Two-loop helicity amplitudes for diphoton plus jet production in full colour

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Theory Seminar
DESY Zeuthen and Humboldt Universität Berlin
10th June 2021

based on [2103.02671, 2105.04585]

in collaboration with: Bakul Agarwal, Andreas von Manteuffel and Lorenzo Tancredi

Outline

Recent developments in higher-order calculations

- drawbacks of standard approaches and potential improvements
- case study: massless 2-loop 5-point amplitudes
- how to go about increasing complexity

Diphoton plus jet production in full colour. The $q\bar{q}$, qg and $g\bar{q}$ channels

- general structure of the amplitude
- reduction of the algebraic complexity
- results, analytic and numerical

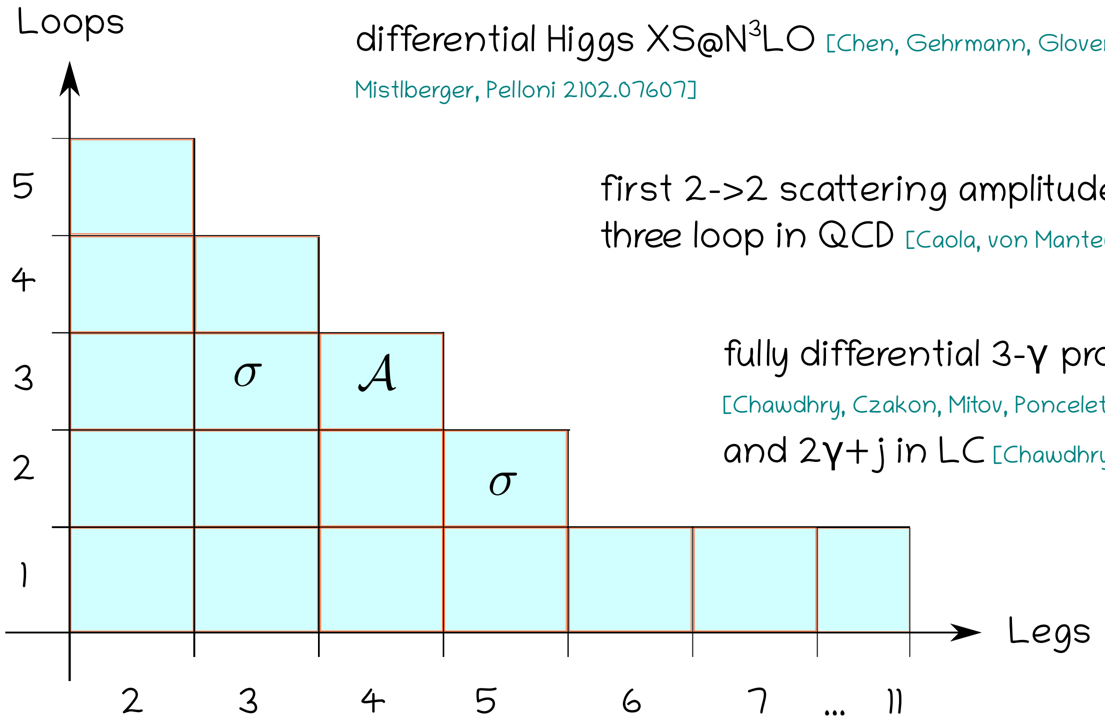
Outlook and conclusions

Amplitudes and precision phenomenology

$$\sigma = \sigma_{\mathrm{LO}} + \boxed{\alpha_s \sigma_{\mathrm{NLO}}} + \boxed{\alpha_s^2 \sigma_{\mathrm{NNLO}}} + \alpha_s^3 \sigma_{\mathrm{N^3LO}} + \dots$$

10-20%   1-10%

Not only, loops. Need to include more legs for relevant LHC pheno



differential Higgs XS@N^{3}LO [Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni 2102.07607]

first 2->2 scattering amplitudes at three loop in QCD [Caola, von Manteuffel, Tancredi 2011.13946]

fully differential 3-γ production in pp collisions in LC [Chawdhry, Czakon, Mitov, Poncelet 1911.00479, Kallweit, Sotnikov, Wiesemann 2010.04681]
and 2γ+j in LC [Chawdhry, Czakon, Mitov, Poncelet 2105.06940]

@Fixed order:

- Multi-loop, multi-leg Amplitudes
- IR Subtraction scheme

This talk

For real-life, pheno applications: EW effects, Parton Showers, PDFs, Resummation

LO and NLO automation

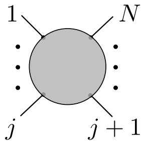
Tree-level and 1-loop calculations are problems solved.

Tree-level

Feynman diagrams, spinor-helicity formalism

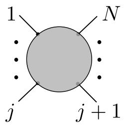
Recursive relations, e.g. Berends Giele off-shell recursion relations [Berends, Giele '88] or BCFW [Britto, Cachazo, Feng, Witten '05].
Efficient, elegant, generalisable.

One-loop

$\mathcal{A}_N =$ 

$$= \sum_{\Omega_0} \int \mathrm{d}^D q \frac{\mathcal{N}^{(\Omega)}(q)}{D_0^{(\Omega)} \dots D_{n-1}^{(\Omega)}},$$

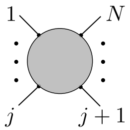
Unitarity-based or On-shell methods



$$= \sum_i d_i \mathcal{I}_{4,i} + \sum_i c_i \mathcal{I}_{3,i} + \sum_i b_i \mathcal{I}_{2,i} + \sum_i a_i \mathcal{I}_{1,i} + \mathcal{R}$$

Rocket [Giele, Zanderighi '08]
Black Hat [Berger et al. '08]
NJet [Badger, Biedermann, Uwer, Yundin '12,'13]

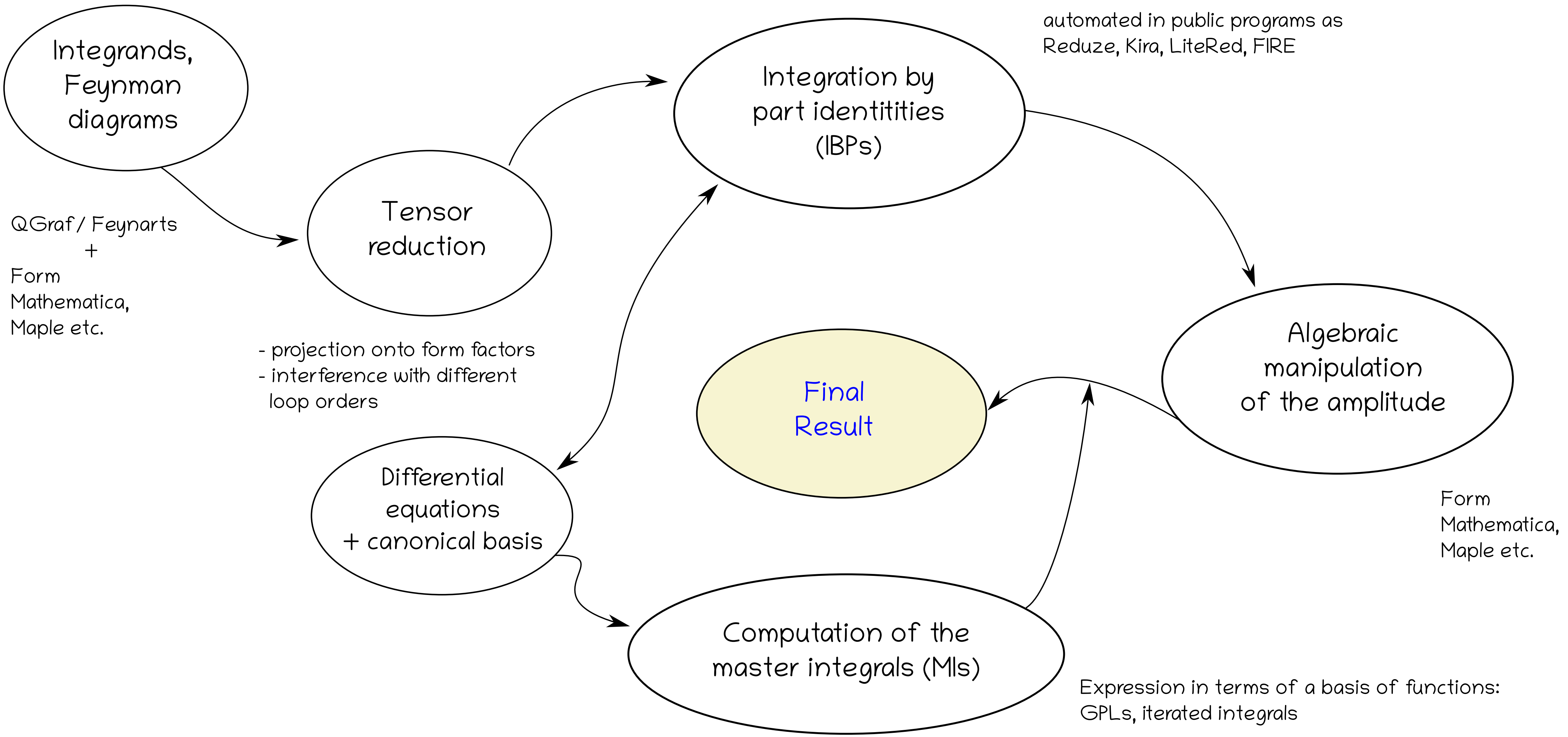
Tensor reduction or Off-shell methods



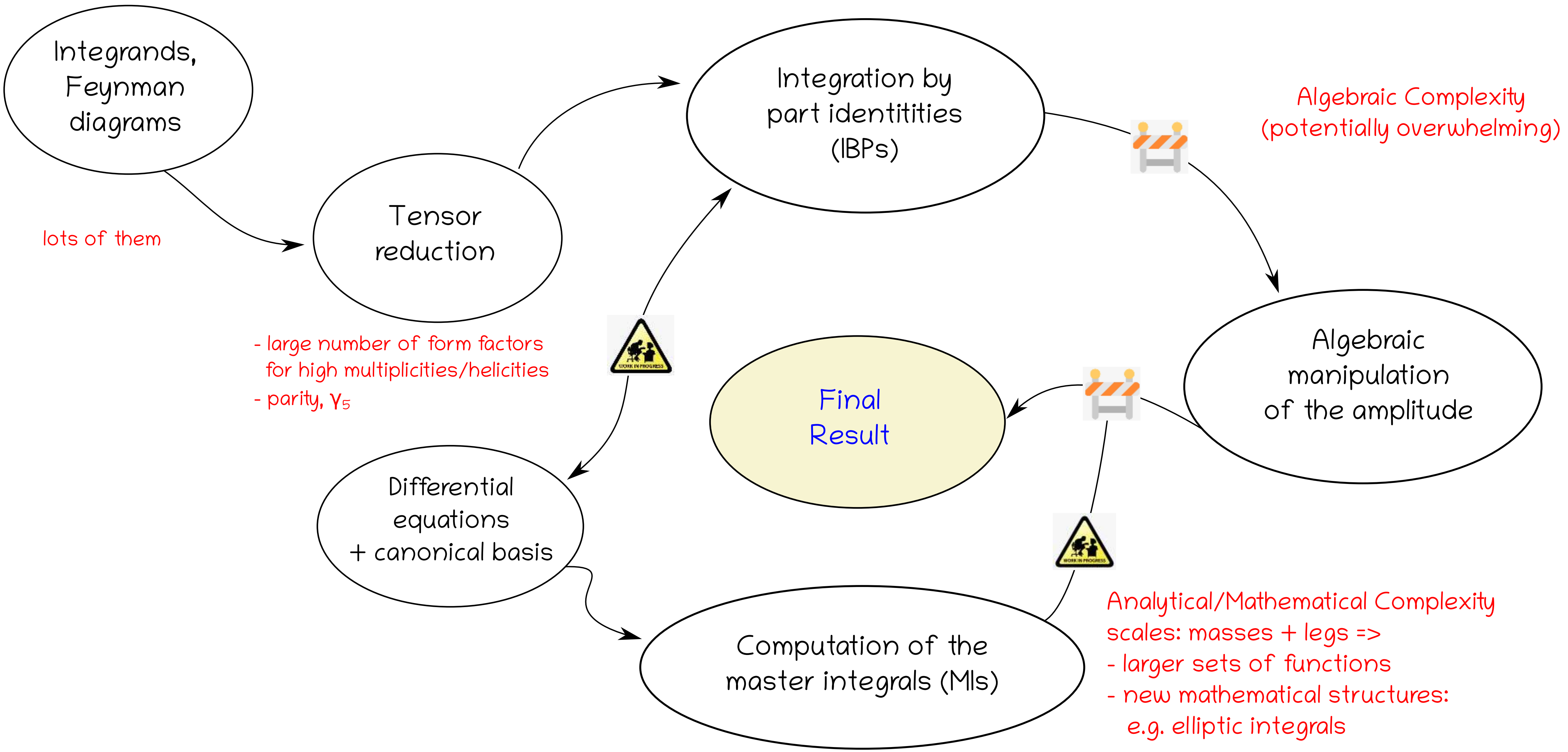
$$= \sum_{\Omega} \sum_{r=0}^{n} \mathcal{N}^{(\Omega)}_{\mu_1 \dots \mu_r} \int \mathrm{d}^D q \frac{q^{\mu_1} \cdots q^{\mu_r}}{D_0^{(\Omega)} \dots D_{N-1}^{(\Omega)}} + R_2 ,$$

GoSam [Cullen et al' '08]
MadLoop [Hirschi et al. '11]
Helac-NLO [Bevilacqua et al '13]
Recola [Actis, et al. '08, Denner, Lang, Uccirati '17]
OpenLoops [Cascioli, Pozzorini, Maiheroefer '12, F.B., Lang, Lindert, Maiheroefer, Pozzorini, Zhang, Zoller '19]

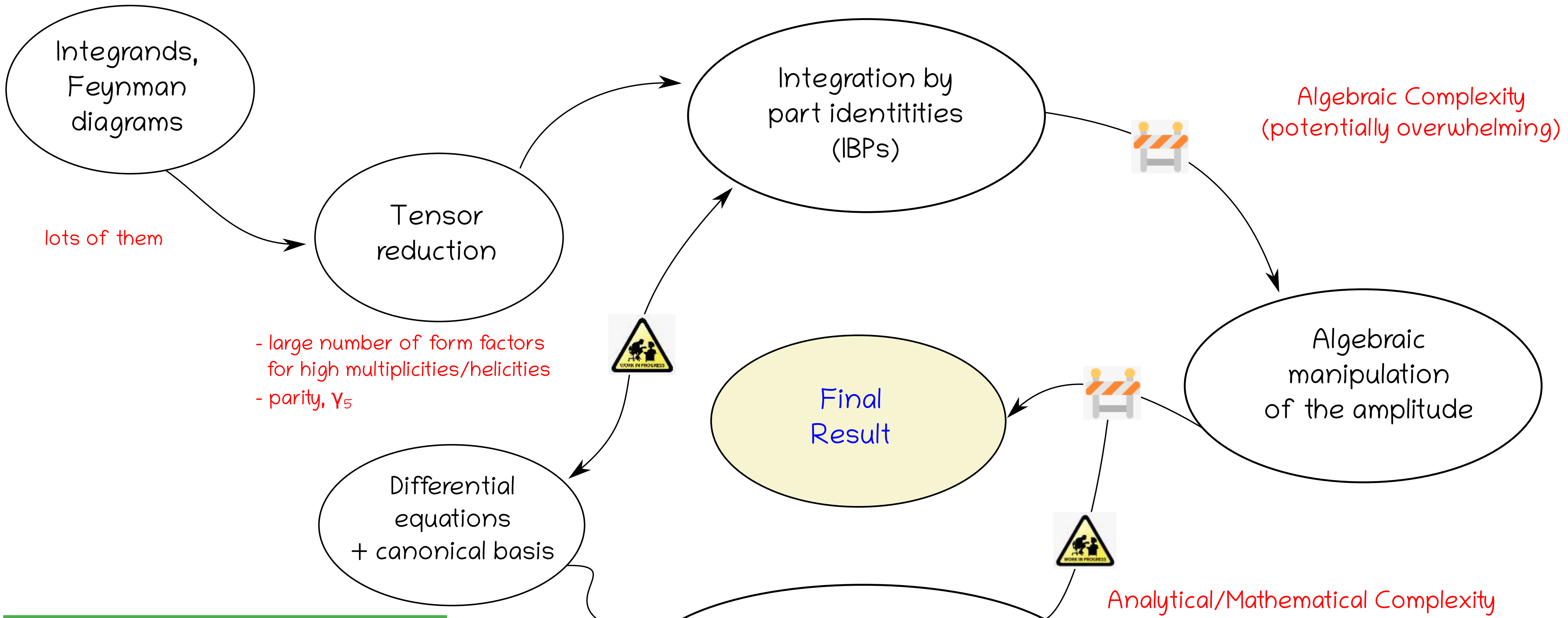
Traditional approach to multiloop amplitudes



Challenges and complexity



Challenges and complexity



- are these steps improvable?
- can one get mathematical insight?
- can one by-pass some of them?

Integration by part identities

Linear relations between multiloop Feynman integrals: reduction to a basis of Master Integrals [Chetyrkin, Tkachov, 1981]

It quickly becomes a very large linear algebra problem. Standard approach (until not too long ago): Laporta algorithm [Laporta 2000]

Reduze 2 [von Manteuffel, Studerus], Fire [Smirnov], LiteRed [Lee], Kira [Maierhoefer, Usovitsch, Uwer]

Start to show its limits for cutting edge pheno applications: many scales and higher loops

New ideas to tackle the IBP reduction

1) By-pass expensive symbolic operations
exploiting Finite field arithmetic

FinRed [von Manteuffel, Schabinger 1406.4513] (+ syzygies)

FiniteFlow [Peraro, 1905.08019]

Kira 2.0 [Klappert, Lange, Maierhoefer, Usovitsch, 2008.06494]

Alternative strategies, e.g.
[Chawhdry, Mitov, Lim 1805.09182]

2) Unitarity-based approaches:

First ideas in [Gluza, Kajda, Kosower, 1009.0472]

Master Integrals + Surface terms [Ita 1510.05626]

And related work in

[Zeng 1702.02355]
[Abreu, Febres-Cordero, Ita, Jacquier, Page, Zeng, 1703.05273,]
[Abreu, Page, Zeng, 1807.11522]

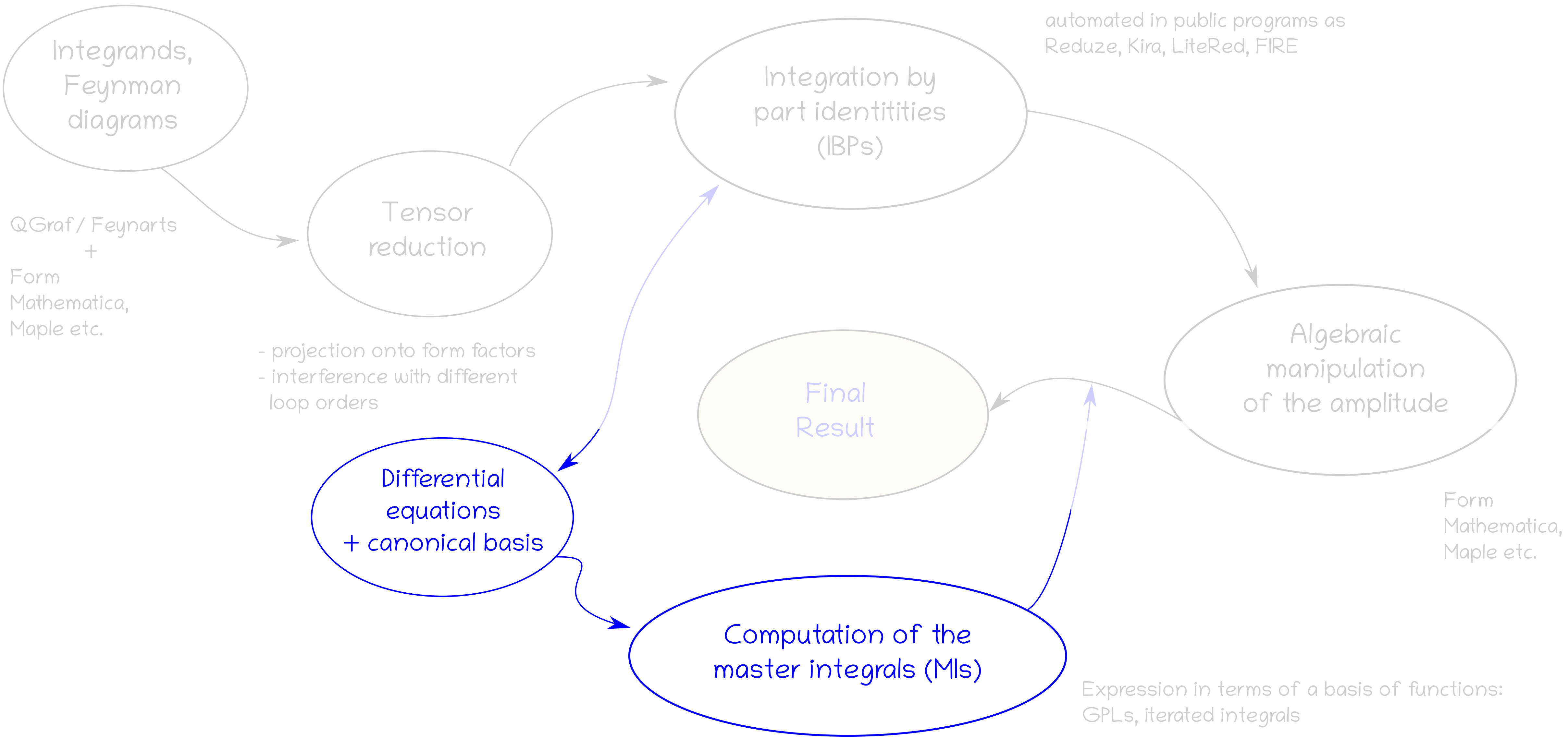
Developed into
Caravel ++ [Abreu et al 2009.11957]

3) Module intersection
for IBP reduction +
Multivariate partial fraction
decomposition

[Bendle, Boehm, Heymann, Ma, Rahn, Ristau, Wittmann, Wu, Zhang 2104.06866]

reduction of most complicated massless non-planar 5-pt 2-loop integrals

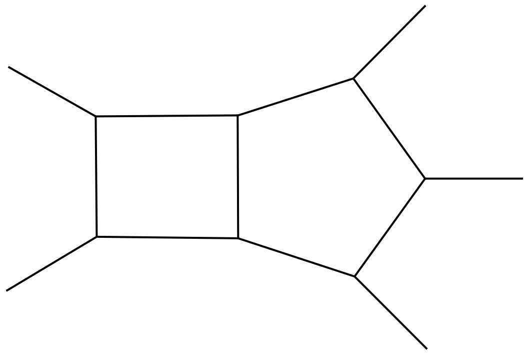
Traditional approach to multiloop amplitudes



Pentagon functions for 2→3 masless scattering amplitudes

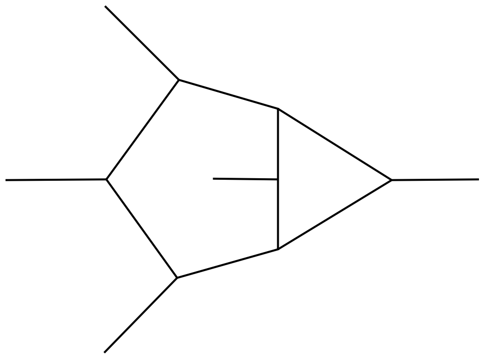
All master integrals known

Pentagon-Box



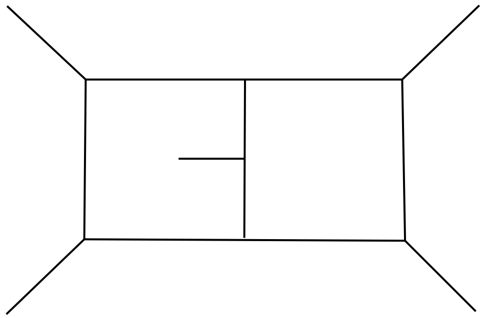
[Gehrmann, Henn, Lo Presti 1511.05409, 1807.09812],
[Papadopoulos, Tommasini, Wever 1511.09404]

Hexagon-Box



[Boehm, Georgoudis, Larsen, Schoenemann, Zhang],
[Abreu, Page, Zeng, 1807.11522]
[Chicherin, Gehrmann, Henn, Lo Presti, Mitev, Wasser 1809.06240]

Double-Pentagon



[Abreu, Dixon, Herrmann, Page, Zeng 1901.08563],
[Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia 1812.11160]

MIs through Pentagon Functions
Expressed (and evaluated) as iterated Chen integrals along a path γ

$$f^{(\omega)}(\vec{x}) = \int_\gamma \underbrace{\mathrm{d}\log W_{i_1} \ldots \mathrm{d}\log W_{i_n}}_{\omega \text{ integrations}}$$

Pentagon functions for planar integrals
[Gehrmann, Henn, Lo Presti 1807.09812]

Full set made available recently
[Chicherin, Sotnikov 2009.07803]

Results in the whole physical region

They can be used for *all* massless 5-pt amplitudes

Tackling the complexity of five-point scattering amplitudes

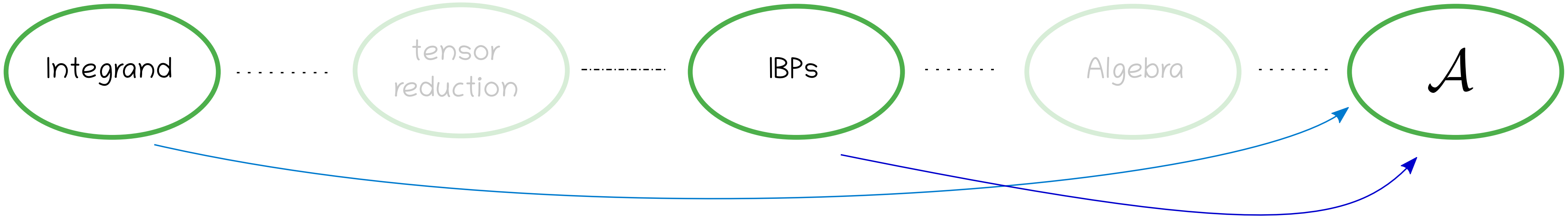
Consider a UV renormalised and IR subtracted 2-loop scattering amplitude

$$\mathcal{A} = \sum_k c_k(s_{ij}) f_k(s_{ij})$$

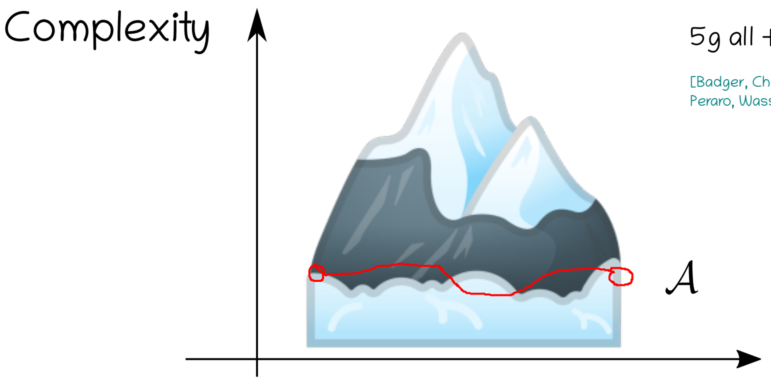
transcendental functions

rational functions

final result much simpler than intermediate steps!



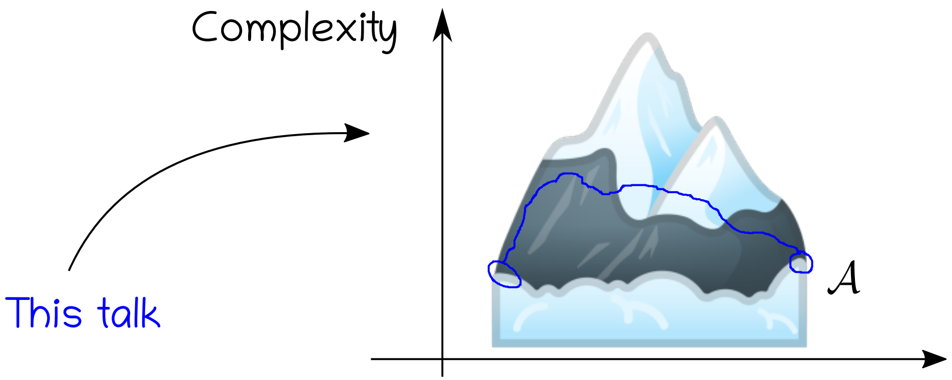
Can we reconstruct directly the final, physical, result?



5g all +

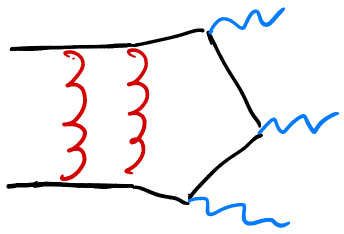
[Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zoia 1905.03733]

Can we keep complexity under control?



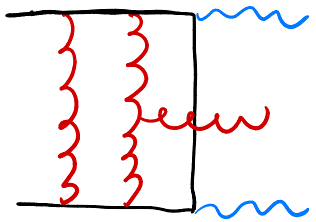
State-of-the-art of two-loop 2→3 scattering amplitudes

qq→3γ



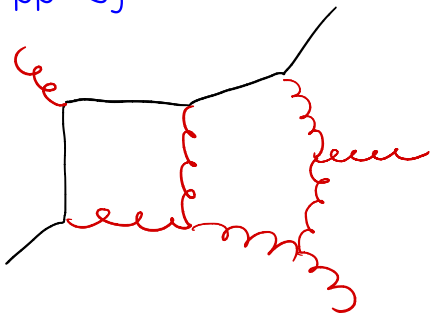
LC [Abreu, Page, Pascual, Sotnikov, 2010.15834]
LC [Chawdhry, Czakon, Mitov, Poncelet 2012.13553]

qq(g)→γγg(q)



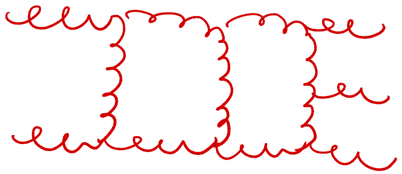
LC [Agarwal, F.B., von Manteuffel, Tancredi 2102.01820]
LC [Chawdhry, Czakon, Mitov, Poncelet 2103.04319]
FC [Agarwal, F.B., von Manteuffel, Tancredi 2105.04585]

pp→3j



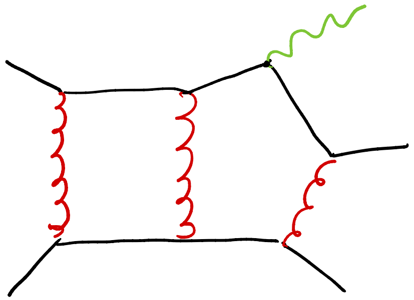
LC [Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov 1904.00945]
LC [Abreu, Febres Cordero, Ita, Page, Sotnikov 2102.13609]

5g all+ helicities, YM



FC [Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zoia 1905.03733]

qq'→Wbb



LC [Badger, Hartanto, Zoia, 2102.02516]

This talk

all helicity configurations

Kinematics of massless 5-point amplitudes

A massles 5-pt amplitude is described by a set of five independent kinematic invariants

$$s_{12} = (p_1 + p_2)^2, \quad s_{23} = (p_2 - p_3)^2, \quad s_{34} = (p_3 + p_4)^2,$$
$$s_{45} = (p_4 + p_5)^2, \quad s_{15} = (p_1 - p_5)^2$$

Together with the parity-odd invariant ε_5

$$\epsilon_5 = 4 i \epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma$$

Don't forget permutations. Lots of them (5! = 120)

ε_5 is related to the determinant of the Gram matrix via

$$(\epsilon_5)^2 = \Delta \equiv \det G_{ij} = \det(2 p_i \cdot p_j)\,, \qquad i, j \in \{1, \ldots, 4\}\,,$$

The physical scattering region 1 2 -> 3 4 5

is defined by

$$s_{12}, s_{34}, s_{45} > 0$$
$$s_{23}, s_{15} < 0$$
$$\Delta < 0$$

with further constraints [Gehrmann, Henn, Lo Presti '18]

$$s_{12} \geq s_{34}$$
$$s_{12} - s_{34} \geq s_{45}$$
$$0 \geq s_{23} \geq s_{45} - s_{12}$$

$$\delta \equiv \mathrm{Im}(\epsilon_5)$$

The pentagon functions are evaluated in the analiticity region δ>0

[Chicherin, Sotnikov, 2009.07803]

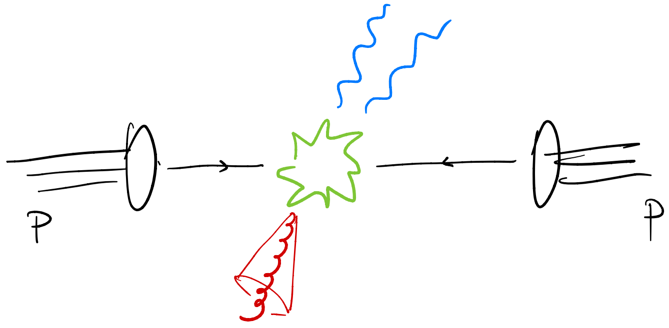
$$q\bar{q}(g) \to \gamma + \gamma + g(q)$$

Diphoton+jet production in pp collisions

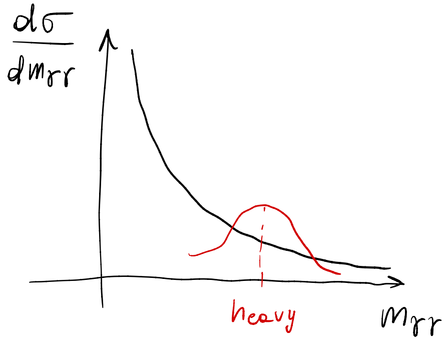
Diphoton production: important class of processes at the LHC

Irreducible background to SM and BSM processes. Most notably $H \to \gamma\gamma$

First pheno studies in NNLO QCD at LC [Chawdhry, Czakon, Mitov, Poncelet 2105.06940]



Invariant mass:
relevant for direct searches of resonances



pT distribution:
unique probe to investigate the properties of the decaying particles

Recoil against hard QCD radiation

Relevance of higher-order QCD corrrections for target accuracy

N^3LO QCD corrections to the cross section for pp -> γγ
di-photon + jet amplitudes necessary ingredient

Partonic channels:

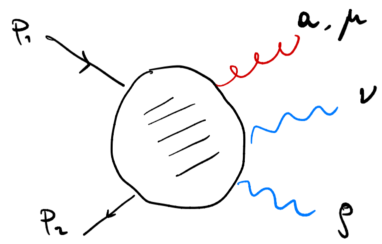
qq(g) -> γγg(q) ← this talk

gg -> γγg ← loop induced

Structure of the scattering amplitude

Let us consider the scattering process:

$$q(p_1) + \bar{q}(p_2) \to g(p_3) + \gamma(p_4) + \gamma(p_5)$$



The helicity amplitudes are given by:

$$A^a_{ij}(\boldsymbol{\lambda}) = i\,(4\pi\alpha)Q_q^2\sqrt{4\pi\alpha_s}\mathbf{T}^a_{ij}\mathcal{A}(\boldsymbol{\lambda})$$

Three independent helicity configurations

$$\boldsymbol{\lambda}_A = \{L,+,+,+\}\,, \qquad \boldsymbol{\lambda}_B = \{L,-,+,+\}\,, \qquad \boldsymbol{\lambda}_C = \{L,-,-,+\}$$

Factor out spinor phase as

$$\mathcal{A}(\boldsymbol{\lambda}) = \Phi(\boldsymbol{\lambda})\mathcal{B}(\boldsymbol{\lambda})$$

$$\Phi(\boldsymbol{\lambda}_A) = 2\sqrt{2}\frac{[31]\langle 12\rangle^3\langle 13\rangle}{\langle 14\rangle^2\langle 15\rangle^2\langle 23\rangle^2}$$

$$\Phi(\boldsymbol{\lambda}_B) = 2\sqrt{2}\frac{\langle 12\rangle\langle 23\rangle^2}{\langle 14\rangle\langle 42\rangle\langle 25\rangle\langle 51\rangle}$$

$$\Phi(\boldsymbol{\lambda}_C) = 2\sqrt{2}\frac{[51]^2[12]}{[14][42][23][31]}$$

Separate then into an even and an odd component and expand perturbatively

$$\mathcal{B}(\boldsymbol{\lambda}) = \mathcal{B}^E(\boldsymbol{\lambda}) + \epsilon_5\mathcal{B}^O(\boldsymbol{\lambda}) \qquad \mathcal{B}^P(\boldsymbol{\lambda}) = \sum_{k=0}^{2}\left(\frac{\alpha_s^b}{2\pi}\right)^k\mathcal{B}^{P,(k)}(\boldsymbol{\lambda}) + \mathcal{O}((\alpha_s^b)^3)$$

UV renormalisation and IR factorisation

$$\mathcal{A}(\boldsymbol{\lambda}) = \Phi(\boldsymbol{\lambda})\left(\mathcal{B}^{(0)}(\boldsymbol{\lambda}) + \left(\frac{\alpha_s}{2\pi}\right)\mathcal{B}^{(1)}(\boldsymbol{\lambda}) + \left(\frac{\alpha_s}{2\pi}\right)^2\mathcal{B}^{(2)}(\boldsymbol{\lambda})\right) + \mathcal{O}(\alpha_s^3)$$

We renormalise our results in $\overline{\text{MS}}$

$$\alpha_s^b \mu_0^{2\epsilon} S_\epsilon = \alpha_s \mu^{2\epsilon}\left[1 - \frac{\beta_0}{\epsilon}\left(\frac{\alpha_s}{2\pi}\right) + \left(\frac{\beta_0^2}{\epsilon^2} - \frac{\beta_1}{2\epsilon}\right)\left(\frac{\alpha_s}{2\pi}\right)^2 + \mathcal{O}(\alpha_s^3)\right]$$

The IR structure is completely determined [Catani 9802439; Garland, Gehrmann, Glover, Koukoutsakis, Remiddi 0206067]

We subtract the IR poles according to Catani's scheme [Catani 9802439]

What we are interested in

Barred objects are UV renormalised

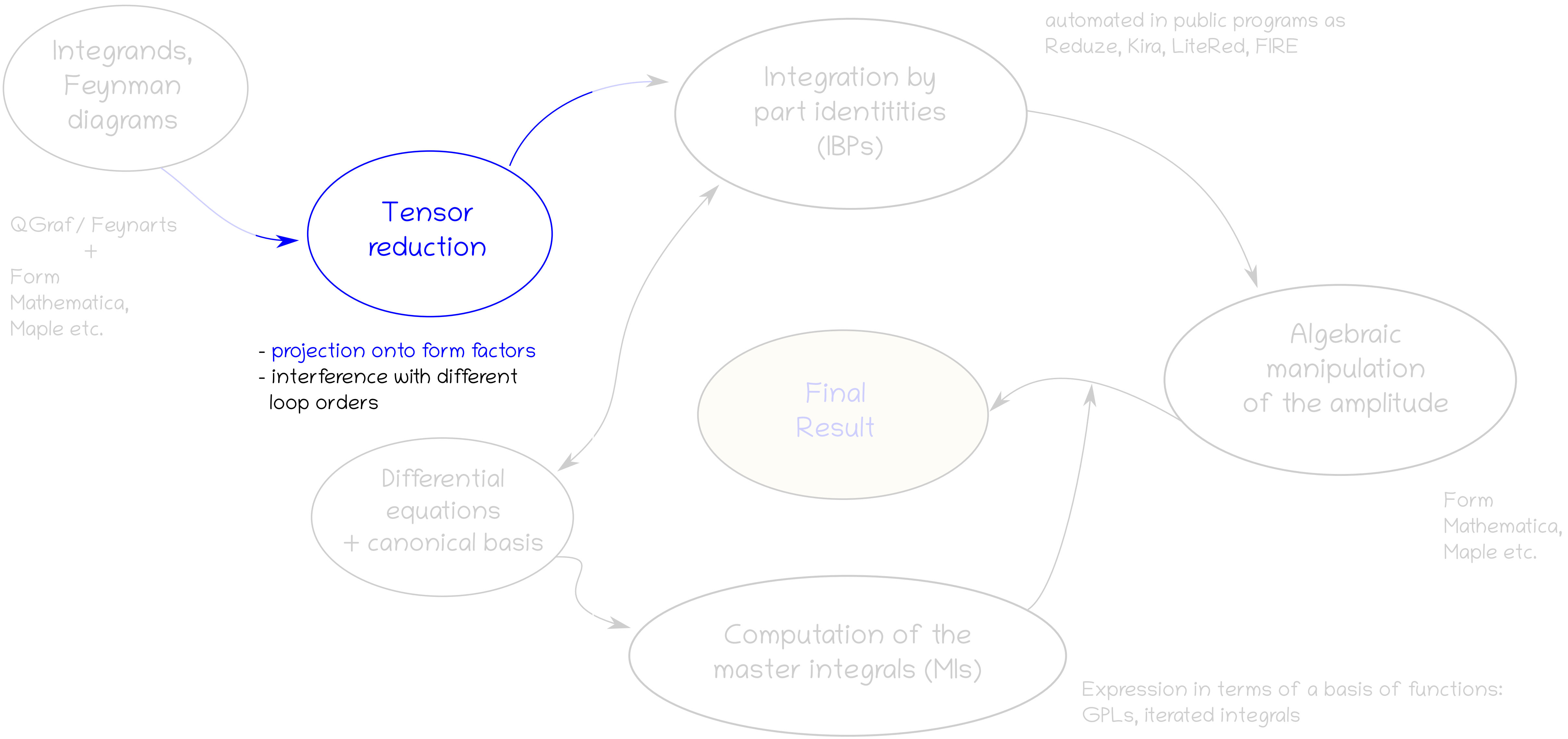
$$\bar{\mathcal{B}}^{(1)}(\boldsymbol{\lambda}) = I_1(\epsilon, \mu^2)\mathcal{B}^{(0)}(\boldsymbol{\lambda}) + \mathcal{R}^{(1)}(\boldsymbol{\lambda})$$

$$\bar{\mathcal{B}}^{(2)}(\boldsymbol{\lambda}) = I_2(\epsilon, \mu^2)\mathcal{B}^{(0)}(\boldsymbol{\lambda}) + I_1(\epsilon, \mu^2)\bar{\mathcal{B}}^{(1)}(\boldsymbol{\lambda}) + \mathcal{R}^{(2)}(\boldsymbol{\lambda})$$

Extract 2-loop finite remainders

$$\mathcal{R}^{P,(k)}(\boldsymbol{\lambda})$$

Traditional approach to multiloop amplitudes



Physical projectors

Project out amplitude onto form factors as suggested in [Peraro, Tancredi 1906.03298,2012.00820]

Main idea: decompose the amplitude into Lorentz structure which are independent in d=4 ← avoid evanescent form factors throughout

In practice: calculate only the physical helicity amplitudes in the t'Hooft-Veltman scheme

Generic tensor structure for our amplitudes:

$$\mathcal{T}_j \sim \bar{u}(p_2)\not{p}_{3,4}u(p_1)p_{i_3}^{\mu}p_{i_4}^{\nu}p_{i_5}^{\rho}\epsilon_{\mu}^{*}(p_3)\epsilon_{\nu}^{*}(p_4)\epsilon_{\rho}^{*}(p_5)$$

Transversality of on-shell bosons + gauge fixing: # of independent tensors in (d=4) = # of helicity configurations

$$\mathcal{A}(\boldsymbol{\lambda}) = \sum_{j=1}^{16} \mathcal{F}_j \mathcal{T}_j(\boldsymbol{\lambda})$$

Each form factor F_j can then be extracted via projectors:

$$\mathcal{P}_j = \sum_{j=1}^{16} c_k^j \mathcal{T}_k^{\dagger}$$

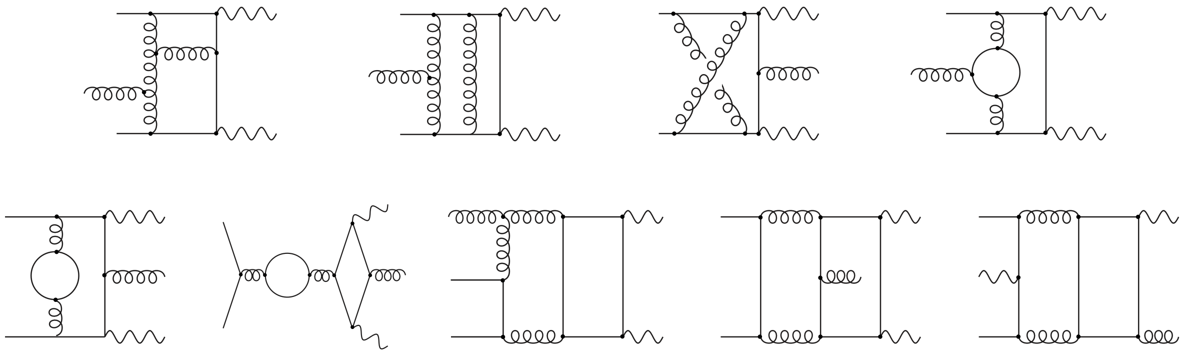
$$\mathcal{F}_j = \sum_{\text{pol}} \mathcal{P}_j \mathcal{A}$$

↑ no d-dependence in c^j_k for n > 4

Colour structure

Colour structure of the (UV and IR renormalised) 2-loop finite remainder

$$\mathcal{R}^{P,(2)}(\boldsymbol{\lambda}) = \sum_{i=1}^{10} \tilde{c}_i \, \mathcal{R}_i^{P,(2)}(\boldsymbol{\lambda})$$



$$n_f^{\gamma\gamma} = \frac{1}{Q_q^2} \sum_i^{n_f} Q_i^2 \qquad n_f^{\gamma} = \frac{1}{Q_q} \sum_i^{n_f} Q_i,$$

For reference/example:

$d\bar{d} \to g\gamma\gamma$: $N = 3$ $n_f = 5$ $n_f^{\gamma\gamma} = 11$ $n_f^{\gamma} = -1$

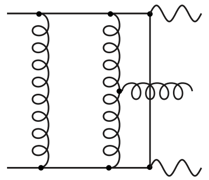
Complexity

DP:

$\tilde{c}_3 = N^{-2}$

$\tilde{c}_2 = 1$

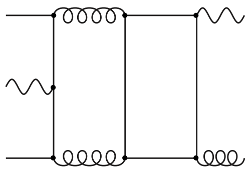
$\tilde{c}_7 = N \, n_f^{\gamma\gamma}$



HB:

$\tilde{c}_9 = d_{abc}d_{abc}n_f^{\gamma}$

$\tilde{c}_8 = N^{-1} \, n_f^{\gamma\gamma}$

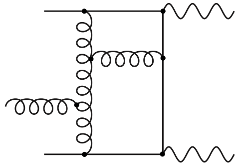


PB:

$\tilde{c}_1 = N^2$

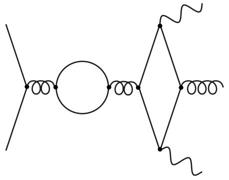
$\tilde{c}_4 = N n_f$

$\tilde{c}_5 = N^{-1} n_f$

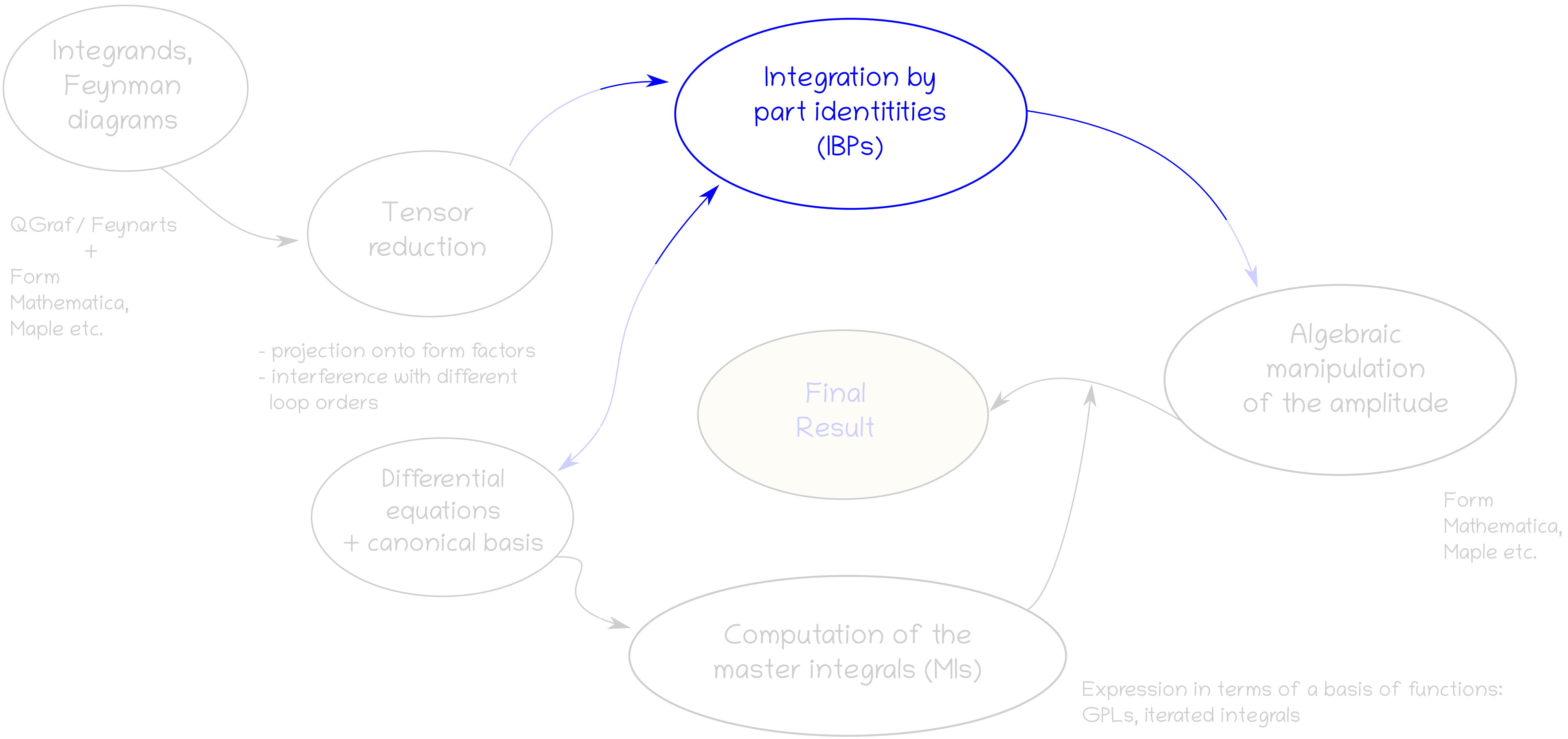


1-loop:

$\tilde{c}_6 = n_f^{\gamma\gamma} n_f$



Traditional approach to multiloop amplitudes



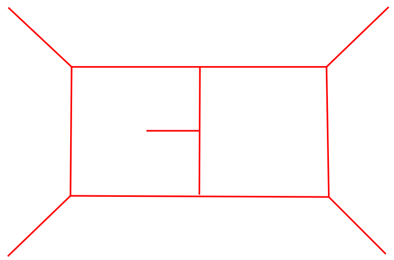
Reduction to master integrals

For the planar topologies using Kira 1.2 was sufficient to get all the relevant IBP identities. O(2k) CPU hours

We tried with FinRed too and timings were comparable

IBP identities for non-planar topologies obtained using FinRed: capable of reducing the non-planar integral families completely

- Finite-fields arithmetics [von Manteuffel, Schabinger 1406.4513; Peraro 1905.08019]
- Syzygy techniques [Gluza, Kadja, Kosower 1009.0472, Ita 1510.05626; Larsen, Zhang, 1511.01071, Agarwal, Jones, von Manteuffel 2011.15113]
- Denominators guessing [Abreu, Dormans, Febres Cordero, Ita, Page 1812.04586; Heller, von Manteuffel 2101.08283]



Most complicated integrals: 8-line denominators + 5 scalar products, ($t=8$, $s=5$) for DP topology

A good choice of MIs basis is crucial: Canonical basis/UT weight integrals

Two possibilities

1) use a precanoical basis, extracted from [Gehrmann, Henn, Lo Presti, 1807.09812].
"Simpler" for practical reasons

2) directly use a canonical basis [Gehrmann, Henn, Lo Presti, 1807.09812] or [Chicherin, Sotnikov 2009.07803].
Preferrable

Pros:

Exposes physical cuts of the integrals

simpler rational coefficients

extra bonus: d-dependence factorised

Integration by part identities: formal structure

Suppose we have a canonical basis of MIs, a generic IBP identity will look like

$$I(s_{ij};d) = \sum_{k=1}^{M} a_k(s_{ij},d)\mathcal{J}(s_{ij};d) \quad \text{with} \quad \mathcal{J}(s_{ij};d) = \sum_{i=m_1}^{m_2} \mathcal{J}^{(i)}(s_{ij})(4-d)^i$$

omitting ε_5 for simplicity now

rational function, over a common denominator

pure function, i.e. no rational dependence on s_{ij}

Note:
d-dependence completely factorises in the denominator:

$$a_k(s_{ij};d) = \frac{\mathcal{N}(s_{ij};d)}{\mathcal{Q}(d)\mathcal{D}(s_{ij})} \qquad \mathcal{D}(s_{ij}) = \prod_{n=1}^{N_d} \mathcal{D}_n^{p_n}(s_{ij})$$

in our case $N_d = 25$

Natural to make the association:

Rational function → partial-fraction decomposition

1) univariate partial-fraction decomposition wrt d (trivial)

2) multivariate partial-fraction decomposition wrt s_{ij} (hard)

Examples:
$D_6 = s_{12} + s_{23} - s_{45}$
$D_{20} = s_{12} + s_{23} - s_{45} - s_{51}$

In the planar case, D_i linear functions of s_{ij}. For non-planar IBPs one can have a Gram determinant

Multivariate partial fraction decomposition (MVPFD)

It has been long known that a MVPFD simplifies significantly the IBP reductions

$$a_k(s_{ij};d) = \frac{\mathcal{N}(s_{ij};d)}{\mathcal{Q}(d)\mathcal{D}(s_{ij})} \xrightarrow[\text{pf in d}]{\text{after}} a_k(s_{ij};d) = \sum_l g_l(d)\mathcal{R}_l(s_{ij})$$

How should we go about this?

Proposals/approaches for MVPFD:
[Pak 1111.0868], [Abreu et al, 1904.00945],
[Boehm, Wittmann, Wu, Xu, Zhang, 2008.13194]

Systematic study of reduction of IBPs: [2008.13194; Bendle et al 2104.06866]

We employ the algorithm implemented in the recently published package MultivariateApart [Heller, von Manteuffel, 2101.08283]

Big advantages of MultivariateApart:

1) systematically avoids spurious denominator factors
2) produces unique results also when applied to terms of a sum separately

we exploit both

Drastic reduction of algebraic complexity. IBPs tractable in a fully symbolic fashion

Examples:

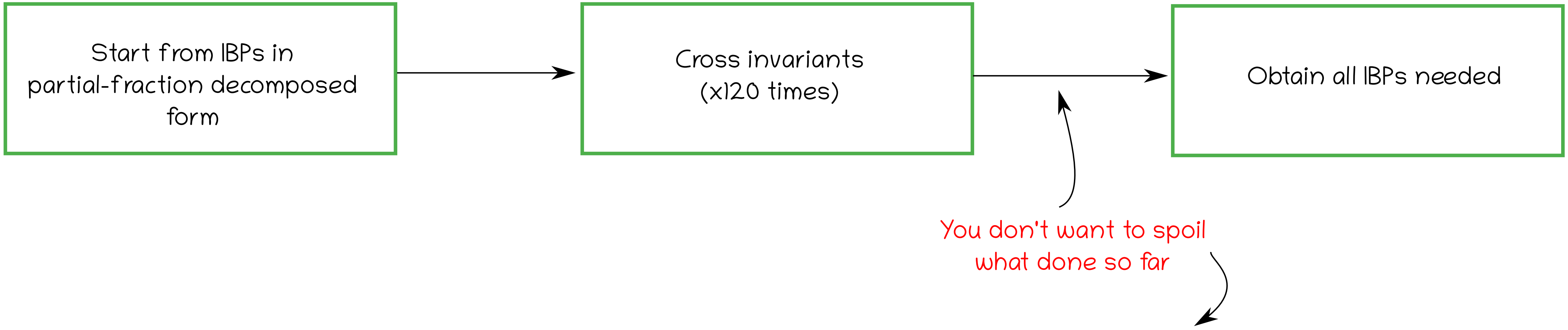
		on GCD		MVPFD
PB:	INT[TA,8,255,8,5,{1,1,1,1,1,1,1,1,-5,0,0}]	162 mb	→	3.9 mb
HB:	INT[TB,8,255,8,5,{1,1,1,1,1,1,1,1,-4,0,-1}]	513 mb	→	9.9 mb
DP:	INT[TB,8,510,8,5,{0,1,1,1,1,1,1,1,1,0,-5}]	1.2 gb	→	12 mb

The largest simplifications occur for the most complicated integrals: up to a factor ~ 100 in reduction size!

Crossing of IBP identities

For the complete reduction we need (potentially) all permutations of the external momenta

Being able to treat the IBPs in a fully symbolic fashion, this becomes extremely cheap (wrt other steps)



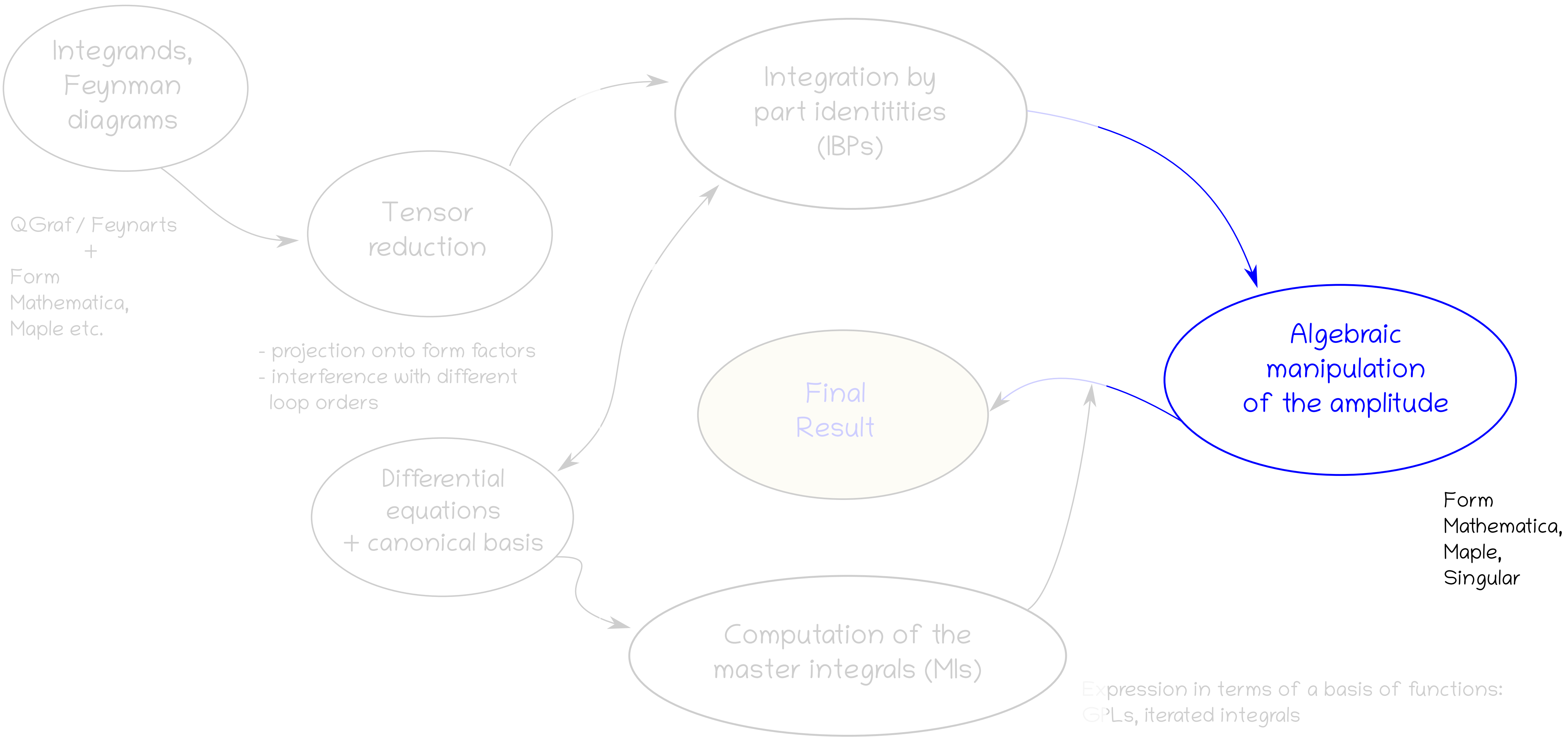
After crossing the invariants: second partial fraction decomposition according to a prefixed global Groebner basis

In practice: all terms in the sum decomposed locally but a unique representation of the rational functions across all IBP identities guaranteed

Crucial for the many (very many indeed) cancellations in the final result.

No need for expensive GCD operations

Traditional approach to multiloop amplitudes



Finite remainder of the amplitude

Insert IBPs into the amplitude, then further partial fraction decomposition: Multivariate Apart + Singular [Decker, Greuel, Pfister, Schoenemann] as backend

No GCD needed to see cancellations!

think of a linear combination with f_k elements of the basis

$$\mathcal{R}(\lambda) = \sum_k r_k(\{s_{ij}, \epsilon_5\}) f_k(\{s_{ij}, \epsilon_5\})$$

in partial fraction decomposed form: i.e. sum of a large number of monomials

Already observed in:
[Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov 1904.00945]
[De Laurentis, Maitre, arXiv:2010.14525]
[Chawdhry, Czakon, Mitov, Poncelet, 2012.13553, 2103.04319]
[Abreu, Cordero, Ita, Page,Sotnikov, 2102.13609]

rational functions are not independent

$$r_k = \sum_{m_1+\ldots+m_n \leq p} a_{k,m_1\ldots m_{30}} M_{m_1\ldots m_{30}}, \qquad M_{m_1\ldots m_{30}} \equiv q_1^{m_1} \cdots q_{25}^{m_{25}} s_{12}^{m_{26}} \cdots s_{51}^{m_{30}}$$

monomials are independent objects

We look for linear relations among the various rational functions:

$$0 = \sum_k r_k b_k$$

as many equations as independent monomials

$$0 = \sum_k a_{k,m_1\ldots m_{30}} b_k$$

monomials >> # rational functions
but this linear system is over constrained thus admits a solution

(similar in spirit to IBP reduction)

drastic reduction of final expressions

Choice of denominator basis in MVPFD

think of a linear combination
with r_k elements of the basis

$$\mathcal{R}(\lambda) = \sum_k r_k(\{s_{ij}, \epsilon_5\}) f_k(\{s_{ij}, \epsilon_5\})$$

A global Groebner basis exposes all cancellations, but might introduce spurious denominators.

Easy (but important) fix

1) Look for all and only the "physical" denominators and the exponents thereof

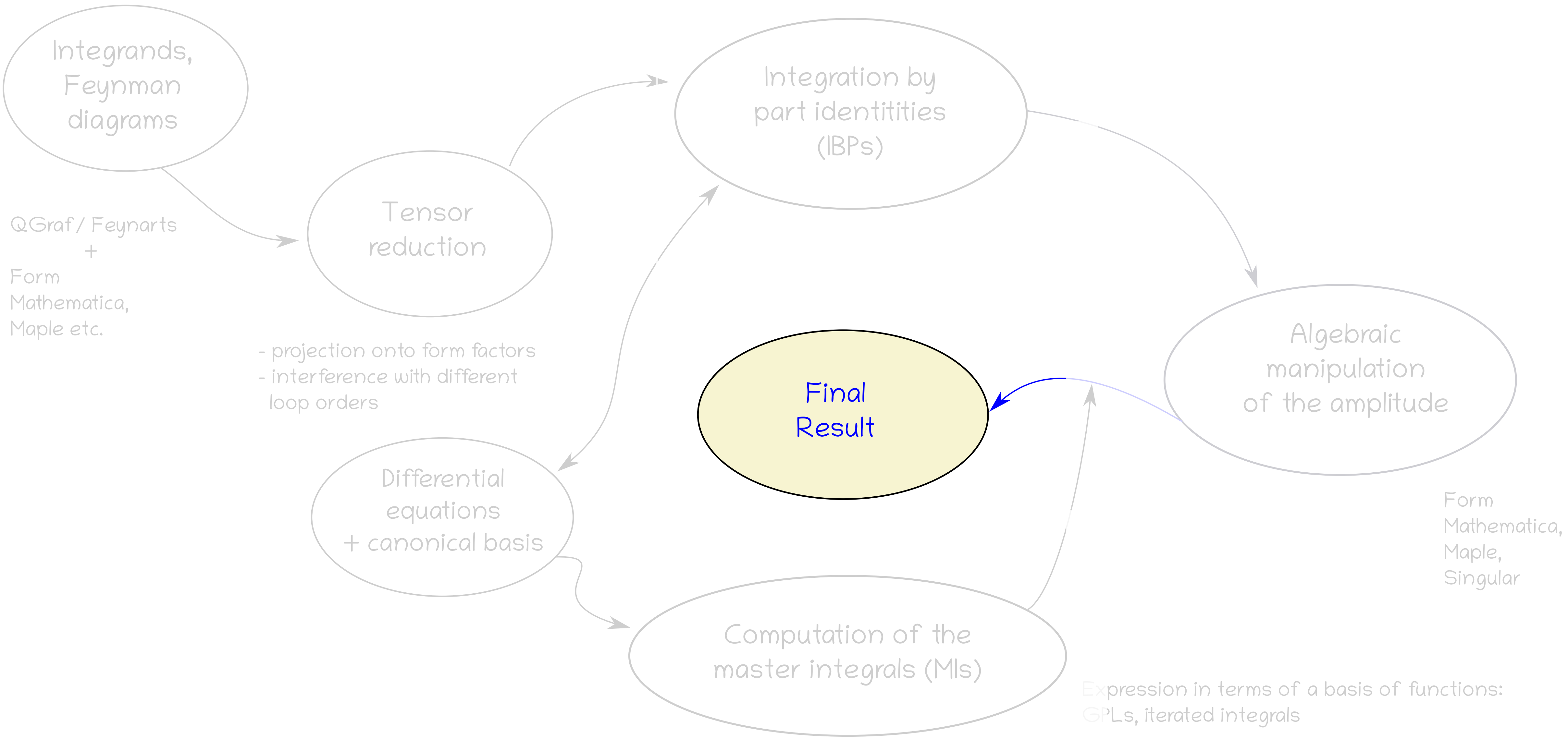
- perform a numerical evaluation (prime numbers)
- perform a prime factors decomposition to identify denominators

2) move to a representation which avoids spurious denominators/exponents

- ie, choose a suitable monomial ordering
- No need for expensive GCD operations!

- More compact results
- Improved numerical stability of rational functions

Traditional approach to multiloop amplitudes

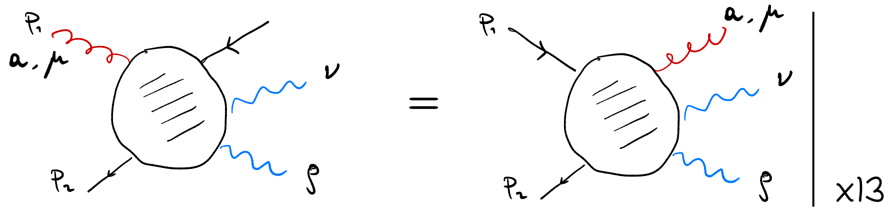


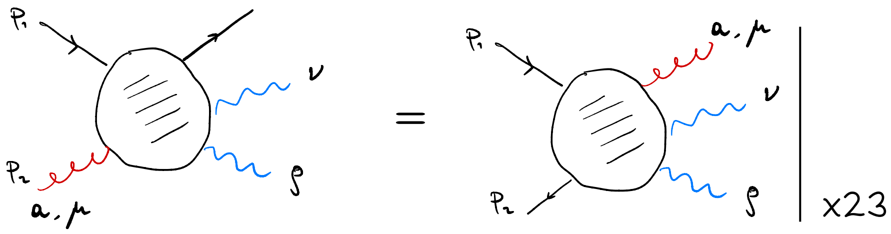
Crossed partonic channels

think of a linear combination
with r_k elements of the basis

$$\mathcal{R}(\lambda) = \sum_k r_k(\{s_{ij}, \epsilon_5\}) f_k(\{s_{ij}, \epsilon_5\})$$

In the position to derive results for crossed partonic channels: qg and $g\bar{q}$ channels





No need to perform any heavy step again: only need $1 \leftrightarrow 3$ and $2 \leftrightarrow 3$ permutations

Crossing r_k is trivial

Crossing f_k more involved

1) Express 2-loop MIs and crossings thereof in terms of pentagon functions

2) Exploit the fact that the full set of MIs is mapped onto itself under permutations

3) Obtain a formal system of linear equations for crossed pentagon functions

4) Solve the system (using FinRed). Solutions are enough to cross the whole amplitude

Checks on the finite remainders of the helicity amplitudes

- We checked that the IR poles of the UV-renormalised helicity amplitudes reproduce those predicted by Catani's factorisation formula

- Check against the LC part of the amplitude published in [Chawhdhry et al. 2103.04319] finding complete agreement.

- Strongest of all checks: we performed an independent calculation of the tree-two-loop interference

  Independent as in:

  - No projectors are used: direct interference of the 2-loop amplitude with the tree-level one summed over polarisations
  - Calculation of the interference fully in CDR
  - The qg channel is derived by crossing the $q\bar{q}$ interference prior to IBP reduction (not at the final level of pentagon functions)

  After UV renormalisation and IR factorisation: finite remainder in CDR and t'Hooft-Veltman are equivalent

Direct interference vs interference from amplitudes: complete agreement for all colour factors

Full color results for the helicity amplitudes

Benchmark results for the complete helicity amplitudes

	$u\bar{u} \to g\gamma\gamma$	$ug \to u\gamma\gamma$
$\mathcal{R}^{(1)}(\boldsymbol{\lambda}_A)$	$0.08637873 + 0.6505825\,i$	$-0.05575262 + 1.282163\,i$
$\mathcal{R}^{(1)}(\boldsymbol{\lambda}_B)$	$4.812087 + 0.8811173\,i$	$-5.332701 - 6.518506\,i$
$\mathcal{R}^{(1)}(\boldsymbol{\lambda}_C)$	$0.05297897 - 4.432186\,i$	$-2.497722 - 22.42864\,i$
$\mathcal{R}^{(2)}(\boldsymbol{\lambda}_A)$	$-2.385158 + 18.22971\,i$	$-28.12588 + 26.67761\,i$
$\mathcal{R}^{(2)}_{\mathrm{LC}}(\boldsymbol{\lambda}_A)$	$0.4123777 + 22.64313\,i$	$-1.450073 + 7.396238\,i$
$\mathcal{R}^{(2)}(\boldsymbol{\lambda}_B)$	$115.9528 + 18.71704\,i$	$17.16557 - 102.3377\,i$
$\mathcal{R}^{(2)}_{\mathrm{LC}}(\boldsymbol{\lambda}_B)$	$144.2892 - 3.600533\,i$	$33.14649 - 134.9655\,i$
$\mathcal{R}^{(2)}(\boldsymbol{\lambda}_C)$	$-36.87656 - 153.3540\,i$	$-26.92189 - 508.2138\,i$
$\mathcal{R}^{(2)}_{\mathrm{LC}}(\boldsymbol{\lambda}_C)$	$-55.57522 - 190.2039\,i$	$76.13565 - 214.1456\,i$

$s_{12} = 157,\ s_{23} = -43,\ s_{34} = 83,\ s_{45} = 61,\ s_{15} = -37,\ \mu^2 = 100$

Numerical evaluation performed using PentagonMI [Chicherin, Sotnikov 2009.07803]

Here, just for comparison/reference: full vs LC

My point of view:

for a reliable assessment of impact of sub-LC,
need to look into a (statistically) large set of MC events

Our analytic results are publicly available at
https://gitlab.msu.edu/vmante/aajamp-symb

README.md

aajamp-symb

Bakul Agarwal, Federico Buccioni, Andreas von Manteuffel, Lorenzo Tancredi

`aajamp-symb` is a repository which provides analytic results for one-loop and two-loop QCD corrections to diphoton production in association with an extra jet in full colour.

If you use the results distributed with `aajamp-symb` in your research work, please cite 2105.04585 along with its external dependency 2009.07803.

External dependencies

The results distributed through this repository are in *Mathematica* readable format. Therefore, all the relevant symbolic manipulations and numerical evaluations can be carried out using *Mathematica*.

The evaluation of the transcendental functions relies on the *Mathematica* package PentagonMI by D. Chicherin and V. Sotnikov, so we strongly recommend to have this available. Further details on how to install and use the package can be found in the git repository PentagonMI.

Structure of the repository

The main object of this repository are the results for the one- and two-loop finite remainders of the helicity amplitudes for diphoton plus jet production. They are located in **helicity_remainders/**. See **helicity_remainders/README.md** for further details on the actual content of the files and the naming scheme adopted.

The **aux/** directory contains auxiliary files needed for the symbolic manipulation and numerical evaluation of the results in **helicity_remainders/**. Further, we provide files with the explicit expressions for the Catani I_1 and I_2 operators for the processes at hand (see hep-ph/9802439 and the supplemental material in 2105.04585).

In **integral_families/** we list the choice of integral families we adopted in our calculation of the one- and two-loop helicity amplitudes. Files are in *yaml* format.

Finally, in **examples/** we provide a few demos which show how to evaluate numerically the finite remainders of the helicity amplitudes, and how to construct the interference with the corresponding tree level. See **examples/README.md** for more details on each example file.

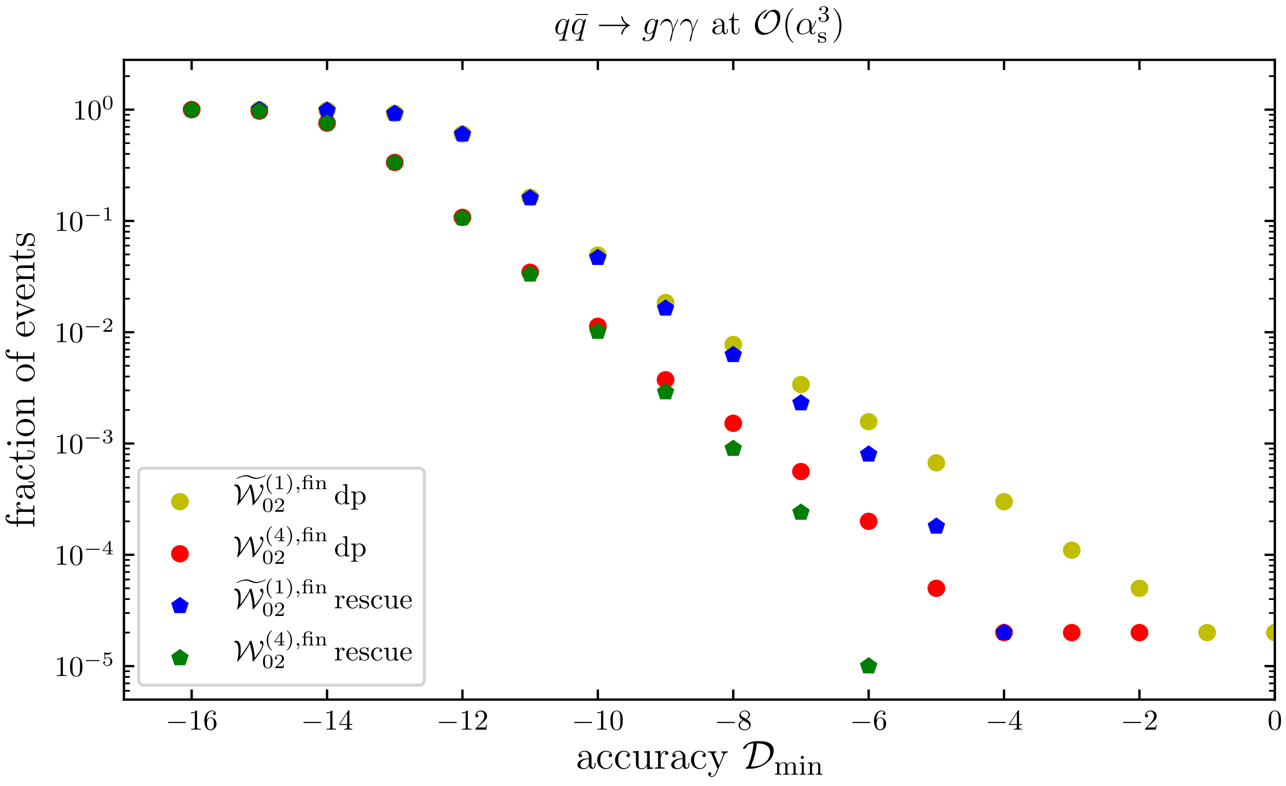
Numerical implementation (LC)

Our LC results implemented in the public code aajamp, available @ https://gitlab.msu.edu/vmante/aajamp

Planning on implementing full-colour helicity amplitudes as well

It relies on the external library Pentagon Functions++ https://gitlab.com/pentagon-functions/PentagonFunctions-cpp [Chicherin, Sotnikov]

Performances: $\sim$ 1.2 s/point.



Allows for evaluations in both double and quadruple precision arithmetic

We have implemented a (simple) rescue system: automatic QP activation if

$$\mathcal{D}_i < \chi\, s_{12}$$

$$E_{\mathrm{com}} = 1\,\mathrm{TeV}, \quad p_{\mathrm{T},g} > 30\,\mathrm{GeV},$$

$$p_{\mathrm{T},\gamma_1} > 30\,\mathrm{GeV}, \quad p_{\mathrm{T},\gamma_2} > 30\,\mathrm{GeV}$$

Outlook

Amplitudes:

Tackle complexity coming from non-planar topologies: our method can be applied to any massless 2-loop 5-point amplitude

all relevant technology now in place (arguably/partly also for IBPs [Guan, Liu, Ma 1912.09294, Bendle et al 2104.06866])

Physical projectors + systematic use of Multivariate Partial Fraction Decomposition:

we expect this to be beneficial for a wider class of multiscale processes: e.g. Vjj, VVj, ttH

Cross sections (pheno):

First pheno studies on $pp \to \gamma\gamma j$ through NNLO QCD in LC [Chawdhry et al 2105.06940] (using STRIPPER).
Desirable an independent calculation, including all colour structures.

Various alternative subtraction schemes can deal with NNLO $\gamma\gamma j$ production: e.g.
N-jettiness, Antenna Subtraction, Nested Soft Collinear Subtraction

All relevant amplitudes for $pp \to \gamma\gamma$@N^{3}LO QCD now available:

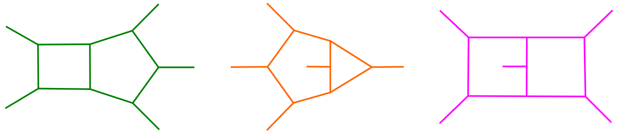
approaching the frontier of fixed-order collider pheno

Summary

- Computation of higher-order QCD corrections crucial for state-of-the-art collider pheno studies

  Among many important contributions: multi-loop, multi-leg amplitudes play a special role and often are the bottleneck

- More generally: amplitudes are an extremely fascinating subject on its own



- We are witnessing dramatic progress in the computation of 2->3 NNLO QCD amplitudes and.
  New results available every month! Triggered first differential results for 2->3 in LC.

- I presented the NNLO QCD corrections to diphoton + jet amplitudes in full colour.
  Focus on amplitudes with a fermionic pair

  Analytic results are publicly available.

First time a massless 5-pt 2-loop amplitude computed exactly for all helicity configurations

- Made it possible thanks to very recent advances:

  physical projectors, pentagon functions, IBP reduction, multivariate partial fraction decomposition

- Main focus of this talk: how to go about algebraic complexity and how to reduce and tame it.

Final remarks

Some more technical remarks/ideas:

except for IBP reduction, the whole calculation has been carried out symbolically

This method can be applied to any massless 5-point 2-loop amplitude

great advantages from MVPFD at basically any step

- drastic reduction of complexity of IBP identities
- choice of a unique Groebner basis: immediate cancellations, never need to do expensive GCD operations
- natural way to look for independent rational functions and physical set of denominators (again, no GCD needed!)

Final results are extremely compact: max 4mb for one helicity configuration for the most complicated colour factor

We implemented our LC results in a computer program ready to use for pheno applications.
Keeping an eye on numerical precision.

Same strategy applicable to full colour helicity amplitudes

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Backup

Integral families

Prop. den.	Family A	Family B
D_1	k_1^2	k_1^2
D_2	$(k_1+p_1)^2$	$(k_1-p_1)^2$
D_3	$(k_1+p_1+p_2)^2$	$(k_1-p_1-p_2)^2$
D_4	$(k_1+p_1+p_2+p_3)^2$	$(k_1-p_1-p_2-p_3)^2$
D_5	k_2^2	k_2^2
D_6	$(k_2+p_1+p_2+p_3)^2$	$(k_2-p_1-p_2-p_3-p_4)^2$
D_7	$(k_2+p_1+p_2+p_3+p_4)^2$	$(k_1-k_2)^2$
D_8	$(k_1-k_2)^2$	$(k_1-k_2+p_4)^2$
D_9	$(k_1+p_1+p_2+p_3+p_4)^2$	$(k_2-p_1)^2$
D_{10}	$(k_2+p_1)^2$	$(k_2-p_1-p_2)^2$
D_{11}	$(k_2+p_1+p_2)^2$	$(k_2-p_1-p_2-p_3)^2$

TC/DP can be obtained from TB as:

TC = TBx12435 + $\{k_2 \to k_1+p_1+p_3,\ k_1 \to p_1+k_2\}$

MultivariateApart, example

Demo on PF decomposition with Multivariate Apart

```
In[-]:= Get[HomeDirectory[] <> "/hep_tools/MultivariateApart.wl"]
```

```
MultivariateApart -- Multivariate partial fractions. By Matthias Heller (maheller@students.uni-mainz.de) and Andreas von Manteuffel (vmante@
```

Univariate PF

```
In[-]:= fx = (x^2 + 3 x - 2)/(x^2 (x - 1) (x + 1)^2);
        Apart[fx]
```

$$\text{Out[-]=}\ \frac{1}{2(-1+x)}+\frac{2}{x^2}-\frac{5}{x}+\frac{2}{(1+x)^2}+\frac{9}{2(1+x)}$$

Multivariate PF

Spurious poles

```
In[-]:= fxy = (2 x + y)/(y (x - y) (x + y));
        Apart[fxy]
```

$$\text{Out[-]=}\ \frac{2}{x\,y}-\frac{3}{2x(-x+y)}-\frac{1}{2x(x+y)}$$

```
In[-]:= Apart[fxy, x] (* Treat y as constant: no spurious poles *)
        Apart[fxy, y] (* Treat x as constant: introduce spurious poles *)
```

$$\text{Out[-]=}\ \frac{3}{2(x-y)\,y}+\frac{1}{2y(x+y)}$$

$$\text{Out[-]=}\ \frac{2}{x\,y}-\frac{3}{2x(-x+y)}-\frac{1}{2x(x+y)}$$

Employ a multivariate partial fraction decomposition

```
In[-]:= MultivariateApart[fxy]
```

$$\text{Out[-]=}\ \frac{3}{2(x-y)\,y}+\frac{1}{2y(x+y)}$$

Unique representation when applied to terms in a sum (commutes with summation)

```
In[-]:= gxy = 1/(x y) + 1/(2 x (x - y)) - 1/(2 x (x + y));

        (* The following GCD operation can be extremely expensive for large and complicated rational functions *)
        gxy // Together
```

$$\text{Out[-]=}\ \frac{x}{(x-y)\,y\,(x+y)}$$

```
In[-]:= (* We could try to apply the PF to each term individually and expand the sum, but with univariate PF:
           - different answers,
           - spurious poles,
           - cancellation not complete *)
        Map[Apart[#, x] &, gxy] // Expand
        Map[Apart[#, y] &, gxy] // Expand
```

$$\text{Out[-]=}\ \frac{1}{2(x-y)\,y}+\frac{1}{2y(x+y)}$$

$$\text{Out[-]=}\ \frac{1}{x\,y}-\frac{1}{2x(-x+y)}-\frac{1}{2x(x+y)}$$

Multivariate Apart

```
In[-]:= DenominatorFactors = {x, y, x - y, x + y};
        qis = {q1, q2, q3, q4};
        DenominatorsToQs = {1/x -> q1, 1/y -> q2, 1/(x - y) -> q3, 1/(x + y) -> q4};
        QsToDenominators = Map[Reverse, DenominatorsToQs];

        Gxy = gxy /. DenominatorsToQs
```

$$\text{Out[-]=}\ q1\,q2+\frac{q1\,q3}{2}-\frac{q1\,q4}{2}$$

Ordering choice 1

```
In[-]:= ord = {{q4}, {q3}, {q2}, {q1}, {x, y}};
        GB = ApartBasis[DenominatorFactors, qis, ord];
        Map[ApartReduce[#, GB, ord] &, Gxy] /. QsToDenominators // Expand
```

$$\text{Out[-]=}\ \frac{1}{2x(x-y)}+\frac{1}{x\,y}-\frac{1}{2x(x+y)}$$

Ordering. Choice 2

```
In[-]:= ord = {{q1}, {q2}, {q3}, {q4}, {x, y}};
        GB = ApartBasis[DenominatorFactors, qis, ord];
        Map[ApartReduce[#, GB, ord] &, Gxy] /. QsToDenominators // Expand
```

$$\text{Out[-]=}\ \frac{1}{(x-y)(x+y)}+\frac{1}{y(x+y)}$$