New dual representation of scattering amplitudes through the LTD: *causality to multi-loop level*

German F. R. SBORLINI

in collaboration with **G. Rodrigo,** J. Aguilera Verdugo, F. Driencourt-Mangin, R. Hernández-Pinto, J. Plenter, S. Ramírez Uribe, A. Rentería Olivo and W. Torres Bobadilla

HU/DESY Seminar – Nov. 5th 2020

Index

1. What is LTD?
2. Brief history of LTD-based methods
3. Nested residues
    1. Displaced poles
    2. Compact representations
4. Manifestly Causal representation
    1. Implementation
5. Conclusions and outlook

1. arXiv:2010.12971 [pdf, other] hep-ph hep-th math-ph

Mathematical properties of nested residues and their application to multi-loop scattering amplitudes

Authors: J. Jesus Aguilera-Verdugo, Roger J. Hernandez-Pinto, German Rodrigo, German F. R. Sborlini, William J. Torres Bobadilla

Abstract: The computation of multi-loop multi-leg scattering amplitudes plays a key role to improve the precision of theoretical predictions for particle physics at high-energy colliders. In this work, we focus on the mathematical properties of the novel integrand-level representation of Feynman integrals, which is based on the Loop-Tree Duality (LTD). We explore the behaviour of the multi-loop iterated res... ▽ More

Submitted 24 October, 2020; **originally announced** October 2020.

Comments: 29 pages + appendices, 11 figures

Report number: IFIC/20-30; DESY 20-172; MPP-2020-184

2. arXiv:2006.13818 [pdf, other] hep-ph hep-th

Universal opening of four-loop scattering amplitudes to trees

Authors: Selomit Ramirez-Uribe, Roger J. Hernandez-Pinto, German Rodrigo, German F. R. Sborlini, William J. Torres Bobadilla

Abstract: The perturbative approach to quantum field theories has made it possible to obtain incredibly accurate theoretical predictions in high-energy physics. Although various techniques have been developed to boost the efficiency of these calculations, some ingredients remain specially challenging. This is the case of multiloop scattering amplitudes that constitute a hard bottleneck to solve. In this Let... ▽ More

Submitted 24 June, 2020; **originally announced** June 2020.

Comments: 7 pages, 4 figures

Report number: IFIC/20-29

3. arXiv:2006.11217 [pdf, other] hep-ph hep-th

Causal representation of multi-loop amplitudes within the loop-tree duality

Authors: J. Jesus Aguilera-Verdugo, Roger J. Hernandez-Pinto, German Rodrigo, German F. R. Sborlini, William J. Torres Bobadilla

Abstract: The numerical evaluation of multi-loop scattering amplitudes in the Feynman representation usually requires to deal with both physical (causal) and unphysical (non-causal) singularities. The loop-tree duality (LTD) offers a powerful framework to easily characterise and distinguish these two types of singularities, and then simplify analytically the underling expressions. In this paper, we work exp... ▽ More

Submitted 19 June, 2020; **originally announced** June 2020.

Comments: 24 pages, 8 figures

Report number: IFIC/20-27

4. arXiv:2001.03564 [pdf, other] hep-ph hep-th doi 10.1103/PhysRevLett.124.211602

Open loop amplitudes and causality to all orders and powers from the loop-tree duality

Authors: J. Jesus Aguilera-Verdugo, Felix Driencourt-Mangin, Roger J. Hernandez-Pinto, Judith Plenter, Selomit Ramirez-Uribe, Andres E. Renteria-Olivo, German Rodrigo, German F. R. Sborlini, William J. Torres Bobadilla, Szymon Tracz

Abstract: Multiloop scattering amplitudes describing the quantum fluctuations at high-energy scattering processes are the main bottleneck in perturbative quantum field theory. The loop-tree duality is a novel method aimed at overcoming this bottleneck by opening the loop amplitudes into trees and combining them at integrand level with the real-emission matrix elements. In this Letter, we generalize the loop... ▽ More

Submitted 19 May, 2020; **v1** submitted 10 January, 2020; **originally announced** January 2020.

Comments: Final version to appear in Physical Review Letters

Report number: IFIC/20-02

arXiv papers in 2020

What is LTD?: Context

- Loop amplitudes are a bottleneck in current high-precision computations
- Presence of singularities and thresholds prevents direct numerical implementations
- Well-known theorems (KLN) guarantee the cancellation of singularities for physical observables
- Real-radiation contributions are defined in Euclidean space (i.e. phase-space integrals)

LOOP AMPLITUDES

- ***Virtual internal momenta***
- ***Defined in Minkowski space-time***

⇨

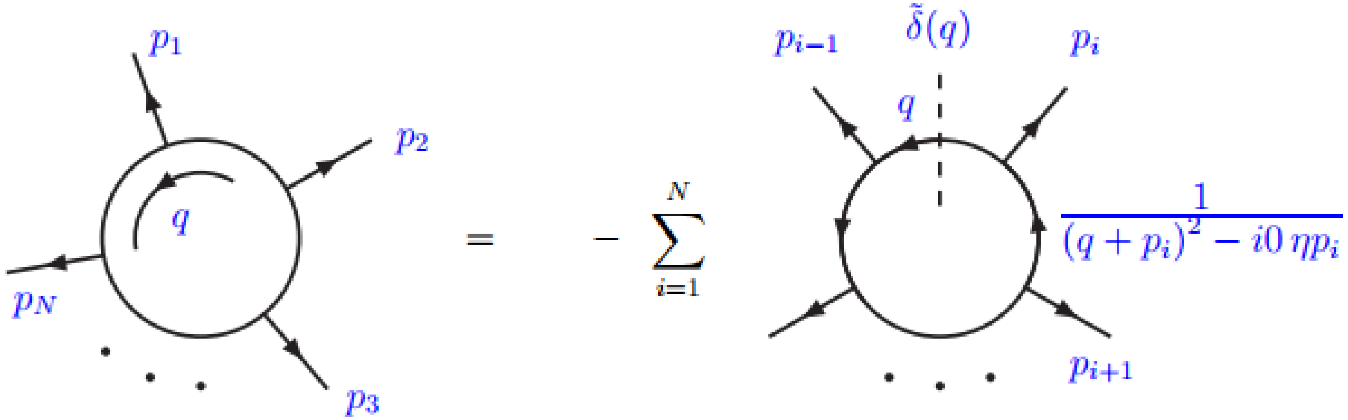
Loop-Tree Duality

⇨

DUAL AMPLITUDES

- ***On-shell cut momenta***
- ***Defined in Euclidean space-time***

Graphical representation of one-loop opening into trees



What is LTD?: General ideas

- **Removal of one component of the loop momenta, by using Cauchy residue theorem**
- Transforms Feynman propagators into dual ones (modified prescription)
- Equivalent to Feynman Tree Theorem (FTT):

Modified prescription ⟷ Sum of multi-cuts

- Arbitrary future-like momenta to define the new prescription (*not necessary in new representation!*)
- Feynman integrals transformed into sum of tree-level-like objects in Euclidean space
- **Number of cuts equal to number of loops (*strong point to support efficiency*)**

Feynman integral

$$L^{(1)}(p_1,\ldots,p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i) = \int_\ell \prod_{i=1}^{N} \frac{1}{q_i^2 - m_i^2 + i0}$$

Dual integral

$$L^{(1)}(p_1,\ldots,p_N) = -\sum_{i=1}^{N} \int_\ell \tilde{\delta}(q_i) \prod_{j=1, j\neq i}^{N} G_D(q_i;q_j)$$

Sum of phase-space integrals!

$$G_D(q_i,q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta(q_j - q_i)}$$

Modified propagator and measure

$$\tilde{\delta}(q_i) = i2\pi\,\theta(q_{i,0})\,\delta(q_i^2 - m_i^2)$$

Brief history of LTD-based methods

- **Foundational paper: a new way to decompose loop amplitudes**

JHEP PUBLISHED BY INSTITUTE OF PHYSICS PUBLISHING FOR SISSA

RECEIVED: *May 6, 2008*
REVISED: *August 14, 2008*
ACCEPTED: *August 26, 2008*
PUBLISHED: *September 11, 2008*

From loops to trees by-passing Feynman's theorem

Stefano Catani
INFN, Sezione di Firenze and Dipartimento di Fisica, Università di Firenze, I-50019 Sesto Fiorentino, Florence, Italy
E-mail: stefano.catani@fi.infn.it

Tanju Gleisberg
Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94309, U.S.A.
E-mail: tanju@slac.stanford.edu

Frank Krauss
Institute for Particle Physics Phenomenology, Durham University, Durham DH1 3LE, U.K.
E-mail: frank.krauss@durham.ac.uk

Germán Rodrigo
Instituto de Física Corpuscular, CSIC-Universitat de València, Apartado de Correos 22085, E-46071 Valencia, Spain
E-mail: german.rodrigo@ific.uv.es

Jan-Christopher Winter
Fermi National Accelerator Laboratory, Batavia, IL 60510, U.S.A.
E-mail: jwinter@fnal.gov

ABSTRACT: We derive a duality relation between one-loop integrals and phase-space integrals emerging from them through single cuts. The duality relation is realized by a modification of the customary $+i0$ prescription of the Feynman propagators. The new prescription regularizing the propagators, which we write in a Lorentz covariant form, compensates for the absence of multiple-cut contributions that appear in the Feynman Tree Theorem. The duality relation can be applied to generic one-loop quantities in any relativistic, local and unitary field theories. We discuss in detail the duality that relates one-loop and tree-level Green's functions. We comment on applications to the analytical calculation of one-loop scattering amplitudes, and to the numerical evaluation of cross-sections at next-to-leading order.

JHEP09(2008)065

- Application of Cauchy theorem taking care of Feynman prescription: leads to a new prescription!
- **Derivation of a duality relation starting from Feynman integrals, extended to scattering amplitudes**
- Applicable to any gauge theory

2008
2010-2012
2014
2015
2016
2017
2018-2019
2019
2020

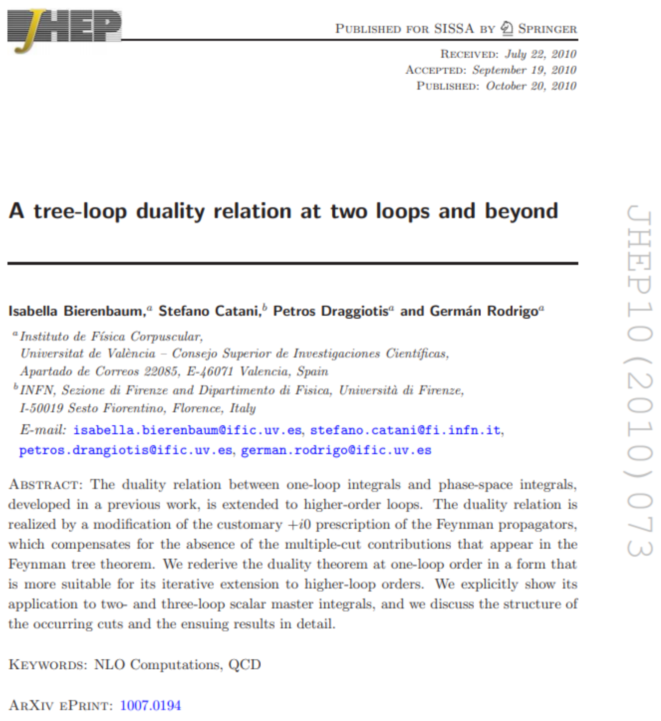
Brief history of LTD-based methods

- Extension to more general amplitudes, including possible local UV counter-terms…
- **Two-loop formula (2010)**

$$L^{(2)}(p_1, p_2, \ldots, p_N) = \int_{\ell_1}\int_{\ell_2} \{-G_D(\alpha_1)\,G_F(\alpha_2)\,G_D(\alpha_3) + G_D(\alpha_1)G_D(\alpha_2 \cup \alpha_3) + G_D(\alpha_3)G_D(-\alpha_1 \cup \alpha_2)\}$$

Uses only double-cuts!

- **Formalism for dealing with higher-order poles (2012)**

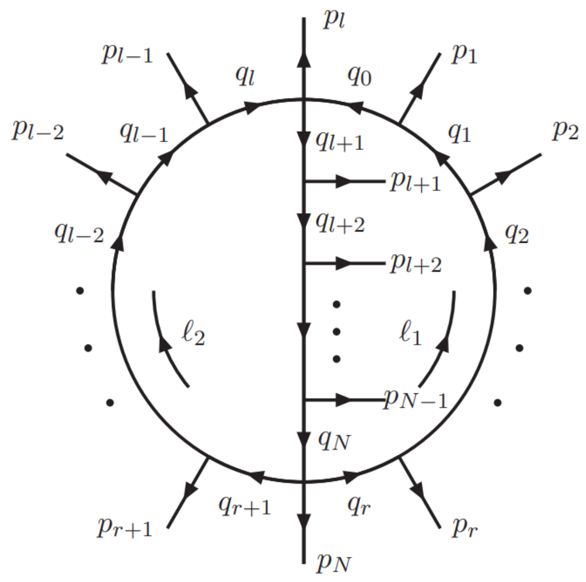

JHEP
Published for SISSA by Springer
Received: July 22, 2010
Accepted: September 19, 2010
Published: October 20, 2010

A tree-loop duality relation at two loops and beyond

Isabella Bierenbaum,[a] Stefano Catani,[b] Petros Draggiotis[a] and Germán Rodrigo[a]
[a] Instituto de Física Corpuscular, Universitat de València – Consejo Superior de Investigaciones Científicas, Apartado de Correos 22085, E-46071 Valencia, Spain
[b] INFN, Sezione di Firenze and Dipartimento di Fisica, Università di Firenze, I-50019 Sesto Fiorentino, Florence, Italy
E-mail: isabella.bierenbaum@ific.uv.es, stefano.catani@fi.infn.it, petros.draggiotis@ific.uv.es, german.rodrigo@ific.uv.es

Abstract: The duality relation between one-loop integrals and phase-space integrals, developed in a previous work, is extended to higher-order loops. The duality relation is realized by a modification of the customary $+i0$ prescription of the Feynman propagators, which compensates for the absence of the multiple-cut contributions that appear in the Feynman tree theorem. We rederive the duality theorem at one-loop order in a form that is more suitable for its iterative extension to higher-loop orders. We explicitly show its application to two- and three-loop scalar master integrals, and we discuss the structure of the occurring cuts and the ensuing results in detail.

Keywords: NLO Computations, QCD
ArXiv ePrint: 1007.0194
JHEP10(2010)073



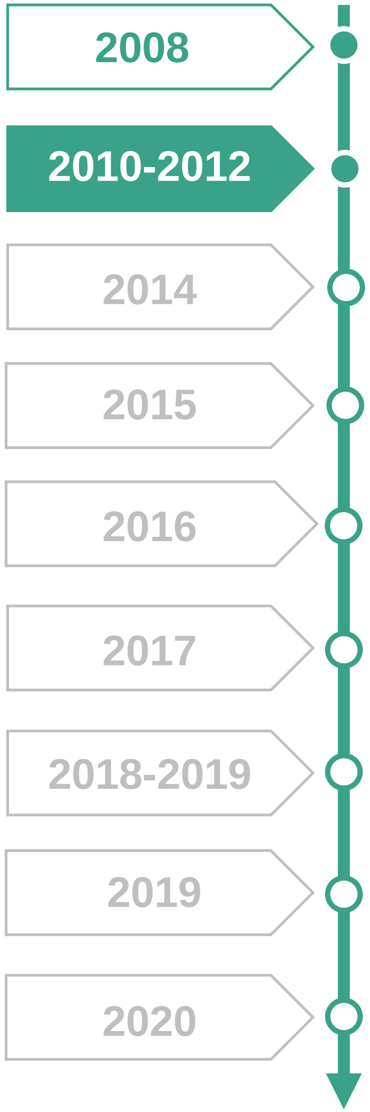

JHEP
Published for SISSA by Springer
Received: November 29, 2012
Accepted: February 13, 2013
Published: March 5, 2013

Tree-loop duality relation beyond single poles

Isabella Bierenbaum,[a] Sebastian Buchta,[b] Petros Draggiotis,[b] Ioannis Malamos[b] and Germán Rodrigo[b]
[a] II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, 22761, Hamburg, Germany
[b] Instituto de Física Corpuscular, Universitat de València, Consejo Superior de Investigaciones Científicas, Parc Científic, E-46980 Paterna (Valencia), Spain
E-mail: isabella.bierenbaum@desy.de, sbuchta@ific.uv.es, petros.draggiotis@ific.uv.es, ioannis.malamos@ific.uv.es, german.rodrigo@ific.uv.es

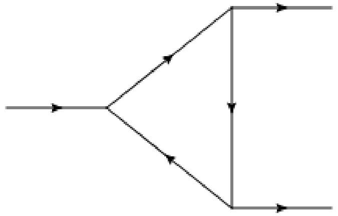
Abstract: We develop the Tree-Loop Duality Relation for two- and three-loop integrals with multiple identical propagators (multiple poles). This is the extension of the Duality Relation for single poles and multi-loop integrals derived in previous publications. We prove a generalization of the formula for single poles to multiple poles and we develop a strategy for dealing with higher-order pole integrals by reducing them to single pole integrals using Integration By Parts.

Keywords: QCD Phenomenology, NLO Computations
ArXiv ePrint: 1211.5048
JHEP03(2013)025

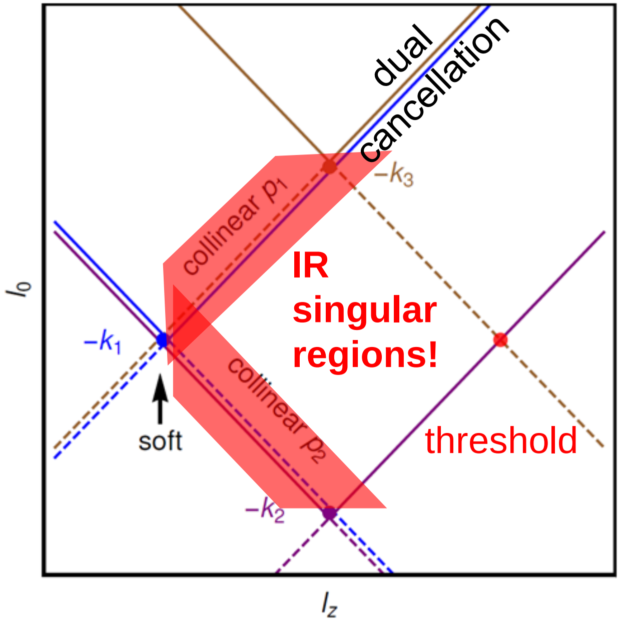


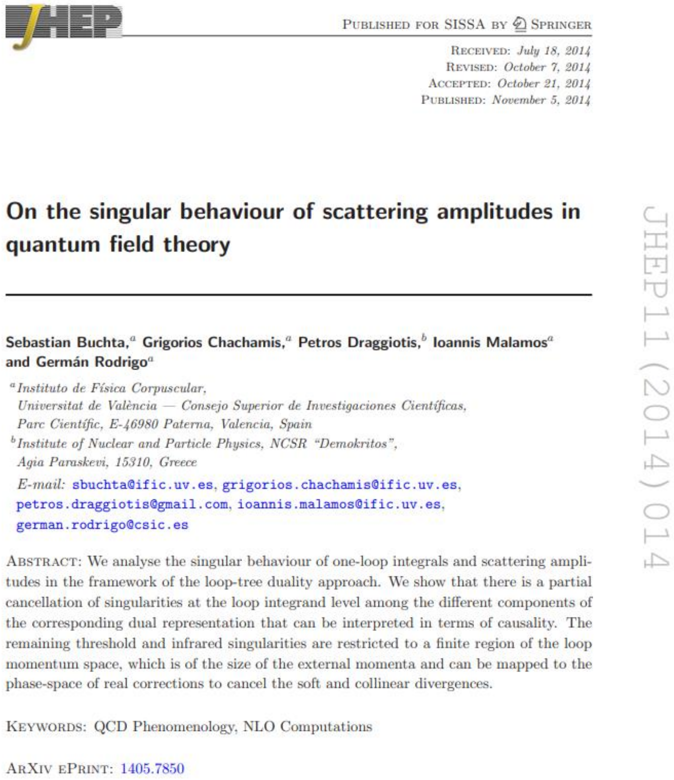
Brief history of LTD-based methods

- Analysis of singular structures of loop amplitudes in LTD representation
- **First clues for real-dual integrand level combination**



Analysis of singularities in triangles



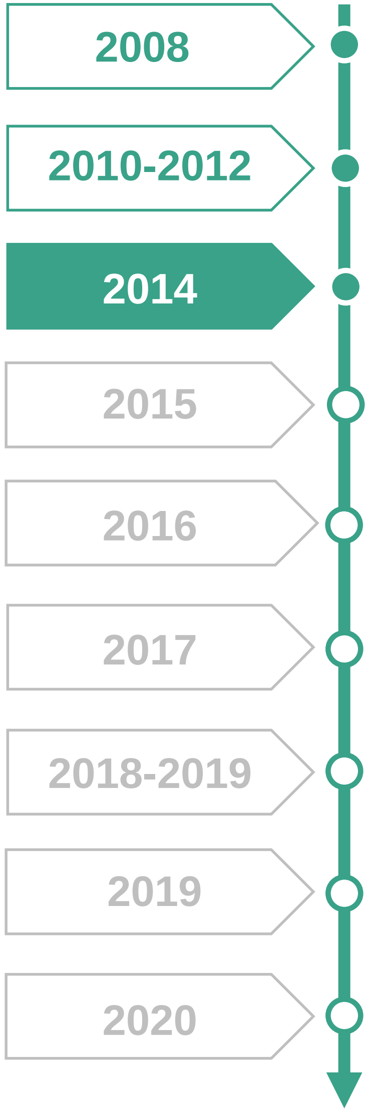

JHEP
Published for SISSA by Springer
Received: July 18, 2014
Revised: October 7, 2014
Accepted: October 21, 2014
Published: November 5, 2014

On the singular behaviour of scattering amplitudes in quantum field theory

Sebastian Buchta,[a] Grigorios Chachamis,[a] Petros Draggiotis,[b] Ioannis Malamos[a] and Germán Rodrigo[a]
[a] Instituto de Física Corpuscular, Universitat de València — Consejo Superior de Investigaciones Científicas, Parc Científic, E-46980 Paterna, Valencia, Spain
[b] Institute of Nuclear and Particle Physics, NCSR "Demokritos", Agia Paraskevi, 15310, Greece
E-mail: sbuchta@ific.uv.es, grigorios.chachamis@ific.uv.es, petros.draggiotis@gmail.com, ioannis.malamos@ific.uv.es, german.rodrigo@csic.es

Abstract: We analyse the singular behaviour of one-loop integrals and scattering amplitudes in the framework of the loop-tree duality approach. We show that there is a partial cancellation of singularities at the loop integrand level among the different components of the corresponding dual representation that can be interpreted in terms of causality. The remaining threshold and infrared singularities are restricted to a finite region of the loop momentum space, which is of the size of the external momenta and can be mapped to the phase-space of real corrections to cancel the soft and collinear divergences.

Keywords: QCD Phenomenology, NLO Computations
ArXiv ePrint: 1405.7850
JHEP11(2014)014



- **Forward (backward) on-shell hyperboloids associated with positive (negative) energy solutions**
- Forward-backward intersections are physical divergences; FF cancel among them

Brief history of LTD-based methods

- Towards the computation of physical observables in four space-time dimensions
- **Tested on toy scalar model; point-by-point cancellation of IR divergences**

JHEP
PUBLISHED FOR SISSA BY SPRINGER
RECEIVED: *September 2, 2015*
REVISED: *December 6, 2015*
ACCEPTED: *January 15, 2016*
PUBLISHED: *February 5, 2016*

Towards gauge theories in four dimensions

Roger J. Hernández-Pinto,[a] Germán F.R. Sborlini[a,b] and Germán Rodrigo[a]

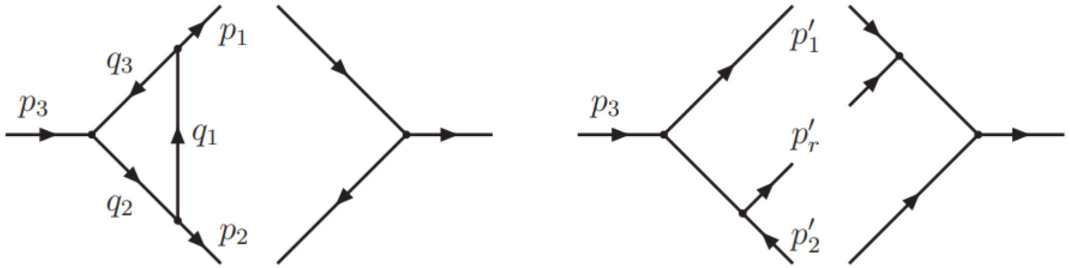
[a] *Instituto de Física Corpuscular, Universitat de València – Consejo Superior de Investigaciones Científicas, Parc Científic, E-46980 Paterna, Valencia, Spain*
[b] *Departamento de Física and IFIBA, FCEyN, Universidad de Buenos Aires, Pabellón 1 Ciudad Universitaria, 1428, Capital Federal, Argentina*
E-mail: rogerjose.hernandez@ific.uv.es, german.sborlini@ific.uv.es, german.rodrigo@csic.es

ABSTRACT: The abundance of infrared singularities in gauge theories due to unresolved emission of massless particles (soft and collinear) represents the main difficulty in perturbative calculations. They are typically regularized in dimensional regularization, and their subtraction is usually achieved independently for virtual and real corrections. In this paper, we introduce a new method based on the loop-tree duality (LTD) theorem to accomplish the summation over degenerate infrared states directly at the integrand level such that the cancellation of the infrared divergences is achieved simultaneously, and apply it to reference examples as a proof of concept. Ultraviolet divergences, which are the consequence of the point-like nature of the theory, are also reinterpreted physically in this framework. The proposed method opens the intriguing possibility of carrying out purely four-dimensional implementations of higher-order perturbative calculations at next-to-leading order (NLO) and beyond free of soft and final-state collinear subtractions.

KEYWORDS: NLO Computations

ARXIV EPRINT: 1506.04617

JHEP02(2016)044



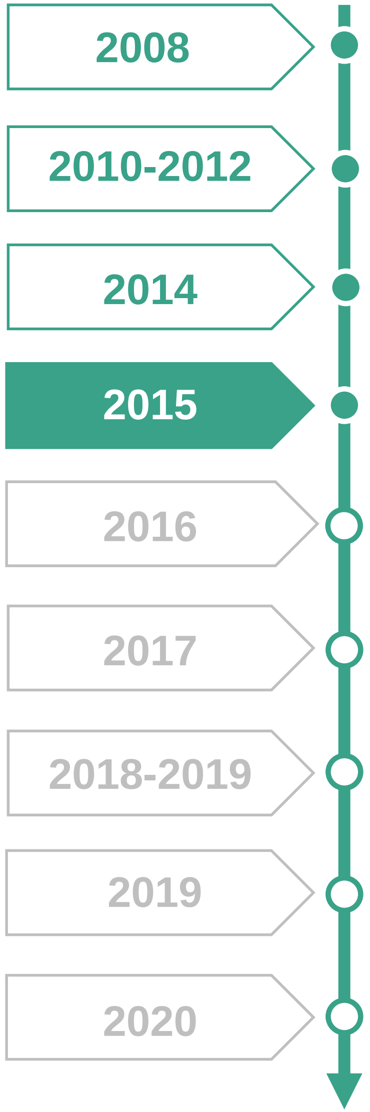
- **Introduction of real-dual mappings, to achieve a local cancellation of IR singularities!**

$$p_r'^{\mu} = q_1^{\mu}\,, \qquad p_1'^{\mu} = -q_3^{\mu} + \alpha_1\, p_2^{\mu} = p_1^{\mu} - q_1^{\mu} + \alpha_1\, p_2^{\mu}\,,$$

$$p_2'^{\mu} = (1-\alpha_1)\, p_2^{\mu}\,, \qquad \alpha_1 = \frac{q_3^2}{2 q_3 \cdot p_2}\,,$$

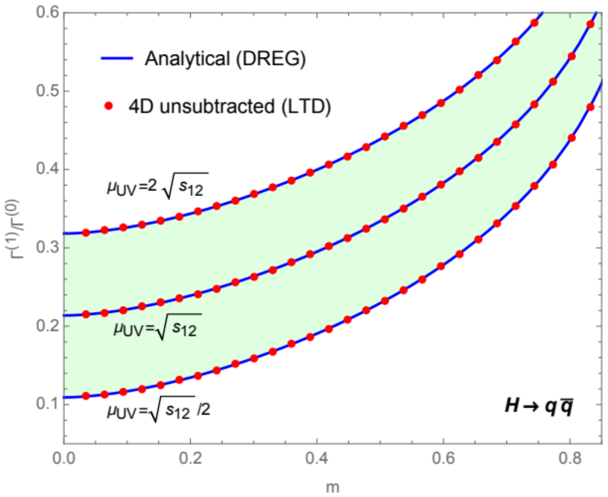
- Purely four-dimensional representation of cross-sections
- First study of dual UV *local* counter-terms:

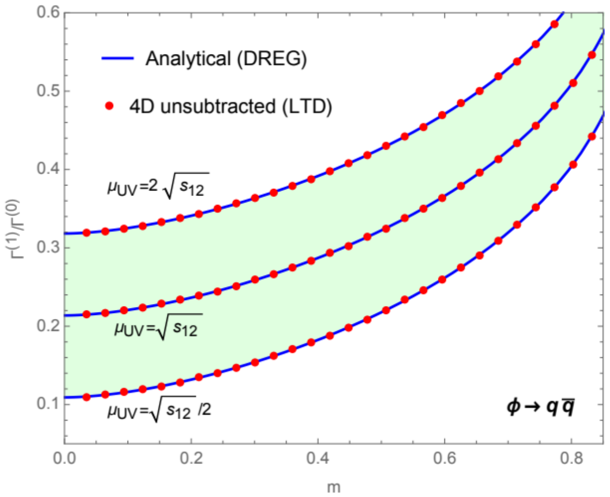
$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{1}{\left(q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0\right)^2}$$



Brief history of LTD-based methods

- Development of the **Four Dimensional Unsubtraction (FDU)** framework @ NLO
- **Ingredients for local cancellation of IR singularities**
- Benchmark case: decay into quarks (both, massive and massless cases) → **Smooth m=0 transition!**





- **Full control of IR/UV singularities in self-energies**

$$\Delta Z_2(p_1) = g_S^2 C_F \int_\ell \left[\frac{\tilde\delta(q_1)}{-2q_1\cdot p_1}\left((d-2)\frac{q_1\cdot p_2}{p_1\cdot p_2} - \frac{4M^2}{2q_1\cdot p_1}\left(1-\frac{q_1\cdot p_2}{p_1\cdot p_2}\right)\right) + \frac{\tilde\delta(q_3)}{2M^2+2q_3\cdot p_1} \times \left((d-2)\left(1+\frac{q_3\cdot p_2}{p_1\cdot p_2}\right) + \frac{4M^2}{p_1\cdot p_2}\left(-\frac{\mathbf{q}_3\cdot\mathbf{p}_2}{2\left(q_{3,0}^{(+)}\right)^2} + \frac{\left(q_{3,0}^{(+)}+p_{1,0}\right)q_3\cdot p_2}{q_{3,0}^{(+)}(2M^2+2q_3\cdot p_1)}\right)\right)\right]$$

$$\Delta Z_2^{\rm UV} = -(d-2)\, g_S^2\, C_F \int_\ell \frac{\tilde\delta\left(q_{\rm UV}\right)}{2\left(q_{{\rm UV},0}^{(+)}\right)^2} \left[\left(1-\frac{\mathbf{q}_{\rm UV}\cdot\mathbf{p}_2}{p_1\cdot p_2}\right) \times \left(1-\frac{3(2\mathbf{q}_{\rm UV}\cdot\mathbf{p}_1-\mu_{\rm UV}^2)}{4\left(q_{{\rm UV},0}^{(+)}\right)^2}\right) - \frac{p_{1,0}\,p_{2,0}}{2p_1\cdot p_2}\right]$$

IR (left) and UV (right) part of fermion wavefunction renormalization

JHEP — PUBLISHED FOR SISSA BY SPRINGER
RECEIVED: April 27, 2016
REVISED: July 13, 2016
ACCEPTED: August 20, 2016
PUBLISHED: August 29, 2016

Four-dimensional unsubtraction from the loop-tree duality

Germán F.R. Sborlini,[a,b] Félix Driencourt-Mangin,[a] Roger J. Hernández-Pinto[a,c] and Germán Rodrigo[a]

JHEP08(2016)160

JHEP — PUBLISHED FOR SISSA BY SPRINGER
RECEIVED: September 19, 2016
ACCEPTED: October 18, 2016
PUBLISHED: October 28, 2016

Four-dimensional unsubtraction with massive particles

Germán F.R. Sborlini, Félix Driencourt-Mangin and Germán Rodrigo
Instituto de Física Corpuscular, Universitat de València, Consejo Superior de Investigaciones Científicas, Parc Científic, Paterna, Valencia, E-46980 Spain
E-mail: german.sborlini@ific.uv.es, felix.dm@ific.uv.es, german.rodrigo@csic.es

ABSTRACT: We extend the four-dimensional unsubtraction method, which is based on the loop-tree duality (LTD), to deal with processes involving heavy particles. The method allows to perform the summation over degenerate IR configurations directly at integrand level in such a way that NLO corrections can be implemented directly in four space-time dimensions. We define a general momentum mapping between the real and virtual kinematics that accounts properly for the quasi-collinear configurations, and leads to an smooth massless limit. We illustrate the method first with a scalar toy example, and then analyse the case of the decay of a scalar or vector boson into a pair of massive quarks. The results presented in this paper are suitable for the application of the method to any multipartonic process.

KEYWORDS: NLO Computations
ARXIV EPRINT: 1608.01584
JHEP10(2016)162

- 2008
- 2010-2012
- 2014
- 2015
- **2016**
- 2017
- 2018-2019
- 2019
- 2020

Brief history of LTD-based methods

- Infinite-mass limit used to define effective vertices in *Hgg* interactions
- Equivalent to explore asymptotic expansions (large mass limit)
- Expansions at integrand level are non-trivial in Minkowski space (i.e. within Feynman integrals) and additional factors are necessary
- **Dual amplitudes defined in Euclidean space → Simplifies expansions**
- **Universal structure for Higgs amplitudes *(i.e. independent of internal particles)***

$$\tilde{\delta}(q_3)\, G_D(q_3;q_2) = \frac{\tilde{\delta}(q_3)}{s_{12}+2q_3\cdot p_{12}-\imath 0} \quad \xrightarrow{M_f^2 \gg s_{12}} \quad \tilde{\delta}(q_3)\, G_D(q_3;q_2) = \frac{\tilde{\delta}(q_3)}{2q_3\cdot p_{12}} \sum_{n=0}^{\infty} \left(\frac{-s_{12}}{2q_3\cdot p_{12}} \right)^n$$

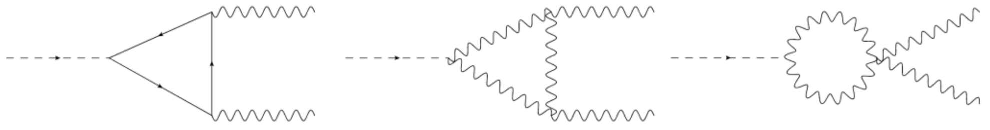
Expansion of the dual propagator (q_3 on-shell)

Eur. Phys. J. C (2018) 78:231
https://doi.org/10.1140/epjc/s10052-018-5692-5
The European Physical Journal C
CrossMark
Regular Article - Theoretical Physics

Universal dual amplitudes and asymptotic expansions for $gg \to H$ and $H \to \gamma\gamma$ in four dimensions

Félix Driencourt-Mangin[1,a], Germán Rodrigo[1,b], Germán F. R. Sborlini[1,2,c]
[1] Instituto de Física Corpuscular, Universitat de València, Consejo Superior de Investigaciones Científicas, Parc Científic, 46980 Paterna, Valencia, Spain
[2] Dipartimento di Fisica, Università di Milano and INFN Sezione di Milano, 20133 Milan, Italy

Received: 13 October 2017 / Accepted: 3 March 2018 / Published online: 17 March 2018



$H \to \gamma\gamma$

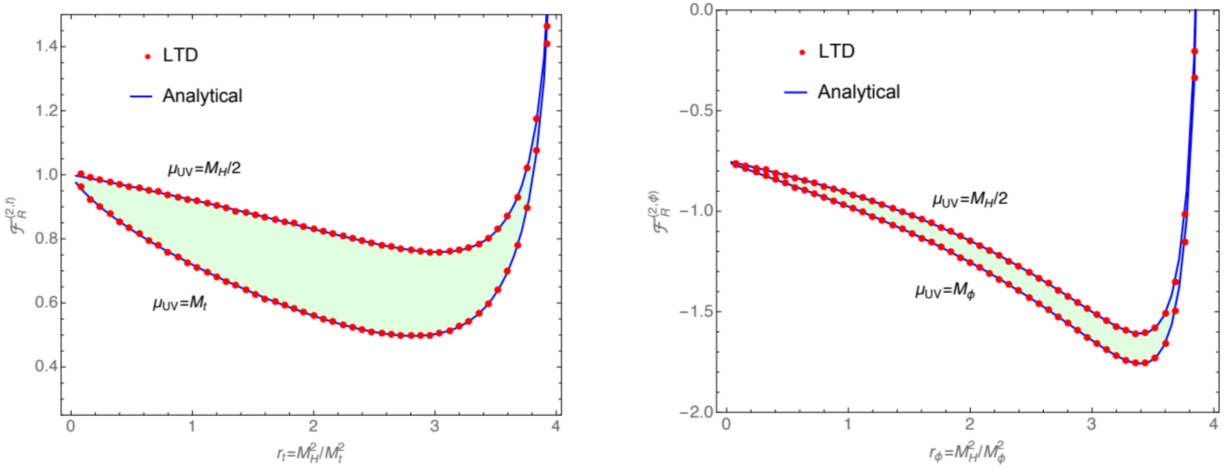
Selected case of study: Higgs decay into photons in four dimensions

FINITE IN D=4 BUT NOT (TRIVIALLY) REGULAR!

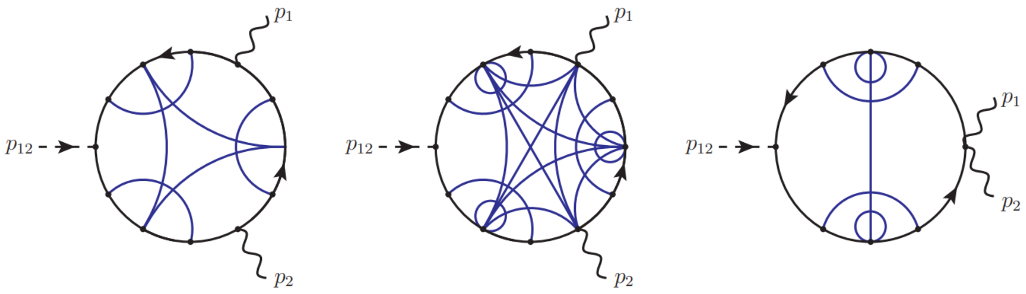
2008 | 2010-2012 | 2014 | 2015 | 2016 | **2017** | 2018-2019 | 2019 | 2020

Brief history of LTD-based methods

- Full analysis of Higgs decays at two-loop (inclusion of EW effects)
- **Known results recovered! Purely four dimensional implementation!**
- Universal structure recovered also at two loops (*is it true for N loops?*)
- **First realization of local UV counter-terms at two-loop level**



Comparison with analytical results for fermion (right) and scalar (left) insertions



Two-loop diagrams considered for EW corrections

JHEP

PUBLISHED FOR SISSA BY SPRINGER

RECEIVED: February 12, 2019
ACCEPTED: February 18, 2019
PUBLISHED: February 21, 2019

Universal four-dimensional representation of $H \to \gamma\gamma$ at two loops through the Loop-Tree Duality

Félix Driencourt-Mangin,[a] Germán Rodrigo,[a] Germán F.R. Sborlini[a,b,c] and William J. Torres Bobadilla[a]

[a] IFIC, Universitat de València-CSIC, Apt. Correus 22085, E-46071 València, Spain
[b] Dipartimento di Fisica, Università di Milano and INFN Sezione di Milano, I-20133 Milano, Italy
[c] International Center for Advanced Studies (ICAS), ECyT-UNSAM, Campus Miguelete, 25 de Mayo y Francia, (1650) Buenos Aires, Argentina

E-mail: felix.dm@ific.uv.es, german.rodrigo@csic.es, german.sborlini@unimi.it, william.torres@ific.uv.es

ABSTRACT: We extend useful properties of the $H \to \gamma\gamma$ unintegrated dual amplitudes from one- to two-loop level, using the Loop-Tree Duality formalism. In particular, we show that the universality of the functional form — regardless of the nature of the internal particle — still holds at this order. We also present an algorithmic way to renormalise two-loop amplitudes, by locally cancelling the ultraviolet singularities at integrand level, thus allowing a full four-dimensional numerical implementation of the method. Our results are compared with analytic expressions already available in the literature, finding a perfect numerical agreement. The success of this computation plays a crucial role for the development of a fully local four-dimensional framework to compute physical observables at Next-to-Next-to Leading order and beyond.

KEYWORDS: Scattering Amplitudes, Higgs Physics, Perturbative QCD

ARXIV EPRINT: 1901.09853

JHEP02(2019)143

- 2008
- 2010-2012
- 2014
- 2015
- 2016
- 2017
- **2018-2019**
- 2019
- 2020

Brief history of LTD-based methods

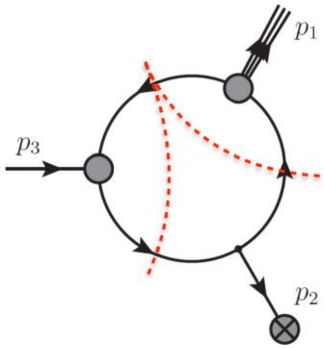
- **Novel approach to deal with threshold singularities, including anomalous ones**
- *Singularities are contained within a compact region in the LTD representation*

Crucial for extending FDU to higher-orders!

- Singularities in terms of novel (suggestive) variables:

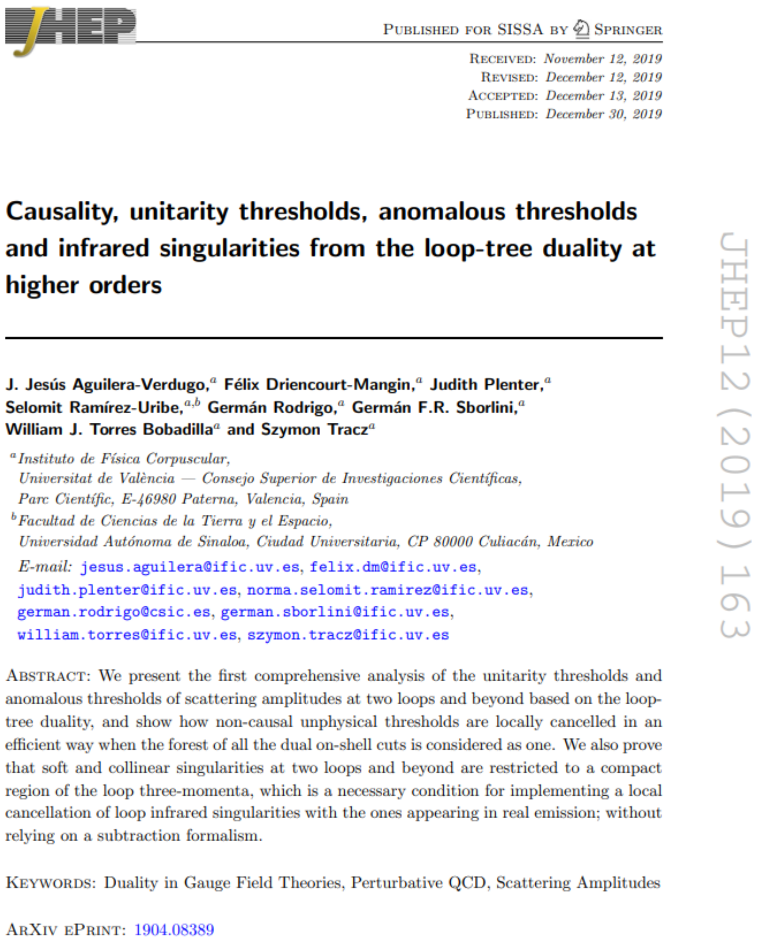
$$\lambda_{ij}^{\pm\pm} = \pm q_{i,0}^{(+)} \pm q_{j,0}^{(+)} + k_{ji,0} = 0$$

- Description of thresholds: **unveiling the causal structure of loop scattering amplitudes!**



Anomalous thresholds arising from causal (time-like) singularities in multiple on-shell propagators simultaneously

Special thanks to D. Broadhurst for suggesting this interesting and illuminating problem!!



JHEP

Published for SISSA by Springer

Received: November 12, 2019
Revised: December 12, 2019
Accepted: December 13, 2019
Published: December 30, 2019

Causality, unitarity thresholds, anomalous thresholds and infrared singularities from the loop-tree duality at higher orders

J. Jesús Aguilera-Verdugo,[a] Félix Driencourt-Mangin,[a] Judith Plenter,[a] Selomit Ramírez-Uribe,[a,b] Germán Rodrigo,[a] Germán F.R. Sborlini,[a] William J. Torres Bobadilla[a] and Szymon Tracz[a]

[a] *Instituto de Física Corpuscular, Universitat de València — Consejo Superior de Investigaciones Científicas, Parc Científic, E-46980 Paterna, Valencia, Spain*

[b] *Facultad de Ciencias de la Tierra y el Espacio, Universidad Autónoma de Sinaloa, Ciudad Universitaria, CP 80000 Culiacán, Mexico*

E-mail: jesus.aguilera@ific.uv.es, felix.dm@ific.uv.es, judith.plenter@ific.uv.es, norma.selomit.ramirez@ific.uv.es, german.rodrigo@csic.es, german.sborlini@ific.uv.es, william.torres@ific.uv.es, szymon.tracz@ific.uv.es

Abstract: We present the first comprehensive analysis of the unitarity thresholds and anomalous thresholds of scattering amplitudes at two loops and beyond based on the loop-tree duality, and show how non-causal unphysical thresholds are locally cancelled in an efficient way when the forest of all the dual on-shell cuts is considered as one. We also prove that soft and collinear singularities at two loops and beyond are restricted to a compact region of the loop three-momenta, which is a necessary condition for implementing a local cancellation of loop infrared singularities with the ones appearing in real emission; without relying on a subtraction formalism.

Keywords: Duality in Gauge Field Theories, Perturbative QCD, Scattering Amplitudes

ArXiv ePrint: 1904.08389

JHEP12(2019)163

2008
2010-2012
2014
2015
2016
2017
2018-2019
2019
2020

PHYSICAL REVIEW LETTERS **124,** 211602 (2020)

Open Loop Amplitudes and Causality to All Orders and Powers from the Loop-Tree Duality

J. Jesús Aguilera-Verdugo,[1,*] Félix Driencourt-Mangin,[1,†] Roger J. Hernández-Pinto,[2,‡] Judith Plenter,[1,§] Selomit Ramírez-Uribe,[1,2,3,∥] Andrés E. Rentería-Olivo,[1,¶] Germán Rodrigo,[1,**] Germán F. R. Sborlini,[1,††] William J. Torres Bobadilla,[1,‡‡] and Szymon Tracz[1,§§]

[1]*Instituto de Física Corpuscular, Universitat de València—Consejo Superior de Investigaciones Científicas, Parc Científic, E-46980 Paterna, Valencia, Spain*

[2]*Facultad de Ciencias Físico-Matemáticas, Universidad Autónoma de Sinaloa, Ciudad Universitaria, CP 80000 Culiacán, Mexico*

[3]*Facultad de Ciencias de la Tierra y el Espacio, Universidad Autónoma de Sinaloa, Ciudad Universitaria, CP 80000 Culiacán, Mexico*

(Received 16 January 2020; revised manuscript received 27 March 2020; accepted 1 May 2020; published 28 May 2020)

Multiloop scattering amplitudes describing the quantum fluctuations at high-energy scattering processes are the main bottleneck in perturbative quantum field theory. The loop-tree duality is a novel method aimed at overcoming this bottleneck by opening the loop amplitudes into trees and combining them at integrand level with the real-emission matrix elements. In this Letter, we generalize the loop-tree duality to all orders in the perturbative expansion by using the complex Lorentz-covariant prescription of the original one-loop formulation. We introduce a series of mutiloop topologies with arbitrary internal configurations and derive very compact and factorizable expressions of their open-to-trees representation in the loop-tree duality formalism. Furthermore, these expressions are entirely independent at integrand level of the initial assignments of momentum flows in the Feynman representation and remarkably free of noncausal singularities. These properties, that we conjecture to hold to other topologies at all orders, provide integrand representations of scattering amplitudes that exhibit manifest causal singular structures and better numerical stability than in other representations.

DOI: 10.1103/PhysRevLett.124.211602

Jan. '20

arXiv:2006.11217 [pdf, other] hep-ph hep-th

Causal representation of multi-loop amplitudes within the loop-tree duality

Authors: J. Jesus Aguilera-Verdugo, Roger J. Hernandez-Pinto, German Rodrigo, German F. R. Sborlini, William J. Torres Bobadilla

Abstract: The numerical evaluation of multi-loop scattering amplitudes in the Feynman representation usually requires to deal with both physical (causal) and unphysical (non-causal) singularities. The loop-tree duality (LTD) offers a powerful framework to easily characterise and distinguish these two types of singularities, and then simplify analytically the underling expressions. In this paper, we work exp... ▽ More

Submitted 19 June, 2020; **originally announced** June 2020.

Comments: 24 pages, 8 figures

Report number: IFIC/20-27

Jun. '20

arXiv:2006.13818 [pdf, other] hep-ph hep-th

Universal opening of four-loop scattering amplitudes to trees

Authors: Selomit Ramirez-Uribe, Roger J. Hernandez-Pinto, German Rodrigo, German F. R. Sborlini, William J. Torres Bobadilla

Abstract: The perturbative approach to quantum field theories has made it possible to obtain incredibly accurate theoretical predictions in high-energy physics. Although various techniques have been developed to boost the efficiency of these calculations, some ingredients remain specially challenging. This is the case of multiloop scattering amplitudes that constitute a hard bottleneck to solve. In this Let... ▽ More

Submitted 24 June, 2020; **originally announced** June 2020.

Comments: 7 pages, 4 figures

Report number: IFIC/20-29

Jun. '20

arXiv:2010.12971 [pdf, other] hep-ph hep-th math-ph

Mathematical properties of nested residues and their application to multi-loop scattering amplitudes

Authors: J. Jesus Aguilera-Verdugo, Roger J. Hernandez-Pinto, German Rodrigo, German F. R. Sborlini, William J. Torres Bobadilla

Abstract: The computation of multi-loop multi-leg scattering amplitudes plays a key role to improve the precision of theoretical predictions for particle physics at high-energy colliders. In this work, we focus on the mathematical properties of the novel integrand-level representation of Feynman integrals, which is based on the Loop-Tree Duality (LTD). We explore the behaviour of the multi-loop iterated res... ▽ More

Submitted 24 October, 2020; **originally announced** October 2020.

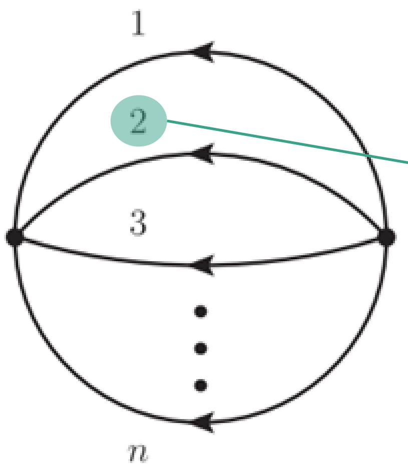
Comments: 29 pages + appendices, 11 figures

Report number: IFIC/20-30; DESY 20-172; MPP-2020-184

Oct. '20

- 2008
- 2010-2012
- 2014
- 2015
- 2016
- 2017
- 2018-2019
- 2019
- **2020**

- *Starting point:* multi-loop Feynman integrals and scattering amplitudes
- ***Iterated** application of the Cauchy residue theorem to remove one DOF for each loop momenta*



Multi-loop diagram

Notation setup

Sets of momenta $i_s \in s$

$$q_{i_s} = \ell_s + k_{i_s}$$

Combination of external momenta (k_{i_s})

Loop momentum (integration) (ℓ_s)

$$G_F(q_{i_s}) = \frac{1}{q_{i_s,0}^2 - (q_{i_s,0}^{(+)})^2}$$

Loop-energy component ($q_{i_s,0}$)

Standard propagator

$$q_{i_s,0}^{(+)} = \sqrt{\mathbf{q}_{i_s}^2 + m_{i_s}^2 - \imath 0}$$

On-shell energy (*complex number!*)

Loop space-vector (*Euclidean*) ($\mathbf{q}_{i_s}$)

Feynman i0 prescription ($-\imath 0$)

- Using this notation, we write *any* L-loop N-particle scattering amplitude:

$$\mathcal{A}_N^{(L)}(1,\ldots,n) = \int_{\ell_1,\ldots,\ell_L} \mathcal{N}(\{\ell_i\}_L, \{p_j\}_N) G_F(1,\ldots,n) \quad \text{with} \quad G_F(1,\ldots,n) = \prod_{i \in 1 \cup \cdots \cup n} (G_F(q_i))^{a_i}$$

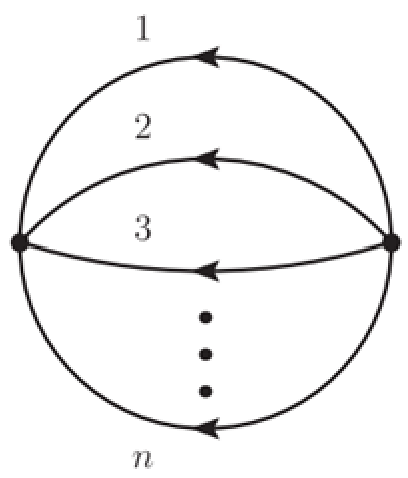
D-dimensional loop momenta (*Minkowski*) ($\ell_1,\ldots,\ell_L$)

Sets of momenta ($i \in 1 \cup \cdots \cup n$)

Aguilera-Verdugo et al (2020) Phys. Rev. Lett. 124, 211602

Nested residues

- *Starting point:* multi-loop Feynman integrals and scattering amplitudes
- ***Iterated** application of the Cauchy residue theorem to remove one DOF for each loop momenta*



Multi-loop diagram

Application of Cauchy's theorem

Dual function (1st step)

$$G_F(1,\ldots,n) = \prod_{i\in 1\cup\cdots\cup n} (G_F(q_i))^{a_i} \Rightarrow G_D(s;t) = -2\pi\imath \sum_{i_s\in s} \mathrm{Res}(G_F(s,t), \mathrm{Im}(\eta\cdot q_{i_s}) < 0)$$

Select one set "s" and compute the residue

Other sets (no residue computation)

Sum over all the elements of the set

Pole selection criteria! **(IMPORTANT)**

- **Observation 1:** For single powers and $\eta = (1,\mathbf{0})$ we get the well-know one-loop LTD formula:

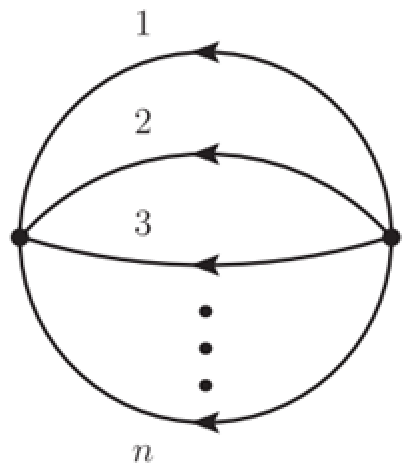
$$G_D(s) = -\sum_{i_s\in s} \tilde{\delta}(q_{i_s}) \prod_{\substack{j_s\neq i_s \\ j_s\in s}} \frac{1}{(q_{i_s,0}^{(+)} + k_{j_s i_s,0})^2 - (q_{j_s,0}^{(+)})^2}$$

- **Observation 2:** The equivalence with previous LTD representation is encoded in $\mathrm{Im}(\eta\cdot q_{i_s}) < 0$ for the integration contour selection (*"dual prescription"*)

Aguilera-Verdugo et al (2020) Phys. Rev. Lett. 124, 211602

Nested residues

- *Starting point:* multi-loop Feynman integrals and scattering amplitudes
- ***Iterated** application of the Cauchy residue theorem to remove one DOF for each loop momenta*



Multi-loop diagram

Iterated application of Cauchy's theorem

Remaining sets (no residue evaluation)

$$G_D(1,\ldots,r;n) = -2\pi\imath \sum_{i_r \in r} \mathrm{Res}(G_D(1,\ldots,r-1;r,n), \mathrm{Im}(\eta \cdot q_{i_r}) < 0)$$

r^{th} residue evaluation

Sum over all the elements of the r^{th} set

$(r-1)^{th}$ dual function

Depends on integration variables (q_i)

Poles could be in-or-out depending on specific momenta…

- Dual representation for L-loop amplitudes is obtained after the L^{th} residue evaluation
- *Equivalent to: **"Number of cuts equal number of loops"***
- ***Sum over all possible poles is implicit: some contributions vanish inside each iteration***

Iterated residues (all the poles) → cancellations, **"Displaced poles"** → **Nested residues *(only physical ones)***

Aguilera-Verdugo et al (2020) Phys. Rev. Lett. 124, 211602

Nested residues: Displaced poles

- Iterated application of the Cauchy residue theorem involves summing over all the poles inside the integration contour ➡ Selection criteria imposed $\text{Im}(\eta \cdot q_{i_s}) < 0$ (to simplify: $\eta = (1, \mathbf{0})$)
- **Displaced poles:** Poles with non-trivially negative imaginary part (i.e. it depends on the kinematics)
- **Important result:**

> **"Contributions associated to displaced poles are vanishing after each iteration of residue evaluation"**

- **Consequences & observations**
  1. Definition of **nested** residues: compact results (compared to **iterated** residues)
  2. Direct consequence of the quadratic structure of propagators
  3. After the **cancellation of displaced poles**, all the remaining contributions can be mapped into cut diagrams (**physical contributions**)
  4. **Deep connection with causality** (more… later!)

Aguilera-Verdugo et al (2020) arXiv:2010.12971 [hep-ph]

Nested residues: Displaced poles

- Practical (mathematical) example:

$$f(\vec{x}) = \frac{1}{(x_1^2 - y_1^2)\dots(x_L^2 - y_L^2)\,(z_{L+1}^2 - y_{L+1}^2)}$$

to calculate

$$I = \left(\prod_{i=1}^{L} \int \frac{dx_i}{2\pi\imath}\right) f(\vec{x})$$

Complex coefficients $y_i \to \tilde{y}_i = \sqrt{y_i^2 - \imath 0}$

$$z_{L+1} = -\sum_{j=1}^{L} x_j + k_{L+1}$$

Sum of integration variables (real)

- **1st step:** Apply C.R.T. in x_1, by promoting $x_1 \in \mathbb{R} \to \mathbb{C}$ (the other *x*'s remain real)

$$I = -\left(\prod_{i=2}^{L} \int \frac{dx_i}{2\pi\imath}\right) \sum_{x_{1,j}\in \text{Poles}[f,x_1]} \text{Res}\left(f(\vec{x}), \{x_1, x_{1,j}\}\right) \theta(-\text{Im}(x_{1,j})) \Rightarrow I = -\left(\prod_{i=2}^{L} \int \frac{dx_i}{2\pi\imath}\right) \sum_{x_{1,j}\in \text{Poles}^{(+)}[f,x_1]} \text{Res}\left(f(\vec{x}), \{x_1, x_{1,j}\}\right)$$

Theta functions removed

$$\text{Poles}^{(+)}[f,x_1] = \{y_1, y_{L+1} - k_{L+1} - x_2 - \ldots - x_L\}$$

Subset of poles with negative imaginary part

IMPORTANT! x's are real, y's are complex

Aguilera-Verdugo et al (2020) arXiv:2010.12971 [hep-ph]

Nested residues: Displaced poles

- Practical (mathematical) example:

$$I = -\left(\prod_{i=2}^{L}\int \frac{dx_i}{2\pi\imath}\right) \sum_{x_{1,j}\in \text{Poles}^{(+)}[f,x_1]} \text{Res}\left(f(\vec{x}), \{x_1, x_{1,j}\}\right)$$

$$\text{Poles}^{(+)}[f, x_1] = \{y_1, y_{L+1} - k_{L+1} - x_2 - \ldots - x_L\}$$

$$\text{Res}(f, \{x_1, \text{Im}(x_1) < 0\}) = \frac{1}{2y_1\,(x_2^2 - y_2^2)\ldots(x_L^2 - y_L^2)\left((y_1 + x_2 + \ldots + x_L - k_{L+1})^2 - y_{L+1}^2\right)} + \frac{1}{2y_{L+1}\left((y_{L+1} + k_{L+1} - x_2 - \ldots - x_L)^2 - y_1^2\right)(x_2^2 - y_2^2)\ldots(x_L^2 - y_L^2)}$$

Sum of the residues in x_1 (negative imaginary part)

- **2nd step:** Apply C.R.T. in x_2, by promoting $x_2 \in \mathbb{R} \to \mathbb{C}$ (the other *x*'s remain real)

$$\text{Res}(\text{Res}(f, \{x_1, \text{Im}(x_1) < 0\}), \{x_2, \text{Im}(x_2) < 0\}) = \sum_{x_{2,l}\in \text{Poles}[f,x_1,x_2]} \text{Res}((\text{Res}(f, \{x_1, \text{Im}(x_1) < 0\}), \{x_2, x_{2,l}\})\,\theta(-\text{Im}(x_{2,l}))$$

Theta functions remain!

$$\text{Poles}[f, x_1; x_2] = \{\pm y_2, \pm y_1 + y_{L+1} - x_3 - \ldots - x_L + k_{L+1}, \pm y_{L+1} - y_1 - x_3 - \ldots - x_L + k_{L+1}\}$$

All the possible poles:
SIGN OF IMAGINARY PART + or - !!!

Aguilera-Verdugo et al (2020) arXiv:2010.12971 [hep-ph]

Nested residues: Displaced poles

- Practical (mathematical) example:

$$\text{Res}(\,\text{Res}(f,\{x_1,\text{Im}(x_1)<0\})\,,\{x_2,\text{Im}(x_2)<0\})\;=\sum_{x_{2,l}\in\text{Poles}[f,x_1,x_2]}\text{Res}(\,(\text{Res}(f,\{x_1,\text{Im}(x_1)<0\}),\{x_2,x_{2,l}\}\,)\,\theta(-\text{Im}(x_{2,l}))$$

- **3rd step:** Collect the different contributions according to $\theta(-\text{Im}(x_{2,l}))$:

$$\text{Res}(\,(\text{Res}(f,\{x_1,\text{Im}(x_1)<0\}),\{x_2,y_2\}\,)$$
$$=\frac{1}{4y_1y_2\,(x_3^2-y_3^2)\ldots(x_L^2-y_L^2)((y_1+y_2+x_3+\ldots+x_L-k_{L+1})^2-y_{L+1}^2)}$$
$$+\frac{1}{4y_{L+1}y_2\,((y_{L+1}-y_2-x_3-\ldots-x_L+k_{L+1})^2-y_1^2)\ldots(x_L^2-y_L^2)}$$

$$\text{Res}(\,\text{Res}(f,\{x_1,\text{Im}(x_1)<0\}),\{x_2,y_1+y_{L+1}-x_3-\ldots-x_L+k_{L+1}\}\,)$$
$$=\frac{1}{4y_1y_3\,((y_1+y_{L+1}-x_3-\ldots-x_L+k_{L+1})^2-y_2^2)(x_3^2-y_3^2)\ldots(x_L^2-y_L^2)}$$

Theta functions are trivially 1: y's have negative imaginary part, x's are real → **Only sums of y's!!! ALIGNED CONTRIBUTIONS**

$$[\text{Res}(\,\text{Res}(f,\{x_1,y_1\}),\{x_2,y_{L+1}-y_1-x_3-\ldots-x_L+k_{L+1}\}\,)$$
$$+\text{Res}(\,\text{Res}(f,\{x_1,y_{L+1}-x_2-\ldots-x_L+k_{L+1}\}),$$
$$\{x_2,y_{L+1}-y_1-x_3-\ldots-x_L+k_{L+1}\}\,)]\;\theta(\text{Im}(y_1-y_{L+1}))$$

Different-sign combinations of y's: **NON-TRIVIAL THETA!** → **DISPLACED POLES: VANISH!!**

Aguilera-Verdugo et al (2020) arXiv:2010.12971 [hep-ph]

Nested residues: Displaced poles

- *Theorem*: Given a generic* rational function $F(x_i, x_j) = \dfrac{P(x_i, x_j)}{((x_i - a_i)^2 - y_i^2)^{\gamma_i}((x_i + x_j - a_{ij})^2 - y_k^2)^{\gamma_k}}$

  then: $$\mathrm{Res}(\mathrm{Res}(F(x_i,x_j), \{x_i, y_i + a_i\}), \{x_j, y_k - y_i + a_{ij} - a_i\})$$
  $$= -\mathrm{Res}\left(\mathrm{Res}\left(F(x_i, x_j), \{x_i, y_k - x_j + a_{ij}\}\right), \{x_j, y_k - y_i + a_{ij} - a_i\}\right)$$

- **Mathematical consequences:**
  1. In each iteration of C.R.T., contributions with **different sign combinations of *y*'s vanish**
  2. Thus, after iterating over all integration variables, **only same-sign combinations of *y*'s remain**

Example: $L = 2$

$$\mathrm{Res}(\,\mathrm{Res}(f, \{x_1, \mathrm{Im}(x_1) < 0\}), \{x_2, \mathrm{Im}(x_2) < 0\}\,)$$
$$= \frac{1}{4y_1y_2\left((y_1 + y_2 - k_3)^2 - y_3^2\right)} + \frac{1}{4y_2y_3\left((y_3 + y_1 + k_3)^2 - y_2^2\right)}$$
$$+ \frac{1}{4y_1y_3\left((y_3 - y_2 + k_3)^2 - y_1^2\right)}$$
$$= -\frac{1}{8y_1y_2y_3}\left(\frac{1}{y_1 + y_2 + y_3 - k_3} + \frac{1}{y_1 + y_2 + y_3 + k_3}\right)$$

Connection to QFT

$y_i \Leftrightarrow q_{i,0}^{(+)} = \sqrt{\boldsymbol{q}_i^2 + m_i^2 - \imath 0}$

$x_i \Leftrightarrow q_{i,0}$

$a_i \Leftrightarrow \{k_{m,0}\}$

Aguilera-Verdugo et al (2020) arXiv:2010.12971 [hep-ph]

- *Theorem*: Given a generic* rational function $F(x_i,x_j)=\frac{P(x_i,x_j)}{((x_i-a_i)^2-y_i^2)^{\gamma_i}((x_i+x_j-a_{ij})^2-y_k^2)^{\gamma_k}}$

then:

$$\text{Res}(\text{Res}(F(x_i,x_j),\{x_i,y_i+a_i\}),\{x_j,y_k-y_i+a_{ij}-a_i\})$$
$$=-\text{Res}\left(\text{Res}\left(F(x_i,x_j),\{x_i,y_k-x_j+a_{ij}\}\right),\{x_j,y_k-y_i+a_{ij}-a_i\}\right)$$

- **Physical consequences:**

1. **Displaced poles** are associated to **un-physical** contributions:

"they can not be mapped into cuts"

2. After applying C.R.T. to all the loop momenta and **summing over the physical poles**:

"only same-sign combinations of $q_{k,0}^{(+)}$ *remain"*

Cancellation of displaced poles —**"Aligned contributions"**→ **Causal propagators**

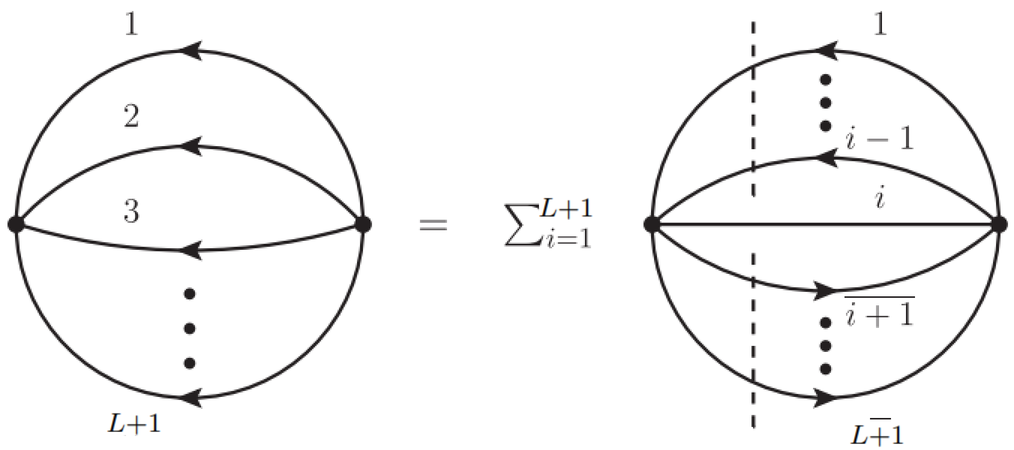
Aguilera-Verdugo et al (2020) arXiv:2010.12971 [hep-ph]

Nested residues: Compact representations

- Cancellation of displaced poles leads to very compact formulae for the dual representation:

REMARK: External particles can be attached to each momenta set

Maximal Loop Topology (2 vertices, L+1 lines)



$$\mathcal{A}^{(L)}_{\mathrm{MLT}}(1,2,\ldots,L+1) = \int_{\vec{\ell}_1,\cdots,\vec{\ell}_L} \sum_{i=1}^{L+1} \mathcal{A}_D(1,\ldots,i-1,\overline{i+1},\ldots,\overline{L+1};i)$$

Defined in Minkowski space

Defined in Euclidean space

On-shell lines

On-shell lines with reversed momenta

1 off-shell line

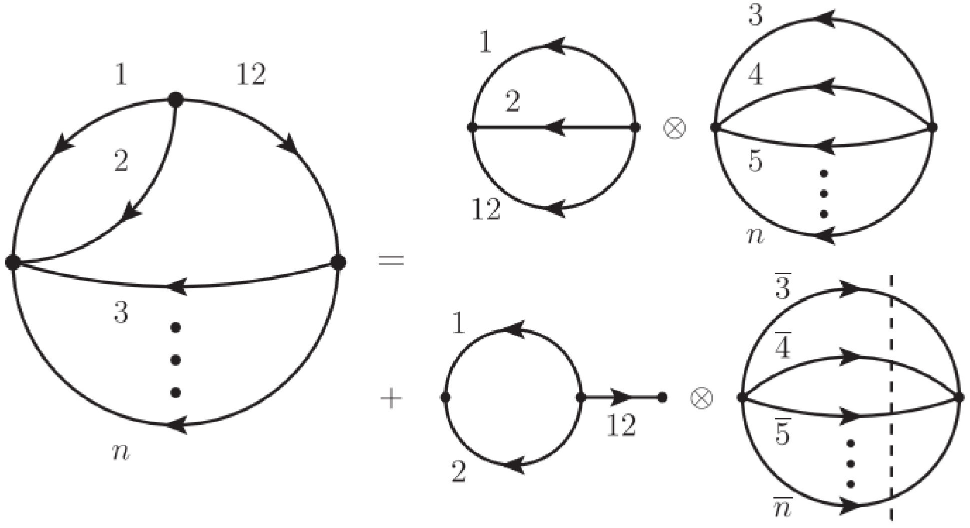
- We define the Maximal Loop Topology (MLT) as a building block to describe multi-loop amplitudes
- **Important:** “*Any one and two-loop amplitude can be described by MLT topologies*”

Aguilera-Verdugo et al (2020) Phys. Rev. Lett. 124, 211602 & arXiv:2006.11217 [hep-ph]

Nested residues: Compact representations

- More complicated topologies can be described by convolutions with MLT-like diagrams

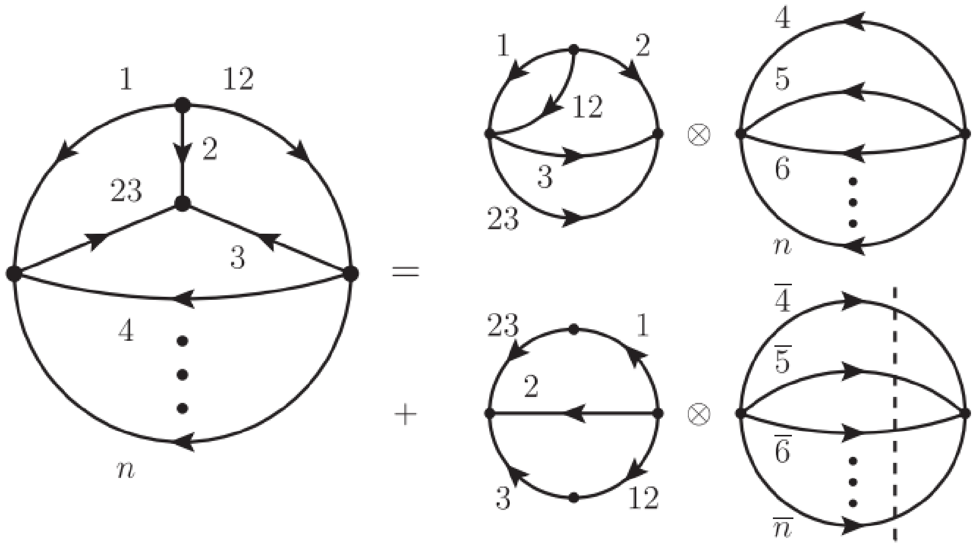
Next-to Maximal Loop Topology (3 vertices, L+2 lines)



$$\mathcal{A}^{(L)}_{\rm NMLT}(1,\ldots,n,12) = \mathcal{A}^{(2)}_{\rm MLT}(1,2,12)\otimes\mathcal{A}^{(L-2)}_{\rm MLT}(3,\ldots,n) + \mathcal{A}^{(1)}_{\rm MLT}(1,2)\otimes\mathcal{A}^{(0)}(12) \otimes\mathcal{A}^{(L-1)}_{\rm MLT}(\bar{3},\ldots,\bar{n})$$

IMPORTANT FACTORIZATION FORMULAE
Singular and causal structure is determined by the corresponding sub-topologies

Next-to-Next-to Maximal Loop Topology (4 vertices, L+3 lines)



$$\mathcal{A}^{(L)}_{\rm NNMLT}(1,\ldots,n,12,23) = \mathcal{A}^{(3)}_{\rm NMLT}(1,2,3,12,23)\otimes\mathcal{A}^{(L-3)}_{\rm MLT}(4,\ldots,n) + \mathcal{A}^{(2)}_{\rm MLT}(1\cup 23,2,3\cup 12)\otimes\mathcal{A}^{(L-2)}_{\rm MLT}(\bar{4},\ldots,\bar{n})$$

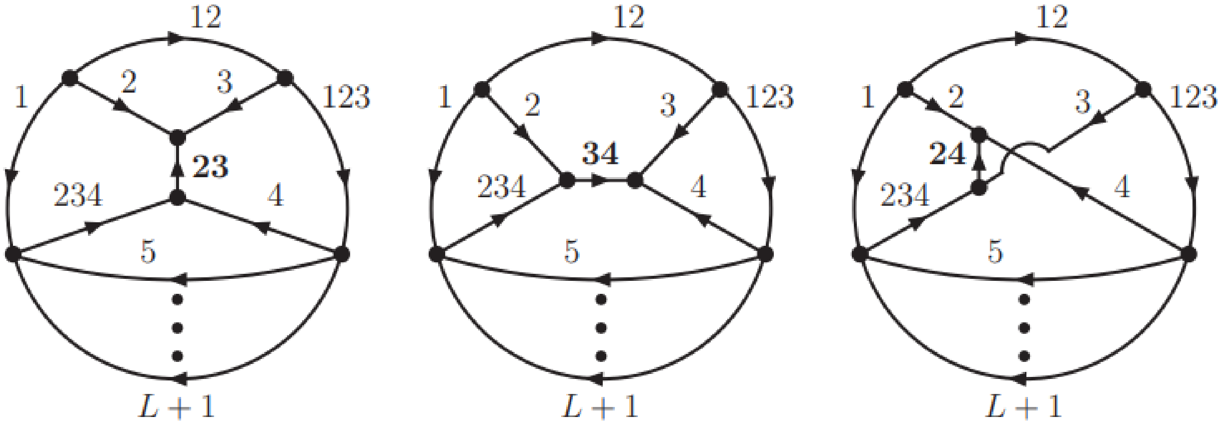
Inductive proofs of these formulae to all-loop orders available in 2010.12971 [hep-ph]

Aguilera-Verdugo et al (2020) Phys. Rev. Lett. 124, 211602

Nested residues: Compact representations

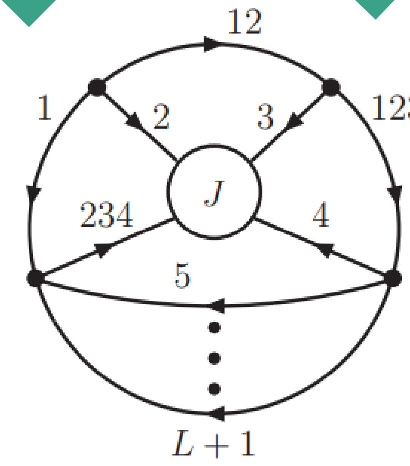
- And even (much) more complicated topologies can be treated within this formalism

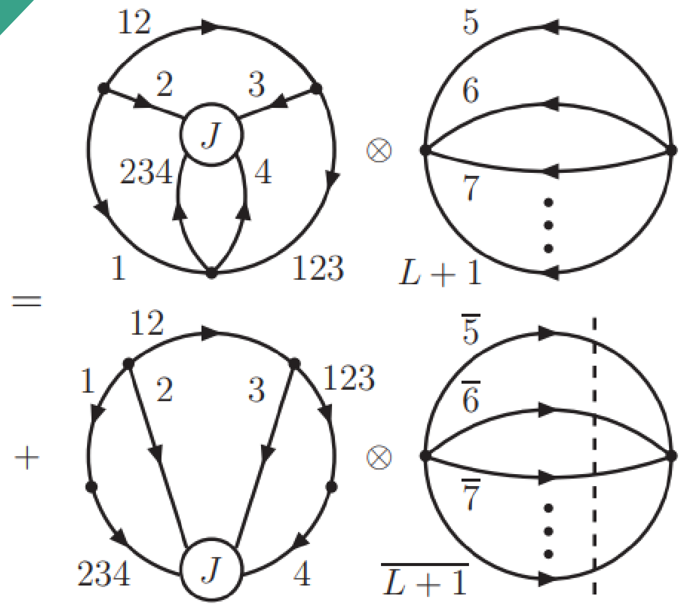
NNNN Maximal Loop Topologies (6 vertices, L+5 lines)



$$
\begin{aligned}
\mathcal{A}^{(L)}_{\mathrm{N^4MLT}}&(1,\ldots,L+1,12,123,234,J) \\
&= \mathcal{A}^{(4)}_{\mathrm{N^4MLT}}(1,2,3,4,12,123,234,J) \\
&\quad \otimes \mathcal{A}^{(L-4)}_{\mathrm{MLT}}(5,\ldots,L+1) \\
&+ \mathcal{A}^{(3)}_{\mathrm{N^2MLT}}(1\cup 234,2,3,4\cup 123,12,J) \\
&\quad \otimes \mathcal{A}^{(L-3)}_{\mathrm{MLT}}(\overline{5},\ldots,\overline{L+1})
\end{aligned}
$$

N⁴MLT universal topology

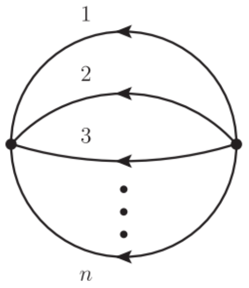




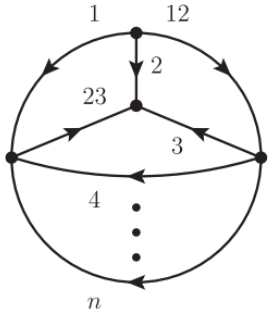
Ramírez Uribe et al (2020) arXiv:2006.13818 [hep-ph]

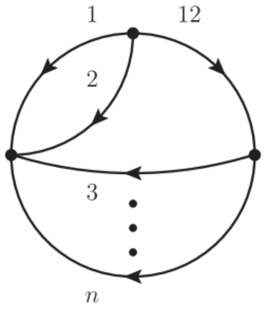
Nested residues: Compact representations

- Why is our topological classification so useful?

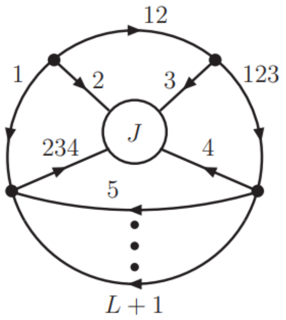


1. MLT topology describes all possible one and two-loop amplitudes





2. N^2MLT ($\supset$ NMLT) topology describes any possible three-loop amplitude

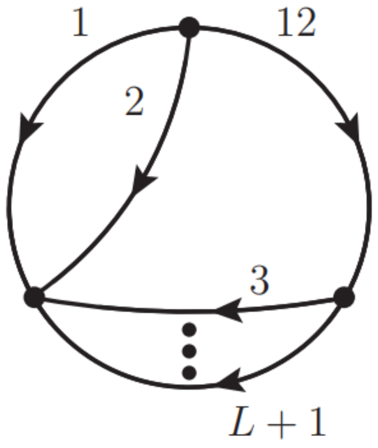


3. N^4MLT ($\supset N^3$MLT) topology describes any possible four-loop amplitude

Thanks to factorization properties, the singular and **causal** structure of complicated multi-loop amplitudes can be understand in terms of simpler objects

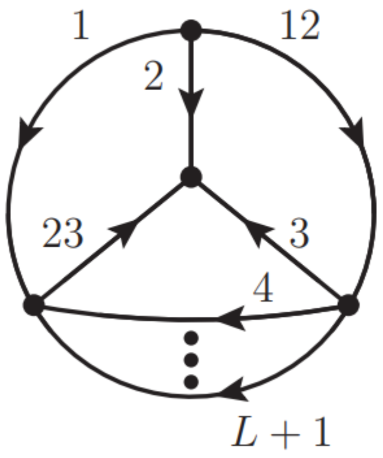
Ramírez Uribe et al (2020) arXiv:2006.13818 [hep-ph]

- Similar formulae can be found for NMLT and NNMLT to all loop orders!



$$\mathcal{A}^{(L)}_{\rm NMLT}(1,2,\ldots,L+2) = \int_{\vec{\ell}_1,\ldots,\vec{\ell}_L} \frac{2}{x_{L+2}} \left(\frac{1}{\lambda_1\lambda_2} + \frac{1}{\lambda_2\lambda_3} + \frac{1}{\lambda_3\lambda_1} \right)$$

with $\lambda_1 = \sum_{i=1}^{L+1} q_{i,0}^{(+)}$ $\quad \lambda_2 = q_{1,0}^{(+)} + q_{2,0}^{(+)} + q_{L+2,0}^{(+)}$ $\quad \lambda_3 = \sum_{i=3}^{L+2} q_{i,0}^{(+)}$



$$\mathcal{A}^{(L)}_{\rm N^2MLT}(1,2,\ldots,L+3) = -\int_{\vec{\ell}_1,\cdots,\vec{\ell}_L} \frac{2}{x_{L+3}} \left[\frac{1}{\lambda_1}\left(\frac{1}{\lambda_2}+\frac{1}{\lambda_3}\right)\left(\frac{1}{\lambda_4}+\frac{1}{\lambda_5}\right) + \frac{1}{\lambda_6}\left(\frac{1}{\lambda_2}+\frac{1}{\lambda_4}\right)\left(\frac{1}{\lambda_3}+\frac{1}{\lambda_5}\right) + \frac{1}{\lambda_7}\left(\frac{1}{\lambda_2}+\frac{1}{\lambda_5}\right)\left(\frac{1}{\lambda_3}+\frac{1}{\lambda_4}\right) \right]$$

with

$$\lambda_4 = q_{2,0}^{(+)} + q_{3,0}^{(+)} + q_{L+3,0}^{(+)} \qquad \lambda_6 = q_{1,0}^{(+)} + q_{3,0}^{(+)} + q_{L+2,0}^{(+)} + q_{L+3,0}^{(+)}$$

$$\lambda_5 = q_{1,0}^{(+)} + q_{L+3,0}^{(+)} + \sum_{i=4}^{L+1} q_{i,0}^{(+)} \qquad \lambda_7 = q_{2,0}^{(+)} + \sum_{i=4}^{L+3} q_{i,0}^{(+)}$$

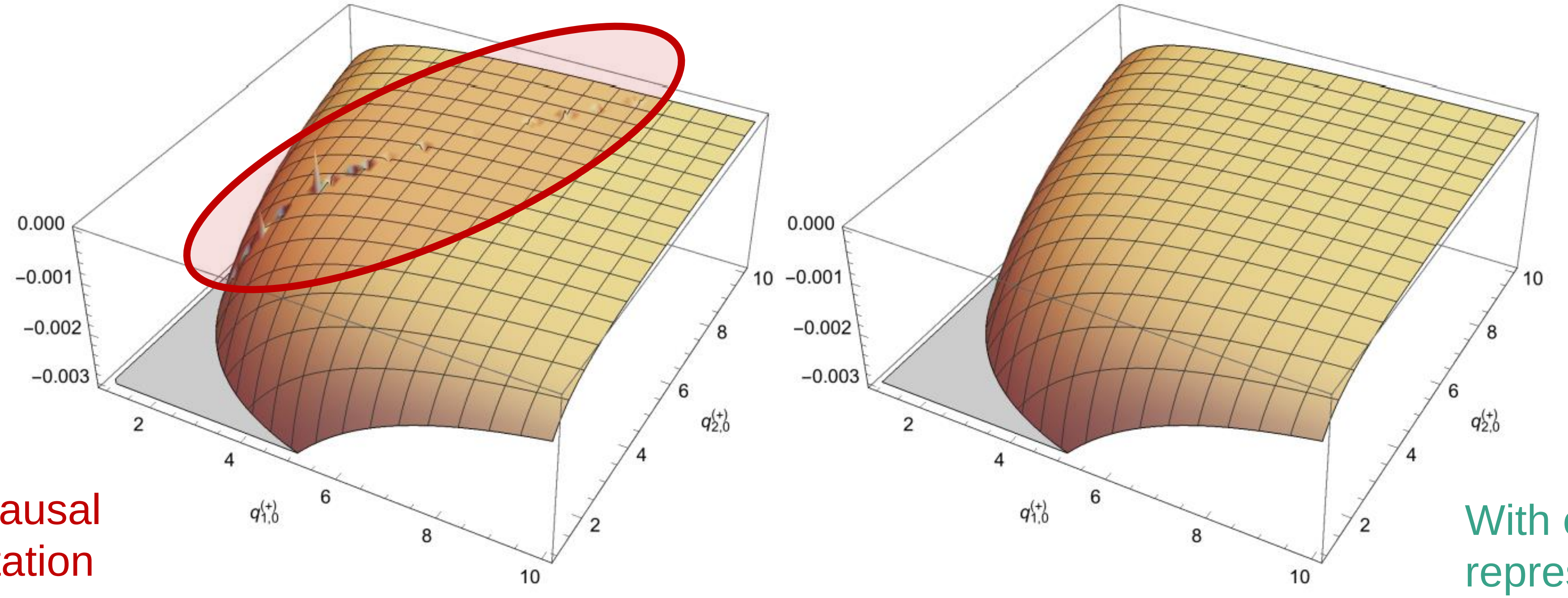
Aguilera-Verdugo et al (2020) arXiv:2006.11217 [hep-ph]

Manifestly Causal representation

- **A similar representation can be found when external particles are present!**
- **Advantages:**

  1. Causal denominators have same-sign combinations of on-shell energies (positive numbers), thus are **more stable numerically!**

  2. **Only physical thresholds remain**; spurious un-physical instabilities are removed!



Without causal representation

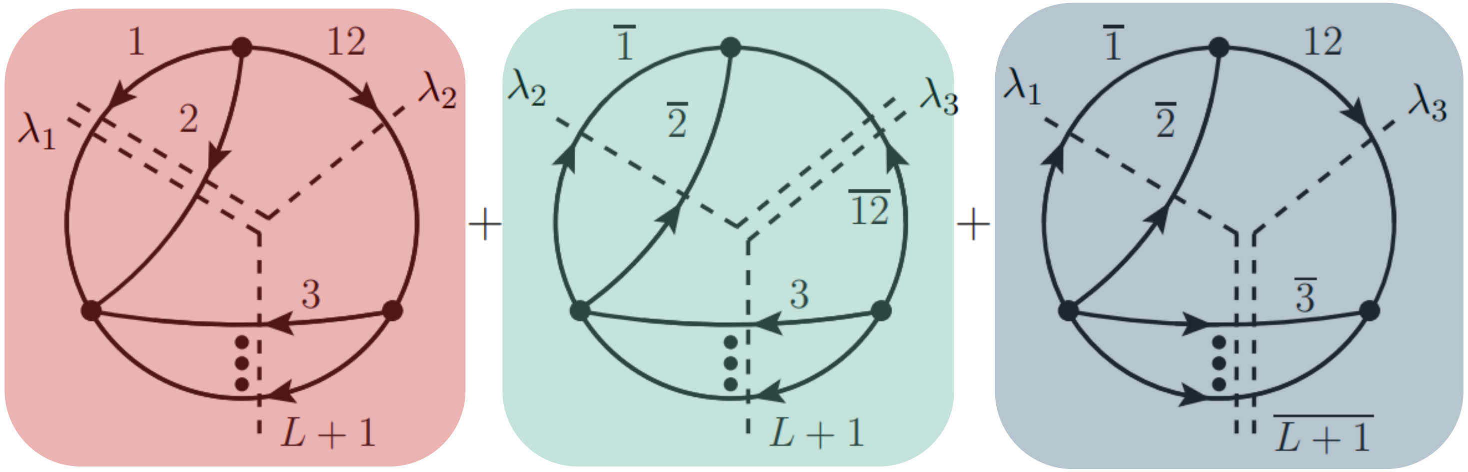
With causal representation

Aguilera-Verdugo et al (2020) arXiv:2006.11217 [hep-ph]

- **A similar representation can be found when external particles are present!**
- **Advantages:**

  1. Causal denominators have same-sign combinations of on-shell energies (positive numbers), thus are **more stable numerically**
  2. **Only physical thresholds remain**; spurious un-physical instabilities are removed
  3. Nice physical interpretation in terms of **entangled thresholds**!!

Causal denominators (λ) are associated to *cut lines* in the diagrams: momenta flow must be adjusted to be compatible



$$\mathcal{A}^{(L)}_{\rm NMLT}(1,2,\ldots,L+2) = \int_{\vec{\ell}_1,\ldots,\vec{\ell}_L} \frac{2}{x_{L+2}} \left(\frac{1}{\lambda_1\lambda_2} + \frac{1}{\lambda_2\lambda_3} + \frac{1}{\lambda_3\lambda_1} \right)$$

Aguilera-Verdugo et al (2020) arXiv:2006.11217 [hep-ph]

Manifestly Causal representation: Implementation

- We profit from compact causal formulae for integrals with higher-powers:

$$\mathcal{A}^{(L)}_{\mathrm{N}^{k-1}\mathrm{MLT}}(1^2, 2^2, \ldots, L^2, L+1, \ldots, L+k) = \prod_{i=1}^{L} \frac{\partial}{\partial (q_{i,0}^{(+)})^2} \mathcal{A}^{(L)}_{\mathrm{N}^{k-1}\mathrm{MLT}}(1, 2, \ldots, L+1, \ldots, L+k)$$

Is also causal by construction! (*derivatives preserve denominators*)

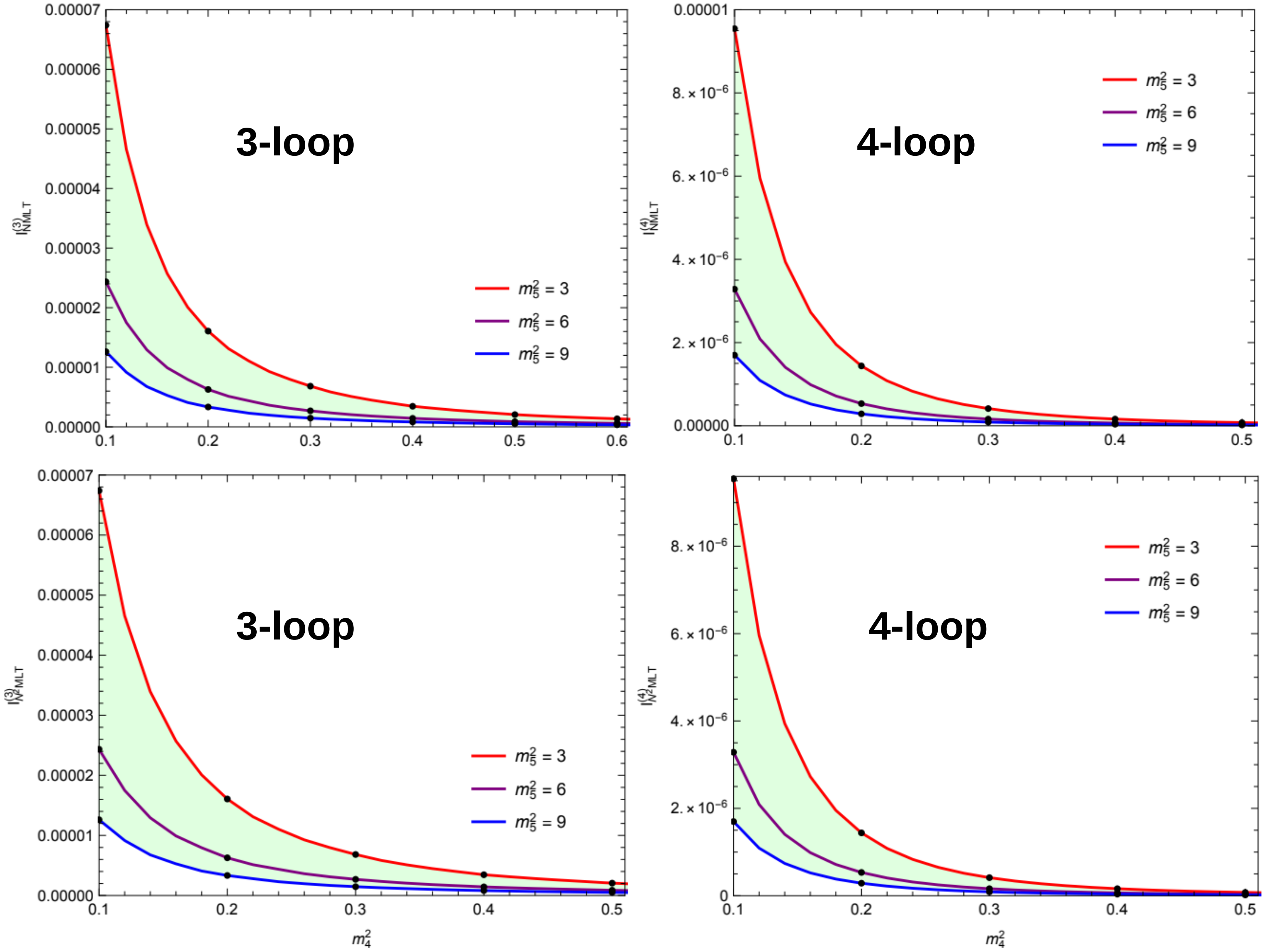
Causal representation available!

- *Setup of the numerical implementation:*

  1. Tested for MLT, NMLT and NNMLT integrals, at 3 and 4 loops
  2. Arbitrary masses, and with different numbers of space-time dimensions (D=2,3,4)
  3. Compared with numerical results from FIESTA 4.2 and SecDec 3.0

Aguilera-Verdugo et al (2020) arXiv:2006.11217 [hep-ph]

- Numerical results in D=3:



NMLT

NNMLT

Solid lines: LTD
Dots: FIESTA

Setup:

$$\mathcal{A}^{(L)}_{\mathrm{N}^{k-1}\mathrm{MLT}}(1^2, 2^2, \ldots, L^2, L+1, \ldots, L+k)$$

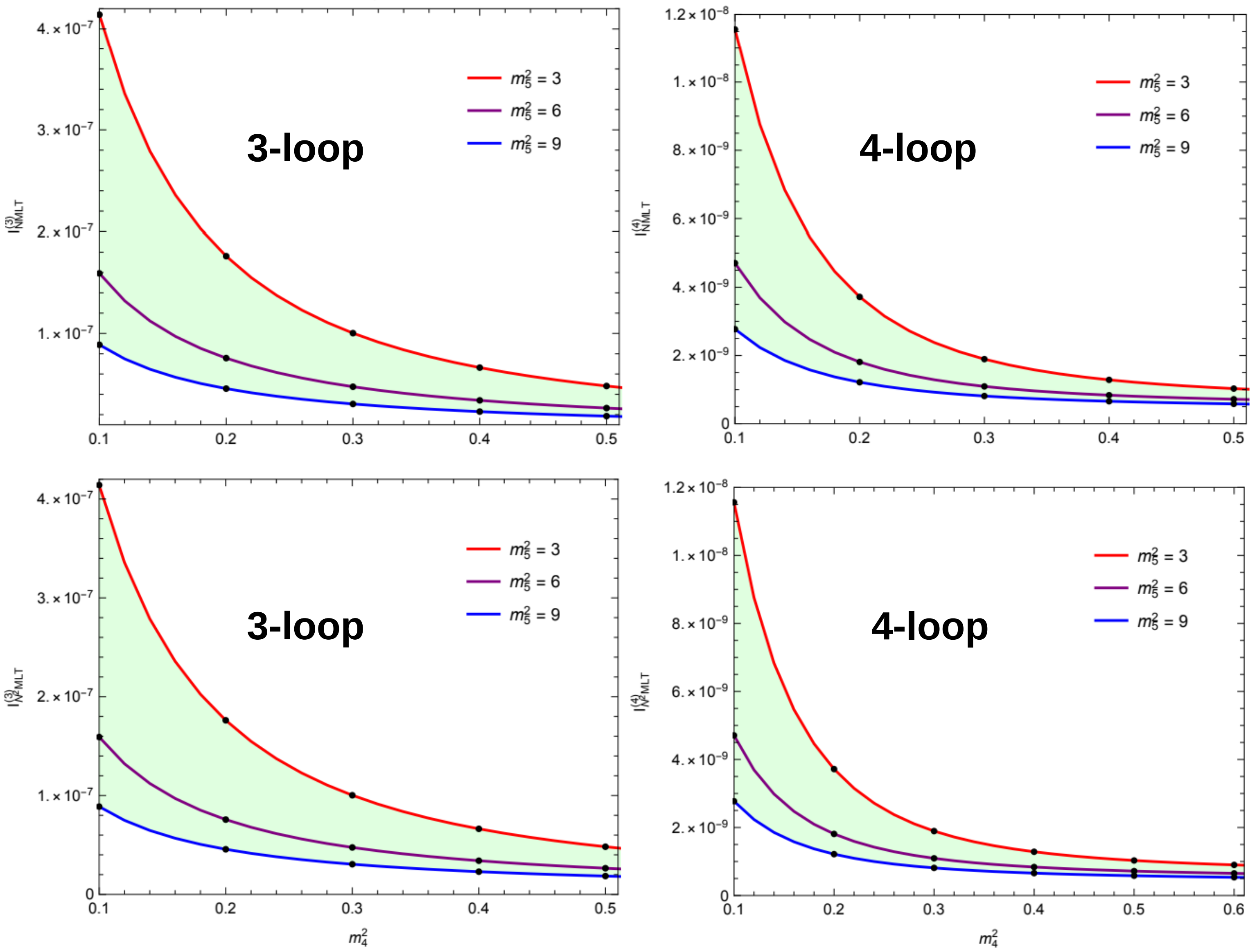
Mases:

$\{1, 2, \ldots, L\} \Longleftrightarrow m_4^2$

$\{L+1, \ldots, L+k\} \Longleftrightarrow m_5^2$

Aguilera-Verdugo et al (2020) arXiv:2006.11217 [hep-ph]

- **Numerical results in D=4:**



NMLT

NNMLT

Solid lines: LTD
Dots: FIESTA

Setup:

$$\mathcal{A}^{(L)}_{\mathrm{N}^{k-1}\mathrm{MLT}}(1^2, 2^2, \dots, L^2, L+1, \dots, L+k)$$

Mases: $\{1, 2, \dots, L\} \Longleftrightarrow m_4^2$

$\{L+1, \dots, L+k\} \Longleftrightarrow m_5^2$

Aguilera-Verdugo et al (2020) arXiv:2006.11217 [hep-ph]

Conclusions and outlook

- LTD-based methods lead to a **novel understanding of singular structure** of multi-loop scattering amplitudes
- Transformed integration domain from Minkowski to **Euclidean**
- Allows a more natural **integrand-level combination with real radiation** through kinematical mappings (for cross-section computation)
- Novel LTD approach based on **nested residues leads to manifestly causal representation** of multi-loop scattering amplitudes
- Interpretation in terms of **entangled causal thresholds**
- **More stable numerical implementation** (absence of spurious singularities)

- **Outlook:**

  1. Deepen into the **interpretation** of causal propagators
  2. Tackle the **calculation of physical observables** with this new representation
  3. Test the **efficiency for cross-section calculations**

An index of submitted letters can be viewed using this direct URL link. The letters will be stored permanently in the Fermilab archive Doc.db shortly after August 31, 2020. The current LOIs files organized in the directories corresponding to the primary frontiers used during submissions are shown here.

Manifestly Causal Scattering Amplitudes

J. Jesús Aguilera-Verdugo[a], Roger J. Hernández-Pinto[b], Selomit Ramírez-Uribe[a,b], Andres Renteria[a], Germán Rodrigo[a], German F. R. Sborlini[a], and William J. Torres Bobadilla*[a]

[a] *Instituto de Física Corpuscular, Universitat de València – Consejo Superior de Investigaciones Científicas, Parc Científic, E-46980 Paterna, Valencia, Spain.*
[b] *Facultad de Ciencias Físico-Matemáticas, Universidad Autónoma de Sinaloa, Ciudad Universitaria, CP 80000 Culiacán, Mexico.*

August 30, 2020

LoI for Snowmass 2021 (sent on 30.08.2020)

Vielen Dank!
Gracias!