Some issues in the computation of 2-loop amplitudes $q\bar{q} \to ZH$

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$\Delta p \cdot \Delta q \geq \frac{1}{2} \hbar$

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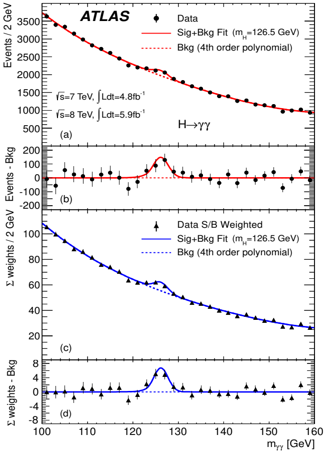
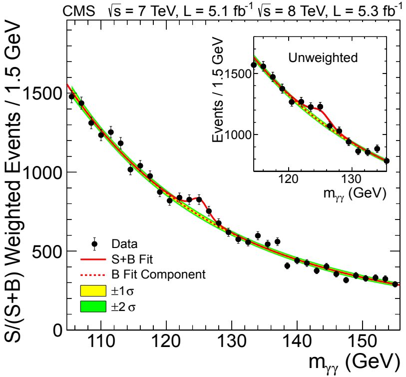
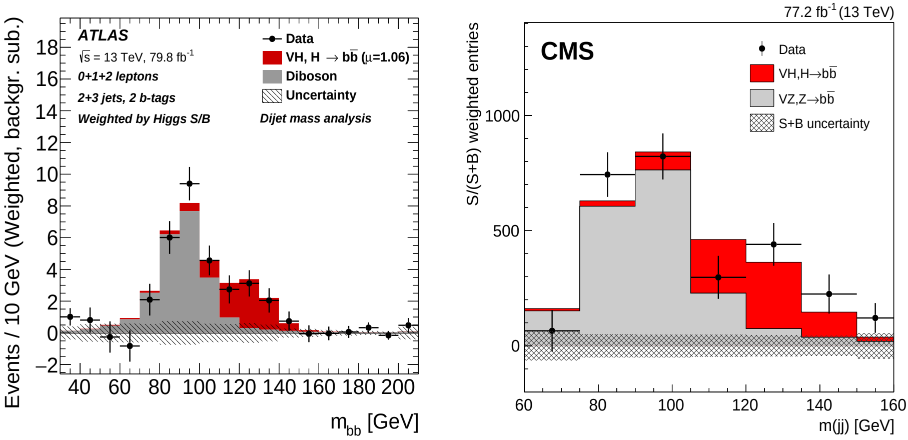
T. Ahmed, W. Bernreuther, A. H. Ajjath, P. K. Dhani, P. Mukherjee, V. Ravindran,
based on JHEP01(2020)030 [arXiv: 1910.06347] and work in progress

The discovery of the $h(125)$ at the LHC (2012)





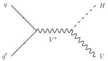
$h \to b\bar{b}$ finally observed recently at the LHC in the VH-events!



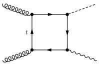
The measured signal strength $\mu = 1.04 \pm 0.20$.

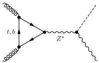
Much theoretical work done already ...

Much work done on the ZH production at LHC: $\mathbf{P} + \mathbf{P} \to Z(l\,l') + H(b\bar{b})$

- $q\bar{q} \to ZH$: 

Starting from $\mathcal{O}(\alpha_e\,\alpha_s^0)$
  - The Higgs-bremsstrahlung (Drell-Yan) part up to $NNLO$ in massless QCD [Brein, Harlander,Wiesemann,Zirke, 2012; Ferrera,Grazzini,Tramontano, 2015/2018; Campbell,Ellis,Williams, 2016]
  - The top-loop induced $NNLO$ (non-Drell-Yan type) QCD corrections in the heavy-top limit [Brein,Djouadi,Harlander, 2004; Brein,Harlander,Wiesemann,Zirke, 2012]
  - N^3LO corrections in massless QCD [Ahmed,Mahakhud,Rana,Ravindran, 2014; Li,von Manteuffel, Schabinger,Zhu, 2014; Catani,Cieri,Florian,Ferrera,Grazzini, 2014; Kumar,Mandal,Ravindran, 2015]
  - NLO electroweak corrections [Ciccolini,Dittmaier,Kramer, 2003; Denner,Dittmaier,Kallweit,Mueck, 2011]
  - ......

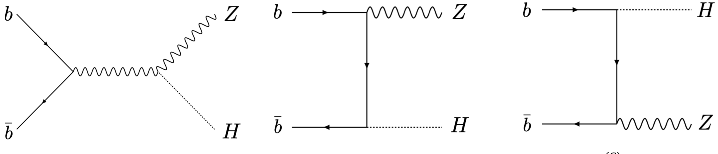
- $g\bar{g} \to ZH$: 



Starting from $\mathcal{O}(\alpha_e\,\alpha_s)$
  - The exact LO (with full m_t dependence) [Kniehl, 1990]
  - NLO QCD corrections to the Higgs-bremsstrahlung (Drell-Yan) part in the heavy-top limit [Brein, Harlander,Wiesemann,Zirke, 2012; Altenkamp, Dittmaier, Harlander, Rzehak, Zirke, 2013]
  - ......

Focus of the talk

Here we focus on a part of the b-quark-induced ZH process that involves a non-vanishing Yukawa coupling λ_b (i.e. (B) and (C)) but with $m_b = 0$.



(A) (B) (C)

Aims of the work:

- Computing the 2-loop (massless) QCD corrections in analytic form
- Addressing a subtlety appearing in the conventional FF decomposition of amplitudes involving axial currents regularised in D dimensions (with a non-anticommuting γ_5)
- Verifying the *unitarity* of a particular regularisation prescription implied by projectors prescribed recently [Chen, 2019]]
- The "same" loop amplitudes built up from just vector FFs of properly grouped classes of diagrams (bypassing completely the need of explicitly manipulating γ_5)

We consider

$$b(p_1) + \bar{b}(p_2) \to Z(q_1) + H(q_2)$$

in $n_f = 5$ massless QCD.

The Mandelstam variables:

$$s \equiv (p_1 + p_2)^2 \,, \quad t \equiv (p_1 - q_1)^2 \quad \text{and} \quad u \equiv (p_2 - q_1)^2$$

satisfying $s + t + u = q_1^2 + q_2^2 = m_z^2 + m_h^2$.

Regarding the definition of γ_5 in dimensional regularisation (HV/BM and Larin):

$$\gamma_5 = \frac{i}{4!} \varepsilon_{\mu\nu\rho\sigma} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma$$

$$\gamma_\mu \gamma_5 \to \frac{1}{2} \Big(\gamma_\mu \gamma_5 - \gamma_5 \gamma_\mu \Big) = \frac{i}{6} \varepsilon_{\mu\nu\rho\sigma} \gamma^\nu \gamma^\rho \gamma^\sigma \,.$$

Form Factor Decomposition: the vector part

By **Lorentz covariance**, the amplitude $b\bar{b}ZH$ can be expressed as **a linear combination of a finite basis of Lorentz structures at any finite order**.

For the vector part :

$$\bar{v}(p_2)\,\Gamma^{\mu}_{vec}\,u(p_1) = F_{1,vec}\,\bar{v}(p_2)\,u(p_1)\,p_1^{\mu} + F_{2,vec}\,\bar{v}(p_2)\,u(p_1)\,p_2^{\mu} + F_{3,vec}\,\bar{v}(p_2)\,u(p_1)\,q_1^{\mu} + F_{4,vec}\,\bar{v}(p_2)\,\gamma^{\mu}\,\not{q}_1\,u(p_1)\,,$$

under the following constraints:

- Even power γ-matrices (one Yukawa vertex on the massless fermion line);
- Parity even for the vector coupling;
- Equations of motion for the on-shell massless spinors: $\not{p}_1 u(p_1) = 0\,, \not{p}_2 v(p_2) = 0$.

Form Factor Decomposition: Gram matrix and Projectors

For a scattering amplitude $\hat{\mathcal{M}}$ at a fixed perturbative order,

$$\hat{\mathcal{M}} = \sum_{n=1}^{N} F_n \, \hat{T}_n \,,$$

compute the **Gram matrix** $\hat{\mathrm{G}}$

$$\mathbf{G}_{ij} = \langle \hat{T}_i^\dagger, \hat{T}_j \rangle \,,$$

whose inverse gives us the formula for the projector

$$\hat{\mathbb{P}}_n = \hat{\mathbf{G}}_{nj}^{-1} \, \hat{T}_j^\dagger$$

to project out the *Lorentz-invariant* coefficient (form-factor) F_n.

Projectors for vector form factors

The projectors for vector form factors are still compact enough to be documented explicitly:

$$\mathbb{P}^{\mu}_{1,vec} = \bar{u}(p_1)\Big\{(-2+D)t^2 p_1^{\mu} + \Big(2(-3+D)m_z^2 s - (-2+D)tu\Big)p_2^{\mu} + (2-D)stq_1^{\mu} + st\not{q}_1\gamma^{\mu}\Big\}v(p_2)\frac{1}{\mathcal{K}_{vec}}\,,$$

$$\mathbb{P}^{\mu}_{2,vec} = \bar{u}(p_1)\Big\{\Big(2(-3+D)m_z^2 s - (-2+D)tu\Big)p_1^{\mu} + (-2+D)u^2 p_2^{\mu} + (4-D)suq_1^{\mu} - su\not{q}_1\gamma^{\mu}\Big\}v(p_2)\frac{1}{\mathcal{K}_{vec}}\,,$$

$$\mathbb{P}^{\mu}_{3,vec} = \bar{u}(p_1)\Big\{-(-2+D)stp_1^{\mu} - (-4+D)sup_2^{\mu} - (2-D)s^2 q_1^{\mu} - s^2\not{q}_1\gamma^{\mu}\Big\}v(p_2)\frac{1}{\mathcal{K}_{vec}}\,,$$

$$\mathbb{P}^{\mu}_{4,vec} = \bar{u}(p_1)\Big\{stp_1^{\mu} - sup_2^{\mu} - s^2 q_1^{\mu} + s^2\not{q}_1\gamma^{\mu}\Big\}v(p_2)\frac{1}{\mathcal{K}_{vec}}\,,$$

where

$$\mathcal{K}_{vec} = 2(-3+D)s(m_z^2 s - tu)\,.$$

Form Factor Decomposition: issues with the axial part

A set of Lorentz structure basis for the axial part of $b\bar{b} \to ZH$ *linearly complete* in 4 dimensions, but **not** in D dimensions (unless a *fully anticommutating* γ_5 is used):

$$\left\{\bar{v}(p_2)\,\gamma_5\,u(p_1)\,p_1^\mu\,,\,\bar{v}(p_2)\,\gamma_5\,u(p_1)\,p_2^\mu\,,\,\bar{v}(p_2)\,\gamma_5\,u(p_1)\,q_1^\mu\,,\,\bar{v}(p_2)\,\gamma^\mu\gamma_5\,\not{q}_1 u(p_1)\right\}.$$

Whether $\mathcal{M}_{axi}$ lives in a space linearly spanned by these 4 structures in D dimensions:

- Enlarge this list by appending the (tree-level) $\mathcal{M}_{axi}$;
- and then compute the **Gram matrix** of this enlarged list of 5 elements;
- one finds out that the matrix rank is increased to 5 rather than staying at 4!

Form Factor Decomposition: issues with the axial part

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Therefore, if a non-anticommuting γ_5 is used in the *dimensional regularisation*, we may face the following issues regarding the axial FF-decomposition :

1. It is not easy to construct the full D-dimensional linearly complete basis for the FF-decomposition of $\bar{v}(p_2)\,\Gamma^\mu_{axi}\,u(p_1)$ at loop orders.
2. Even with just an incomplete basis, keeping the full D-dependence could lead to expressions with too complicated D-dependence to use.
3. Setting D=4 could simplify the expressions a bit, which, however, is not known to be legitimate in general (for FF-decomposition).

Projectors for axial form factors

The projectors for 4 axial FFs (corresponding to the basis of 4 structures) read as:

$$\mathbb{P}^{\mu}_{1,axi} = -\bar{u}(p_1)\epsilon_{\gamma\gamma\gamma\gamma}v(p_2)\Big\{(-1+D)^2(48-14D+D^2)(m_z^2-u)^2 p_1^{\mu}$$
$$+\Big((176+22D-69D^2+16D^3-D^4)m_z^4$$
$$+2(-148-54D+73D^2-16D^3+D^4)m_z^2 s$$
$$+(176+22D-69D^2+16D^3-D^4)tu$$
$$-(176+22D-69D^2+16D^3-D^4)m_z^2(t+u)\Big)p_2^{\mu}$$
$$-(-1+D)^2(48-14D+D^2)s(m_z^2-u)q_1^{\mu}\Big\}\frac{12}{(8-9D+D^2)s^2\mathcal{K}_{axi}}$$
$$+\bar{u}(p_1)\not{q}_1\epsilon^{\gamma\gamma\gamma\mu}v(p_2)\Big\{(-56+71D-16D^2+D^3)s(m_z^2-u)\Big\}\frac{12}{(8-9D+D^2)s^2\mathcal{K}_{axi}},$$

$$\mathbb{P}^{\mu}_{2,axi} = \bar{u}(p_1)\epsilon_{\gamma\gamma\gamma\gamma}v(p_2)\Big\{\Big((-176-22D+69D^2-16D^3+D^4)m_z^4$$
$$--2(-148-54D+73D^2-16D^3+D^4)m_z^2 s$$
$$+(-176-22D+69D^2-16D^3+D^4)tu$$
$$-(-176-22D+69D^2-16D^3+D^4)m_z^2(t+u)\Big)p_1^{\mu}$$
$$-(m_z^2-t)^2(-144-62D+77D^2-16D^3+D^4)p_2^{\mu}$$
$$-(m_z^2-t)(120+86D-77D^2+16D^3-D^4)sq_1^{\mu}\Big\}\frac{12}{(8-9D+D^2)s^2\mathcal{K}_{axi}}$$
$$+\bar{u}(p_1)\not{q}_1\epsilon^{\gamma\gamma\gamma\mu}v(p_2)\Big\{(-1+D)(m_z^2-t)\Big\}\frac{12}{s\mathcal{K}_{axi}},$$

$$\mathbb{P}^{\mu}_{3,axi} = -\bar{u}(p_1)\epsilon_{\gamma\gamma\gamma\gamma}v(p_2)\Big\{-(-1+D)^2(48-14D+D^2)(m_z^2-u)p_1^{\mu}$$
$$-(-120-86D+77D^2-16D^3+D^4)(m_z^2-t)p_2^{\mu}$$
$$+(-1+D)^2(48-14D+D^2)sq_1^{\mu}\Big\}\frac{12}{(8-9D+D^2)s\mathcal{K}_{axi}}$$
$$-\bar{u}(p_1)\not{q}_1\epsilon^{\gamma\gamma\gamma\mu}v(p_2)\Big\{(-56+71D-16D^2+D^3)s\Big\}\frac{12}{(8-9D+D^2)s\mathcal{K}_{axi}},$$

$$\mathbb{P}^{\mu}_{4,axi} = -\bar{u}(p_1)\epsilon_{\gamma\gamma\gamma\gamma}v(p_2)\Big\{-(-7+D)(m_z^2-u)p_1^{\mu}$$
$$-(-1+D)(m_z^2-t)p_2^{\mu}+(-7+D)sq_1^{\mu}\Big\}\frac{12}{s\mathcal{K}_{axi}}$$
$$-\bar{u}(p_1)\not{q}_1\epsilon^{\gamma\gamma\gamma\mu}v(p_2)(8-9D+D^2)\frac{3}{\mathcal{K}_{axi}},$$

$$\mathcal{K}_{axi} = (-888+416D+560D^2-515D^3+159D^4-21D^5+D^6)$$
$$(m_z^4+tu-m_z^2(s+t+u)),$$

$$\epsilon^{\gamma\gamma\gamma\mu} \equiv -\frac{i}{6}\epsilon^{\nu\rho\sigma\mu}\gamma_\nu\gamma_\rho\gamma_\sigma .$$

An ***acrobatic*** form factor decomposition for the axial part

Due to the aforementioned theoretical issues, we ***define*** our "axial form factors" as

$$F_{1,axi} \equiv \mathbb{P}^{[4],\,\mu}_{1,axi}\,\bar{v}(p_2)\,\Gamma^{\nu}_{axi}\,u(p_1)\,g_{\mu\nu}$$
$$F_{2,axi} \equiv \mathbb{P}^{[4],\,\mu}_{2,axi}\,\bar{v}(p_2)\,\Gamma^{\nu}_{axi}\,u(p_1)\,g_{\mu\nu}$$
$$F_{3,axi} \equiv \mathbb{P}^{[4],\,\mu}_{3,axi}\,\bar{v}(p_2)\,\Gamma^{\nu}_{axi}\,u(p_1)\,g_{\mu\nu}$$
$$F_{4,axi} \equiv \mathbb{P}^{[4],\,\mu}_{4,axi}\,\bar{v}(p_2)\,\Gamma^{\nu}_{axi}\,u(p_1)\,g_{\mu\nu}$$

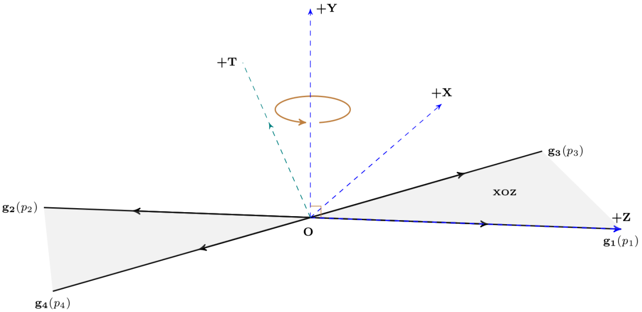
where the $[4]$ in the superscript denotes the setting D = 4 in the original $\mathbb{P}^{\mu}_{i,axi}$.

Subsequently, we build up an intermediate axial amplitude $\tilde{\mathcal{M}}^{\mu}_{axi}$ defined as

$$\tilde{\mathcal{M}}^{\mu}_{axi} \equiv F_{1,axi}\,\bar{v}(p_2)\,\gamma_5\,u(p_1)\,p_1^{\mu} + F_{2,axi}\,\bar{v}(p_2)\,\gamma_5\,u(p_1)\,p_2^{\mu} + F_{3,axi}\,\bar{v}(p_2)\,\gamma_5\,u(p_1)\,q_1^{\mu} + F_{4,axi}\,\bar{v}(p_2)\,\gamma^{\mu}\,\gamma_5\,\not{q}_1 u(p_1)\,.$$

which is not *algebraically* identical to the original Feynman amplitude.

Projecting D-dimensional amplitudes directly onto a *linear polarisation basis* [LC, 19 (arXiv: 1904.00705)] **:**



Momentum basis representations of elementary linear polarisation vectors:

$$\varepsilon_X^\mu = c_1^X p_1^\mu + c_2^X p_2^\mu + c_3^X p_3^\mu ,$$
$$\varepsilon_T^\mu = c_1^T p_1^\mu + c_2^T p_2^\mu + c_3^T p_3^\mu ,$$
$$\varepsilon_Y^\mu = \begin{cases} \mathcal{N}_Y \, \epsilon^{\nu\rho\sigma\mu} p_{1\nu} p_{2\rho} p_{3\sigma} & \text{if } N \leq 4 \\ c_1^Y p_1^\mu + c_2^Y p_2^\mu + c_3^Y p_3^\mu + c_4^Y p_4^\mu & \text{if } N > 5 \end{cases}$$

Projectors for linearly polarised amplitudes

- For $b\bar{b}ZH$, we need only

$$\bar{u}(p_1)\,\mathbf{N}_i\,v(p_2)\,\varepsilon_j^{\mu}\,, \quad \text{for } i = s, p \text{ and } j = T, Y, L$$

  where all open Lorentz indices are *D-dimensional* by definition and all pairs of $\epsilon^{\nu\rho\sigma\mu}$ should be contracted first (in one definite ordering).

- Upon pulling out the overall normalization factors, all projectors so constructed have only *polynomial* dependence in kinematics, and it is always $g_{\mu\nu}$ used in index contraction.

- Resulting (bare) amplitudes are different from those defined in CDR, HV, FDH, DRED, $\cdots$, albeit *unitarity* is still preserved.

- The usual helicity amplitudes can be constructed optionally, as circular polarisation states from the linear ones, e.g.

$$\varepsilon_{\pm}^{\mu}(p_1) = \frac{1}{\sqrt{2}}\left(\varepsilon_X^{\mu} \pm i\varepsilon_Y^{\mu}\right).$$

Projectors for linearly polarised amplitudes

Up to overall normalization factors, our “linear polarisation” projectors for bbZH read:

$$
\begin{aligned}
\mathcal{P}_1^\mu &= \bar{u}(p_1)\, v(p_2) \Big(- \big(2m_z^4 + u(t+u) - m_z^2(2s+t+3u)\big)\ p_1^\mu \\
&\qquad\qquad + \big(2m_z^4 + t(t+u) - m_z^2(2s+3t+u)\big)\ p_2^\mu\ + s(t-u)\, q_1^\mu \Big)\,, \\
\mathcal{P}_2^\mu &= \bar{u}(p_1)\, v(p_2) \Big(-\epsilon^{\mu\nu\rho\sigma} p_{1\,\nu} p_{2\,\rho} q_{1\,\sigma} \Big)\,, \\
\mathcal{P}_3^\mu &= \bar{u}(p_1)\, v(p_2) \Big((2m_z^2 - t - u)\, q_1^\mu - 2m_z^2 (p_1^\mu + p_2^\mu) \Big)\,, \\
\mathcal{P}_4^\mu &= \bar{u}(p_1) \epsilon_{\gamma\gamma\gamma\gamma} v(p_2) \Big(- \big(2m_z^4 + u(t+u) - m_z^2(2s+t+3u)\big)\ p_1^\mu \\
&\qquad\qquad + \big(2m_z^4 + t(t+u) - m_z^2(2s+3t+u)\big)\ p_2^\mu\ + s(t-u)\, q_1^\mu \Big)\,, \\
\mathcal{P}_5^\mu &= \bar{u}(p_1) \frac{i}{8} \Big((-2m_z^2 + t + u) \Big(\not{p}_2 \gamma_\mu + \gamma_\mu \not{p}_1 \Big) + 2s \Big(\gamma_\mu \not{q}_1 - \not{q}_1 \gamma_\mu \Big) \\
&\qquad + 2(u-t) \Big(p_{1\mu} + p_{2\mu} \Big) \Big) v(p_2)\,, \\
\mathcal{P}_6^\mu &= \bar{u}(p_1) \epsilon_{\gamma\gamma\gamma\gamma} v(p_2) \Big((2m_z^2 - t - u)\, q_1^\mu - 2m_z^2 (p_1^\mu + p_2^\mu) \Big)\,.
\end{aligned}
$$

The tool chain employed for computing (projected) amplitudes

- Unreduced (projected) amplitudes: **QGRAF** [P. Nogueir, 1993] + **FORM** [J. Vermaseren, 2000]

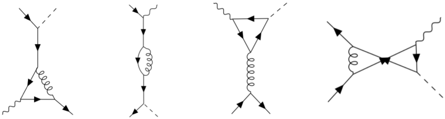
- IBP-tables: **Kira** [P. Maierhofer, J. Usovitsch, P. Uwer; 17/18]
  - 8 one-loop masters known to $\mathcal{O}(\epsilon^2)$
  - 134 two-loop masters known to $\mathcal{O}(\epsilon^0)$.

  All these masters are available in *HepForge* in computer readable format [J. Henn, K. Melnikov, V. A. Smirnov, 2014; T. Gehrmann, A. von Manteuffel, L. Tancre, 2015]

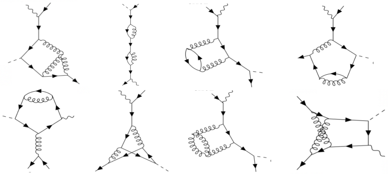
- Simplifying rational coefficients of masters: **mathematica** + **fermat** [R. Lewis; 2009]

Samples of loop diagrams

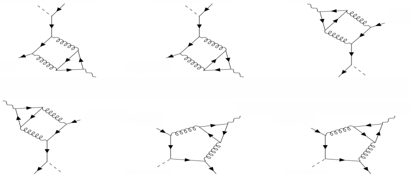
- 1-loop:



- 2-loop flavor-nonsinglet (non-anomalous):



- 2-loop flavor-singlet (anomalous):



UV renormalisation

- Vector part: the $\overline{\text{MS}}$ scheme.
  - The QCD coupling:

$$\hat{a}_s S_\epsilon = a_s(\mu_R^2) Z_{a_s}(\mu_R^2) \left(\frac{\mu^2}{\mu_R^2}\right)^{-\epsilon}$$

  - The Yukawa coupling:

$$\hat{\lambda}_b S_\epsilon = \lambda_b(\mu_R^2) Z_\lambda(\mu_R^2) \left(\frac{\mu^2}{\mu_R^2}\right)^{-\epsilon}$$

- Axial part: additional axial-current ren. $J^{ns(s)}_{\mu,A}(x) = Z^{ns(s)}_{5,A}\, Z^{ns}_A\, \hat{J}^{ns(s)}_{\mu,A}(x)$ [M. Chanowitz, M. Furman, I. Hinchliffe, 1979; T. Trueman, 1979].
  - Flavor-Nonsinglet (non-anomalous) [S. Larin, J. Vermaseren, 1991]:

$$Z^{ns}_A = 1 + a_s^2(\mu_R^2)\frac{1}{\epsilon}\left(\frac{22}{3}C_F C_A - \frac{4}{3}C_F n_f\right),$$

$$Z^{ns}_{5,A} = 1 + a_s(\mu_R^2)\left(-4C_F\right) + a_s^2(\mu_R^2)\left(22C_F^2 - \frac{107}{9}C_F C_A + \frac{2}{9}C_F n_f\right).$$

  - Flavor-Singlet (anomalous) [S. Larin, 1993]:

$$Z^s_A = 1 + a_s^2(\mu_R^2)\frac{3}{\epsilon}C_F\,,$$

$$Z^s_{5,A} = 1 + a_s^2(\mu_R^2)\frac{3}{2}C_F\,.$$

UV renormalisation

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  - The Yukawa coupling:

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  UV renormalised amplitudes:

$$\begin{aligned}
\mathcal{M}^{[j]} &= \mathcal{M}^{[j]}_{vec}(a_s(\mu_R^2)) + \mathcal{M}^{[j]}_{axi}(a_s(\mu_R^2)) \\
&= \mathcal{M}^{[j]}_{vec}(a_s(\mu_R^2)) + \mathcal{M}^{[j],ns}_{axi}(a_s(\mu_R^2)) + \mathcal{M}^{[j],s}_{axi}(a_s(\mu_R^2)) \\
&= \hat{\mathcal{M}}^{[j]}_{vec}(\hat{a}_s, \mu^2) + Z^{ns}_5(a_s(\mu_R^2)) Z^{ns}_A(a_s(\mu_R^2)) \hat{\mathcal{M}}^{[j],ns}_{axi}(\hat{a}_s, \mu^2) \\
&\qquad + Z^{s}_5(a_s(\mu_R^2)) Z^{s}_A(a_s(\mu_R^2)) \hat{\mathcal{M}}^{[j],s}_{axi}(\hat{a}_s, \mu^2)\,,
\end{aligned}$$

  where $[j]$ runs over all six polarisation configurations.

IR factorisation formulae

The IR pole structures in the UV renormalised bbZH amplitudes can be exhibited through a factorisation formula in terms of "universal" $\mathbf{I}^{(i)}(\epsilon)$ [S. Catani, 1998]

$$\mathcal{M}^{[j],(1)} = 2\mathbf{I}^{(1)}(\epsilon)\mathcal{M}^{[j],(0)} + \mathcal{M}_{\text{fin}}^{[j],(1)},$$
$$\mathcal{M}^{[j],(2)} = 4\mathbf{I}^{(2)}(\epsilon)\mathcal{M}^{[j],(0)} + 2\mathbf{I}^{(1)}(\epsilon)\mathcal{M}^{[j],(1)} + \mathcal{M}_{\text{fin}}^{[j],(2)}.$$

$\mathcal{M}_{\text{fin}}^{[j],(1)}$ and $\mathcal{M}_{\text{fin}}^{[j],(2)}$ (in 4 dimensions) are defined as the *finite remainders*.

The explicit expressions of the $\mathbf{I}^{(i)}(\epsilon)$ (needed for bbZH) are given (in CDR) by [S. Catani, 1998; T. Becher, M. Neubert, 2009]

$$\mathbf{I}^{(1)}(\epsilon) = -C_F \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \left(\frac{1}{\epsilon^2} + \frac{3}{2\epsilon}\right) \left(-\frac{\mu_R^2}{s}\right)^{\epsilon},$$
$$\mathbf{I}^{(2)}(\epsilon) = -\frac{1}{2}\mathbf{I}^{(1)}(\epsilon)\left(\mathbf{I}^{(1)}(\epsilon) + \frac{1}{\epsilon}\beta_0\right) + \frac{e^{-\epsilon\gamma_E}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{2}{\epsilon}\beta_0 + K\right)\mathbf{I}^{(1)}(2\epsilon) + 2H^{(2)}(\epsilon)$$

IR poles contained in our renormalised amplitudes match with those predicted by these formulae.

The same RS-independent finite remainders

The linear transformation connecting $F_{i,vec}$ to $\mathcal{P}_i^{\mu}\,\bar{v}(p_2)\,\Gamma_{vec,\mu}\,u(p_1)$ reads as

$$\mathcal{P}_i^{\mu}\,\bar{v}(p_2)\,\Gamma_{vec,\mu}\,u(p_1) = F_{1,vec}\left(\mathcal{P}_i^{\mu}\,\bar{v}(p_2)\,u(p_1)\,p_{1,\mu}\right) + F_{2,vec}\left(\mathcal{P}_i^{\mu}\,\bar{v}(p_2)\,u(p_1)\,p_{2,\mu}\right) + F_{3,vec}\left(\mathcal{P}_i^{\mu}\,\bar{v}(p_2)\,u(p_1)\,q_{1,\mu}\right) + F_{4,vec}\left(\mathcal{P}_i^{\mu}\,\bar{v}(p_2)\,\gamma_\mu \not{q}_1 u(p_1)\right),$$

and similarly for the axial part,

$$\left[\mathcal{P}_i^{\mu}\,\bar{v}(p_2)\,\Gamma_{axi,\mu}\,u(p_1)\right]_{\text{fin}} = F_{1,axi,\text{fin}}\left[\mathcal{P}_i^{\mu}\,\bar{v}(p_2)\,\gamma_5\,u(p_1)\,p_{1,\mu}\right] + F_{2,axi,\text{fin}}\left[\mathcal{P}_i^{\mu}\,\bar{v}(p_2)\,\gamma_5\,u(p_1)\,p_{2,\mu}\right] + F_{3,axi,\text{fin}}\left[\mathcal{P}_i^{\mu}\,\bar{v}(p_2)\,\gamma_5\,u(p_1)\,q_{1,\mu}\right] + F_{4,axi,\text{fin}}\left[\mathcal{P}_i^{\mu}\,\bar{v}(p_2)\,\gamma_\mu\gamma_5\,\not{q}_1 u(p_1)\right],$$

where i runs from 1 to 6 linear polarisation configurations.

Through the verified equality between the *finite remainders* computed following different approaches,

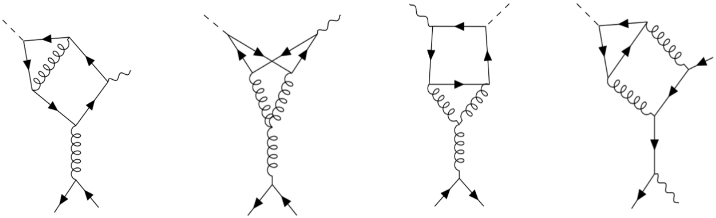
we confirm:

projectors derived in "four" dimensions can be used also in calculations in D dimensions and lead to correct results (for physical observables), irrespective of whether the quantity projected out is a form factor or a linearly polarised amplitude.

(even though the resulting amplitudes may not be regularised strictly in the HV scheme.)

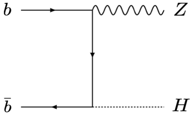
Axial Form factors restored from Vector Form Factors

With $m_b = 0$, all 2-loop diagrams with the Higgs (and Z) radiated from a closed fermion loop vanish, e.g.

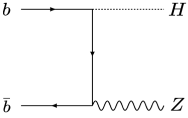


due to odd number of Dirac γ matrices.

Consequently, all non-vanishing *non-anomalous* Feynman diagrams can be divided into the class-HZ and class-ZH



(A)

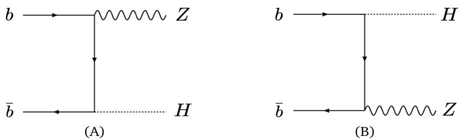


(B)

Axial Form factors restored from Vector Form Factors

Turning off completely the axial coupling of the Z boson,

$$\mathcal{M}_{vec} = \bar{v}(p_2)\,\Gamma^{\mu}_{ZH}\,u(p_1)\,\varepsilon^{*}_{\mu}(q_1) + \bar{v}(p_2)\,\Gamma^{\mu}_{HZ}\,u(p_1)\,\varepsilon^{*}_{\mu}(q_1)$$



$$\begin{aligned}
\bar{v}(p_2)\,\Gamma^{\mu}_{ZH}\,u(p_1) &= F_{1,ZH}\,\bar{v}(p_2)\,u(p_1)\,p_1^{\mu} + F_{2,ZH}\,\bar{v}(p_2)\,u(p_1)\,p_2^{\mu} \\
&\quad + F_{3,ZH}\,\bar{v}(p_2)\,u(p_1)\,q_1^{\mu} + F_{4,ZH}\,\bar{v}(p_2)\,\gamma^{\mu}\not{q}_1 u(p_1)\,, \\
\bar{v}(p_2)\,\Gamma^{\mu}_{HZ}\,u(p_1) &= F_{1,HZ}\,\bar{v}(p_2)\,u(p_1)\,p_1^{\mu} + F_{2,HZ}\,\bar{v}(p_2)\,u(p_1)\,p_2^{\mu} \\
&\quad + F_{3,HZ}\,\bar{v}(p_2)\,u(p_1)\,q_1^{\mu} + F_{4,HZ}\,\bar{v}(p_2)\,\gamma^{\mu}\not{q}_1 u(p_1)\,.
\end{aligned}$$

Axial (and vector) Form factors can be restored as:

$$\begin{aligned}
F_{i,vec} &= F_{i,HZ} + F_{i,ZH}\,, \\
F_{i,axi(ns)} &= F_{i,HZ} - F_{i,ZH}\,.
\end{aligned}$$

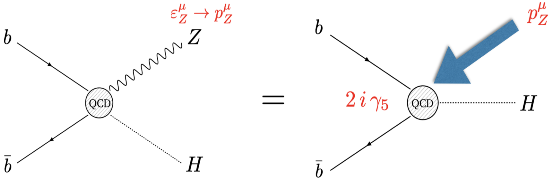
The Ward identity for bbZH: the non-anomalous part

Starting from the classical Lagrangian,

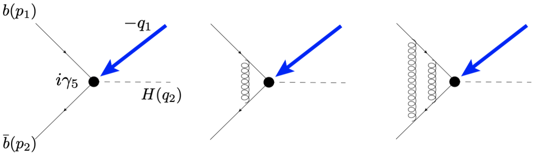
$$\mathcal{L}_c = -\tfrac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \bar{b}i\gamma^\mu D_\mu b - J^\mu_Z Z_\mu - \lambda_b \bar{b}bH\,,$$

one obtains the Ward identity for the non-anomalous diagrams:

$$q_1^\mu \mathcal{M}_\mu = -2g_{A,b}\lambda_b \langle H(q_2)|\bar{b}(0)i\gamma_5 b(0)H(0)|b(p_1)\bar{b}(p_2)\rangle\,.$$

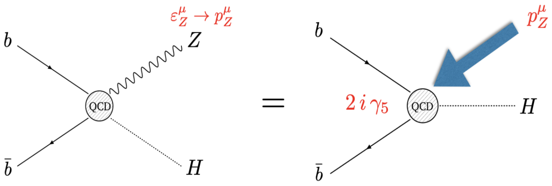


whose RHS can be perturbatively expanded as



Checking the Ward identity for bbZH using $F_{i,ZH}$ and $F_{i,HZ}$

$$q_1^\mu \mathcal{M}_\mu = -2 g_{A,b} \lambda_b \langle H(q_2) | \bar{b}(0) i\gamma_5 b(0) H(0) | b(p_1) \bar{b}(p_2) \rangle \,.$$



The LHS of the Ward identity can be composed in terms of “split” vector form factors:

$$q_1 \cdot \mathcal{M}_{vec} = \bar{v}(p_2)\, u(p_1) \Bigg(\left(F_{1,HZ} + F_{1,ZH}\right) \frac{m_Z^2 - t}{2} + \left(F_{2,HZ} + F_{2,ZH}\right) \frac{m_Z^2 - u}{2} + \left(F_{3,HZ} + F_{3,ZH}\right) m_Z^2 + \left(F_{4,HZ} + F_{4,ZH}\right) m_Z^2 \Bigg),$$

$$q_1 \cdot \mathcal{M}_{axi} = \bar{v}(p_2)\, \gamma_5\, u(p_1) \Bigg(\left(F_{1,HZ} - F_{1,ZH}\right) \frac{m_Z^2 - t}{2} + \left(F_{2,HZ} - F_{2,ZH}\right) \frac{m_Z^2 - u}{2} + \left(F_{3,HZ} - F_{3,ZH}\right) m_Z^2 + \left(F_{4,HZ} - F_{4,ZH}\right) m_Z^2 \Bigg).$$

The Ward identity for bbZH: the anomalous part

The Adler-Bell-Jackiw (ABJ) anomaly equation:

$$\left(\partial^{\mu} J^{s}_{\mu,A}\right)_R = a_s \frac{1}{2} \left(G\tilde{G}\right)_R \,,$$

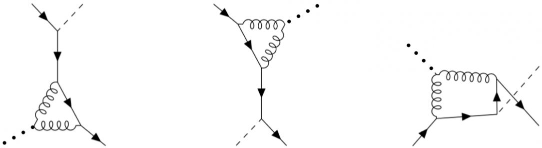
where $G\tilde{G} \equiv \epsilon_{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma}$ with the gluonic field strength tensor $G^a_{\mu\nu}$.

At the level of matrix elements (regarding the anomalous part of $b\bar{b} \to ZH$),

$$\bar{v}(p_2)\, \Gamma^{\mu}_{\text{ABJ}}\, u(p_1)\, q_{1,\mu} = g_{A,q} \frac{a_s}{2} \langle H(q_2) \big| \left[G\tilde{G}(0)\right]_R \big| q(p_1) \bar{q}(p_2) \rangle \,,$$

(with the kinematics $p_1 + p_2 - q_2 = q_1$).

Diagrammatically, the RHS consists of



The numerical results of the total cross section of ZH production at LHC@13TeV (in unit pb):

Order	s-channel	EW	σ_{gg}^{ZH}	$\sigma_{q\bar{q}}^{ZH}$(top)	$(t+u)$-channel
LO	$5.897\ 10^{-1}$	-$3.111\ 10^{-2}$	-	-	$2.989\ 10^{-4}$
NLO	$7.756\ 10^{-1}$	-	-	-	$2.934\ 10^{-4}$
NNLO	$8.015\ 10^{-1}$	-	$5.051\ 10^{-2}$	$9.442\ 10^{-3}$	$3.027\ 10^{-4}$
N^3LO_{SV}	$8.013\ 10^{-1}$	-	-	-	-

Setting: $\mu_R = \mu_F = m_h + m_z,\ G_F = 1.16637 \times 10^{-5}$ GeV^{-2}, $\overline{m}_b(\mu_R = m_b) = 4.18$ GeV.

- The s-channel contributions (Higgs-bremsstrahlung) are obtained using **vh@nnlo** [Brein,Harlander,Zirke, 2012].
- The σ_{gg}^{ZH} refers to the contribution coming from the gluon initiated sub-processes.
- The top quark loop contribution is denoted by $\sigma_{q\bar{q}}^{ZH}$(top).
- The $(t+u)$-channel contribution at NLO is obtained using **Madgraph** [Alwall,Frederix,Frixione,Hirschi,Maltoni,Mattelaer et al.,2014], and at NNLO is under the *Soft-Virtual* approximation [Ravindran,2005].

Summary

- ☑ Computed the 2-loop QCD corrections to $b\bar{b} \to ZH$ amplitude via directly projecting onto a linear polarisation basis, with the analytic results expressed in terms of multiple polylogarithms;

- ☑ Addressed an interesting subtlety appearing in the conventional form-factor(FF) decomposition of amplitudes involving axial currents regularised in D dimensions;

- ☑ Revealed a relation between axial and vector FFs of the non-Drell-Yan $b\bar{b} \to ZH$, which enables us to restore axial FFs from "split" vector ones;

- ☑ Derived the Ward identities for $b\bar{b} \to ZH$ in the presence of λ_b, which are checked using the axial FFs restored from their vector counterparts;

- ☑ Computed the *SV* cross section at NNLO, in order to make a quantitative analysis of the contribution from these non-Drell-type processes.

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THANK YOU