Muon g-2 ⇔ $\Delta \alpha$ connection

Massimo Passera
INFN Padova

DESY Zeuthen Theory Seminar
December 2 2021
Muon g-2: FNAL confirms BNL

\[ a_\mu^{\text{EXP}} = (116592089 \pm 63) \times 10^{-11} [0.54\text{ppm}] \quad \text{BNL E821} \]
\[ a_\mu^{\text{EXP}} = (116592040 \pm 54) \times 10^{-11} [0.46\text{ppm}] \quad \text{FNAL E989 Run 1} \]
\[ a_\mu^{\text{EXP}} = (116592061 \pm 41) \times 10^{-11} [0.35\text{ppm}] \quad \text{WA} \]

- FNAL aims at 16 x 10^{-11}. First 4 runs completed, 5th just started.
- Muon g-2 proposal at J-PARC: Phase-1 with ~ BNL precision.
Muon g-2: the Standard Model prediction

Muon g-2 $\iff \Delta \alpha$ connection

The MUonE project
Muon g-2: the Standard Model prediction

\[ a_\mu^{\text{QED}} = \left(\frac{1}{2}\right)\left(\frac{\alpha}{\pi}\right) \quad \text{Schwinger 1948} \]

\[ + \ 0.765857426 \ (16) \left(\frac{\alpha}{\pi}\right)^2 \]

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04

\[ + \ 24.05050988 \ (28) \left(\frac{\alpha}{\pi}\right)^3 \]

Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek '99; MP '04; Friot, Greynat & de Rafael '05, Ananthanarayan, Friot, Ghosh 2020

\[ + \ 130.8780 \ (60) \left(\frac{\alpha}{\pi}\right)^4 \]

Kinoshita & Lindquist '81, …, Kinoshita & Nio '04, '05; Aoyama, Hayakawa, Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015; Steinhauser et al. 2013, 2015 & 2016 (all electron & \(\tau\) loops, analytic); Laporta, PLB 2017 (mass independent term) COMPLETED\(^2\)!

\[ + \ 750.86 \ (88) \left(\frac{\alpha}{\pi}\right)^5 \] COMPLETED!


Volkov 1909.08015: \(A_1^{(10)}\) [no lept loops] at variance, but negligible \(\delta a_\mu \sim 6 \times 10^{-14}\)

**Adding up, we get:**

\[ a_\mu^{\text{QED}} = 116584718.931 \ (19)(100)(23) \times 10^{-11} \]

mainly from 4-loop coeff. unc. \(\leftarrow\) 6-loop \(\rightarrow\) from \(\alpha\) (Cs)

\[ \alpha = 1/137.035999046(27) \ [0.2 \text{ppb}] \] Parker et al 2018

WP20 value

\[ a_\mu^{\text{QED}} \sim 1.4 \times 10^{-12} \] with new LKB Paris \(\alpha\) (Rb) value (Morel et al 2020)

---

\[ a = 1/137.035999046(27) \ [0.2 \text{ppb}] \] Parker et al 2018

WP20 value

M Passera  DESY  02.12.2021

Shift down in \(a_\mu^{\text{QED}} \sim 1.4 \times 10^{-12} \) with new LKB Paris \(\alpha\) (Rb) value (Morel et al 2020)
The electroweak contribution

- **One-loop term:**

\[
a_{\mu}^{\text{EW}}(1\text{-loop}) = \frac{5G_{\mu}m_{\mu}^2}{24\sqrt{2}\pi^2} \left[ 1 + \frac{1}{5} \left( 1 - 4\sin^2\theta_W \right)^2 + O\left( \frac{m_{\mu}^2}{M_{Z,W,H}^2} \right) \right] \approx 195 \times 10^{-11}
\]

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda; Studenikin et al. '80s

- **One-loop plus higher-order terms:**

\[
a_{\mu}^{\text{EW}} = 153.6 \text{ (1.0) } \times 10^{-11}
\]

Kukhito et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013, Ishikawa, Nakazawa, Yasui, 2019.

Hadronic loop uncertainties (and 3-loop nonleading logs).

WP20 value
The hadronic LO contribution

\[ a^\text{HLO}_\mu = \frac{1}{4\pi^3} \int_{m^2_\pi}^{\infty} ds K(s) \sigma_{\text{had}}^{(0)}(s) \]

\[ K(s) = \int_0^1 dx \frac{x^2 (1 - x)}{x^2 + (1 - x) \left( s/m^2_\mu \right)} \]

\[
\begin{align*}
a^\text{HLO}_\mu &= 6895 (33) \times 10^{-11} \\
&= 6939 (40) \times 10^{-11} \\
&= 6928 (24) \times 10^{-11} \\
&= 6931 (40) \times 10^{-11} (0.6\%) 
\end{align*}
\]

WP20 value obtained merging conservatively DHMZ + KNT + constraints from CHHKS

Colangelo, Hoferichter, Hoid, Kubis, Stoffer 2018-19

Radiative Corrections to \( \sigma(s) \) are crucial.

The low-energy hadronic cross section

\[ R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\frac{4\pi\alpha(s)^2}{3s}} \]

Keshavarzi, Nomura Teubner
PRD 2018

Davier, Hoecker, Malaescu, Zhang
EPJC 2020
Great progress in lattice QCD results. The BMW collaboration reached 0.8% precision:

\[ a_{\mu}^{HLO} = 7075(23)_{\text{stat}}(50)_{\text{syst}} [55]_{\text{tot}} \times 10^{-11} \]

2–2.5σ tension with the dispersive evaluations. BMW collaboration 2021

Borsanyi et al (BMWc), Nature 2021
The hadronic HO VP contribution

- $O(\alpha^3)$ contributions of diagrams containing HVP insertions:

\[
a_\mu^{\text{HNLO}}(vp) = -98.3 (7) \times 10^{-11}
\]

Krause '96; Keshavarzi, Nomura, Teubner 2019; WP20.

- $O(\alpha^4)$ contributions of diagrams containing HVP insertions:

\[
a_\mu^{\text{HNNLO}}(vp) = 12.4 (1) \times 10^{-11}
\]

Kurz, Liu, Marquard, Steinhauser 2014
Hadronic light-by-light at $O(\alpha^3)$

This term had a troubled life! But nowadays:

\[ a_\mu^{HNLO}(lbl) = 80 \pm 40 \times 10^{-11} \quad \text{Knecht & Nyffeler '02} \]
\[ = 136 \pm 25 \times 10^{-11} \quad \text{Melnikov & Vainshtein '03} \]
\[ = 105 \pm 26 \times 10^{-11} \quad \text{Prades, de Rafael, Vainshtein '09} \]
\[ = 100 \pm 29 \times 10^{-11} \quad \text{Jegerlehner, arXiv:1705.00263} \]
\[ = 92 \pm 19 \times 10^{-11} \quad \text{WP20 (phenomenology)} \]

Significant improvements due to data-driven dispersive approach.

Lattice: RBC: $82(35)\times10^{-11}$ 1911.08123  Mainz: $110(15)\times10^{-11}$ 2104.02632

Hadronic light-by-light at $O(\alpha^4)$

\[ a_\mu^{NNLO}(lbl) = 2 \pm 1 \times 10^{-11} \]

Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014; WP20
Comparing the SM prediction with the measured muon g-2 value:

\[ a_{\mu}^{\text{EXP}} = 116592061 (41) \times 10^{-11} \quad \text{BNL+FNAL} \]

\[ a_{\mu}^{\text{SM}} = 116591810 (43) \times 10^{-11} \quad \text{WP20} \]

\[ \Delta a_{\mu} = a_{\mu}^{\text{EXP}} - a_{\mu}^{\text{SM}} = 251 (59) \times 10^{-11} \quad 4.2 \sigma \]

If BMW 2021 HLO instead of WP20, EXP & SM differ only by 1.6\( \sigma \)

Is \( \Delta a_{\mu} \) due to new physics beyond the SM? Could be due to:

- NP at the weak scale and weakly coupled to SM particles
- NP very heavy and strongly coupled to SM particles
- NP very light (\( \Lambda \lesssim 1 \text{ GeV} \)) and feebly coupled to SM particles
Muon $g-2 \iff \Delta \alpha$ connection

Marciano, MP, Sirlin 2008 & 2010
Keshavarzi, Marciano, MP, Sirlin 2020
Can $\Delta a_\mu$ be due to missing contributions in the hadronic $\sigma(s)$?

An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$.

Consider:

$$\begin{align*}
ad_{\text{HLO}} & \rightarrow \\
\int_{4m^2_\pi}^{s_u} ds \ f(s) \ \sigma(s), & \quad f(s) = \frac{K(s)}{4\pi^3}, \ s_u < M_Z^2, \\
\Delta\alpha_{\text{had}}^{(5)} & \rightarrow \\
\int_{4m^2_\pi}^{s_u} ds \ g(s) \ \sigma(s), & \quad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},
\end{align*}$$

and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

$\epsilon > 0$, in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$$
How much does the $M_H$ upper bound from the EW fit change when we shift up $\sigma(s)$ by $\Delta \sigma(s)$ [and thus $\Delta \alpha_{\text{had}}^{(5)}(M_Z)$] to fix $\Delta a_\mu$?
### Major update: Higgs discovered, improved EW observables ($M_W$, $\sin^2\theta$, $M_{\text{top}}$, …), updates to $\sigma(s)$, theory improvements, global fit, …

Keshavarzi, Marciano, MP, Sirlin, PRD 2020 (using Gfitter)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Input value</th>
<th>Reference</th>
<th>Fit result</th>
<th>Result w/o input value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_W$ (GeV)</td>
<td>80.379(12)</td>
<td>[5]</td>
<td>80.359(3)</td>
<td>80.357(4)(5)</td>
</tr>
<tr>
<td>$M_H$ (GeV)</td>
<td>125.10(14)</td>
<td>[5]</td>
<td>125.10(14)</td>
<td></td>
</tr>
<tr>
<td>$\Delta a_{\text{had}}(M_Z^2) \times 10^4$</td>
<td>276.1(1.1)</td>
<td>[23]</td>
<td>275.8(1.1)</td>
<td>94( ^{+18}_{-18} )</td>
</tr>
<tr>
<td>$m_t$ (GeV)</td>
<td>172.9(4)</td>
<td>[5]</td>
<td>173.0(4)</td>
<td></td>
</tr>
<tr>
<td>$\alpha_s(M_Z^2)$</td>
<td>0.1179(10)</td>
<td>[5]</td>
<td>0.1180(7)</td>
<td></td>
</tr>
<tr>
<td>$M_Z$ (GeV)</td>
<td>91.1876(21)</td>
<td>[5]</td>
<td>91.1883(20)</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_Z$ (GeV)</td>
<td>2.4952(23)</td>
<td>[5]</td>
<td>2.4940(4)</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_W$ (GeV)</td>
<td>2.085(42)</td>
<td>[5]</td>
<td>2.090(34)</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{had}}^0$ (nb)</td>
<td>41.541(37)</td>
<td>[108]</td>
<td>41.490(4)</td>
<td></td>
</tr>
<tr>
<td>$R_l^0$</td>
<td>20.767(25)</td>
<td>[108]</td>
<td>20.732(4)</td>
<td></td>
</tr>
<tr>
<td>$R_c^0$</td>
<td>0.1721(30)</td>
<td>[108]</td>
<td>0.17222(8)</td>
<td></td>
</tr>
<tr>
<td>$R_b^0$</td>
<td>0.21629(66)</td>
<td>[108]</td>
<td>0.21581(8)</td>
<td></td>
</tr>
<tr>
<td>$\bar{m}_t$ (GeV)</td>
<td>1.27(2)</td>
<td>[5]</td>
<td>1.27(2)</td>
<td></td>
</tr>
<tr>
<td>$\bar{m}_b$ (GeV)</td>
<td>4.18( ^{+0.03}_{-0.02} )</td>
<td>[5]</td>
<td>4.18( ^{+0.03}_{-0.02} )</td>
<td></td>
</tr>
<tr>
<td>$A_{FB}^{0,\text{J}}$</td>
<td>0.0171(10)</td>
<td>[108]</td>
<td>0.01622(7)</td>
<td></td>
</tr>
<tr>
<td>$A_{FB}^{0,\text{c}}$</td>
<td>0.0707(35)</td>
<td>[108]</td>
<td>0.0737(2)</td>
<td></td>
</tr>
<tr>
<td>$A_{FB}^{0,\text{b}}$</td>
<td>0.0992(16)</td>
<td>[108]</td>
<td>0.1031(2)</td>
<td></td>
</tr>
<tr>
<td>$A_{\epsilon}$</td>
<td>0.1499(18)</td>
<td>[75,108]</td>
<td>0.1471(3)</td>
<td></td>
</tr>
<tr>
<td>$A_{\epsilon}$</td>
<td>0.670(27)</td>
<td>[108]</td>
<td>0.6679(2)</td>
<td></td>
</tr>
<tr>
<td>$A_{\epsilon}$</td>
<td>0.923(20)</td>
<td>[108]</td>
<td>0.93462(7)</td>
<td></td>
</tr>
<tr>
<td>$\sin^2\theta_{\text{eff}}(Q_{FB})$</td>
<td>0.2324(12)</td>
<td>[108]</td>
<td>0.23152(4)</td>
<td>0.23152(4)(4)</td>
</tr>
<tr>
<td>$\sin^2\theta_{\text{eff}}(\text{Had Coll})$</td>
<td>0.23140(23)</td>
<td>[100]</td>
<td>0.23152(4)</td>
<td>0.23152(4)(4)</td>
</tr>
</tbody>
</table>
Shifts $\Delta \sigma(s)$ to fix $\Delta a_\mu$ are possible, but conflict with the EW fit if they occur above $\sim 1$ GeV.
How large are the required shifts $\Delta\sigma(s)$?

Shifts below $\sim 1$ GeV conflict with the quoted exp. precision of $\sigma(s)$.

Keshavarzi, Marciano, MP, Sirlin, PRD 2020 (updated 2021)
What happens to the electron g-2?
The electron g-2 no longer provides the best value of $\alpha$

- The 2008 measurement of the electron g-2 is:
  $$a_e^{\text{EXP}} = 11596521807.3 \times 10^{-13}$$  
  Hanneke et al, PRL100 (2008) 120801
  vs. old (factor of 15 improvement, 1.8\$ difference):
  $$a_e^{\text{EXP}} = 11596521883 \times 10^{-13}$$  
  Van Dyck et al, PRL59 (1987) 26

- Equate $a_e^{\text{SM}}(\alpha) = a_e^{\text{EXP}}$ → “g-e-2” determination of alpha:
  $$\alpha^{-1} = 137.035\,999\,151\,(33) \quad [0.24\,\text{ppb}]$$

- The best determination of $\alpha$ is obtained via atomic interferometry:
  $$\alpha^{-1} = 137.035\,999\,046\,(27) \quad [0.20\,\text{ppb}]$$  
  Parker et al, Science 360 (2018) 192 (Cs)
  $$\alpha^{-1} = 137.035\,999\,206\,(11) \quad [0.08\,\text{ppb}]$$  
  Morel et al, Nature 588 (2020) 61 (Rb)

2018→2020: improvement in precision, but 5.4\$ difference!
Morel et al, Nature 588 (2020) 61
Using the best determinations of $\alpha$ (which differ by 5.4\sigma!):

$\alpha = 1/137.035\,999\,046\ (27) \ [\text{Cs}\ 2018]$

$\alpha = 1/137.035\,999\,206\ (11) \ [\text{Rb}\ 2020]$

\[a_e^{\text{SM}} = 115\,965\,218\ 16.16\ (0.11)\ (0.08)\ (2.28) \times 10^{-13} \ [\text{Cs}\ 18]\]

\[= 115\,965\,218\ 02.64\ (0.11)\ (0.08)\ (0.93) \times 10^{-13} \ [\text{Rb}\ 20]\]

The (EXP – SM) difference is:

\[\Delta a_e = a_e^{\text{EXP}} - a_e^{\text{SM}} = -8.9\ (3.6) \times 10^{-13} \ [2.5\sigma] \ [\text{Cs}\ 18]\]

\[= +4.7\ (3.0) \times 10^{-13} \ [1.6\sigma] \ [\text{Rb}\ 20]\]

QED 5-loop: $a_e^{\text{QED5}} = 4.6 \times 10^{-13}$

NP sensitivity limited only by the experimental errors in $\alpha$ and $a_e$. May soon play a pivotal role in probing NP in the leptonic sector.
Using $\alpha(Rb2020)$, the sensitivity is $\delta \Delta a_e = 3.0 \times 10^{-13}$, ie $(\times 10^{-13})$: 

$$(0.1)_{\text{QED5}}, \quad (0.1)_{\text{HAD}}, \quad (0.9)_{\delta \alpha}, \quad (2.8)_{\delta a_e}\text{EXP}$$

$$(0.2)_{\text{TH}}$$

The $(g-2)_e$ experimental error may soon drop below $10^{-13} \rightarrow a_e$ sensitivity below $10^{-13}$ may soon be reached!

In a broad class of BSM theories, contributions to $a_\perp$ scale as

$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}}\right)^2$$

This Naive Scaling leads to:

$$\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}; \quad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.8 \times 10^{-6}$$

Giudice, Paradisi & MP, JHEP 2012
Shifts $\Delta \sigma(s)$ to fix $\Delta a_\mu$ only slightly change $\Delta a_e$

Keshavarzi, Marciano, MP, Sirlin, PRD 2020
Good agreement between lattice [Giusti & Simula 2020] and KNT19. Possible future bounds on very low energy shifts $\Delta \sigma(s)$?

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

Eduardo de Rafael, “On Constraints Between \(\Delta\alpha_{\text{had}}(M_Z^2)\) and \((g_\mu-2)_{\text{HVP}}\),” arXiv:2006.13880.

Malaescu and Schott, “Impact of correlations between \(a_\mu\) and \(\alpha_{\text{QED}}\) on the EW fit,” arXiv:2008.08107.

The MUonE project
The spacelike method for $a_\mu^{\text{HLO}}$

- Leading hadronic contribution computed via the usual dispersive (timelike) formula:

  \[
  a_\mu^{\text{HLO}} = \frac{1}{4\pi^3} \int_{m_\pi^2}^\infty ds \, K(s) \sigma^{(0)}_{\text{had}}(s)
  \]

  \[
  K(s) = \int_0^1 dx \, \frac{x^2 (1 - x)}{x^2 + (1 - x) (s/m_\mu^2)}
  \]

- Alternatively, simply exchanging the $x$ and $s$ integrations:

  \[
  a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx \, (1 - x) \Delta\alpha_{\text{had}}[t(x)]
  \]

  \[
  t(x) = \frac{x^2 m_\mu^2}{x - 1} < 0
  \]

$Lautrup, Peterman, de Rafael, 1972$

$\Delta\alpha_{\text{had}}(t)$ is the hadronic contribution to the space-like running of $\alpha$: proposal to measure $a_\mu^{\text{HLO}}$ via scattering data!

$Carloni Calame, MP, Trentadue, Venanzoni, 2015$
$a_{\mu}^{HLO}$: timelike vs spacelike method

**Timelike**

**Spacelike**

F. Jegerlehner, arXiv:1511.04473

Carloni Calame, MP, Trentadue, Venanzoni, PLB 2015

- Inclusive measurement
- Smooth integrand
- Direct interplay with lattice QCD
$
abla \alpha_{\text{had}}(t)$ can be measured via the elastic scattering $\mu e \to \mu e$.

We propose to scatter a 150 GeV muon beam, available at CERN’s North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by several thin Silicon strip detectors.

Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna, Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni

EPJC 2017 - arXiv:1609.08987
For a 150 GeV muon beam ($\sqrt{s} \approx 400$ MeV), MUonE’s scan region extends up to $x=0.932$, ie beyond the $x=0.914$ peak!

It looks like an ideal process!
Statistics: With CERN’s 150 GeV muon beam M2 ($1.3 \times 10^7 \mu/s$), incident on 40 15mm Be targets (total Be thickness: 60cm), 2-3 years of data taking ($2 \times 10^7$ s/yr) $\Rightarrow$ $L_{int} \sim 1.5 \times 10^7$ nb$^{-1}$.

With this $L_{int}$ we estimate that measuring the shape of $d\sigma/dt$ we can reach a statistical sensitivity of $\sim 0.3\%$ on $a_{\mu}^{HLO}$, ie $\sim 20 \times 10^{-11}$.

Systematic effects must be known at $\leq 10$ppm!

Test beams performed at CERN in 2017 & 2018 arXiv:1905.11677, 2102.11111


If test run successful, intermediate run hopefully in 2023–24.
MUonE — Getting ready for the Test Run
To extract $\Delta \alpha_{\text{had}}(t)$ from MUonE’s measurement, the ratio of the SM cross sections in the signal and normalisation regions must be known at $\leq 10\text{ppm}$!

- Fully differential fixed-order MC @ NLO ready Pavia and PSI 2018-19
- NNLO QED: Master Integrals for 2-loop box diagrams computed. Full 2-loop amplitude completed! ($m_e=0$) Padova 2017 - present
- Two MC built including partial subsets of the NNLO QED corrections due to electron and muon radiation Pavia and PSI 2020
- NNLO hadronic effects computed Padova and KIT 2019
- Extraction of the leading electron mass effects from the massless muon-electron scattering amplitudes PSI 2019-present
- New Physics extracting $\Delta \alpha_{\text{had}}(t)$ at MUonE? Padova and Heidelberg 2020
- ...

Theory for muon-electron scattering @ 10 ppm:
MUonE — Theory workshops

Muon-electron scattering: Theory kickoff workshop

4-5 September 2017

MUonE theory workshops: Padova 2017, Mainz 2018, Zurich 2019
Next MUonE theory workshop: MITP Mainz 2020-21 postponed to 2022
Conclusions

- Fermilab’s Muon g-2 experiment confirms BNL's result: the discrepancy between experiment and SM increases to 4.2σ.

- The BMWc lattice QCD result weakens the exp-SM discrepancy. It must be confirmed or refuted by other lattice calculations.

- Is $\Delta a_\mu$ due to missed contributions in the hadronic cross section? Shifts above 1 GeV to fix $\Delta a_\mu$ conflict with the electroweak fit.

- Leading hadronic contribution to $a_\mu$: dispersive vs lattice. MUonE will provide a new independent & alternative determination.