A NEW METHOD TO COMPUTE PHYSICAL OBSERVABLES

THE FDU FRAMEWORK

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STARTING CONCEPT:

0

Inside a “0” there COULD BE many hidden things

Basic introduction

Theoretical motivation

- Theoretical framework: **Standard Model + factorization theorem**

$$\frac{d\sigma}{d^2\vec{q}_T\, dM^2 d\Omega dy} = \sum_{a,b} \int dx_1 dx_2\, f_a^{h_1}(x_1)\, f_b^{h_2}(x_2)\, \frac{d\hat{\sigma}_{ab\to V+X}}{d^2\vec{q}_T\, dM^2 d\Omega dy}$$

PDFs (non-perturbative)

Partonic cross-section (perturbative)

What we want to compute!

- Deal with **ill-defined expressions** in intermediate steps → **DREG!!!**
  - Proposed by {Giambiagi&Bollini, t'Hooft&Veltman, Cicuta&Montaldi, Ashmore,…}, it becomes a standard in HEP since it preserves gauge invariance
  - **Abstract idea:** «Change the dimension of the loop-momentum space»
  - **Reality:** «Introduce a parameter ε in order to make everything integrable»

$$\mathcal{O}_d[F] = \int d^d\mathrm{x}\, F(\mathrm{x})$$

$$d = 4 - 2\varepsilon$$

1. linearity $\int d^d\mathrm{x}\,(aF(\mathrm{x}) + bG(\mathrm{x})) = a\int d^d\mathrm{x}\, F(\mathrm{x}) + b\int d^d\mathrm{x}\, G(\mathrm{x})$
2. scaling $\int d^d\mathrm{x}\, F(s\mathrm{x}) = s^{-d}\int d^d\mathrm{x}\, F(\mathrm{x})$
3. translational invariance $\int d^d\mathrm{x}\, F(\mathrm{x}+\mathrm{y}) = \int d^d\mathrm{x}\, F(\mathrm{x})$

Nice mathematical properties!! **("Todo bonito")**

Basic introduction

Theoretical motivation

- Singularities in perturbative theories with DREG → **Poles in ε!!!**
- Two kind of physical singularities:
  - **Ultraviolet poles** coming from the high-energy region in loop integrals

    SOLUTION: *Add proper counterterms obtained from RENORMALIZATION procedure. These counterterms have EXPLICIT ε–poles and are proportional to lower-order amplitudes.*

  - **Infrared poles** associated with degenerate configurations: extra-particle radiation in soft (i.e. low energy) or collinear (i.e. parallel) configurations

    SOLUTION: *Kinoshita-Lee-Nauenberg theorem states that adding real-emission processes and computing IR-safe observables guarantees the cancellation of all the IR poles present in renormalized virtual amplitudes and INTEGRATED real-radiation contributions.*

  - **Loop integrals** could contain **UV and IR** singularities

Basic introduction

Theoretical motivation

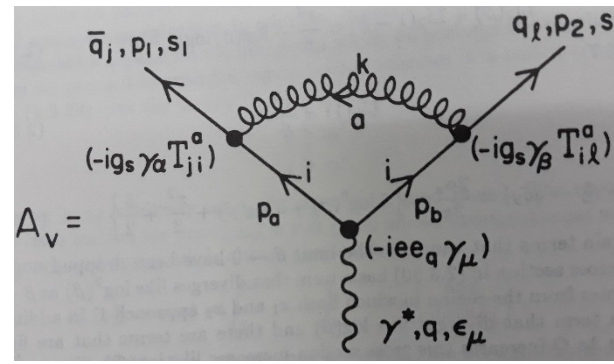
- **Summary scheme:** obtaining finite physical results at higher-orders

Vacuum quantum fluctuations (no experimental signature)

Virtual corrections (loop integrals)
IR/UV divergences

+

Renormalization counter-terms
UV divergences



Renormalization procedure

KLN theorem + IR safe observables

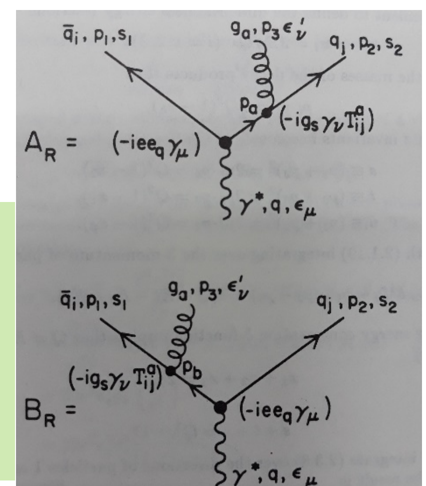
Finite physical observable

Renormalized virtual corrections
IR divergences

+

Real corrections (PS integrals)
IR divergences

Contributions with extra-radiation (included in the definition of the observable)



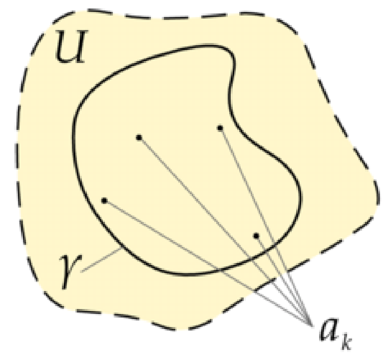
WE WANT INTEGRAND LEVEL CANCELLATION!!!

Towards Loop-Tree Duality

Feynman integrals and propagators

Residue theorem
(*from Wikipedia*)

$$\oint_\gamma f(z)\,dz = 2\pi i \sum \mathrm{Res}(f, a_k)$$

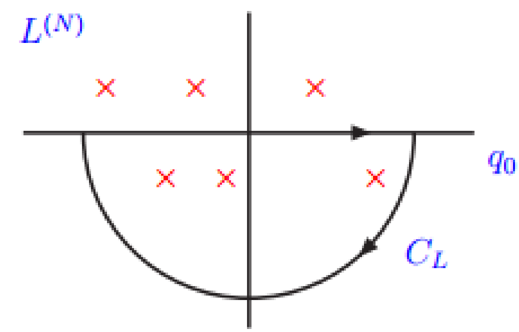


«If f is a holomorphic function in U/{a_i}, and g a simple positively oriented curve, then the integral is given by the sum of the residues at each singular point a_i»

Residue theorem can be used to compute integrals involving propagators: the prescription and the contour that we choose determine the result!

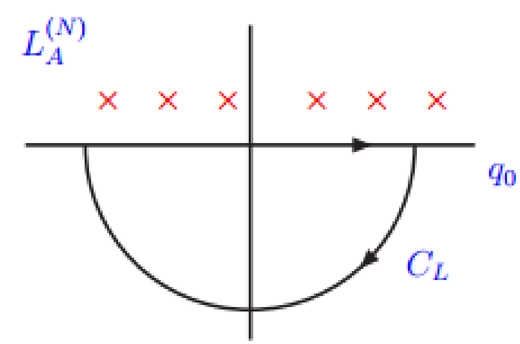
Feynman propagator

$$[G(q)]^{-1} = 0 \quad \Longrightarrow \quad q_0 = \pm\sqrt{\mathbf{q}^2 - i0}$$



Advanced propagator

$$[G_A(q)]^{-1} = 0 \quad \Longrightarrow \quad q_0 \simeq \pm\sqrt{\mathbf{q}^2} + i0$$



NO POLES CLOSED BY C_L!

Towards Loop-Tree Duality

Dual representation of one-loop integrals

Loop Feynman integral

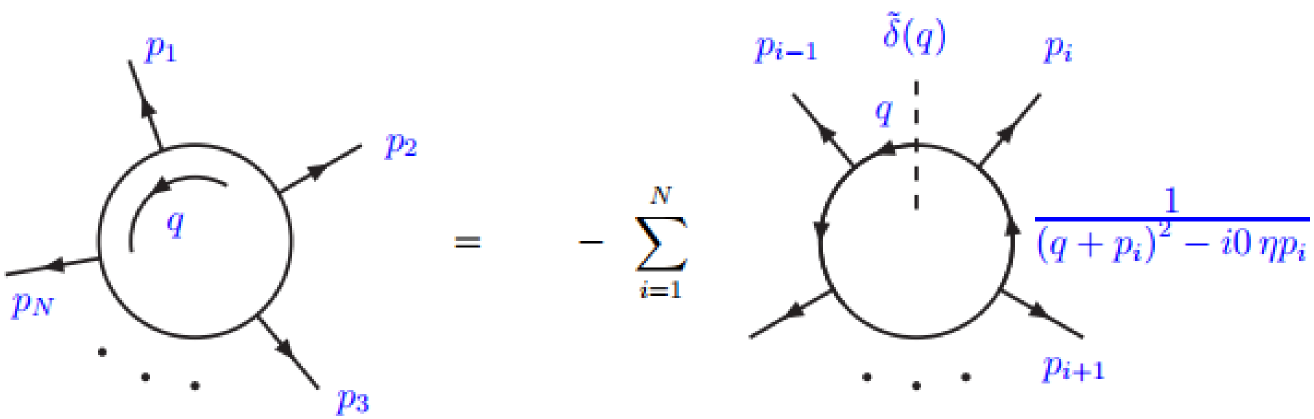
$$L^{(1)}(p_1,\dots,p_N) = \int_\ell \prod_{i=1}^{N} G_F(q_i) = \int_\ell \prod_{i=1}^{N} \frac{1}{q_i^2 - m_i^2 + i0}$$

Dual integral

$$L^{(1)}(p_1,\dots,p_N) = -\sum_{i=1}^{N} \int_\ell \tilde{\delta}(q_i) \prod_{j=1, j\neq i}^{N} G_D(q_i;q_j)$$

Sum of phase-space integrals!

$$G_D(q_i,q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta(q_j - q_i)} \qquad \tilde{\delta}(q_i) = i2\pi\,\theta(q_{i,0})\,\delta(q_i^2 - m_i^2)$$



Even at higher-orders, the number of cuts is equal the number of loops

Catani et al, JHEP09(2008)065; Rodrigo et al, JHEP02(2016)044

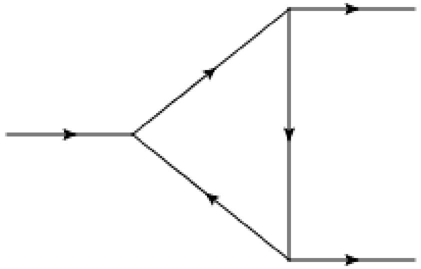
Part I: Applications of LTD

- **I)- Deal with massless Feynman integrals**
- **II)- Analysis of IR-divergent integrals**
- **III)- Study of UV-divergent integrals and local UV counter-terms**

LTD for Feynman integrals

- Two different kinds of physical singularities: **UV and IR**
  - IR divergences: *massless triangle*

$$L^{(1)}(p_1, p_2, -p_3) = \int_\ell \prod_{i=1}^{3} G_F(q_i) = -\frac{c_\Gamma}{\epsilon^2 s_{12}} \left(\frac{-s_{12} - \imath 0}{\mu^2} \right)^{-\epsilon}$$

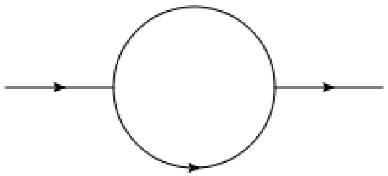


IR pole

IDEA: Define a proper MOMENTUM MAPPING to generate REAL EMISSION KINEMATICS, and use REAL TERMS as fully local IR counter-terms!

  - UV divergences: *bubble with massless propagators*

$$L^{(1)}(p, -p) = \int_l \prod_{i=1}^{2} G_F(q_i) = c_\Gamma \frac{\mu^{2\epsilon}}{\epsilon (1 - 2\epsilon)} (-p^2 - \imath 0)^{-\epsilon}$$



UV pole

IDEA: Define an INTEGRAND LEVEL REPRESENTATION of standard UV counter-terms, and combine it with the DUAL REPRESENTATION of virtual terms!

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

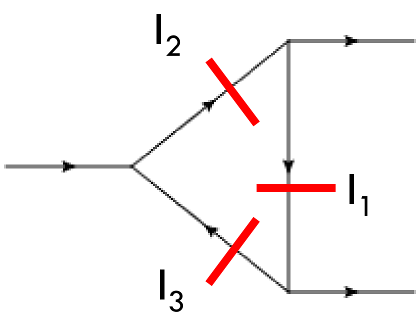
LTD for Feynman integrals: IR case

IR singularities

- *Reference example: Massless scalar three-point function in the time-like region*

$$L^{(1)}(p_1, p_2, -p_3) = \int_\ell \prod_{i=1}^{3} G_F(q_i) = -\frac{c_\Gamma}{\epsilon^2}\left(-\frac{s_{12}}{\mu^2} - i0\right)^{-1-\epsilon} = \sum_{i=1}^{3} I_i$$

LTD



$$I_1 = \frac{1}{s_{12}} \int d[\xi_{1,0}]\, d[v_1]\, \xi_{1,0}^{-1}\, (v_1(1-v_1))^{-1}$$

$$I_2 = \frac{1}{s_{12}} \int d[\xi_{2,0}]\, d[v_2]\, \frac{(1-v_2)^{-1}}{1-\xi_{2,0} + i0}$$

$$I_3 = \frac{1}{s_{12}} \int d[\xi_{3,0}]\, d[v_3]\, \frac{v_3^{-1}}{1+\xi_{3,0} - i0}$$

To regularize threshold singularity

- This integral is UV-finite (power counting); there are only IR-singularities, associated to soft and collinear regions
- **OBJECTIVE:** Define a *IR-regularized* loop integral by adding real corrections at integrand level (i.e. no epsilon should appear, 4D representation)

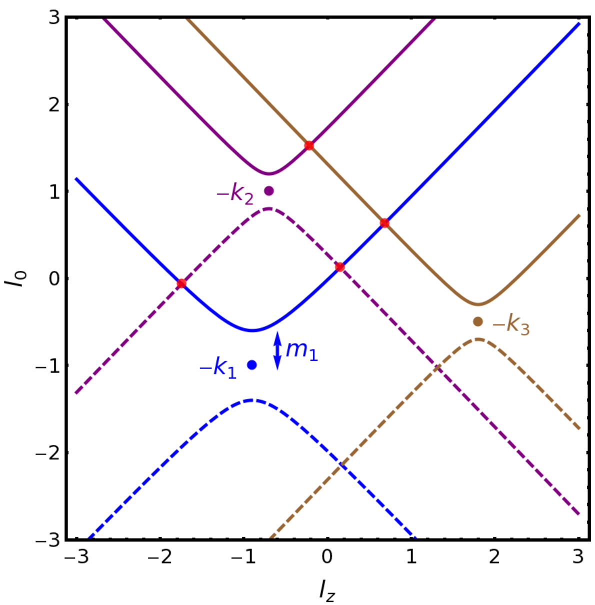
Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD for Feynman integrals: IR case

Location of IR singularities in the dual-space

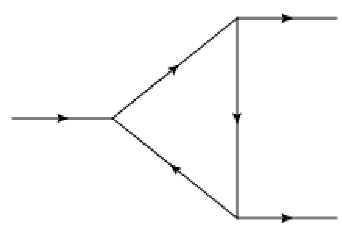
- Analize the dual integration region. It is obtained as the positive energy solution of the on-shell condition:

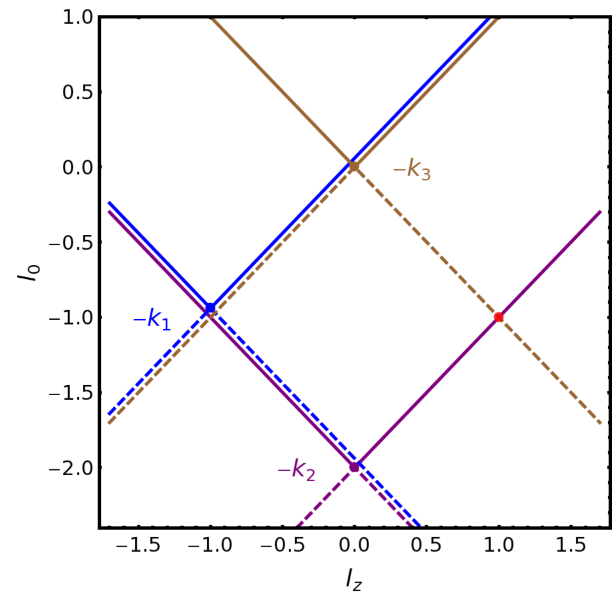
$$G_F^{-1}(q_i) = q_i^2 - m_i^2 + i0 = 0 \quad \Rightarrow \quad q_{i,0}^{(\pm)} = \pm\sqrt{\mathbf{q}_i^2 + m_i^2 - i0}$$



Massive case: hyperboloids

- **Forward** (backward) on-shell hyperboloids associated with **positive** (negative) energy solutions.
- Degenerate to light-cones for massless propagators.
- *Dual integrands become singular at intersections* (two or more on-shell propagators)





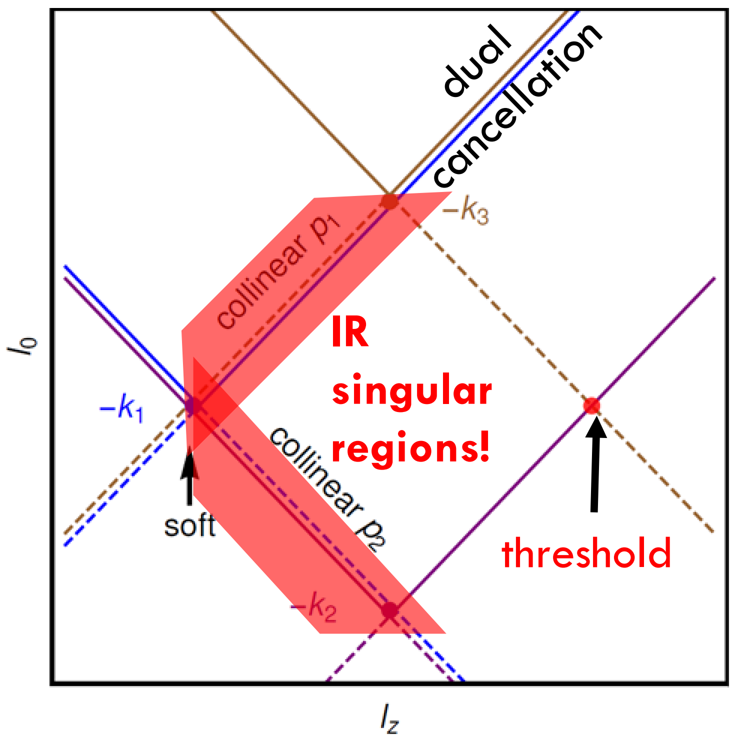
Massless case: light-cones

Rodrigo et al, JHEP11(2014)014, JHEP02(2016)044, JHEP08(2016)160

LTD for Feynman integrals: IR case

Location of IR singularities in the dual-space

- The application of LTD converts loop-integrals into PS ones: **integration over forward light-cones**.



- Only **forward-backward** interferences originate **threshold or IR poles** (*other propagators become singular in the integration domain*)
- **Forward-forward** singularities cancel among dual contributions
- Threshold and IR singularities associated with finite regions (i.e. contained in a **compact region**)
- **No threshold or IR singularity at large loop momentum**

- This structure suggests how to perform real-virtual combination! Also, how to overcome threshold singularities (integrable but numerically unstable)

Rodrigo et al, JHEP11(2014)014, JHEP02(2016)044, JHEP08(2016)160

LTD for Feynman integrals: UV case

UV singularities

- *Reference example: two-point function with massless propagators*

$$L^{(1)}(p,-p) = \int_\ell \prod_{i=1}^{2} G_F(q_i) = \frac{c_\Gamma}{\epsilon(1-2\epsilon)} \left(-\frac{p^2}{\mu^2} - i0\right)^{-\epsilon} = \sum_{i=1}^{2} I_i$$

LTD

$$I_1 = -\int_\ell \frac{\tilde{\delta}(q_1)}{-2q_1 \cdot p + p^2 + i0}$$

$$I_2 = -\int_\ell \frac{\tilde{\delta}(q_2)}{2q_2 \cdot p + p^2 - i0}$$

To regularize threshold singularity

- In this case, the integration regions of dual integrals are two energy-displaced forward light-cones. This integral contains UV poles only
- **OBJETIVE:** Define a *UV-regularized* loop integral by adding unintegrated UV counter-terms, and find a purely 4-dimensional representation of the loop integral

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

LTD for Feynman integrals: UV case

UV counter-term

- Divergences arise from the high-energy region (UV poles) and can be cancelled with a suitable renormalization counter-term. For the scalar case, we use

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{1}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + i0)^2}$$

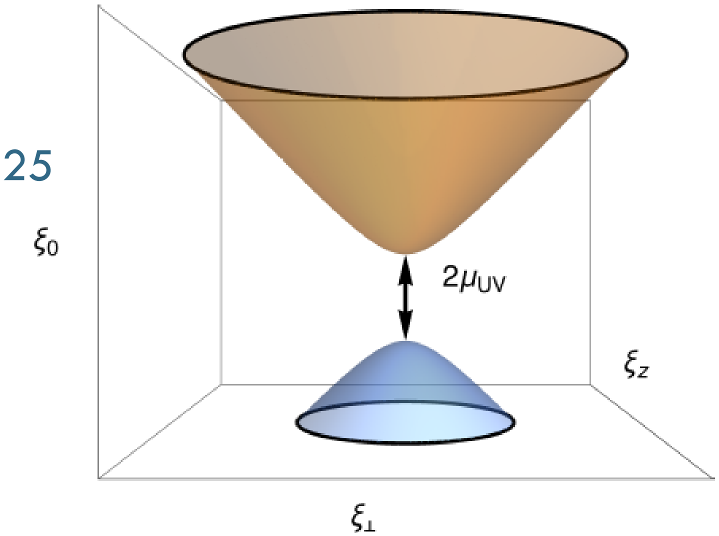
Becker, Reuschle, Weinzierl, JHEP 12 (2010) 013

- Dual representation (**new: double poles in the loop energy**)

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{\tilde{\delta}(q_{\rm UV})}{2\left(q_{{\rm UV},0}^{(+)}\right)^2}$$

Bierenbaum *et al.* JHEP 03 (2013) 025

$$q_{{\rm UV},0}^{(+)} = \sqrt{\mathbf{q}_{\rm UV}^2 + \mu_{\rm UV}^2 - i0}$$



- Loop integration for loop energies larger than $\mu_{\rm UV}$

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

Part II: FDU formalism

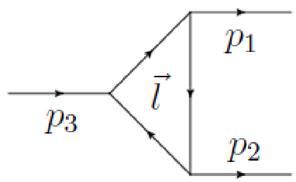
- **I)- Adding real contributions to locally cancel IR singularities: Universal kinematical mappings**
- **II)- Local four-dimensional representation of renormalization counter-terms**
- **OBJETIVE: Avoid using DREG (or any other regularization) through a purely 4D representation of physical observables**

Towards local IR regularization

Finite real+virtual integration

- According to KLN theorem **real** contributions. Suppose **one-loop** scalar scattering amplitude given by the triangle

Virtual

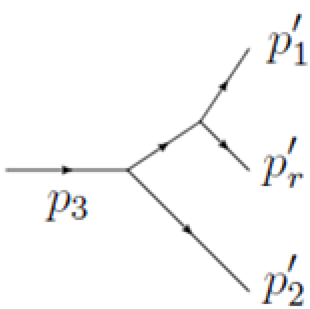


$$|\mathcal{M}^{(0)}(p_1,p_2;p_3)\rangle = ig$$
$$|\mathcal{M}^{(1)}(p_1,p_2;p_3)\rangle = -i\,g^3\,L^{(1)}(p_1,p_2,-p_3) \quad \Rightarrow \mathrm{Re}\,\langle \mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle$$

- **1->2 one-loop process** ➡ **1->3 with unresolved extra-parton**
- Add scalar tree-level contributions with one extra-particle; consider interference terms:

Opposite sign!

Real



$$|\mathcal{M}^{(0)}_{ir}(p'_1,p'_2,p'_r;p_3)\rangle = -ig^2/s'_{ir} \Rightarrow \mathrm{Re}\,\langle \mathcal{M}^{(0)}_{ir}|\mathcal{M}^{(0)}_{jr}\rangle = \frac{g^4}{s'_{ir}\,s'_{jr}}$$

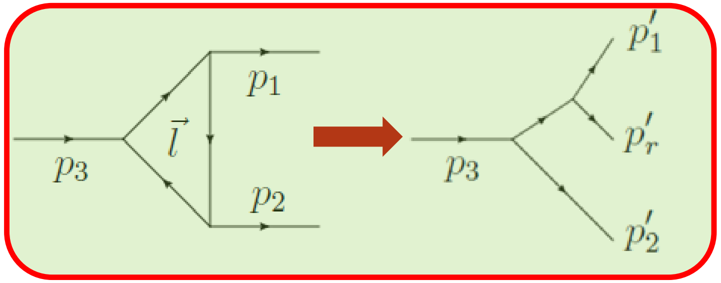
- Generate 1->3 kinematics starting from 1->2 configuration plus the loop three-momentum $\vec{l}$!!!

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

Generalization of mappings

- **Real-virtual momentum mapping with massive particles:**
  - Consider **1** the **emitter**, **r** the **radiated particle** and **2** the **spectator**
  - Apply the PS partition and restrict to the only region where **1//r** is allowed (i.e. $\mathcal{R}_1 = \{y'_{1r} < \min y'_{kj}\}$)
  - Propose the following mapping:

$$
\begin{aligned}
p_r'^{\mu} &= q_1^{\mu} \\
p_1'^{\mu} &= (1-\alpha_1)\,\hat{p}_1^{\mu} + (1-\gamma_1)\,\hat{p}_2^{\mu} - q_1^{\mu} \\
p_2'^{\mu} &= \alpha_1\,\hat{p}_1^{\mu} + \gamma_1\,\hat{p}_2^{\mu}
\end{aligned}
$$



Impose on-shell conditions to determine mapping parameters

with $\hat{p}_i$ massless four-vectors build using p_i (simplify the expressions)

  - *Express the loop three-momentum with the same parameterization used for describing the dual contributions!*

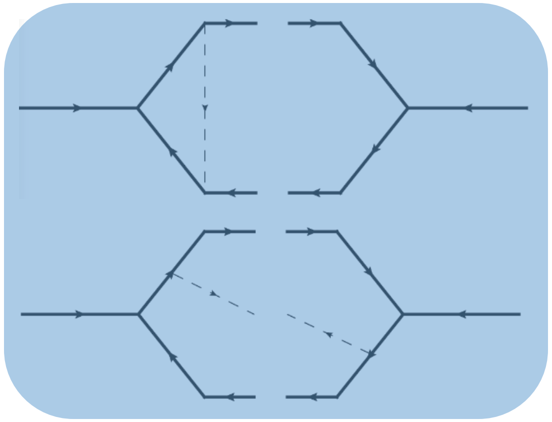
Repeat in each region of the partition…

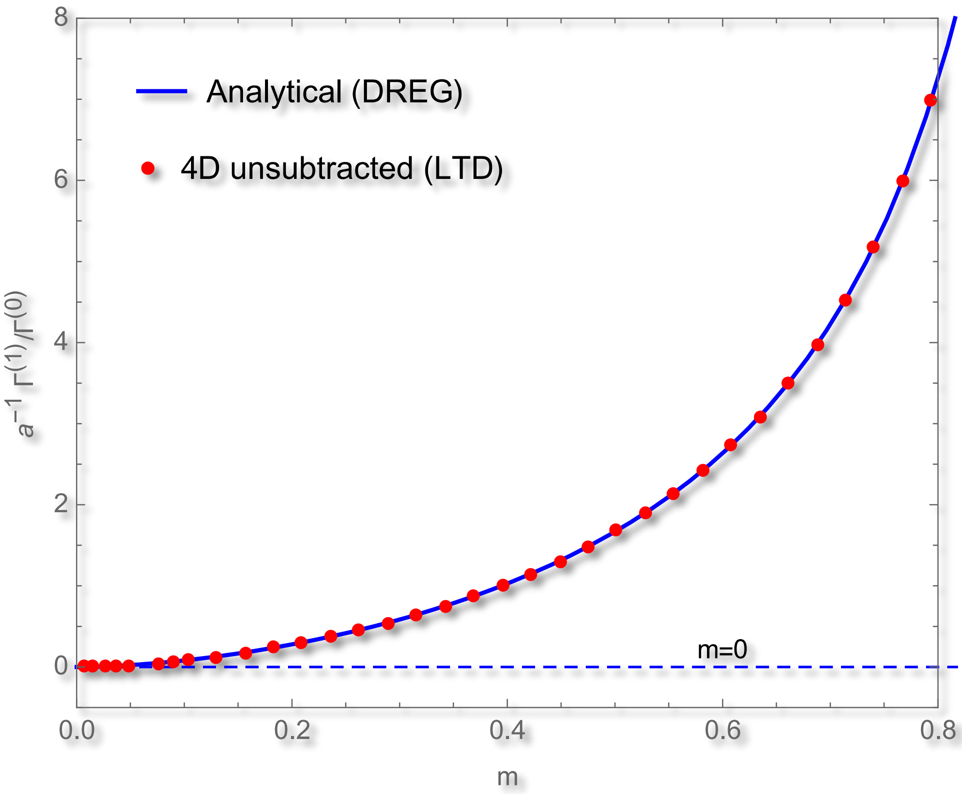
Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

Generalization of mappings

Example: massive scalar three-point function (DREG vs LTD)

- We combine the dual contributions with the real terms (after applying the proper mapping) to get the total decay rate in the scalar toy-model.
  - The result agrees ***perfectly*** with standard DREG.
  - **Massless limit** is **smoothly** approached due to proper treatment of **quasi-collinear** configurations in the **RV mapping**





Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

Local renormalization within LTD

- LTD can also deal with **UV singularities** by building **local** versions of the usual UV counterterms.
- **1: Expand** internal propagators around the *"UV propagator"*

$$\frac{1}{q_i^2 - m_i^2 + \imath 0} = \frac{1}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0} \times \left[1 - \frac{2q_{\rm UV}\cdot k_{i,{\rm UV}} + k_{i,{\rm UV}}^2 - m_i^2 + \mu_{\rm UV}^2}{q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0} + \frac{(2q_{\rm UV}\cdot k_{i,{\rm UV}})^2}{(q_{\rm UV}^2 - \mu_{\rm UV}^2 + \imath 0)^2}\right] + \mathcal{O}\left((q_{\rm UV}^2)^{-5/2}\right)$$

Becker, Reuschle, Weinzierl, JHEP 12 (2010) 013

- **2:** Apply LTD to get the **dual representation** for the expanded UV expression, and **subtract** it from the **dual+real** combined integrand.

$$I_{\rm UV}^{\rm cnt} = \int_\ell \frac{\tilde{\delta}(q_{\rm UV})}{2\left(q_{{\rm UV},0}^{(+)}\right)^2}$$

$$q_{{\rm UV},0}^{(+)} = \sqrt{\mathbf{q}_{\rm UV}^2 + \mu_{\rm UV}^2 - i0}$$

LTD extended to deal with multiple poles
(use residue formula to obtain the dual representation)

Bierenbaum *et al.* JHEP 03 (2013) 025

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

0

In the massless case, the renormalization factors are usually ignored because they are "0": but they hide a cancelation between UV and IR singularities…

Local renormalization within LTD

UV counterterms and renormalization

- Requires **unintegrated** wave-function, mass and vertex renormalization constants
- Self-energy corrections with **on-shell renormalization** conditions

$$\Sigma_R(\not{p}_1 = M) = 0 \qquad\qquad \left.\frac{d\Sigma_R(\not{p}_1)}{d\not{p}_1}\right|_{\not{p}_1=M} = 0$$

- Wave function renormalization constant, **both IR and UV poles**

$$\Delta Z_2(p_1) \;=\; -g_S^2 C_F \int_\ell G_F(q_1) G_F(q_3) \left((d-2)\frac{q_1\cdot p_2}{p_1\cdot p_2} + 4M^2 \left(1 - \frac{q_1\cdot p_2}{p_1\cdot p_2}\right) G_F(q_3)\right)$$

- ***Remove UV poles*** *by expanding around the UV-propagator (same for the* ***vertex counterterm****)*
- Integrated form of local counterterms agrees with standard UV counterterms

Rodrigo et al, JHEP02(2016)044; JHEP08(2016)160; JHEP10(2016)162

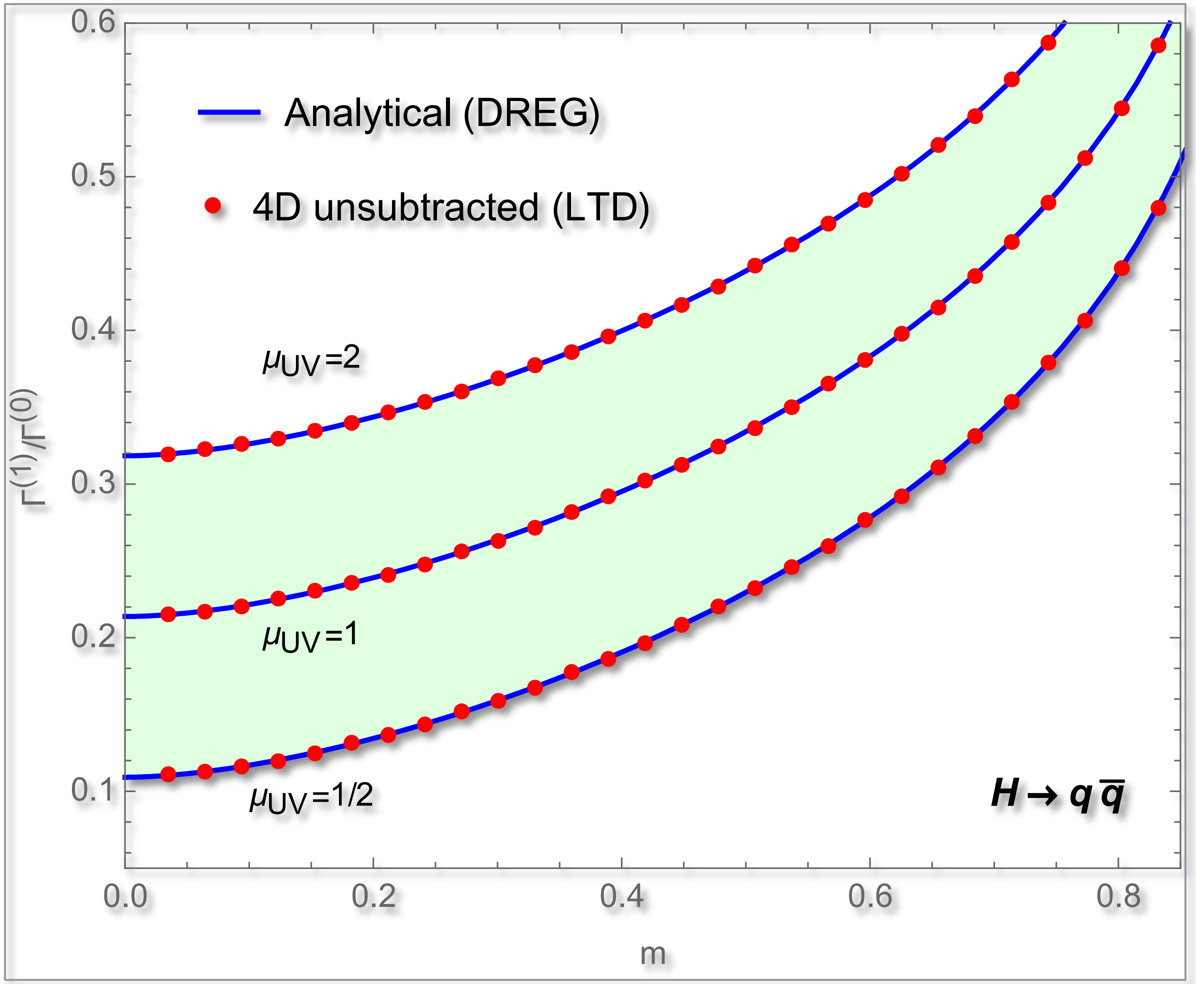
Part III: Physical examples

- **Application of the FDU/LTD formalism to express amplitudes and observables in four space-time dimensions.**
- **I)- Vector boson decays at NLO**
- **II)- Higgs amplitudes at one and two loop**
- **REMARK: First application of two-loop local renormalization in 4D!!!**

Driencourt-Mangin et al, JHEP 02 (2019) 143

Physical example: $A^* \to q\bar{q}(g)$@NLO

Results and comparison with DREG

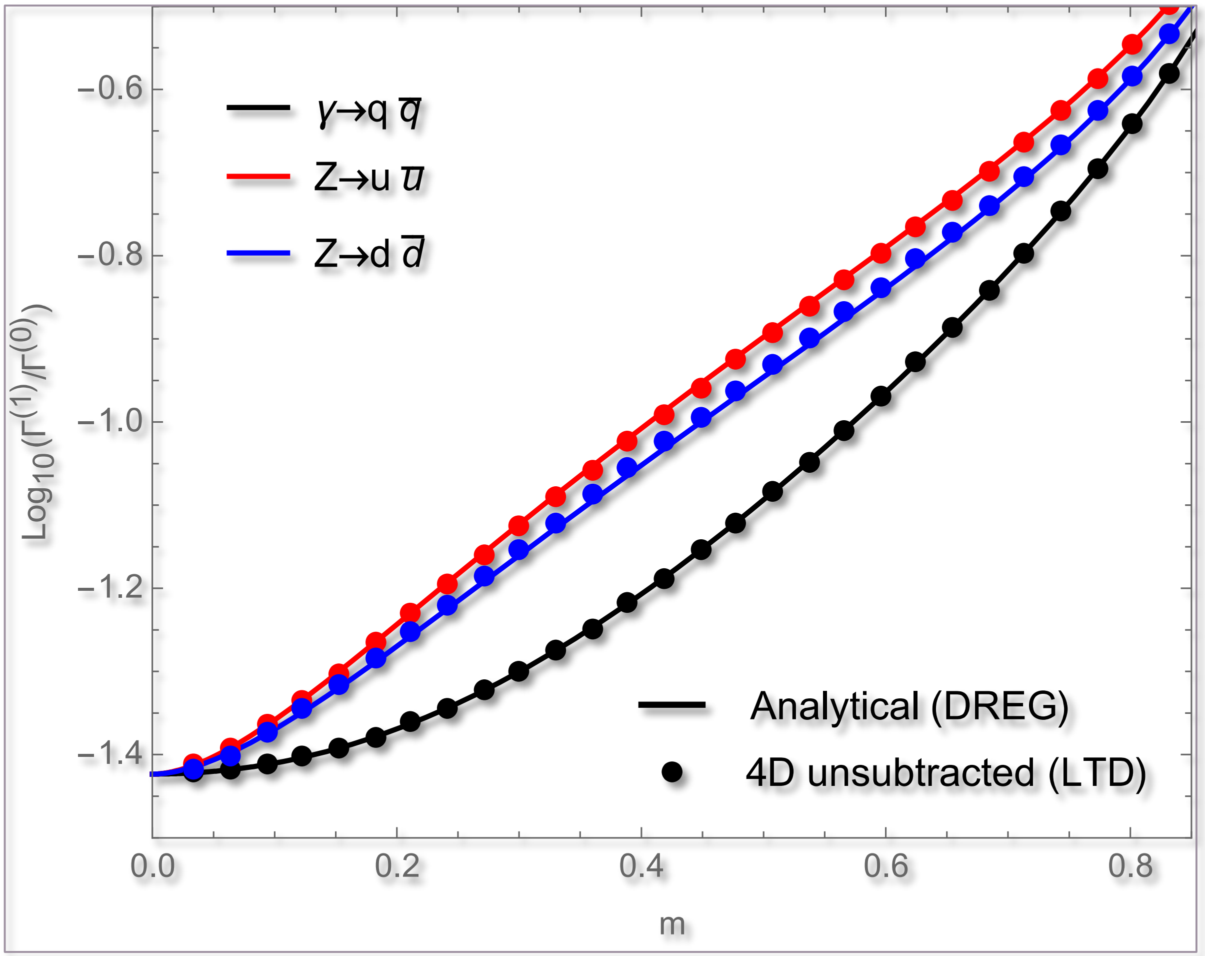


- *Total decay rate for Higgs into a pair of massive quarks:*
  - Agreement with the standard DREG result
  - Smoothly achieves the massless limit
  - Local version of UV counterterms succesfully reproduces the expected behaviour
  - Efficient numerical implementation

Rodrigo et al, JHEP10(2016)162

Physical example: $A^* \to q\bar{q}(g)$@NLO

Results and comparison with DREG



- *Total decay rate for a vector particle into a pair of massive quarks:*
  - Agreement with the standard DREG result
  - Smoothly achieves the massless limit
  - Efficient numerical implementation

Rodrigo et al, JHEP10(2016)162

Physical example: $A^* \to q\bar{q}(g)$@NLO

Important remarks

- The total decay-rate can be expressed using purely **four-dimensional integrands** (which are **integrable** functions!!)
- We recover the total NLO correction, **avoiding to deal with** DREG (ONLY used for comparison with known results)
- **Main advantages:**
  - ✓ Direct **numerical** implementation (integrable functions for **ε=0**)

    Finite integral for ε=0 $\not\Leftrightarrow$ Integrability with ε=0

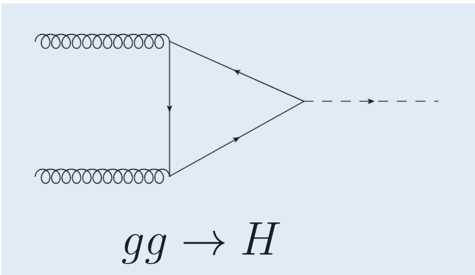
    With FDU is true!
  - ✓ No need of tensor reduction (**avoids the presence of Gram determinants,** which could introduce numerical instabilities)
  - ✓ **Smooth transition** to the massless limit (due to the efficient treatment of **quasi-collinear** configurations)
  - ✓ **Mapped real-contribution used as a fully local IR counter-term for the dual contribution!**

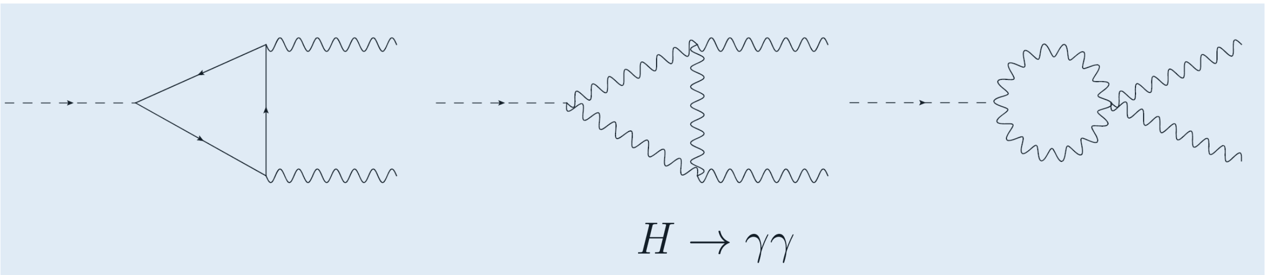
Rodrigo et al, JHEP10(2016)162

Physical example: Higgs@(N)NLO

Using LTD to regularize finite amplitudes

- Application of LTD to compute one-loop Higgs amplitudes:





- They are IR/UV finite BUT still not well-defined in 4D!!! Hidden cancellation of singularities leads to potentially undefined results (scheme dependence!!!)
- We start by defining a tensor basis and projecting (amplitude level!):

$$\mathcal{A}_{\mu\nu}^{(1,f)} = \sum_{i=1}^{5} \mathcal{A}_i^{(1,f)} T_{\mu\nu}^i \qquad \text{with}$$

$$T_i^{\mu\nu} = \left\{ g^{\mu\nu} - \frac{2\,p_1^\nu\,p_2^\mu}{s_{12}}, g^{\mu\nu}, \frac{2\,p_1^\mu\,p_2^\nu}{s_{12}}, \frac{2\,p_1^\mu\,p_1^\nu}{s_{12}}, \frac{2\,p_2^\mu\,p_2^\nu}{s_{12}} \right\}$$

$$\Rightarrow \boxed{\begin{aligned} P_1^{\mu\nu} &= \frac{1}{d-2}\left(g^{\mu\nu} - \frac{2\,p_1^\nu p_2^\mu}{s_{12}} - (d-1)P_2^{\mu\nu}\right) \\ P_2^{\mu\nu} &= \frac{2\,p_1^\mu p_2^\nu}{s_{12}} \qquad \text{Projectors} \end{aligned}}$$

- Then, scalar coefficients $P_i^{\mu\nu}\mathcal{A}_{\mu\nu}^{(1,f)} = \mathcal{A}_i^{(1,f)}$ are dualized.
- **IMPORTANT:** Take into account 1-2 exchange symmetry (different cuts and non-trivial cancellations!!!)

Driencourt-Mangin et al, Eur.Phys.J. C78 (2018) no.3 231; JHEP 02 (2019) 143

Physical example: Higgs@(N)NLO

Using LTD to regularize finite amplitudes

- Combine expressions (use "zero integrals" in DREG associated with Ward identities):

$$\mathcal{A}_1^{(1,f)} = g_f \int_\ell \tilde{\delta}(\ell) \left[\left(\frac{\ell_0^{(+)}}{q_{1,0}^{(+)}} + \frac{\ell_0^{(+)}}{q_{4,0}^{(+)}} + \frac{2(2\ell\cdot p_{12})^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2} \right) \frac{s_{12} M_f^2}{(2\,\ell\cdot p_1)(2\,\ell\cdot p_2)} c_1^{(f)} \right.$$

Well defined in 4-d!!

$$\left. + \frac{2\, s_{12}^2}{s_{12}^2 - (2\ell\cdot p_{12} - \imath 0)^2} c_{23}^{(f)} \right]$$

UV divergent

$\mathcal{O}(\epsilon)$

Non-commutativity of limit and integration!!!!

- Use local renormalization (equivalent to Dyson's prescription…)

$$\mathcal{A}_{1,\mathrm{R}}^{(1,f)}\Big|_{d=4} = \left(\mathcal{A}_1^{(1,f)} - \mathcal{A}_{1,\mathrm{UV}}^{(1,f)} \right)_{d=4}$$

$$\mathcal{A}_{1,\mathrm{UV}}^{(1,f)} = -g_f \int_\ell \frac{\tilde{\delta}(\ell)\, \ell_0^{(+)} s_{12}}{2(q_{\mathrm{UV},0}^{(+)})^3} \left(1 + \frac{1}{(q_{\mathrm{UV},0}^{(+)})^2} \frac{3\,\mu_{\mathrm{UV}}^2}{d-4} \right) c_{23}^{(f)}$$

  - Counter-term mimics UV behaviour at integrand level.
  - Term proportional to μ_{UV}^2 used to fix DREG scheme (vanishing counter-term in d-dim!!)
  - Valid also for W amplitudes in unitary-gauge (naive Dyson's prescription fails to subtract subleading terms due to enhanced UV divergences)

$$-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2} \right) \frac{1}{q_i^2 - M_W^2 + \imath 0}$$

Driencourt-Mangin et al, Eur.Phys.J. C78 (2018) no.3 231; JHEP 02 (2019) 143

Physical example: Higgs@(N)NLO

- Infinite-mass limit used to define effective vertices. Equivalent to explore asymptotic expansions!
- Expansions at **integrand level** are non-trivial in **Minkowski** space (i.e. within Feynman integrals) and additional factors are neccesary
- **Dual amplitudes** are expressed as **phase-space** integrals ➡ **Euclidean** space!!

$$\tilde{\delta}(q_3)\, G_D(q_3;q_2) = \frac{\tilde{\delta}(q_3)}{s_{12}+2q_3\cdot p_{12}-\imath 0} \xrightarrow{M_f^2 \gg s_{12}} \tilde{\delta}(q_3)\, G_D(q_3;q_2) = \frac{\tilde{\delta}(q_3)}{2q_3\cdot p_{12}} \sum_{n=0}^{\infty} \left(\frac{-s_{12}}{2q_3\cdot p_{12}}\right)^n$$

Expansion of the dual propagator (q_3 on-shell)

- *Example*: Higgs amplitudes with heavy-particles within the loop

$$\mathcal{A}^{(1,f)}_{1,\mathrm{R}}(s_{12}<4M_f^2)\Big|_{d=4} = \frac{M_f^2}{\langle v\rangle} \int_\ell \tilde{\delta}(\ell) \left[\frac{3\,\mu_{\mathrm{UV}}^2\,\ell_0^{(+)}}{(q_{\mathrm{UV},0}^{(+)})^5}\,\hat{c}_{23}^{(f)} + \frac{M_f^2}{(\ell_0^{(+)})^4}\left(\sum_{n=0}^{\infty} Q_n(z)\left(\frac{s_{12}}{(2\ell_0^{(+)})^2}\right)^n\right) c_1^{(f)}\right]$$

$$z = (2\boldsymbol{\ell}\cdot\mathbf{p}_1)/(\ell_0^{(+)}\sqrt{s_{12}}) \ \text{ and } \ Q_n(z) = \frac{1}{1-z^2}\left(P_{2n}(z)-1\right)$$

Reproduces all the known-results!!

Driencourt-Mangin et al, Eur.Phys.J. C78 (2018) no.3 231; JHEP 02 (2019) 143

About Higgs@NNLO…

Introducing the notation

- Dual amplitudes can be defined at **higher-orders** (even with multiple poles)

  Bierenbaum, Catani, Draggiotis, Rodrigo; JHEP 10 (2010) 073

- *Standard example:* two-loop N-point scalar amplitude

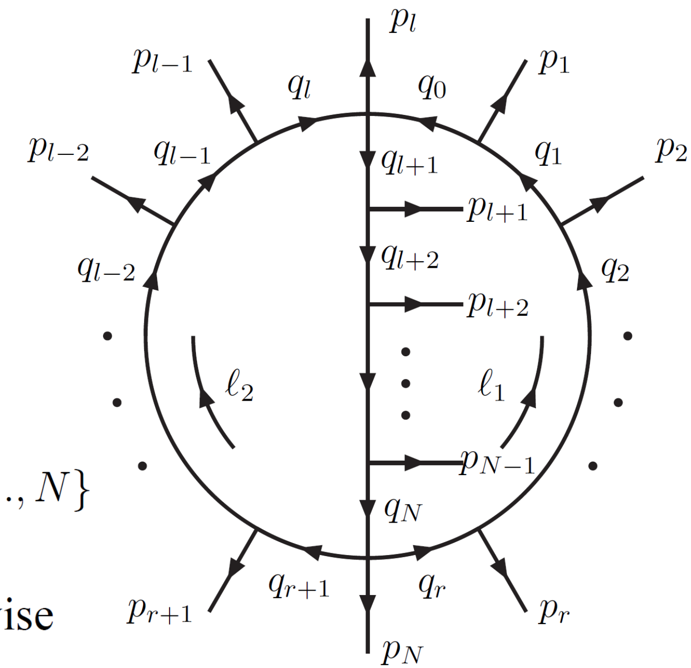
$$L^{(2)}(p_1,p_2,\ldots,p_N) = \int_{\ell_1}\int_{\ell_2} G_F(\alpha_1 \cup \alpha_2 \cup \alpha_3)$$

$$\int_{\ell_i} \bullet = -\imath \int \frac{d^d\ell_i}{(2\pi)^d} \bullet \quad , \quad G_F(\alpha_k) = \prod_{i\in\alpha_k} G_F(q_i)$$

- Three possible sets of momenta, according to their dependence on l_1, l_2 or l_1+l_2 (*integration variables*)

$$\alpha_1 \equiv \{0,1,...,r\}\ ,\ \alpha_2 \equiv \{r+1,r+2,...,l\}\ ,\ \alpha_3 \equiv \{l+1,l+2,...,N\}$$

$$q_i = \begin{cases} \ell_1 + p_{1,i} & , i\in\alpha_1 \\ \ell_2 + p_{i,l-1} & , i\in\alpha_2 \\ \ell_1+\ell_2+p_{i,l-1} & , i\in\alpha_3 \end{cases} \quad \text{with} \quad \begin{array}{ll} \ell_1 & \text{anti–clockwise} \\ \ell_2 & \text{clockwise} \end{array}$$



Generic two-loop diagram

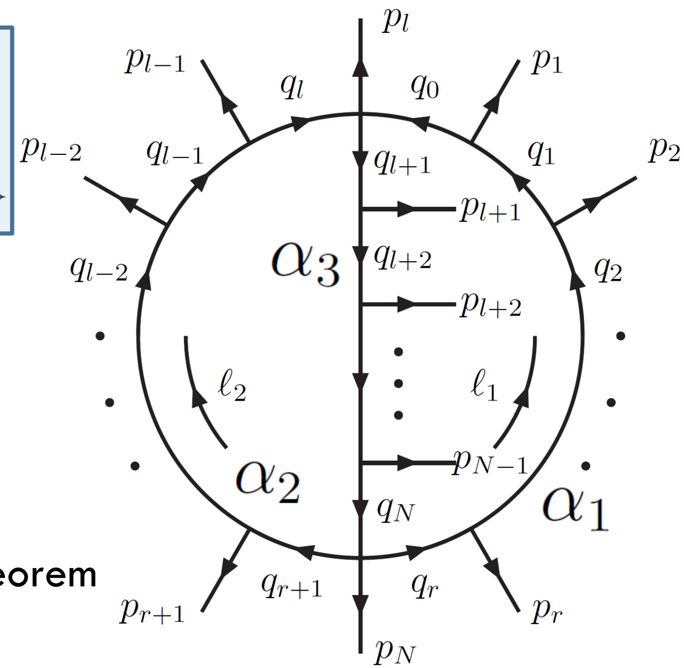
Driencourt-Mangin, Rodrigo, G.S., Torres Bobadilla, JHEP 02 (2019) 143

About Higgs@NNLO…

Cuts and LTD formula

- **"The number of cuts equals the number of loops"**
- *Derivation:* "Iterate" the one-loop formula and use propagator properties
- *Standard example:* two-loop N-point scalar amplitude

$$L^{(2)}(p_1, p_2, \ldots, p_N) = \int_{\ell_1} \int_{\ell_2} \{ G_D(\alpha_2)\, G_D(\alpha_1 \cup \alpha_3) + G_D(-\alpha_2 \cup \alpha_1)\, G_D(\alpha_3) - G_F(\alpha_1)\, G_D(\alpha_2)\, G_D(\alpha_3) \}$$



where we used $G_D(\alpha_k) = \sum_{i \in \alpha_k} \tilde{\delta}(q_i) \prod_{\substack{j \in \alpha_k \\ j \neq i}} G_D(q_i; q_j)$

- **Remarks and subtleties:**
  - Modified prescription depends on loop momenta.
  - Not a "trivial" iteration: connection with Cauchy's theorem and multivariable residues.
  - *Thesis:* "Virtual-real amplitudes mapped with one-loop formulae" (*partial cancellations*), but a new mapping required for double-real emission.

Driencourt-Mangin, Rodrigo, G.S., Torres Bobadilla, JHEP 02 (2019) 143

Part IV: Singular structures

- **LTD formalism recasts Minkowski integrals into Euclidean ones, thus leading to some "nice" mathematical properties.**

- **I)- Characterization of threshold singularities within the LTD approach**

Characterization of singularities with LTD

Description of threshold singularities @ 1-loop

- In general, the location of the singularities is given by the solutions of

$$\lambda_{ij}^{\pm\pm} = \pm q_{i,0}^{(+)} \pm q_{j,0}^{(+)} + k_{ji,0} = 0$$

with q_i on-shell and $k_{ji} = q_j - q_i$.

- We consider the following test functions

$$\mathcal{S}_{ij}^{(1)} = (2\pi\imath)^{-1}\, G_D(q_i; q_j)\, \tilde{\delta}\,(q_i) + (i \leftrightarrow j)$$

Up to 2 on-shell states (standard thresholds)

$$\mathcal{S}_{ijk}^{(1)} = (2\pi\imath)^{-1}\, G_D(q_i; q_k)\, G_D(q_i; q_j)\, \tilde{\delta}\,(q_i) + \text{perm.}$$

Up to 3 on-shell states (*anomalous thresholds*)

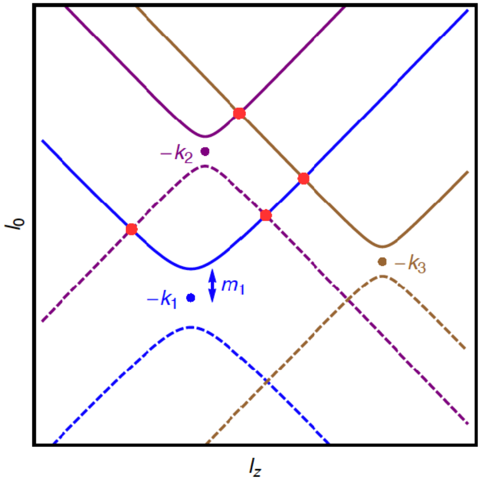
- **IMPORTANT:** *The singular structure of scattering amplitudes is dictated by their propagators. So, the proposed test functions are general enough to do a proper analysis of threshold singularities.*

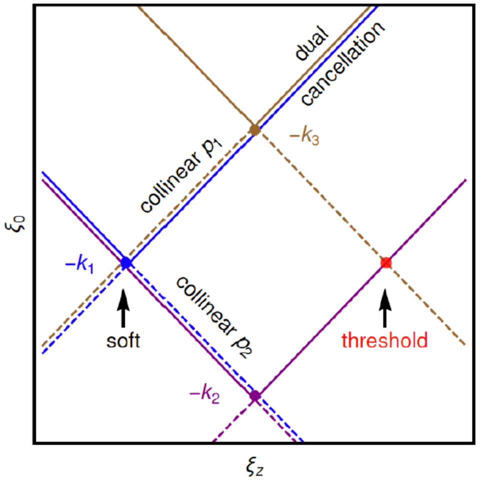
Rodrigo et al, arXiv:1904.08389 [hep-ph]

Characterization of singularities with LTD

 Description of threshold singularities @ 1-loop

- The singular structure depends on the separation among momenta:





- **Time-like separation (causal connection):**

$$k_{ji}^2 - (m_j + m_i)^2 \geq 0$$

Physical threshold singularities are originated.

$$\lim_{\lambda_{ij}^{++} \to 0} \mathcal{S}_{ij}^{(1)} = \frac{\theta(-k_{ji,0})\theta(k_{ji}^2 - (m_i + m_j)^2)}{x_{ij}(-\lambda_{ij}^{++} - \imath 0 k_{ji,0})} + \mathcal{O}\left((\lambda_{ij}^{++})^0\right)$$

Always +i0 !!!

$$x_{ij} = 4\, q_{i,0}^{(+)}\, q_{j,0}^{(+)}$$

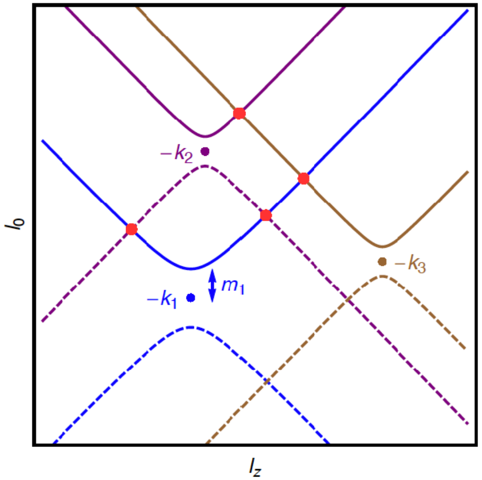
The prescription is crucial to determine the imaginary part: it is always **+i0** and corresponds to the usual Feynman prescription! For this configuration, LTD and FTT give equivalent descriptions!

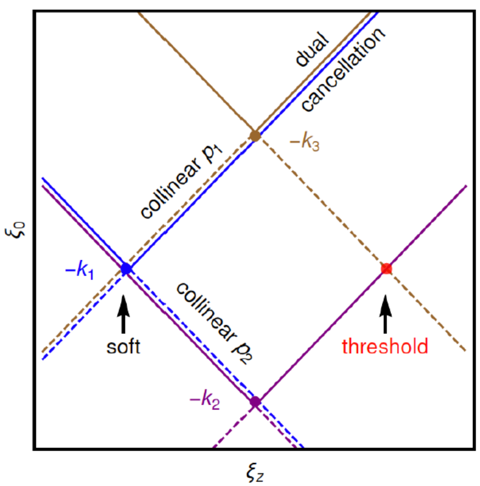
Rodrigo et al, arXiv:1904.08389 [hep-ph]

Characterization of singularities with LTD

Description of threshold singularities @ 1-loop

- The singular structure depends on the separation among momenta:





- **Space-like separation:**

$$k_{ji}^2 - (m_j - m_i)^2 \leq 0$$

The dual-prescription changes sign within the different contributions, which allows a perfect cancellation of any singular behaviour.

$$q_{j,0}^{(+)}\, G_D(q_i;q_j)|_{\lambda_{ij}^{+-}\to 0} = -q_{i,0}^{(+)}\, G_D(q_j;q_i)|_{\lambda_{ij}^{+-}\to 0}$$

$$\lim_{\lambda_{ij}^{+-}\to 0} \mathcal{S}_{ij}^{(1)} = \mathcal{O}\left((\lambda_{ij}^{+-})^0\right)$$

Cancellation codified by multiple-cuts in FTT!!

- **Light-like separation:**

It originates IR and threshold singularities that remain in a compact region of the integration domain. There is a partial cancellation among dual contributions, *but IR might remain!*

Rodrigo et al, arXiv:1904.08389 [hep-ph]

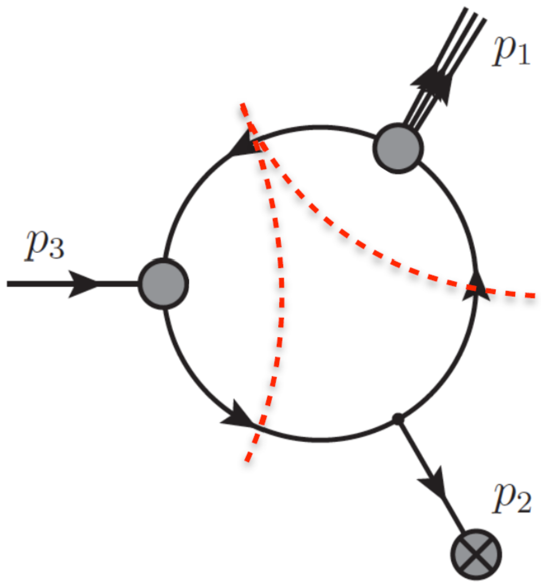
Characterization of singularities with LTD

Description of threshold singularities @ 1-loop

- **Anomalous thresholds:** *causal (i.e. time-like separated) singularities originated by multiple propagators going on-shell.*

$$\lim_{\lambda_{ij}^{++},\lambda_{ik}^{++}\to 0} \mathcal{S}_{ijk}^{(1)} = \frac{1}{x_{ijk}} \prod_{r=j,k} \frac{\theta(-k_{ri,0})\,\theta(k_{ri}^2-(m_i+m_r)^2)}{(-\lambda_{ir}^{++}-\imath 0 k_{ri,0})} + \mathcal{O}\left((\lambda_{ij}^{++})^{-1},(\lambda_{ik}^{++})^{-1}\right)$$

$$x_{ijk} = 8\, q_{i,0}^{(+)}\, q_{j,0}^{(+)}\, q_{k,0}^{(+)}$$



- Intersections of two hyperboloids lead to the standard IR and threshold singularities.

- Anomalous thresholds are originated from the intersection of **two** forward (backward) and **one** backward (forward) hyperboloids.

- **There are not singularities for** $\lambda_{jk}^{-+} = \lambda_{ik}^{++} - \lambda_{ij}^{++} \to 0$ **!!!**

Rodrigo et al, arXiv:1904.08389 [hep-ph]

NEW!

Part V: Multiloop numerical approach

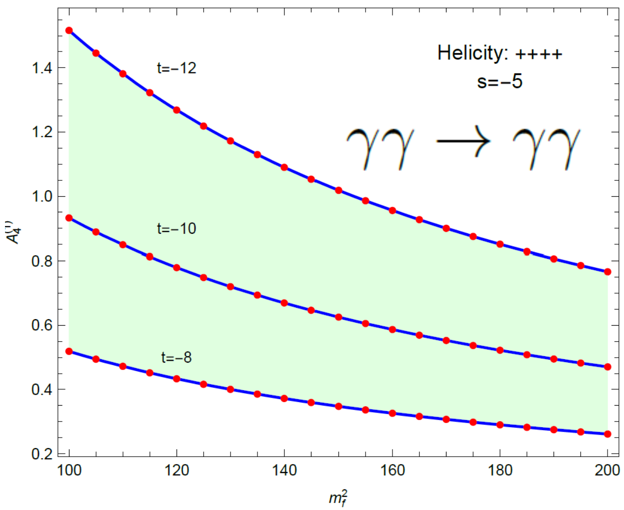
- **LTD allows to transform loop into phase-space integrals: *we integrate on space components*!**

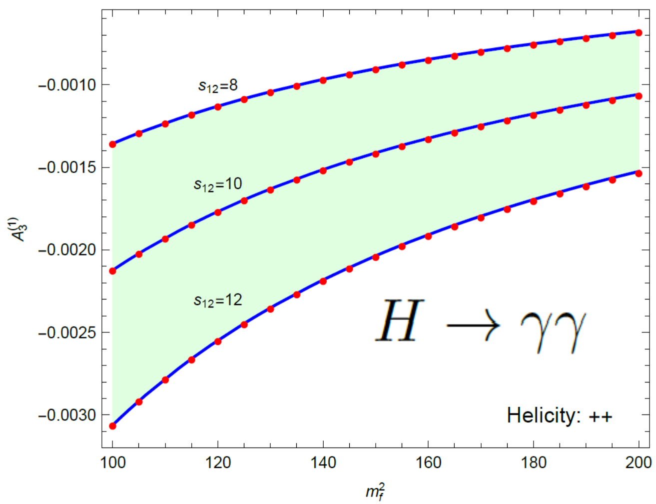
- **I)- Combination with spinor-helicity formalism in four-dimensions (NEW!)**
- **II)- Novel multiloop formulae based on iterative LTD!!! (ONGOING, RESULTS TO APPEAR SOON!)**

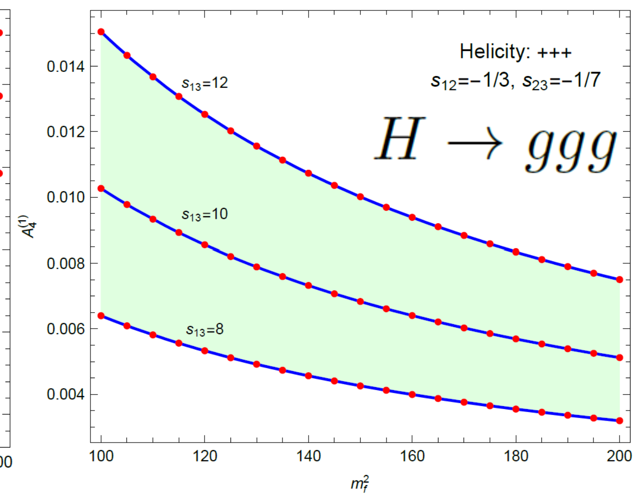
Towards multiloop&multileg numerical LTD

1-loop examples in four space-time dimensions

- We consider three processes which are UV and IR finite, at one-loop.







- **Purely 4D numerical implementation** for fixed helicity configurations!
- Complete **agreement** with well-known analytical results (solid blue lines)
- **Smooth dependence** on the external parameters (kinematical invariants & masses)
- Competitive computational performance (*vs. SecDec and standard tools*)

Rodrigo et al, arXiv:1911.11125 [hep-ph]

Conclusions and perspectives

- ✓ Loop-tree duality allows to treat **virtual and real** contributions **in the same way** (implementation simplified)
- ✓ Physical interpretation of **IR/UV singularities** in loop integrals (light-cone diagrams) and **proper disentanglement**
  - ✓ **Combined virtual-real terms are integrable in 4D!!**
  - ✓ **Local 4-dimensional 2-loop results (Higgs/heavy quarks)**
  - ✓ **Efficient numerical implementation for multiloop multileg processes!!** → ongoing!!! ("*paso a paso*")

- **Perspectives:**
  - Automation of multileg processes at NLO and beyond!!!
  - *Carefull comparison with other schemes* → "WorkStop-ThinkStart" *Eur.Phys.J. C77 (2017) no.7, 471* ***NEW! Firenze Meeting 2019***

0

Inside a “0” there ARE many hidden things

GRACIAS!!!