{ijk}^{[S]}(q_1,q_2) + d^{abc}\left(F_{ijk}^{[A]}(q_1,q_2) - \frac{1}{2}F_{iji}^{[A]}(q_1,q_2) - \frac{1}{2}F_{ijj}^{[A]}(q_1,q_2)\right)\right]$$

Three hard-parton correlations

Sum over distinct hard-parton indices

Valid to arbitrary orders in the ϵ expansion

$$w_{ij}^{[S]}(q_1,q_2) = w_{ij}^{[S]}(q_2,q_1)\;, \qquad F_{ijk}^{[S]}(q_1,q_2) = F_{ijk}^{[S]}(q_2,q_1)$$

$$w_{ij}^{[A]}(q_1,q_2) = -w_{ij}^{[A]}(q_2,q_1)\;, \qquad F_{ijk}^{[A]}(q_1,q_2) = -F_{ijk}^{[A]}(q_2,q_1)$$

Therefore these contributions produce a qqbar charge asymmetry in the one-loop squared current

Soft qqbar radiation: squared amplitude and current

Catani, LC (2021)

Results

- The one-loop squared current

Explicit results of the ε expansion of the functions w and F

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(1\ell)} = -\frac{1}{2} T_R \sum_{\substack{i,j\in H\\ i\neq j}} \left[\boldsymbol{T}_i\cdot\boldsymbol{T}_j \;\; w_{ij}^{[S]}(q_1,q_2) + \widetilde{\boldsymbol{D}}_i\cdot\boldsymbol{T}_j \;\; w_{ij}^{[A]}(q_1,q_2)\right]$$
$$- T_R \sum_{\substack{i,j,k\in H\\ \mathrm{dist.}\{i,j,k\}}} T_i^a T_j^b T_k^c \left[f^{abc} F_{ijk}^{[S]}(q_1,q_2) + d^{abc}\left(F_{ijk}^{[A]}(q_1,q_2) - \frac{1}{2} F_{iji}^{[A]}(q_1,q_2) - \frac{1}{2} F_{ijj}^{[A]}(q_1,q_2)\right)\right]$$

$$w_{ij}^{[S]}(q_1,q_2) = \Big\{ w_{ij}(q_1,q_2) \Big[-C_F \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \pi^2 + 8 + (\delta_R - 1)\right) - \frac{4}{3} T_R N_f \left(\frac{1}{\epsilon} + \frac{5}{3}\right)$$
$$+ \frac{1}{3} C_A \left(\frac{11}{\epsilon} + \frac{76}{3} - \pi^2 + (\delta_R - 1)\right) + \frac{1}{2} C_A \left(\frac{2}{\epsilon} (L_{ijR} + \ell_{i1} + \ell_{j2}) - L_{ijR}^2 - (\ell_{i1} - \ell_{j2})^2 \right)\Big]$$
$$- C_A \left[\mathcal{I}_{ii}(q_1,q_2) - \mathcal{I}_{jj}(q_1,q_2)\right] \frac{q_{12}^2}{q_{12\perp ij}^2} L_{ijR}\left(\ell_{i1} - \ell_{j2}\right) + \mathcal{O}(\epsilon) \Big\} + (q_1 \leftrightarrow q_2) \;. \qquad (q_1^0, q_2^0 > 0)$$

It controls the size of the tree-level colour dipole correlations **Ti . Tj**. It is symmetric under the exchange i <-> j

$$F_{ijk}^{[S]}(q_2,q_1) = 2\pi\, \mathcal{I}_{ki}(q_1,q_2) \Big\{ L_{ijR} + \ell_{i1} + \ell_{j2} \qquad (q_1^0, q_2^0 > 0)$$
$$+ \Theta_{ij}^{(\mathrm{in})} \left[2\left(\frac{1}{\epsilon} - L_{ijR}\right) - 2\frac{q_{12}^2}{q_{12\perp ij}^2} (\ell_{i1} - \ell_{j2})\right] + \mathcal{O}(\epsilon) \Big\} + (q_1 \leftrightarrow q_2)$$

It is associated with non-abelian three-particle correlations

$$F_{ijk}^{[A]}(q_2,q_1) = \Big\{ \mathcal{I}_{ki}(q_1,q_2) \left[-\frac{2}{\epsilon} (\ell_{i1} + \ell_{j2}) + (\ell_{i1} - \ell_{j2})^2 + 2\frac{q_{12}^2}{q_{12\perp ij}^2} L_{ijR} (\ell_{i1} - \ell_{j2})\right]$$
$$+ \mathcal{O}(\epsilon) \Big\} - (q_1 \leftrightarrow q_2) \;. \qquad (q_1^0, q_2^0 > 0)$$

The charge-asymmetry contributions to the one-loop soft current

$$w_{ij}^{[A]}(q_1,q_2) = \Big\{ w_{ij}(q_1,q_2) \left[-\frac{2}{\epsilon} (\ell_{i1} + \ell_{j2}) + (\ell_{i1} - \ell_{j2})^2 \right]$$
$$+ \left[\mathcal{I}_{ii}(q_1,q_2) - \mathcal{I}_{jj}(q_1,q_2)\right] \frac{2\, q_{12}^2}{q_{12\perp ij}^2} L_{ijR} (\ell_{i1} - \ell_{j2}) + \mathcal{O}(\epsilon) \Big\} - (q_1 \leftrightarrow q_2)$$
$$(q_1^0, q_2^0 > 0)$$

It is antisymmetric under the exchange i <-> j

$$w_{ij}^{[A]}(q_1,q_2) = \left[F_{iji}^{[A]}(q_1,q_2) + F_{jii}^{[A]}(q_1,q_2)\right] - (i \leftrightarrow j)$$

Valid to arbitrary orders in the ε expansion

Soft qqbar radiation: squared amplitude and current

Catani, LC (2021)

Results

- **The one-loop squared current**

  A few comments about the charge-asymmetry

$$|\boldsymbol{J}(q_1,q_2)|^2_{(1\ell)} = -\frac{1}{2} T_R \sum_{\substack{i,j\in H\\ i\neq j}} \Big[\boldsymbol{T}_i \cdot \boldsymbol{T}_j \;\; w_{ij}^{[S]}(q_1,q_2) + \widetilde{\boldsymbol{D}}_i \cdot \boldsymbol{T}_j \;\; w_{ij}^{[A]}(q_1,q_2) \Big]$$
$$- T_R \sum_{\substack{i,j,k\in H\\ \mathrm{dist.}\{i,j,k\}}} T_i^a \, T_j^b \, T_k^c \left[f^{abc} \, F_{ijk}^{[S]}(q_1,q_2) + d^{abc} \Big(F_{ijk}^{[A]}(q_1,q_2) - \frac{1}{2} F_{iji}^{[A]}(q_1,q_2) - \frac{1}{2} F_{ijj}^{[A]}(q_1,q_2) \Big) \right]$$

$$w_{ij}^{[S]}(q_1,q_2) = \Big\{ w_{ij}(q_1,q_2) \Big[-C_F \Big(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \pi^2 + 8 + (\delta_R - 1) \Big) - \frac{4}{3} T_R \, N_f \Big(\frac{1}{\epsilon} + \frac{5}{3} \Big)$$
$$+ \frac{1}{3} C_A \Big(\frac{11}{\epsilon} + \frac{76}{3} - \pi^2 + (\delta_R - 1) \Big) + \frac{1}{2} C_A \Big(\frac{2}{\epsilon} (L_{ijR} + \ell_{i1} + \ell_{j2}) - L_{ijR}^2 - (\ell_{i1} - \ell_{j2})^2 \Big) \Big]$$
$$- C_A \left[\mathcal{I}_{ii}(q_1,q_2) - \mathcal{I}_{jj}(q_1,q_2) \right] \frac{q_{12}^2}{q_{12\perp ij}^2} \, L_{ijR} \left(\ell_{i1} - \ell_{j2} \right) + \mathcal{O}(\epsilon) \Big\} + (q_1 \leftrightarrow q_2) \;. \qquad (q_1^0, q_2^0 > 0)$$

$$F_{ijk}^{[S]}(q_2,q_1) = 2\pi \, \mathcal{I}_{ki}(q_1,q_2) \Big\{ L_{ijR} + \ell_{i1} + \ell_{j2} \qquad (q_1^0, q_2^0 > 0)$$
$$+ \Theta_{ij}^{(\mathrm{in})} \Big[2 \Big(\frac{1}{\epsilon} - L_{ijR} \Big) - 2 \frac{q_{12}^2}{q_{12\perp ij}^2} (\ell_{i1} - \ell_{j2}) \Big] + \mathcal{O}(\epsilon) \Big\} + (q_1 \leftrightarrow q_2)$$

$$\boxed{F_{ijk}^{[A]}(q_2,q_1)} = \Big\{ \mathcal{I}_{ki}(q_1,q_2) \Big[-\frac{2}{\epsilon} (\ell_{i1} + \ell_{j2}) + (\ell_{i1} - \ell_{j2})^2 + 2 \, \frac{q_{12}^2}{q_{12\perp ij}^2} \, L_{ijR} \left(\ell_{i1} - \ell_{j2} \right) \Big]$$
$$+ \; \mathcal{O}(\epsilon) \Big\} - (q_1 \leftrightarrow q_2) \;. \qquad (q_1^0, q_2^0 > 0)$$

$$\boxed{w_{ij}^{[A]}(q_1,q_2)} = \Big\{ w_{ij}(q_1,q_2) \Big[-\frac{2}{\epsilon} (\ell_{i1} + \ell_{j2}) + (\ell_{i1} - \ell_{j2})^2 \Big]$$
$$+ \; \left[\mathcal{I}_{ii}(q_1,q_2) - \mathcal{I}_{jj}(q_1,q_2) \right] \frac{2 \, q_{12}^2}{q_{12\perp ij}^2} \, L_{ijR} \left(\ell_{i1} - \ell_{j2} \right) + \mathcal{O}(\epsilon) \Big\} - (q_1 \leftrightarrow q_2)$$
$$(q_1^0, q_2^0 > 0)$$

The charge-asymmetry contributions are not vanishing only for specific classes of scattering amplitudes and quantities that are not invariant under charge conjugation

The charge-asymmetry contributions give a vanishing effect after phase-space symmetric integration over q1 and q2

Soft qqbar radiation: squared amplitude and current

Catani, LC (2021)

Results

- **The one-loop squared current**

  A few comments about the charge-asymmetry

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(1\ell)} = -\frac{1}{2}T_R \sum_{\substack{i,j\in H\\ i\neq j}} \left[\boldsymbol{T}_i\cdot\boldsymbol{T}_j \;\; w_{ij}^{[S]}(q_1,q_2) + \widetilde{\boldsymbol{D}}_i\cdot\boldsymbol{T}_j \;\; w_{ij}^{[A]}(q_1,q_2)\right]$$

$$-T_R \sum_{\substack{i,j,k\in H\\ \text{dist.}\{i,j,k\}}} T_i^a\,T_j^b\,T_k^c \left[f^{abc}\,F_{ijk}^{[S]}(q_1,q_2) + d^{abc}\left(F_{ijk}^{[A]}(q_1,q_2) - \frac{1}{2}F_{iji}^{[A]}(q_1,q_2) - \frac{1}{2}F_{ijj}^{[A]}(q_1,q_2)\right)\right]$$

$$w_{ij}^{[S]}(q_1,q_2) = \Big\{ w_{ij}(q_1,q_2)\,\Big[-C_F\left(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-\pi^2+8+(\delta_R-1)\right) - \frac{4}{3}T_R\,N_f\left(\frac{1}{\epsilon}+\frac{5}{3}\right)$$
$$+\frac{1}{3}C_A\left(\frac{11}{\epsilon}+\frac{76}{3}-\pi^2+(\delta_R-1)\right) + \frac{1}{2}C_A\left(\frac{2}{\epsilon}\left(L_{ijR}+\ell_{i1}+\ell_{j2}\right) - L_{ijR}^2 - (\ell_{i1}-\ell_{j2})^2\right)\Big]$$
$$-C_A\left[\mathcal{I}_{ii}(q_1,q_2)-\mathcal{I}_{jj}(q_1,q_2)\right]\frac{q_{12}^2}{q_{12\perp ij}^2}\,L_{ijR}\left(\ell_{i1}-\ell_{j2}\right) + \mathcal{O}(\epsilon)\Big\} + (q_1\leftrightarrow q_2)\;. \qquad (q_1^0,q_2^0>0)$$

$$F_{ijk}^{[S]}(q_2,q_1) = 2\pi\,\mathcal{I}_{ki}(q_1,q_2)\,\Big\{ L_{ijR}+\ell_{i1}+\ell_{j2} \qquad (q_1^0,q_2^0>0)$$
$$+\;\Theta_{ij}^{(\text{in})}\left[2\left(\frac{1}{\epsilon}-L_{ijR}\right) - 2\frac{q_{12}^2}{q_{12\perp ij}^2}(\ell_{i1}-\ell_{j2})\right] + \mathcal{O}(\epsilon)\Big\} + (q_1\leftrightarrow q_2)$$

$$\boxed{F_{ijk}^{[A]}(q_2,q_1)} = \Big\{ \mathcal{I}_{ki}(q_1,q_2)\left[-\frac{2}{\epsilon}\left(\ell_{i1}+\ell_{j2}\right) + (\ell_{i1}-\ell_{j2})^2 + 2\,\frac{q_{12}^2}{q_{12\perp ij}^2}\,L_{ijR}\left(\ell_{i1}-\ell_{j2}\right)\right]$$
$$+\;\mathcal{O}(\epsilon)\Big\} - (q_1\leftrightarrow q_2)\;. \qquad (q_1^0,q_2^0>0)$$

$$\boxed{w_{ij}^{[A]}(q_1,q_2)} = \Big\{ w_{ij}(q_1,q_2)\left[-\frac{2}{\epsilon}\left(\ell_{i1}+\ell_{j2}\right) + (\ell_{i1}-\ell_{j2})^2\right]$$
$$+\;\left[\mathcal{I}_{ii}(q_1,q_2)-\mathcal{I}_{jj}(q_1,q_2)\right]\frac{2\,q_{12}^2}{q_{12\perp ij}^2}\,L_{ijR}\left(\ell_{i1}-\ell_{j2}\right) + \mathcal{O}(\epsilon)\Big\} - (q_1\leftrightarrow q_2)$$
$$(q_1^0,q_2^0>0)$$

The charge-asymmetry contributions give non-vanishing effects to quantities in which the soft quark (or antiquark) is triggered either directly (bottom or charm quark) or indirectly (e.g., through its fragmentation function), in the final state

The Altarelli-Parisi splitting functions for collinear evolution of parton densities and fragmentation functions have a qqbar charge asymmetry which starts at $\mathcal{O}(\alpha^3_S)$

Catani, de Florian, Rodrigo (2003)
Catani, de Florian Rodrigo, Vogelsang (2004)
Moch, Vermaseren, Vogt (2004)
Mitov, Moch, Vogt (2004)

Soft qqbar radiation: squared amplitude and current

Catani, LC (2021)

Results

- **The one-loop squared current**

  A few comments about the charge-asymmetry

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(1\ell)} = -\frac{1}{2} T_R \sum_{\substack{i,j \in H \\ i \neq j}} \left[\boldsymbol{T}_i \cdot \boldsymbol{T}_j \;\; w_{ij}^{[S]}(q_1,q_2) + \widetilde{\boldsymbol{D}}_i \cdot \boldsymbol{T}_j \;\; w_{ij}^{[A]}(q_1,q_2) \right]$$

$$- T_R \sum_{\substack{i,j,k \in H \\ \text{dist.}\{i,j,k\}}} T_i^a \, T_j^b \, T_k^c \left[f^{abc} \, F_{ijk}^{[S]}(q_1,q_2) + d^{abc} \Big(F_{ijk}^{[A]}(q_1,q_2) - \frac{1}{2} F_{iji}^{[A]}(q_1,q_2) - \frac{1}{2} F_{ijj}^{[A]}(q_1,q_2) \Big) \right]$$

$$w_{ij}^{[S]}(q_1,q_2) = \Big\{ w_{ij}(q_1,q_2) \left[-C_F \left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \pi^2 + 8 + (\delta_R - 1) \right) - \frac{4}{3} T_R \, N_f \left(\frac{1}{\epsilon} + \frac{5}{3} \right) \right.$$
$$\left. + \frac{1}{3} C_A \left(\frac{11}{\epsilon} + \frac{76}{3} - \pi^2 + (\delta_R - 1) \right) + \frac{1}{2} C_A \left(\frac{2}{\epsilon} (L_{ijR} + \ell_{i1} + \ell_{j2}) - L_{ijR}^2 - (\ell_{i1} - \ell_{j2})^2 \right) \right]$$
$$- C_A \left[\mathcal{I}_{ii}(q_1,q_2) - \mathcal{I}_{jj}(q_1,q_2) \right] \frac{q_{12}^2}{q_{12\perp ij}^2} \, L_{ijR} \, (\ell_{i1} - \ell_{j2}) + \mathcal{O}(\epsilon) \Big\} + (q_1 \leftrightarrow q_2) \;. \qquad (q_1^0, q_2^0 > 0)$$

$$F_{ijk}^{[S]}(q_2,q_1) = 2\pi \, \mathcal{I}_{ki}(q_1,q_2) \Big\{ L_{ijR} + \ell_{i1} + \ell_{j2} \qquad (q_1^0, q_2^0 > 0)$$
$$+ \; \Theta_{ij}^{(\text{in})} \left[2 \left(\frac{1}{\epsilon} - L_{ijR} \right) - 2 \frac{q_{12}^2}{q_{12\perp ij}^2} (\ell_{i1} - \ell_{j2}) \right] + \mathcal{O}(\epsilon) \Big\} + (q_1 \leftrightarrow q_2)$$

$$\boxed{F_{ijk}^{[A]}(q_2,q_1)} = \Big\{ \mathcal{I}_{ki}(q_1,q_2) \left[-\frac{2}{\epsilon} (\ell_{i1} + \ell_{j2}) + (\ell_{i1} - \ell_{j2})^2 + 2 \frac{q_{12}^2}{q_{12\perp ij}^2} L_{ijR} \, (\ell_{i1} - \ell_{j2}) \right]$$
$$+ \; \mathcal{O}(\epsilon) \Big\} - (q_1 \leftrightarrow q_2) \;. \qquad (q_1^0, q_2^0 > 0)$$

$$\boxed{w_{ij}^{[A]}(q_1,q_2)} = \Big\{ w_{ij}(q_1,q_2) \left[-\frac{2}{\epsilon} (\ell_{i1} + \ell_{j2}) + (\ell_{i1} - \ell_{j2})^2 \right]$$
$$+ \; \left[\mathcal{I}_{ii}(q_1,q_2) - \mathcal{I}_{jj}(q_1,q_2) \right] \frac{2\, q_{12}^2}{q_{12\perp ij}^2} L_{ijR} \, (\ell_{i1} - \ell_{j2}) + \mathcal{O}(\epsilon) \Big\} - (q_1 \leftrightarrow q_2)$$
$$(q_1^0, q_2^0 > 0)$$

The charge-asymmetry contributions give non-vanishing effects to quantities in which the soft quark (or antiquark) is triggered either directly (bottom or charm quark) or indirectly (e.g., through its fragmentation function), in the final state

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Catani, de Florian, Rodrigo (2003)
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Moch, Vermaseren, Vogt (2004)
Mitov, Moch, Vogt (2004)

The charge-asymmetry contributions vanish if $\mathcal{M}$ is a pure multi gluon scattering amplitude (only gluon external lines), with no additional external qqbar pairs or colourless particles

Soft qqbar radiation: squared amplitude and current

Catani, LC (2021)

Results

- **The one-loop squared current**

  A few comments about the singularity

$$|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(1\ell)} = -\frac{1}{2} T_R \sum_{\substack{i,j\in H \\ i\neq j}} \Big[\boldsymbol{T}_i \cdot \boldsymbol{T}_j \;\; w_{ij}^{[S]}(q_1,q_2) + \widetilde{\boldsymbol{D}}_i \cdot \boldsymbol{T}_j \;\; w_{ij}^{[A]}(q_1,q_2) \Big]$$
$$- T_R \sum_{\substack{i,j,k\in H \\ \mathrm{dist.}\{i,j,k\}}} T_i^a \, T_j^b \, T_k^c \left[f^{abc} \, F_{ijk}^{[S]}(q_1,q_2) + d^{abc} \Big(F_{ijk}^{[A]}(q_1,q_2) - \frac{1}{2} F_{iji}^{[A]}(q_1,q_2) - \frac{1}{2} F_{ijj}^{[A]}(q_1,q_2) \Big) \right]$$

$$w_{ij}^{[S]}(q_1,q_2) = \Big\{ w_{ij}(q_1,q_2) \; \Big[-C_F \Big(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \pi^2 + 8 + (\delta_R - 1) \Big) - \frac{4}{3} T_R \, N_f \Big(\frac{1}{\epsilon} + \frac{5}{3} \Big)$$
$$+ \frac{1}{3} C_A \Big(\frac{11}{\epsilon} + \frac{76}{3} - \pi^2 + (\delta_R - 1) \Big) + \frac{1}{2} C_A \Big(\frac{2}{\epsilon} \left(L_{ijR} + \ell_{i1} + \ell_{j2} \right) - L_{ijR}^2 - (\ell_{i1} - \ell_{j2})^2 \Big) \Big]$$
$$- C_A \left[\mathcal{I}_{ii}(q_1,q_2) - \mathcal{I}_{jj}(q_1,q_2) \right] \boxed{\frac{q_{12}^2}{q_{12\perp ij}^2} \, L_{ijR}} (\ell_{i1} - \ell_{j2}) + \mathcal{O}(\epsilon) \Big\} + (q_1 \leftrightarrow q_2) \;\; . \qquad (q_1^0, q_2^0 > 0)$$

$$F_{ijk}^{[S]}(q_2,q_1) = 2\pi \, \mathcal{I}_{ki}(q_1,q_2) \, \Big\{ L_{ijR} + \ell_{i1} + \ell_{j2} \qquad (q_1^0, q_2^0 > 0)$$
$$+ \; \Theta_{ij}^{(\mathrm{in})} \Big[2 \Big(\frac{1}{\epsilon} - L_{ijR} \Big) - 2 \boxed{\frac{q_{12}^2}{q_{12\perp ij}^2} (\ell_{i1} - \ell_{j2})} \Big] + \mathcal{O}(\epsilon) \Big\} + (q_1 \leftrightarrow q_2)$$

$$\ell_{i1} + \ell_{j2} = \ln \frac{(p_i \cdot q_1)(p_j \cdot q_2)}{(p_i \cdot q_{12})(p_j \cdot q_{12})} \, , \;\; \ell_{i1} - \ell_{j2} = \ln \frac{(p_i \cdot q_1)(p_j \cdot q_{12})}{(p_i \cdot q_{12})(p_j \cdot q_2)} \, , \;\; L_{ij} = L_{ijR} + 2 i \pi \, \Theta_{ij}^{(\mathrm{in})}$$

$$F_{ijk}^{[A]}(q_2,q_1) = \Big\{ \mathcal{I}_{ki}(q_1,q_2) \, \Big[-\frac{2}{\epsilon} \left(\ell_{i1} + \ell_{j2} \right) + (\ell_{i1} - \ell_{j2})^2 + 2 \boxed{\frac{q_{12}^2}{q_{12\perp ij}^2} \, L_{ijR}} (\ell_{i1} - \ell_{j2}) \Big]$$
$$+ \; \mathcal{O}(\epsilon) \Big\} - (q_1 \leftrightarrow q_2) \;\; . \qquad (q_1^0, q_2^0 > 0)$$

$$L_{ijR} = \ln \frac{(p_i \cdot q_{12})(p_j \cdot q_{12})}{(p_i \cdot p_j)(q_1 \cdot q_2)} = \ln \Big(1 + \frac{q_{12\perp ij}^2}{q_{12}^2} \Big) \;\; , \qquad \Theta_{ij}^{(\mathrm{in})} \equiv \Theta(-p_i^0) \, \Theta(-p_j^0)$$

$$w_{ij}^{[A]}(q_1,q_2) = \Big\{ w_{ij}(q_1,q_2) \, \Big[-\frac{2}{\epsilon} \left(\ell_{i1} + \ell_{j2} \right) + (\ell_{i1} - \ell_{j2})^2 \Big]$$
$$+ \; \left[\mathcal{I}_{ii}(q_1,q_2) - \mathcal{I}_{jj}(q_1,q_2) \right] \boxed{\frac{2 \, q_{12}^2}{q_{12\perp ij}^2} \, L_{ijR}} (\ell_{i1} - \ell_{j2}) + \mathcal{O}(\epsilon) \Big\} - (q_1 \leftrightarrow q_2)$$
$$(q_1^0, q_2^0 > 0)$$

Soft qqbar radiation: squared amplitude and current

Catani, LC (2021)

Results

- **The one-loop squared current**

A few comments about the singularity

$$|\boldsymbol{\hat{J}}(q_1,q_2)|^2_{(1\ell)} = -\frac{1}{2}T_R \sum_{\substack{i,j\in H\\ i\neq j}} \Big[\boldsymbol{T}_i\cdot\boldsymbol{T}_j\;\; w_{ij}^{[S]}(q_1,q_2) + \widetilde{\boldsymbol{D}}_i\cdot\boldsymbol{T}_j\;\; w_{ij}^{[A]}(q_1,q_2)\Big]$$
$$-T_R \sum_{\substack{i,j,k\in H\\ \mathrm{dist.}\{i,j,k\}}} T_i^a\,T_j^b\,T_k^c \left[f^{abc}\,F_{ijk}^{[S]}(q_1,q_2) + d^{abc}\left(F_{ijk}^{[A]}(q_1,q_2) - \frac{1}{2}F_{iji}^{[A]}(q_1,q_2) - \frac{1}{2}F_{ijj}^{[A]}(q_1,q_2)\right)\right]$$

$$w_{ij}^{[S]}(q_1,q_2) = \Big\{w_{ij}(q_1,q_2)\,\Big[-C_F\Big(\frac{2}{\epsilon^2}+\frac{3}{\epsilon}-\pi^2+8+(\delta_R-1)\Big) - \frac{4}{3}T_R\,N_f\Big(\frac{1}{\epsilon}+\frac{5}{3}\Big)$$
$$+\frac{1}{3}C_A\Big(\frac{11}{\epsilon}+\frac{76}{3}-\pi^2+(\delta_R-1)\Big) + \frac{1}{2}C_A\Big(\frac{2}{\epsilon}(L_{ijR}+\ell_{i1}+\ell_{j2}) - L_{ijR}^2 - (\ell_{i1}-\ell_{j2})^2\Big)\Big]$$
$$-C_A\left[\mathcal{I}_{ii}(q_1,q_2)-\mathcal{I}_{jj}(q_1,q_2)\right]\boxed{\frac{q_{12}^2}{q_{12\perp ij}^2}\,L_{ijR}}\,(\ell_{i1}-\ell_{j2}) + \mathcal{O}(\epsilon)\Big\} + (q_1\leftrightarrow q_2)\;. \qquad (q_1^0,q_2^0>0)$$

$$(q^2_{12\perp ij})^{-1}\,L_{ijR} \to (q^2_{12})^{-1}$$

$$F_{ijk}^{[S]}(q_2,q_1) = 2\pi\,\mathcal{I}_{ki}(q_1,q_2)\,\Big\{L_{ijR}+\ell_{i1}+\ell_{j2}$$
$$+\;\Theta_{ij}^{(\mathrm{in})}\Big[2\Big(\frac{1}{\epsilon}-L_{ijR}\Big) - \boxed{2\frac{q_{12}^2}{q_{12\perp ij}^2}(\ell_{i1}-\ell_{j2})}\Big] + \mathcal{O}(\epsilon)\Big\} + (q_1\leftrightarrow q_2) \qquad (q_1^0,q_2^0>0)$$

It is the only contribution to present transverse-momentum singularity

$$\ell_{i1}+\ell_{j2} = \ln\frac{(p_i\cdot q_1)(p_j\cdot q_2)}{(p_i\cdot q_{12})(p_j\cdot q_{12})}\,,\quad \ell_{i1}-\ell_{j2} = \ln\frac{(p_i\cdot q_1)(p_j\cdot q_{12})}{(p_i\cdot q_{12})(p_j\cdot q_2)}\,,\quad L_{ij} = L_{ijR}+2i\pi\,\Theta_{ij}^{(\mathrm{in})}$$

$$F_{ijk}^{[A]}(q_2,q_1) = \Big\{\mathcal{I}_{ki}(q_1,q_2)\,\Big[-\frac{2}{\epsilon}(\ell_{i1}+\ell_{j2}) + (\ell_{i1}-\ell_{j2})^2 + \boxed{2\frac{q_{12}^2}{q_{12\perp ij}^2}\,L_{ijR}}(\ell_{i1}-\ell_{j2})\Big]$$
$$+\;\mathcal{O}(\epsilon)\Big\} - (q_1\leftrightarrow q_2)\;. \qquad (q_1^0,q_2^0>0)$$

$$(q^2_{12\perp ij})^{-1}\,L_{ijR} \to (q^2_{12})^{-1}$$

$$L_{ijR} = \ln\frac{(p_i\cdot q_{12})(p_j\cdot q_{12})}{(p_i\cdot p_j)(q_1\cdot q_2)} = \ln\Big(1+\frac{q^2_{12\perp ij}}{q^2_{12}}\Big)\,,\qquad \Theta_{ij}^{(\mathrm{in})} \equiv \Theta(-p_i^0)\,\Theta(-p_j^0)$$

$$w_{ij}^{[A]}(q_1,q_2) = \Big\{w_{ij}(q_1,q_2)\,\Big[-\frac{2}{\epsilon}(\ell_{i1}+\ell_{j2}) + (\ell_{i1}-\ell_{j2})^2\Big]$$
$$+\;\left[\mathcal{I}_{ii}(q_1,q_2)-\mathcal{I}_{jj}(q_1,q_2)\right]\boxed{\frac{2\,q_{12}^2}{q_{12\perp ij}^2}\,L_{ijR}}\,(\ell_{i1}-\ell_{j2}) + \mathcal{O}(\epsilon)\Big\} - (q_1\leftrightarrow q_2)$$
$$(q_1^0,q_2^0>0)$$

$$(q^2_{12\perp ij})^{-1}\,L_{ijR} \to (q^2_{12})^{-1}$$

Soft qqbar radiation: squared amplitude and current

Catani, LC (2021)

Results

- **The one-loop squared current**

  A few comments about the singularity

$$
|\hat{\boldsymbol{J}}(q_1,q_2)|^2_{(1\ell)} = -\frac{1}{2}\, T_R \sum_{\substack{i,j\in H\\ i\neq j}} \Big[\boldsymbol{T}_i\cdot\boldsymbol{T}_j \;\; w^{[S]}_{ij}(q_1,q_2) + \widetilde{\boldsymbol{D}}_i\cdot\boldsymbol{T}_j \;\; w^{[A]}_{ij}(q_1,q_2) \Big]
$$
$$
- T_R \sum_{\substack{i,j,k\in H\\ \mathrm{dist.}\{i,j,k\}}} T_i^a\, T_j^b\, T_k^c \Big[f^{abc}\, F^{[S]}_{ijk}(q_1,q_2) + d^{abc}\Big(F^{[A]}_{ijk}(q_1,q_2) - \frac{1}{2} F^{[A]}_{iji}(q_1,q_2) - \frac{1}{2} F^{[A]}_{ijj}(q_1,q_2)\Big)\Big]
$$

$$
\begin{aligned}
w^{[S]}_{ij}(q_1,q_2) = \Big\{ & w_{ij}(q_1,q_2)\, \Big[-C_F \Big(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} - \pi^2 + 8 + (\delta_R - 1) \Big) - \frac{4}{3} T_R\, N_f \Big(\frac{1}{\epsilon} + \frac{5}{3} \Big) \\
& + \frac{1}{3} C_A \Big(\frac{11}{\epsilon} + \frac{76}{3} - \pi^2 + (\delta_R - 1) \Big) + \frac{1}{2} C_A \Big(\frac{2}{\epsilon} (L_{ijR} + \ell_{i1} + \ell_{j2}) - L^2_{ijR} - (\ell_{i1} - \ell_{j2})^2 \Big) \Big] \\
& - C_A \left[\mathcal{I}_{ii}(q_1,q_2) - \mathcal{I}_{jj}(q_1,q_2) \right] \frac{q^2_{12}}{q^2_{12\perp ij}}\, L_{ijR}\, (\ell_{i1} - \ell_{j2}) + \mathcal{O}(\epsilon) \Big\} + (q_1 \leftrightarrow q_2) \; . \\
& \qquad (q_1^0, q_2^0 > 0)
\end{aligned}
$$

$$
\begin{aligned}
F^{[S]}_{ijk}(q_2,q_1) \;=\; & 2\pi\, \mathcal{I}_{ki}(q_1,q_2) \Big\{ L_{ijR} + \ell_{i1} + \ell_{j2} \qquad (q_1^0, q_2^0 > 0) \\
& + \; \Theta^{(\mathrm{in})}_{ij} \Big[2 \Big(\frac{1}{\epsilon} - L_{ijR} \Big) - \boxed{2 \frac{q^2_{12}}{q^2_{12\perp ij}} (\ell_{i1} - \ell_{j2})} + \mathcal{O}(\epsilon) \Big\} + (q_1 \leftrightarrow q_2)
\end{aligned}
$$

It is the only contribution to present transverse-momentum singularity

This singularity contributes for the class of processes with initial-state colliding partons i and j and two or more final-state hard partons (e.g. dijet or heavy-quark production)

$$
\begin{aligned}
F^{[A]}_{ijk}(q_2,q_1) \;=\; & \Big\{ \mathcal{I}_{ki}(q_1,q_2) \Big[-\frac{2}{\epsilon} (\ell_{i1} + \ell_{j2}) + (\ell_{i1} - \ell_{j2})^2 + 2\, \frac{q^2_{12}}{q^2_{12\perp ij}}\, L_{ijR}\, (\ell_{i1} - \ell_{j2}) \Big] \\
& + \; \mathcal{O}(\epsilon) \Big\} - (q_1 \leftrightarrow q_2) \; . \qquad (q_1^0, q_2^0 > 0)
\end{aligned}
$$

$$
\begin{aligned}
w^{[A]}_{ij}(q_1,q_2) \;=\; & \Big\{ w_{ij}(q_1,q_2) \Big[-\frac{2}{\epsilon} (\ell_{i1} + \ell_{j2}) + (\ell_{i1} - \ell_{j2})^2 \Big] \\
& + \; \left[\mathcal{I}_{ii}(q_1,q_2) - \mathcal{I}_{jj}(q_1,q_2) \right] \frac{2\, q^2_{12}}{q^2_{12\perp ij}}\, L_{ijR}\, (\ell_{i1} - \ell_{j2}) + \mathcal{O}(\epsilon) \Big\} - (q_1 \leftrightarrow q_2) \\
& (q_1^0, q_2^0 > 0)
\end{aligned}
$$

It contributes in the same class of processes that is sensitive to effects due to the violation of strict collinear factorization

Catani, de Florian, Rodrigo (2011)

Outlook

- We have derived the explicit form of the ε-pole (divergent) contributions of the multi parton soft current.

- We have presented the one-loop soft current for the emission of a soft qqbar pair, considering arbitrary kinematical regions of the soft-parton and the hard-parton momenta. We have included all the finite terms at $\mathcal{O}(\varepsilon^0)$.

- The one-loop qqbar soft current includes powers of logarithmic functions but no dilog functions.

- The one-loop soft current produces a new type of singularity if the soft-qqbar pair is radiated with a vanishing transverse momentum with respect to the directions of two colliding hard partons in the initial state <- pure non-abelian character. It can appear also in the double soft-gluon emission.

- At the squared amplitude level, the transverse momentum singularity contributes to the cross section of processes with two initial-state colliding partons and two (or more) hard partons in the final state.

- At variance with the case of multi-gluon radiation, the emission of soft fermions and anti fermions lead to charge asymmetry effects.

Thank you!!!

Backup slides

The sketchy form of the factorization formula

Catani, LC (2021)

Catani, Grazzini (2000)