(g-2)\(\mu\): Is the end of the Standard Model nigh?

- Introduction
- SM prediction from the Muon g-2 Theory Initiative:
  - Hadronic Vacuum Polarisation & Light-by-Light contributions
- Discussion, outlook & paths to further progress
Introduction

• **Muons** are like electrons, but about 200 times heavier, and they **decay**: \( \mu^- \rightarrow e^- \nu_\mu \overline{\nu}_e \)

• Like other matter particles, they have intrinsic angular momentum, **spin** = \( \frac{1}{2} \)

• As they are also **charged**, they have a magnetic moment: \( \vec{\mu} = g \frac{Qe}{2m} \vec{s} \)

• The **Dirac equation** (1928) not only implied antiparticles, but also tells us that the gyromagnetic factor \( g = 2 \)

• If put in a magnetic field, muons precess (like a spinning top)

• This **g-2 precession** can be **measured very precisely** (Brendan Casey’s seminar) and can be **calculated very precisely** (this talk)
Introduction

• 1947: small deviations from predictions in hydrogen and deuterium hyperfine structure; Kusch & Foley propose explanation with \( g = 2.00229 \pm 0.00008 \)

• 1948: Schwinger calculates the famous radiative correction:

\[
\Rightarrow g = 2 (1 + a), \text{ with the anomaly}
\]

\[
a = \frac{g - 2}{2} = \frac{\alpha}{2\pi} \approx 0.001161
\]

This explained the discrepancy and was a crucial step in the development of perturbative QFT and QED

``If you can’t join ‘em, beat ‘em”

• In terms of an effective Lagrangian, the anomaly is from the Pauli term:

\[
\delta L_{\text{off}}^{\text{AMM}} = -\frac{Q e}{4m} a \bar{\psi}_L \sigma^{\mu\nu} \psi_R F_{\mu\nu} + (L \leftrightarrow R)
\]

Note: This is a dimension 5 operator and NOT part of the fundamental (QED) Lagrangian, but occurs through radiative corrections and is calculable in (Standard Model) theory:

\[
a^{\text{SM}}_\mu = a^{\text{QED}}_\mu + a^{\text{weak}}_\mu + a^{\text{hadronic}}_\mu
\]
\(a_e\) vs. \(a_\mu\)

\[a_e = 1\,159\,652\,180.73 (0.28) \times 10^{-12} \quad [0.24\text{ppb}]\]

Hanneke, Fogwell, Gabrielse, PRL 100(2008)120801

\[a_\mu = 116\,592\,089(63) \times 10^{-11} \quad [0.54\text{ppm}]\]

Bennet et al., PRD 73(2006)072003

BNL

- \(a_{e,\text{EXP}}\) more than 2000 times more precise than \(a_{\mu,\text{EXP}}\), but for e\(^-\) loop contributions come from very small photon virtualities, whereas muon `tests’ higher scales

- dimensional analysis: sensitivity to NP (at high scale \(\Lambda_{\text{NP}}\)): \(a_{\mu,\text{NP}} \sim C m_{\mu}^2 / \Lambda_{\text{NP}}^2\)

\(\mu\) wins by \(m_{\mu}^2 / m_e^2 \sim 43000\) for NP, but \(a_e\) determines \(\alpha\), tests QED & low scales

[Notes: \(\tau\) too short-lived for storage-rings. Unclear exp situation with \(\alpha\) from Cs vs Rb: 5.4σ]
\( a_\mu \): back to the future

- CERN started it nearly 40 years ago
- Brookhaven delivered 0.5ppm precision
- E989 at FNAL and J-PARC’s g-2/EDM experiments are happening and should give us certainty

The closer you look the more there is to see'
SM theory vs. Experiment (before 7.4.2021)

If the two don’t match, something may be missing in the SM

Precision measurements + precision theory

discovery potential for New Physics

need for consolidated & reliable SM prediction

\[ a_\mu = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{hadronic}} + a_\mu^{\text{NP?}} \]
Theory vs. Experiment: sensitivity chart

\[ a_\mu = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{weak}} + a_{\mu}^{\text{hadronic}} + a_{\mu}^{\text{NP?}} \]

Plot from Fred Jegerlehner

Need to control the hadronic contributions
SM theory vs. Experiment (after FNAL on 7.4.2021)

Measurement of the Positive Muon Anomalous Magnetic Moment to 0.46 ppm

- Unblinding of Run 1 analyses: 25 February ’21
- FNAL confirms BNL
- Release of result: 7 April ’21
- As of today, PRL has 158 citations (most of them BSM)
- Run 1 is only 6% of total expected statistics

➤ But what about the Standard Model prediction?
Muon g-2 Theory Initiative est. 2017

```
... map out strategies for obtaining the **best theoretical predictions for these hadronic corrections** in advance of the experimental result."
```

- Organised 6 int. workshops in 2017-2020, (virtual) plenary workshop June 28 – July 2, 2021 hosted by KEK (Japan)
- **White Paper** posted 10 June 2020 (132 authors, from 82 institutions, in 21 countries)

```
```

```
The anomalous magnetic moment of the muon in the Standard Model"
```

Group photo from the Seattle workshop in September 2019
SM WP20 prediction from the TI White Paper (0.37 ppm)

- **QED**
  
  \[ a_{\mu}^{\text{QED}} = 116584718.9 \times 10^{-11} \]
  
  0.001 ppm

- **Weak**
  
  \[ a_{\mu}^{\text{Weak}} = 153.6 \times 10^{-11} \]
  
  0.01 ppm

- **Hadronic…**
  - **Vacuum Polarization (HVP)**
    
    \[ a_{\mu}^{\text{HVP}} = 6845 (40) \times 10^{-11} \]
    
    0.34 ppm

  - **Light-by-Light (HLbL)**
    
    \[ a_{\mu}^{\text{HLbL}} = 92 (18) \times 10^{-11} \]
    
    0.15 ppm

- **Uncertainty dominated by hadronic contributions, now**
  \[ \delta_{\text{HVP}} > \delta_{\text{HLbL}} \]
\(a_{\mu}^{\text{QED}}\) & \(a_{\mu}^{\text{weak}}\): a triumph for perturbative QFT

**QED:** Kinoshita et al. + many tests

- \(g-2\) @ 1, 2, 3, 4 & 5 loops
- Subset of 12672 5-loop diagrams:
  - code-generating code, including
  - renormalisation
  - multi-dim. numerical integrations

\[
a_{\mu}^{\text{QED}} = 116\,584\,718.9\ (1) \times 10^{-11} \quad \checkmark
\]

**Weak:** (several groups agree)

- done to 2-loop order, 1650 diagrams
- the first full 2-loop weak calculation

\[
a_{\mu}^{\text{weak}} = 153.6\ (1.0) \times 10^{-11} \quad \checkmark
\]
\( a_\mu^{\text{hadronic}} \): non-perturbative, the limiting factor of the SM prediction

- **Q:** What’s in the hadronic (Vacuum Polarisation & Light-by-Light scattering) blobs?
  - **A:** Anything `hadronic` the virtual photons couple to, i.e. quarks + gluons + photons
  - **But:** low \( q^2 \) photons dominate loop integral(s) \( \implies \) cannot calculate blobs with perturbation theory

- **Two very different strategies:**
  1. use wealth of hadronic data, `data-driven dispersive methods`:
     - data combination from many experiments, radiative corrections required
  2. simulate the strong interaction (+photons) w. discretised Euclidean space-time, `lattice QCD`:
     - finite size, finite lattice spacing, artifacts from lattice actions, QCD + QED needed
     - numerical Monte Carlo methods require large computer resources

➤ discussed in detail in Laurent Lellouch’s recent talk
HVP from:

- LM20
- BMW20
- ETM18/19
- Mainz/CLS19
- FHM19
- PACS19
- RBC/UKQCD18
- BMW17
- RBC/UKQCD data/lattice
- BDJ19
- J17
- DHMZ19
- KNT19
- WP20

TI White Paper 2020 value:

\[ a_{\mu}^{\text{HVP}} = 6845 (40) \times 10^{-11} \]

- **TI WP20** prediction uses dispersive data-driven evaluations with **minimal model dependence**
- **\( a_{\mu}^{\text{HVP}} \) value and error** obtained by merging procedure \[\Rightarrow\] accounts for tensions in input data and differences in data treatment & combination (going beyond usual \( \chi^2_{\text{min}} \) inflation)

**Lattice QCD + QED**
- impressive progress, but...
- large spread between results
- tensions when looking at ‘Euclidean time window’ comparisons
- large systematic uncertainties

**Dispersive/lattice hybrid**

(`window’ method)

For WP20: Dispersive data-driven from DHMZ and KNT
\( a_{\mu}^{\text{HVP}} \): Basic principles of dispersive method

One-loop diagram with hadronic blob =
integral over \( q^2 \) of virtual photon, 1 HVP insertion

Causality \( \Rightarrow \) analyticity \( \Rightarrow \) dispersion integral:
obtain HVP from its imaginary part only

Unitarity \( \Rightarrow \) Optical Theorem:
imaginary part (‘cut diagram’) =
sum over \(|\text{cut diagram}|^2\), i.e.
\( \propto \) sum over all total hadronic cross sections

- Weight function \( \hat{K}(s)/s = \mathcal{O}(1)/s \)
  \( \Rightarrow \) Lower energies more important
  \( \Rightarrow \) \( \pi^+\pi^- \) channel: 73\% of total \( a_{\mu}^{\text{had,LO}} \)

- Total hadronic cross section \( \sigma_{\text{had}} \) from >100 data sets for \( e^+e^- \rightarrow \text{hadrons} \) in >35 final states
- Uncertainty of \( a_{\mu}^{\text{HVP}} \) prediction from statistical & systematic uncertainties of input data
- Pert. QCD used only at large \( s \), **no modelling** of \( \sigma_{\text{had}}(s) \) **required**, direct data integration
\(a_{\mu}^{\text{HLbL}}:\) Hadronic Light-by-Light: Dispersive approach

For HVP \(\Rightarrow\)
\[2 \text{Im} \sum_{\text{had.}} \left| \Phi \right|^2 \quad \Rightarrow \quad \text{Im} \Pi_{\text{had}}(s) = \left( \frac{s}{4\pi\alpha} \right) \sigma_{\text{had}}(s)\]

For HLbL \(\Rightarrow\)
\[\Pi_{\mu\nu\lambda\sigma} = \Pi_{\mu\nu\lambda\sigma}^{\text{pole}} + \Pi_{\mu\nu\lambda\sigma}^{\text{box}} + \Pi_{\mu\nu\lambda\sigma} + \ldots\]

\(\Rightarrow\) Dominated by pole (pseudoscalar exchange) contributions

\(\Rightarrow\) Sum all possible diagrams to get \(a_{\mu}^{\text{HLbL}}\)

- See also review by Danilkin+Redmer+Vanderhaeghen using dispersive techniques estimates \((8.7 \pm 1.3) \times 10^{-10}\) [Prog. Part. Nucl. Phys. 107 (2019) 20]

- With new results & progress, L-by-L can now be reliably predicted! ✓
• **data-driven dispersive** & **lattice** results have confirmed the earlier model-based predictions

• **uncertainty much better under control** and at 0.15ppm already **sub-leading compared to HVP**

• **lattice** predictions now competitive, good prospects for combination and error reduction to ≤ 10%

WP20 data-driven dispersive

hadronic models + pQCD

very new lattice QCD + QED

lattice QCD + QED

data-driven

TI White Paper 2020 value:

\[ a_\mu^{\text{HLbL}} = 92 (18) \times 10^{-11} \]

✓
\( a_\mu^{\text{HVP}} \): Higher orders & QED power counting; WP20 values in \( 10^{-11} \)

All hadronic blobs also contain photons, i.e. real + virtual corrections in \( \sigma_{\text{had}}(s) \)

- **LO:** 6931(40)
- **NLO:** -98.3(7) from three classes of graphs:
  \[-207.7(7) + 105.9(4) + 3.4(1)\] [KNT19]
  (photonic, extra e-loop, 2 h-loops)
- **NNLO:** 12.4(1) [Kurz et al, PLB 734(2014)144, see also F Jegerlehner]
  from five classes of graphs:
  \[8.0 - 4.1 + 9.1 - 0.6 + 0.005\]

\( \Rightarrow \) good convergence,
iterations of hadronic blobs very small
$a_{\mu, \text{LO}}^{\text{had}} = \frac{\alpha^2}{3\pi^2} \int_{s_{th}}^{\infty} \frac{ds}{s} R(s) K(s)$, where $R(s) = \frac{\sigma_{\text{had}, \gamma}^0(s)}{4\pi \alpha^2 / 3s}$
$a_\mu^{\text{HVP}}$: Recent (of 25+ years) experiments providing input $\sigma_{\text{had}(s)}$ data.

S. Serednyakov (for SND) @ HVP KEK workshop

- Different methods: 'Direct Scan' (tunable $e^+e^-$ beams) & 'Radiative Return' (Initial State Radiation scan at fixed cm energy)
- Over last decades detailed studies of radiative corrections & Monte Carlo Generators for $\sigma_{\text{had}(s)}$
  - full NLO radiative corrections in ISR MC Phokhara: Campanario et al, PRD 100(2019)7,076004
HVP dispersive: cross section compilation

How to get the most precise $\sigma^0_{\text{had}}$? Use of $e^+e^- \rightarrow \text{hadrons (+}\gamma\text{)}$ data:

- **Low energies**: $\text{sum } \sim 35 \text{ exclusive channels}$, $2\pi$, $3\pi$, $4\pi$, $5\pi$, $6\pi$, KK, KK$\pi$, KK$\pi\pi$, $\eta\pi$, ...,
  [now very limited use iso-spin relations for missing channels]

- **Above } \sim 1.8 \text{ GeV}**: use of inclusive data or $\text{pQCD}$ (away from flavour thresholds), supplemented by narrow resonances ($J/\Psi$, $\Upsilon$)

- **Challenge of data combination** (locally in $\sqrt{s}$, with error inflation if tensions):
  - many experiments, different energy ranges and bins,
  - statistical + systematic errors from many different sources,
  - use of correlations; must avoid inconsistencies, bias

  ➤ Significant differences between DHMZ and KNT in use of correlated errors:
  - KNT allow non-local correlations to influence mean values,
  - DHMZ restrict this but retain correlations for errors and also betw. channels

- $\sigma^0_{\text{had}}$ means the `bare' cross section, i.e. excluding `running coupling' (VP) effects, but including Final State ($\gamma$) Radiation: data subject to Radiative Corrections
Rad. Corrs.: HVP for running $\alpha(q^2)$. Undressing

- Dyson summation of Real part of one-particle irreducible blobs $\Pi$ into the effective, real running coupling $\alpha_{\text{QED}}$:

$$\Pi = \gamma^*_{\text{q}} \quad \text{Full photon propagator} \sim 1 + \Pi + \Pi \cdot \Pi + \Pi \cdot \Pi \cdot \Pi + \ldots$$

$$\sim \quad \alpha(q^2) = \frac{\alpha}{1 - \text{Re}\Pi(q^2)} = \alpha / \left(1 - \Delta \alpha_{\text{lep}}(q^2) - \Delta \alpha_{\text{had}}(q^2)\right)$$

- The Real part of the VP, $\text{Re}\Pi$, is obtained from the Imaginary part, which via the Optical Theorem is directly related to the cross section, $\text{Im}\Pi \sim \sigma(e^+e^- \rightarrow \text{hadrons})$:

$$\Delta \alpha_{\text{had}}^{(5)}(q^2) = \frac{-q^2}{4\pi^2\alpha} \mathcal{P} \int_{m^2_\pi}^{\infty} \frac{\sigma_{\text{had}}^0(s)}{s - q^2} \, ds, \quad \sigma_{\text{had}}(s) = \frac{\sigma_{\text{had}}^0(s)}{|1 - \Pi|^2}$$

$$\rightarrow \sigma^0 \text{ requires ‘undressing’, e.g. via } \cdot \left(\alpha/\alpha(s)\right)^2 \sim \text{ iteration needed}$$

- Observable cross sections $\sigma_{\text{had}}$ contain the $|\text{full photon propagator}|^2$, i.e. $|\text{infinite sum}|^2$. 

$$\rightarrow \text{To include the subleading Imaginary part, use dressing factor } \frac{1}{|1-\Pi|^2}.$$
Rad. Corrs.: HVP for running $\alpha(q^2)$. Undressing

- $\Delta\alpha(q^2)$ in the time-like: HLMNT compared to Fred Jegerlehner’s new routines

For demonstration only, results $>10$ years old!

Different groups use their own HVP routines:
- Fred Jegerlehner,
- DHMZ,
- KNT,
- Novosibirsk (Fedor Ignatov)

$\rightarrow$ with new version big differences (with 2003 version) gone
- smaller differences remain and reflect different choices, smoothing etc.
Rad. Corrs.: Final State $\gamma$ Radiation

- Real + virtual, must be included in $\sigma_\text{had}^0$ as part of the hadronic dynamics,

- but some events with real radiation will have been cut-off by experimental analyses (no problem if $\gamma$ just missed but event counted. Possible problem of mis-identifies)

- Experiments (or compilations) account for this and add FSR back;
  - based on MC and scalar QED for pions (detailed studies, checked to work well)
  - contributes to systematic uncertainties
  - intrinsic part of Radiative Return analyses of many recent data sets

- Notes:
  - at low energies and at resonances, hard radiation is limited by phase space
  - different compilations apply additional uncertainty to cover possible problems of the FSR (& VP/undressing) treatment, e.g.

  - KNT: $\delta a_\mu^{\text{had, FSR}} = 7.0 \times 10^{-11}$, and also $\delta a_\mu^{\text{had, VP}} = 2.1 \times 10^{-11}$
<table>
<thead>
<tr>
<th>Channel</th>
<th>Energy range [GeV]</th>
<th>$\alpha_\mu^{\text{had,LLO VP}} \times 10^{10}$</th>
<th>$\Delta a_\mu^{(S)}(M_\mu^2) \times 10^4$</th>
<th>New data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0\gamma$</td>
<td>$m_\pi \leq \sqrt{s} \leq 0.600$</td>
<td>0.12 ± 0.01</td>
<td>0.00 ± 0.00</td>
<td>...</td>
</tr>
<tr>
<td>$\pi^+\pi^-$</td>
<td>$2m_\pi \leq \sqrt{s} \leq 0.305$</td>
<td>0.07 ± 0.02</td>
<td>0.00 ± 0.00</td>
<td>...</td>
</tr>
<tr>
<td>$\pi^+\pi^-\pi^0$</td>
<td>$3m_\pi \leq \sqrt{s} \leq 0.660$</td>
<td>0.01 ± 0.00</td>
<td>0.00 ± 0.00</td>
<td>...</td>
</tr>
<tr>
<td>$\eta\eta'$</td>
<td>$m_\eta \leq \sqrt{s} \leq 0.660$</td>
<td>0.00 ± 0.00</td>
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### Chiral perturbation theory (ChPT) threshold contributions

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<td>$0.600 \leq \sqrt{s} \leq 1.350$</td>
<td>4.46 ± 0.10</td>
<td>0.36 ± 0.01</td>
<td>[65]</td>
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<td>$\pi^+\pi^-\pi^0$</td>
<td>$0.305 \leq \sqrt{s} \leq 1.937$</td>
<td>502.97 ± 1.97</td>
<td>34.26 ± 0.12</td>
<td>[34,35]</td>
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<td>$\pi^+\pi^-\pi^0$</td>
<td>$0.660 \leq \sqrt{s} \leq 1.937$</td>
<td>47.79 ± 0.89</td>
<td>4.77 ± 0.08</td>
<td>[36]</td>
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### Data based channels ($\sqrt{s} \leq 1.937$ GeV)

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### Estimated contributions ($\sqrt{s} \leq 1.937$ GeV)

<table>
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<td>$\eta\eta'$</td>
<td>$m_\eta \leq \sqrt{s} \leq 0.660$</td>
<td>0.00 ± 0.00</td>
<td>0.00 ± 0.00</td>
<td>...</td>
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</table>

### Other contributions ($\sqrt{s} > 1.937$ GeV)

<table>
<thead>
<tr>
<th>Channel</th>
<th>Energy range [GeV]</th>
<th>$\alpha_\mu^{\text{had,LLO VP}} \times 10^{10}$</th>
<th>$\Delta a_\mu^{(S)}(M_\mu^2) \times 10^4$</th>
<th>New data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J/\psi$</td>
<td>$...$</td>
<td>6.26 ± 0.19</td>
<td>7.07 ± 0.22</td>
<td>...</td>
</tr>
<tr>
<td>$\psi'$</td>
<td>$...$</td>
<td>1.58 ± 0.04</td>
<td>2.51 ± 0.06</td>
<td>...</td>
</tr>
<tr>
<td>$\Upsilon(1S - 4S)$</td>
<td>$...$</td>
<td>0.09 ± 0.00</td>
<td>1.06 ± 0.02</td>
<td>...</td>
</tr>
<tr>
<td>pQCD</td>
<td>$11.199 \leq \sqrt{s} &lt; \infty$</td>
<td>2.07 ± 0.00</td>
<td>124.79 ± 0.10</td>
<td>...</td>
</tr>
</tbody>
</table>

**Total**

| $m_\pi \leq \sqrt{s} \leq \infty$ | 693.26 ± 2.46 | 276.11 ± 1.11 | ... |

**Update: KNT19**

LO+NLO HVP for $\alpha_{e,\mu,\tau}$ & hyperfine splitting of muonium

**PRD101(2020)014029**

**Breakdown of HVP contributions in $\sim 35$ hadronic channels**

From 2-11 GeV, use of inclusive data, pQCD only beyond 11 GeV
$a_\mu^{\text{HVP}}$: Landscape of $\sigma_{\text{had}}(s)$ data & most important $\pi^+\pi^-$ channel

$\pi^+\pi^-$:

- Combination of >30 data sets, >1000 points, contributing >70% of total HVP
- Precise measurements from 6 independent experiments with different systematics and different radiative corrections
- Data sets from Radiative Return dominate
- Some tension in data accounted for by local $\chi^2_{\text{min}}$ inflation and via WP merging procedure

[KNT18, PRD97, 114025]

- hadronic channels for energies below 2 GeV
- dominance of $2\pi$

[KNT19, PRD101, 014029]
HVP: $\pi^+\pi^-$ channel [KNT19, Phys. Rev. D 101(2020)1, 014029]

- Tension between different sets, especially between the most precise 4 sets from BaBar and KLOE
- Inflation of error with local $\chi^2_{\text{min}}$ accounts for tensions, leading to a $\sim15\%$ error inflation
- Important role of correlations; their treatment in the data combination is crucial and can lead to significant differences between different combination methods
**HVP: \( \pi^+\pi^- \) channel**  [DHMZ, Eur. Phys. J. C 80(2020)3, 241]

- In addition they employ a fit, based on analyticity + unitarity + crossing symmetry, similar to Colangelo et al. and Ananthanarayan+Caprini+Das, leading to stronger constraints/lower errors at low energies.

- For 2\( \pi \), based on difference between result for \( a_\mu^\pi\pi \) w/out KLOE and BaBar, sizeable additional systematic error is applied and mean value adjusted.

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**arXiv:1908.00921** Figure 5:

<table>
<thead>
<tr>
<th>SM predictions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEO</td>
<td>376.9 ± 6.3</td>
</tr>
<tr>
<td>SND</td>
<td>371.7 ± 5.0</td>
</tr>
<tr>
<td>BESIII</td>
<td>368.2 ± 4.2</td>
</tr>
<tr>
<td>CMD-2</td>
<td>372.4 ± 3.0</td>
</tr>
<tr>
<td>BABAR</td>
<td>376.7 ± 2.7</td>
</tr>
<tr>
<td>KLOE</td>
<td>366.9 ± 2.1</td>
</tr>
</tbody>
</table>

**arXiv:1908.00921** Figure 6:

Graph showing relative difference to fit for all data with various fits and data points from different experiments.
HVP: KK channels \[\text{[KNT18, PRD97, 114025]}\]

**K^+ K^-**

\[
\sigma(e^+e^- \rightarrow K^+K^-) \quad \text{[nb]} \\
\sqrt{s} \quad \text{[GeV]}
\]

New data:
- CMD-3: [arXiv:1710.02989.]


\[ a_{\mu}^{K^+K^-} = 23.03 \pm 0.22_{\text{tot}} \]

HLMNT11: 22.15 ± 0.46_{\text{tot}}

Large increase in mean value

**K_S^0 K_L^0**

\[
\sigma(e^+e^- \rightarrow K_S^0K_L^0) \quad \text{[nb]} \\
\sqrt{s} \quad \text{[GeV]}
\]

New data:

\[ a_{\mu}^{K_S^0K_L^0} = 13.04 \pm 0.19_{\text{tot}} \]

HLMNT11: 13.33 ± 0.16_{\text{tot}}

Large changes due to new precise measurements on $\phi$
**HVP: $\sigma_{\text{had}}$ inclusive region [KNT18]**


**Results**

From individual channels

Inclusive $\mu=4.367^{+0.017}_{-0.017}^{\text{stat}} \pm 0.48^{\text{sys}} \pm 0.04^{\text{vp}} \pm 0.44^{\text{fsr}} \equiv 43.67 \pm 0.67^{\text{tot}}$
History plot of $a_\mu^{\text{HVP}}$ w. min. model dep. Pies.

- Stability and consolidation over two decades thanks to more and better data input and improved compilation procedures

- Compare with `merged’ DHMZ & KNT WP20 value:

\[ a_\mu^{\text{had, LO VP (WP20)}} = 693.1(4.0) \times 10^{-10} \]

Pie diagrams [KNT]:

- error still dominated by two pion channel

- significant contribution to error from additional uncertainty from radiative corrections
### Detailed comparisons by-channel and energy range between direct integration results:

<table>
<thead>
<tr>
<th></th>
<th>DHMZ19</th>
<th>KNT19</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+\pi^-$</td>
<td>507.85(0.83)(3.23)(0.55)</td>
<td>504.23(1.90)</td>
<td>3.62</td>
</tr>
<tr>
<td>$\pi^+\pi^-\pi^0$</td>
<td>46.21(0.40)(1.10)(0.86)</td>
<td>46.63(94)</td>
<td>-0.42</td>
</tr>
<tr>
<td>$\pi^+\pi^-\pi^+\pi^-$</td>
<td>13.68(0.03)(0.27)(0.14)</td>
<td>13.99(19)</td>
<td>-0.31</td>
</tr>
<tr>
<td>$\pi^+\pi^0\pi^0$</td>
<td>18.03(0.06)(0.48)(0.26)</td>
<td>18.15(74)</td>
<td>-0.12</td>
</tr>
<tr>
<td>$K^+K^-$</td>
<td>23.08(0.20)(0.33)(0.21)</td>
<td>23.00(22)</td>
<td>0.08</td>
</tr>
<tr>
<td>$K_S K_L$</td>
<td>12.82(0.06)(0.18)(0.15)</td>
<td>13.04(19)</td>
<td>-0.22</td>
</tr>
<tr>
<td>$\pi^0\gamma$</td>
<td>4.41(0.06)(0.04)(0.07)</td>
<td>4.58(10)</td>
<td>-0.17</td>
</tr>
<tr>
<td>Sum of the above</td>
<td>626.08(0.95)(3.48)(1.47)</td>
<td>623.62(2.27)</td>
<td>2.46</td>
</tr>
<tr>
<td>[1.8, 3.7] GeV (without $c\bar{c}$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$J/\psi, \psi(2S)$</td>
<td>33.45(71)</td>
<td>34.45(56)</td>
<td>-1.00</td>
</tr>
<tr>
<td>[3.7, $\infty$) GeV</td>
<td>7.76(12)</td>
<td>7.84(19)</td>
<td>-0.08</td>
</tr>
<tr>
<td>Total $a_H^{HVP,LO} \mu$</td>
<td>17.15(31)</td>
<td>16.95(19)</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>694.0(1.0)(3.5)(1.6)(0.1)$\psi(0.7)_{DV+QCD}$</td>
<td>692.8(2.4)</td>
<td>1.2</td>
</tr>
</tbody>
</table>

+ evaluations using unitarity & analyticity constraints for $\pi\pi$ and $\pi\pi\pi$ channels
[CHS 2018, HHKS 2019]
Conservative merging procedure developed during 2019 Seattle TI workshop:

- Accounts for the different results obtained by different groups based on the same or similar experimental input
- Includes correlations and their different treatment as much as possible
- Allows to give one recommended (merged) result, which is conservative w.r.t. the underlying (and possibly underestimated) uncertainties
- Note: Merging leads to a bigger error estimate compared to individual evaluations

$$a_\mu^\text{HVP, LO} = 693.1 \ (4.0) \times 10^{-10}$$ is the result used in the WP `SM2020’ value

- This result does not include lattice, but is compatible with published lattice results apart from the BMW prediction:
  $$a_\mu^\text{HVP, LO (BMW)} = 707.5 \ (5.5) \times 10^{-10} \ [\text{Nature}]$$

Efforts are ongoing in the community to check their result, with a topical online workshop from the g-2 Theory Initiative in November 2020 shedding first light.
**BMW20**: large systematics from *continuum limit*,
large taste-breaking corrections (‘SRHO’)

- upper right panel: limit and uncertainty estimation
- lower right panel: limit for central ‘window’ compared to other lattice and data-driven results (3.7σ tension!)
This is experiment vs. theory with the new FNAL g-2 Run-1 result announced 7\textsuperscript{th} April.
HVP from electron-muon scattering in the space-like region.

\[ a_{\mu}^{HLO} = \frac{\alpha}{\pi} \int_0^1 dx \ (1-x) \Delta \alpha_{\text{had}}[t(x)] \]

\[ t(x) = \frac{x^2 m_{\mu}^2}{x - 1} < 0 \]

\( \Delta \alpha_{\text{had}}(t) \) is the hadronic contribution to the running of \( \alpha \) in the space-like region. It can be extracted from scattering data!

- use CERN M2 muon beam (150 GeV)
- Physics beyond colliders program @ CERN
- LOI June 2019
- Jan 2020: SPSC recommends pilot run in 2021
- goal: run with full apparatus in 2023-2024
Summary & Perspective

- The still **unresolved muon g-2 discrepancy** has triggered new experiments and a lot of theory activities, including and helped by the Muon g-2 Theory Initiative.

- **Much progress** has been made for **HLbL** which previously was seen as the bottleneck. *New data driven dispersive approaches & lattice* have confirmed earlier model estimates and now allow a **reliable error estimate**, and more work is in progress.

- For **HVP dispersive**, the **TI published a conservative & robust consensus**. Soon **new hadronic data for 2π** will come from **BaBar, CMD-3, BESIII and Belle-II**.

- Longer term: direct HVP measurement planned with e-μ scattering: **MUonE at CERN**.

- **Lattice** has started to deliver impressive results with **high precision**. **Further work is needed** and ongoing to scrutinize, check and improve different approaches, and lattice is expected to play an important role in the future.

- The **Muon g-2 Theory Initiative** will continue to facilitate this work and to publish **agreed & conservative SM predictions** for g-2 prior to new experimental results.

- With the **WP20 SM** prediction and the new first g-2 result from **FNAL**, the discrepancy stands at 4.2σ and is more intriguing than ever.
Extras/discussion
Lattice HVP: Cross checks, window method (I)

\[ a_{\mu}^{\text{HVP,LO}} = \left( \frac{\alpha}{\pi} \right)^2 \int_0^\infty dt \tilde{w}(t) C(t) \]

- Use windows in Euclidean time to consider the different time regions separately.

  Short Distance (SD) \( t : 0 \rightarrow t_0 \)
  Intermediate (W) \( t : t_0 \rightarrow t_1 \)
  Long Distance (LD) \( t : t_1 \rightarrow \infty \)

- Compute each window separately (in continuum, infinite volume limits, ...) and combine

\[ a_{\mu} = a_{\mu}^{\text{SD}} + a_{\mu}^{W} + a_{\mu}^{\text{LD}} \]
Lattice HVP: Cross checks, window method (II)

H. Wittig @ Lattice HVP workshop

\[ a_\mu = a_\mu^{SD} + a_\mu^W + a_\mu^{LD} \]

\[ t_0 = 0.4 \text{ fm}, \quad t_1 = 1.0 \text{ fm} \]
\[ \Delta = 0.15 \text{ fm} \]

**“Window” quantities**

\((t_0, \Delta) = (0.4, 1.0, 0.15) \text{ fm}\)

- Aubin et al. 19
- Aubin et al. 19 - finest \(\mu\)s
- LM 20
- BMW 20
- FHM 20 (prelim., stat only)
- RBC/UKQCD 18
- ETMC 20 (prelim.)
- Mainz/CLS 20 (prelim.)
- R-ratio & lattice

\[ a_\mu^W (\text{ud, conn, iso}) * 10^{10} \]

\((t_0, \Delta) = (0.4, 0.15) \text{ fm}\)

- FHM 20 (prelim., stat. only)
- RBC/UKQCD 20 (prelim., stat. only)

\[ a_\mu^{SD} (\text{ud, conn, iso}) * 10^{10} \]

\((t, \Delta) = (1.0, 0.15) \text{ fm}\)

- FHM 20 (prelim., stat. only)
- ETMC 20 (prelim.)
- Mainz/CLS 20 (prelim.)

\[ a_\mu^{LD} (\text{ud, conn, iso}) * 10^{10} \]

- Straightforward reference quantities
- Can be applied to individual contributions (light, strange, charm, disconnected, ...)
- **Large discrepancies** between different results, also with data-driven: BMW vs KNT: 3.7\(\sigma\)
- Individual results **must sum up**, and different groups & discretisations **must agree** (universality)
HVP: $\Phi$ in different final states $K^+K^-$, $K_S^0K_L^0$, $\pi^+\pi^-\pi^0$

- Direct data integration automatically accounts for all hadronic dynamics, no resonance fits/parametrisations or estimates of mixing effects needed.

For demo. only, does not include latest data.
Historically, hadronic tau decay data, e.g. $\tau^- \rightarrow \pi^0 \pi^- \nu_\tau$, were used to improve precision of $e^+e^-$ based evaluations.

However, with the increased precision of the $e^+e^-$ data there is now limited merit in this (DHMZ have dropped it), and the required iso-spin breaking corrections re-introduce a model-dependence and connected systematic uncertainty (there is, e.g., no $\rho-\omega$ mixing in $\tau$ decays).

Quote from the WP, where this approach is discussed in detail:

"Concluding this part, it appears that, at the required precision to match the $e^+e^-$ data, the present understanding of the IB corrections to $\tau$ data is unfortunately not yet at a level allowing their use for the HVP dispersion integrals. It remains a possibility, however, that the alternate lattice approach, discussed in Sec. 3.4.2, may provide a solution to this problem.”
HVP from electron muon scattering in the space-like

C. Carloni @ g-2 INT workshop [A. Abbiendi et al, arXiv:1609.08987, EPJC 2017]

- requires calculations of radiative corrections [M. Fael @ g-2 INT workshop]
- complement region not accessible to experiment with LQCD calculation [M. Marinkovic @ g-2 INT workshop]
The goal first is to improve the experimental measurement of $a_\mu$ at resolving whether new physics is being revealed in the precision evaluation of the muon’s magnetic moment. To this end, a group was formed—the Muon g-2 Theory Initiative—to holistically evaluate all aspects of the Standard Model (SM) and to recommend a single value against which new experimental results should be compared. This White Paper (WP) is an update to T. Aoyama et al, arXiv:2006.04822, with contributions from Secs. 2.3.8 and 7.4

### Summary Table

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Value $\times 10^{11}$</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment (E821)</td>
<td>116 592 089(63)</td>
<td>Ref. [1]</td>
</tr>
<tr>
<td>HVP LO ($e^+e^-$)</td>
<td>6931(40)</td>
<td>Refs. [2–7]</td>
</tr>
<tr>
<td>HVP NLO ($e^+e^-$)</td>
<td>$-98.3(7)$</td>
<td>Ref. [7]</td>
</tr>
<tr>
<td>HVP NNLO ($e^+e^-$)</td>
<td>12.4(1)</td>
<td>Ref. [8]</td>
</tr>
<tr>
<td>HVP LO (lattice, $udsc$)</td>
<td>7116(184)</td>
<td>Refs. [9–17]</td>
</tr>
<tr>
<td>HlbL (phenomenology)</td>
<td>92(19)</td>
<td>Refs. [18–30]</td>
</tr>
<tr>
<td>HlbL NLO (phenomenology)</td>
<td>2(1)</td>
<td>Ref. [31]</td>
</tr>
<tr>
<td>HlbL (lattice, $uds$)</td>
<td>79(35)</td>
<td>Ref. [32]</td>
</tr>
<tr>
<td>HlbL (phenomenology + lattice)</td>
<td>90(17)</td>
<td>Refs. [18–30, 32]</td>
</tr>
</tbody>
</table>

### References


### Executive Summary

The final result from the Brookhaven National Laboratory (BNL) experiment E821, published in 2004, has a precision of 0.7 ppm. The lattice QCD calculation of the HVP LO ($e^+e^-$) contribution builds on the reference between $\mu^+\mu^-$ and $e^+e^-$, generated significant interest in the particle physics community because it might arise from new physics. In the 2006–2018 period, there was a lot of activity to produce dedicated, precise, and extensive measurements of $a_\mu$. Concerns over the reliability of the model-dependent HLbL estimates. On the theoretical side, there was a lot of activity to develop new model-independent approaches, including dispersive methods for HLbL and lattice-QCD methods for QED. A comprehensive experimental effort to produce dedicated, precise, and extensive measurements of $a_\mu$ of as yet undiscovered particles contributing through virtual loops. The final result from the Brookhaven National Laboratory (BNL) experiment E821, published in 2004, has a precision of 0.7 ppm.

### Table 1: Summary of the contributions to $a_\mu$

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Value $\times 10^{11}$</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED</td>
<td>116 584 718.931(104)</td>
<td>Refs. [33, 34]</td>
</tr>
<tr>
<td>Electroweak</td>
<td>153.6(1.0)</td>
<td>Refs. [35, 36]</td>
</tr>
<tr>
<td>HVP ($e^+e^-$, LO + NLO + NNLO)</td>
<td>6845(40)</td>
<td>Refs. [2–8]</td>
</tr>
<tr>
<td>HlbL (phenomenology + lattice + NLO)</td>
<td>92(18)</td>
<td>Refs. [18–32]</td>
</tr>
<tr>
<td>Total SM Value</td>
<td>116 591 810(43)</td>
<td>Refs. [2–8, 18–24, 31–36]</td>
</tr>
<tr>
<td>Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$</td>
<td>279(76)</td>
<td></td>
</tr>
</tbody>
</table>

w.r.t. BNL
Precision observable $\alpha(M_Z^2) = \alpha/(1 - \Delta\alpha(M_Z^2))$ as a sensitive test of HVP

- Can $\Delta a_\mu$ be due to hypothetical mistakes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{\text{had}}^{(5)}(M_Z)$.
- Consider:

$$a = \int_{4m^2}^{s_u} ds \, f(s) \sigma(s), \quad f(s) = \frac{K(s)}{4\pi^3}, \quad s_u < M_Z^2;$$

$$b = \int_{4m^2}^{s_u} ds \, g(s) \sigma(s), \quad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

and the increase

$$\Delta \sigma(s) = \epsilon \sigma(s)$$

$\epsilon > 0$, in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$$

Note the very different energy-dependent weighting of the integrands…
Marciano, Passera, Sirlin (2008):

- changing the hadronic cross section at higher energies significantly upwards leads to tensions in EW precision fits of the SM.
- not easy to reconcile g-2 without running into problems with $\Delta \alpha(M_Z^2)$

Recent studies by several groups, e.g.

- **Crivellin et al, PRL125(2020)9,091801**: Shifts in HVP make fit based on HEPFitter worse, but they can not rule out shifts at low energies as obtained by the BMW lattice analysis
- **Keshavarzi et al, PRD102(2020)3,033002**: updating Marciano et al, again find significant tensions with Gfitter if shifts in HVP were to explain g-2, unless they are below ~0.7 GeV
- However, the low energies hadronic cross section measurements (mainly 2pi) are most precise there.