Some issues in the computation of 2-loop amplitudes $q\bar{q} \to ZH$

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The discovery of the $h(125)$ at the LHC (2012)
$h(125) \rightarrow b\bar{b}$ observed through $VH(b\bar{b})$

$h \rightarrow b\bar{b}$ finally observed recently at the LHC in the $VH$-events!

The measured signal strength $\mu = 1.04 \pm 0.20$. 
Much theoretical work done already...

Much work done on the ZH production at LHC: \( P + P \rightarrow Z(ll') + H(b\bar{b}) \)

- \( q\bar{q} \rightarrow ZH: \) Starting from \( \mathcal{O}(\alpha_e \alpha_s^0) \)
  - The Higgs-bremsstrahlung (Drell-Yan) part up to \( \text{NNLO} \) in massless QCD [Brein, Harlander, Wiesemann, Zirke, 2012; Ferrera, Grazzini, Tramontano, 2015/2018; Campbell, Ellis, Williams, 2016]
  - The top-loop induced \( \text{NNLO} \) (non-Drell-Yan type) QCD corrections in the heavy-top limit [Brein, Djouadi, Harlander, 2004; Brein, Harlander, Wiesemann, Zirke, 2012]
  - \( \text{N}^3\text{LO} \) corrections in massless QCD [Ahmed, Mahakhud, Rana, Ravindran, 2014; Li, von Manteuffel, Schabinger, Zhu, 2014; Catani, Cieri, Florian, Ferrera, Grazzini, 2014; Kumar, Mandal, Ravindran, 2015]
  - \( \text{NLO} \) electroweak corrections [Ciccolini, Dittmaier, Kramer, 2003; Denner, Dittmaier, Kallweit, Mueck, 2011]
  - . . . . .

- \( g\bar{g} \rightarrow ZH: \) Starting from \( \mathcal{O}(\alpha_e \alpha_s) \)
  - The exact \( \text{LO} \) (with full \( m_t \) dependence) [Kniehl, 1990]
  - \( \text{NLO} \) QCD corrections to the Higgs-bremsstrahlung (Drell-Yan) part in the heavy-top limit [Brein, Harlander, Wiesemann, Zirke, 2012; Altenkamp, Dittmaier, Harlander, Rzehak, Zirke, 2013]
  - . . . . .
Focus of the talk

Here we focus on a part of the $b$-quark-induced $ZH$ process that involves a non-vanishing Yukawa coupling $\lambda_b$ (i.e. (B) and (C)) but with $m_b = 0$.

Aims of the work:

- Computing the 2-loop (massless) QCD corrections in analytic form
- Addressing a subtlety appearing in the conventional FF decomposition of amplitudes involving axial currents regularised in D dimensions (with a non-anticommuting $\gamma_5$)
- Verifying the unitarity of a particular regularisation prescription implied by projectors prescribed recently [Chen, 2019]
- The “same” loop amplitudes built up from just vector FFs of properly grouped classes of diagrams (bypassing completely the need of explicitly manipulating $\gamma_5$)
Kinematics and the $\gamma_5$ prescription

We consider

$$b(p_1) + \bar{b}(p_2) \rightarrow Z(q_1) + H(q_2)$$

in $n_f = 5$ massless QCD.

The Mandelstam variables:

$$s \equiv (p_1 + p_2)^2, \quad t \equiv (p_1 - q_1)^2 \quad \text{and} \quad u \equiv (p_2 - q_1)^2$$

satisfying $s + t + u = q_1^2 + q_2^2 = m_z^2 + m_h^2$.

Regarding the definition of $\gamma_5$ in dimensional regularisation (HV/BM and Larin):

$$\gamma_5 = \frac{i}{4!} \epsilon_{\mu \nu \rho \sigma} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma$$

$$\gamma_\mu \gamma_5 \rightarrow \frac{1}{2} \left( \gamma_\mu \gamma_5 - \gamma_5 \gamma_\mu \right) = \frac{i}{6} \epsilon_{\mu \nu \rho \sigma} \gamma^\nu \gamma^\rho \gamma^\sigma.$$
Form Factor Decomposition: the vector part

By Lorentz covariance, the amplitude \( b\bar{b}ZH \) can be expressed as a linear combination of a finite basis of Lorentz structures at any finite order.

For the vector part:

\[
\bar{v}(p_2) \Gamma_{vec}^\mu u(p_1) = F_{1,vec} \bar{v}(p_2) u(p_1) p_1^\mu + F_{2,vec} \bar{v}(p_2) u(p_1) p_2^\mu + F_{3,vec} \bar{v}(p_2) u(p_1) q_1^\mu + F_{4,vec} \bar{v}(p_2) \gamma^\mu q_1 u(p_1),
\]

under the following constraints:

- Even power \( \gamma \)-matrices (one Yukawa vertex on the massless fermion line);
- Parity even for the vector coupling;
- Equations of motion for the on-shell massless spinors: \( p_1 u(p_1) = 0, p_2 v(p_2) = 0 \).
Form Factor Decomposition: Gram matrix and Projectors

For a scattering amplitude $\hat{\mathcal{M}}$ at a fixed perturbative order,

$$\hat{\mathcal{M}} = \sum_{n=1}^{N} F_n \hat{T}_n,$$

compute the **Gram matrix** $\hat{G}$

$$G_{ij} = \langle \hat{T}_i^\dagger, \hat{T}_j \rangle,$$

whose inverse gives us the formula for the projector

$$\hat{P}_n = \hat{G}^{-1}_{nj} \hat{T}_j^\dagger$$

to project out the **Lorentz-invariant** coefficient (form-factor) $F_n$. 
The projectors for vector form factors are still compact enough to be documented explicitly:

\[
\mathbf{P}_{1,\text{vec}}^\mu = \bar{u}(p_1) \begin{array}{l}
\{(-2 + D)t^2 p_1^\mu + \left(2(-3 + D)m_z^2 s - (-2 + D)tu\right)p_2^\mu + (2 - D)stq_1^\mu \\
+ stq_1 \gamma^\mu \} v(p_2) \frac{1}{\mathcal{K}_{\text{vec}}},
\end{array}
\]

\[
\mathbf{P}_{2,\text{vec}}^\mu = \bar{u}(p_1) \begin{array}{l}
\left\{2(-3 + D)m_z^2 s - (-2 + D)tu\right)p_1^\mu + (-2 + D)u^2 p_2^\mu + (4 - D)suq_1^\mu \\
- suq_1 \gamma^\mu \} v(p_2) \frac{1}{\mathcal{K}_{\text{vec}}},
\end{array}
\]

\[
\mathbf{P}_{3,\text{vec}}^\mu = \bar{u}(p_1) \begin{array}{l}
\{-(2 + D)stp_1^\mu + (-4 + D)sup_2^\mu - (2 - D)s^2 q_1^\mu - s^2 q_1 \gamma^\mu \} v(p_2) \frac{1}{\mathcal{K}_{\text{vec}}},
\end{array}
\]

\[
\mathbf{P}_{4,\text{vec}}^\mu = \bar{u}(p_1) \begin{array}{l}
\{stp_1^\mu - sup_2^\mu - s^2 q_1^\mu + s^2 q_1 \gamma^\mu \} v(p_2) \frac{1}{\mathcal{K}_{\text{vec}}},
\end{array}
\]

where

\[
\mathcal{K}_{\text{vec}} = 2(-3 + D)s(m_z^2 s - tu).
\]
Form Factor Decomposition: issues with the axial part

A set of Lorentz structure basis for the axial part of $b\bar{b} \rightarrow ZH$ linearly complete in 4 dimensions, but not in D dimensions (unless a fully anticommutating $\gamma_5$ is used):

$$\left\{ \bar{\nu}(p_2) \gamma_5 u(p_1) p_1^\mu, \bar{\nu}(p_2) \gamma_5 u(p_1) p_2^\mu, \bar{\nu}(p_2) \gamma_5 u(p_1) q_1^\mu, \bar{\nu}(p_2) \gamma^\mu \gamma_5 q_1 u(p_1) \right\}.$$ 

Whether $M_{axi}$ lives in a space linearly spanned by these 4 structures in D dimensions:

- Enlarge this list by appending the (tree-level) $M_{axi}$;
- and then compute the **Gram matrix** of this enlarged list of 5 elements;
- one finds out that the matrix rank is increased to 5 rather than staying at 4!
A set of Lorentz structure basis for the axial part of $b\bar{b} \to ZH$ linearly complete in 4 dimensions, but not in D dimensions (unless a fully anticommutating $\gamma_5$ is used):

$$\{ \bar{v}(p_2) \gamma_5 u(p_1) p_1^\mu, \bar{v}(p_2) \gamma_5 u(p_1) p_2^\mu, \bar{v}(p_2) \gamma_5 u(p_1) q_1^\mu, \bar{v}(p_2) \gamma_5 q_1 u(p_1) \}.$$ 

Therefore, if a non-anticommuting $\gamma_5$ is used in the dimensional regularisation, we may face the following issues regarding the axial FF-decomposition:

1. It is not easy to construct the full D-dimensional linearly complete basis for the FF-decomposition of $\bar{v}(p_2) \Gamma^\mu_{\text{axi}} u(p_1)$ at loop orders.

2. Even with just an incomplete basis, keeping the full D-dependence could lead to expressions with too complicated D-dependence to use.

3. Setting D=4 could simplify the expressions a bit, which, however, is not known to be legitimate in general (for FF-decomposition).
The projectors for 4 axial FFs (corresponding to the basis of 4 structures) read as:

$$P^\mu_{1,axi} = -\bar{u}(p_1)\epsilon^{\nu\rho\sigma\tau}v(p_2) \begin{pmatrix}
(1 + D)^2 (48 - 14D + D^2) (m_2^2 - u)^2 p_1^\mu \\
+ (176 + 22D - 69D^2 + 16D^3 - D^4) m_2^2 \\
+ 2(-148 - 54D + 73D^2 - 16D^3 + D^4) m_2^2 s \\
+ (176 + 22D - 69D^2 + 16D^3 - D^4) tu \\
- (176 + 22D - 69D^2 + 16D^3 - D^4) m_2^2 (t + u) \\
-(1 + D)^2 (48 - 14D + D^2) s (m_2^2 - u) q_1^\mu \\
\end{pmatrix} \frac{12}{(8 - 9D + D^2) s^2 K_{axi}},$$

$$P^\mu_{2,axi} = -\bar{u}(p_1)\epsilon^{\nu\rho\sigma\tau}v(p_2) \begin{pmatrix}
(176 - 22D + 69D^2 - 16D^3 + D^4) m_2^2 \\
- 2(-148 - 54D + 73D^2 - 16D^3 + D^4) m_2^2 s \\
+ (176 - 22D + 69D^2 - 16D^3 + D^4) tu \\
- (176 - 22D + 69D^2 - 16D^3 + D^4) m_2^2 (t + u) \\
-(m_2^2 - t)^2 (144 - 62D + 77D^2 - 16D^3 + D^4) p_2^\mu \\
- (m_2^2 - t) (120 + 86D - 77D^2 + 16D^3 - D^4) s q_1^\mu \\
\end{pmatrix} \frac{12}{(8 - 9D + D^2) s^2 K_{axi}},$$

$$P^\mu_{3,axi} = -\bar{u}(p_1)\epsilon^{\nu\rho\sigma\tau}v(p_2) \begin{pmatrix}
(-1 + D)^2 (48 - 14D + D^2) (m_2^2 - u) p_1^\mu \\
- (120 - 86D + 77D^2 - 16D^3 + D^4) m_2^2 t \\
+ (1 + D)^2 (48 - 14D + D^2) s q_1^\mu \\
\end{pmatrix} \frac{12}{(8 - 9D + D^2) s K_{axi}},$$

$$P^\mu_{4,axi} = -\bar{u}(p_1)\epsilon^{\nu\rho\sigma\tau}v(p_2) \begin{pmatrix}
-(7 + D) (m_2^2 - u) p_1^\mu \\
-(1 + D) (m_2^2 - t) p_2^\mu + (7 + D) s q_1^\mu \\
\end{pmatrix} \frac{12}{s K_{axi}},$$

$$K_{axi} = \frac{(-888 + 416D + 560D^2 - 515D^3 + 159D^4 - 21D^5 + D^6) (m_2^2 + tu - m_2^2 (s + t + u)),}{\epsilon^{\nu\rho\sigma\tau} \equiv \frac{i}{6} \gamma^\nu \gamma^\rho \gamma^\sigma \gamma^\tau},$$
An *acrobatic* form factor decomposition for the axial part

Due to the aforementioned theoretical issues, we **define** our “axial form factors” as

\[
F_{1,\text{axi}} \equiv \mathbb{P}_{1,\text{axi}}^{[4],\mu} \bar{\nu}(p_2) \Gamma_{\text{axi}}^\nu u(p_1) g_{\mu\nu} \\
F_{2,\text{axi}} \equiv \mathbb{P}_{2,\text{axi}}^{[4],\mu} \bar{\nu}(p_2) \Gamma_{\text{axi}}^\nu u(p_1) g_{\mu\nu} \\
F_{3,\text{axi}} \equiv \mathbb{P}_{3,\text{axi}}^{[4],\mu} \bar{\nu}(p_2) \Gamma_{\text{axi}}^\nu u(p_1) g_{\mu\nu} \\
F_{4,\text{axi}} \equiv \mathbb{P}_{4,\text{axi}}^{[4],\mu} \bar{\nu}(p_2) \Gamma_{\text{axi}}^\nu u(p_1) g_{\mu\nu}
\]

where the [4] in the superscript denotes the setting $D = 4$ in the original $\mathbb{P}^{\mu}_{i,\text{axi}}$.

Subsequently, we build up an intermediate axial amplitude $\tilde{\mathcal{M}}^{\mu}_{\text{axi}}$ defined as

\[
\tilde{\mathcal{M}}^{\mu}_{\text{axi}} \equiv F_{1,\text{axi}} \bar{\nu}(p_2) \gamma_5 u(p_1) p_1^{\mu} + F_{2,\text{axi}} \bar{\nu}(p_2) \gamma_5 u(p_1) p_2^{\mu} \\
+ F_{3,\text{axi}} \bar{\nu}(p_2) \gamma_5 u(p_1) q_1^{\mu} + F_{4,\text{axi}} \bar{\nu}(p_2) \gamma^{\mu} \gamma_5 q_1 u(p_1).
\]

which is **not algebraically identical** to the original Feynman amplitude.
Projectors for linearly polarised amplitudes

Projecting D-dimensional amplitudes directly onto a *linear polarisation basis* [LC, 19 (arXiv: 1904.00705)]:

**Momentum basis representations** of elementary linear polarisation vectors:

\[ \varepsilon_X^\mu = c_1 X p_1^\mu + c_2 X p_2^\mu + c_3 X p_3^\mu, \]
\[ \varepsilon_T^\mu = c_1 T p_1^\mu + c_2 T p_2^\mu + c_3 T p_3^\mu, \]
\[ \varepsilon_Y^\mu = \begin{cases} \mathcal{N}_Y \varepsilon^{\nu\rho\sigma\mu} p_1^\nu p_2^\rho p_3^\sigma & \text{if } N \leq 4 \\ c_1 Y p_1^\mu + c_2 Y p_2^\mu + c_3 Y p_3^\mu + c_4 Y p_4^\mu & \text{if } N > 5 \end{cases} \]
For $b\bar{b}ZH$, we need only

$$\bar{u}(p_1) \not\!N_i \, \nu(p_2) \, \epsilon^\mu_j, \quad \text{for } i = s, p \text{ and } j = T, Y, L$$

where all open Lorentz indices are \textit{D-dimensional by definition} and all pairs of $\epsilon^{\nu\rho\sigma\mu}$ should be contracted first (in one definite ordering).

Upon pulling out the overall normalization factors, all projectors so constructed have \textit{only polynomial dependence in kinematics}, and it is always $g_{\mu\nu}$ used in index contraction.

Resulting (bare) amplitudes are different from those defined in CDR, HV, FDH, DRED, \cdots, albeit \textit{unitarity} is still preserved.

The usual helicity amplitudes can be constructed optionally, as circular polarisation states from the linear ones, e.g.

$$\epsilon^\mu_{\pm}(p_1) = \frac{1}{\sqrt{2}} \left( \epsilon^\mu_X \pm i\epsilon^\mu_Y \right).$$
Projectors for linearly polarised amplitudes

Up to overall normalization factors, our “linear polarisation” projectors for bbZH read:

\[ P_1^\mu = \bar{u}(p_1) v(p_2) \left( - (2m_A^4 + u(t + u) - m_Z^2(2s + t + 3u)) \ p_1^\mu \\
+ (2m_A^4 + t(t + u) - m_Z^2(2s + 3t + u)) \ p_2^\mu + s(t - u) q_1^\mu \right), \]

\[ P_2^\mu = \bar{u}(p_1) v(p_2) \left( - \epsilon^{\mu\nu\rho\sigma} p_1^\nu p_2^\rho q_1^\sigma \right), \]

\[ P_3^\mu = \bar{u}(p_1) v(p_2) \left( (2m_A^2 - t - u) q_1^\mu - 2m_Z^2(p_1^\mu + p_2^\mu) \right), \]

\[ P_4^\mu = \bar{u}(p_1) \epsilon_{\gamma\gamma\gamma\gamma} v(p_2) \left( - (2m_A^4 + u(t + u) - m_Z^2(2s + t + 3u)) \ p_1^\mu \\
+ (2m_A^4 + t(t + u) - m_Z^2(2s + 3t + u)) \ p_2^\mu + s(t - u) q_1^\mu \right), \]

\[ P_5^\mu = \bar{u}(p_1) \frac{i}{8} \left( (-2m_A^2 + t + u) \left( p_2 \gamma_\mu + \gamma_\mu p_1 \right) + 2s \left( \gamma_\mu q_1 - q_1 \gamma_\mu \right) \\
+ 2(u - t) \left( p_{1\mu} + p_{2\mu} \right) \right) v(p_2), \]

\[ P_6^\mu = \bar{u}(p_1) \epsilon_{\gamma\gamma\gamma\gamma} v(p_2) \left( (2m_A^2 - t - u) q_1^\mu - 2m_Z^2(p_1^\mu + p_2^\mu) \right). \]
The tool chain employed for computing (projected) amplitudes

- **Unreduced (projected) amplitudes:** **QGRAF** [P. Nogueir, 1993] + **FORM** [J. Vermaseren, 2000]

- **IBP-tables:** **Kira** [P. Maierhofer, J. Usovitsch, P. Uwer; 17/18]
  - 8 one-loop masters known to $O(\epsilon^2)$
  - 134 two-loop masters known to $O(\epsilon^0)$.

All these masters are available in **HepForge** in computer readable format [J. Henn, K. Melnikov, V. A. Smirnov, 2014; T. Gehrmann, A. von Manteuffel, L. Tancre, 2015]

- **Simplifying rational coefficients of masters:** **mathematica** + **fermat** [R. Lewis; 2009]
Samples of loop diagrams

- 1-loop:

- 2-loop flavor-nonsinglet (non-anomalous):

- 2-loop flavor-singlet (anomalous):
UV renormalisation

- **Vector part:** the $\overline{MS}$ scheme.
  - The QCD coupling:
    \[
    \hat{a}_s S_\epsilon = a_s(\mu^2_R) Z_{a_5} (\mu^2_R) \left( \frac{\mu^2}{\mu^2_R} \right)^{-\epsilon}
    \]
  - The Yukawa coupling:
    \[
    \hat{\lambda}_b S_\epsilon = \lambda_b(\mu^2_R) Z_{\lambda} (\mu^2_R) \left( \frac{\mu^2}{\mu^2_R} \right)^{-\epsilon}
    \]

- **Axial part:** additional axial-current ren. $j^{ns(s)}_{\mu,A}(x) = Z_{5,\mu}^{ns(s)} Z_{A}^{ns} j^{ns(s)}_{\mu,A}(x)$ [M. Chanowitz, M. Furman, I. Hinchliffe, 1979; T. Trueman, 1979].
  - Flavor-Nonsinglet (non-anomalous) [S. Larin, J. Vermaseren, 1991]:
    \[
    Z_{5,\mu}^{ns} = 1 + a_s^2 (\mu^2_R) \frac{1}{\epsilon} \left( \frac{22}{3} C_F C_A - \frac{4}{3} C_F n_f \right),
    \]
    \[
    Z_{\mu}^{ns} = 1 + a_s (\mu^2_R) (-4C_F) + a_s^2 (\mu^2_R) \left( 22C_F^2 - \frac{107}{9} C_F C_A + \frac{2}{9} C_F n_f \right) .
    \]
  - Flavor-Singlet (anomalous) [S. Larin, 1993]:
    \[
    Z_{\mu}^{s} = 1 + a_s^2 (\mu^2_R) \frac{3}{\epsilon} C_F ,
    \]
    \[
    Z_{5,\mu}^{s} = 1 + a_s^2 (\mu^2_R) \frac{3}{2} C_F .
    \]
UV renormalisation

- **Vector part:** the $\overline{\text{MS}}$ scheme.
  - The QCD coupling:
    \[
    \hat{a}_s S_\epsilon = a_s(\mu_R^2) Z_{a_s}(\mu_R^2) \left( \frac{\mu^2}{\mu_R^2} \right)^{-\epsilon}
    \]
  - The Yukawa coupling:
    \[
    \hat{\lambda}_b S_\epsilon = \lambda_b(\mu_R^2) Z_{\lambda}(\mu_R^2) \left( \frac{\mu^2}{\mu_R^2} \right)^{-\epsilon}
    \]

- **Axial part:** additional axial-current ren.
  \[
  J_{\mu,A}^{ns(s)}(x) = Z_{5,A}^{ns(s)} \hat{J}_{\mu,A}^{ns(s)}(x)
  \]

UV renormalised amplitudes:

\[
\mathcal{M}^{[j]} = \mathcal{M}_{\text{vec}}^{[j]}(a_s(\mu_R^2)) + \mathcal{M}_{\text{axi}}^{[j]}(a_s(\mu_R^2)) \\
= \mathcal{M}_{\text{vec}}^{[j]}(a_s(\mu_R^2)) + \mathcal{M}_{\text{axi}}^{[j],ns}(a_s(\mu_R^2)) + \mathcal{M}_{\text{axi}}^{[j],s}(a_s(\mu_R^2)) \\
= \hat{\mathcal{M}}_{\text{vec}}^{[j]}(\hat{a}_s, \mu^2) + Z_{5}^{ns}(a_s(\mu_R^2)) Z_{A}^{ns}(a_s(\mu_R^2)) \hat{\mathcal{M}}_{\text{axi}}^{[j],ns}(\hat{a}_s, \mu^2) \\
+ Z_{5}^{s}(a_s(\mu_R^2)) Z_{A}^{s}(a_s(\mu_R^2)) \hat{\mathcal{M}}_{\text{axi}}^{[j],s}(\hat{a}_s, \mu^2),
\]

where $[j]$ runs over all six polarisation configurations.
IR factorisation formulae

The IR pole structures in the UV renormalised bbZH amplitudes can be exhibited through a factorisation formula in terms of “universal” \( I^{(i)}(\epsilon) \) [S. Catani, 1998]

\[
\mathcal{M}^{[j],(1)} = 2I^{(1)}(\epsilon)\mathcal{M}^{[j],(0)} + \mathcal{M}^{[j],(1)}_{\text{fin}},
\]
\[
\mathcal{M}^{[j],(2)} = 4I^{(2)}(\epsilon)\mathcal{M}^{[j],(0)} + 2I^{(1)}(\epsilon)\mathcal{M}^{[j],(1)} + \mathcal{M}^{[j],(2)}_{\text{fin}}.
\]

\( \mathcal{M}^{[j],(1)}_{\text{fin}} \) and \( \mathcal{M}^{[j],(2)}_{\text{fin}} \) (in 4 dimensions) are defined as the finite remainders.

The explicit expressions of the \( I^{(i)}(\epsilon) \) (needed for bbZH) are given (in CDR) by [S. Catani, 1998; T. Becher, M. Neubert, 2009]

\[
I^{(1)}(\epsilon) = -C_F \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)} \left( \frac{1}{\epsilon^2} + \frac{3}{2\epsilon} \right) \left( -\frac{\mu_R^2}{s} \right)^\epsilon,
\]
\[
I^{(2)}(\epsilon) = -\frac{1}{2}I^{(1)}(\epsilon) \left( I^{(1)}(\epsilon) + \frac{1}{\epsilon} \beta_0 \right) + \frac{e^{-\epsilon\gamma_E}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \left( \frac{2}{\epsilon} \beta_0 + K \right) I^{(1)}(2\epsilon) + 2H^{(2)}(\epsilon)
\]

IR poles contained in our renormalised amplitudes match with those predicted by these formulae.
The same RS-independent finite remainders

The linear transformation connecting $F_{i,\text{vec}}$ to $\mathcal{P}_{i}^{\mu} \bar{v}(p_2) \Gamma_{\text{vec},\mu} u(p_1)$ reads as

$$
\mathcal{P}_{i}^{\mu} \bar{v}(p_2) \Gamma_{\text{vec},\mu} u(p_1) = F_{1,\text{vec}} \left( \mathcal{P}_{i}^{\mu} \bar{v}(p_2) u(p_1) p_{1,\mu} \right) + F_{2,\text{vec}} \left( \mathcal{P}_{i}^{\mu} \bar{v}(p_2) u(p_1) p_{2,\mu} \right)
$$

$$
+ F_{3,\text{vec}} \left( \mathcal{P}_{i}^{\mu} \bar{v}(p_2) u(p_1) q_{1,\mu} \right) + F_{4,\text{vec}} \left( \mathcal{P}_{i}^{\mu} \bar{v}(p_2) \gamma_{\mu} q_{1,\mu} u(p_1) \right),
$$

and similarly for the axial part,

$$
\left[ \mathcal{P}_{i}^{\mu} \bar{v}(p_2) \Gamma_{\text{axi},\mu} u(p_1) \right]_{\text{fin}} = F_{1,\text{axi},\text{fin}} \left( \mathcal{P}_{i}^{\mu} \bar{v}(p_2) \gamma_{5} u(p_1) p_{1,\mu} \right) + F_{2,\text{axi},\text{fin}} \left( \mathcal{P}_{i}^{\mu} \bar{v}(p_2) \gamma_{5} u(p_1) p_{2,\mu} \right)
$$

$$
+ F_{3,\text{axi},\text{fin}} \left( \mathcal{P}_{i}^{\mu} \bar{v}(p_2) \gamma_{5} u(p_1) q_{1,\mu} \right) + F_{4,\text{axi},\text{fin}} \left( \mathcal{P}_{i}^{\mu} \bar{v}(p_2) \gamma_{\mu} \gamma_{5} q_{1,\mu} u(p_1) \right),
$$

where $i$ runs from 1 to 6 linear polarisation configurations.

Through the verified equality between the finite remainders computed following different approaches,

we confirm:

-projectors derived in “four” dimensions can be used also in calculations in $D$ dimensions and lead to correct results (for physical observables), irrespective of whether the quantity projected out is a form factor or a linearly polarised amplitude.

(even though the resulting amplitudes may not be regularised strictly in the HV scheme.)
Axial Form factors restored from Vector Form Factors

With $m_b = 0$, all 2-loop diagrams with the Higgs (and $Z$) radiated from a closed fermion loop vanish, e.g.

due to odd number of Dirac $\gamma$ matrices.

Consequently, all non-vanishing non-anomalous Feynman diagrams can be divided into the class-$HZ$ and class-$ZH$
Axial Form factors restored from Vector Form Factors

Turning off completely the axial coupling of the $Z$ boson,

$$\mathcal{M}_{\text{vec}} = \bar{v}(p_2) \Gamma^\mu_{ZH} u(p_1) \varepsilon^*_\mu(q_1) + \bar{v}(p_2) \Gamma^\mu_{HZ} u(p_1) \varepsilon^*_\mu(q_1)$$

$$\bar{b} \rightarrow Z \rightarrow b$$ (A)  \hspace{2cm} \bar{b} \rightarrow H \rightarrow b$$ (B)

$$\bar{v}(p_2) \Gamma^\mu_{ZH} u(p_1) = F_{1,ZH} \bar{v}(p_2) u(p_1) p_1^\mu + F_{2,ZH} \bar{v}(p_2) u(p_1) p_2^\mu + F_{3,ZH} \bar{v}(p_2) u(p_1) q_1^\mu + F_{4,ZH} \bar{v}(p_2) \gamma^\mu q_1 u(p_1),$$

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Axial (and vector) Form factors can be restored as:

$$F_{i,\text{vec}} = F_{i,HZ} + F_{i,ZH},$$

$$F_{i,\text{axi}(ns)} = F_{i,HZ} - F_{i,ZH}.$$
The Ward identity for(bbZ)H: the non-anomalous part

Starting from the classical Lagrangian,

\[ \mathcal{L}_c = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{b}i \gamma^\mu D_\mu b - J_Z^\mu Z_\mu - \lambda_b \bar{b}b H, \]

one obtains the Ward identity for the non-anomalous diagrams:

\[ q_1^\mu \mathcal{M}_\mu = -2g_{A,b} \lambda_b \langle H(q_2) | \bar{b}(0) i \gamma_5 b(0) H(0) | b(p_1) \bar{b}(p_2) \rangle. \]

whose RHS can be perturbatively expanded as
Checking the Ward identity for bbZH using $F_{i,ZH}$ and $F_{i,HZ}$

$$q_1^\mu M_\mu = -2g_{A,b}\lambda_b \langle H(q_2) | \bar{b}(0) i\gamma_5 b(0) H(0) | b(p_1) \bar{b}(p_2) \rangle.$$ 

The LHS of the Ward identity can be composed in terms of “split” vector form factors:

$$q_1 \cdot M_{vec} = \bar{v}(p_2) u(p_1) \left( (F_{1,ZH} + F_{1,HZ}) \frac{m_Z^2 - t}{2} + (F_{2,HZ} + F_{2,ZH}) \frac{m_Z^2 - u}{2} ight. \left. + (F_{3,HZ} + F_{3,ZH}) m_Z^2 + (F_{4,HZ} + F_{4,ZH}) m_Z^2 \right) \right).$$ 

$$q_1 \cdot M_{axi} = \bar{v}(p_2) \gamma_5 u(p_1) \left( (F_{1,HZ} - F_{1,ZH}) \frac{m_Z^2 - t}{2} + (F_{2,HZ} - F_{2,ZH}) \frac{m_Z^2 - u}{2} \right. \left. + (F_{3,HZ} - F_{3,ZH}) m_Z^2 + (F_{4,HZ} - F_{4,ZH}) m_Z^2 \right) \right).$$

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The Ward identity for bbZH: the anomalous part

The Adler-Bell-Jackiw (ABJ) anomaly equation:

\[
(\partial^\mu J^s_{\mu,A})_R = a_s \frac{1}{2} (G\tilde{G})_R ,
\]

where \( G\tilde{G} \equiv \epsilon_{\mu\nu\rho\sigma} G^a_{\mu\nu} G^a_{\rho\sigma} \) with the gluonic field strength tensor \( G^a_{\mu\nu} \).

At the level of matrix elements (regarding the anomalous part of \( b\bar{b} \rightarrow ZH \)),

\[
\bar{v}(p_2) \Gamma^\mu_{ABJ} u(p_1) q_{1,\mu} = g_{A,q} \frac{a_s}{2} \langle H(q_2) \mid [G\tilde{G}(0)]_R \mid q(p_1)\bar{q}(p_2) \rangle ,
\]

(with the kinematics \( p_1 + p_2 - q_2 = q_1 \)).

Diagrammatically, the RHS consists of
Numerical results

The numerical results of the total cross section of $ZH$ production at LHC@13TeV (in unit pb):

<table>
<thead>
<tr>
<th>Order</th>
<th>$s$-channel</th>
<th>EW</th>
<th>$\sigma_{gg}^{ZH}$</th>
<th>$\sigma_{q\bar{q}}^{ZH}$ (top)</th>
<th>$(t + u)$-channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>5.897 $10^{-1}$</td>
<td>-3.111 $10^{-2}$</td>
<td>-</td>
<td>-</td>
<td>2.989 $10^{-4}$</td>
</tr>
<tr>
<td>NLO</td>
<td>7.756 $10^{-1}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2.934 $10^{-4}$</td>
</tr>
<tr>
<td>NNLO</td>
<td>8.015 $10^{-1}$</td>
<td>-</td>
<td>5.051 $10^{-2}$</td>
<td>9.442 $10^{-3}$</td>
<td>3.027 $10^{-4}$</td>
</tr>
<tr>
<td>$N^3$LO$_{SV}$</td>
<td>8.013 $10^{-1}$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Setting: $\mu_R = \mu_F = m_h + m_z$, $G_F = 1.16637 \times 10^{-5}$ GeV$^{-2}$, $m_b(\mu_R = m_b) = 4.18$ GeV.

- The $s$-channel contributions (Higgs-bremsstrahlung) are obtained using $vh@nnlo$ [Brein,Harlander,Zirke, 2012].
- The $\sigma_{gg}^{ZH}$ refers to the contribution coming from the gluon initiated sub-processes.
- The top quark loop contribution is denoted by $\sigma_{q\bar{q}}^{ZH}$ (top).
- The $(t + u)$-channel contribution at NLO is obtained using Madgraph [Alwall,Frederix,Frixione,Hirschi, Maltoni,Mattelaer et al.,2014], and at NNLO is under the Soft-Virtual approximation [Ravindran,2005].
Summary

☑ Computed the 2-loop QCD corrections to $b\bar{b} \rightarrow ZH$ amplitude via directly projecting onto a linear polarisation basis, with the analytic results expressed in terms of multiple polylogarithms;

☑ Addressed an interesting subtlety appearing in the conventional form-factor (FF) decomposition of amplitudes involving axial currents regularised in D dimensions;

☑ Revealed a relation between axial and vector FFs of the non-Drell-Yan $b\bar{b} \rightarrow ZH$, which enables us to restore axial FFs from “split” vector ones;

☑ Derived the Ward identities for $b\bar{b} \rightarrow ZH$ in the presence of $\lambda_b$, which are checked using the axial FFs restored from their vector counterparts;

☑ Computed the $SV$ cross section at NNLO, in order to make a quantitative analysis of the contribution from these non-Drell-type processes.
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THANK YOU