MIXED EW-QCD TWO-LOOP AMPLITUDES FOR DRELL-YAN LEPTON PAIR PRODUCTION

Andreas von Manteuffel

Michigan State University

In collaboration with Matthias Heller, Robert M. Schabinger, Hubert Spiesberger
[1907.00491 + 2012.05918]

25 February 2021
Theory Seminar, DESY Zeuthen & Humboldt Universität Berlin

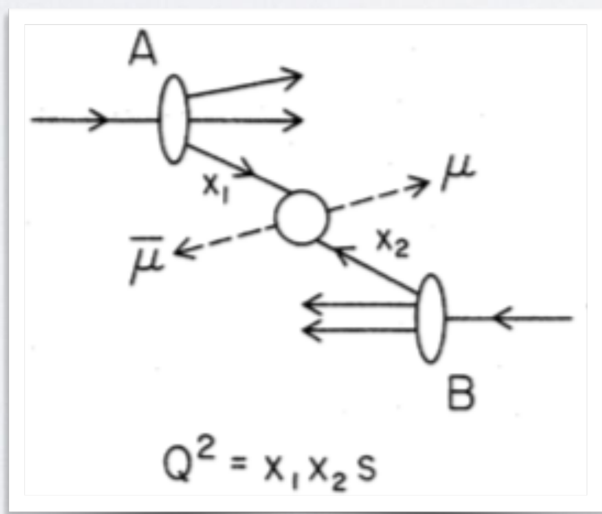
THE DRELL YAN PROCESS

Phys. Rev. Lett. 25 (1970) 316

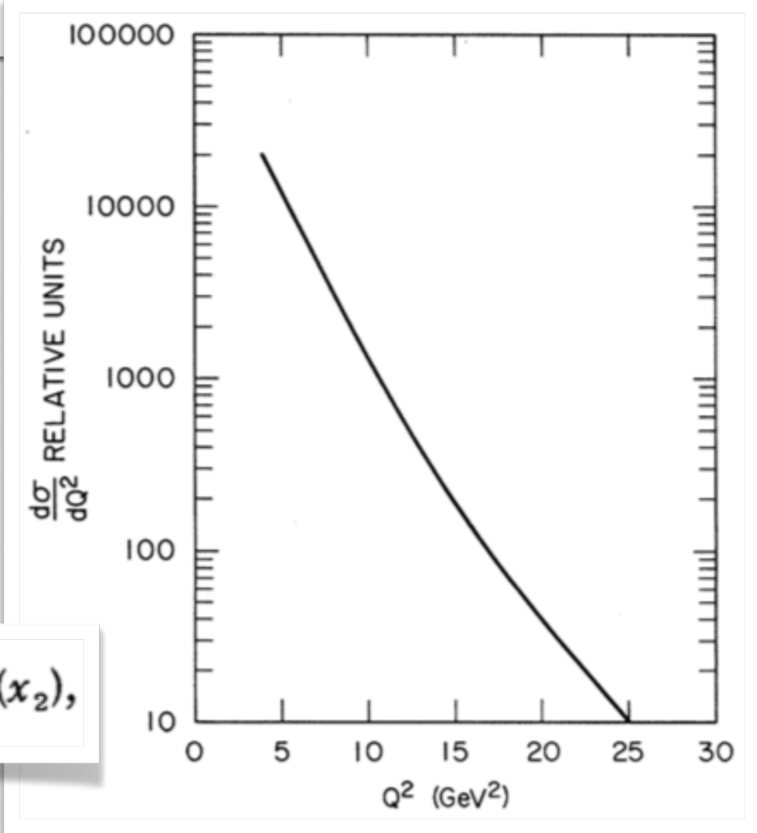
MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron-hadron inelastic collisions in the limiting region, $s \to \infty$, Q^2/s finite, Q^2 and s being the squared invariant masses of the lepton pair and the two initial hadrons, respectively. General scaling properties and connections with deep inelastic electron scattering are discussed. In particular, a rapidly decreasing cross section as $Q^2/s \to 1$ is predicted as a consequence of the observed rapid falloff of the inelastic scattering structure function νW_2 near threshold.



$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha^2}{3Q^2}\right)\left(\frac{1}{Q^2}\right)\mathfrak{F}(\tau) = \left(\frac{4\pi\alpha^2}{3Q^2}\right)\left(\frac{1}{Q^2}\right)\int_0^1 dx_1 \int_0^1 dx_2 \delta(x_1 x_2 - \tau) \sum_a \lambda_a^{-2} F_{2a}(x_1) F_{2\bar{a}}'(x_2),$$



DRELL-YAN PROCESS @ LHC

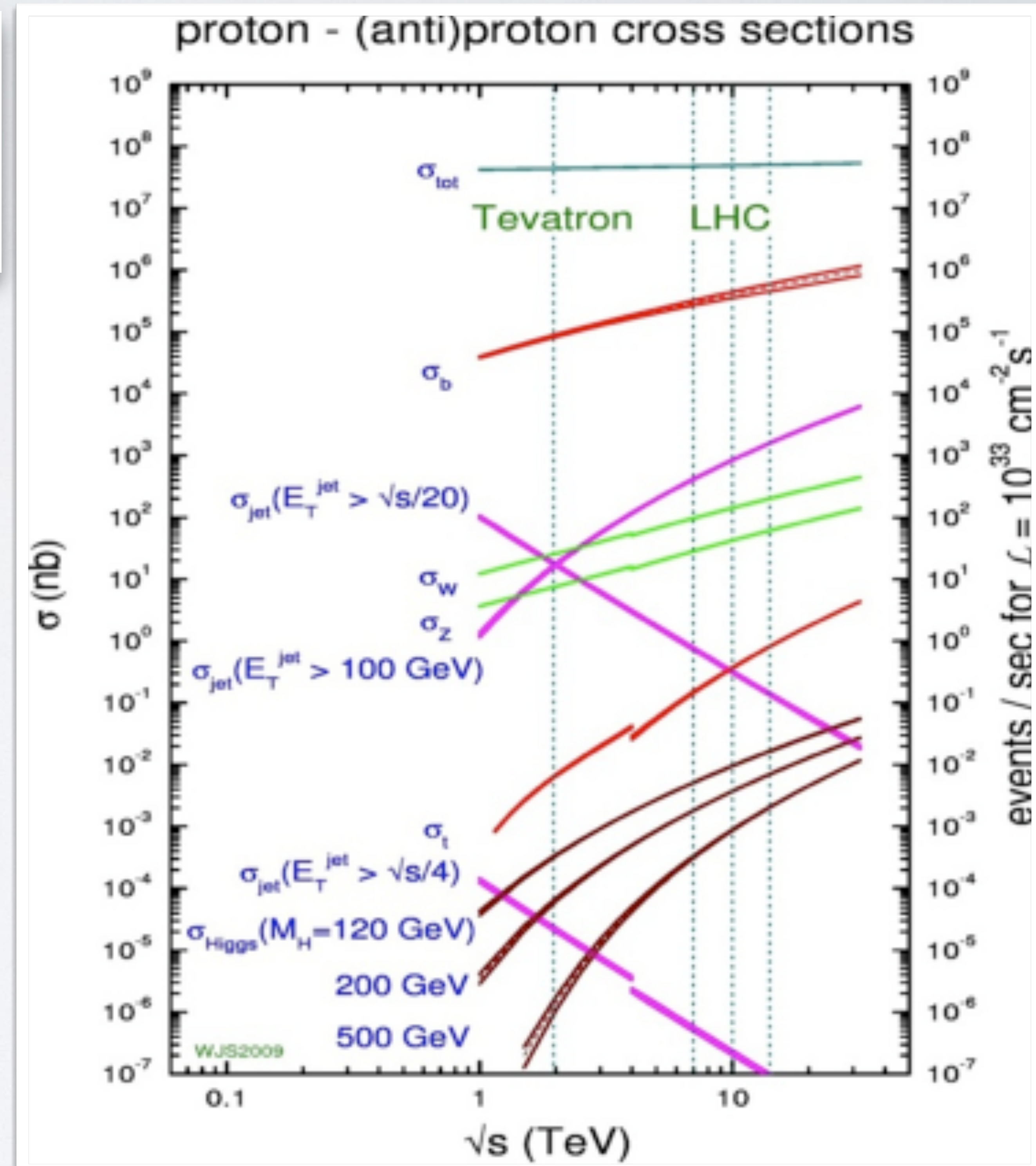
$$pp \to l^- l^+ + X, \quad l = e, \mu \quad \text{(NC)}$$

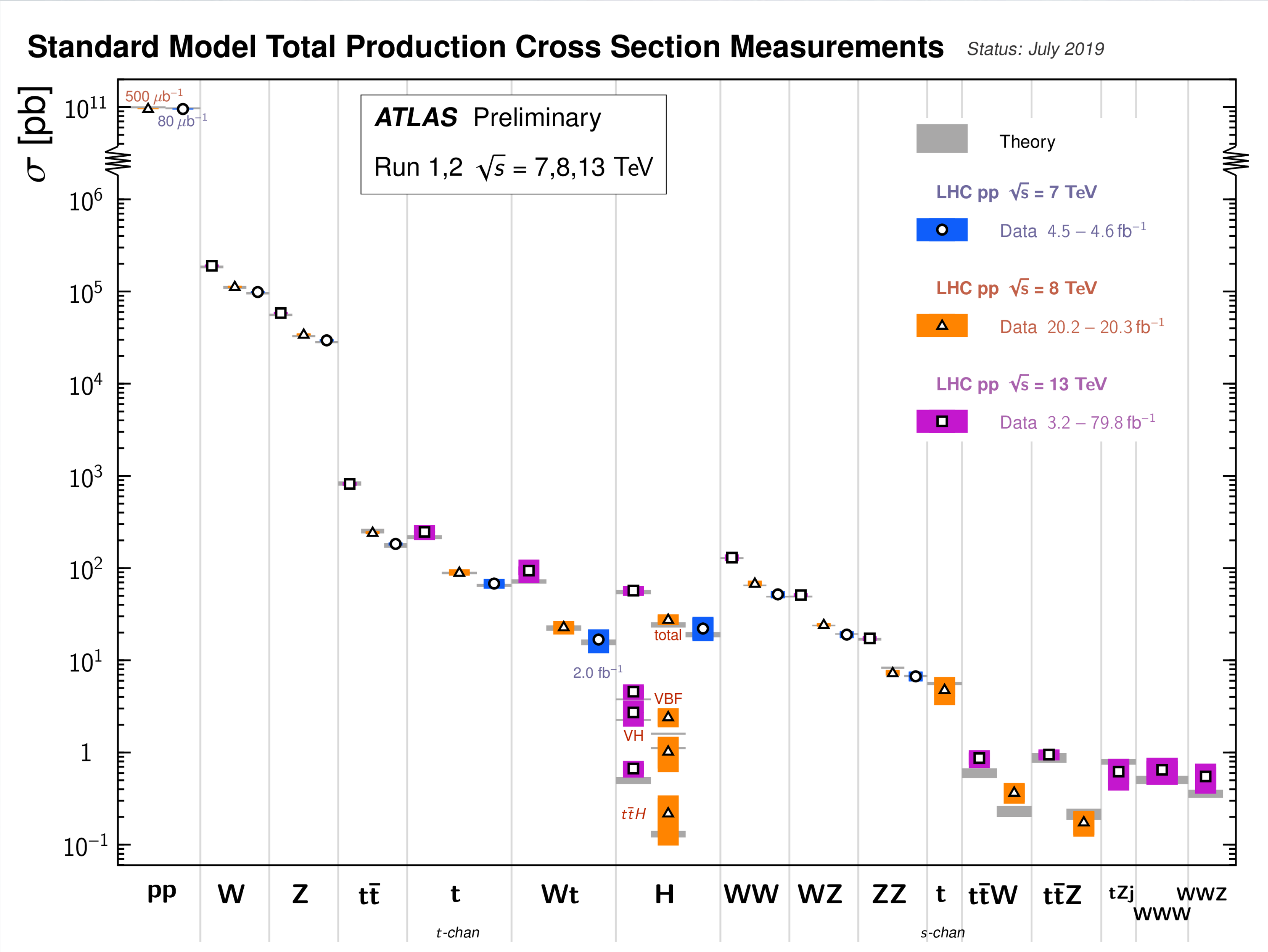
$$pp \to l^{\mp} \overset{(-)}{\nu_l} + X, \quad l = e, \mu \quad \text{(CC)}$$

- ***EW precision measurements:***
  W mass, Z mass, weak mixing
  (resonant W/Z)

- ***New physics searches***
  direct or via SMEFT
  (large dilepton mass)

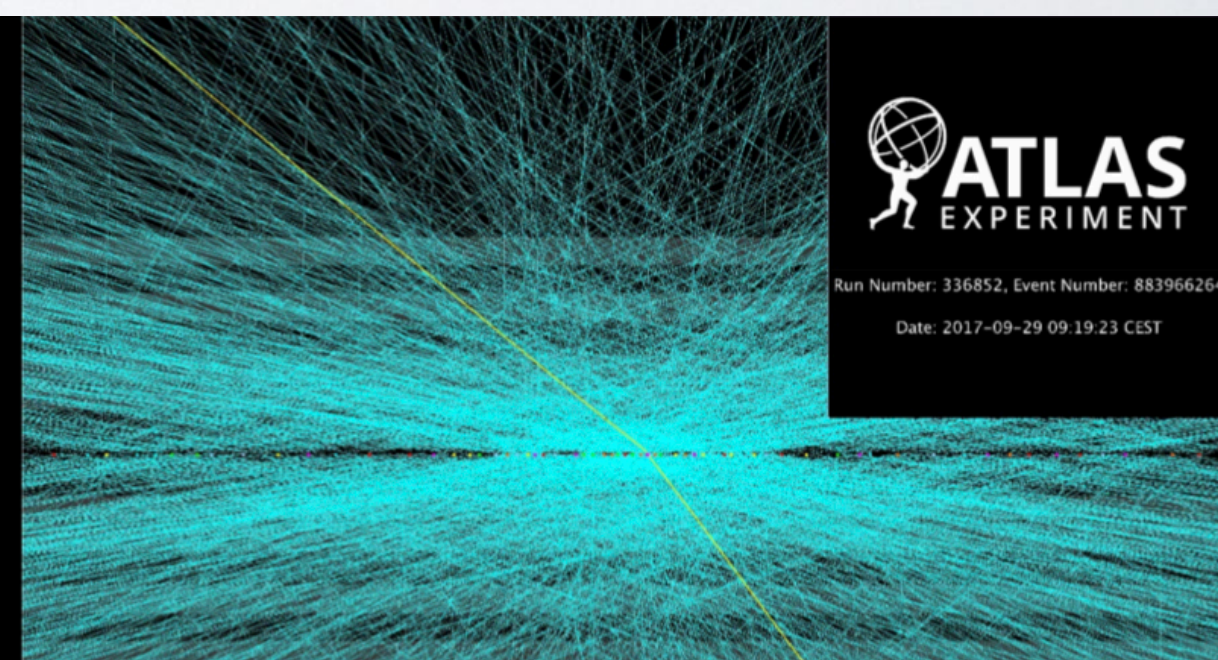
- ***PDF determinations***



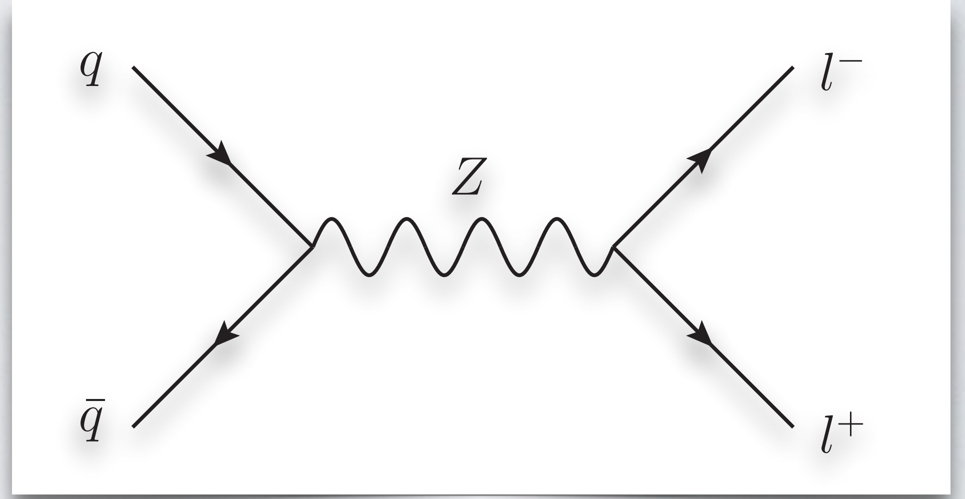


- Measurements with impressive precision
- More data to be analyzed, in particular for high-mass
- "Easy" signature (but see below)

$Z \rightarrow \mu\mu$ candidate event, with 65 **additional** reconstructed primary vertices.



PERTURBATIVE CORRECTIONS



- LO: quark-pair initiated s-channel
- QCD on-shell: NNLO, recently: N3LO
  [Hamberg, van Neerven, Matsuura '91, Harlander, Kilgore '02, Anastasiou, Dixon, Melnikov, Petriello '03, Melnikov, Petriello '06], [Duhr, Dulat, Mistlberger '20,'20]
- QED on-shell NC: NNLO
  [Berends, van Neerven, Burgers '88,
  discrepancies: Blümlein, De Freitas, Raab, Schönwald '19]
- EW: NLO
  [Baur, Brein, Holllik, Schappacher, Wackeroth '01, Dittmaier, Krämer '01, Baur, Wackeroth '04], …
- Mixed EW-QCD resonance region:
  [Dittmaier, Huss, Schwinn '14,'15]
- Mixed QED-QCD:
  [Kilgore, Sturm '11] off-shell; [de Florian, Der, Fabre '18] on-shell

EW-QCD ON-SHELL Z

- Total Z+X cross section *[Bonciani, Buccioni, Rana, Vicini '20]*

	G_μ-scheme	$\alpha(0)$-scheme
A_1	$55787^{+0.26\%}_{-0.99\%}$	$53884^{+0.26\%}_{-0.99\%}$
B_2	$55501^{+0.26\%}_{-0.99\%}$	$55015^{+0.52\%}_{-1.26\%}$
B_3	$55469^{+0.28\%}_{-1.01\%}$	$55340^{+0.37\%}_{-1.13\%}$

$A_1 : LO + \alpha_s + \alpha_s^2$

$B_2 : LO + \alpha_s + \alpha_S^2 + \alpha$

$B_3 : LO + \alpha_s + \alpha_S^2 + \alpha + \alpha_s \alpha$

(all in pb)

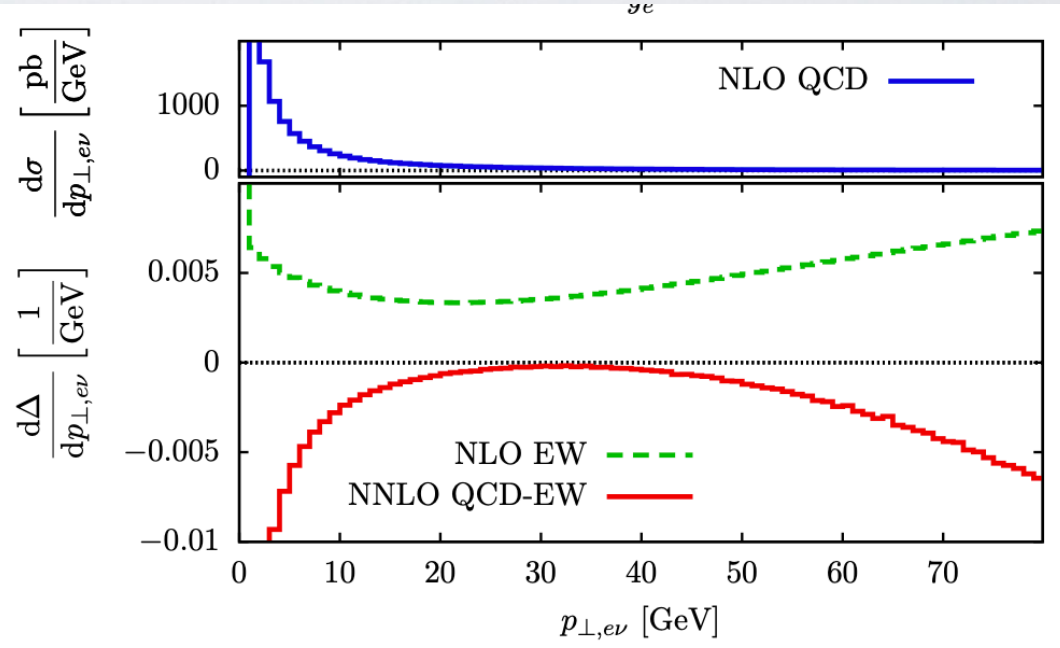
- N_f terms for on-shell Z/W *[Dittmaier, Schmidt, Schwarz '20]*

EW-QCD ON-SHELL W

- Differential W+X cross section (nested subtraction)

  [Behring, Buccioni, Caola, Delta, Jaquier, Melnikov, Röntsch '20]

σ[pb]	channel	$\mu = M_W$	$\mu = M_W/2$	$\mu = M_W/4$
σ_{LO}		6007.6	5195.0	4325.9
$\Delta\sigma_{\mathrm{NLO},\alpha_s}$	all ch.	508.8	1137.0	1782.2
	$q\bar{q}'$	1455.2	1126.7	839.2
	qg/gq	-946.4	10.3	943.0
$\Delta\sigma_{\mathrm{NLO},\alpha}$	all ch.	2.1	-1.0	-2.6
	$q\bar{q}'$	-2.2	-5.2	-6.7
	$q\gamma/\gamma q$	4.2	4.2	4.04
$\Delta\sigma_{\mathrm{NNLO},\alpha_s\alpha}$	all ch.	-2.4	-2.3	-2.8
	$q\bar{q}'/qq'$	-1.0	-1.2	-1.0
	qg/gq	-1.4	-1.2	-2.1
	$q\gamma/\gamma q$	0.06	0.03	-0.04
	$g\gamma/\gamma g$	-0.12	0.04	0.30



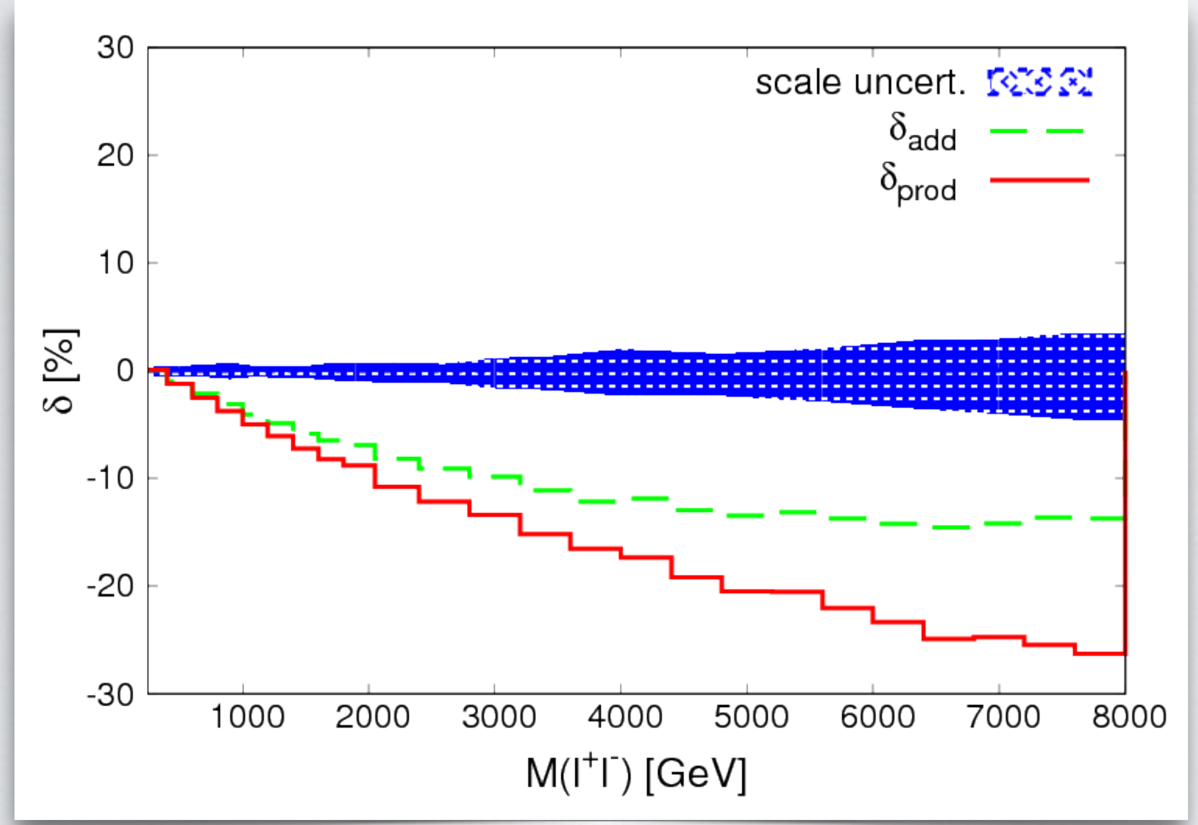
- Note: α and $\alpha\alpha_s$ of similar order

- Note: cancellations between partonic channels (also for N3LO)

- Expect larger contributions at high invariant masses, also from "off-shell" configurations

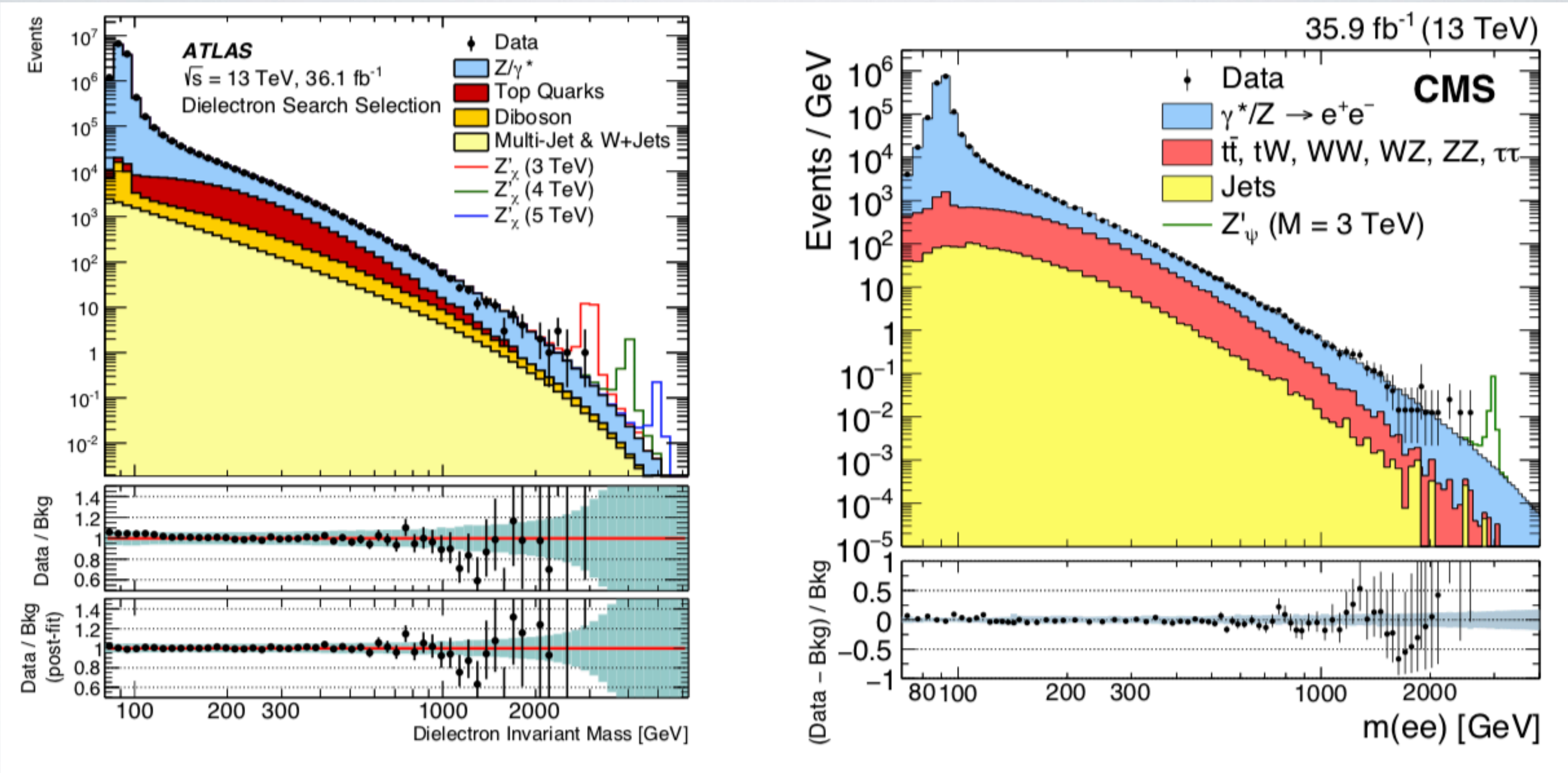
LARGE ENERGIES

- ***Mixed EW-QCD important at higher invariant masses*** uncertainties considered in *[Campbell, Wackeroth, Zhou '16]:*

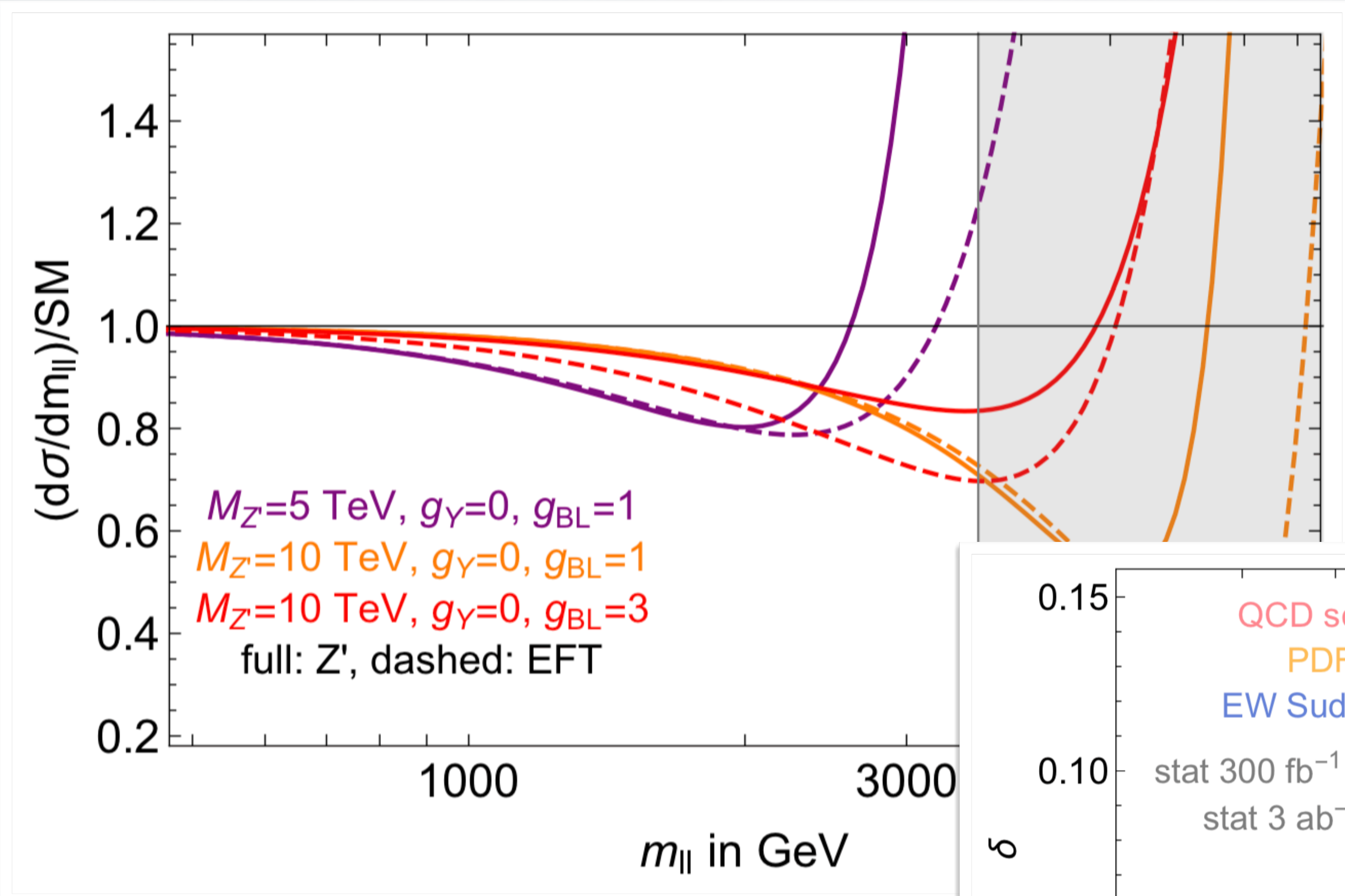


- In that region, no reason to restrict to single W/Z resonance

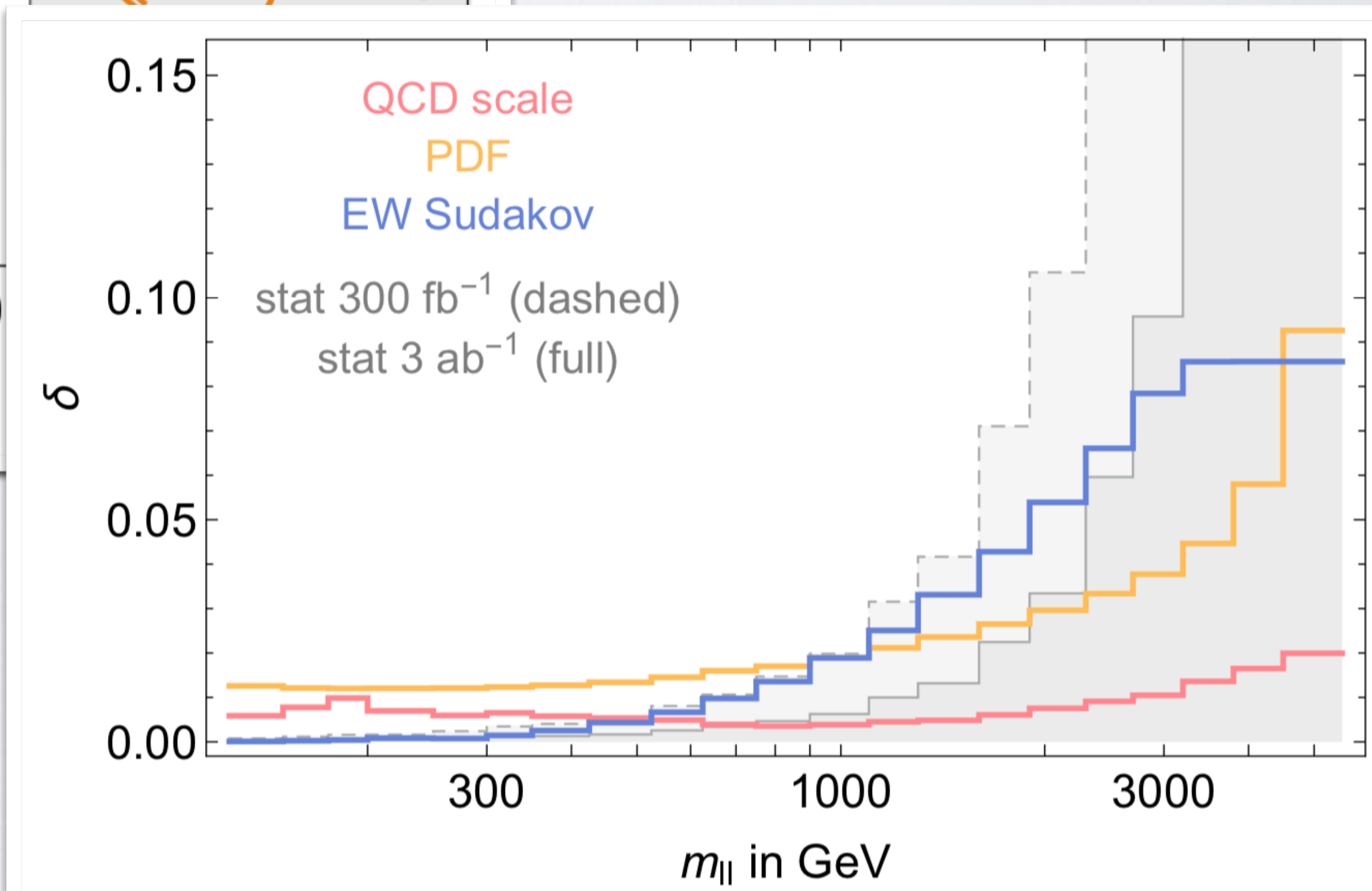
DILEPTON INVARIANT MASS



NEW RESONANCES AND EFTS



- Want sensitivity to > 5 TeV resonances
- Motivates non-bump searches
- Interferences important (diff. Xsec)
- EFT useful



plots from:

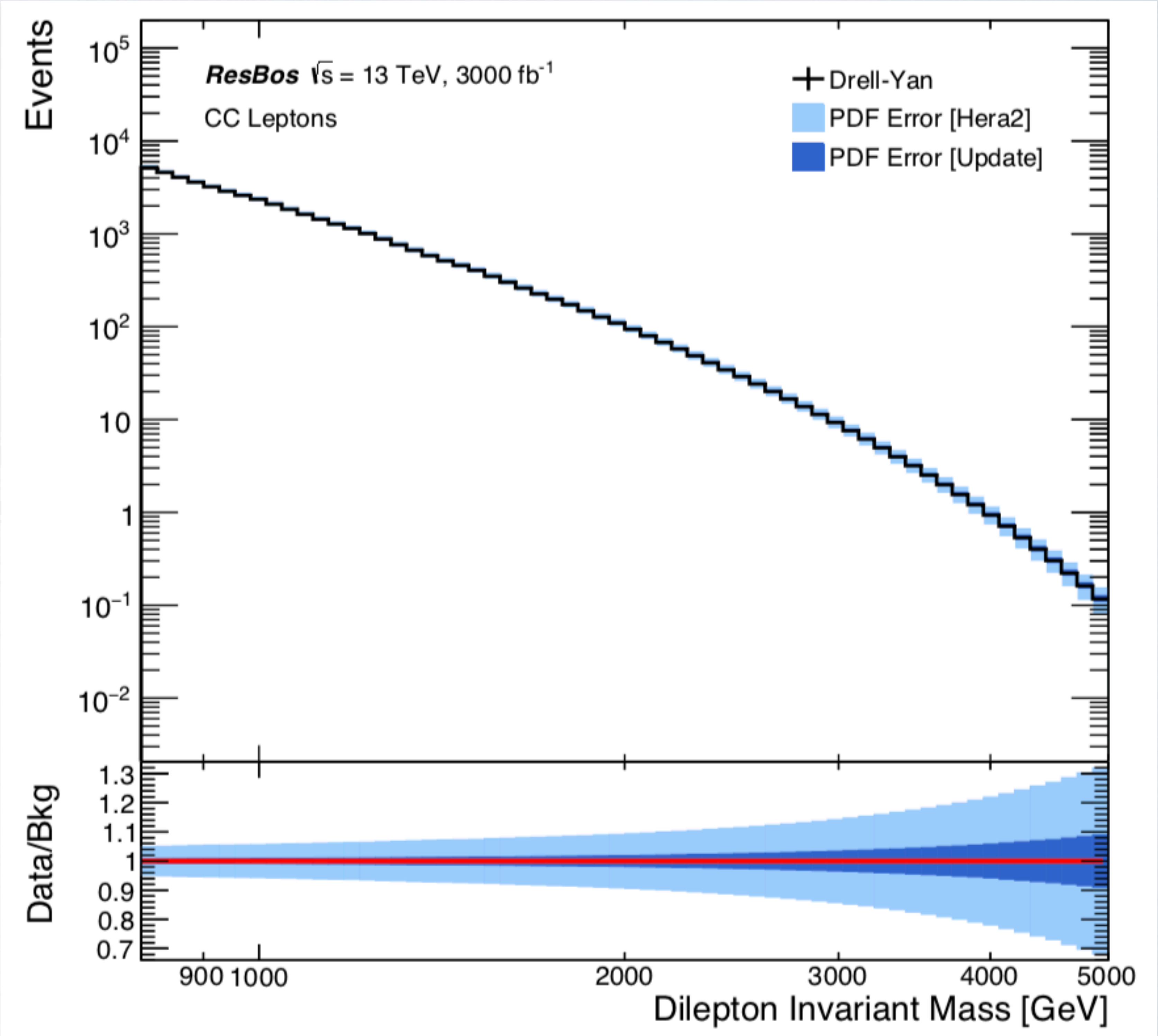
[Alioli, Farina, Pappadopulo, Ruderman '17]

PDF UNCERTAINTIES

(and how to improve them)

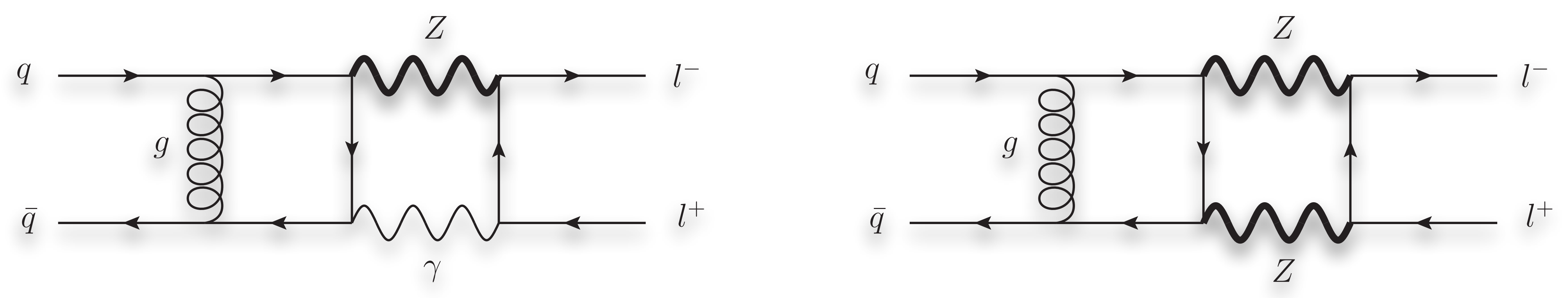
[Willis, Brock, Hayden, Hou, Isaacson, Schmidt, Yuan '19]

- Large PDF uncertainties when extrapolating over large range
- Idea: use control region < 1 TeV to improve multi-TeV signal region
- Make sure radiative corrections under control !



EW-QCD CORRECTIONS TO DILEPTON PRODUCTION

- Don't restrict to singly resonant V production, consider ***dilepton*** final state
- ***Representative Feynman diagrams***



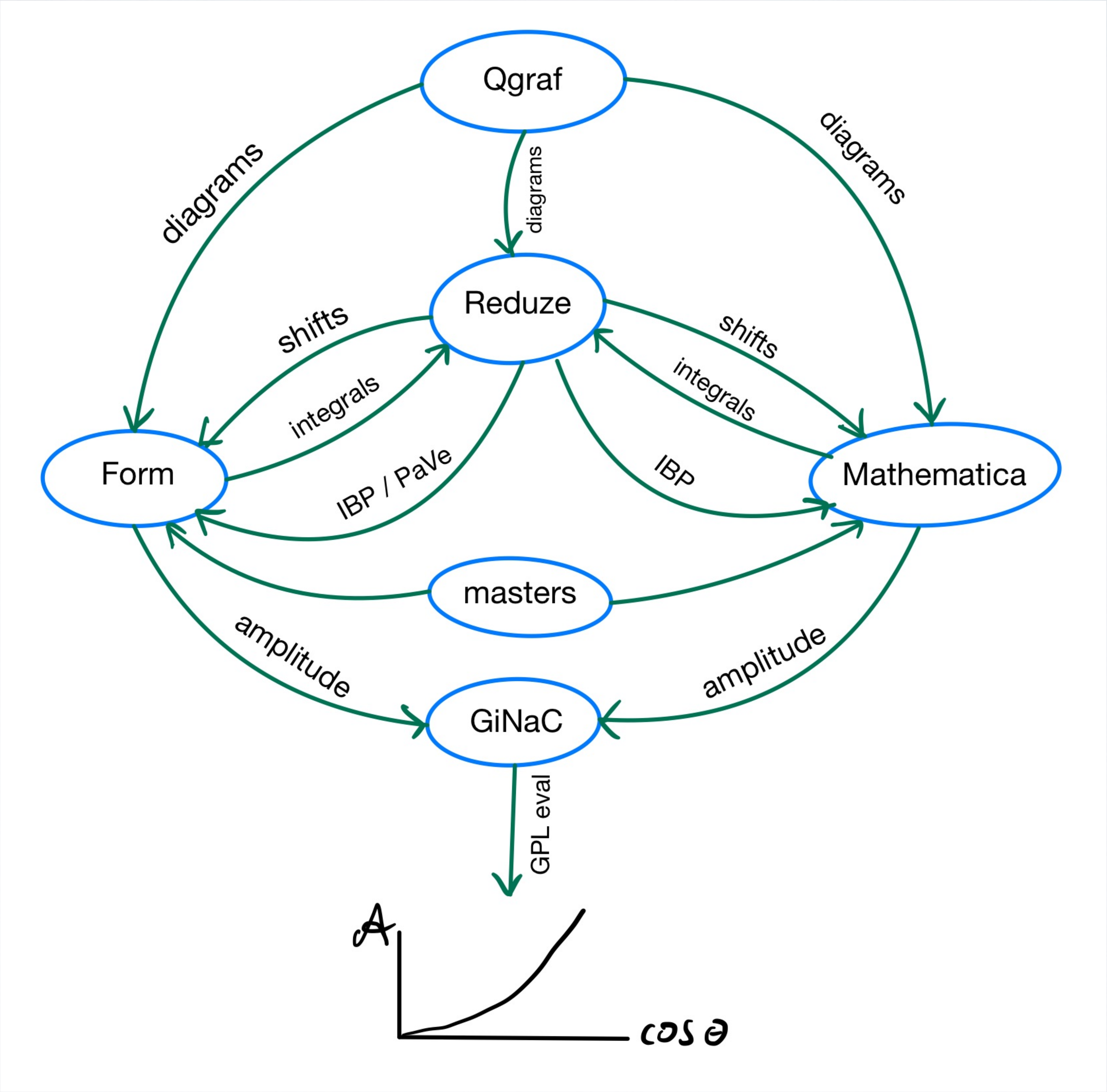
- Collaboration with:

Matthias Heller (Mainz)

Robert Schabinger (MSU)

Hubert Spiesberger (Mainz)

TOOLS



Qgraf
diagrams
diagrams
diagrams
Reduze
shifts
integrals
shifts
integrals
IBP / PaVe
IBP
Form
Mathematica
masters
amplitude
amplitude
GiNaC
GPL eval
A
cos θ

γ_5 AND DIM. REG.

- Conventional dimensional regularization (CDR):

$$\int \frac{\mathrm{d}^4 k_i}{(2\pi)^4} \longrightarrow \left(\mu^2\right)^\epsilon \int \frac{\mathrm{d}^{4-2\epsilon} k_i}{(2\pi)^{4-2\epsilon}}$$

$$g^{\mu\nu} g_{\mu\nu} = 4 - 2\epsilon\,,$$
$$\{\gamma_\mu, \gamma_\nu\} = 2 g_{\mu\nu}\,\mathbf{1}\,,$$
$$\text{and}\quad \gamma^\mu \gamma_\mu = \frac{1}{2} g^{\mu\nu} \{\gamma_\mu, \gamma_\nu\} = g^{\mu\nu} g_{\mu\nu} = 4 - 2\epsilon$$

  all vectors d-dim (internal+external), fermions 2 pol (enters only via $\sum u(p)\bar{u}(p) = p^\mu \gamma_\mu$)

- Problem: γ_5 really a ***4-dimensional*** object

$$\mathrm{tr}\,\{\gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \gamma_5\} = -4i\,\varepsilon_{\mu\nu\rho\sigma}$$

- ***Split*** d dimensional space into 4 dimensional one (bar) + $\epsilon = (4-d)/2$ dimensional one (hat)

$$k^\mu = \bar{k}^\mu + \hat{k}^\mu$$
$$\gamma^\mu = \bar{\gamma}^\mu + \hat{\gamma}^\mu$$

  in particular: $\varepsilon^{\mu\nu\rho}{}_\alpha\, \varepsilon_{\mu\nu\rho\beta} = -6\, \bar{g}_{\alpha\beta}$ etc

- To give meaning to γ_5 in d dims:
  - give up ***anti-commutativity***: *'t Hooft, Veltman, Breitenlohner, Maison* (HVBM)

$$\{\bar{\gamma}_\mu, \gamma_5\} = 0$$
$$[\hat{\gamma}_\mu, \gamma_5] = 0$$

    (violates chiral symmetry)

  - give up ***cyclicity*** of Dirac trace: *Kreimer*

$$\{\gamma_\mu, \gamma_5\} = 0$$

    (requires reading point prescription)

SETUP AND μ TERMS

- We perform the calculation using ***3 different setups***:

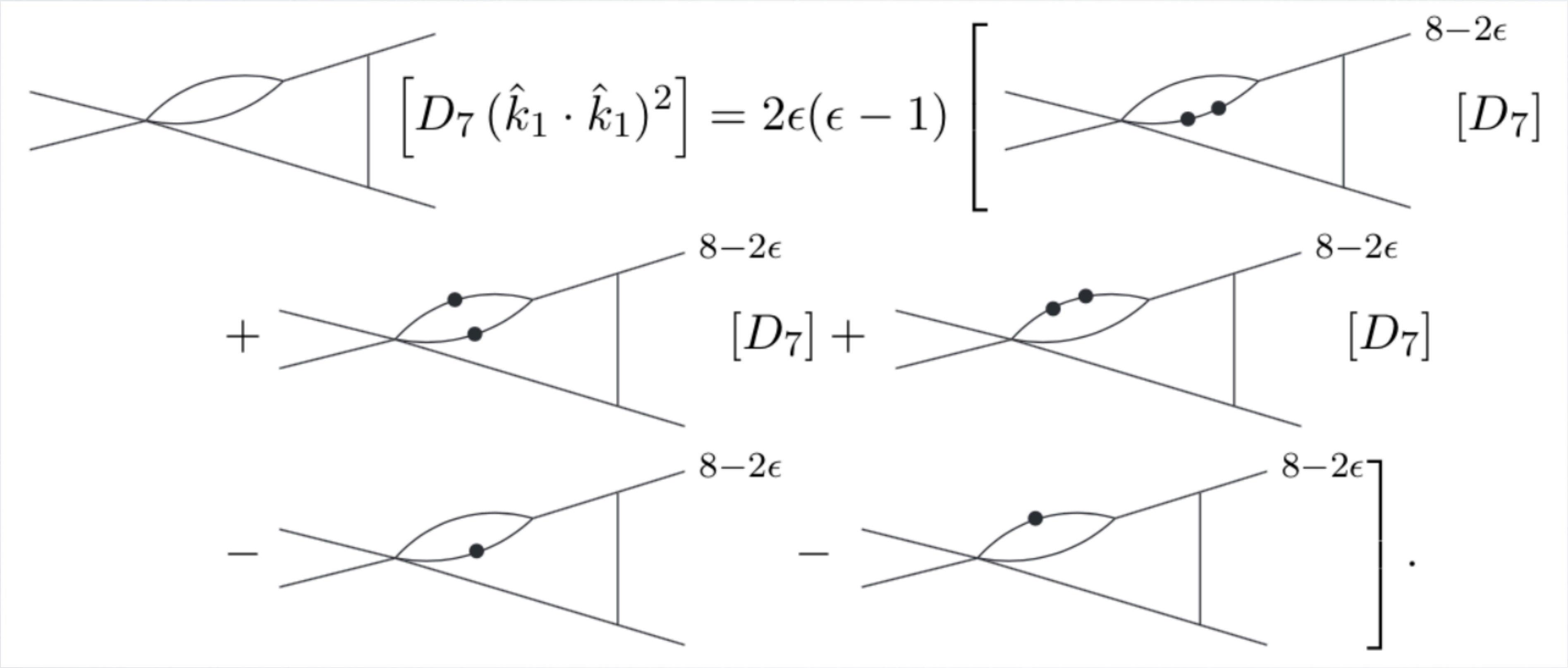
    1. HVBM scheme + projectors + mu-terms

    2. Kreimer's scheme + projectors + mu-terms (boxes)

    3. Kreimer's scheme + PaVe reduction

- HVMB requires ***split of indices*** right away, Kreimer's scheme allows to perform Dirac traces without

- Tensor integrals with ϵ ***dimensional loop momenta*** (μ ***terms***) treated using dimension shifts



CHIRAL SYMMETRY

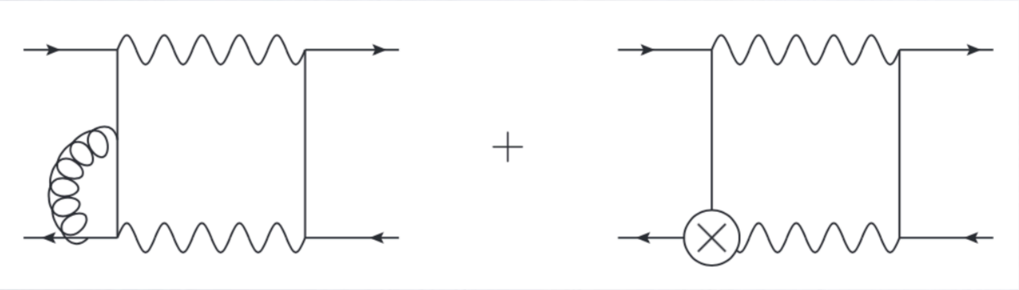
- In HVBM, corrections to ***vector and axial-vector currents*** differ
- Restore ***chiral symmetry*** by adding counter terms
- For vertex, ***require***

$$\bar{\mathcal{A}}^{(0,1)}_{Z\bar{q}q}(s) = -\frac{a_q}{v_q}\,\bar{\mathcal{V}}^{(0,1)}_{Z\bar{q}q}(s)$$

- Implement by adding ***counter terms***

$$\begin{aligned}\delta Z^{(0,1)}_{Z\bar{q}q} &= \bar{\mathcal{A}}^{(0,1)}_{Z\bar{q}q}(s) - \mathcal{A}^{(0,1)}_{Z\bar{q}q}(s) \\ &= 2\,a_q \frac{(2-\epsilon)\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)e^{\gamma_E \epsilon}}{(1-\epsilon)\Gamma(2-2\epsilon)} C_F e^{i\pi\epsilon}\left(\frac{\mu^2}{s}\right)^{\epsilon}\end{aligned}$$

- Note: need also symmetry restoring counter terms in boxes



- We keep also the ***higher order ϵ terms*** for the counter term

TENSOR INTEGRALS

- Consider treatment of tensor integrals

$$I^{\mu_1\dots\mu_n} = \int d^dk_1\dots d^dk_m \frac{k_{i_1}^{\mu_1}\dots k_{i_n}^{\mu_n}}{D_1\dots D_N} = \sum_j I_j p_{j_1}^{\mu_1}\dots g^{\mu_k\mu_l}\dots$$

- One option: compute interference terms
- Here: ***Lorentz decomposition***
  - at level of ***integrals***: Passarino-Veltman reduction
  - at level of ***amplitude***: form factor decomposition

FORM FACTORS

- Start with CDR building blocks *[Glover]*

$$\bar{\mathcal{T}}_1 = \bar{v}(p_2)\gamma^\mu u(p_1)\; \bar{u}(p_3)\gamma_\mu v(p_4),$$
$$\bar{\mathcal{T}}_2 = \bar{v}(p_2)\not{p}_3 u(p_1)\; \bar{u}(p_3)\not{p}_1 v(p_4),$$
$$\bar{\mathcal{T}}_3 = \bar{v}(p_2)\gamma^\mu\gamma^\nu\gamma^\rho u(p_1)\; \bar{u}(p_3)\gamma_\mu\gamma_\nu\gamma_\rho v(p_4),$$
$$\bar{\mathcal{T}}_4 = \bar{v}(p_2)\gamma^\mu\not{p}_3\gamma^\nu u(p_1)\; \bar{u}(p_3)\gamma_\mu\not{p}_1\gamma_\nu v(p_4),$$
$$\bar{\mathcal{T}}_5 = \bar{v}(p_2)\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\tau u(p_1)\; \bar{u}(p_3)\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_\tau v(p_4),$$
$$\bar{\mathcal{T}}_6 = \bar{v}(p_2)\gamma^\mu\gamma^\nu\not{p}_3\gamma^\rho\gamma^\sigma u(p_1)\; \bar{u}(p_3)\gamma_\mu\gamma_\nu\not{p}_1\gamma_\rho\gamma_\sigma v(p_4),$$

- Insert γ_5 , for example in $\mathcal{T}_1$:

$$\bar{\mathcal{T}}_{\mathrm{VV}} = \bar{v}(p_2)\gamma^\mu u(p_1)\; \bar{u}(p_3)\gamma_\mu v(p_4),$$
$$\bar{\mathcal{T}}_{\mathrm{AA}} = \bar{v}(p_2)\gamma^\mu\gamma_5 u(p_1)\; \bar{u}(p_3)\gamma_\mu\gamma_5 v(p_4),$$
$$\bar{\mathcal{T}}_{\mathrm{AV}} = \bar{v}(p_2)\gamma^\mu\gamma_5 u(p_1)\; \bar{u}(p_3)\gamma_\mu v(p_4),$$
$$\bar{\mathcal{T}}_{\mathrm{VA}} = \bar{v}(p_2)\gamma^\mu u(p_1)\; \bar{u}(p_3)\gamma_\mu\gamma_5 v(p_4),$$

(use $\frac{1}{2}[\gamma_\mu, \gamma_5] = \bar{\gamma}_\mu\gamma_5$ in HVBM)

- Amplitude is now

$$i\mathcal{A}_{\mathrm{DY}} = i\sum_{\alpha=1}^{16} \mathbf{C}_\alpha\, T_\alpha,$$

$$T_\alpha = (\bar{\mathcal{T}}_{1,\mathrm{VV}}, \bar{\mathcal{T}}_{1,\mathrm{AA}}, \bar{\mathcal{T}}_{2,\mathrm{VV}}, \bar{\mathcal{T}}_{2,\mathrm{AA}}, \ldots, \bar{\mathcal{T}}_{1,\mathrm{AV}}, \bar{\mathcal{T}}_{1,\mathrm{VA}}, \bar{\mathcal{T}}_{2,\mathrm{AV}}, \bar{\mathcal{T}}_{2,\mathrm{VA}}, \ldots)$$

FORM FACTORS

- Computation of projectors to extract form factors:

$$i\mathcal{A}_{\mathrm{DY}} = i\sum_{\alpha=1}^{16} \mathbf{C}_\alpha T_\alpha, \qquad M_{\alpha\beta} = \sum_{\text{spin,color}} T_\alpha^\dagger T_\beta \qquad \mathbf{C}_\alpha = \sum_{\text{spin,color}} \mathcal{P}_\alpha i\mathcal{A}_{\mathrm{DY}} \qquad \mathcal{P}_\alpha = -i\sum_{\beta=1}^{16} M_{\alpha\beta}^{-1} T_\beta^\dagger$$

- Only ***4 tensors independent*** in $d = 4$, equal to number of ***helicity amplitudes***.

- Would like to ignore other directions, but how ? Note: M^{-1} ***diverges*** for $d \to 4$!

- ***Change basis*** of tensors *[see also Peraro,Tancredi '19,'20; Chen, Ravindran et al '19,'19]*

$$T_1' = T_1, \qquad T_2' = T_2, \qquad T_\alpha' = T_\alpha + \sum_{\beta=1}^{2} R_{\alpha\beta} T_\beta \quad \text{for } \alpha = 3 \ldots 8$$

  such that irrelevant directions ***decouple*** exactly in d dimensions

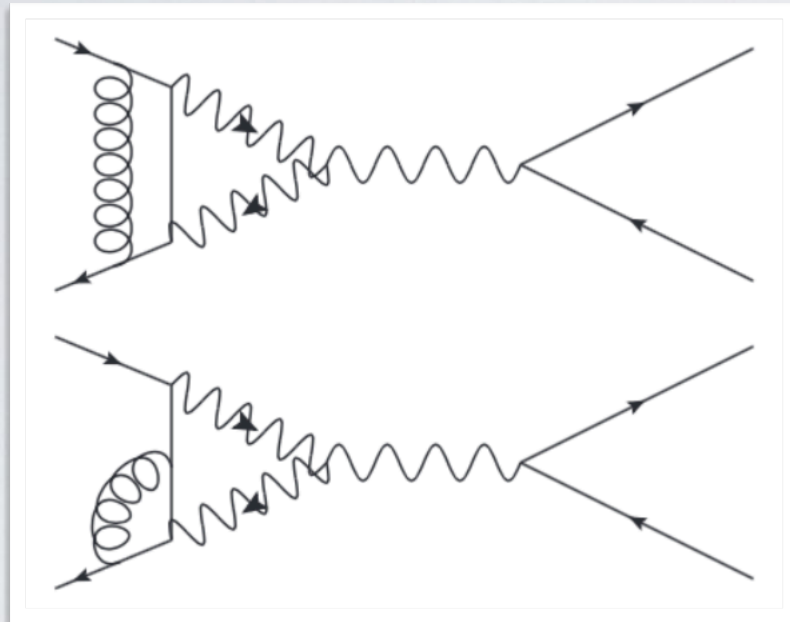
$$(M_{\alpha\beta}') = \sum_{\text{spin,color}} (T_\alpha'^\dagger T_\beta') = (RMR^\dagger) = \begin{pmatrix} M_{2\times2}' & 0 \\ 0 & M_{6\times6}' \end{pmatrix}$$

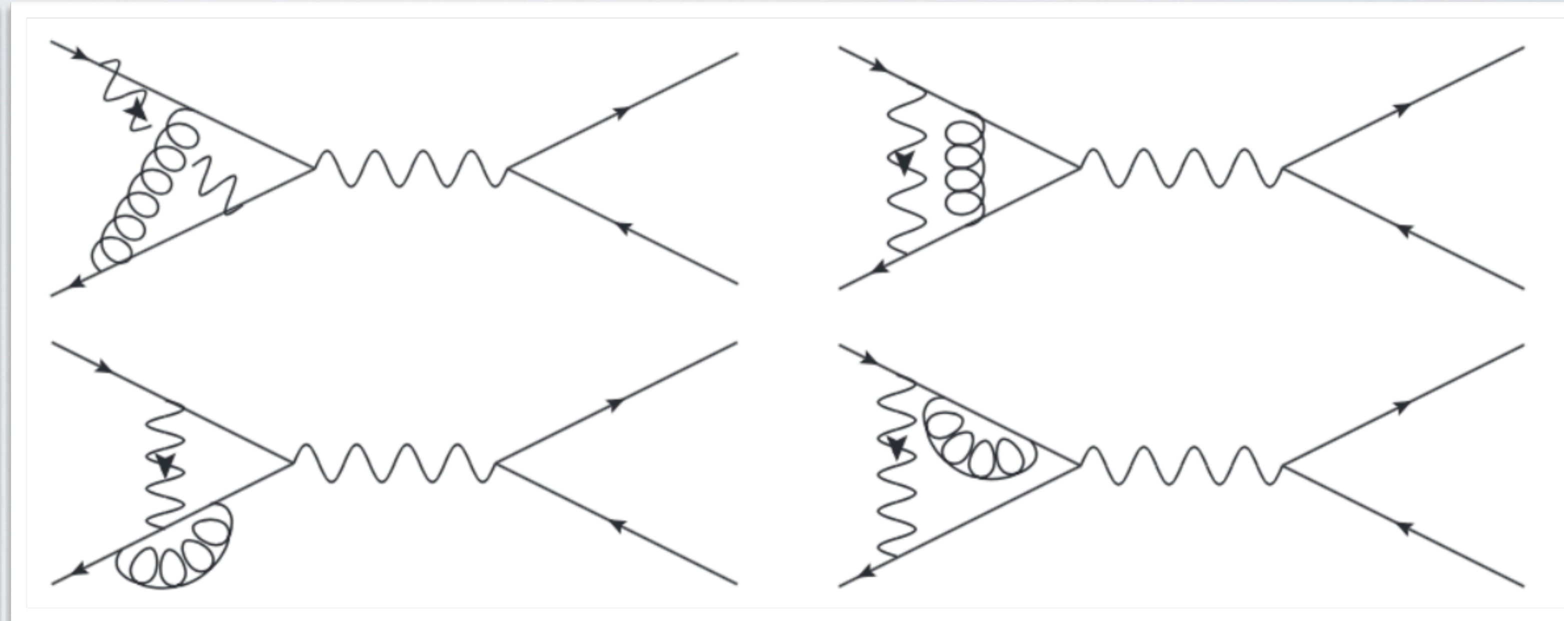
- M_{2x2}^{-1} is ***regular*** for $d \to 4$, irrelevant directions contribute only at order $d - 4$!

- **Result:** exact d dim. projectors for relevant form factors and subtraction terms (γ_5 scheme dep.), irrelevant ones not needed for finite remainder

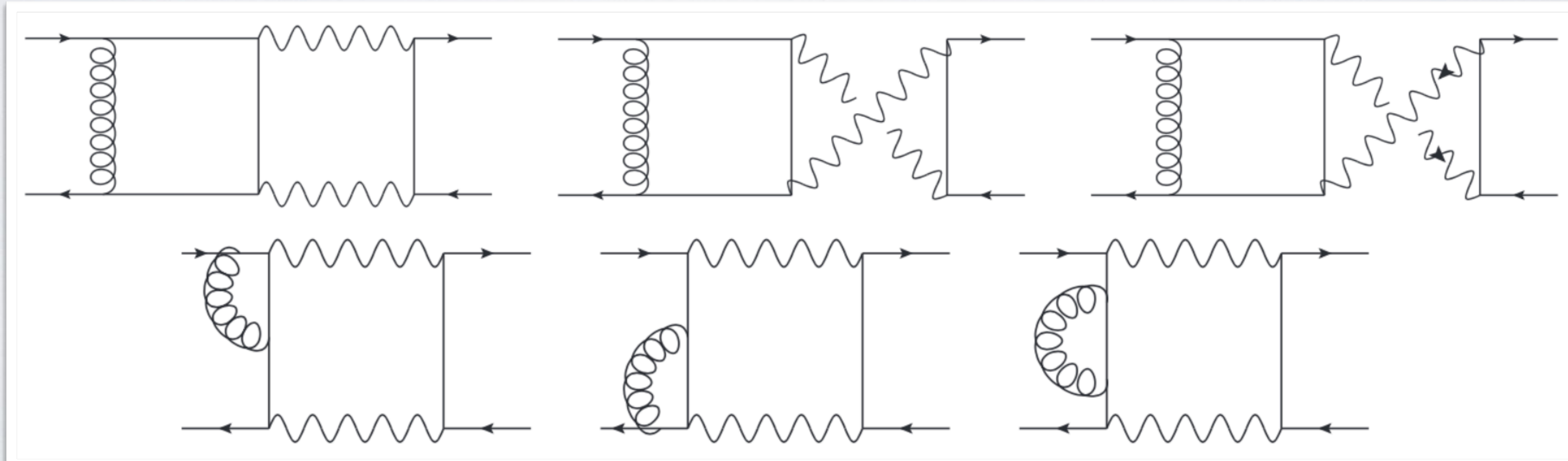
FEYNMAN DIAGRAMS

Examples for vertex corrections at two loops:



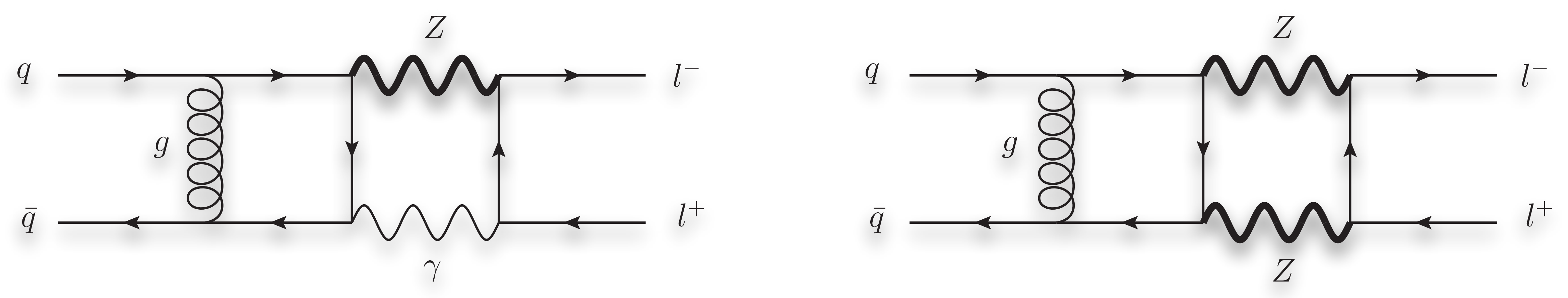


Examples for box corrections at two-loops:



MASTER INTEGRALS

- ***Feynman diagrams*** with one and two masses:



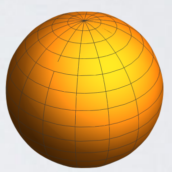
- Master integrals:
  - 1-fold integral over polylogarithms (Euclidean region)
    [Bonciani, Di Vita, Mastrolia, Schubert '16]
  - One-mass: real-valued multiple polylogarithms (physical region)
    [AvM, Schabinger '17]
  - Two-mass: ***optimized representation*** in physical region
    [Heller, AvM, Schabinger '19]

DIFFERENTIAL EQUATIONS

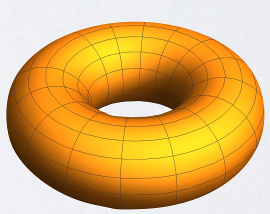
- Need to solve ***master integrals***, use method of differential equations
- Aim: analytical integration of differential equations *[Kotikov '91, Remiddi '97]*:

$$\partial_x \vec{I}(x;\epsilon) = A(x;\epsilon)\vec{I}(x;\epsilon) \quad \text{where} \quad \epsilon = (4-d)/2$$

- Homogeneous solutions for $\epsilon = 0$ (leading singularities):
  - ***Rational number***, e.g. $1/2$



  - ***Rational functions***, e.g. $1/x$
  - ***Algebraic functions***, e.g. $\sqrt{x(x-4)}$
  - ***Elliptic integrals***, e.g. $K(x) = \int_0^1 \frac{\mathrm{d}z}{\sqrt{(1-z^2)(1-xz^2)}}, \ldots$



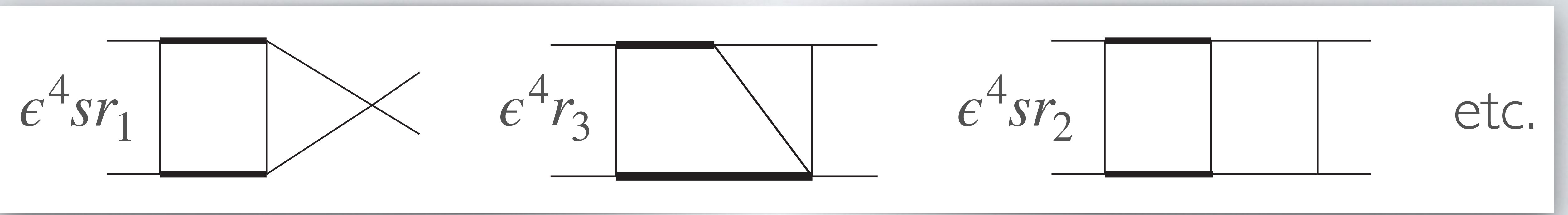
[see also Blümlein, De Freitas et al. '15,'17]

- Basis change involving homogenous solutions may allow to find ***ϵ-form***:

$$d\vec{m} = \epsilon \, \mathrm{dln}(l_a(x)) \, A^{(a)}(x) \, \vec{m}$$

[Kotikov '10, Henn '13, Remiddi, Tancredi '16, AvM, Tancredi '17, Adams, Weinzierl '18]

CANONICAL BASIS



- $r_1 = \sqrt{s(s-4m^2)}$, $r_2 = \sqrt{-st\left(4m^2(t+m^2)-st\right)}$, $r_3 = \sqrt{s\left(t^2(s-4m^2)+sm^2(m^2-2t)\right)}$

- Reparametrization $s = -m^2\dfrac{(1-w)^2}{w}$, $t = -m^2\dfrac{w(1+z)^2}{z(1+w)^2}$ rationalizes 2 out of 3 roots:

$$r_1 = -\frac{m^2(1-w)(1+w)}{w},\ r_2 = -\frac{m^4(1-w)(1-z)(1+z)}{z(1+w)},\ r_3 = \frac{m^4(1-w)}{w\,z(1+w)}r$$

- $r = \sqrt{4(1-w)^2 wz^2 + (w+z)^2(1+wz)^2}$ ***not rationalizable !*** Root will enter diff eq. birationally equivalent to K3 *[van Straten '14, Besier, Festi, Harrison, Naskrecki '19]*

SYMBOL CALCULUS

- ***How to solve the differential equation ?***

$$\mathrm{d}\vec{m} = \epsilon \sum_a \mathrm{d}\ln(l_a)\, A^{(a)}\, \vec{m}$$

- If letters l_i simple: iterated integration gives ***multiple polylogarithms***

e.g. $l_1 = x, \quad l_2 = x - 1, \quad Li_2(x) = -\int_0^x \frac{dt}{t}\int_0^t \frac{dt'}{t'-1}$

- ***Symbol calculus*** *[Duhr, Gangl, Rhodes 2011]*: $S(Li_2(x)) = -(\ln(x) \otimes \ln(x-1))$

- ***Letters*** form ***words*** with grammar (log law):

$$\ln(l_1 l_2) \otimes \ln(l_3) = \ln(l_1) \otimes \ln(l_3) + \ln(l_2) \otimes \ln(l_3)$$

- C***omputer algebra*** allows to automatically derive functional identities, take limits, …

DIFFERENTIAL EQUATION

Diff. eq. $d\vec{m} = \epsilon \sum_a d\ln(l_a)\, A^{(a)}\, \vec{m}$ with root-valued letters l_i

e.g. $l_{13} = -(1-w)(z-w)(1-wz) + (1+w)\sqrt{4(1-w)^2wz^2 + (w+z)^2(1+wz)^2}$

- ***How to solve ?***
  - Can't integrate in terms of GPLs in ***filtration basis***
  - Match against ansatz of ***std. multiple polylogs*** ? Possible at all ?
  - Need to integrate in terms of ***elliptic polylogarithms*** ?

    $r = \sqrt{4(1-w)^2wz^2 + (w+z)^2(1+wz)^2}$ elliptic curve ?

    see *[Broedel, Dulat, Duhr, Penante, Tancredi '19]* for related case

CONSTRUCTION OF ANSATZ

- ***Method*** of *[Duhr, Gangl, Rhodes '11]*
  - Find multiple polylogarithms which do not introduce letters beyond original alphabet
  - Pick a set of functions, e.g. classical polylogs etc.
  - For dilogarithm we see from $\mathrm{Sym}(\mathrm{Li}_2(a)) = -(1-a) \otimes a$ that both a and $1-a$ should factorize over original alphabet
  - Construct power products (pos./neg. powers) of letters $a = \pm\, l_1^{a_1} l_2^{a_2} \cdots$ and check whether $1-a$ factorizes over letters

WHY IS BASIS CONSTRUCTION DIFFICULT ?

- ***General issues:***
  - Combinatorical complexity
  - Best choice of letters (or even just minimal number) not obvious ($\{l_a, l_b\}$ vs. $\{l_a l_b, l_b\}$)
  - Might need numerical letter ($l_a = 2$) in argument, but invisible in derivative
- ***New issues specific to algebraic letters:***
  - No unique factorization
  - Might need non-integer powers of letters in argument ($\sqrt{l_a}, l_a^{-3/4}, \ldots$)
  - No cheap way of factorizing even for given letters
- First weight 4 result in *[AvM, Tancredi '17]*, but roots were secretly rationalizable
- Here: need new idea

A NEW APPROACH

- ***Observation:***
  - Consider a letter $l_a = x + y\sqrt{\ }$ in our alphabet,
    define its conjugate $\bar{l}_a = x - y\sqrt{\ }$,
    then $l_a\bar{l}_a$ factorizes over the rational part of the alphabet !
- ***Construct*** improved letters:
  - Make an ansatz $l_a = x + y\sqrt{\ }$ and require $l_a\bar{l}_a$ factorizes over the rational part of the alphabet. Pick simple candidates.
  - Define the root itself to be a letter
- ***Factorizations*** either using polynomial rings or (cheaper) using numerical sample and integer relation algorithm:
  - $g = c l_1^{a_1} l_2^{a_2} \cdots$ implies integer relation between logs:
    $\ln(g) - \ln(c) - a_1 \ln(l_1) - a_2 \ln(l_2) - \ldots = 0$

OUR IMPROVED LETTERS

- ***Rational letters:***

$$\mathscr{L}_R = \{1-w, -w, 1+w, 1-w+w^2, 1-z, -z, 1+z, \\ 1-wz, 1+w^2 z, -z-w^2, z-w\}$$

- ***Initial algebraic letters:***

$$\mathscr{L}_A = \{r, -(1-w)(z-w)(1-wz)+r(1+w), \\ -(1-w)\big(4wz+(w+z)(1+wz)\big)-r(1+w), \\ r^2-2wz^2(1-w)^2+r(w+z)(1+wz), \\ r^2(1-z)^2+2z^2(z+w^2)(1+w^2z)+r(1-z)(1+z)\big(2wz-(w+z)(1+wz)\big)$$

with $r=\sqrt{4(1-w)^2wz^2+(w+z)^2(1+wz)^2}$

- ***Improved algebraic letters:***

$$\mathscr{L}_{\tilde{A}} = \Big\{r, \tfrac{1}{2}\big(2+z-w+wz(w+z)+r\big), \tfrac{1}{2}\big(2w^2+z-w+wz(w+z)+r\big), \\ \tfrac{1}{2}\big(-(w+z)(1-wz)+r\big), \tfrac{1}{2}\big(-(z-w)(1+wz)+r\big)\Big\}$$

SUCCESS WITH NEW LETTERS

- ***Factorization*** of old letters w.r.t. new alphabet:

$$-(1-w)(z-w)(1-wz)+\mathbf{r}(1+w) = \frac{2(-w)(1+z)(-z-w^2)(2+z-w+wz(w+z)+\mathbf{r})}{2w^2+z-w+wz(w+z)+\mathbf{r}}$$

$$-(1-w)\big(4wz+(w+z)(1+wz)\big)-\mathbf{r}(1+w) = \frac{8(-w)^2(-z)(1+z)^3(1+w^2z)(2w^2+z-w+wz(w+z)+\mathbf{r})}{(2+z-w+wz(w+z)+r)(-(w+z)(1-wz)+\mathbf{r})(-(z-w)(1+wz)+r)}$$

$$r^2-2wz^2(1-w)^2+\mathbf{r}(w+z)(1+wz) = \frac{(-z)^2(2+z-w+wz(w+z)+\mathbf{r})^2(2w^2+z-w+wz(w+z)+r)^2}{8(1+z)^2(1+w^2z)^2(-(w+z)(1-wz)+\mathbf{r})^2(-(z-w)(1+wz)+r)^{-2}}$$

$$r^2(1-z)^2+2z^2(z+w^2)(1+w^2z)+\mathbf{r}(1-z)(1+z)\big(2wz-(w+z)(1+wz)\big) = \frac{2(-z)^2(1+w^2z)^2(-(w+z)(1-wz)+\mathbf{r})^2}{(-(z-w)(1+wz)+\mathbf{r})^2}$$

- ***Result: successful integration*** of symbol !
    - no roots of letters needed, much lower degree of arguments, no numerical letters !

A MATHEMATICAL RESULT
WITH PRACTICAL CONSEQUENCES

- In conclusion, despite the presence of non-rationalizable roots integrable in terms of standard multiple polylogarithms !

- **Algorithm** gives:

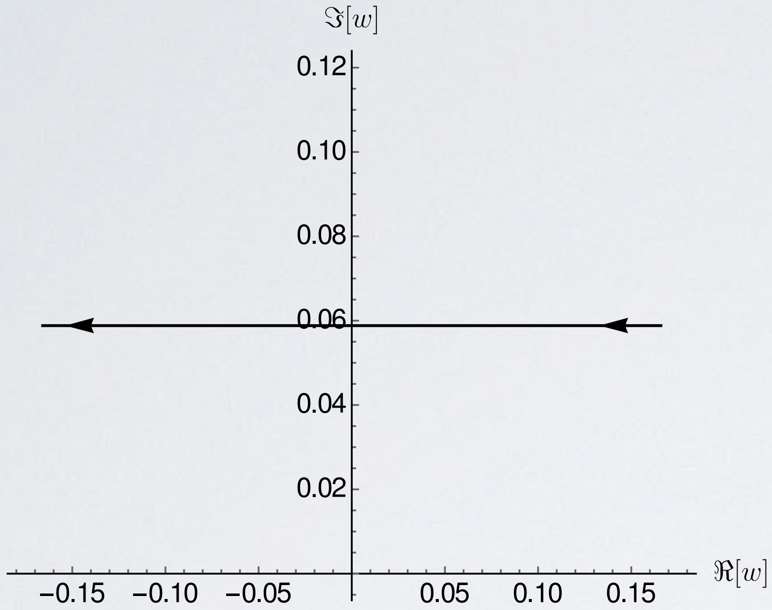
$$m_{32} = \epsilon^3 \left[4\,\mathrm{Li}_3\!\left(\frac{l_1 l_2 l_6 l_7 l_{10} l_{13}}{l_{14} l_{15} l_{16}}\right) - 2\,\mathrm{Li}_3\!\left(\frac{l_2^3 l_6 l_7^2}{l_{15} l_{16}}\right) + \ldots + 4\,\mathrm{Li}_2\!\left(\frac{l_6 l_{14} l_{16}}{l_7 l_9 l_{15}}\right) \ln(l_3) + \ldots \right]$$
$$+\epsilon^4 \left[-\,\mathrm{Li}_{2,2}\!\left(-\frac{l_1^2 l_3 l_{15}}{l_2^2 l_7 l_{14}}, \frac{l_2^2 l_7 l_{15}}{l_1 l_3 l_6 l_1 4}\right) + \ldots + \frac{701}{4}\,\mathrm{Li}_4\!\left(\frac{l_1 l_3^2 l_6^2 l_9 l_{14}}{l_2 l_7 l_{13} l_{15} l_{16}}\right) + \ldots \right] + O(\epsilon^5)$$

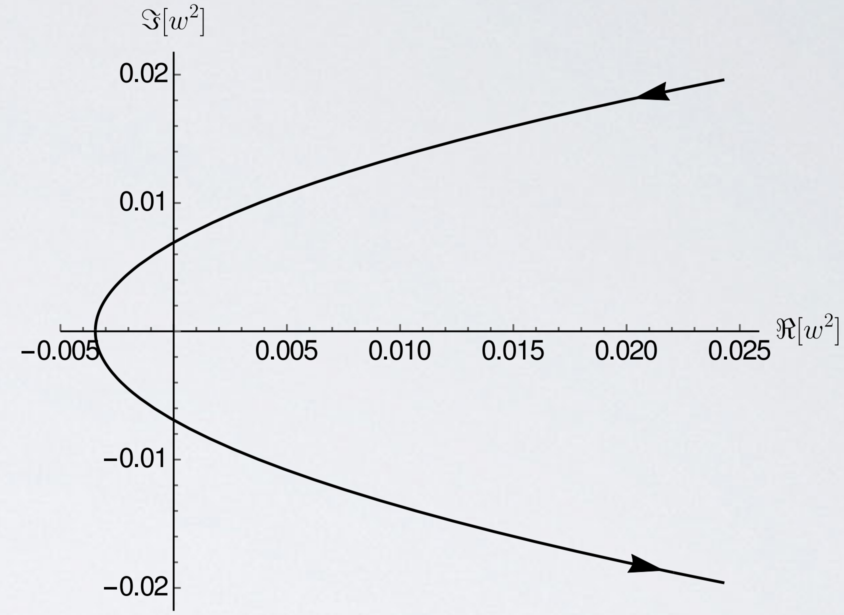
[Heller, AvM, Schabinger 2019]

- ***Fast*** and ***robust*** numerical evaluations in Monte Carlo programs

ANALYTIC CONTINUATION

Option 1:





$$\ln\left(w^2\right) \to \ln\left(w^2\right) + 2\pi i$$

$$\ln\left(w^2\right) = 2\ln\left(w\right) \to 2\ln(w) = 2\ln(-w) + 2\pi i = \ln\left(w^2\right) + 2\pi i$$

$$\mathrm{Li}_2\left(1 - w^2\right) \to \mathrm{Li}_2\left(1 - w^2\right) - 2\pi i \ln\left(1 - w^2\right),$$

monodromies of multiple poylogs: with coproduct *[Goncharov ’01, Duhr ’11]*

Option 2: fix boundaries in each region separately

Option 3: solve diff. eqs. by expansion, fit precise numerics to transport analyt. constants. *[Lee, Smirnov, Smirnov ‘18, Moriello ’19]*

RESTRICTIONS ON FUNCTIONAL BASIS

- Wish to ***avoid i0 prescriptions***, cancelations at pseudo-thresholds
- ***Select functions*** based on absence of cuts (read off from symbol)
- ***Rich structure of pseudo-thresholds*** in physical region
- Already integrals with rationalizable letters require sub-domains:
  $0 < s < m^2, \quad m^2 < s < 2m^2, \quad 2m^2 < s < 4m^2, \quad s > 4m^2$
  (note: only $s = 0,\ m^2,\ 4m^2$ physical thresholds)

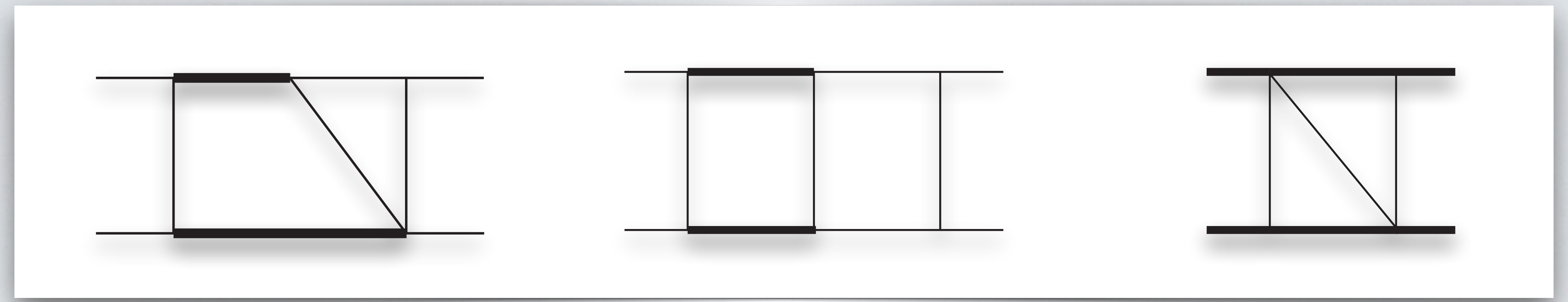
NUMERICAL PERFORMANCE

- Our representation allows ***fast and robust*** usage in Monte-Carlo
- E.g. at $(s, t, m^2) = (17, -7, 6241/1681)$:

$$m_{32} \approx \epsilon^3\big(0.06653798496208053075\ldots - 27.50824587001145752\ldots\ i\big)$$
$$+\epsilon^4\big(51.61560743380638113\ldots - 149.0632661954243719\ldots\ i\big) + \mathcal{O}\left(\epsilon^5\right),$$

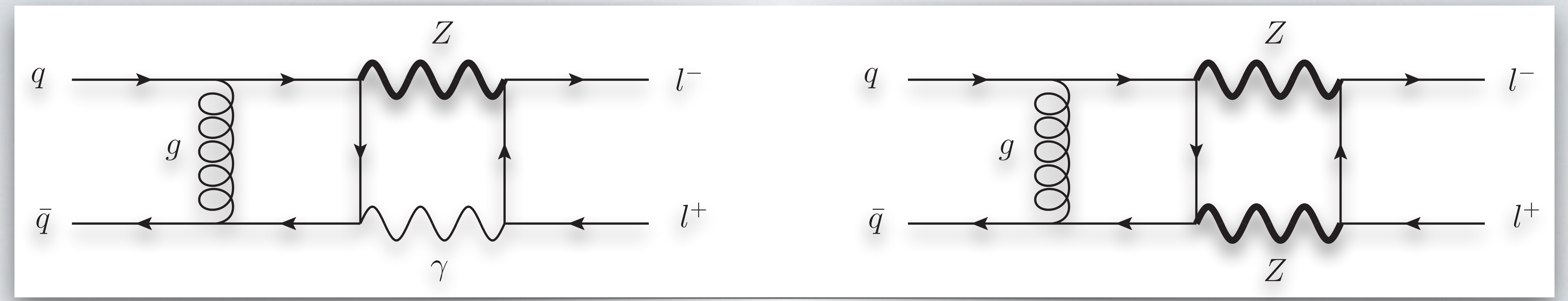
all master integrals: O(second) for double precision with GiNaC's polylogs *[Vollinga, Weinzierl '04]*

SCOPE OF METHOD



- We also considered DY integrals and planar Bhabha integrals in ***direct integration*** approach *[Brown '08, Panzer '14]*
- Found ***variable changes*** to prove multiple polylog solution possible to ***all orders in*** ϵ
- Obtained explicit results, but not as nice as differential equations
- Note: DY and Bhabha K3s not isomorphic *[Besier, Festi, Harrison, Naskrecki '19]*
- Constructed alphabets with up to 5 simultaneous roots for HH/VV production

RESULTS FOR AMPLITUDES



- Calculated $O(\alpha_s), O(\alpha), O(\alpha_s\alpha)$ corrections to sufficient order in ϵ
- Amplitudes ***finite*** after UV renormalizations and IR subtractions
- Confirm known ***QED-QCD*** result *[Kilgore, Sturm '11]*
- γ^5 scheme dependent results, but ***finite remainders coincide*** !

NOTATION

- Perturbative expansion:

$$\overset{(-)}{\mathcal{A}}_{\rm DY} = 4\pi\alpha\left(\overset{(-)}{\mathcal{A}}{}^{(0,0)}_{\rm DY} + \overset{(-)}{\mathcal{A}}{}^{(0,1)}_{\rm DY}\left(\frac{\alpha_s}{4\pi}\right) + \overset{(-)}{\mathcal{A}}{}^{(1,0)}_{\rm DY}\left(\frac{\alpha}{4\pi}\right) + \overset{(-)}{\mathcal{A}}{}^{(1,1)}_{\rm DY}\left(\frac{\alpha}{4\pi}\right)\left(\frac{\alpha_s}{4\pi}\right) + \cdots\right)$$

- From form factors to helicity amplitudes:

$$\begin{aligned}
\mathcal{H}^{(m,n)}_{+-+-} &= -2(s+t)\left(\mathbf{C}^{(m,n),\,\rm fin}_{\rm VV} + \mathbf{C}^{(m,n),\,\rm fin}_{\rm AA} + \mathbf{C}^{(m,n),\,\rm fin}_{\rm VA} + \mathbf{C}^{(m,n),\,\rm fin}_{\rm AV}\right),\\
\mathcal{H}^{(m,n)}_{-+-+} &= -2(s+t)\left(\mathbf{C}^{(m,n),\,\rm fin}_{\rm VV} + \mathbf{C}^{(m,n),\,\rm fin}_{\rm AA} - \mathbf{C}^{(m,n),\,\rm fin}_{\rm VA} - \mathbf{C}^{(m,n),\,\rm fin}_{\rm AV}\right),\\
\mathcal{H}^{(m,n)}_{+--+} &= -2t\left(\mathbf{C}^{(m,n),\,\rm fin}_{\rm VV} - \mathbf{C}^{(m,n),\,\rm fin}_{\rm AA} - \mathbf{C}^{(m,n),\,\rm fin}_{\rm VA} + \mathbf{C}^{(m,n),\,\rm fin}_{\rm AV}\right),\\
\mathcal{H}^{(m,n)}_{-++-} &= -2t\left(\mathbf{C}^{(m,n),\,\rm fin}_{\rm VV} - \mathbf{C}^{(m,n),\,\rm fin}_{\rm AA} + \mathbf{C}^{(m,n),\,\rm fin}_{\rm VA} - \mathbf{C}^{(m,n),\,\rm fin}_{\rm AV}\right).
\end{aligned}$$

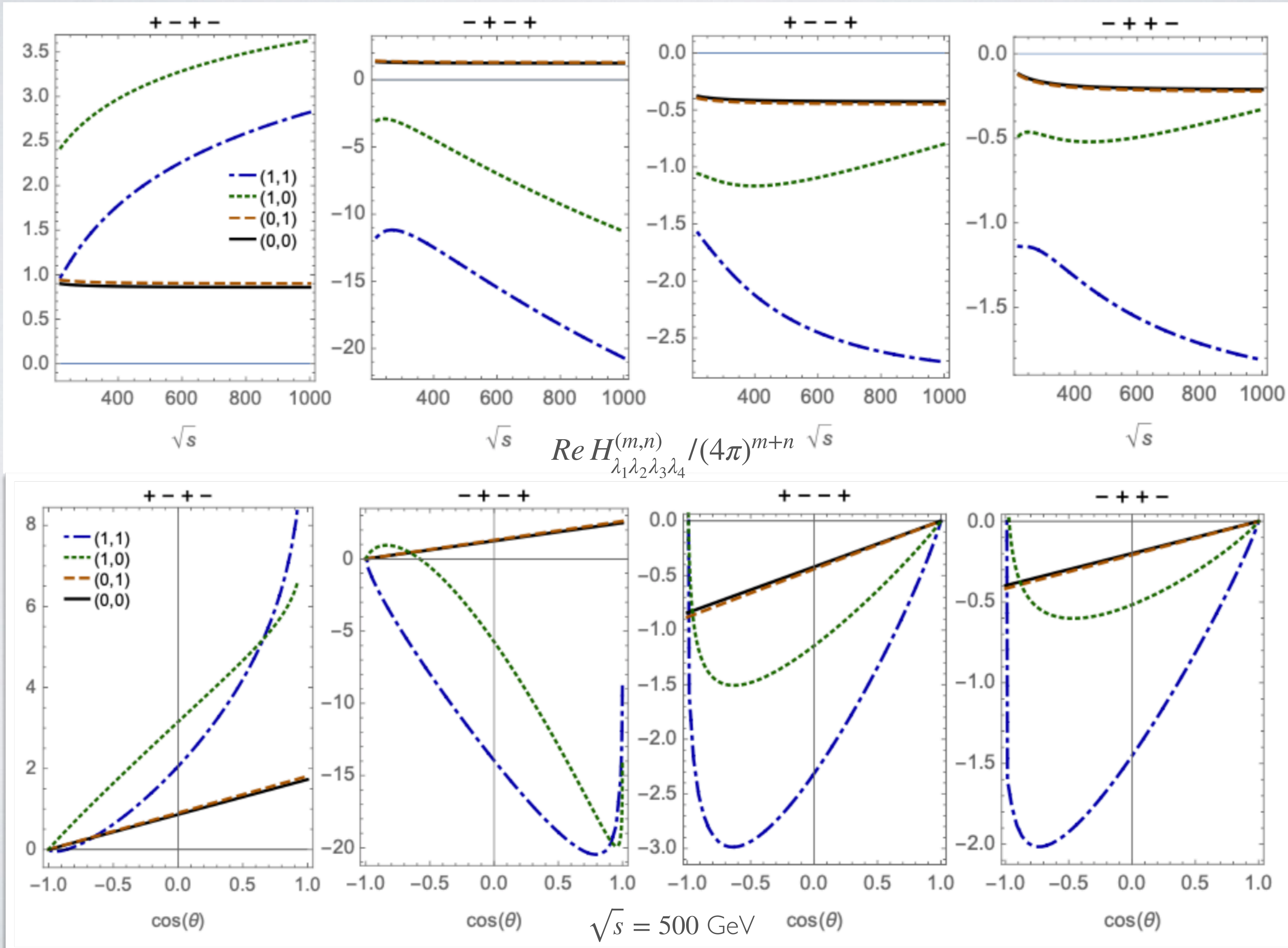
- NLO QCD:

$$\mathcal{H}^{(0,1)}_{\lambda_1\lambda_2\lambda_3\lambda_4}/(4\pi) = \left(\frac{\pi}{3} - i\right)\mathcal{H}^{(0,0)}_{\lambda_1\lambda_2\lambda_3\lambda_4}$$

(note: $\pi/3 \approx 1.05$)

HELICITY AMPLITUDES

[Heller, AvM, Schabinger, Spiesberger 2020]



$Re\, H^{(m,n)}_{\lambda_1\lambda_2\lambda_3\lambda_4} / (4\pi)^{m+n}$

$\sqrt{s} = 500$ GeV

CONCLUSIONS

- ***Drell-Yan process*** important for SM precision physics and BSM searches
  - Want control at highest energies
  - Here: mixed QCD-EW corrections to dilepton production (“off-shell DY")
- ***Two-loop Feynman integrals***
  - ϵ dln basis and root-valued letters
  - possible to solve in terms of standard multiple polylogarithms
  - new method to construct algebraic letters
- ***Two-loop amplitudes***
  - Analytical calculation in two γ_5 schemes: HVBM and Kreimer’s scheme
  - Finite remainders agree
  - MC-friendly compact results, ready-to-go for cross section calculation