Multiloop Feynman diagrams in a quantum computer

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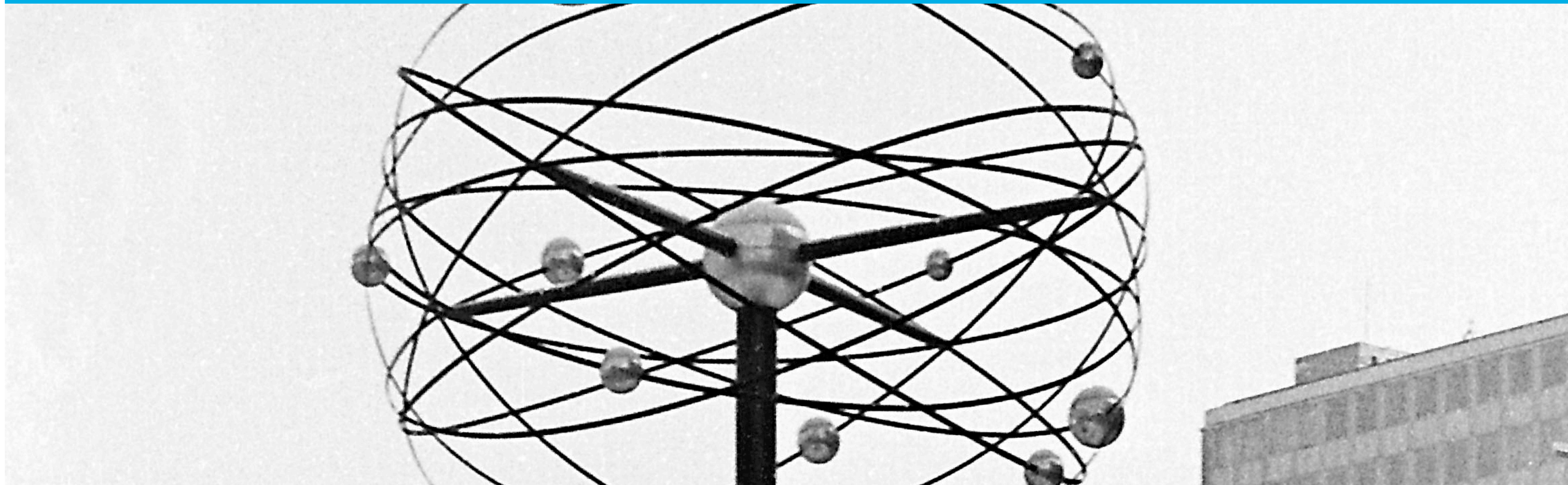
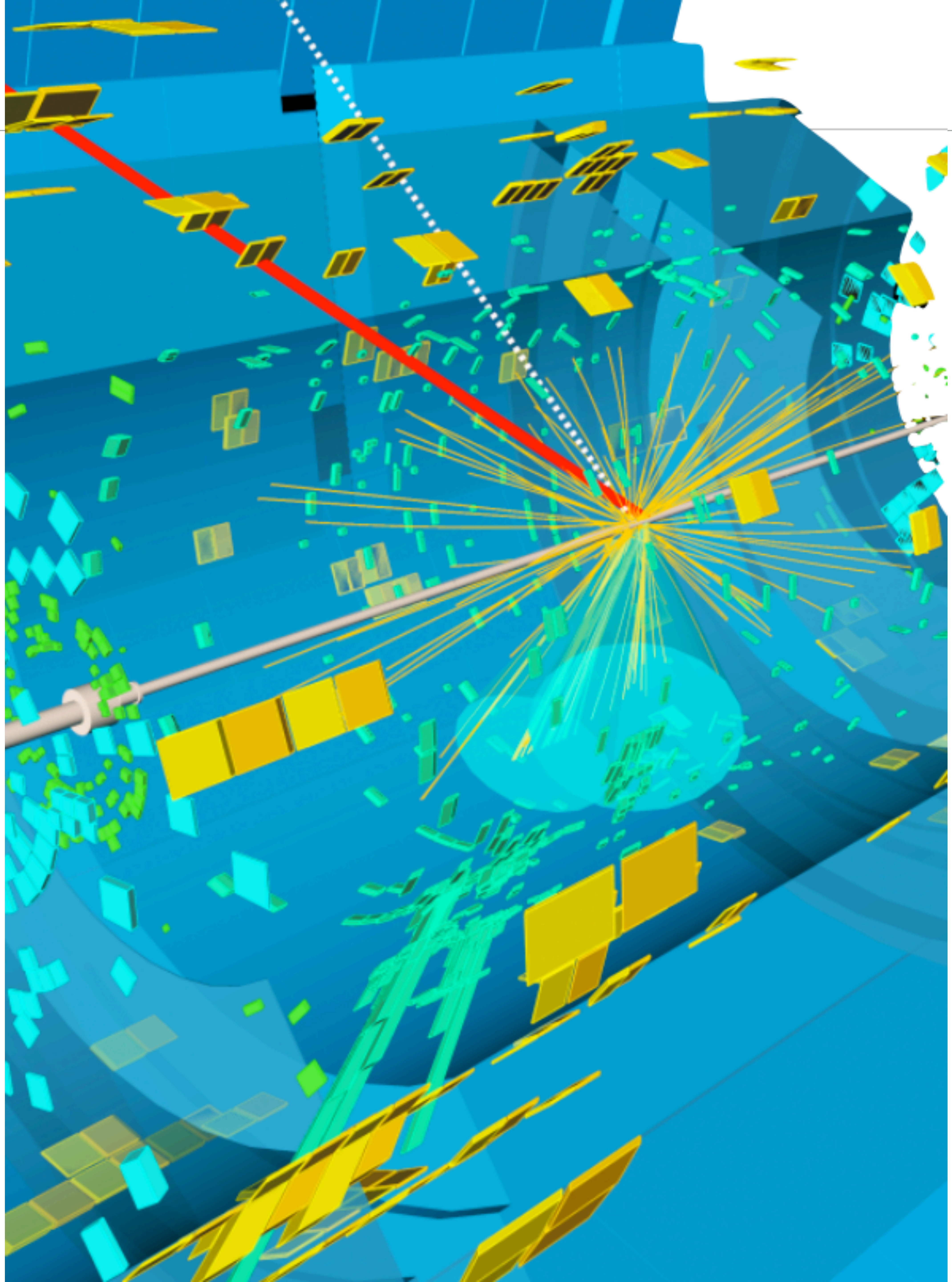
DESY Theory Seminar

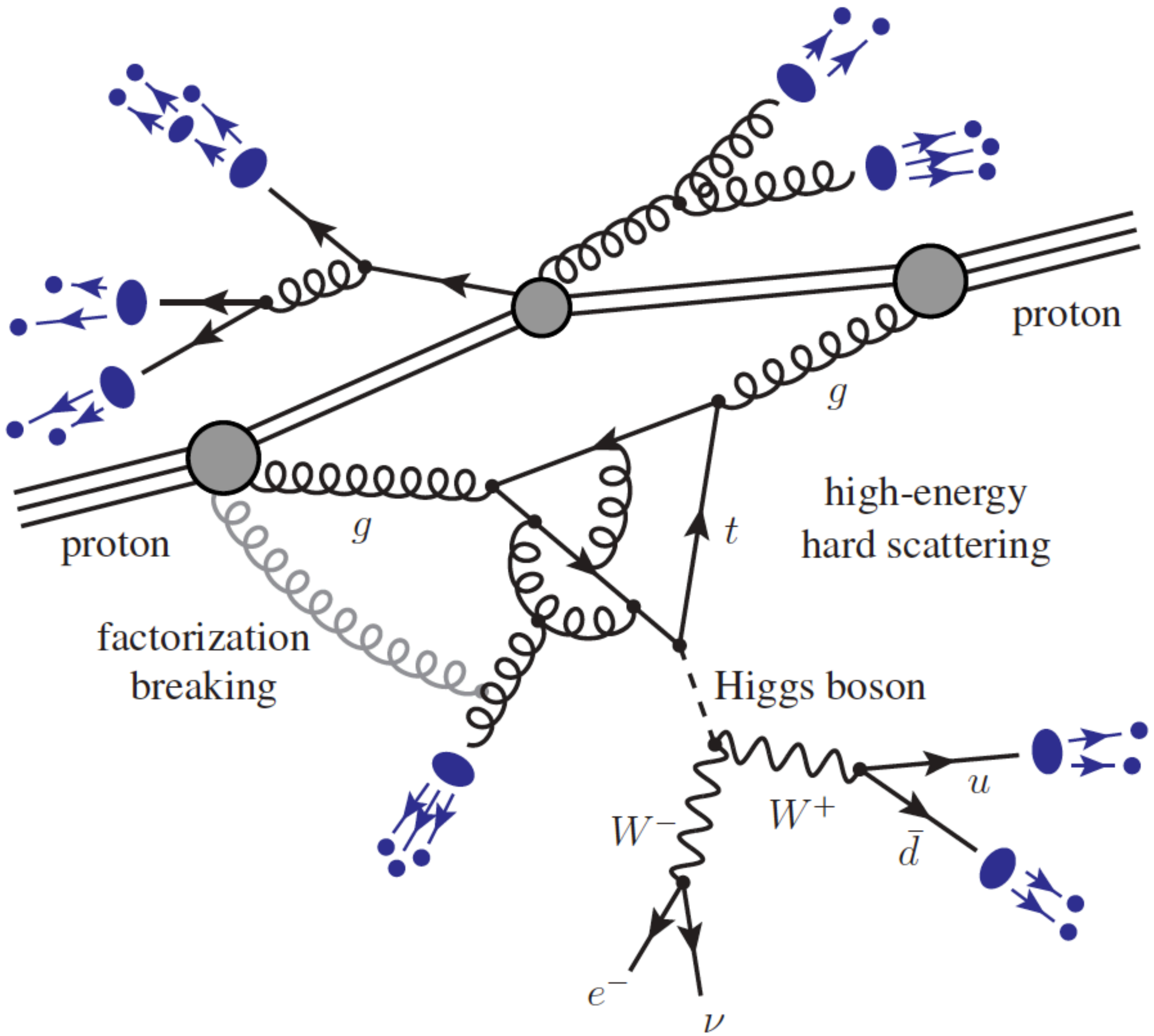
11 November 2021



Based on: S. Ramírez Uribe, A.E. Rentería Olivo, GR, G.F.R. Sborlini, L. Vale Silva, arXiv 2105.08703

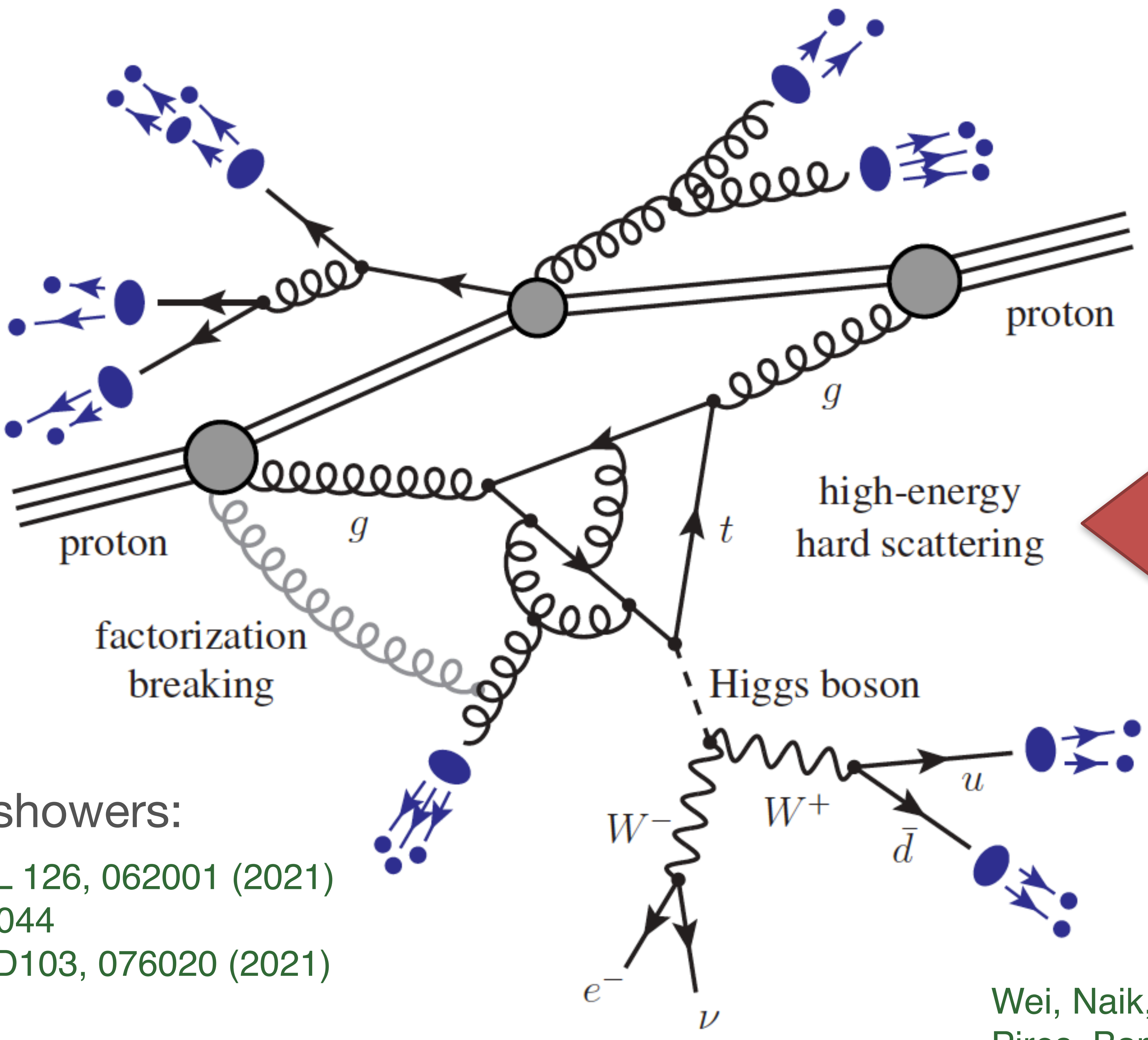
THEORIST VIEW





- **factorization** into short distance (hard scattering) and long distance (initial and final state)
- QFT = **quantum mechanics** + special relativity + creation-annihilation of particles

RECENT QUANTUM APPLICATIONS



- tree-level helicity amplitudes:
  Bepari, Malik, Spannowsky, Williams, PRD103, 076020 (2021)
- parton densities:
  Pérez-Salinas, Cruz-Martínez, Alhajri, Carrazza, PRD 103, 034027 (2021)
- multiloop scattering amplitudes:
  generally accepted to be beyond the reach of quantum computers, since it would require a prohibitive number of qubits
- Parton showers:
  Bauer, de Jong, Nachman, Provasoli, PRL 126, 062001 (2021)
  Bauer, Freytsis, Nachman, arXiv:2102.05044
  Bepari, Malik, Spannowsky, Williams, PRD103, 076020 (2021)
- jet clustering:
  Wei, Naik, Harrow, Thaler, PRD 101, 094015 (2020)
  Pires, Bargassa, Seixas, Omar, arXiv:2101.05618
  Pires, Omar, Seixas, arXiv:2012.14514
- quantum machine learning:
  Guan, Perdue, Pesah, Schuld, Terashi, Vallecorsa, Vlimant, arXiv:2005.08582
  Wu et al., arXiv:2012.11560.
  Trenti, Sestini, Gianelle, Zuliani, Felser, Lucchesi, Montangero, arXiv:2004.13747

Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, Rodrigo, Sborlini, Torres, Tracz, PRL**124** (2020) 21, 211602

LTD TO ALL ORDERS AND POWERS

- **Multi-loop scattering amplitude:** n sets of internal momenta, each set depends on a specific linear combination of the L loop momenta

$$\mathcal{A}_N^{(L)} = \int_{\ell_1 \cdots \ell_L} \mathcal{N}(\{\ell_i\}_L, \{p_j\}_N)\, G_F(1,\ldots,n)$$

$$= \prod_{i \in 1\cup\ldots\cup n} \left(G_F(q_i)\right)^{a_i}$$

- Starting from the integrand in the Feynman representation, take the residues of propagators in the first set (in each loop, integrates out one component of the loop momemtum)

$$\mathcal{A}_D(1;2,\ldots,n) = -2\pi\imath \sum_{i_1\in 1} \text{Res}\left(\mathcal{A}_F(1,\ldots,n), \text{Im}(\eta q_{i_1}) < 0\right)$$

- **Cauchy contour** always from **below** the real axis
- valid for arbitrary powers and **Lorentz invariant** [Catani et al. JHEP **0809**, 065]
- reverse momenta, if necessary, to keep a coherent momentum flow: $s \to \bar{s} \quad (q_{i_s} \to -q_{i_s})$

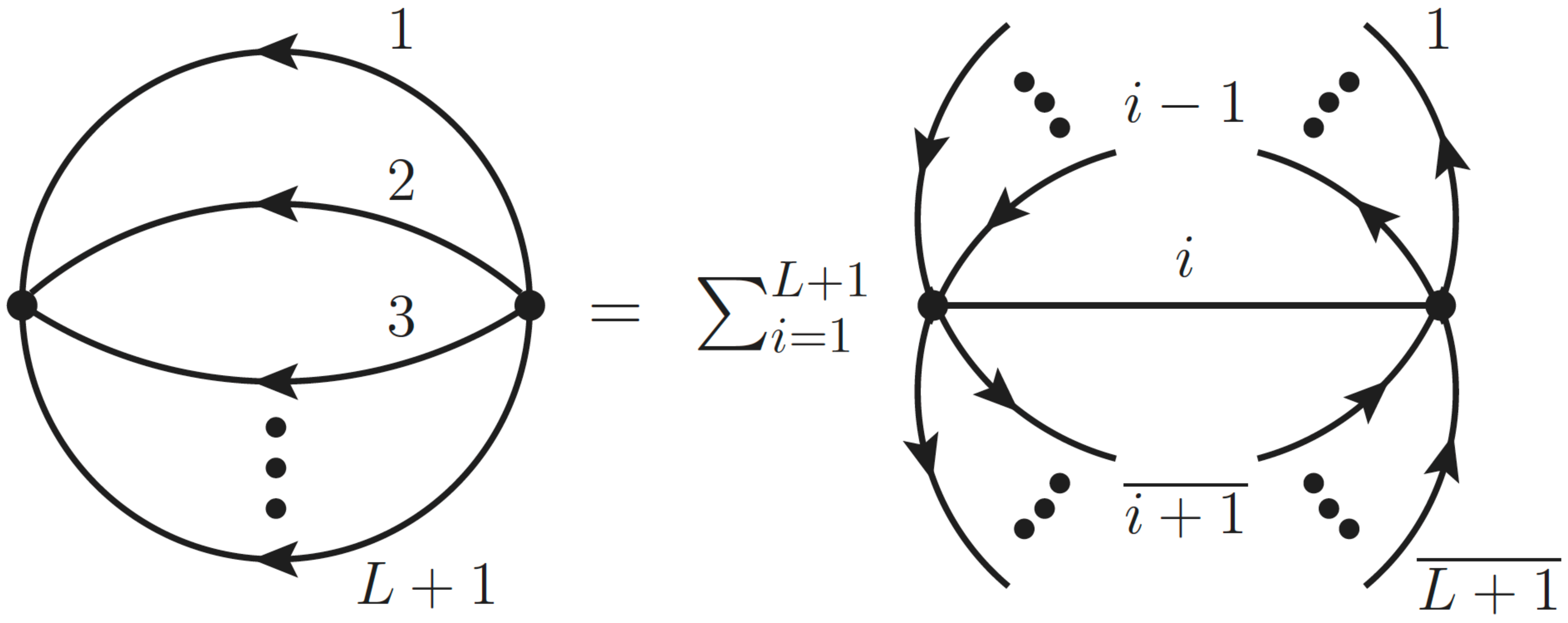
- The **nested residue** involving several sets

$$\mathcal{A}_D(1,\ldots,r;r+1,\ldots) = -2\pi\imath \sum_{i_r\in r} \text{Res}\left(\mathcal{A}_D(1,\ldots,r-1;r,\ldots), \text{Im}(\eta q_{i_r}) < 0\right)$$

- If $\eta = (1,\mathbf{0})$ integrate out the energy component and the integration domain is **Euclidean**

Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, Rodrigo, Sborlini, Torres, Tracz, PRL**124** (2020) 21, 211602

CAUSAL CONJECTURE

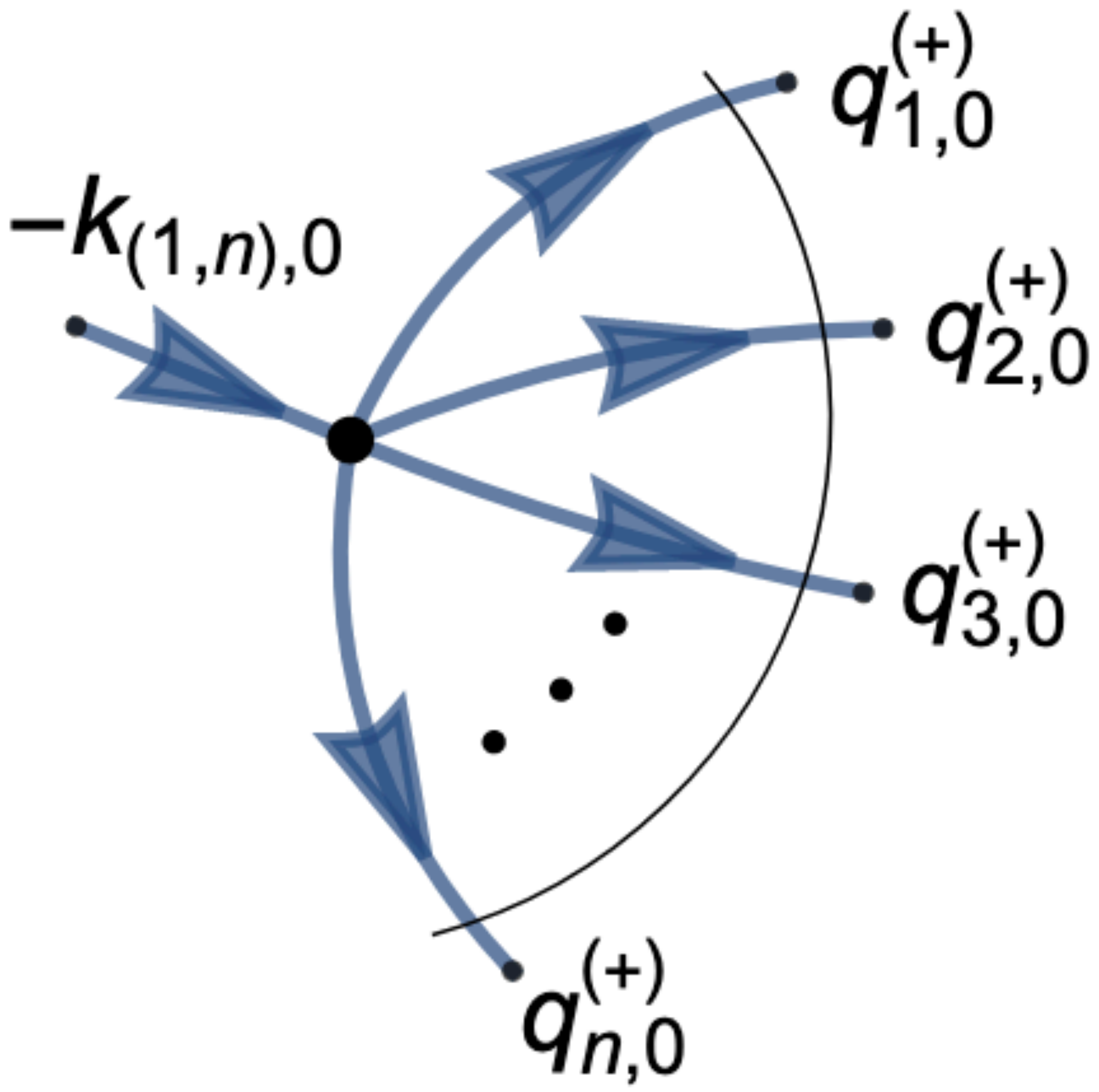


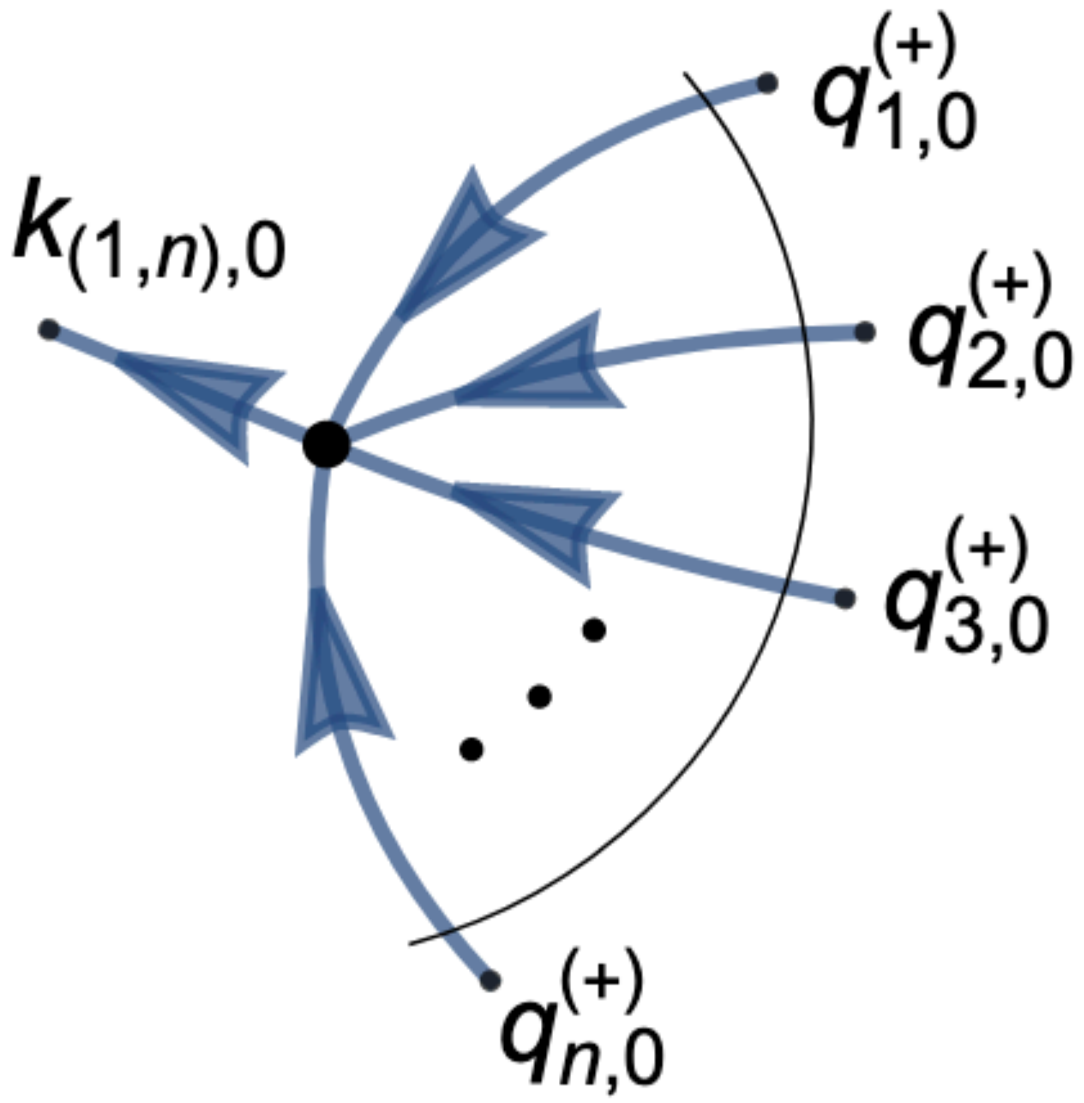
- **What if we reorder the loop lines (order of residues)? Do we get a different representation?**
- In fact no (math vs physics), and if we sum up all the nested residues, e.g. scalar integral

$$\mathscr{A}^{(L)}_{\rm MLT} = \int_{\vec\ell_1 \dots \vec\ell_L} \frac{1}{\prod 2q^{(+)}_{i,0}} \left(\frac{1}{\lambda^+_{1,n}} + \frac{1}{\lambda^-_{1,n}} \right), \qquad \lambda^\pm_{1,n} = \sum q^{(+)}_{i,0} \pm k_{(1,n),0} \qquad q^{(+)}_{i,0} = \sqrt{\boldsymbol{q}_i^2 + m_i^2 - \imath 0}$$

  - Independent of the initial momentum flow assignments
  - **Causal conjecture:** the LTD representation if manifestly free of non-causal singularities for all topologies and internal configurations

CAUSAL PROPAGATORS



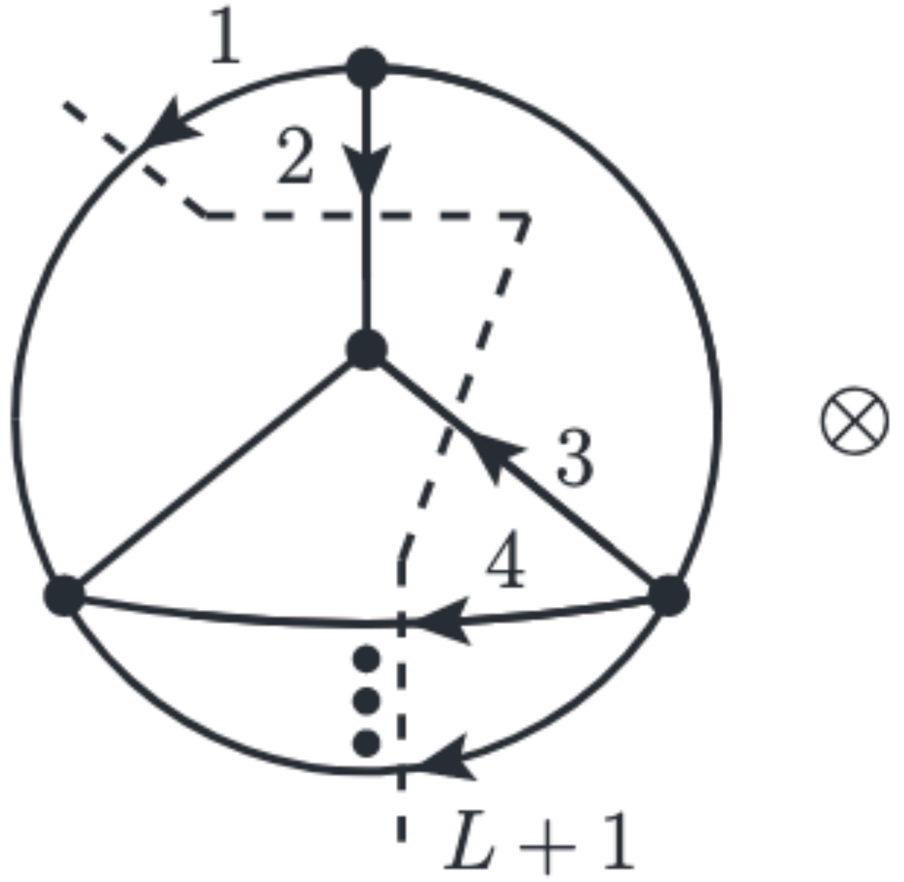


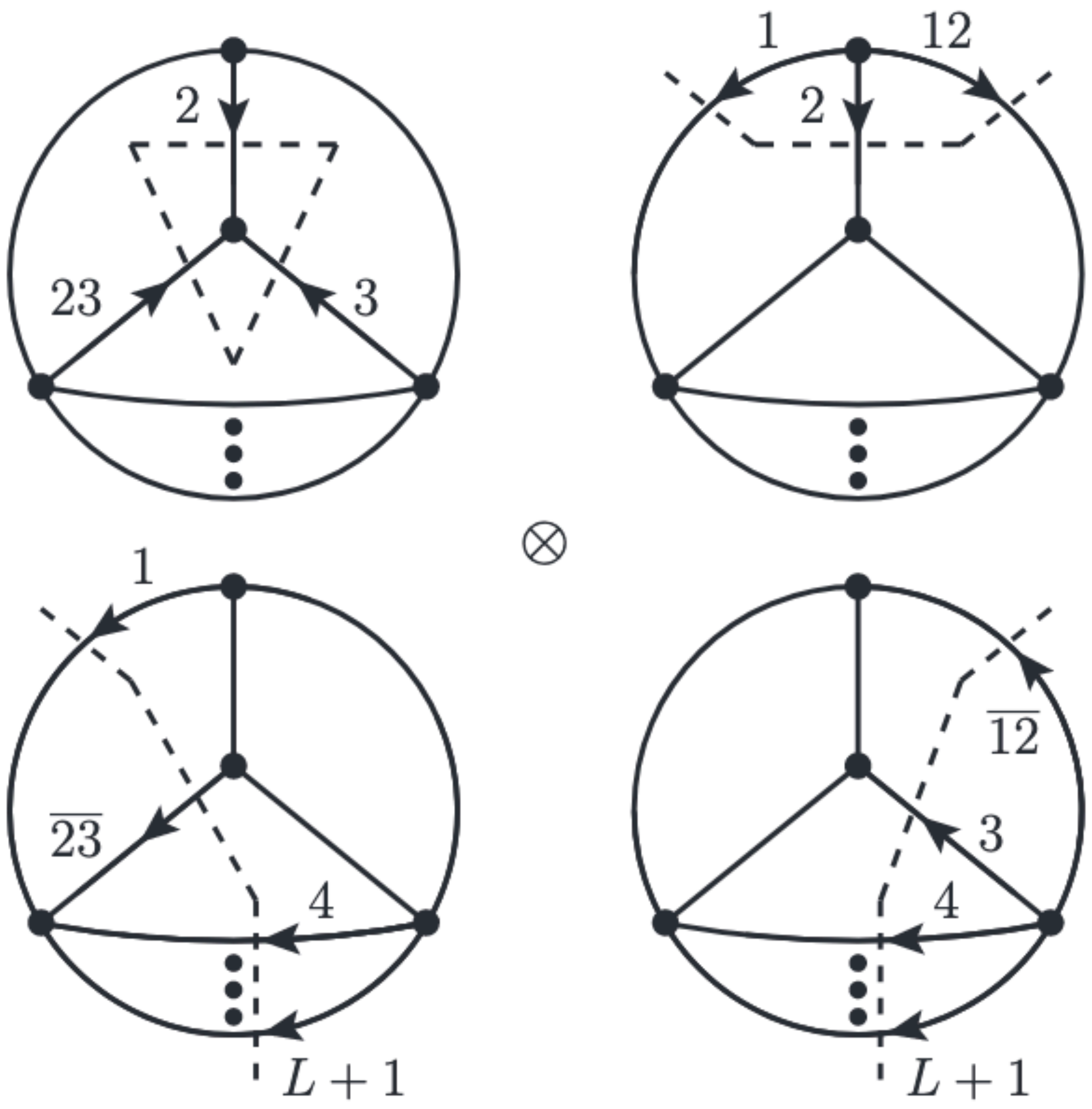
$$\frac{1}{\lambda_{1,n}^{+}} = \frac{1}{\sum_{i=1}^{n} q_{i,0}^{(+)} + k_{(1,n),0}}$$

$$\frac{1}{\lambda_{1,n}^{-}} = \frac{1}{\sum_{i=1}^{n} q_{i,0}^{(+)} - k_{(1,n),0}}$$

- **Incoming** external momenta, $k_{(1,n),0} < 0$, a causal threshold / soft / collinear singularity arises when all the internal momenta are on shell and aligned in the same direction
- **Outgoing** external momenta, $k_{(1,n),0} > 0$, a causal threshold / soft / collinear singularity arises when all the internal momenta are on shell and aligned in the opposite direction

CAUSAL REPRESENTATION





$$\mathcal{A}^{(L)}_{\rm LTD} = \int_{\vec\ell_1\ldots\vec\ell_L} \frac{1}{\prod 2q^{(+)}_{i,0}} \sum_{\sigma\in\Sigma} \frac{\mathcal{N}_{\sigma(i_1,\ldots,i_{n-L})}}{\lambda^{h_1}_{\sigma(i_1)}\cdots\lambda^{h_{n-L}}_{\sigma(i_{n-L})}} + (\lambda_p^+ \leftrightarrow \lambda_p^-)$$

- Each combination of compatible causal propagators in Σ fixes the momentum flows of all the internal momenta
- Conversely, if we fix the causal momentum flows we can **bootstrap the causal LTD representation**

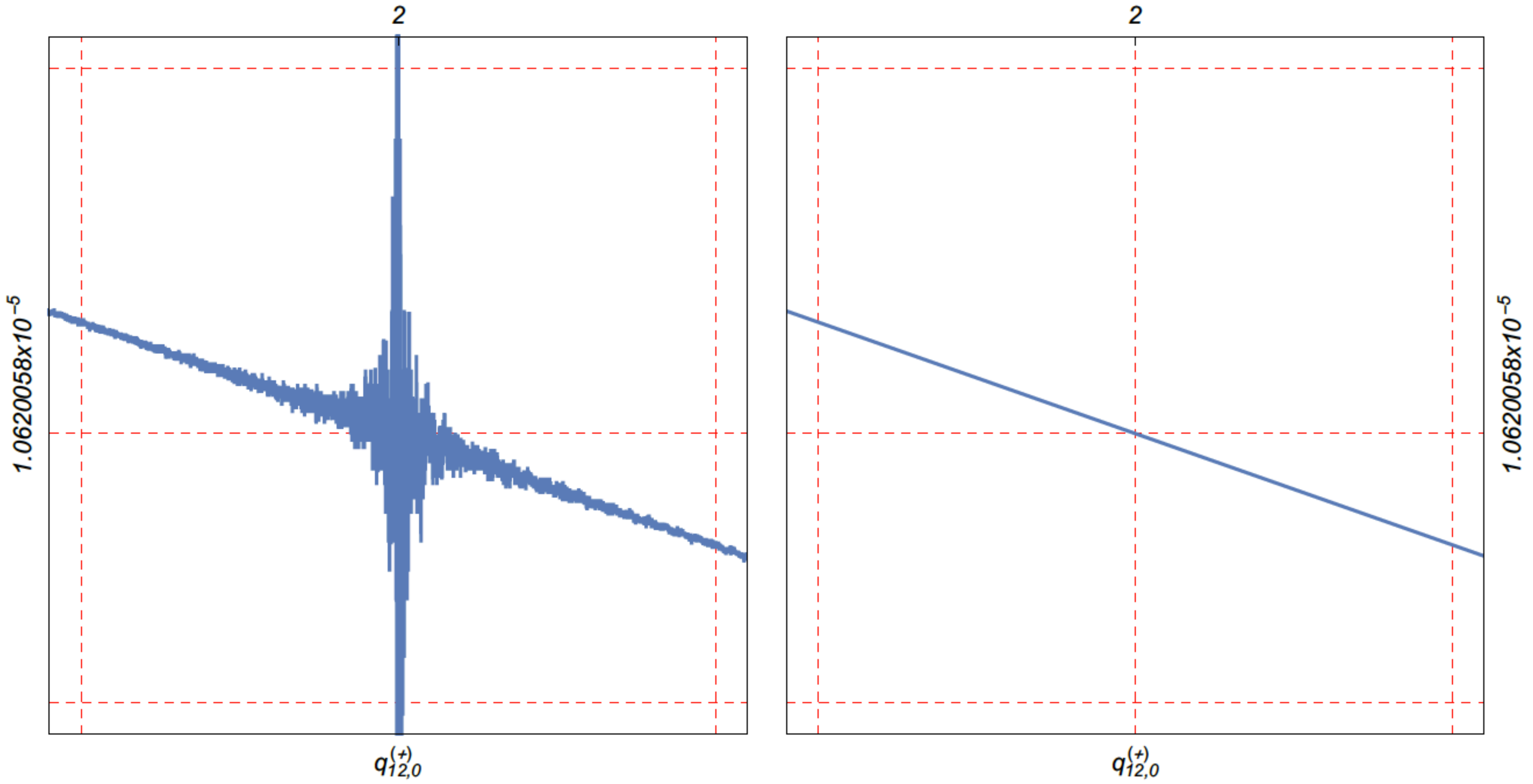
→ **Quantum algorithm**

Aguilera, Driencourt, Hernández, Plenter, Ramírez, Rentería, Rodrigo, Sborlini, Torres, Tracz, PRL**124** (2020) 21, 211602

JHEP **1912**, 063 | JHEP **2101**, 069 | JHEP **2102**, 112 | JHEP **2104**, 129

CAUSALITY IN LOOP-TREE DUALITY

- integrands in the Feynman representation have singularities that are nonphysical ≡ not related to the optical theorem
- LTD leads to **manifestly causal representations** (free of non-causal singularities): more stable numerically



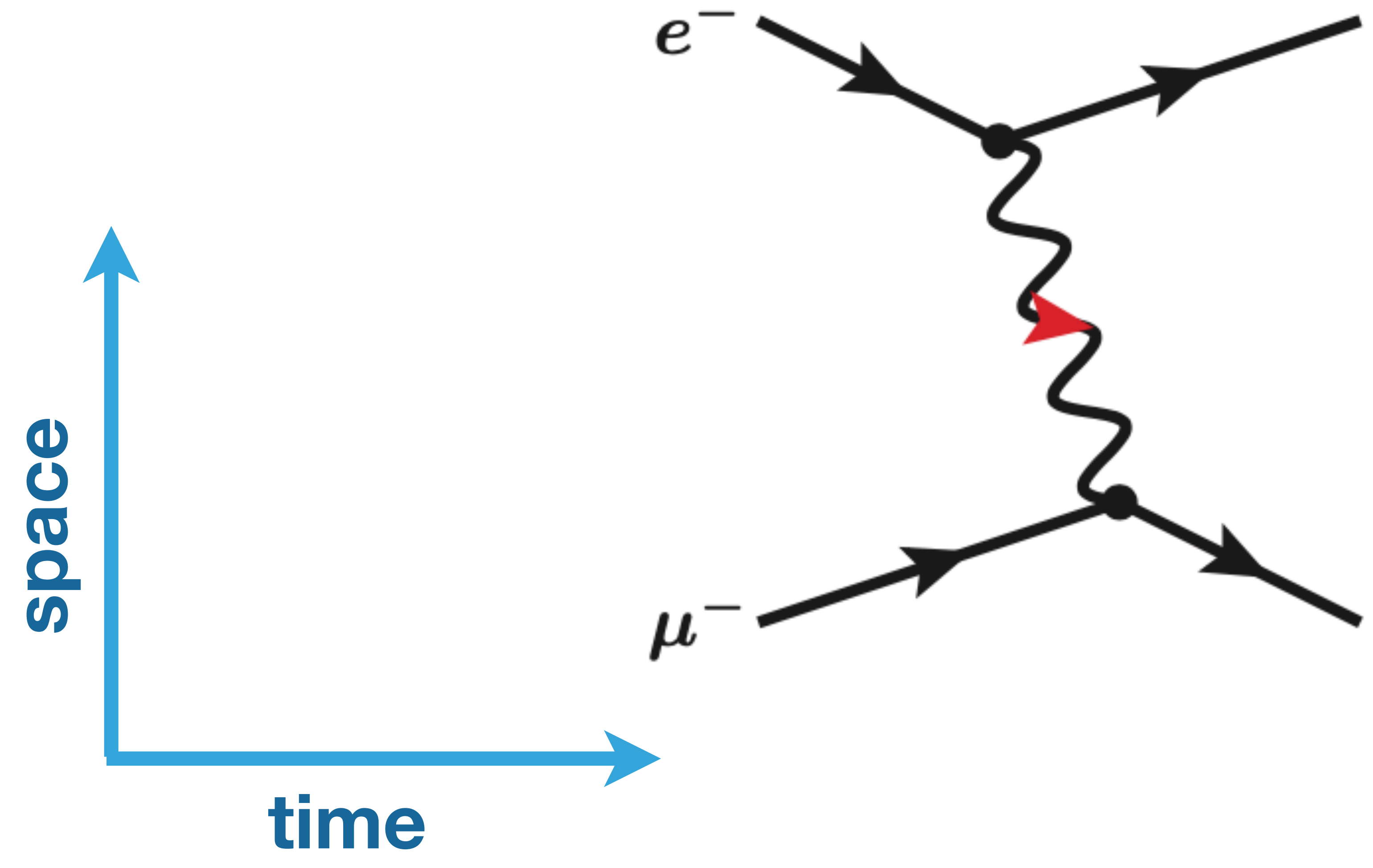
- **Integrand numerical instabilities** across a noncausal threshold
- manifestly causal LTD representation

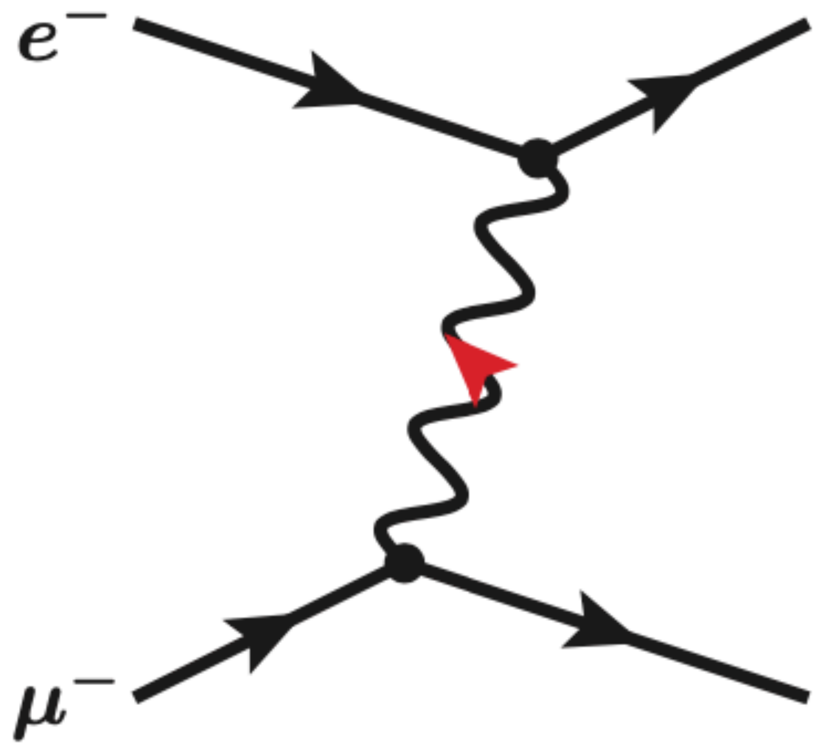
- geometric interpretation: Sborlini PRD **104** (2021) 036014
- Wolfram Mathematica package: Torres Bobadilla JHEP **2104**, 183 and EPJC **81** (2021) 514

Lotty
THE LOOP-TREE DUALITY AUTOMATION

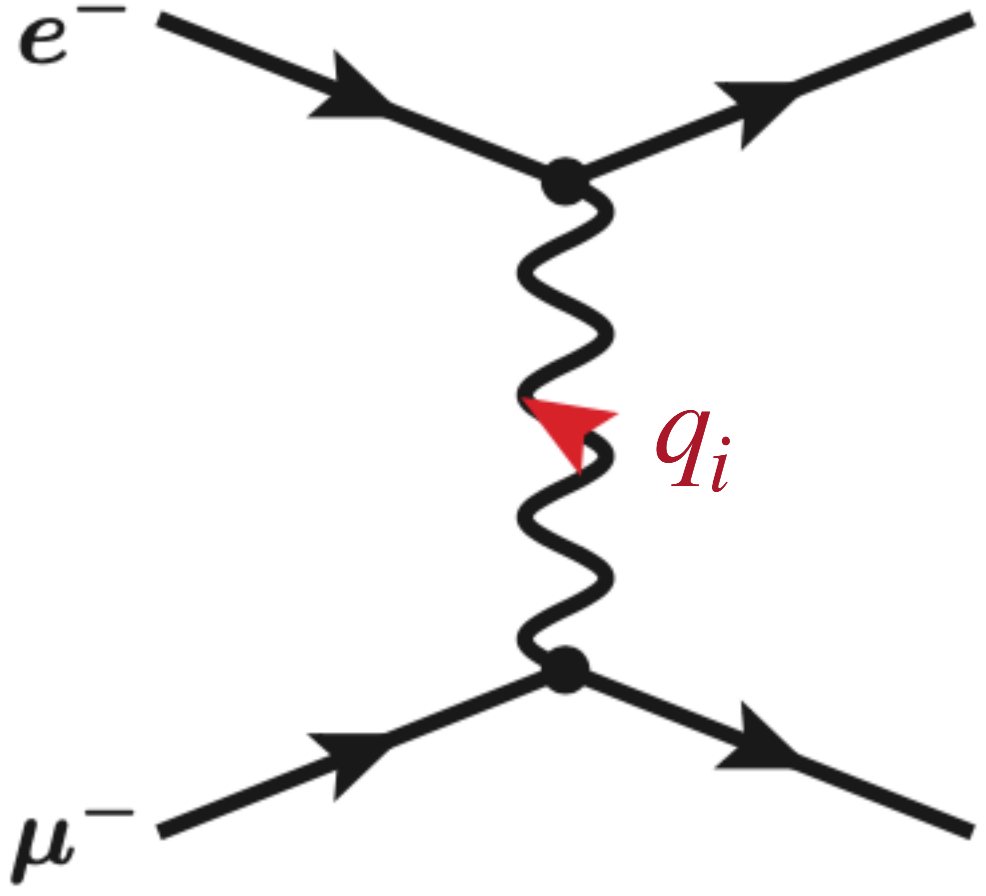
CAUSALITY

Cause-effect relationships are unbreakable. (You cannot be born before your grandfather.) [Symmetry Magazine]





- e^- and μ^- get closer, and the **electron** emits a photon which is absorbed by the muon
- e^- and μ^- get closer, and the **muon** emits a photon which is absorbed by the electron



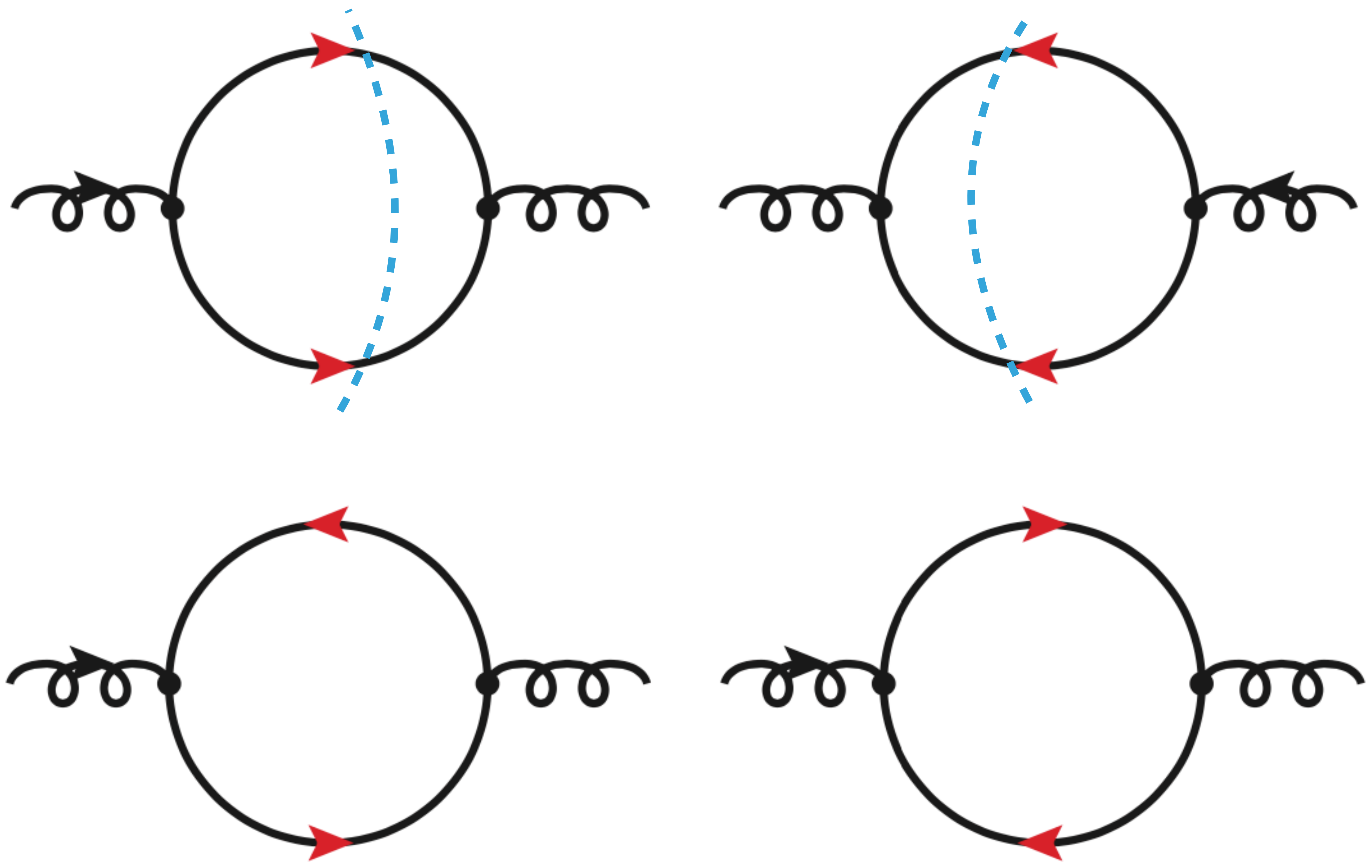
$$G_F(q_i) = \frac{1}{q_i^2 - m_i^2 + \imath 0} = \frac{1}{(q_{i,0} - q_{i,0}^{(+)})(q_{i,0} + q_{i,0}^{(+)})}$$

- positive frequencies are propagated forwards in time, and negative are propagated backwards
- **two on-shell states**, one with positive and another with negative energy:

$$q_{i,0} \to \pm\, q_{i,0}^{(+)} = \pm \sqrt{\boldsymbol{q}_i^2 + m_i^2 - \imath 0}$$

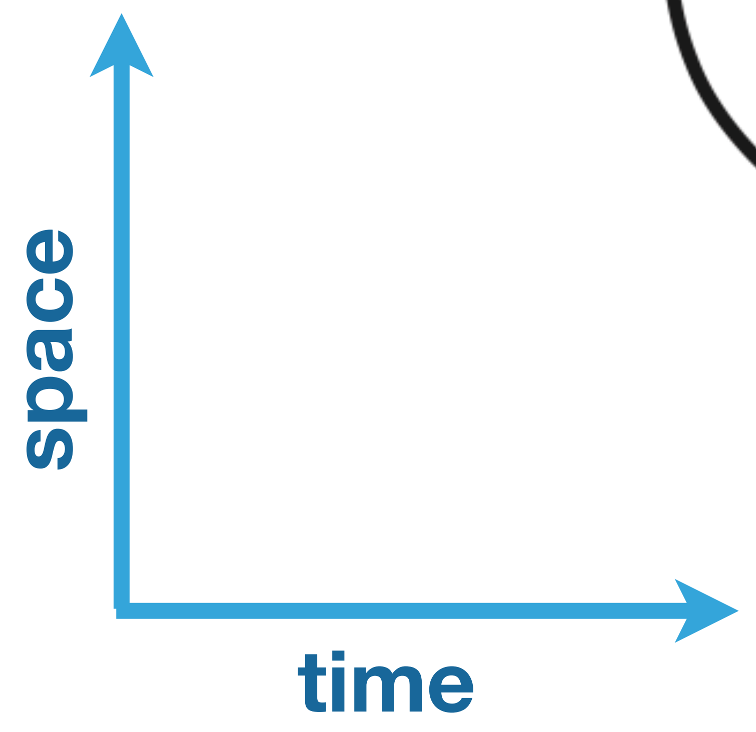
CAUSALITY

$$\mathscr{A}^{(1)} = \int_\ell \mathscr{N}(\ell, p_i)\, G_F(q_1, q_2)$$



- the gluon fluctuates into a quark-antiquark pair, which merges back into a gluon
- a **causal discontinuity** (imaginary part) arises when the quark and antiquark are produced as real (on-shell) ≡ **Cutkosky rule / optical theorem**

- the gluon emits a quark, which travels backwards in time to the point of emission
- each configuration generates a **non-causal singularity** of the integrand: they are nonphysical and must somehow be cancelled out



- there are 2^n configurations for n propagators

Ramírez, Rentería, Rodrigo, Sborlini, Vale Silva, e-Print: [2105.08703](https://arxiv.org/abs/2105.08703)

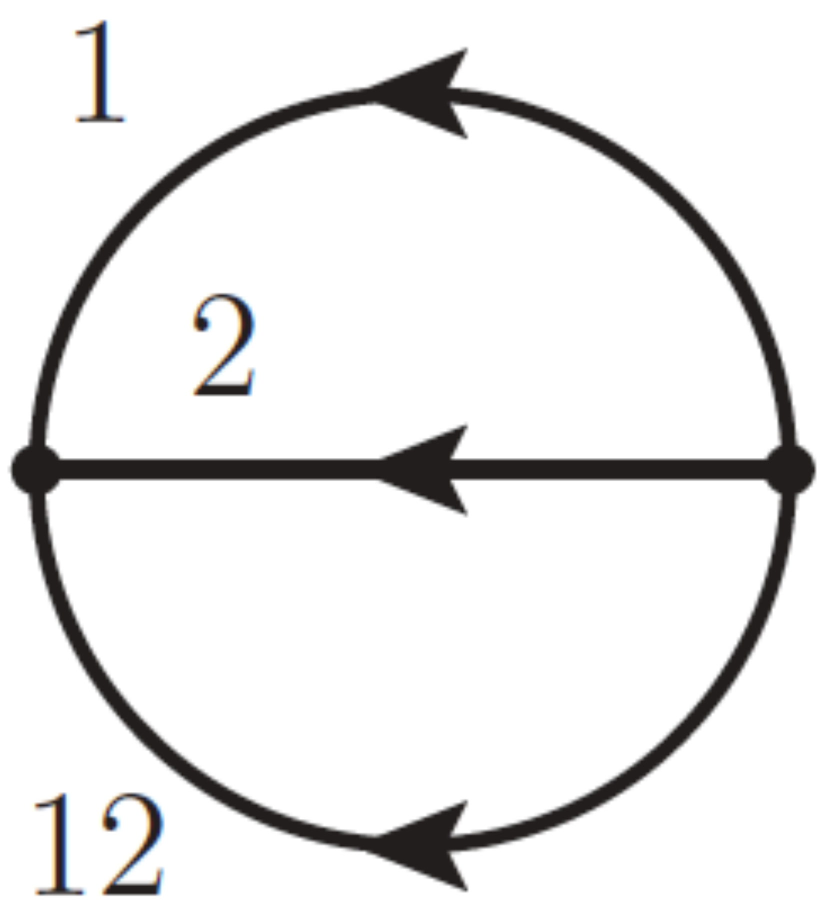
LOOPS IN A QUANTUM COMPUTER

- Each Feynman propagator has **two on-shell states**, one with positive energy $|1\rangle$, and another with negative energy $|0\rangle \equiv$ momentum flow in one direction or the opposite
- A classical algorithm requires to test causal conditions ($\equiv$ **directed acyclic graph**) for $\mathcal{O}(2^n)$ states
- A **quantum algorithm** exploits superposition and entanglement to test causality at once, in $\mathcal{O}(1)$ iterations (still a number of measurements $\mathcal{O}(\text{causal states})$)

$$q_1 = \ell_1$$

$$q_2 = \ell_2$$

$$q_{12} = -\ell_1 - \ell_2 + p$$

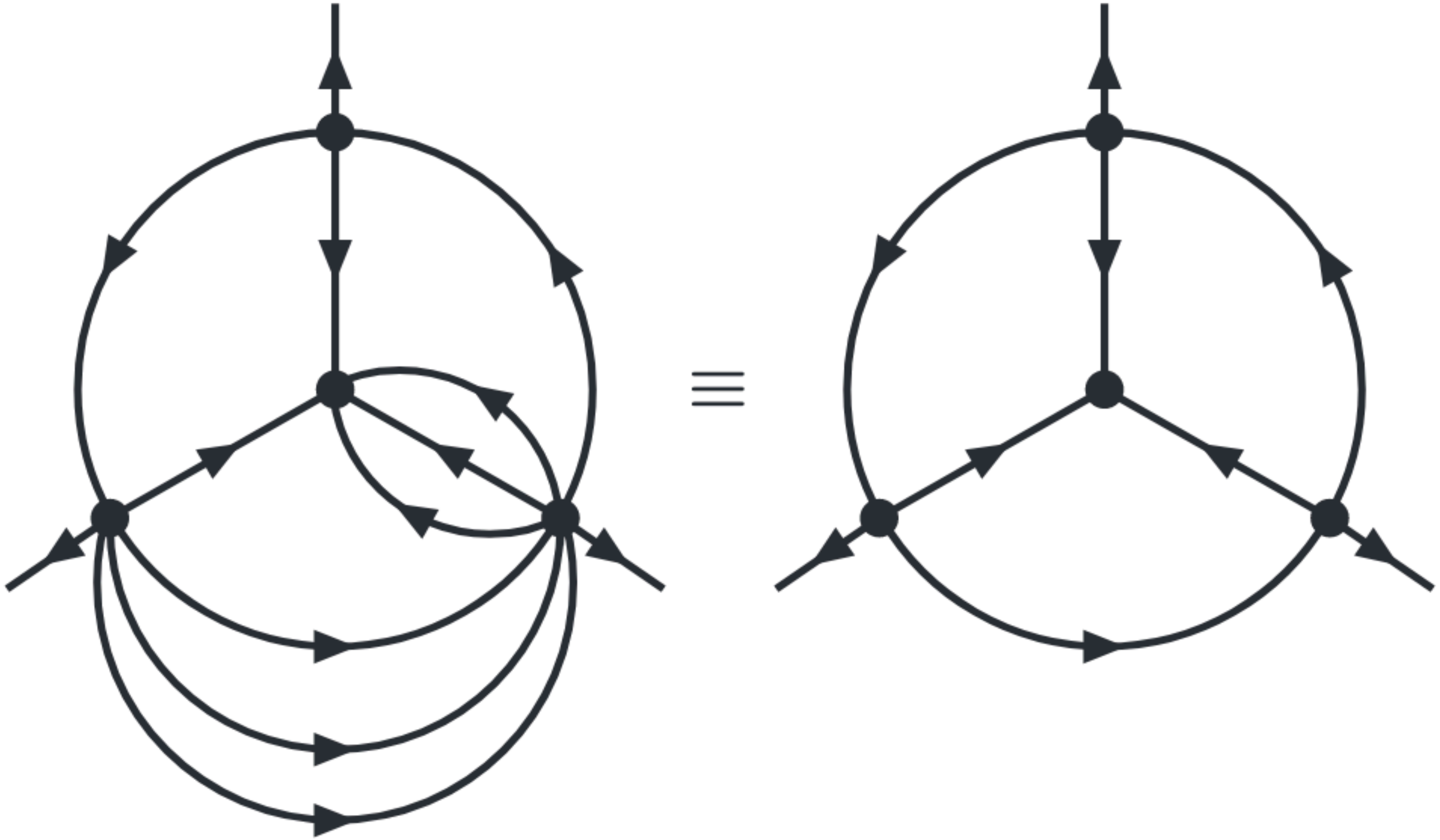


$$\mathcal{A}^{(2)} = \int_{\ell_1 \ell_2} G_F(q_1, q_2, q_{12})$$

$$\mathcal{A}^{(2)}_{\text{LTD}} = -\int_{\vec{\ell}_1 \vec{\ell}_2} \frac{1}{x_{12}} \left(\frac{1}{q^{(+)}_{1,0} + q^{(+)}_{2,0} + q^{(+)}_{12,0} + p_0} + \frac{1}{q^{(+)}_{1,0} + q^{(+)}_{2,0} + q^{(+)}_{12,0} - p_0} \right)$$

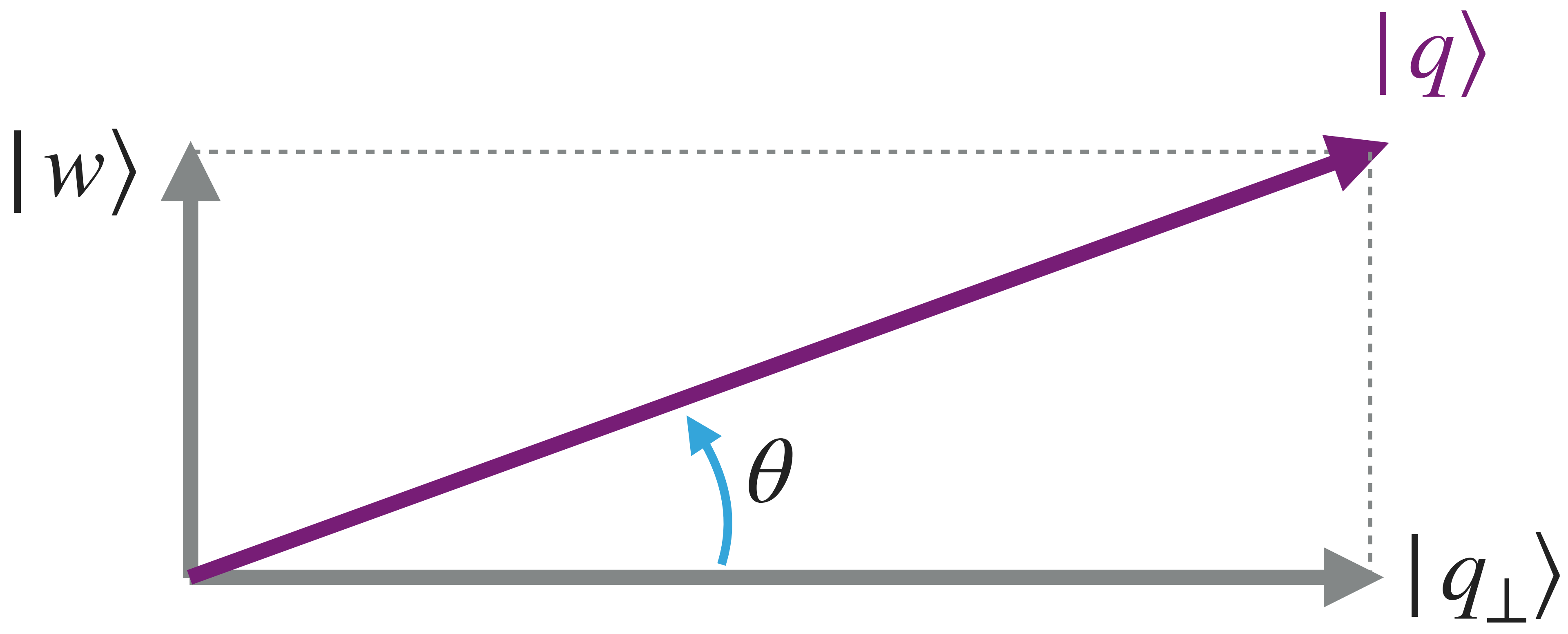
$$x_{12} = \prod_{i=1,2,12} 2q^{(+)}_{i,0}$$

LOOPS VS ELOOPS



- Seven loops $L = 7$
- Equivalent to **3 eLoops (Mercedes)** regarding **causal bootstrapping**

GROVER'S ALGORITHM



- A uniform superposition of $N = 2^n$ states

$$|q\rangle = \frac{1}{\sqrt{N}} \sum_{x=0}^{N-1} |x\rangle$$

- which is a superposition of the winning state $|w\rangle$ encoding the r **causal states**, and the orthogonal state $|q_\perp\rangle$ collecting the noncausal states

$$|q\rangle = \sin\theta |w\rangle + \cos\theta |q_\perp\rangle$$

$$|w\rangle = \frac{1}{\sqrt{r}} \sum_{x\in w} |x\rangle \qquad |q_\perp\rangle = \frac{1}{\sqrt{N-r}} \sum_{x\notin w} |x\rangle$$

- The mixing angle is

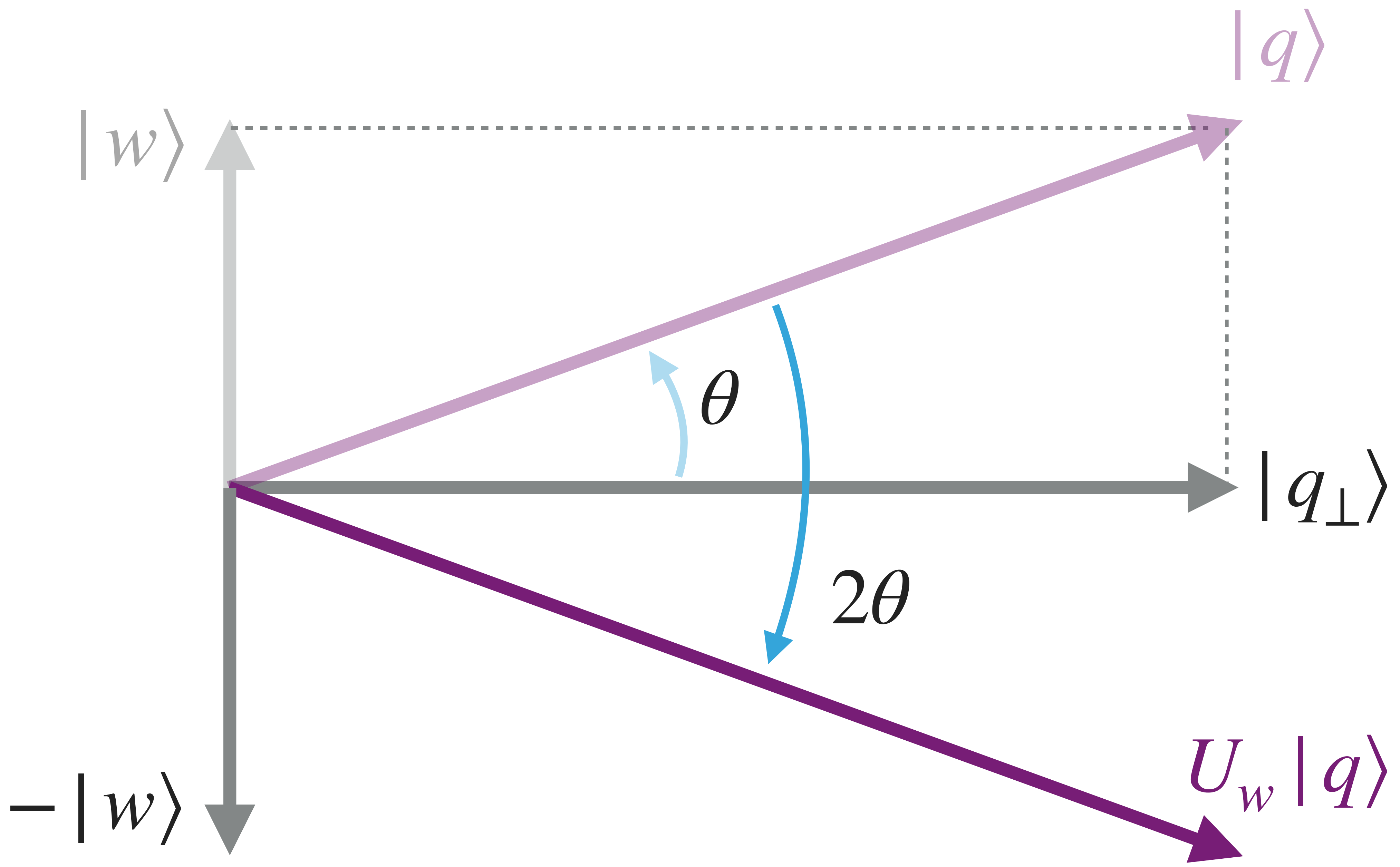
$$\theta = \arcsin\sqrt{r/N}$$

GROVER'S ALGORITHM

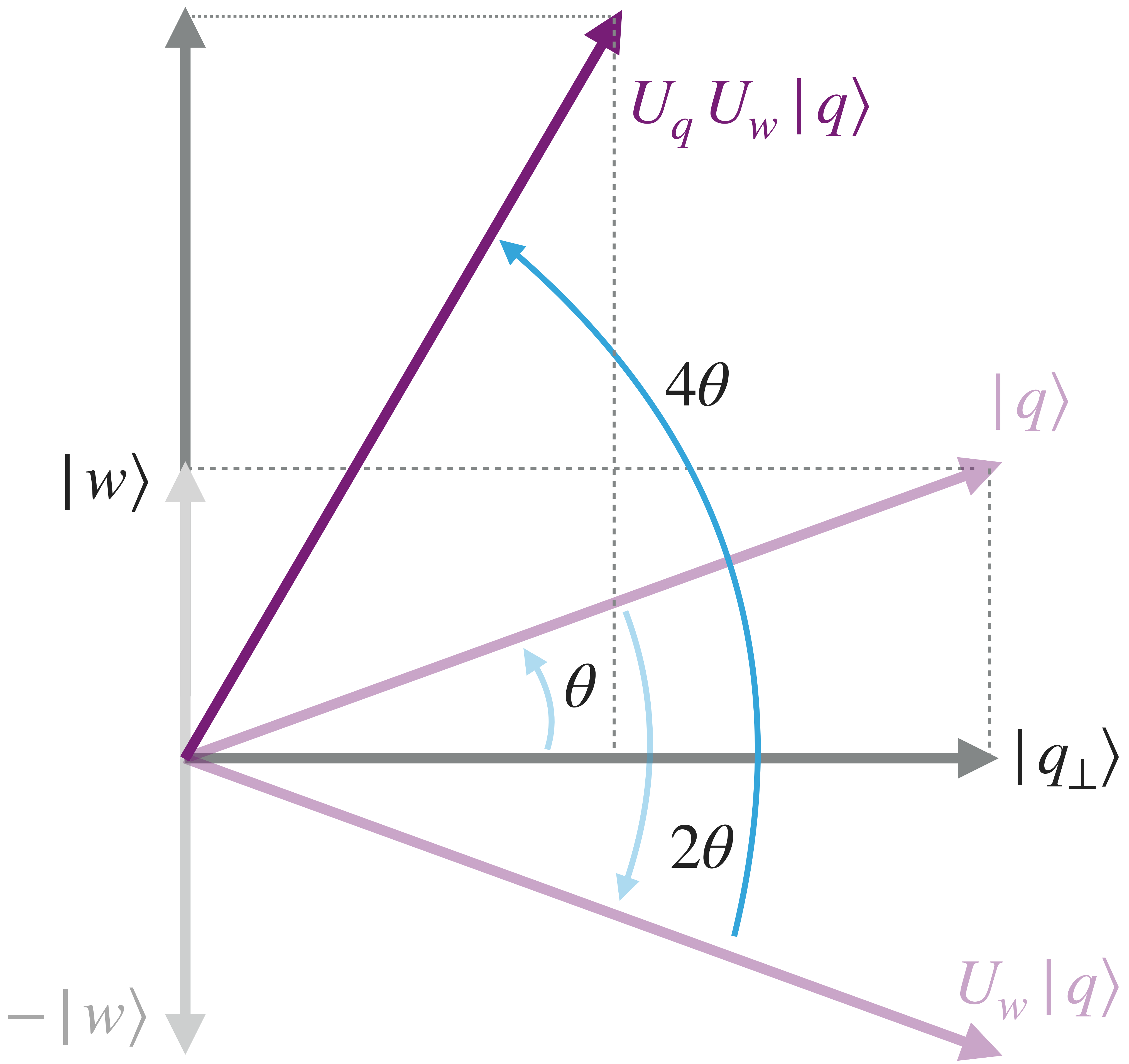
- The **oracle operator**

$$U_w = \boldsymbol{I} - 2|w\rangle\langle w|$$

- **flips** the state $|x\rangle$ if $x \in w$: $U_w|x\rangle = -|x\rangle$
- leaves it unchanged otherwise: $U_w|x\rangle = |x\rangle$ if $x \notin w$



GROVER'S ALGORITHM



- The **oracle operator**

$$U_w = \boldsymbol{I} - 2|w\rangle\langle w|$$

- **flips** the state $|x\rangle$ if $x \in w : U_w |x\rangle = -|x\rangle$
- leaves it unchanged otherwise: $U_w |x\rangle = |x\rangle$ if $x \notin w$

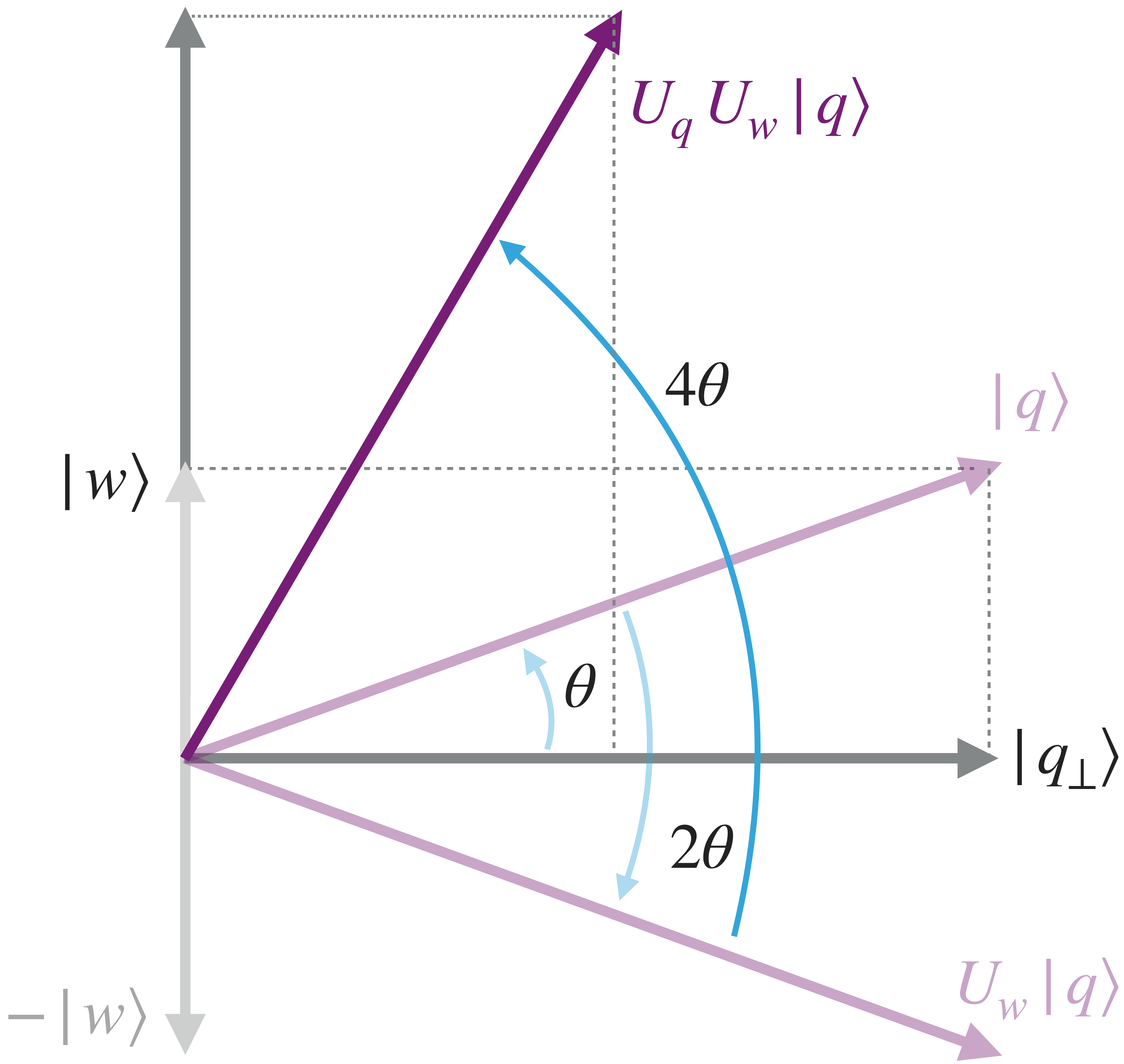
- The **diffusion operator**

$$U_q = 2|q\rangle\langle q| - \boldsymbol{I}$$

- performs a **reflection** around the initial state. The iterative application t times leads to

$$(U_q U_w)^t |q\rangle = \sin\theta_t |w\rangle + \cos\theta_t |q_\perp\rangle \qquad \theta_t = (2t+1)\,\theta$$

GROVER'S ALGORITHM



- if $r \ll N$, requires $\mathcal{O}(\sqrt{N/r})$ iterations instead of $\mathcal{O}(N)$ from a **classical** computation
- if $r = N/4$ or $\theta = 30^o$, then one single iteration brings to $\theta_t = 90^o$

- if $r = 3N/4$ or $\theta = 60^o$, then one single iteration brings to $\theta_t = 180^o$ which suppresses the probability of the winning state
- if $r = N/2$ or $\theta = 45^o$, it does not matter how many iterations are applied, probabilities remain unchanged

- increasing the total number of states, e.g. maximum of two **ancillary** qubits [see also Nielsen-Chuang 2000]
- Given a causal state, the mirror state with all the momentum flows reversed is also causal

LOOP QUANTUM ALGORITHM

- **The** $|q\rangle$ **register** encodes the states of the edges/internal propagators: the qubit q_i is in the state $|1\rangle$ if the momentum flow of the corresponding edge is oriented in the direction of the original assignment, and $|0\rangle$ if it is in the opposite direction
- **The** $|c\rangle$ **register** stores the binary clauses that probe if two qubits representing two adjacent edges are in the same state (oriented in the same direction)

$$c_{ij} \equiv (q_i = q_j) \qquad \bar{c}_{ij} \equiv (q_i \neq q_j)$$

- **The** $|a\rangle$ **register** stores the loop clauses that probe if all the qubits (edges) in each subloop form a cyclic circuit
- **The Grover's marker** initialized to the Bell state $|out_0\rangle = |-\rangle = \big(|0\rangle - |1\rangle\big)/\sqrt{2}$
- **The oracle**

$$U_w|q\rangle|c\rangle|a\rangle|out_0\rangle = |q\rangle|c\rangle|a\rangle|out_0 \otimes f(a,q)\rangle$$

$$|out_0 \otimes 0\rangle = |out_0\rangle$$
$$|out_0 \otimes 1\rangle = -|out_0\rangle$$

- **The diffuser** U_q from IBM Qiskit

LOOP QUANTUM ALGORITHM

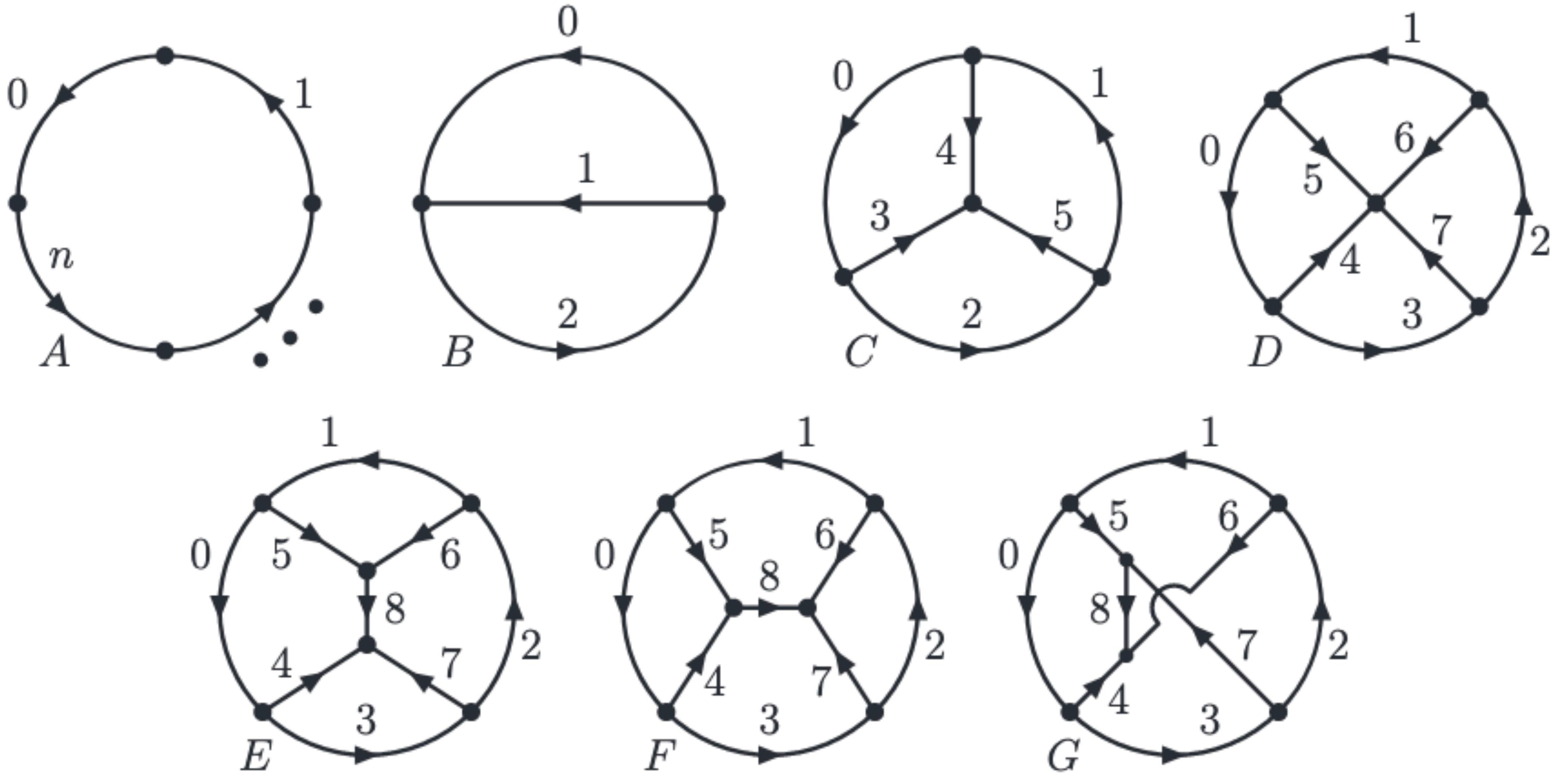


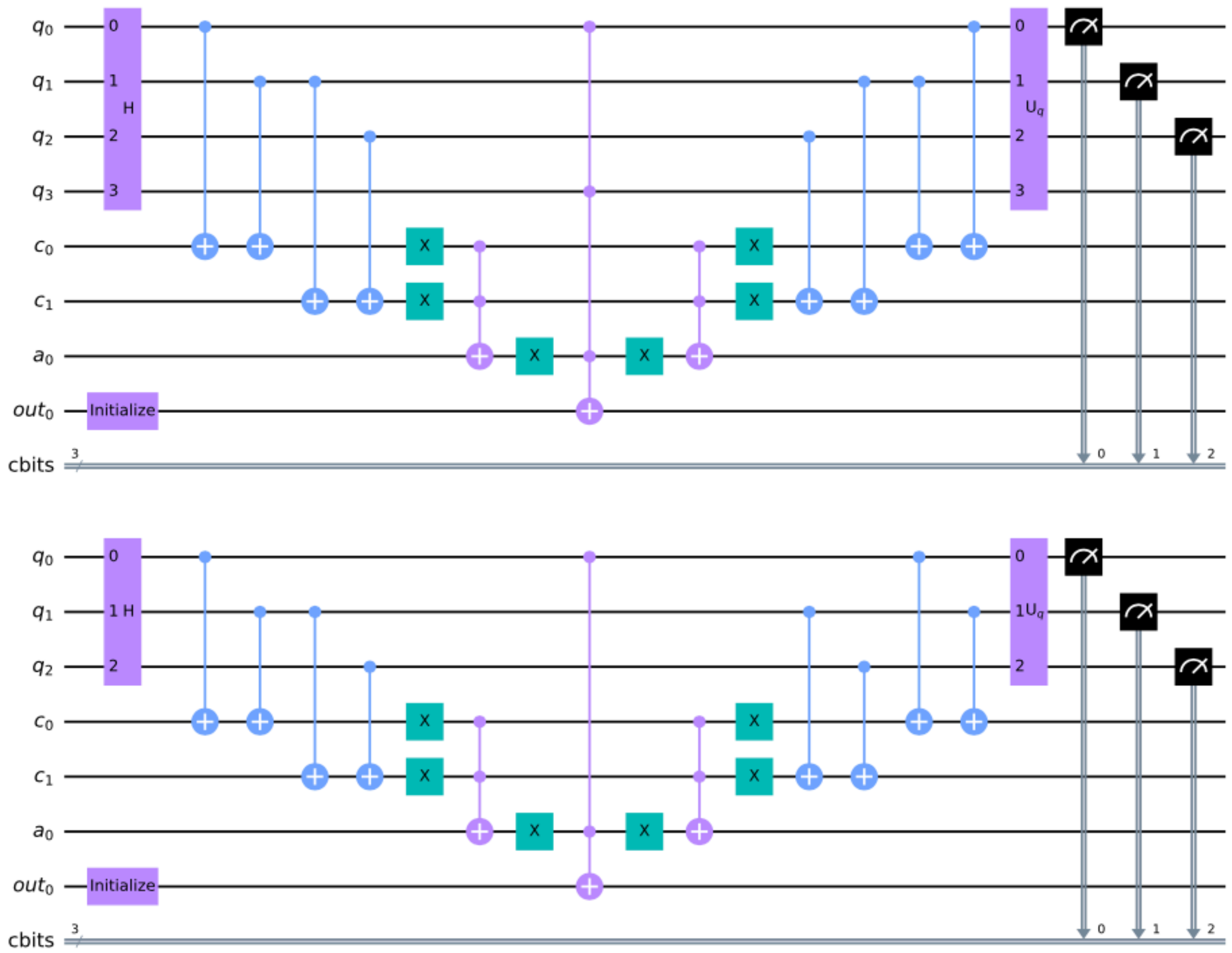
TABLE I. Number of qubits in each of the three main registers. The total number of qubits includes the ancillary qubit which is initialized to $|-\rangle$ to implement Grover's oracle. Measurements are made on $n = \sum n_i$ classical bits.

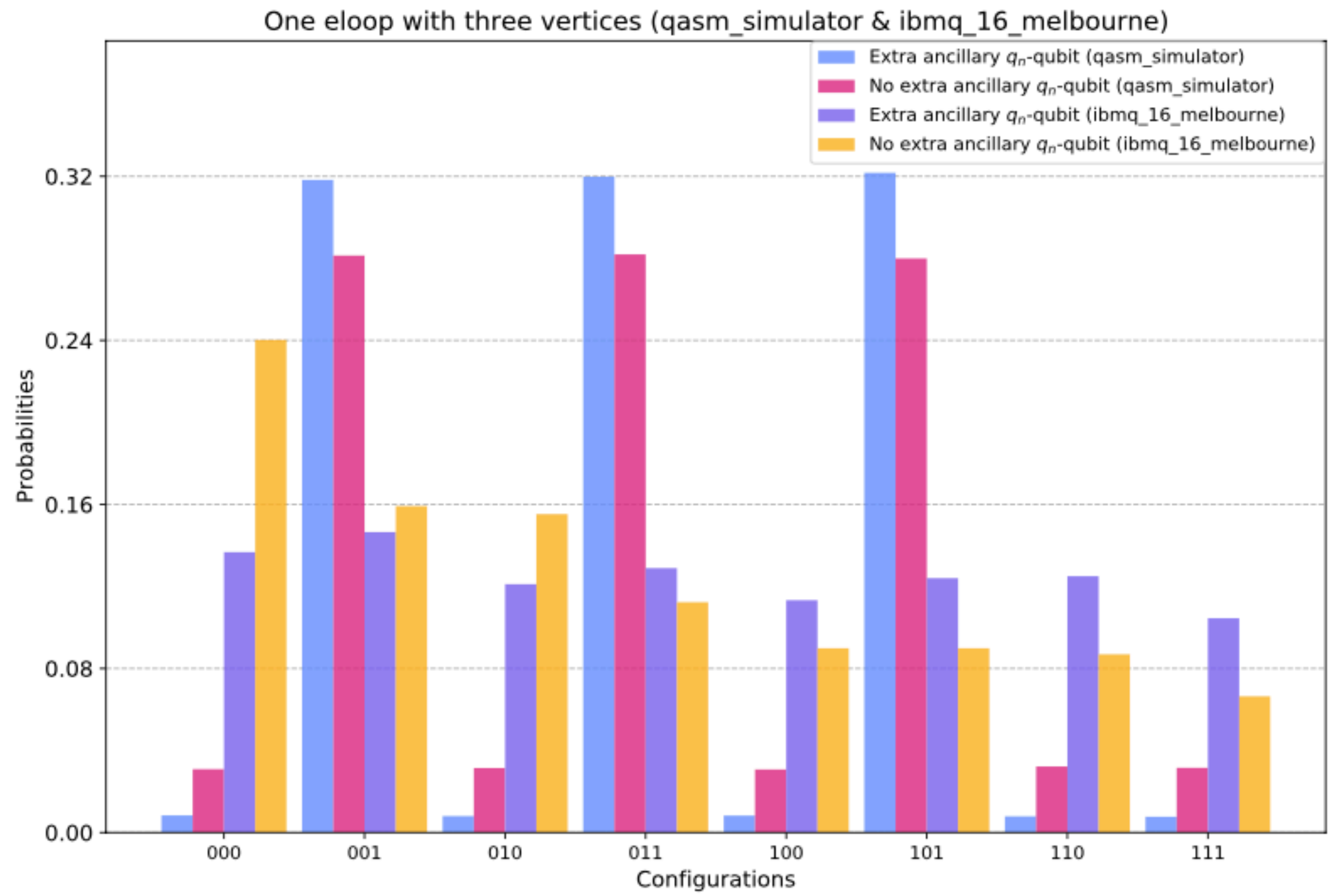
eloops (edges per set)	$\lvert q\rangle$	$\lvert c\rangle$	$\lvert a\rangle$	Total
one (n)	$n+1$	$n-1$	1	$2n+2$
two (n_0, n_1, n_2)	n	n	3	$2n+4$
three ($n_0, \ldots, n_5$)	n	$n+(2 \text{ to } 3)$	4 to 7	$2n+(7 \text{ to } 11)$
four ($n_0, \ldots, n_7$)	n	$n+(3 \text{ to } 6)$	5 to 13	$2n+(9 \text{ to } 20)$
four ($n_0, \ldots, n_8^{(t,s)}$)	n	$n+(4 \text{ to } 7)$	5 to 13	$2n+(10 \text{ to } 21)$
four ($n_0, \ldots, n_8^{(u)}$)	n	$n+(5 \text{ to } 8)$	9 to 13	$2n+(15 \text{ to } 22)$

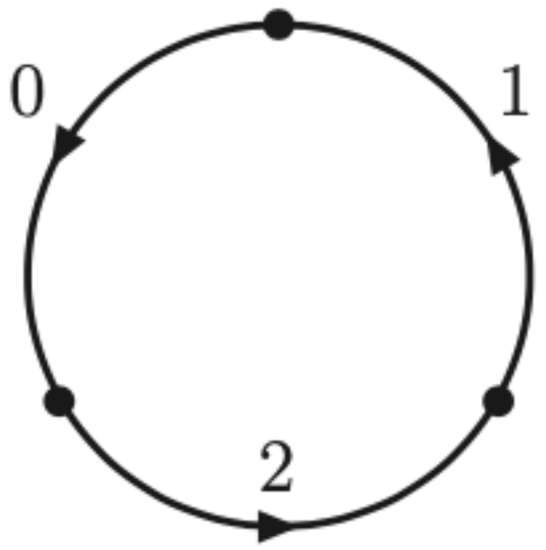
- 19 qubits if $n_i = 1$,
- 25 qubits if $n_i = 1$,
- 28 qubits if $n_i = 1$,
- 33 qubits if $n_i = 1$,

ONE ELOOP Qiskit

- $|q\rangle = H^{\otimes n}|0\rangle$
- The $|c\rangle$ and $|a\rangle$ registers initialized to $|0\rangle$



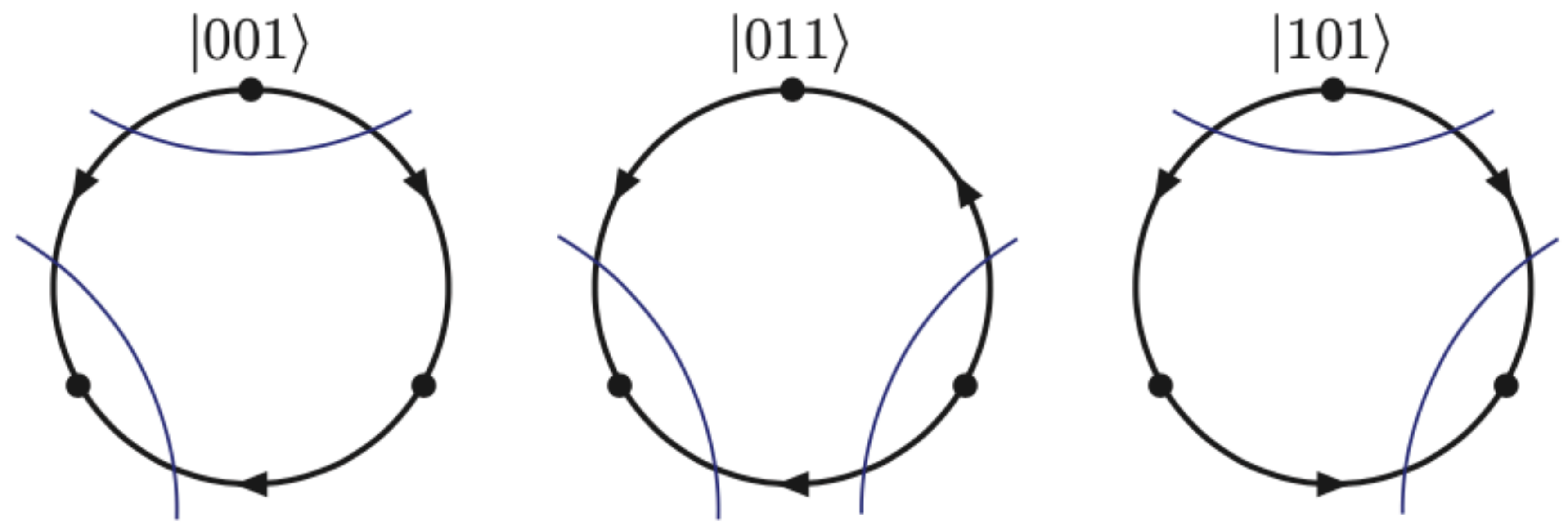




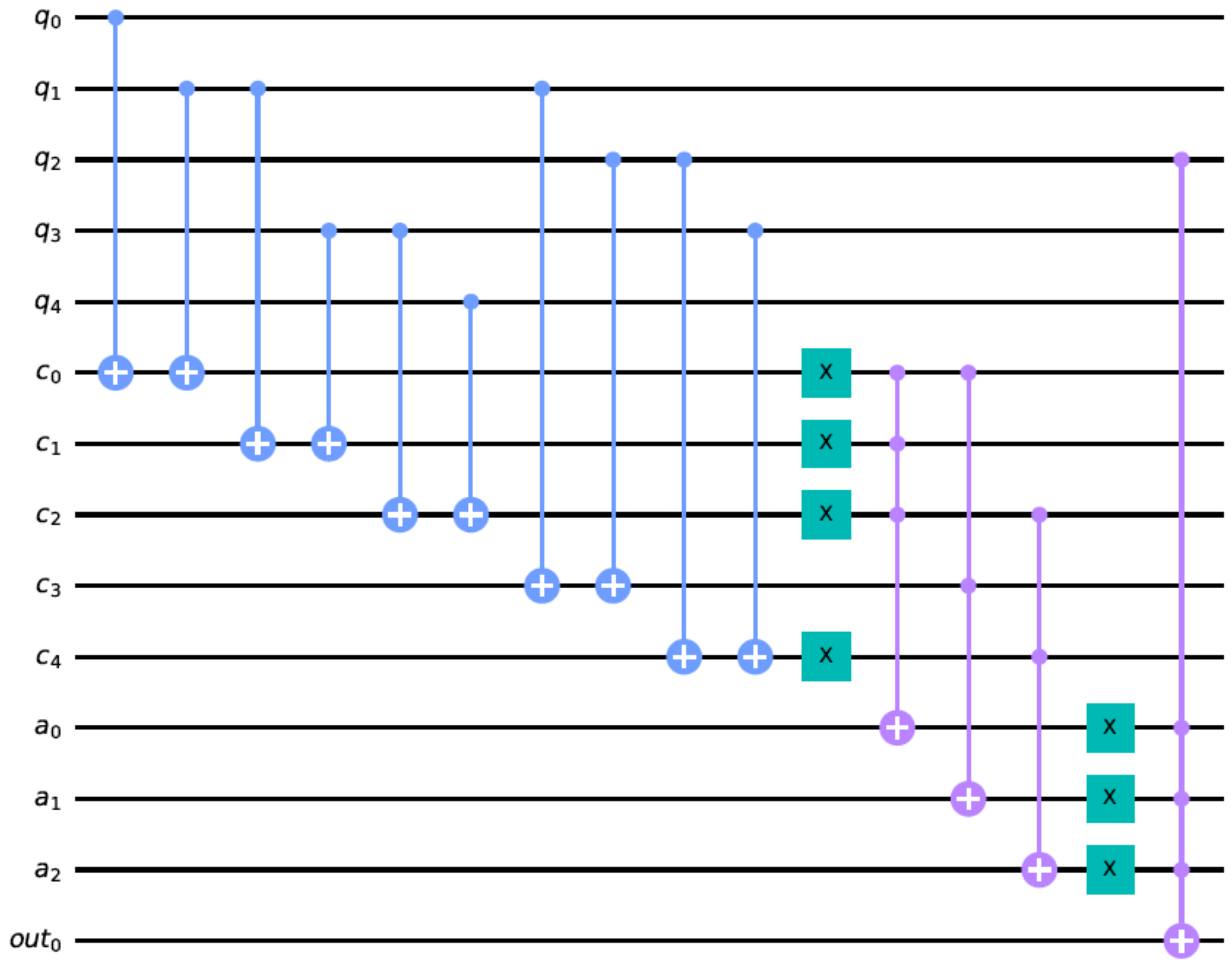
$$a_0 = \neg(c_{01} \wedge c_{12})$$

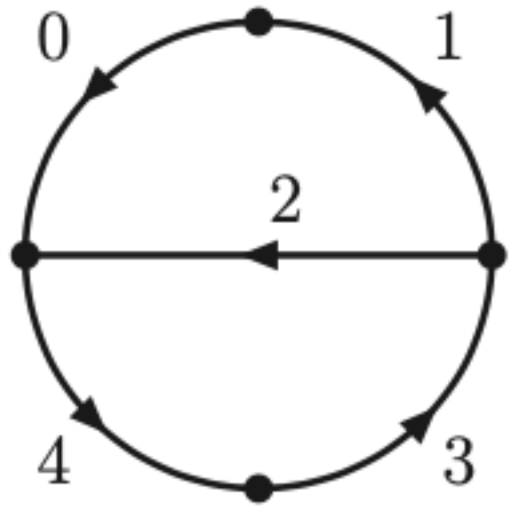
$$f^{(1)}(a,q) = a_0 \wedge q_0 \wedge q_3$$

ancillary qubit



TWO ELOOPS Qiskit



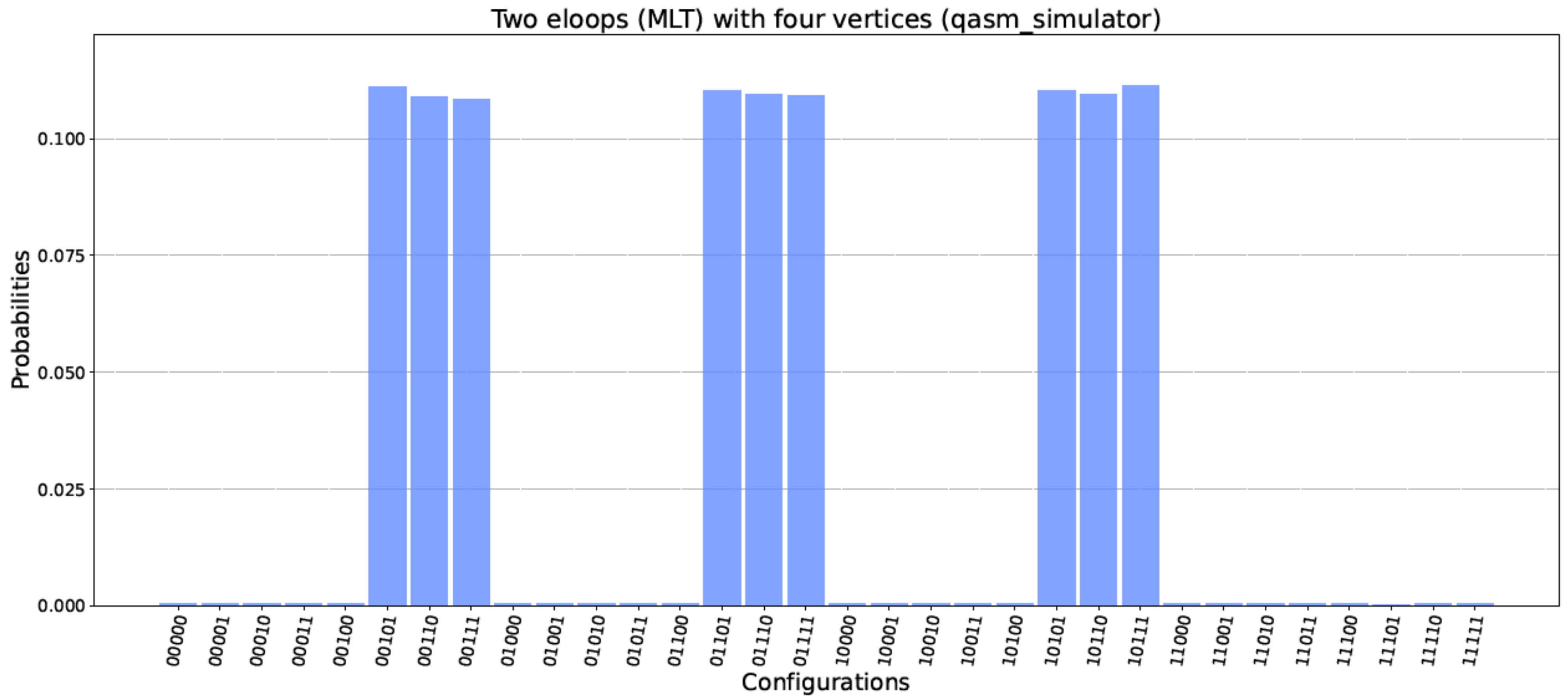


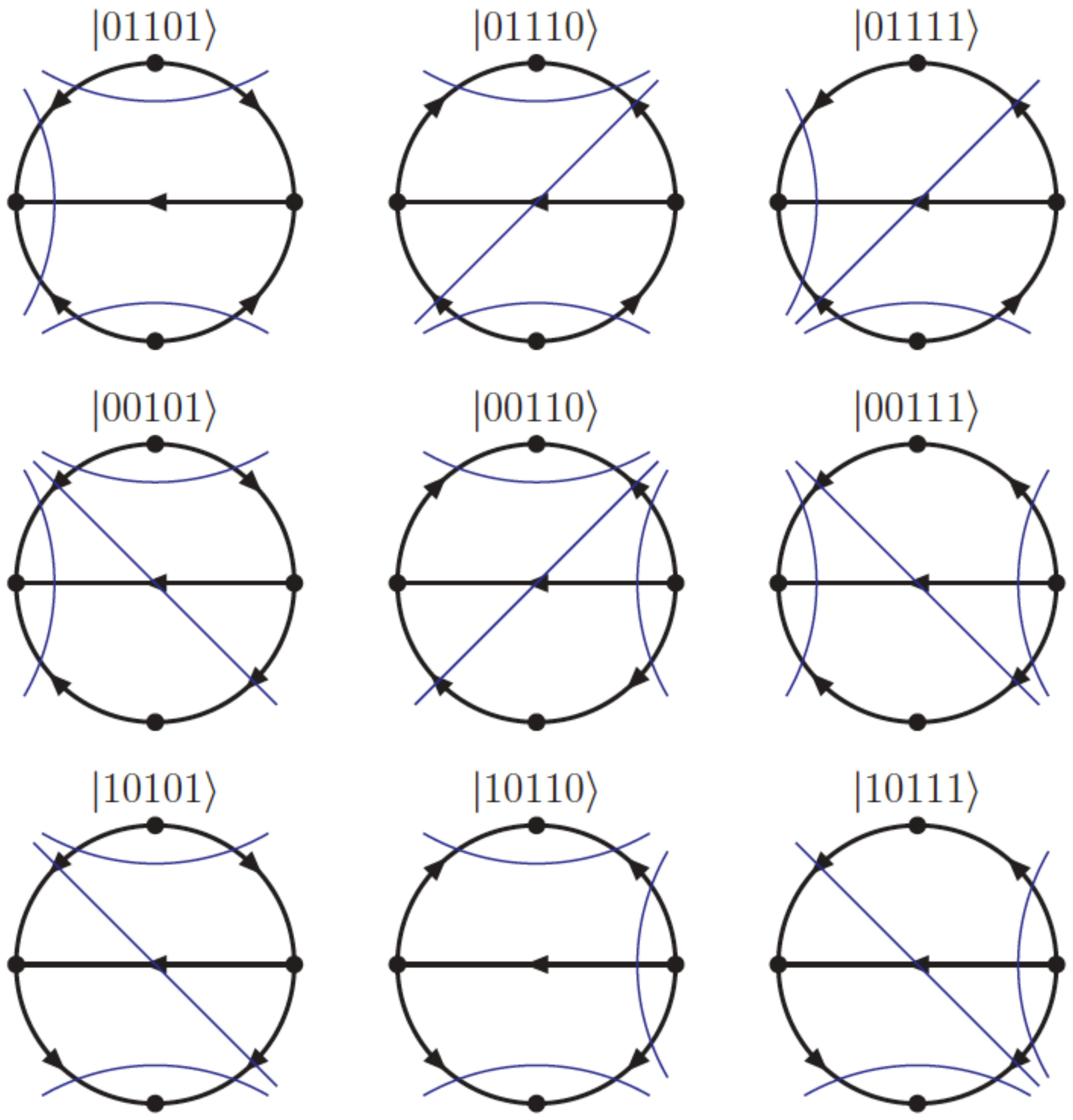
$$a_0 = \neg(c_{01} \wedge c_{13} \wedge c_{34})$$

$$a_1 = \neg(c_{01} \wedge \bar{c}_{12})$$

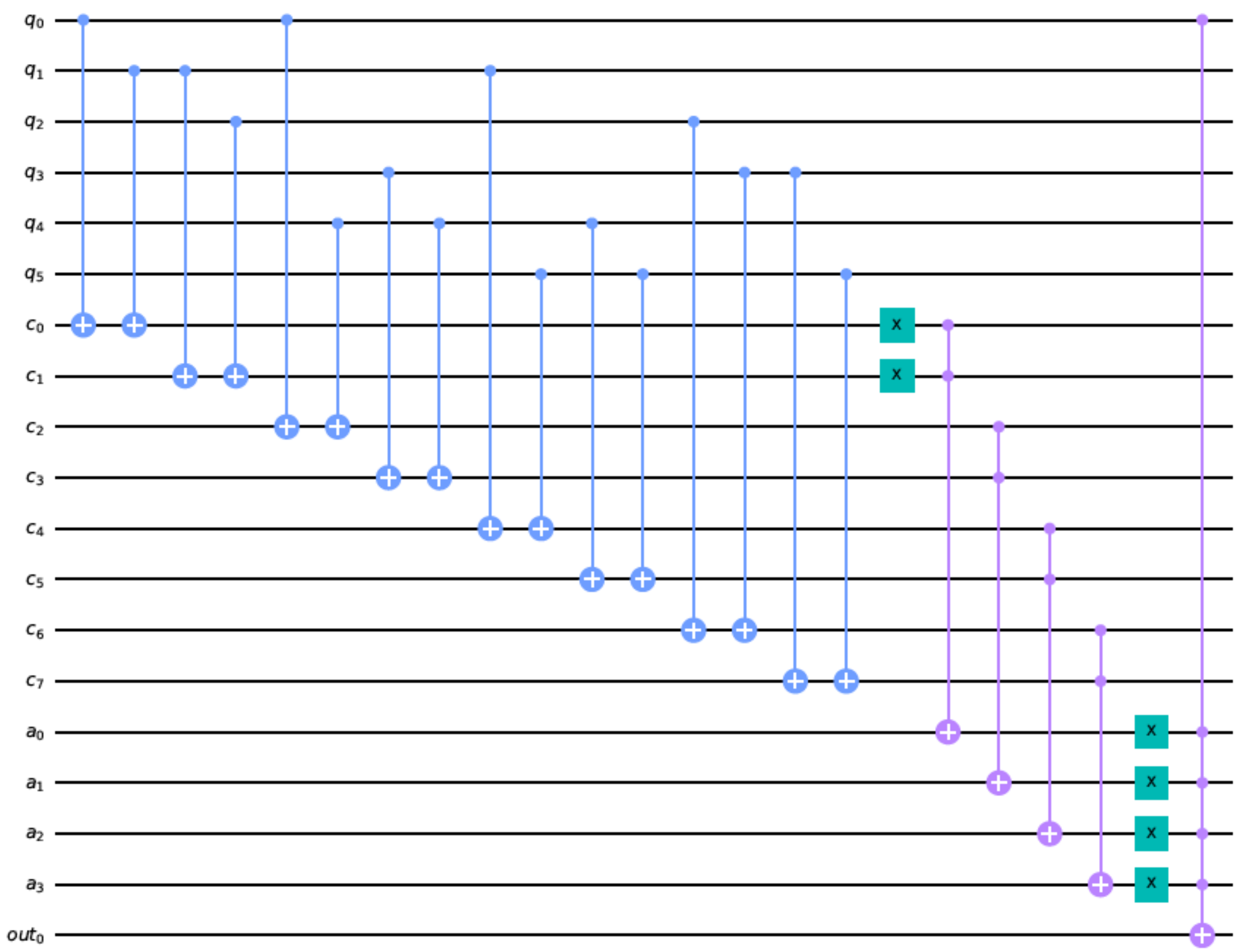
$$a_2 = \neg(c_{23} \wedge c_{34})$$

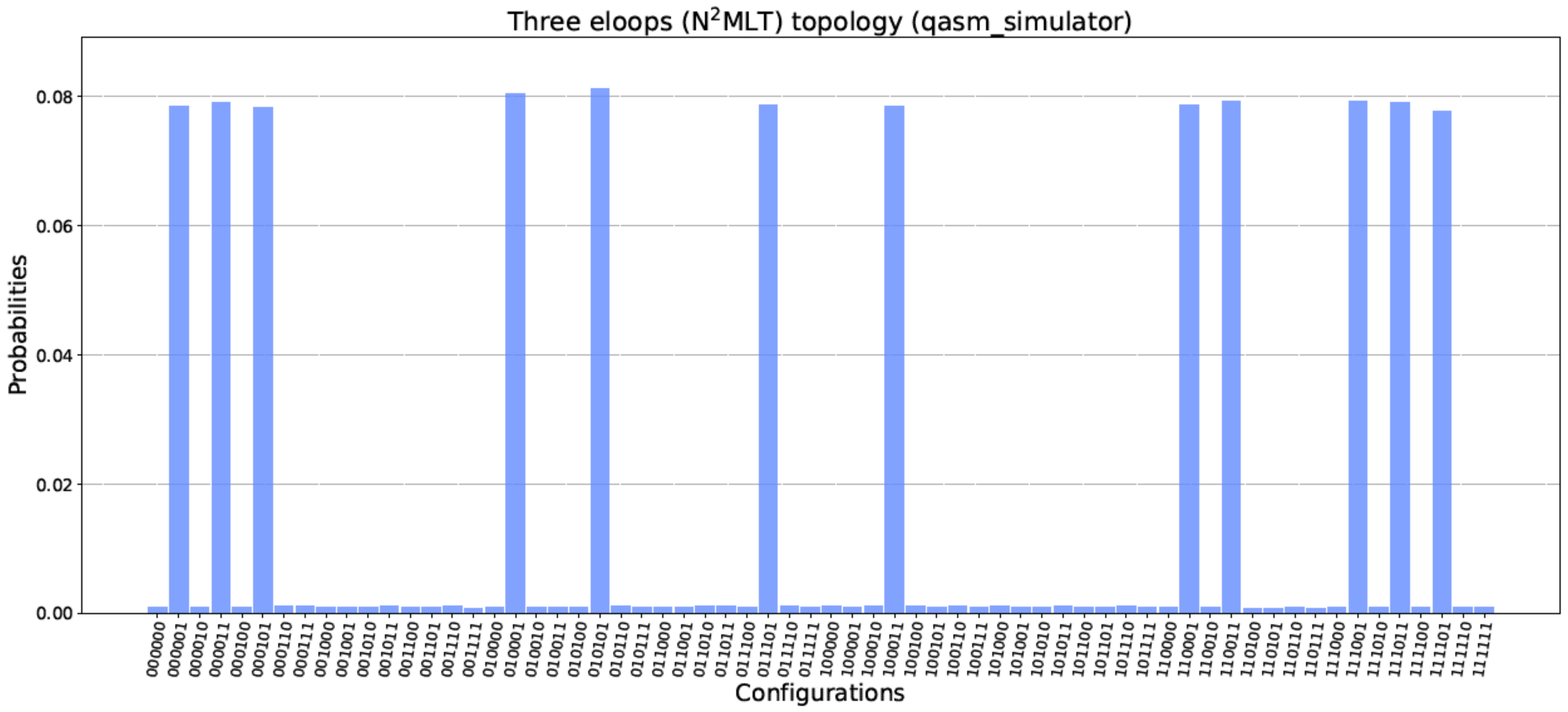
$$f^{(2)}(a,q) = (a_0 \wedge a_1 \wedge a_2) \wedge q_2$$

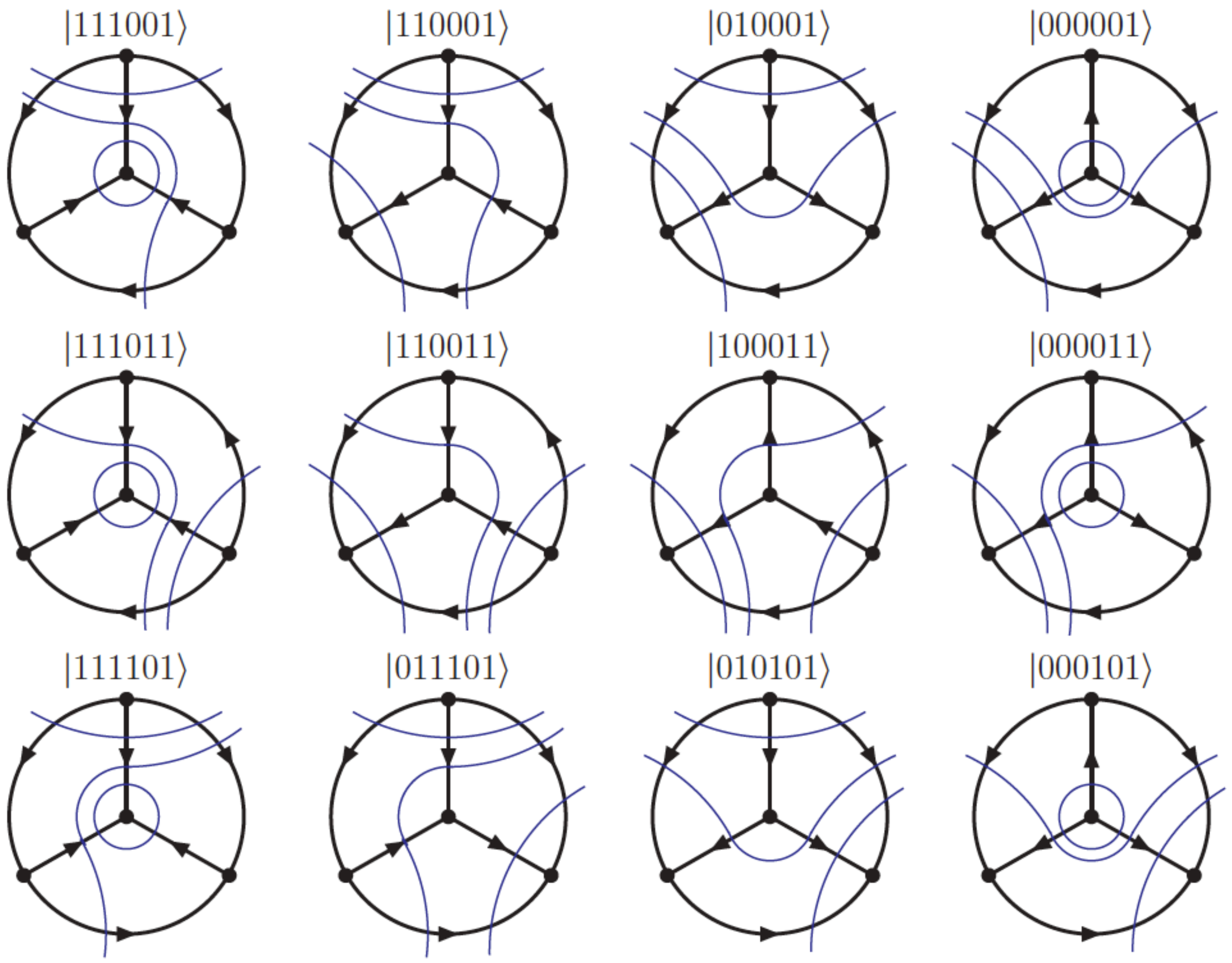


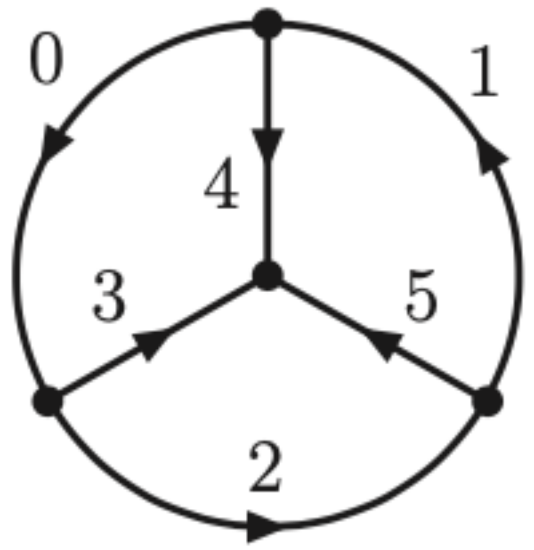


THREE ELOOPS Qiskit







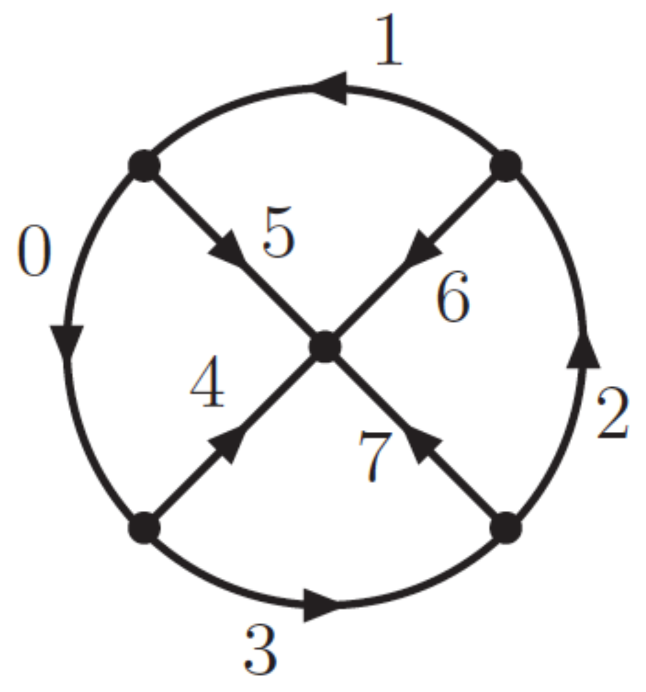


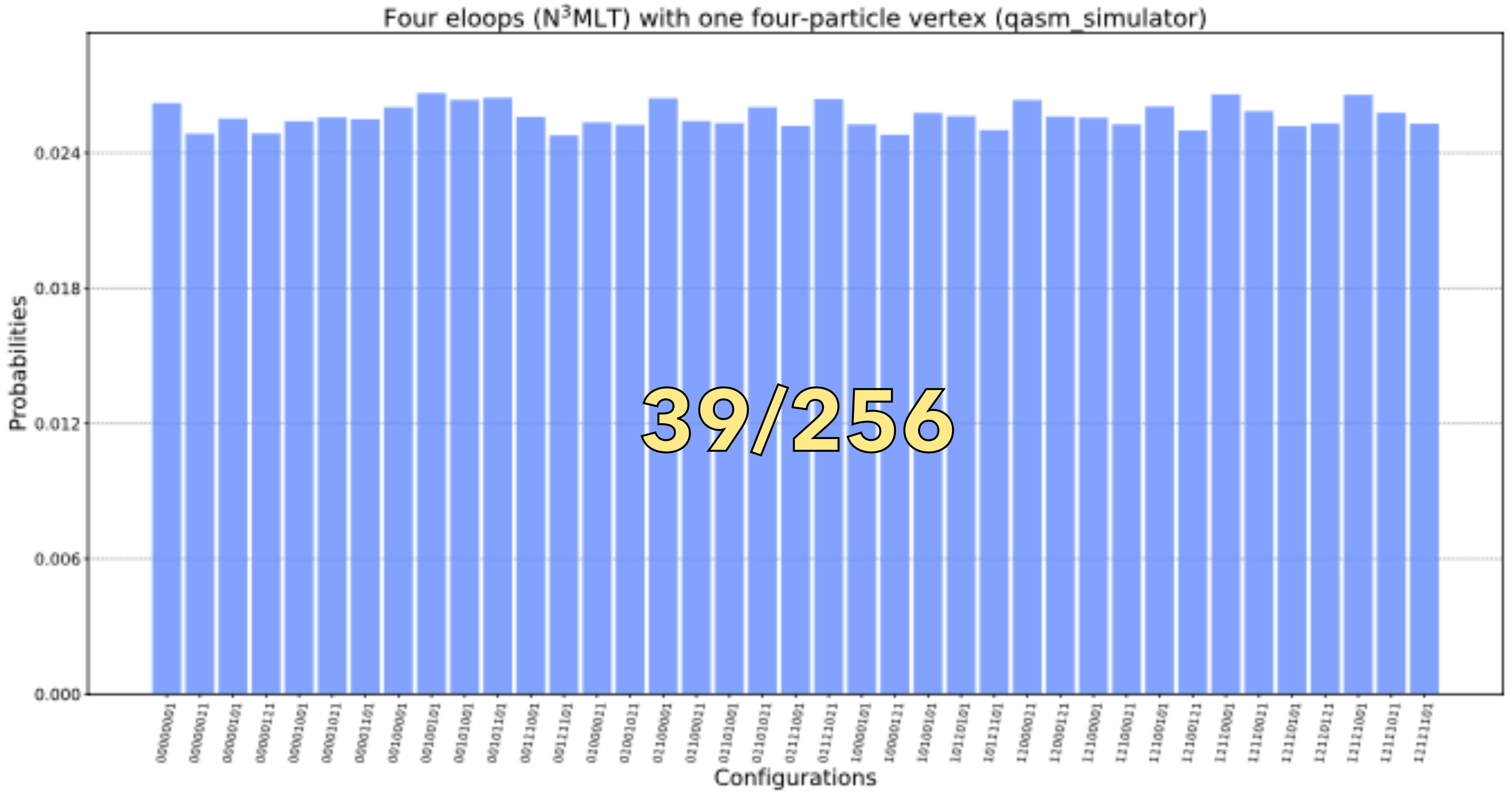
$$a_0 = \neg\left(c_{01} \wedge c_{12}\right) \qquad a_1 = \neg\left(\bar{c}_{04} \wedge \bar{c}_{34}\right)$$

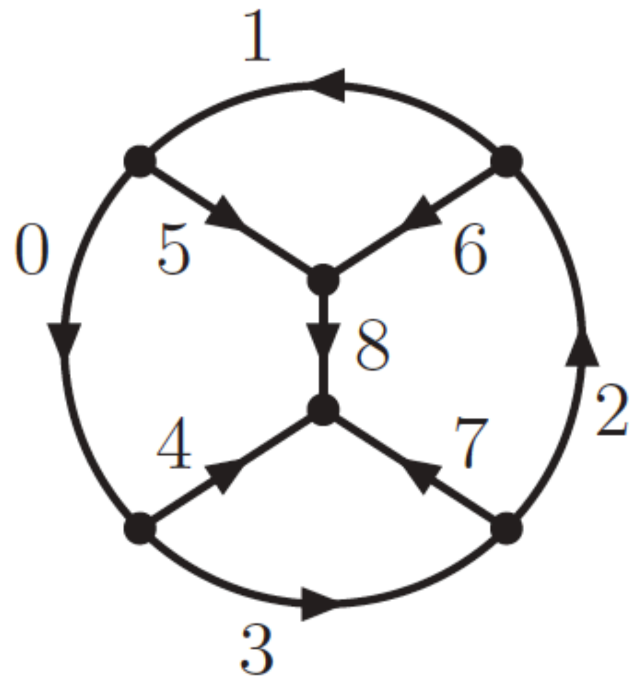
$$a_2 = \neg\left(\bar{c}_{15} \wedge \bar{c}_{45}\right) \qquad a_3 = \neg\left(\bar{c}_{23} \wedge \bar{c}_{35}\right)$$

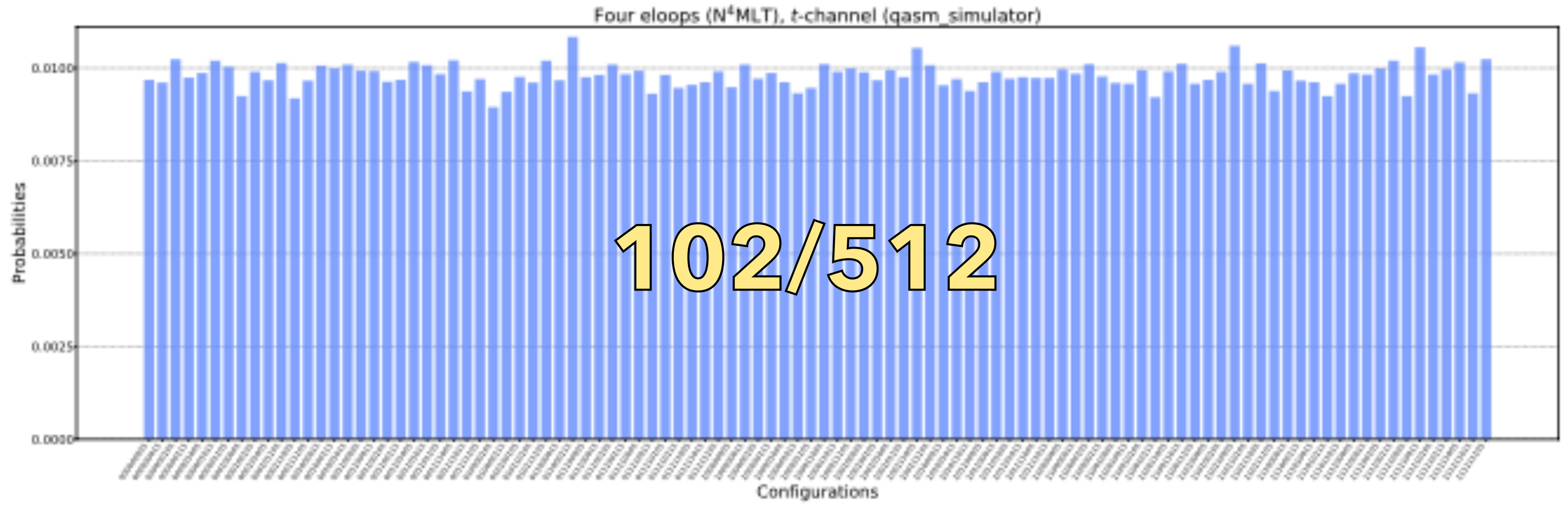
$$f^{(3)}(a, q) = (a_0 \wedge \ldots \wedge a_3) \wedge q_0$$

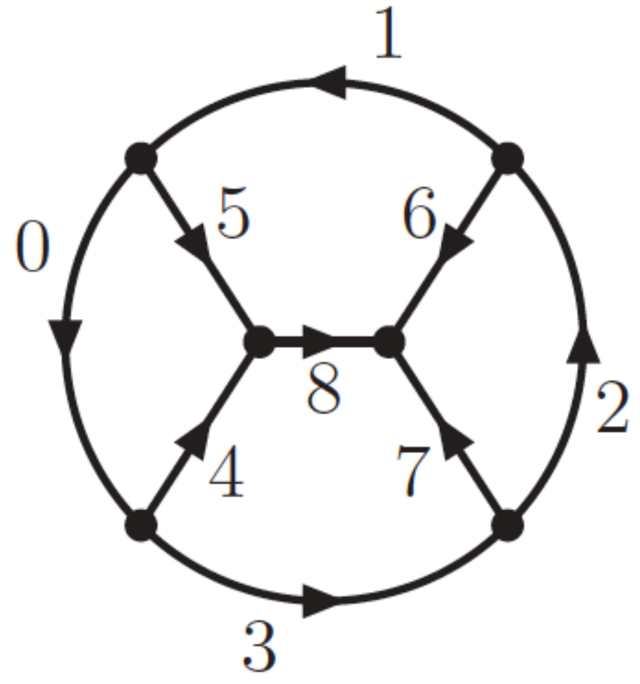
FOUR ELOOPS Qiskit

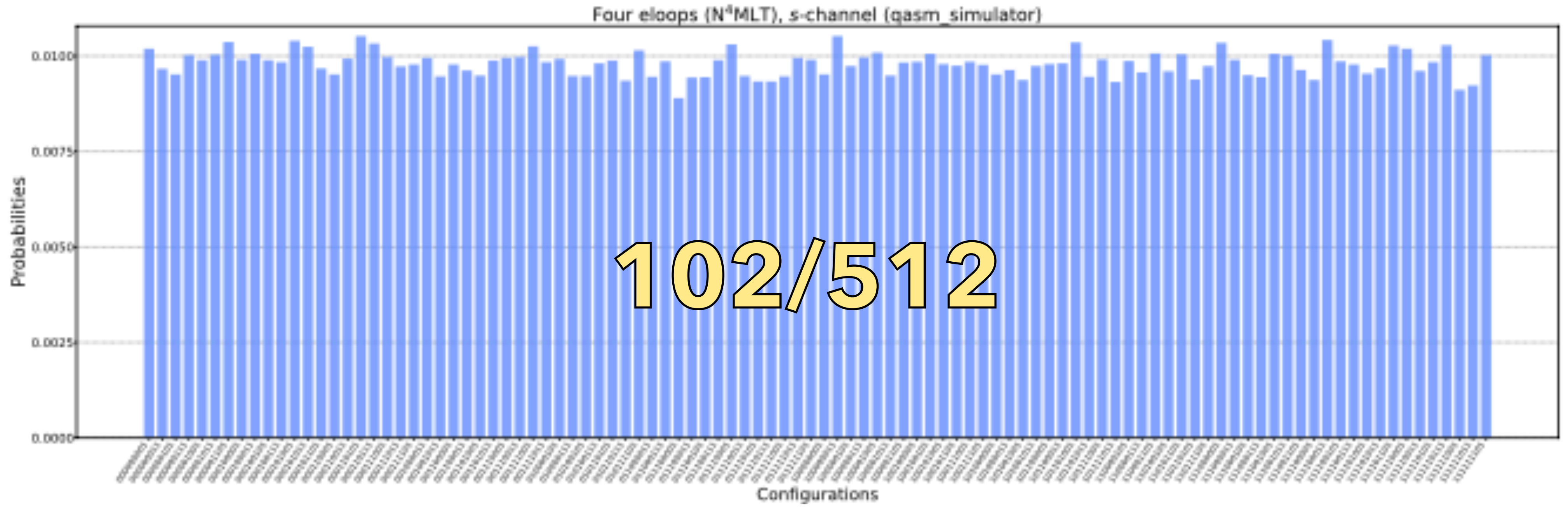


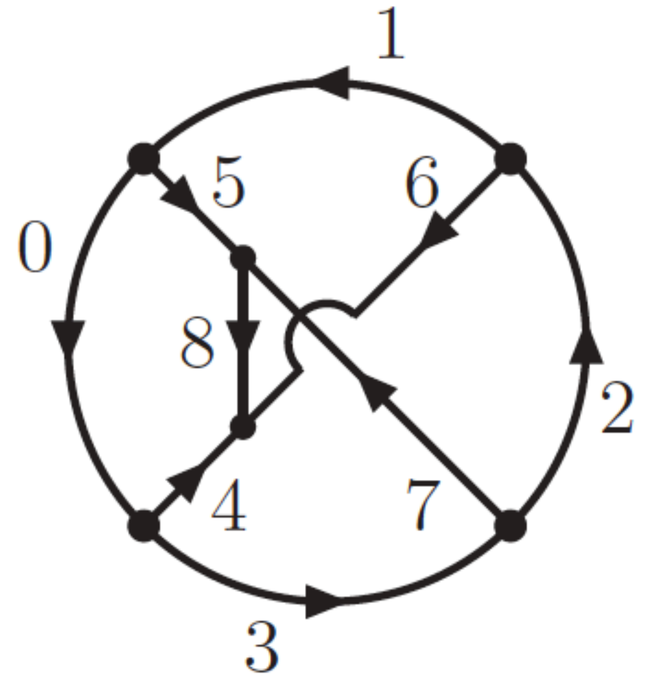






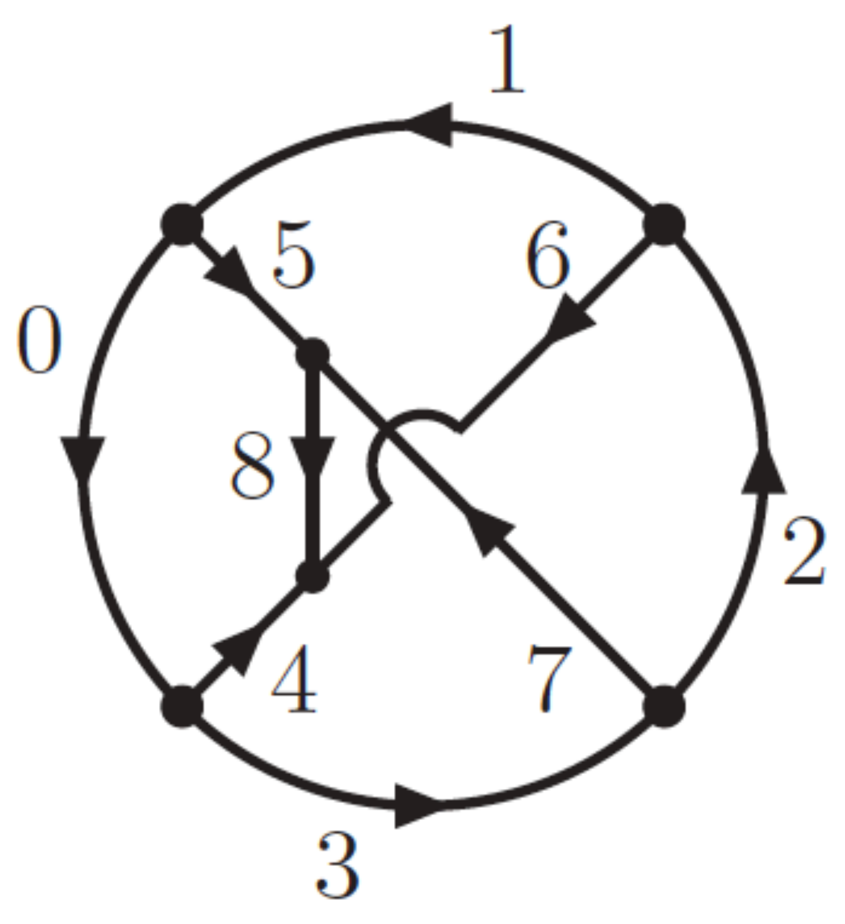






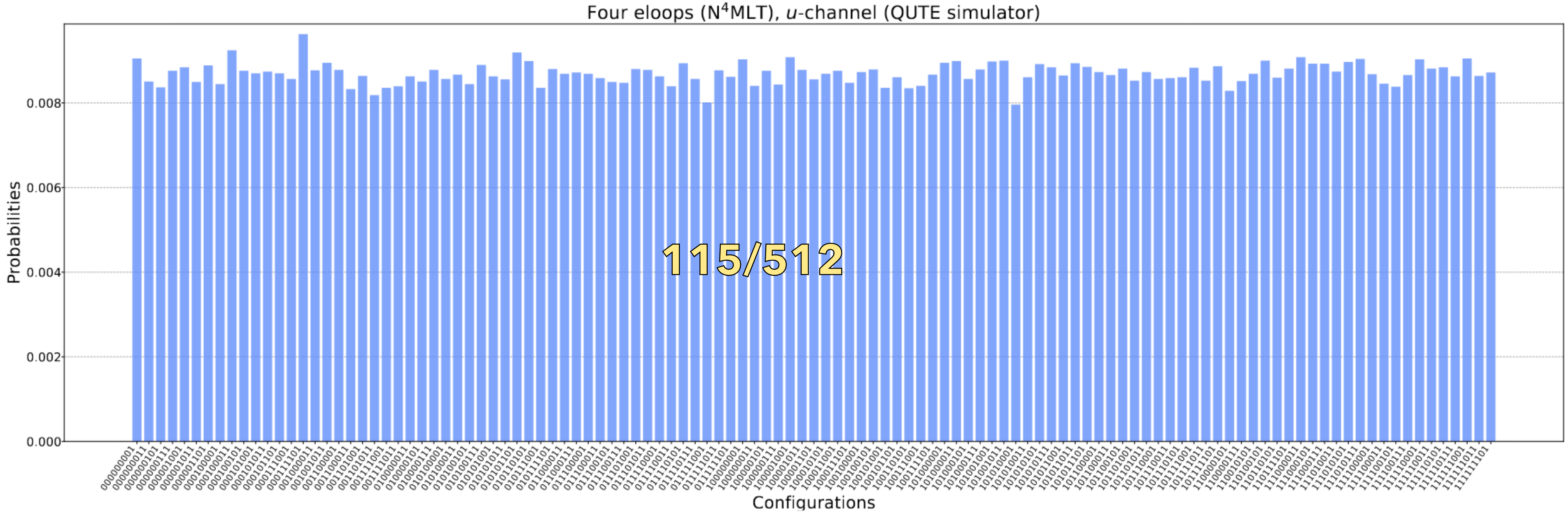
- First **nonplanar** graph starting at four loops
- 115/512 causal states
- requires 33 qubits > IBM Qiskit capacity

FOUR ELOOPS



- First **nonplanar** graph starting at four loops
- 115/512 causal states
- **QUTE simulator**, up to 38 logical qubits
  Fundación Centro Tecnológico de la Información y la Comunicación (CTIC), Gijón, Spain

centro tecnológico



CONCLUSIONS

- **Causal configurations** of multilloop Feynman integrals successfully identified with an efficient modification of Grover's quantum algorithm for querying over unstructured databases
- Beyond particle physics, finds application for **directed acyclic graphs**
- Still limited by quantum volume in real devices, and number of qubits in quantum simulators
- Given the rapid progress in the field, flagship application for future developments