Combining Sudakov and small-x resummation for forward dijet production

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based on:
A. van Hameren, P. Kotko, K. Kutak, SS, Phys.Lett.B 814 (2021) 136078
A. van Hameren, P. Kotko, K. Kutak, SS, E. Żarów, *in preparation*

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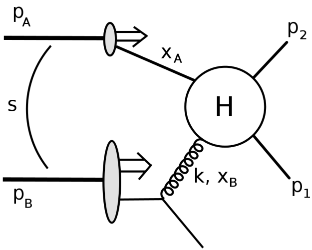
Outline

1. Context: forward jet production
2. Low-x resummation
3. Sudakov resummation
4. Combination
5. Phenomenology for LHC and EIC

Forward jet production in hadronic collisions



Low x_B leads to appearance of large logarithms $\ln x_B$, which need to be resummed

Incoming partons' energy fractions:

$$x_A = \frac{1}{\sqrt{s}} \left(|\vec{p}_{1T}| e^{y_1} + |\vec{p}_{2T}| e^{y_2} \right) , \qquad x_B = \frac{1}{\sqrt{s}} \left(|\vec{p}_{1T}| e^{-y_1} + |\vec{p}_{2T}| e^{-y_2} \right)$$

$$y_1 \sim 0,\; y_2 \gg 0 \quad \longrightarrow \quad x_A \sim 1,\; x_B \lesssim 1 \quad \text{(central-forward)}$$

$$y_1 \gg 0,\; y_2 \gg 0 \quad \longrightarrow \quad x_A \sim 1,\; x_B \ll 1 \quad \text{(forward-forward)}$$

Gluon's transverse momentum (p_{1t}, p_{2t} imbalance):

$$|\vec{k}_T|^2 = |\vec{p}_{1T} + \vec{p}_{2T}|^2 = |\vec{p}_{1T}|^2 + |\vec{p}_{2T}|^2 + 2|\vec{p}_{1T}||\vec{p}_{2T}| \cos \Delta\phi$$

Transverse momentum dependent parton distributions

In collinear factorization one assumes

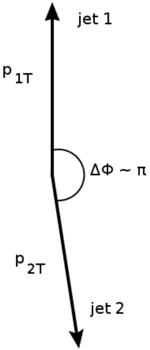
$$k_+ \gg k_-, k_T$$

For some observables, however, k_T should not be neglected, *e.g.*

$$\frac{d\sigma}{dq_T} \quad \text{where} \quad q_T = |p_{1T} + p_{2T}|$$

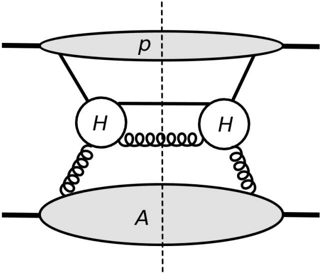
In the back-to-back limit

$$q_T \text{ small} \quad \Longrightarrow \quad q_T \sim k_T$$



Neglecting k_T will have an effect on low-q_T region of $\frac{d\sigma}{dq_T}$. Hence, one should include both k_+ and k_T in the parton distribution functions.

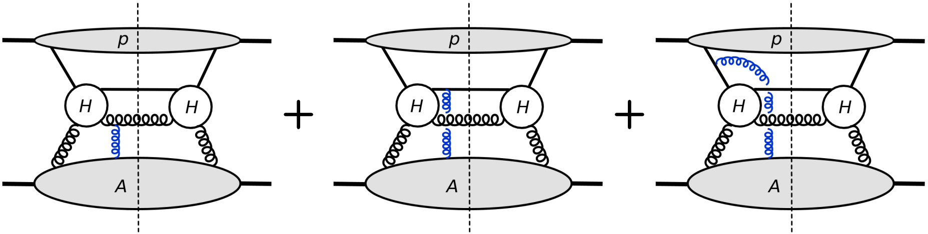
TMD gluon distribution (naive)



$$\mathcal{F}_{g/A}(x_2, k_T) \stackrel{\text{naive}}{=} 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_T\cdot\boldsymbol{\xi}_T} \left\langle A \middle| \mathrm{Tr}\left[F^{i-}\left(\xi^+, \boldsymbol{\xi}_T\right) F^{i-}(0)\right] \middle| A \right\rangle$$

This definition is gauge dependent!

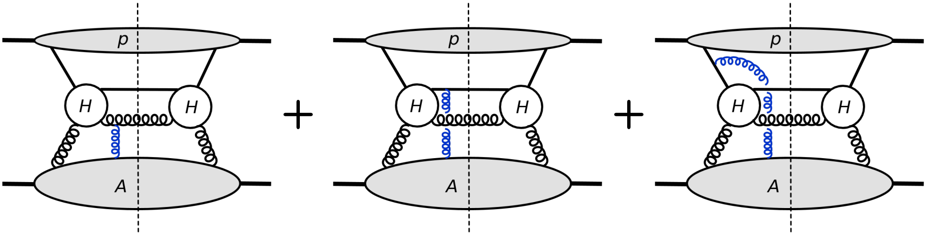
TMD gluon distributions (proper definition)



$+$ similar diagrams with $2, 3, \ldots$ gluon exchanges

They all contribute at leading power and need to be resummed.

TMD gluon distributions (proper definition)



$+$ similar diagrams with $2, 3, \ldots$ gluon exchanges

They all contribute at leading power and need to be resummed.

That is done by gauge links $U_{[\alpha,\beta]}$

$$\mathcal{F}_{g/A}(x_2, k_T) = 2\int \frac{d\xi^+ d^2\boldsymbol{\xi}_T}{(2\pi)^3 p_A^-} e^{ix_2 p_A^- \xi^+ - ik_T\cdot\boldsymbol{\xi}_T} \langle A|\mathrm{Tr}\big[F^{i-}\left(\xi^+, \boldsymbol{\xi}_T\right) U_{[\xi,0]} F^{i-}(0)\big]|A\rangle$$

- $U_{[\alpha,\beta]}$ renders gluon distribution gauge invariant

Gauge links

The operators $U_{[\alpha,\beta]}$ are built from Wilson lines

$$\mathcal{W}^{n}_{[a,b]} = \mathcal{P}\exp\left[-ig\int_a^b dz\, n\cdot A(z)\right],\quad \mathcal{W}^{T}_{[a,b]} = \mathcal{P}\exp\left[-ig\int_a^b dz_T\cdot A_T(z)\right]$$

The two basic gauge links

$$\mathcal{U}^{[\pm]} = \mathcal{W}^{n}_{[(0^-,\mathbf{0}_T);(\pm\infty^-,\mathbf{0}_T)]}\mathcal{W}^{T}_{[(\pm\infty^-,\mathbf{0}_T);(\pm\infty^-,\boldsymbol{\xi}_T)]}\mathcal{W}^{n}_{[(\pm\infty^-,\boldsymbol{\xi}_T);(\xi_-,\boldsymbol{\xi}_T)]}$$

We can also encounter loops

$$\mathcal{U}^{[\Box]} = \mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger} = \mathcal{U}^{[-]}\mathcal{U}^{[+]\dagger}$$

As we see, the path $[\alpha,\beta]$ depends on the hard process. Hence, gluon TMD is in general process-dependent.

Gluon TMDs

$$\mathcal{F}_{qg}^{(1)} \propto \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(2)} \propto \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(1)} \propto \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(2)} \propto \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\right]\mathrm{Tr}\left[\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(3)} \propto \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(4)} \propto \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[-]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(5)} \propto \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(6)} \propto \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

Gluon TMDs

$$\mathcal{F}_{qg}^{(1)} \propto \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

dipole

$$\mathcal{F}_{qg}^{(2)} \propto \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(1)} \propto \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

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$$\mathcal{F}_{gg}^{(3)} \propto \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

Weizsacker-Williams

$$\mathcal{F}_{gg}^{(4)} \propto \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[-]}\right]\right\rangle$$

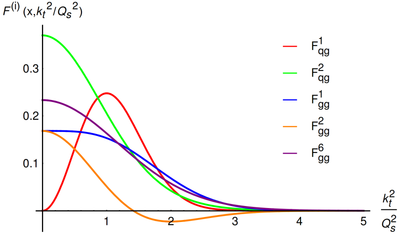
$$\mathcal{F}_{gg}^{(5)} \propto \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(6)} \propto \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

Comparison of TMDs

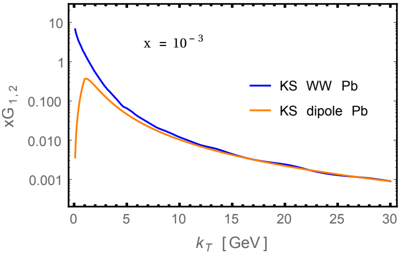
Dijet at LHC

[van Hameren, Kotko, Kutak, Marquet, Petreska, SS '17]



Dijet at EIC

[Kotko, Kutak, SS, Stasto, Strikman '17]



Improved TMD Factorization

- Cross section for dijet production in hadron-hadron collisions cannot be written down with a single gluon TMD [Bomhof, Mulders, Pijlman '06]
- Different processes receive contributions from different TMDs

[Kotko, Kutak, Marquet, Petreska, SS, van Hameren '15] $\boxed{p_t \gg Q_s,\ \text{any } k_T}$

$$d\sigma^{AB\to X} = \sum_{a,X} x_A f_{a/A}(x_A,\mu^2) \sum_i H^{(i)}_{ag^*\to X} \mathcal{F}^{(i)}_{ag\to X}(x_B,k_T)$$

$H^{(i)}_{ag\to cd}$	–	hard factor of i-th type with off-shell gluon
$\mathcal{F}^{(i)}_{ag\to cd}(x_B,k_T)$	–	unintegrated gluon distribution of i-th type in B

$\to$ Later obtained from CGC [Altinoluk, Boussarie, Kotko '19]

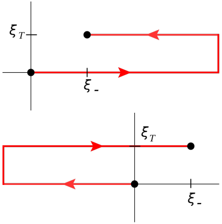
- Dijets at LHC: $\mathcal{F}^{(1)}_{qg}$, $\mathcal{F}^{(2)}_{qg}$, $\mathcal{F}^{(1)}_{gg}$, $\mathcal{F}^{(2)}_{gg}$, $\mathcal{F}^{(3)}_{gg}$, $\mathcal{F}^{(4)}_{gg}$, $\mathcal{F}^{(5)}_{gg}$, $\mathcal{F}^{(6)}_{gg}$
- Dijets at EIC: $\mathcal{F}^{(3)}_{gg}$

Limiting case of ITMD: High Energy Factorization

In the limit $Q_s \ll k_T \lesssim p_T$, $\perp$ part of FT becomes quickly varied

$$\mathcal{F}_g \propto \int d\xi^+ d^2\boldsymbol{\xi}_T e^{ix_2 p_A^- \xi^+ - ik_T\cdot\boldsymbol{\xi}_T} \left\langle \cdots U_{[\xi,0]} \cdots \right\rangle$$

and ξ_T dependence of the gauge links can be neglected.



High Energy Factorization *[Catani, Ciafaloni, Hautmann '91]* $\boxed{Q_s \ll k_T \lesssim p_T}$

$$\frac{d\sigma^{AB\to\text{dijets}+X}}{dy_1 dy_2 d^2p_{1t} d^2p_{2t}} \propto \sum_{a,c,d} x_A f_{a/A}(x_A,\mu^2)\, |\overline{\mathcal{M}_{ag^*\to cd}}|^2 \mathcal{F}^{(1)}_{qg/B}(x_B,k_T)$$

$x_A f_{a/A}(x_A,\mu^2)$	–	collinear PDF in A, suitable for $x_A \sim 1$
$\lvert\overline{\mathcal{M}_{ag^*\to cd}}\rvert^2$	–	matrix element with off-shell incoming gluon
$\mathcal{F}^{(1)}_{qg/B}(x_B,k_T)$	–	unintegrated gluon PDF in B, suitable for $x_B \ll 1$

Balitsky-Fadin-Kuraev-Lipatov equation

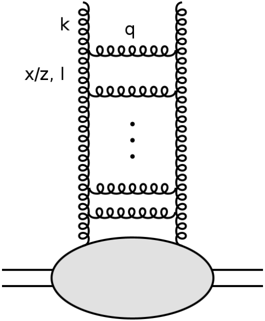
- The small-x resummation is governed by BFKL equation

$$\mathcal{F}_{qg}^{(1)}(x,k^2) \;=\; \mathcal{F}_{qg}^{(1,0)}(x,k^2)$$

$$+\;\frac{\alpha_s(k^2)N_c}{\pi}\int_x^1\frac{dz}{z}\int_{k_0^2}^{\infty}\frac{dl^2}{l^2}\left\{\frac{l^2\mathcal{F}_{qg}^{(1)}(\frac{x}{z},l^2)\;-\;k^2\mathcal{F}_{qg}^{(1)}(\frac{x}{z},k^2)}{|l^2-k^2|}+\frac{k^2\mathcal{F}_{qg}^{(1)}(\frac{x}{z},k^2)}{|4l^4+k^4|^{\frac{1}{2}}}\right\}$$

- LO BFKL predicts fast rise of gluon densities with $x \to 0$
- NLO corrections to BFKL large
- Soon, density becomes high and non-linear effects cannot be ignored: Balitsky-Kovchegov equation (BK)
- This leads to appearance of the new saturation scale Q_s
- Kinematic constraint [Kwieciński, Martin, Sutton '96]

$$l_T^2 < \frac{k_T^2}{z}$$



BK equation + kinematic constraint + DGLAP corrections

[Kwieciński, Martin, Staśto '97; Kwieciński, Kutak '03]

$$
\begin{aligned}
\phi_p(x,k^2) &= \phi_p^{(0)}(x,k^2) \\
&+ \frac{\alpha_s(k^2)N_c}{\pi}\int_x^1 \frac{dz}{z}\int_{k_0^2}^{\infty}\frac{dl^2}{l^2}\left\{\frac{l^2\phi_p(\frac{x}{z},l^2)\,\theta(\frac{k^2}{z}-l^2)\,-\,k^2\phi_p(\frac{x}{z},k^2)}{|l^2-k^2|}+\frac{k^2\phi_p(\frac{x}{z},k^2)}{|4l^4+k^4|^{\frac{1}{2}}}\right\} \\
&+\frac{\alpha_s(k^2)}{2\pi k^2}\int_x^1 dz\left(P_{gg}(z)-\frac{2N_c}{z}\right)\int_{k_0^2}^{k^2} dl^2\,\phi_p\left(\frac{x}{z},l^2\right)+\frac{\alpha_s(k^2)}{2\pi}\int_x^1 dz\,P_{gq}(z)\Sigma\left(\frac{x}{z},k^2\right) \\
&-\frac{2\alpha_s^2(k^2)}{R^2}\left[\left(\int_{k^2}^{\infty}\frac{dl^2}{l^2}\phi_p(x,l^2)\right)^2+\phi_p(x,k^2)\int_{k^2}^{\infty}\frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\phi_p(x,l^2)\right]
\end{aligned}
$$

- For nucleus: $R_A = R\,A^{1/3}$

KS gluon TMD

As a basis for all the following calculations we use the nonlinear KS gluon TMD [Kutak, Sapeta '12], which:

- for $k_\perp^2 > 1$, comes from evolution of the distribution

  $$\mathcal{F}^{(0)}_{g^*/B}(x, k_\perp^2) = \frac{\alpha_S(k_\perp^2)}{2\pi k_\perp^2} \int_x^1 dz P_{gg}(z) \frac{x}{z} g\left(\frac{x}{z}\right) ,$$

  with the initial condition

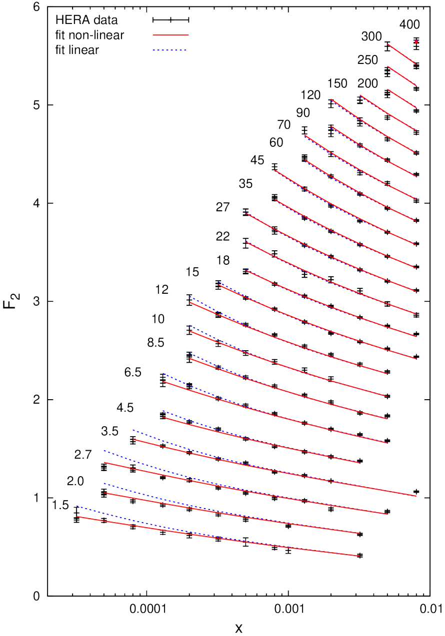
  $$xg(x) = N(1-x)^\beta (1-Dx) ,$$

  with the BK equation with kinematic constraint, non-singular DGLAP pieces and running coupling [Kwieciński, Martin, Stasto '97; Kutak, Kwieciński '03] – fitted to combined F_2 HERA data
- for $k_\perp^2 < 1$, is taken as

  $$\mathcal{F}_{g^*/B}(x, k_\perp^2) = k_\perp^2 \mathcal{F}_{g^*/B}(x, 1),$$

  which is motivated by the shape obtained from the solution of the LO BK equation in the saturation regime [Sergey '08].

KS fits F_2 well



Sudakov resummation

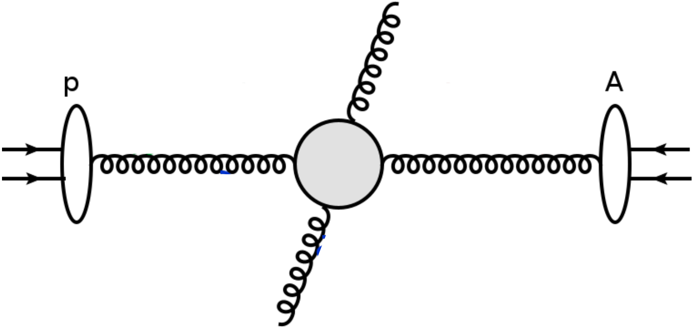
In forward jet production, besides the logarithms $\ln x$, which are resummed in TMDs, there is another class of large logarithms, namely

$$\ln \mu \qquad \text{where} \qquad \mu \sim p_{t,\mathrm{jet}} \,.$$

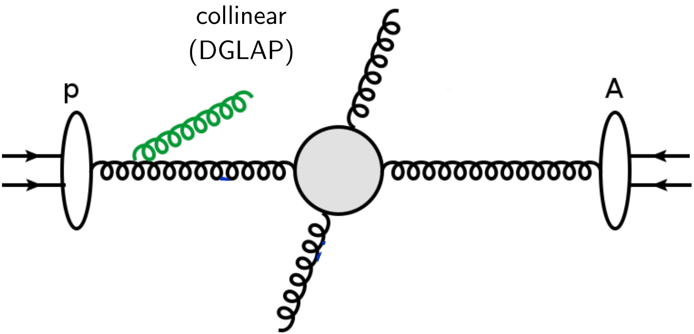
Appearance of those logarithms opens phase space for soft and collinear emissions, which should also be resummed.

This resummation can be accounted for by inclusion of the Sudakov factor.

Kinematics of dijet production



Kinematics of dijet production



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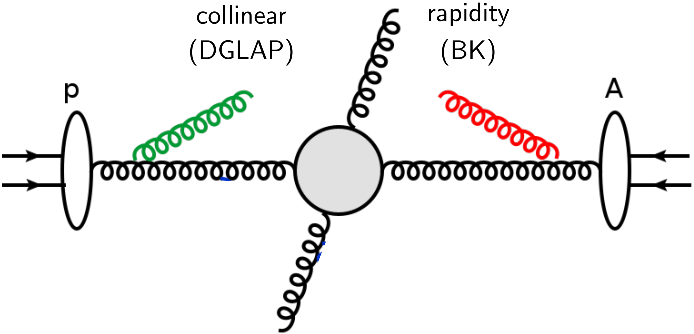
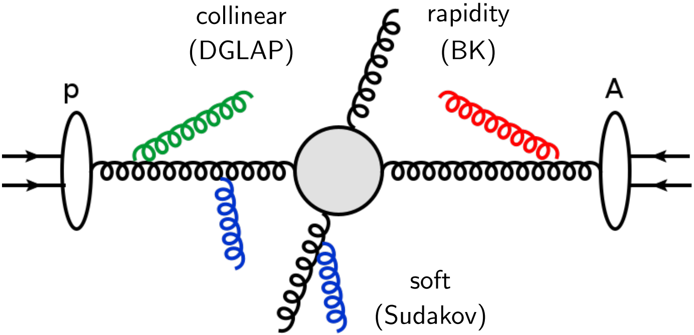


Kinematics of dijet production



Earlier modeling of Sudakov effects

- Model 1: The survival probability model [van Hameren, Kotko, Kutak, SS '14], where the Sudakov factor of the form [Watt, Martin, Ryskin '03]

$$T_s(\mu^2, k_T^2) = \exp\left(-\int_{k_T^2}^{\mu^2} \frac{dk_T'^2}{k_t'^2} \frac{\alpha_s(k_T'^2)}{2\pi} \sum_{a'} \int_0^{1-\Delta} dz' P_{a'a}(z') \right) ,$$

where $\Delta = k_T/(k_T + \mu)$, is imposed at the level of the cross section

$$\sigma \;\sim \sum_{i \in \text{events}} \sigma_i \, T_s(\mu_i^2, k_{Ti}^2) \Theta(\mu_i - k_{Ti}) + \mathcal{W} \sum_{i \in \text{events}} \sigma_i \, \Theta(k_{Ti} - \mu_i) \, ,$$

and $\mathcal{W}$ is constructed such that the total cross section is preserved.

This procedure corresponds to performing DGLAP-type evolution from the scale k_T to μ, decoupled from the small-x evolution.

Earlier modeling of Sudakov effects

- Model 2: The model with a hard scale [Kutak '14]. The Sudakov factor of the same form as above is imposed on top of the gluon distribution in such a way that, after integration of the resulting hard scale dependent gluon TMD, one obtains the same result as by integrating the original gluon distribution

$$\mathcal{F}(x, k_T^2, \mu^2) = \theta(\mu^2 - k_T^2)\, T_s(\mu^2, k_T^2)\, \frac{xg(x,\mu^2)}{xg_{hs}(x,\mu^2)}\, \mathcal{F}(x, k_T^2) \\ + \theta(k_T^2 - \mu^2)\mathcal{F}(x, k_T^2)\,,$$

where

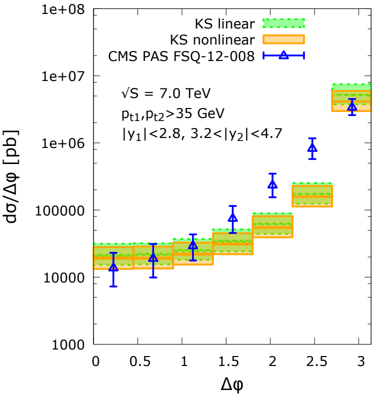
$$xg_{hs}(x,\mu^2) = \int^{\mu^2} dk_T^2\, T_s(\mu^2, k_T^2)\mathcal{F}(x, k_T^2)\,,$$

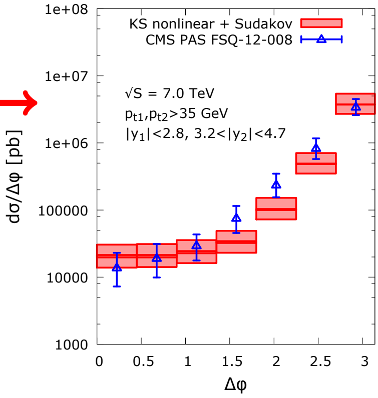
$$xg(x,\mu^2) = \int^{\mu^2} dk_T^2 \mathcal{F}(x, k_T^2)\,.$$

Earlier modeling of Sudakov effects

Those simplistic models significantly improved description of data for azimuthal decorrelations in central-forward dijet production

[van Hameren, Kotko, Kutak, SS '14]

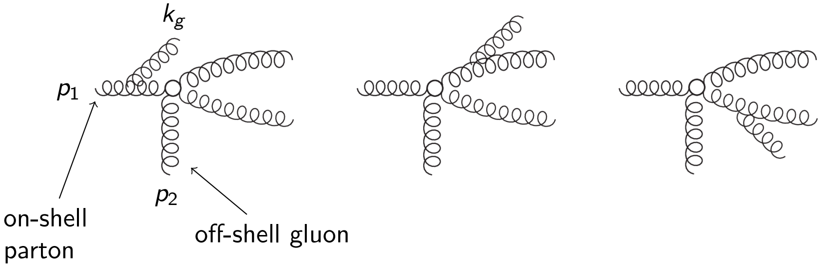




Sudakov resummation in small-x formalism

[Mueller, Xiao, Yuan, PRL 110 '13; Mueller, Xiao, Yuan,PRD 88 '13]

Calculate NLO correction to the $2 \to 2$ process in the soft limit. For example, for the $gg \to gg$ we have



- physical polarization of the radiated gluon along p_2, $\epsilon(k_g) \cdot p_2 = 0$: no need to consider radiation from the off-shell gluon

Sudakov resummation in small-x formalism

- Use the NLO cross section to resum Sudakov radiation following Collins-Soper-Sterman approach

- Write down evolution equation for the hard scale Q^2. By solving it, one obtains the resummed cross section

$$d\sigma^{AB\to X} \propto \sum_{a,X} x_A f_{a/A}(x_A, \mu^2) \mathcal{M}_{ag^*\to X} e^{-S_{\rm Sud}^{ag\to X}(\mu, b_\perp)} \tilde{\mathcal{F}}_{g^*/B}(x_B, b_\perp)$$

  where the Sudakov has the general structure

$$S_{\rm Sud} = \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[A \ln \frac{Q^2}{\mu^2} + B \right]$$

  and A, B have perturbative expansion.

- The result is valid in the limit $k_T \ll Q^2$

Dijets at the LHC

Sudakov: dijets in pA

Sudakov effects are most conveniently included in position space [Stasto, Wei, Xiao, Yuan '18]

$$\mathcal{F}_{g^*/B}^{ag\to cd}(x,q_\perp,\mu) = \frac{-N_c S_\perp}{2\pi\alpha_s}\int \frac{b_\perp db_\perp}{2\pi} J_0(q_\perp b_\perp)\, e^{-S_{\rm Sud}^{ag\to cd}(\mu,b_\perp)}\, \nabla^2_{b_\perp} S(x,b_\perp)\,,$$

where

$S_\perp$	–	transverse area of the target,
$S(x,b_\perp)$	–	dipole scattering amplitude.

We can however express the gluon with Sudakov by the gluon without Sudakov, all in momentum space

$$\mathcal{F}_{g^*/B}^{ab\to cd}(x,k_\perp,\mu) = \int db_\perp \int dk'_\perp\, b_\perp\, k'_\perp\, J_0(b_\perp\, k'_\perp)\, J_0(b_\perp\, k_\perp)$$
$$\times\, \mathcal{F}_{g^*/B}(x,k'_\perp)\, e^{-S_{\rm Sud}^{ab\to cd}(\mu,b_\perp)}$$

For each channel, the Sudakov receives perturbative and non-perturbative contributions

$$S_{\rm Sud}^{ab\to cd}(b_\perp) = \sum_{i=a,b,c,d} S_p^i(b_\perp) + \sum_{i=a,c,d} S_{np}^i(b_\perp)\,.$$

Sudakov: dijets in pA

Perturbative part [Mueller, Xiao, Yuan '13; Stasto, Wei, Xiao, Yuan '18]

$$S_p^{qg\to qg}(Q,b_\perp) = \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left[2(C_F+C_A)\frac{\alpha_s}{2\pi}\ln\left(\frac{Q^2}{\mu^2}\right) - \left(\frac{3}{2}C_F + C_A\beta_0\right)\frac{\alpha_s}{\pi}\right],$$

$$S_p^{gg\to gg}(Q,b_\perp) = \int_{\mu_b^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left[4C_A\frac{\alpha_s}{2\pi}\ln\left(\frac{Q^2}{\mu^2}\right) - 3C_A\beta_0\frac{\alpha_s}{\pi}\right],$$

where $\mu_b = 2e^{-\gamma_E}/b_*$, and $b_* = b_\perp/\sqrt{1+b_\perp^2/b_{\max}^2}$, $b_{\max} = 0.5\,\mathrm{GeV}^{-1}$.

Sudakov: dijets in pA

Non-perturbative part [Sun, Isaacson, Yuan, Yuan '14; Prokudin, Sun, Yuan '15]

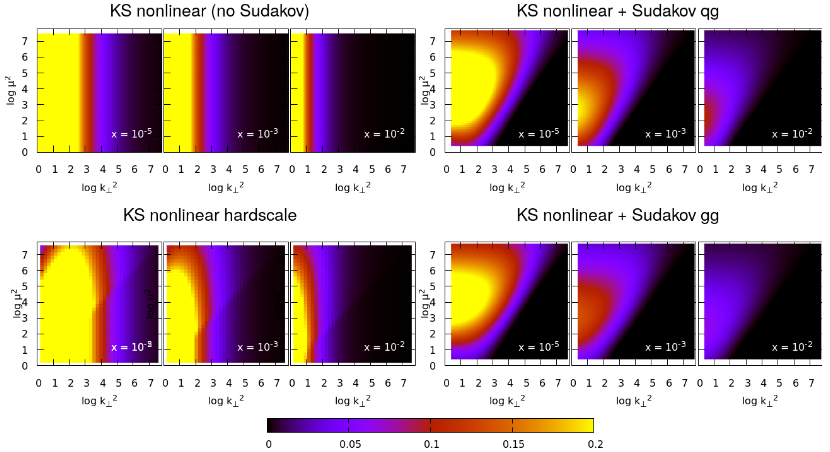
$$S_{np}^{qg\to qg}(Q,b_\perp) = \left(2+\frac{C_A}{C_F}\right)\frac{g_1}{2}b_\perp^2 + \left(2+\frac{C_A}{C_F}\right)\frac{g_2}{2}\ln\frac{Q}{Q_0}\ln\frac{b_\perp}{b_*}\,,$$

$$S_{np}^{gg\to gg}(Q,b_\perp) = \frac{3C_A}{C_F}\frac{g_1}{2}b_\perp^2 + \frac{3C_A}{C_F}\frac{g_2}{2}\ln\frac{Q}{Q_0}\ln\frac{b_\perp}{b_*}\,,$$

with $g_1 = 0.212$, $g_2 = 0.84$ and $Q_0^2 = 2.4\mathrm{GeV}^2$.

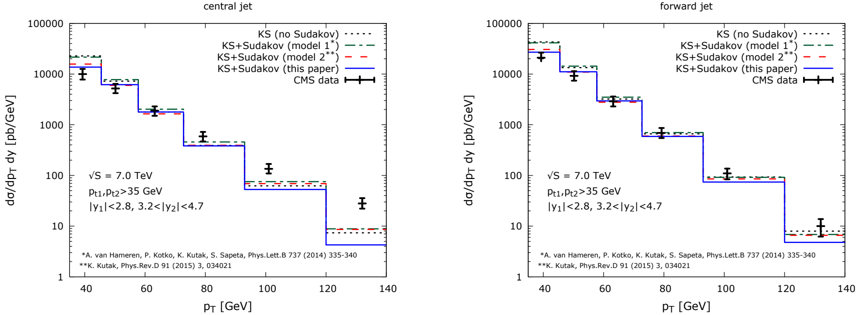
- Since the off-shell gluon comes from fits of small-x data, it already contains non-perturbative information about the target. Hence, we do not include non-perturbative Sudakov for this TMD gluon.

KS (dipole) gluon distributions with and without Sudakov



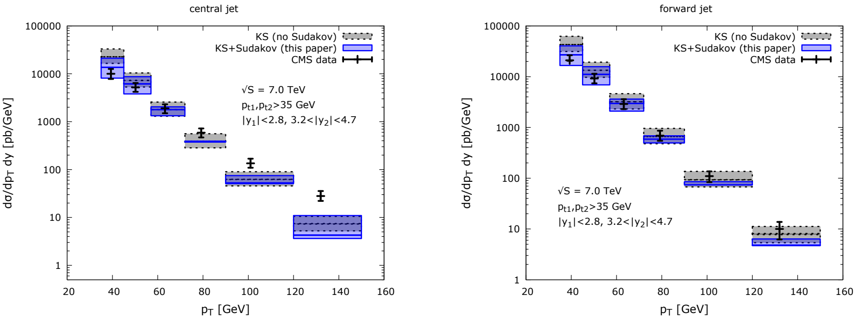
- The new and the old gluon with Sudakov qualitatively similar
- The *qg* gluon broader than *gg* – comes from $C_A > C_F$

p_T spectra: predictions with and without Sudakov



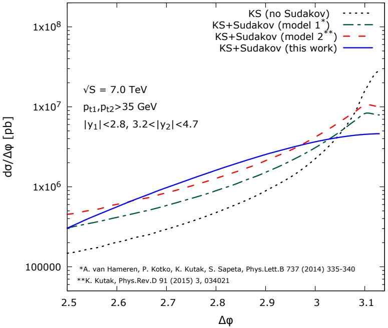
- Calculations performed with LxJet [Kotko] and KaTie [van Hameren]; $\mu_F = \mu_R = \frac{1}{2}(p_{t1} + p_{t2})$, CTEQ18 NLO collinear PDFs
- Predictions with Sudakov tend to describe data better at small p_T
- Overall picture: Sudakov effects not very strong for this observable

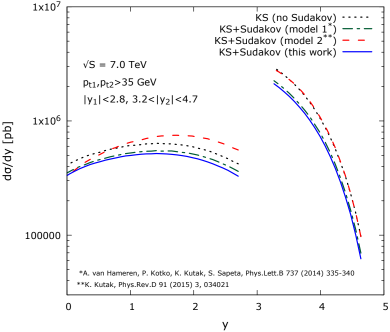
p_T spectra: no Sudakov vs proper QCD Sudakov



- Uncertainty estimated by the usual scale variation by factor $2^{\pm 1}$
- Good agreement with CMS data, except the tail, which is sensitive to large x, not included in our TMDs

Distributions in azimuthal distance and rapidity





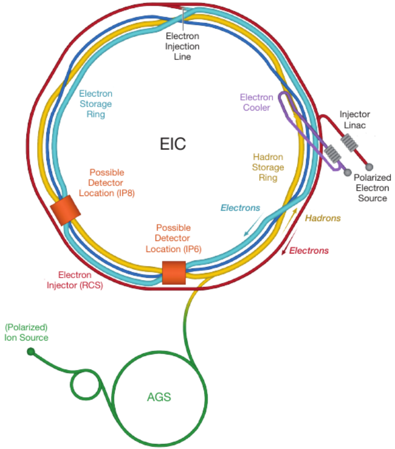
- Qualitatively similar behaviour for predictions with the proper QCD Sudakov and earlier naive models: the region of small $\Delta\phi$ populated at the expense of large $\Delta\phi$ region
- In other words: suppression of the back-to-back peak and broadening of the cross section
- Convex decorrelations from earlier models vs concave from this work
- Marked differences between rapidity distributions from various versions of KS gluon

Dijets at EIC

Electron Ion Collider (EIC)

Will use the existing RHIC complex at Brookhaven National Laboratory

- Variable center of mass energies: 20-140 GeV
- Ion beams: from d to Au, Pb, U
- High luminosity: $10^{33} - 10^{34} cm^{-2} s^{-1}$
- Highly polarized (70%) electron and nucleon beams

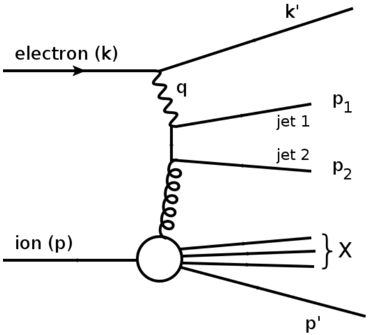


Dijets in DIS

$$x_B = \frac{-q^2}{2p \cdot q} \quad \text{Bjorekn x}$$

$$y = \frac{p \cdot q}{p \cdot k} \quad \text{inelasticity}$$

$$\Delta\phi \quad \measuredangle(p_{1T}, p_{2T})$$



Dijets at EIC: framework

The cross section reads

$$d\sigma_{\gamma^* A\to 2j+X} \propto \int \frac{dx}{x} d^2 k_T \, \mathcal{F}_{gg}^{(3)}(x, k_T, \mu) \, \mathcal{M}_{\gamma^* g^* \to 2j}$$

- Implemented in KaTie Monte Carlo [van Hameren] (`https://bitbucket.org/hameren/katie`)
- $\mathcal{F}_{gg}^{(3)}$ can be obtained from $\mathcal{F}_{qg}^{(1)}$ in the so-called Gaussian approximation

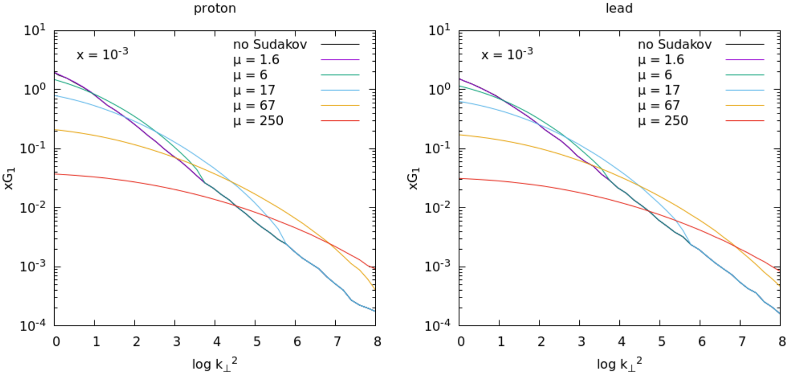
Sudakov resummation included via use of

$$\mathcal{F}_{gg}^{(3)}(x, k_T, \mu) = \int db_T \, dk_T' \, b_T \, k_T' \, J_0(b_T \, k_T') \, J_0(b_T \, k_T) \, \mathcal{F}_{gg}^{(3)}(x, k_T') \, e^{-S_{\text{Sud}}^{g\to q\bar{q}}(\mu, b_T)}$$

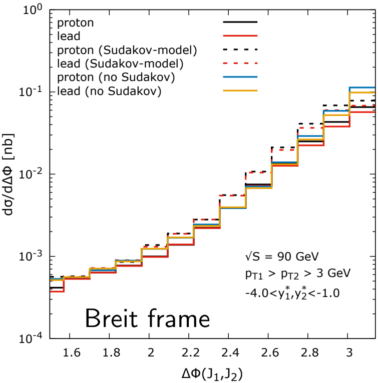
with the form factor [Mueller, Xiao, Yuan '13]

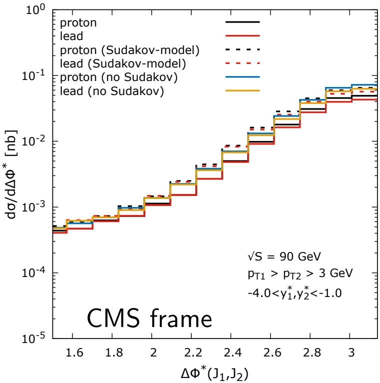
$$S_{\text{Sud}}^{g\to q\bar{q}}(\mu, b_T) = \frac{\alpha_s N_c}{4\pi} \ln^2 \frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}$$

KS WW gluon distributions with and without Sudakov



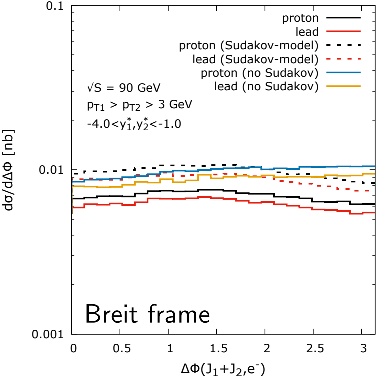
Azimuthal angle between jets

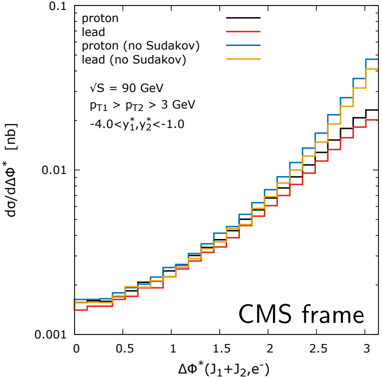




- Non-negligible Sudakov effects
- Similar behaviour of this result and earlier model-Sudakovs near π
- Small saturation effects

Azimuthal angle between jet plane and the electron





- Significant Sudakov effects
- Saturation effects up to 15%

Summary

- We obtained TMD distributions which combine small-x resummation and Sudakov resummation, where the latter comes from proper QCD calculations
- We used the above TMDs to calculate p_T, $\Delta\phi$ and y distributions in central-forward dijet production and forward dijets at EIC

Central-forward dijets at the LHC

- Both the TMDs and the differential distributions are consistent with our earlier calculations based on simple modeling of Sudakov factors
- We achieved good description of CMS data for p_T distributions
- Sudakov resummation has a moderate effect on p_T spectra but sizable effect on the shapes of $\Delta\phi$

Summary

Forward dijets at EIC

- Sudakov resummation has an important effect on the WW TMD and differential cross sections
- Saturation effects moderate for $\Delta\phi(j1, j2)$ and $\Delta\phi(j1 + j2, e^-)$ distribution – up to 15%
- $d\sigma/d\Delta\phi(j1 + j2, e^-)$ well suited to test the framework