Calculating compact binary dynamics

using effective field theory and computer algebra

Jan Steinhoff

Max-Planck-Institute for Gravitational Physics (AEI), Potsdam-Golm, Germany

DESY Zeuthen, January 9th, 2019

Gravity: from Galileo and Newton to Einstein

Newton's theory of gravity:

- things fall down due to a gravitational force
- universality of free fall

London Science Museum

Einstein's theory of gravity
a.k.a. general relativity
a.k.a. our best theory of gravity:

- free fall = force-free motion
- no gravitational force
- space & time are curved

Gravity: from Galileo and Newton to Einstein

Newton's theory of gravity:

- things fall down due to a gravitational force
- universality of free fall

London Science Museum

Einstein's theory of gravity
a.k.a. general relativity
a.k.a. our best theory of gravity:

- free fall $=$ force-free motion
- no gravitational force
- space & time are curved

International Space Station, nasa.gov

Gravitational waves

general relativity predicts gravitational waves:
small ripples in the fabric of spacetime

best source: black hole/neutron star binaries

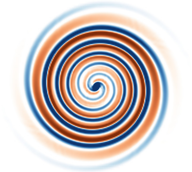
$\rightarrow$ chirp signal

Gravitational waves

general relativity predicts gravitational waves:
small ripples in the fabric of spacetime

best source: black hole/neutron star binaries

→ chirp signal 



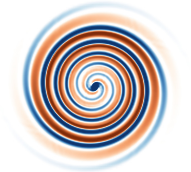
waves from a binary black hole
[simulation with the Einstein Toolkit]

Gravitational waves

general relativity predicts gravitational waves:
small ripples in the fabric of spacetime

best source: black hole/neutron star binaries

→ chirp signal

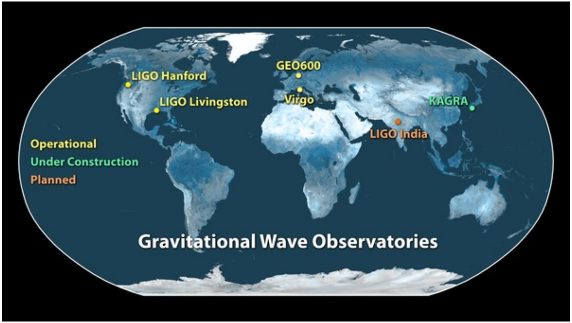


waves from a binary black hole

[simulation with the Einstein Toolkit]

detector network:

Virgo, www.ligo.caltech.edu/images



www.ligo.caltech.edu/images

Detections of gravitational waves (GWs)

https://www.ligo.caltech.edu/page/detection-companion-papers

using LIGO:

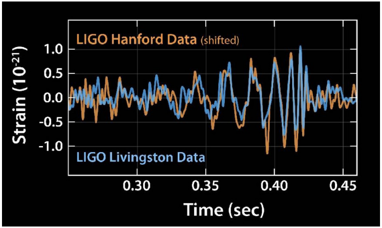
- GW150914
- GW151226
- GW170104

→ Nobel Prize in physics 2017

many more since then. . .

GW170817:
→ (likely) a binary neutron star inspiral!

now: LIGO and Virgo
→ network of detectors still growing
→ accuracy/quantity will improve
e.g., drastically improved sky localization
→ need better predictions, too!



GW150914 (binary black hole), ligo.caltech.edu

Detections of gravitational waves (GWs)

https://www.ligo.caltech.edu/page/detection-companion-papers

using LIGO:

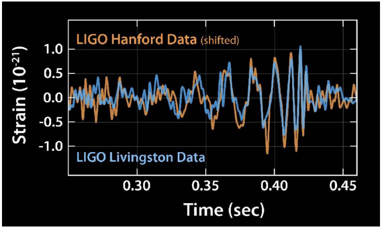
- GW150914
- GW151226
- GW170104

→ Nobel Prize in physics 2017

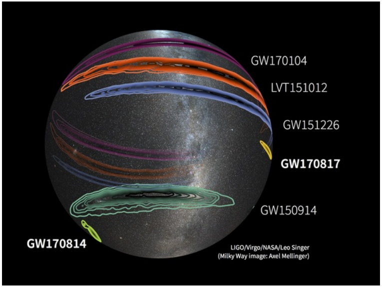
many more since then...

GW170817:
→ (likely) a binary neutron star inspiral!

now: LIGO and Virgo
→ network of detectors still growing
→ accuracy/quantity will improve
e.g., drastically improved sky localization
→ need better predictions, too!

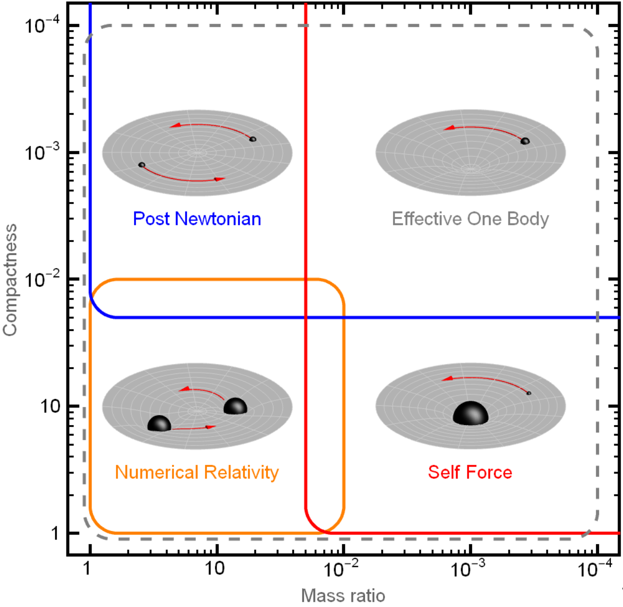


GW150914 (binary black hole), ligo.caltech.edu



sky location of GW detections, ligo.caltech.edu

Method for predicting gravitational waves



wikipedia.org

Effective field theory for post-Newtonian approximation

[Goldberger, Rothstein, PRD **73** (2006) 104029; Goldberger, arXiv:hep-ph/0701129]

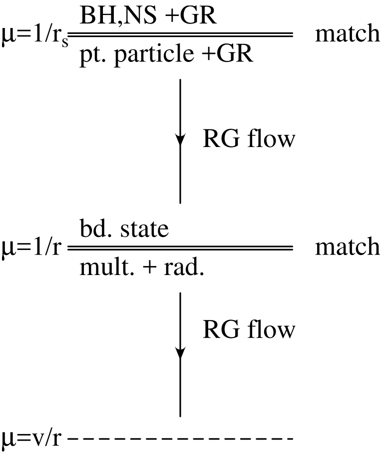
for generic binaries in general relativity (GR):

- black holes (BHs)
- neutron stars (NSs)
- bold astronauts...

separation of scales:

- scale μ
- object size r_s
- orbital size r
- velocity v

  $\rightarrow$ frequency $\sim \frac{v}{r}$



from [Goldberger, arXiv:hep-ph/0701129]

decomposition of the metric:

$$g_{\mu\nu} - \eta_{\mu\nu} = \left[h^s_{\mu\nu} \sim \frac{1}{r_s} \right] + \left[H_{\mu\nu} \sim \frac{1}{r} \right] + \left[h_{\mu\nu} \sim \frac{v}{r} \right]$$

Integrating out the orbital scale

action after integrating out $h^s_{\mu\nu}$: point-particles coupled to GR

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}\bar{R} - \sum_{A=1,2}\int d\sigma_A \sqrt{\bar{g}_{\mu\nu}\frac{dx^\mu_A}{d\sigma_A}\frac{dx^\nu_A}{d\sigma_A}}(m_A + \dots)$$

$$\bar{g}_{\nu\nu} = \eta_{\mu\nu} + H_{\mu\nu} + h_{\mu\nu}$$

from now on: ignore radiation, drop $h_{\mu\nu}$ contributions

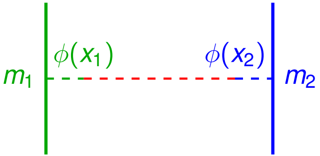
add gauge fixing, post-Newtonian expansion:

$$S = -\frac{1}{16\pi G}\int dt\, d^3x \left[2\delta^{ij}\partial_i\phi\partial_j\phi + \dots\right] - \sum_A m_A \int dt \left[1 - \frac{\dot{\vec{x}}^2_A}{2} + \phi + \dots\right]$$

$$\bar{g}_{\mu\nu}dx^\mu dx^\nu = e^{2\phi}(dt - A_i dx^i)^2 - e^{-2\phi}(\delta_{ij} + \sigma_{ij})dx^i dx^j$$

integrate out ϕ, A_i, σ_{ij} ...

Explicit example: the Newtonian limit



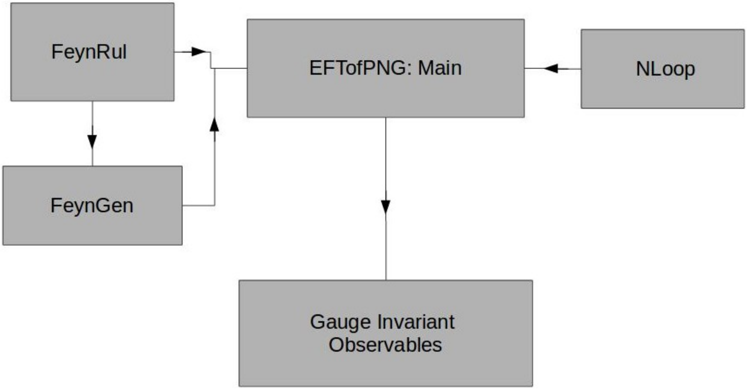
Newtonian potential as graviton exchange

$$\approx \int dt_1\, m_1\, e^{i\vec{k}\cdot\vec{x}_1(t_1)}\, 4\pi G \delta(t_1 - t_2) \frac{d^3k}{(2\pi)^3} \frac{1}{\vec{k}^2}\, dt_2\, m_2\, e^{-i\vec{k}\cdot\vec{x}_2(t_2)}$$

$$= \int dt \frac{G\, m_1\, m_2}{r_{12}(t)} \qquad \text{where } r_{12} = |\vec{x}_1 - \vec{x}_2|$$

Automation using computer algebra: EFTofPNG

M. Levi, JS, CQG **34** (2017) 244001, arXiv:1705.06309



demo time

```
git clone -b talk-zeuthen20 https://github.com/jsteinhoff/pncbc-eftofpng.git
```

TODO

- wrap things into package environments, more abstractions
- need to improve substitution/representation of integrals
  $\rightarrow$ problem solved elsewhere (?)
  $\rightarrow$ delegate to other packages?
- optimal strategy to remove momentum-conserving δ's?
- more sophisticated way of collecting/sorting terms
  $\rightarrow$ keep subexpressions small
  $\rightarrow$ keep dim.-dependent prefactors together
- parallelize
  $\rightarrow$ should be straightforward, but memory is the limiting factor
- run with open source replacement for Mathematica?
  `https://mathics.github.io`