

All-order renormalization of electric charge in the Standard Model and beyond

Stefan Dittmaier

Albert-Ludwigs-Universität Freiburg



mainly based on

S.D., Phys.Rev.D 103 (2021) 5, 053006 [arXiv:2101.05154];

A.Denner, S.D., Phys.Rept. 864 (2020) 1 [arXiv:1912.06823];

A.Denner, S.D., G.Weiglein, Nucl.Phys.B 440 (1995) 95-128 [hep-ph/9410338]

Contents

Introduction

Charge renormalization in QED

Charge renormalization in the SM @ NLO

Renormalization in the background-field method

Charge renormalization in the SM to all orders in R_ξ gauge

Generalization to gauge groups $G \times U(1)$

Conclusions

Introduction



Introduction

Electric unit charge e :

- ▶ crucial input parameters of QED and the SM(+beyond)
- ▶ defined in the *Thomson limit* (low-energy γ interaction with e^-)
- ▶ high-precision measurements of $\alpha(0) = \frac{e^2}{4\pi}$ at low energies
- ▶ links high-energy and low-energy physics
- ▶ enters analysis of running of couplings to high energies (unification?)

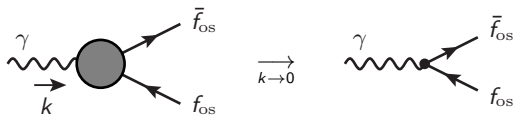
Some features/theorems connected with electric charge Q_e :

- ▶ **charge quantization:**
all relative charges Q are integers or integer multiple of $1/3$
 \hookrightarrow only partial explanation in SM ($Q_e = Q_W$ ok, $Q_e = 3Q_d$ unexplained)
- ▶ **charge universality:**
renormalization of e does not depend on charged particle
(=field-theoretical fact in QED and the SM)
 \hookrightarrow any charged particle can be used for renormalization of e
- ▶ **Thirring's low-energy theorem:** Thirring '50 (SM: S.D. '97)

$$\sigma_{\text{Compton}}^{e^- \gamma \rightarrow e^- \gamma}(E_\gamma) \xrightarrow{E_\gamma \rightarrow 0} \sigma_{\text{Thomson}}^{e^- \gamma \rightarrow e^- \gamma} \quad \text{to all orders!}$$

Renormalization of e :

- ▶ renormalization condition in Thomson limit
 \hookrightarrow no corrections in the low-energy limit for on-shell (OS) electrons



$$\bar{u}(p') \Gamma_{R,\mu}^{A\bar{f}f}(k, -p', p) u(p) \xrightarrow{k \rightarrow 0} -Q_f e \bar{u}(p) \gamma_\mu u(p)$$

- ▶ renormalization transformation: $e_0 = Z_e e = (1 + \delta Z_e) e$
bare ren.
- \hookrightarrow charge ren. constant Z_e fixed by vertex correction $\Gamma_\mu^{A\bar{f}f}(0, -p, p)$
- ▶ QED: Z_e derived from $\gamma\gamma$ self-energy Σ^{AA} via QED Ward identity
- ▶ SM(+beyond): Z_e calculable from $\Sigma^{AA/AZ}$, but underlying gauge-invariance arguments much more complicated!
 \hookrightarrow issue of this talk

Relevance of relating Z_e to (f -independent) self-energies $\Sigma^{AA/AZ}$?

\hookrightarrow General understanding, proving theorems, technical simplifications, ...

On the history of charge renormalization in the SM

Charge renormalization at NLO:

- ▶ pioneering works on electroweak renormalization:
 - ▶ δZ_e from explicit 1-loop vertex correction
Ross, Taylor '73; Sirlin '80; Bardin, Khristova, Fedorenko '80; Aoki et al. '82
 - ▶ δZ_e via Ward identities from explicit calculations
Fleischer, Jegerlehner '81; Böhm, Spiesberger, Hollik '86; Hollik '90; Denner '93
- ▶ more recently: derivation of δZ_e via Lee identities Denner, S.D. '19

Charge renormalization at NNLO:

- ▶ δZ_e from explicit 2-loop vertex correction
Degrassi, Vicini '03; Actis, Passarino et al. '06
- ▶ δZ_e checked against BFM result (see below) Degrassi, Vicini '03

Charge renormalization to all orders:

- ▶ Background-Field Method (BFM):
generalization of QED-like result to SM Denner, S.D., Weiglein '94
- ▶ conventional R_ξ gauge:
 - ▶ correct conjecture, but wrong proofs Bauberger '97; Freitas et al. '02; Awramik et al. '02
 - ▶ new approach via charge universality S.D. '21
 - ↔ confirmation of previous conjecture → this talk!

Charge renormalization in QED

Charge renormalization in QED

Ward identity for the **unrenormalized** $Af\bar{f}$ vertex function:

$$k^\mu \Gamma_\mu^{A\bar{f}f}(k, \bar{p}, p) = -Q_f e_0 \left[\Gamma^{\bar{f}f}(\bar{p}, -\bar{p}) - \Gamma^{\bar{f}f}(-p, p) \right]$$

Renormalization transformation: $\psi_{0,f}(x) = Z_f^{1/2} \psi_f(x)$, $A_0^\mu(x) = Z_{AA}^{1/2} A^\mu(x)$

$$G_\mu^{A\bar{f}f}(k, \bar{p}, p) = Z_{AA}^{1/2} Z_f G_{R,\mu}^{A\bar{f}f}(k, \bar{p}, p) \quad (\text{full, reducible Green function})$$

$$\Gamma_{R,\mu}^{A\bar{f}f}(k, \bar{p}, p) = Z_{AA}^{1/2} Z_f \Gamma_\mu^{A\bar{f}f}(k, \bar{p}, p) \quad (\text{amputated, 1PI vertex function})$$

\Rightarrow Ward identity for the **renormalized** $Af\bar{f}$ vertex function:

$$k^\mu \Gamma_{R,\mu}^{A\bar{f}f}(k, \bar{p}, p) = -Q_f e Z_e Z_{AA}^{1/2} \left[\Gamma_R^{\bar{f}f}(\bar{p}, -\bar{p}) - \Gamma_R^{\bar{f}f}(-p, p) \right]$$

\hookrightarrow Expansion for $k \rightarrow 0$ yields

$$\underbrace{\bar{u}(p) \Gamma_{R,\mu}^{A\bar{f}f}(0, -p, p) u(p)}_{\stackrel{!}{=} -Q_f e \bar{u}(p) \gamma_\mu u(p)} = -Q_f e Z_e Z_{AA}^{1/2} \underbrace{\bar{u}(p) \frac{\partial \Gamma_R^{\bar{f}f}(-p, p)}{\partial p^\mu} u(p)}_{\substack{= \bar{u}(p) \gamma_\mu u(p) \\ (\text{wave-fct. ren. of } \psi_f)}}$$

$\Rightarrow Z_e = Z_{AA}^{-1/2}$ = independent of $f \Rightarrow$ Charge universality of QED!

Charge renormalization in the SM @ NLO

QED

SM

Gauge symmetry:

$U(1)_{\text{em}}$ exact

$U(1)_Y$ spontaneously broken,
 $U(1)_{\text{em}}$ mixes $SU(2)_I$ and $U(1)_Y$ trafo

Neutral gauge bosons:

photon field A_μ

2 neutral gauge fields W_μ^3, B_μ ,
 mass basis via Weinberg rotation:

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = \begin{pmatrix} c_W & -s_W \\ s_W & c_W \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}$$

\hookrightarrow mixing of A_μ, Z_μ in higher orders

BRS symmetry:

fully decoupling FP ghost fields

\hookrightarrow ghost-free Ward identities

FP ghosts non-decoupling

\hookrightarrow ghost contributions in
 Slavnov–Taylor (ST) ids.

for Green fcts. $G^{\Psi_1\Psi_2\dots}$ and

Lee ids. for 1PI vertex fcts. $\Gamma^{\Psi_1\Psi_2\dots}$

Charge renormalization in the SM @ NLO in R_ξ gauge

1-loop Ward identity: derived via Lee identities or ST identities

(Denner, S.D., 1912.06823) (see also backup slides)

$$\begin{aligned} \bar{u}(p) \Gamma_\mu^{A\bar{f}f}(0, -p, p) u(p) &= -Q_f e_0 \bar{u}(p) \frac{\partial \Gamma^{\bar{f}f}(-p, p)}{\partial p^\mu} u(p) \\ &\quad - \frac{I_{W,f}^3 e_0}{s_{W,0} c_{W,0}} \frac{\Sigma_T^{AZ}(0)}{M_Z^2} \bar{u}(p) \gamma_\mu \omega_- u(p), \quad \omega_\pm = \frac{1}{2}(1 \pm \gamma_5) \end{aligned} \quad (*)$$

Generalization beyond 1-loop level not available!

Renormalization: $\begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} Z_{ZZ}^{1/2} & Z_{ZA}^{1/2} \\ Z_{AZ}^{1/2} & Z_{AA}^{1/2} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}, \quad \omega_\sigma f_0 = f_0^\sigma = (Z^{f,\sigma})^{1/2} f^\sigma$

$$\begin{aligned} \underbrace{\bar{u}(p) \Gamma_{R,\mu}^{A\bar{f}f}(0, -p, p) u(p)}_{\stackrel{!}{=} -Q_f e \bar{u}(p) \gamma_\mu u(p)} &= \bar{u}(p) \Gamma_\mu^{A\bar{f}f}(0, -p, p) u(p) + \frac{1}{2} \delta Z_{ZA} \bar{u}(p) \Gamma_{\mu,0}^{Z\bar{f}f} u(p) \\ &\stackrel{(*)}{=} -Q_f e \left(1 + \delta Z_e + \frac{1}{2} \delta Z_{AA} + \frac{s_W}{2c_W} \delta Z_{ZA} \right) \bar{u}(p) \gamma_\mu u(p) \\ &\quad + \frac{1}{2} (\delta Z_{AA} + \delta Z^{f,+} + \delta Z^{f,-}) \bar{u}(p) \Gamma_{\mu,0}^{A\bar{f}f} u(p) \\ &\quad + \frac{I_{W,f}^3 e}{s_W c_W} \underbrace{\left(\frac{1}{2} \delta Z_{ZA} - \frac{\Sigma_T^{AZ}(0)}{M_Z^2} \right)}_{=0 \text{ (OS renormalization)}} \bar{u}(p) \gamma_\mu \omega_- u(p) \end{aligned}$$

$$\Rightarrow \delta Z_e = -\frac{1}{2} \delta Z_{AA} - \frac{s_W}{2c_W} \delta Z_{ZA}$$

Renormalization in the background-field method

BFM Ward identity for the **unrenormalized** $\hat{A}\bar{f}f$ vertex function:

$$k^\mu \hat{\Gamma}_{\mu\nu}^{\hat{A}\hat{V}}(k, -k) = 0, \quad \hat{V} = \hat{A}, \hat{Z},$$

$$k^\mu \hat{\Gamma}_{\mu}^{\hat{A}\bar{f}f_j}(k, \bar{p}, p) = -Q_f e_0 \left[\hat{\Gamma}_{\bar{p}, -\bar{p}}^{\bar{f}f_j} - \hat{\Gamma}_{-p, p}^{\bar{f}f_j} \right]$$

Renormalization transformation: $\begin{pmatrix} \hat{Z}_0 \\ \hat{A}_0 \end{pmatrix} = \begin{pmatrix} Z_{\hat{Z}\hat{Z}}^{1/2} & Z_{\hat{Z}\hat{A}}^{1/2} \\ Z_{\hat{A}\hat{Z}}^{1/2} & Z_{\hat{A}\hat{A}}^{1/2} \end{pmatrix} \begin{pmatrix} \hat{Z} \\ \hat{A} \end{pmatrix}$, $f_{0,n}^\sigma = \sum_j (Z_{nj}^{f,\sigma})^{1/2} f_j^\sigma$

$$\hat{\Gamma}_R^{\bar{f}f_j}(-p, p) = \sum_{l,n} (Z_{li}^{f,\sigma^*})^{1/2} (Z_{nj}^{f,\sigma})^{1/2} \hat{\Gamma}_R^{\bar{f}f_n}(-p, p),$$

$$\hat{\Gamma}_{R,\mu}^{\hat{A}\bar{f}f_j}(k, \bar{p}, p) = \sum_{\hat{V}=\hat{A},\hat{Z}} \sum_{l,n} \underbrace{Z_{\hat{V}\hat{A}}^{1/2}}_{Z_{\hat{Z}\hat{A}}=0 \text{ due to BFM gauge invariance}} (Z_{li}^{f,\sigma^*})^{1/2} (Z_{nj}^{f,\sigma})^{1/2} \hat{\Gamma}_R^{\hat{V}\bar{f}f_n}(k, \bar{p}, p)$$

BFM Ward identity for the **renormalized** $\hat{A}\bar{f}f$ vertex function:

$$k^\mu \hat{\Gamma}_{R,\mu}^{\hat{A}\bar{f}f_j}(k, \bar{p}, p) = -Q_f e Z_e Z_{\hat{A}\hat{A}}^{1/2} \left[\hat{\Gamma}_R^{\bar{f}f_j}(\bar{p}, -\bar{p}) - \hat{\Gamma}_R^{\bar{f}f_j}(-p, p) \right]$$

$\Rightarrow Z_e = Z_{\hat{A}\hat{A}}^{-1/2}$ analogously to QED \Rightarrow **Charge universality of the SM!**

Charge renormalization in the SM to all orders in R_ξ gauge

Charge renormalization with a fake fermion

Idea: exploit charge universality and introduce **fake fermion** η with infinitesimal weak hypercharge:

$$\frac{1}{2} Y_{W,\eta} = Q_\eta \rightarrow 0, \quad k_{W,\eta} = 0, \quad m_\eta = \text{arbitrary}$$

Lagrangian:

$$\mathcal{L}_\eta = \bar{\eta} \left(i \not{\partial} - \frac{1}{2} g_1 Y_{W,\eta} \not{B} - m_\eta \right) \eta = \bar{\eta} \left[i \not{\partial} - Q_\eta e \left(\not{A} + \frac{s_W}{c_W} \not{Z} \right) - m_\eta \right] \eta$$

Charge renormalization condition for η :

$$\bar{u}(p) \Gamma_{R,\mu}^{A\bar{\eta}\eta}(0, -p, p) u(p) \Big|_{p^2=m_\eta^2} = -Q_\eta e \bar{u}(p) \gamma_\mu u(p)$$

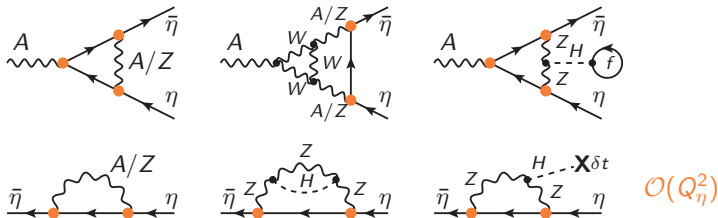
Renormalization transformation: $\eta_0 = Z_\eta^{1/2} \eta$

$$\Gamma_{R,\mu}^{A\bar{\eta}\eta}(k, \bar{p}, p) = Z_\eta Z_{AA}^{1/2} \underbrace{\Gamma_\mu^{A\bar{\eta}\eta}(k, \bar{p}, p)}_{= -Q_\eta e_0 \gamma_\mu + \text{h.o.}} + Z_\eta Z_{ZA}^{1/2} \underbrace{\Gamma_\mu^{Z\bar{\eta}\eta}(k, \bar{p}, p)}_{= -Q_\eta e_0 \frac{s_{W,0}}{c_{W,0}} \gamma_\mu + \text{h.o.}}$$

But: higher-order contributions to Z_η and $\Gamma_\mu^{V\bar{\eta}\eta}$ are of $\mathcal{O}(Q_\eta^2)$!

Charge renormalization with a fake fermion (continued)

Some sample diagrams for corrections to Z_η and $\Gamma_\mu^{V\bar{\eta}\eta}$:



⇒ Direct calculation of $\Gamma_{R,\mu}^{A\bar{\eta}\eta}(0, -p, p)$ (without Ward identities, etc.):

$$\Gamma_{R,\mu}^{A\bar{\eta}\eta}(0, -p, p) = -Q_\eta e \gamma_\mu Z_e \left[Z_{AA}^{1/2} + Z_{ZA}^{1/2} \frac{S_{W,0}}{C_{W,0}} \right] + \mathcal{O}(Q_\eta^2)$$

$$\stackrel{!}{=} -Q_\eta e \gamma_\mu$$

$$\Rightarrow Z_e = \left[Z_{AA}^{1/2} + Z_{ZA}^{1/2} \frac{S_{W,0}}{C_{W,0}} \right]^{-1} = \left[Z_{AA}^{1/2} + Z_{ZA}^{1/2} \sqrt{\frac{S_W^2 - \delta C_W^2}{C_W^2 + \delta C_W^2}} \right]^{-1}$$

= function of gauge-boson self-energies only

and in agreement with previously “conjectured” results of
 Bauberger '97; Freitas et al. '02; Awramik et al. '02

Generalization to gauge groups $G \times U(1)$

The class of $G \times U(1)_Y$ models

- ▶ Gauge group $G \times U(1)_Y$: G = any Lie group of rank r ,
 $U(1)_Y$ = “weak hypercharge” subgroup as in the SM,
 so that $U(1)_{em} =$ mixture of G and $U(1)_Y$
- ▶ Neutral gauge fields: B^μ and C_k^μ ($k = 1, \dots, r$),
 (C_k correspond to diagonal generators of G)

Mass basis of neutral gauge fields: photon A^μ , r bosons Z_k^μ

$$\begin{pmatrix} B^\mu \\ C_1^\mu \\ \vdots \\ C_r^\mu \end{pmatrix} = R \begin{pmatrix} A^\mu \\ Z_1^\mu \\ \vdots \\ Z_r^\mu \end{pmatrix}, \quad R = \begin{pmatrix} R_{BA} & R_{BZ_1} & \cdots & R_{BZ_r} \\ R_{C_1A} & R_{C_1Z_1} & \cdots & R_{C_1Z_r} \\ \vdots & \vdots & \ddots & \vdots \\ R_{C_rA} & R_{C_rZ_1} & \cdots & R_{C_rZ_r} \end{pmatrix}, \quad \text{generalization of Weinberg rotation}$$

▶ Examples:

- ▶ $SU(2)_I \times U(1)_Y \rightarrow$ SM
- ▶ $SU(3) \times U(1)_Y$ Pisano, Pleitez '91; ...
- ▶ $SU(2) \times U(1) \times U(1)_Y$ Babu, Kolda, March-Russell '97; ...

Special feature: kinetic mixing

$$\mathcal{L}_{YM} \supset \kappa C_1^{\mu\nu} B_{\mu\nu} \text{ for } U(1) \text{ gauge fields } C_1^\mu, B_1^\mu$$

Renormalization of the $G \times U(1)_Y$ model

OS renormalization in the photon-Z-boson sector

$$\begin{pmatrix} A_0^\mu \\ Z_{0,1}^\mu \\ \vdots \\ Z_{0,r}^\mu \end{pmatrix} = \begin{pmatrix} Z_{AA}^{1/2} & Z_{AZ_1}^{1/2} & \cdots & Z_{AZ_r}^{1/2} \\ Z_{Z_1A}^{1/2} & Z_{Z_1Z_1}^{1/2} & \cdots & Z_{Z_1Z_r}^{1/2} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{Z_rA}^{1/2} & Z_{Z_rZ_1}^{1/2} & \cdots & Z_{Z_rZ_r}^{1/2} \end{pmatrix} \begin{pmatrix} A^\mu \\ Z_1^\mu \\ \vdots \\ Z_r^\mu \end{pmatrix}$$

↪ order-by-order calculation of OS field ren. constants:

$$Z_{Z_kA}^{1/2} = \frac{\Sigma_{SR,T}^{Z_kA}(0) - \sum_{l(l \neq k)} Z_{Z_lZ_k}^{1/2} Z_{Z_lA}^{1/2} (M_{Z_l}^2 + \delta M_{Z_l}^2)}{Z_{Z_kZ_k}^{1/2} (M_{Z_k}^2 + \delta M_{Z_k}^2)} = \mathcal{O}(\alpha)$$

$$Z_{AA} = 1 - \sum_k Z_{Z_kA} - \Sigma_{SR,T}^{AA}(0) = 1 + \mathcal{O}(\alpha), \quad \dots$$

BFM renormalization → in full analogy to the SM!

- ▶ gauge-invariant BFM invariant action $\hat{\Gamma}$
- ▶ QED-like Ward identities, $Z_{\hat{Z}_k \hat{A}} = 0$
- ▶ charge renormalization constant: $Z_e = Z_{\hat{A}\hat{A}}^{-1/2}$
- ▶ charge universality ⇒ method of fake fermion applicable!

Charge renormalization of the $G \times U(1)_Y$ model with a fake fermion η

Properties of η :

- ▶ gauge charges: $\frac{1}{2} Y_{W,\eta} = Q_\eta \rightarrow 0, \quad e = g_1 R_{BA}$
- ▶ Yukawa couplings to Higgs fields S_i : $y_i \rightarrow 0$

Lagrangian:

$$\mathcal{L}_\eta = \bar{\eta} \left[i \not{\partial} - Q_\eta e \left(A + \sum_k \frac{R_{BZ_k}}{R_{BA}} Z_k \right) - m_\eta - \sum_i y_i S_i \right] \eta$$

Renormalized $A\bar{\eta}\eta$ vertex function:

$$\begin{aligned} \Gamma_{R,\mu}^{A\bar{\eta}\eta}(k, \bar{p}, p) &= Z_\eta Z_{AA}^{1/2} \Gamma_\mu^{A\bar{\eta}\eta}(k, \bar{p}, p) + \sum_k Z_\eta Z_{Z_k A}^{1/2} \Gamma_\mu^{Z_k \bar{\eta}\eta}(k, \bar{p}, p) \\ &= -i Q_\eta e \gamma_\mu Z_e \left[Z_{AA}^{1/2} + \sum_k Z_{Z_k A}^{1/2} \frac{R_{BZ_k,0}}{R_{BA,0}} \right] + \mathcal{O}(Q_\eta^2) \\ &\stackrel{!}{=} -i Q_\eta e \gamma_\mu \quad \text{for } k \rightarrow 0 \text{ and } p^2 = m_\eta^2 \end{aligned}$$

Charge renormalization:

$$Z_e = \left[Z_{AA}^{1/2} + \sum_k Z_{Z_k A}^{1/2} \frac{R_{BZ_k} + \delta R_{BZ_k}}{R_{BA} + \delta R_{BA}} \right]^{-1}$$

Conclusions



Conclusions

Electric charge in QED, the SM, and beyond:

- ▶ Unit charge e is a crucial model parameter
↪ defined (renormalized) in Thomson's low-energy limit
- ▶ Charge quantization → grand unification required for full explanation
- ▶ Charge universality → charge ren. does not distinguish specific particle

Renormalization of electric charge (ren. constant Z_e):

↪ desirable: express Z_e in terms of (gauge-boson) self-energies $\Sigma^{VV'}$

- ▶ **QED**: Z_e fixed by Σ^{AA} to all orders (Ward identity)
- ▶ **SM**: situation more complicated (AZ-mixing, BRS invariance)
 - ▶ NLO: Z_e fixed by $\Sigma^{AA/AZ}$ (laborious proof via Slavnov–Taylor/Lee ids.)
 - ▶ all orders: situation “QED-like” in BFM renormalization,
new: exploit charge universality in conventional R_ξ gauge and
employ “fake fermion” η with $Q_\eta \rightarrow 0$ for renormalization
(concept adapted from ren. of mixing angles) Denner, S.D., Lang '18
- ▶ **$G \times U(1)_Y$** gauge theories: all-order renormalization as in SM
- ▶ Outlook: general gauge theories?
BFM renormalization similar to SM/QCD; R_ξ gauge laborious?!

Backup slides



Derivation of the QED Ward identity for the $Af\bar{f}$ vertex

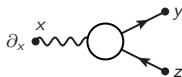
Becchi–Rouet–Stora (BRS) invariance of Green fcts.

+ decoupling of U(1) Faddeev–Popov fields $u(x)$, $\bar{u}(x)$:

$$\delta_{\text{BRS}}\psi_f(y) = -iQ_f e_0 u(y)\psi_f(y), \quad \delta_{\text{BRS}}\bar{u}(x) = -\frac{1}{\xi}\partial A(x) \quad (R_\xi \text{ gauge})$$

$$\Rightarrow 0 = \delta_{\text{BRS}}\langle 0 | T \bar{u}(x) \psi_f(y) \bar{\psi}_f(z) | 0 \rangle$$

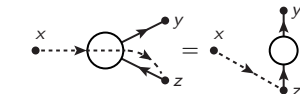
$$= -\frac{1}{\xi} \langle 0 | T \partial_x^\mu A_\mu(x) \psi_f(y) \bar{\psi}_f(z) | 0 \rangle$$



$$-iQ_f e_0 \langle 0 | T u(y) \bar{u}(x) \psi_f(y) \bar{\psi}_f(z) | 0 \rangle$$



$$+iQ_f e_0 \langle 0 | T u(z) \bar{u}(x) \psi_f(y) \bar{\psi}_f(z) | 0 \rangle$$



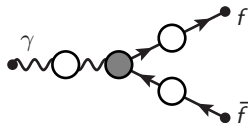
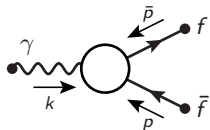
$$= -\frac{1}{\xi} \partial_x^\mu G_\mu^{Af\bar{f}}(x, y, z) - iQ_f e_0 G_0^{u\bar{u}}(y, x) G^{f\bar{f}}(y, z) + iQ_f e_0 G_0^{u\bar{u}}(z, x) G^{f\bar{f}}(y, z)$$

\hookrightarrow Ward in. for $G_\mu^{Af\bar{f}}$ in momentum space:

$$0 = -\frac{1}{\xi} k^\mu G_\mu^{Af\bar{f}}(k, \bar{p}, p) - \frac{iQ_f e_0}{k^2} \left[G^{f\bar{f}}(-p, p) - G^{f\bar{f}}(\bar{p}, -\bar{p}) \right]$$

Amputation of external propagators: Green fcts. $G \rightarrow$ 1PI vertex fcts. Γ

$$G_{\mu}^{A\bar{f}f}(k, \bar{p}, p) = G_{\mu\nu}^{AA}(k, -k) G^{f\bar{f}}(\bar{p}, -\bar{p}) i\Gamma^{A\bar{f}f, \nu}(k, \bar{p}, p) G^{f\bar{f}}(-p, p)$$



$$i\Gamma_{\mu\nu}^{AA}(k, -k) = - \left[G_{\mu\nu}^{AA}(k, -k) \right]^{-1}, \quad i\Gamma^{\bar{f}f}(-p, p) = - \left[G^{f\bar{f}}(-p, p) \right]^{-1}$$

+ use Ward identity for γ propagator: $k^{\mu} G_{\mu\nu}^{AA}(k, -k) = -\frac{i\xi k_{\nu}}{k^2}$

\Rightarrow Ward id. for $\Gamma^{A\bar{f}f, \nu}(k, \bar{p}, p)$:

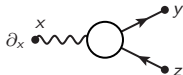
$$k^{\mu} \Gamma_{\mu}^{A\bar{f}f}(k, \bar{p}, p) = -Q_f e_0 \left[\Gamma^{\bar{f}f}(\bar{p}, -\bar{p}) - \Gamma^{\bar{f}f}(-p, p) \right]$$

Deriving Ward identity (*) for the $Af\bar{f}$ vertex in the SM from BRS invariance

$$\delta_{\text{BRS}}\psi_f(y) = -iQ_f e_0 u^A(y)\psi_f(y) + \dots, \quad \delta_{\text{BRS}}\bar{u}^A(x) = -\frac{1}{\xi_A}\partial A(x) \quad (R_\xi \text{ gauge})$$

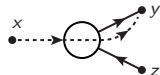
$$\Rightarrow 0 = \delta_{\text{BRS}}\langle 0 | T \bar{u}^A(x) \psi_f(y) \bar{\psi}_f(z) | 0 \rangle$$

$$= -\frac{1}{\xi_A} \partial_x^\mu G_\mu^{Af\bar{f}}(x, y, z)$$



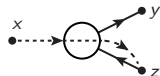
$$+ ie_0 \left[-\frac{Q_f}{c_{W,0}} G^{u^B \bar{u}^A f \bar{f}}(y, x, y, z) + \frac{I_{W,f}^3}{s_{W,0} c_{W,0}} \omega_- G^{u^Z \bar{u}^A f \bar{f}}(y, x, y, z) \right.$$

$$\left. + \frac{1}{\sqrt{2} s_{W,0}} \omega_- G^{u^\rho \bar{u}^A f' \bar{f}}(y, x, y, z) \right]$$



$$+ ie_0 \left[\frac{Q_f}{c_{W,0}} G^{u^B \bar{u}^A f \bar{f}}(z, x, y, z) - \frac{I_{W,f}^3}{s_{W,0} c_{W,0}} G^{u^Z \bar{u}^A f \bar{f}}(z, x, y, z) \omega_+ \right.$$

$$\left. - \frac{1}{\sqrt{2} s_{W,0}} G^{u^{-\rho} \bar{u}^A f \bar{f}'}(z, x, y, z) \omega_+ \right]$$



$$I_{W,f} = \text{weak isospin}, \quad \rho = \text{sgn}(I_{W,f}^3), \quad u^B = c_W u^A + s_W u^Z$$

$$(f, f')_L = \text{weak isospin doublet}, \quad \omega_\pm = \frac{1}{2}(1 \pm \gamma_5)$$

Sketch of the evaluation of the ST identity

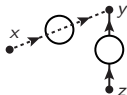
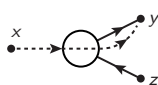
Further manipulations:

- ▶ “Project away” weak isospin part: $G_{\mu}^{Af\bar{f}}(x, y, z) \rightarrow \omega_{+} G_{\mu}^{Af\bar{f}}(x, y, z) \omega_{-}$

$\hookrightarrow G^{u^B \bar{u}^A f \bar{f}}$ terms are the only surviving ghost contributions:

- ▶ no 1-loop graphs with links between $\langle u^B \bar{u}^A \rangle$ and $\langle f \bar{f} \rangle$ lines

$\hookrightarrow G^{u^B \bar{u}^A f \bar{f}}(y, x, y, z) = G^{u^B \bar{u}^A}(y, x) G^{f \bar{f}}(y, z)$ at one loop



- ▶ non-vanishing contributions are major obstacle beyond 1-loop level!

- ▶ Transition to momentum space \rightarrow projected 1-loop identity

$$0 = \omega_{+} k^{\mu} G_{\mu}^{Af\bar{f}}(k, \bar{p}, p) \omega_{-} + \xi_A \frac{Q_f e_0}{c_{W,0}} G^{u^B \bar{u}^A}(k^2) \omega_{+} \left[G^{f\bar{f}}(-p, p) - G^{f\bar{f}}(\bar{p}, -\bar{p}) \right] \omega_{-} \quad (**)$$

Next steps:

- Determination of $G^{u^B \bar{u}^A}(k^2)$ via ST identity: $(\delta_{\text{BRS}} B^\nu = \partial^\nu u^B)$

$$\begin{aligned}
 0 &= \delta_{\text{BRS}} \langle 0 | T \bar{u}^A(x) B_\nu(y) | 0 \rangle = -\frac{1}{\xi_A} \partial_x^\mu G_{\mu\nu}^{AB}(x, y) + \partial_y^\nu G^{u^B \bar{u}^A}(x, y) \\
 \Rightarrow G^{u^B \bar{u}^A}(k^2) &= -\frac{k^\mu k^\nu}{\xi_A k^2} G_{\mu\nu}^{AB}(k, -k) = -\frac{1}{\xi_A} \left[c_{W,0} \underbrace{G_L^{AA}(k^2)}_{= -i\xi_A/k^2 \text{ (Slavnov identity)}} + s_{W,0} G_L^{AZ}(k^2) \right] \\
 &= \frac{ic_{W,0}}{k^2} \left[1 - \frac{s_{W,0}}{c_{W,0}} \frac{\xi_Z \Sigma_L^{AZ}(k^2)}{k^2 - \xi_Z M_Z^2} \right] \quad \text{at one loop}
 \end{aligned}$$

where $G_{\mu\nu}^{VV'}(k, -k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) G_T^{VV'}(k^2) + \frac{k_\mu k_\nu}{k^2} G_L^{VV'}(k^2)$

at one loop: $G_T^{AZ}(k^2) = \frac{i\Sigma_T^{AZ}(k^2)}{k^2(k^2 - M_Z^2)}$, $G_L^{AZ}(k^2) = \frac{i\xi_A \xi_Z \Sigma_L^{AZ}(k^2)}{k^2(k^2 - \xi_Z M_Z^2)}$

$\Sigma_T^{AZ}(0) = \Sigma_L^{AZ}(0)$ (analyticity!)

- Amputation of the photon leg: (underlining = amputation; χ = Goldstone boson)

$$\begin{aligned}
 k^\mu G_\mu^{A\bar{f}\bar{f}}(k, \bar{p}, p) &= \sum_{V=A,Z} \underbrace{k^\mu G_{\mu\nu}^{AV}}_{= k_\nu G_L^{AV}} G^{\underline{V}f\bar{f},\nu} + \underbrace{k^\mu G_\mu^{A\chi}}_{= k^2 G_L^{AZ} / (i\xi_Z M_Z)} G^{\underline{\chi}f\bar{f}} \\
 &= -\frac{i\xi_A}{k^2} k^\nu G_\nu^{A\bar{f}\bar{f}} + G_L^{AZ} \left[k^\nu G_\nu^{\underline{Z}f\bar{f}} - \frac{ik^2}{\xi_Z M_Z} G^{\underline{\chi}f\bar{f}} \right]
 \end{aligned}$$

(ST identity)

⇒ Intermediate 1-loop result from (**):

$$\begin{aligned}
 \omega_+ k^\mu G_\mu^{A\bar{f}\bar{f}} \omega_- &= Q_f e_0 \omega_+ \left[G^{f\bar{f}}(-p, p) - G^{f\bar{f}}(\bar{p}, -\bar{p}) \right] \omega_- \\
 &\quad - \frac{ik^2}{\xi_A} G_L^{AZ} \omega_+ \left\{ k^\mu G_{0,\mu}^{\underline{Z}f\bar{f}} - \frac{ik^2}{\xi_Z M_Z} G_0^{\underline{\chi}f\bar{f}} \right. \\
 &\quad \left. - Q_f e_0 \frac{s_{W,0}}{c_{W,0}} \left[G_0^{f\bar{f}}(-p, p) - G_0^{f\bar{f}}(\bar{p}, -\bar{p}) \right] \right\} \omega_-
 \end{aligned}$$

Sketch of the evaluation of the ST identity (continued)

Final cumbersome steps:

- Low-energy limit:

$$\text{calculate } \frac{\partial}{\partial k^\mu} \left[\omega_+ k^\nu G_{\nu}^{Af\bar{f}}(k, -p - k, p) \omega_- \right]_{k=0} \text{ using } G_L^{AZ}(k^2) = \frac{i\xi_A \xi_Z \Sigma_L^{AZ}(k^2)}{k^2(k^2 - \xi_Z M_Z^2)}$$

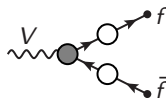
$$\begin{aligned} \Rightarrow \omega_+ G_{\mu}^{Af\bar{f}}(0, -p, p) \omega_- &= -Q_f e_0 \omega_+ \frac{\partial G^{f\bar{f}}}{\partial p^\mu}(-p, p) \omega_- \\ &\quad - \frac{\Sigma_L^{AZ}(0)}{M_Z^2} \omega_+ \left[G_{0,\mu}^{Zf\bar{f}}(0, -p, p) + Q_f e_0 \frac{s_{W,0}}{c_{W,0}} \frac{\partial G_0^{f\bar{f}}}{\partial p^\mu}(-p, p) \right] \omega_- \end{aligned}$$

- Amputation of fermion lines:

$$\Gamma^{\bar{f}f}(-p, p) = i \left[G^{f\bar{f}}(-p, p) \right]^{-1} = \not{p} - m_f + \text{1-loop corrections}$$

$$\frac{\partial G^{f\bar{f}}}{\partial p^\mu}(-p, p) = G^{f\bar{f}}(-p, p) i \frac{\partial \Gamma^{\bar{f}f}}{\partial p^\mu}(-p, p) G^{f\bar{f}}(-p, p)$$

$$G_{\mu}^{Vf\bar{f}}(0, -p, p) = G^{f\bar{f}}(-p, p) i \Gamma_{\mu}^{V\bar{f}f}(0, -p, p) G^{f\bar{f}}(-p, p)$$



- On-shell limit: isolate terms $\propto (p^2 - m_f^2)^{-2}$ for $p^2 \rightarrow m_f^2$
(m_f = renormalized OS mass!)

2-point vertex functions:

$$\hat{\Gamma}_{\mu\nu}^{\hat{V}'\hat{V}}(-k, k) = \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \hat{\Gamma}_T^{\hat{V}'\hat{V}}(k^2) + \frac{k_\mu k_\nu}{k^2} \hat{\Gamma}_L^{\hat{V}'\hat{V}}(k^2), \quad \hat{V}, \hat{V}' = \hat{A}, \hat{Z}$$

BFM Ward identity for the **unrenormalized** $\hat{A}\hat{V}$ vertex functions:

$$\begin{aligned} k^\mu \hat{\Gamma}_{\mu\nu}^{\hat{A}\hat{V}}(k, -k) &= 0 \quad \Rightarrow \quad \hat{\Gamma}_L^{\hat{A}\hat{V}}(k^2) = 0 \\ \hat{\Gamma}_{\mu\nu}^{\hat{A}\hat{V}} &= \text{regular at } k^2=0 \quad \Rightarrow \quad \hat{\Gamma}_T^{\hat{A}\hat{V}}(0) = \hat{\Gamma}_L^{\hat{A}\hat{V}}(0) = 0 \end{aligned}$$

OS renormalization transformation:

$$\begin{pmatrix} \hat{Z}_0 \\ \hat{A}_0 \end{pmatrix} = \begin{pmatrix} Z_{\hat{Z}\hat{Z}}^{1/2} & Z_{\hat{Z}\hat{A}}^{1/2} \\ Z_{\hat{A}\hat{Z}}^{1/2} & Z_{\hat{A}\hat{A}}^{1/2} \end{pmatrix} \begin{pmatrix} \hat{Z} \\ \hat{A} \end{pmatrix}, \quad M_{Z,0}^2 = M_Z^2 + \delta M_Z^2$$

$$\Gamma_{R,T}^{\hat{V}'\hat{V}}(k^2) = -Z_{\hat{A}\hat{V}'}^{1/2} Z_{\hat{A}\hat{V}}^{1/2} k^2 - Z_{\hat{Z}\hat{V}'}^{1/2} Z_{\hat{Z}\hat{V}}^{1/2} (k^2 - M_Z^2 - \delta M_Z^2) - \underbrace{\Sigma_{SR,T}^{\hat{V}'\hat{V}}(k^2)}_{\text{"subgraph-ren." self-energy}}$$

OS renormalization conditions:

- ▶ AA wave-fct. ren.: $1 + \Gamma_{R,T}^{\hat{A}\hat{A}'}(0) = 0 \Rightarrow Z_{\hat{A}\hat{A}} = 1 - Z_{\hat{Z}\hat{A}} - \Sigma_{SR,T}^{\hat{A}\hat{A}'}(0)$
- ▶ no AZ mixing for $k^2=0$: $\Gamma_{R,T}^{\hat{Z}\hat{A}}(0) = 0 \Rightarrow Z_{\hat{Z}\hat{A}}^{1/2} = \frac{\Sigma_{SR,T}^{\hat{Z}\hat{A}}(0)}{Z_{\hat{Z}\hat{Z}}^{1/2} (M_Z^2 + \delta M_Z^2)} = 0$