Threshold effects for heavy-particle production at hadron and lepton colliders

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12.12.2019

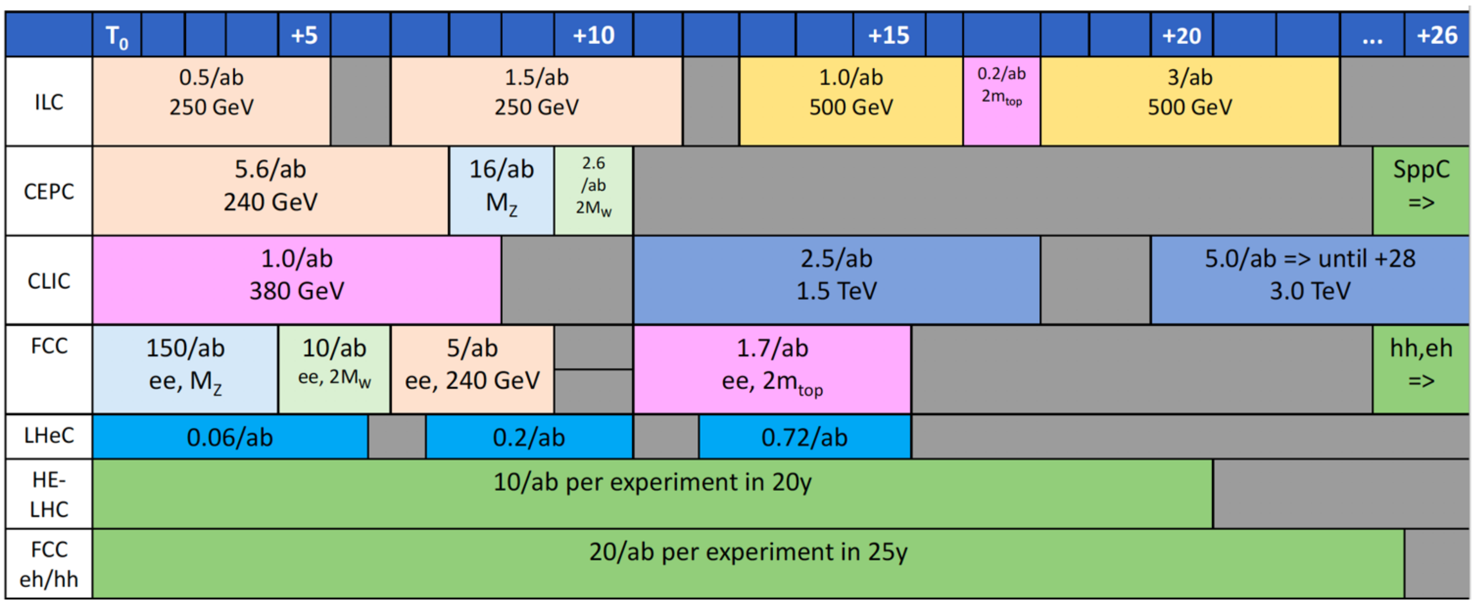
Proposed future collider projects:

ILC in Japan: $\sqrt{s} = 250\,\mathrm{GeV}$, extensions to $350, 500, 1000\,\mathrm{GeV}$
GigaZ run at Z-threshold (re)considered

CLIC at CERN: $\sqrt{s} = 380\,\mathrm{GeV}$, extensions to $1.5, 3\,\mathrm{TeV}$

FCC at CERN: FCC-ee: $\sqrt{s} = 90 - 360\,\mathrm{GeV}$; FCC-pp: $\sqrt{s} = 100\,\mathrm{TeV}$.

CEPC/SppC in China: $\sqrt{s} = 90 - 240\,\mathrm{GeV}$ (ee); $75\,\mathrm{TeV}$ (pp)

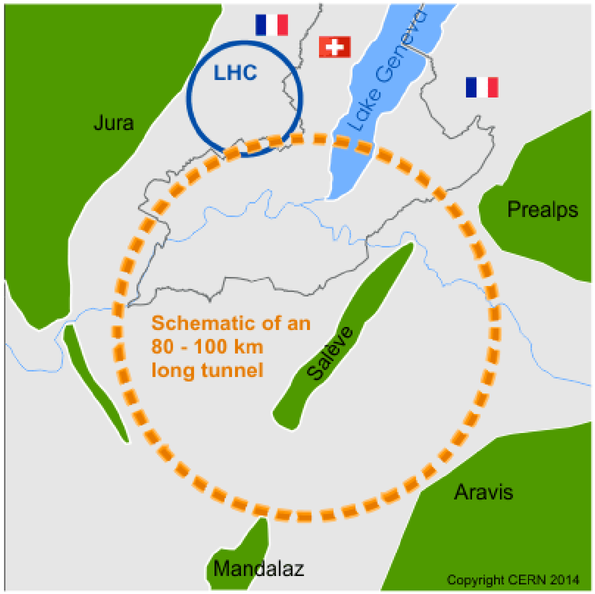


(Physics Briefing Book, European Strategy for Particle Physics; arXiv:1910.11775)

FCC-ee

- 100 km tunnel at CERN
- Higgs factory using $e^-e^+ \to ZH$
- precision measurements at Z-pole, WW and $t\bar{t}$ threshold

$\Rightarrow$ order-of-magnitude improvement of EW precision data



Phase	run time (years)	$\sqrt{s}$ [GeV]	$\mathcal{L}$ (ab^{-1})	# events
FCC-ee-Z	4	88-95	150	$3 \cdot 10^{12}$ visible Z decays
FCC-ee-W	2	158-162	12	10^8 WW events
FCC-ee-H	3	240	5	10^6 ZH events
FCC-ee-tt	5	345-365	1.7	10^6 $t\bar{t}$ events

(Theory report on the 11th FCC-ee workshop; arXiv:1905.05078 [hep-ph])

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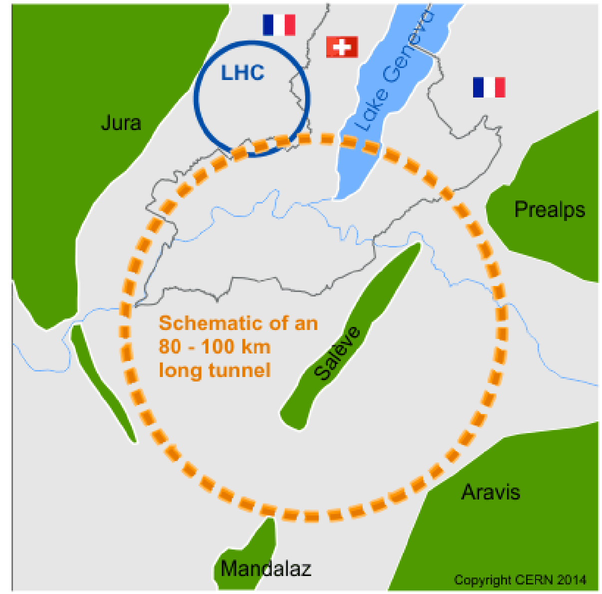


Observable	present value			FCC-ee stat.	FCC-ee syst.
m_Z (keV)	91186700	$\pm$	2200	4	100
Γ_Z (keV)	2495200	$\pm$	2300	7	100
$\alpha_s(m_Z)$ $(\times 10^4)$	1196	$\pm$	30	0.1	0.4-1.6
$\sin^2\theta_W^{eff}(\times 10^6)$	231480	$\pm$	160	3	2 - 5
$1/\alpha_{QED}(m_Z)(\times 10^3)$	128952	$\pm$	14	4	small
m_W (MeV)	80350	$\pm$	15	0.5	0.3
Γ_W (MeV)	2085	$\pm$	42	1.2	0.3
m_{top}	172740	$\pm$	500	17	small

Precision measurements of m_t, M_W at FCC-ee/CEPC

- $t\bar{t}$ threshold scan

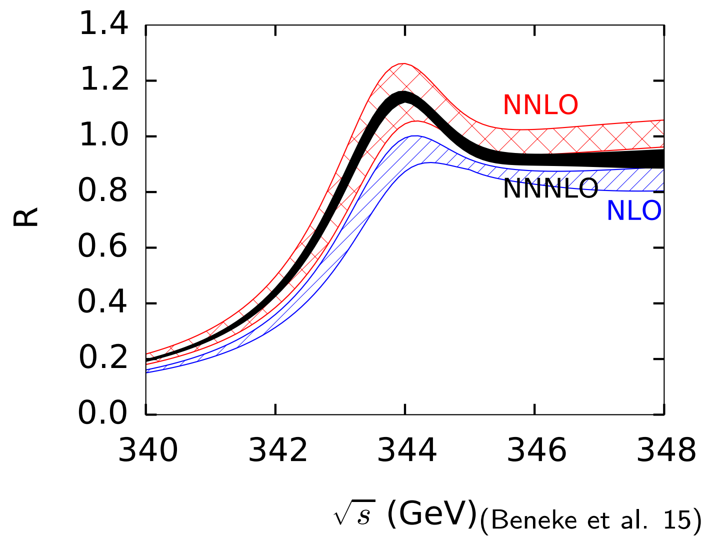
  Experimental uncertainty

  $$\Delta m_t \simeq 17\,\text{MeV},\ \Delta\Gamma_t \simeq 45\,\text{MeV}$$

  from eight points with $25\,\text{fb}^{-1}$

  Current theory uncertainty

  $$\Delta m_t \simeq 50\,\text{MeV}$$



(Beneke et al. 15)

- WW threshold scan

  $$\Delta M_W \simeq 0.5\,\text{MeV},\ \Delta\Gamma_W \simeq 1.2\,\text{MeV}$$

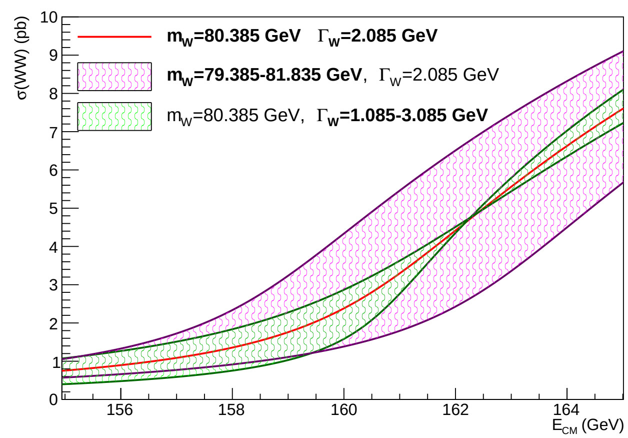
  from two points

  $$\sqrt{s} = 157.5,\ 162.5\,\text{GeV}$$

  with $15\,\text{ab}^{-1}$ if

  $$\delta\sigma^{\text{th.}}_{WW} < 0.6\,\text{fb}(\approx 0.01\%)$$

  Realistic uncertainty goal?



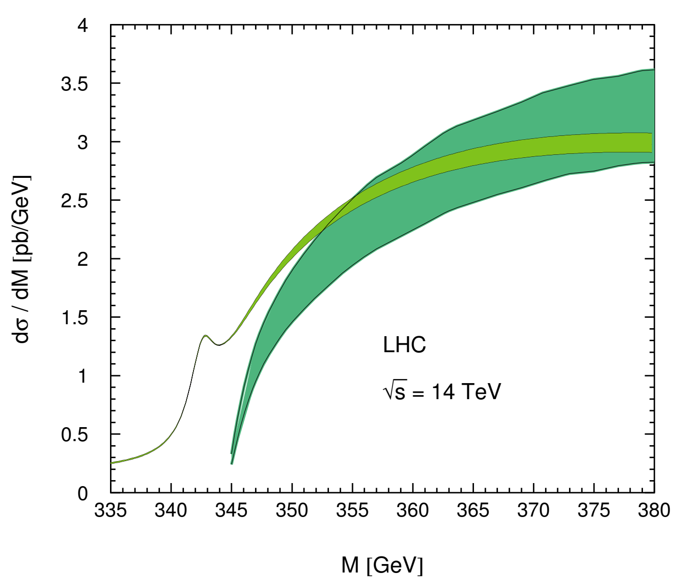
(1703.01626 [hep-ph])

Threshold effects at hadron colliders?

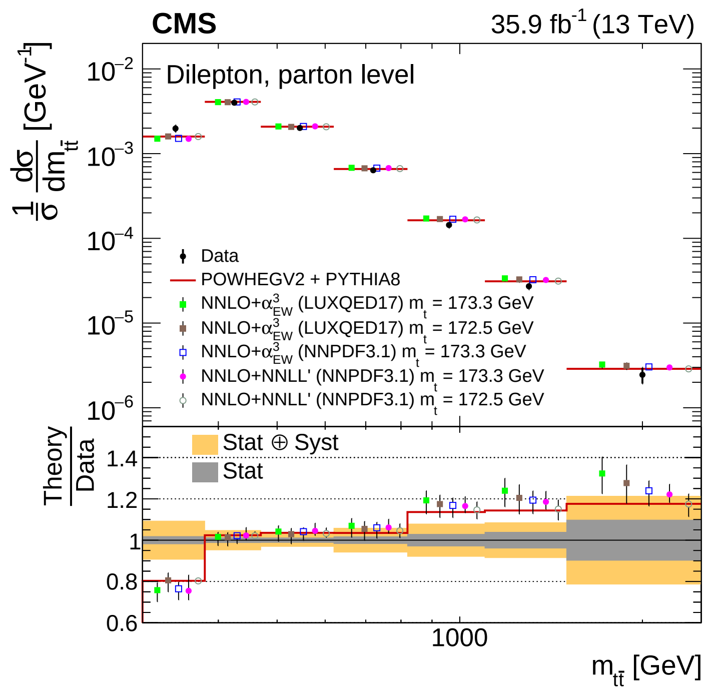
- Convolution with parton luminosity

$\Rightarrow$ threshold not directly accessible

- Effects on invariant mass spectrum visible?



(Kiyo et.al. 08)

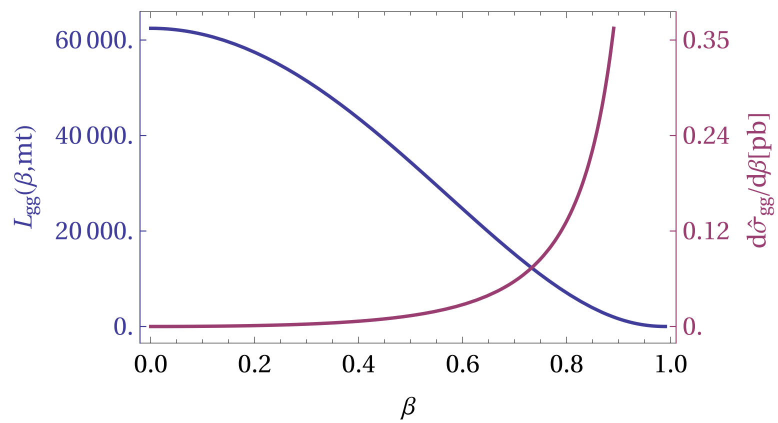


(arXiv:1811.06625)

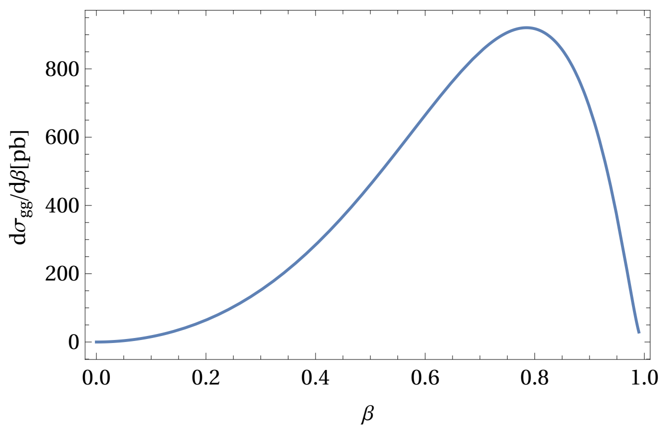
Total cross section: parton luminosities enhance threshold region

$$\sigma = \int d\beta \frac{d\sigma(\beta)}{d\beta} = \int d\beta L_{ij}(\beta,\mu_f) \frac{8\beta m_t^2}{s(1-\beta^2)^2} \hat{\sigma}(\beta,\mu_f)$$

- Top-pair production dominated by $\beta > 0.5$
  $\Rightarrow$ justification of threshold approximation?



$\Rightarrow$



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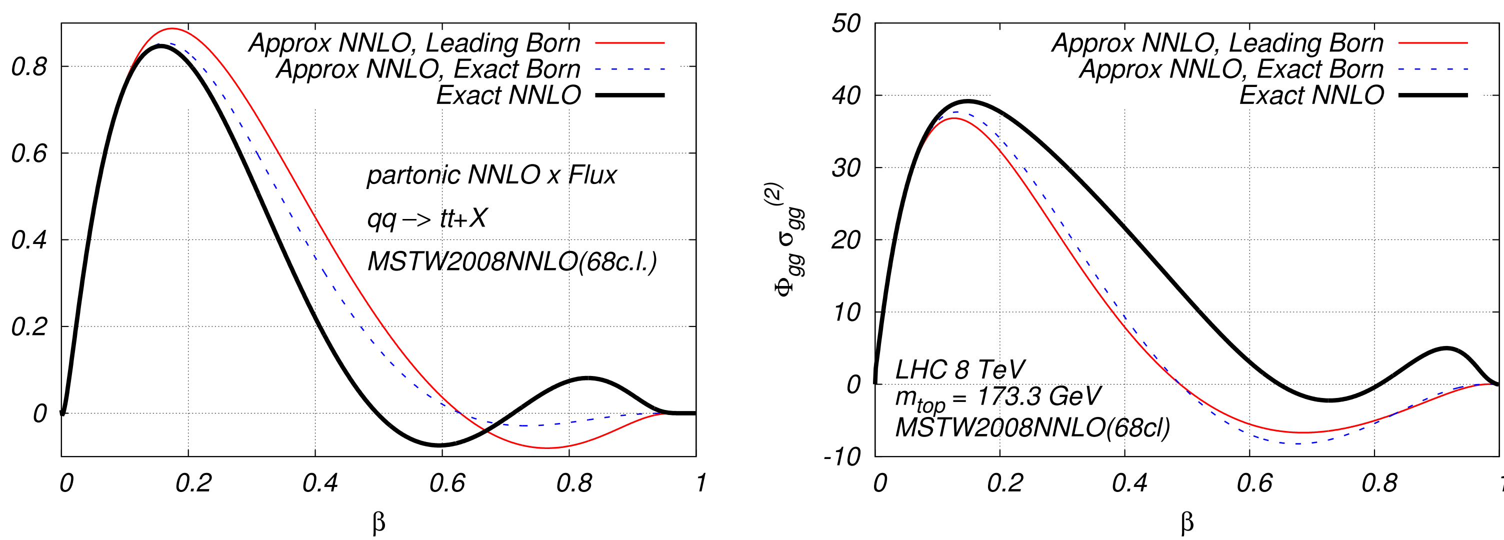


(Bärnreuther/Czakon/Mitov 12; Czakon/Fiedler/Mitov 13)

$\Rightarrow$ threshold corrections give estimate of higher-order corrections

$\Rightarrow$ careful estimate of uncertainties necessary

Overview of talk

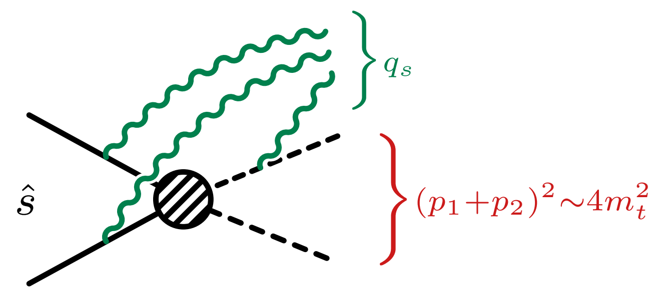
- Threshold effects in heavy particle production
  - Finite width effects
  - soft and Coulomb photon/gluon corrections '
  - factorization and resummation
- Threshold-enhanced N^{3}LO corrections for $pp \to t\bar{t}$ (Piclum/CS 18)
- $e^-e^+ \to WW$ at threshold
  - NLO and leading NNLO corrections in threshold expansion (Beneke et al. 07, Actis et al. 08)
  - prospects towards FCC-ee precision (CS 19)

Production of non-relativistic heavy particles H (W, t, SUSY partners,...)

$$E \sim M_H \beta^2 \,, \quad |\vec{p}| \sim M_H \beta, \quad \beta = \sqrt{1 - \frac{4M_H^2}{s}}$$

Finite-width effects:

off-shell effects of order $E \sim |\vec{p}|^2 \sim \Gamma_H$



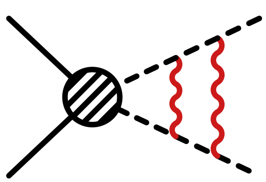
Initial/final-state radiation:

soft ($q_s \sim M_H \beta$) and collinear ($q_c \sim M_H$, $q_c^2 = 0$)

photon/gluon radiation dominant

Coulomb and bound-state effects:

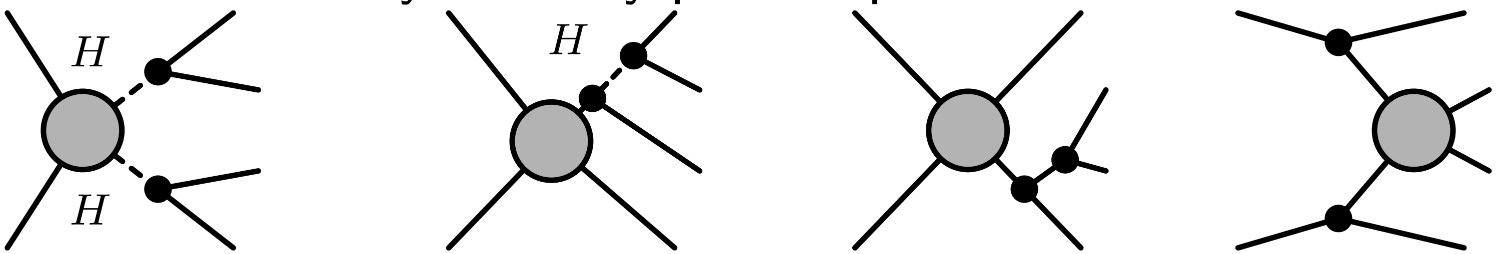
- photon/gluon exchange between heavy particles:
  effective Coulomb potential



$\Rightarrow V(r) = \frac{\alpha_i D_C}{r}$, $\alpha_i \in \{\alpha, \alpha_s\}$, D_C: charge/colour factor

- bound-state formation possible for $D_C < 0$ with $E_n = -\frac{\alpha_i^2 D_C^2 m_t}{4n^2}$

Production and decay of heavy particle pair:

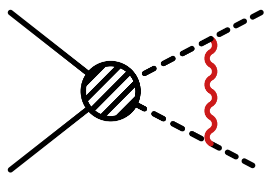


double resonant   single resonant   non-resonant   non-resonant

Separation of double resonant contributions:

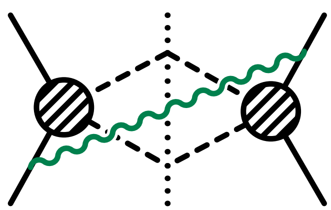
- expansion of amplitude around propagator poles (Stuart 91; Aeppli/v.Oldenbourgh/Wyler 93)
- generic kinematics: non-res. contributions suppressed by Γ/M
- near threshold: EFT approach (Beneke et al. 03/04)
- $\Gamma_t/m_t < 1\%$: neglect $\sigma_{\text{non-res.}}$ for $t\bar{t}$ production at hadron colliders (justified for total cross section: Bevilaqua et al; Denner et al. 10)
- $\Gamma_W/M_W \approx 3\%$: relevant for threshold scan in e^-e^+

Coulomb corrections from "potential" momenta $q \sim m(\beta^2, \beta)$

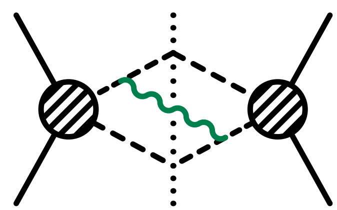


$$\sim \alpha_i \int dq^0 d^3\vec{q}\, \frac{1}{E_2 - \frac{\vec{p}_2^{\,2}}{2m_t}}\, \frac{1}{\vec{q}^{\,2}}\, \frac{1}{E_1 - \frac{\vec{p}_1^{\,2}}{2m_t}} \sim \frac{\alpha_i}{\beta}$$

Threshold logarithms: remnants of cancellation of soft/collinear divergences between real and virtual corrections



$\Rightarrow \alpha_i \ \log^2(\beta)$



$\Rightarrow \alpha_i \ \log(\beta)$

Relevance of threshold enhancement:

$$e^- e^+ \to W^- W^+ : \quad \beta \sim \sqrt{\Gamma_W / M_W} \sim \alpha^{1/2}, \quad \Rightarrow \quad \alpha/\beta \sim \alpha^{1/2}, \quad \alpha \ln \beta \sim \mathcal{O}(1\%)$$

$\Rightarrow$ Coulomb corrections enhanced, soft corrections not dominant

$$pp \to t\bar{t} : \quad \beta \sim \sqrt{\Gamma_t / M_t} \sim \mathcal{O}(10\%) \sim \alpha_s, \quad \Rightarrow \quad \alpha_s/\beta \sim \mathcal{O}(1), \quad \alpha_s \ln \beta \sim \mathcal{O}(10\%)$$

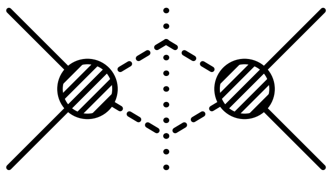
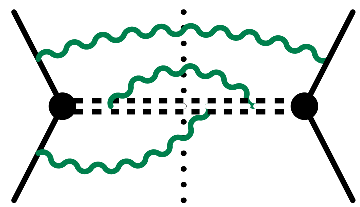
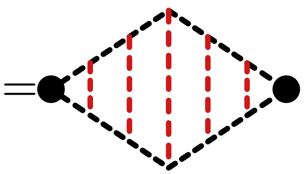
$\Rightarrow$ Coulomb and soft corrections can be large

Soft and Coulomb gluons

Factorization of resonant part of cross section (Beneke, Falgari, CS 09/10)

$$\Rightarrow \hat{\sigma}_{\text{res}}|_{\hat{s}\to 4M^2} = \sum_R H_R(M,\mu) \int d\omega \, J_R(\sqrt{\hat{s}} - 2M - \frac{\omega}{2}) W^R(\omega,\mu)$$

Hard, soft and Coulomb functions:

$H_R =$  , $W^R =$  , $J^R =$ 

- derived using soft-collinear and non-relativistic effective field theories
- soft radiation universal (eikonal approximation)
  "sees" only total (colour) charge R (Singlet, octet,...)
- Coulomb resummation in potential function using non-relativistic field theory

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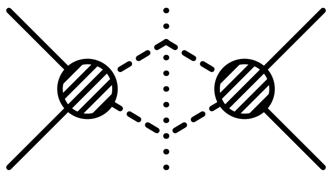
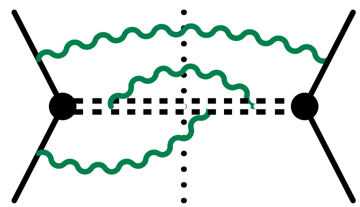
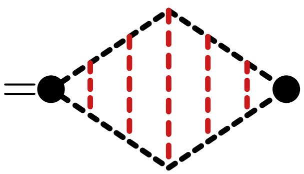
$H_R =$  , $W^R =$  , $J^R =$ 

Factorization scale dependence of H, W cancels against PDFs:

$$\frac{d\sigma}{d\mu} = \frac{d}{d\mu}\left(f_1 \otimes f_2 \otimes H \otimes W \otimes J\right) = 0$$

- $\frac{df_i}{d\mu} \Rightarrow$ Altarelli-Parisi equation (3-loop: Moch/Vermaseren/Vogt 04/05)
- $\frac{dH}{d\mu} \Rightarrow$ IR singularities (2-loop: Becher, Neubert; Ferroglia et.al. 09)

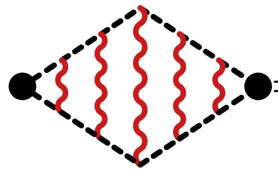
$\Rightarrow$ RGE for soft function (NNLL: Beneke/Falgari/CS; Czakon/Mitov/Sterman 09)

- Resummation in Mellin-space (Sterman 87; Catani/Trentadue 89)
  Momentum-space solution to RGE (Becher/Neubert/Pecjak 07)

Soft and Coulomb gluons

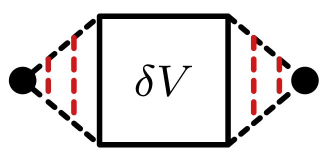
Resummation of $\frac{\alpha_i}{\beta}$ corrections: (Fadin, Khoze 87; Peskin, Strassler 90)

- Green's function for non.rel. Schrödinger equation

$\mathrm{Im}G_{\mathrm{C}}^{R}(0,0;E) =$  $= \begin{cases} \frac{m_t^2 \pi D_C \alpha_i}{2\pi} \left(e^{\pi D_C \alpha_i \sqrt{\frac{m_t}{E}}} - 1 \right)^{-1} & E > 0 \\ 2\pi \sum_{n=1}^{\infty} \delta(E - E_n) |\psi_n(0)|^2 & E < 0 \end{cases}$

sums all (α_i/β) corrections

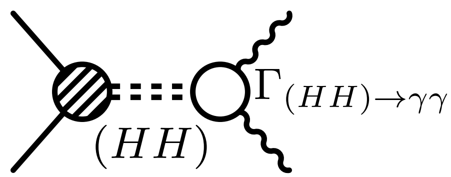
- Finite width effects: shift $E \to E + \mathrm{i}\Gamma$
- Bound-state poles at $E_n = -\frac{\alpha_i^2 D_C^2 m_t}{4n^2}$ smeared out by finite Γ_t.
- Non-relativistic perturbation theory with insertions of higher-order potentials δV with radiative/kinematic corrections
- NLO potential function sums all terms $\alpha_s(\alpha_s/\beta)^n$

$\delta G_R^{(1)}(0,0,E) =$ 

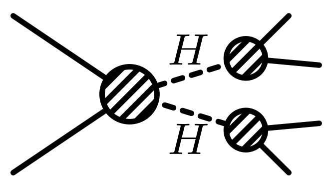
$= \int d^3 z \, G_R^{(0)}(0, \vec{z}, E) \, (i\delta V^R(\vec{z})) \, iG_R^{(0)}(\vec{z}, 0, E)$

Different scenarios for phenomenology: (e.g. Hagiwara/Yokoya 09)

- $\Gamma_H < \Gamma_{(HH)\to X} \propto \alpha_i |\psi(0)|^2$: formation and decay of (HH) bound state



- $|E_1| \gg \Gamma_H > \Gamma_{(HH)\to X}$: formation of bound-states with constituent H decay
- $|E_1| > \Gamma_H \gg \Gamma_{(HH)\to X}$: Coulomb-enhanced continuum HH production
- $\Gamma_H \gtrsim |E_1|$: Continuum HH production and decay



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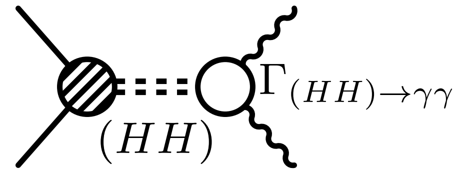

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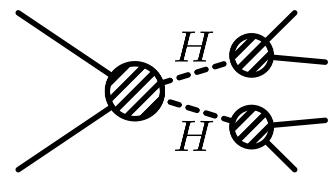


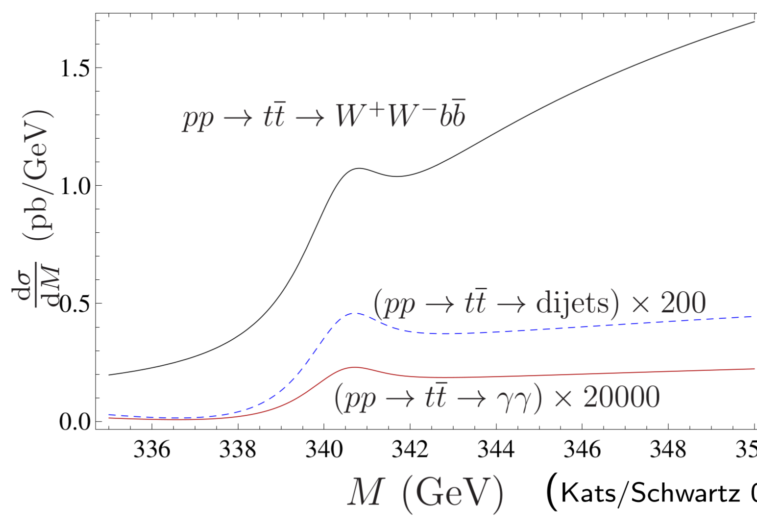
Example: "Toponium":

$$\Gamma_t = 1.3\,\mathrm{GeV}$$

$$\approx E_1 = \frac{1}{4} C_F^2 \alpha_s^2(\mu_C) m_t \approx 1.5\,\mathrm{GeV}$$

$$\gg \Gamma_{(t\bar{t})\to gg} \propto \frac{\alpha_s(m_t)}{m_t^2} (\alpha_s(\mu_C) C_F m_t/2)^3$$

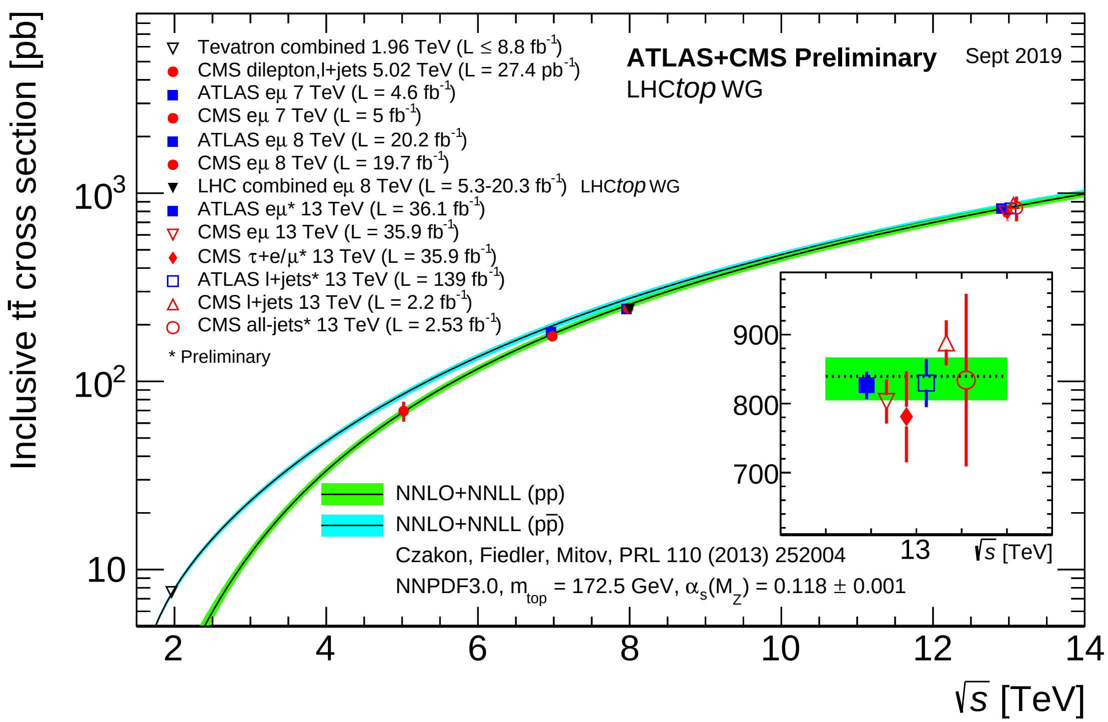
$$\sim \mathcal{O}(10^{-3}\,\mathrm{GeV})$$



(Kats/Schwartz 09)

$t\bar{t}$ **production at LHC** test of QCD and nature of top-quark:

- Sensitive to m_t, α_s, gluon PDF
- Experimental precision $\Delta\sigma_{t\bar{t}} \sim 3 - 4\%$ comparable to uncertainty of NNLO+NNLL prediction (Bärnreuther/Czakon/Fiedler/Mitov 12–13)



Resummation of threshold-enhanced corrections, $\beta = \sqrt{1 - \frac{4m_t^2}{\hat{s}}} \to 0$

$$\hat{\sigma}_{pp'} \propto \sigma^{(0)} \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \underbrace{\alpha_s^2 g_3(\alpha_s \ln\beta)}_{\text{(N}^3\text{LL)}} + \ldots\Big]$$

$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \times \Big\{ \underbrace{1}_{\text{(LL,NLL)}} \;;\; \underbrace{\alpha_s, \beta}_{\text{(NNLL)}} \;;\; \underbrace{\alpha_s^2, \alpha_s\beta, \beta^2}_{\text{(NNLL', N}^3\text{LL)}} \;; \ldots \Big\} :$$

- Mellin-space NNLL resummation of threshold logarithms

  (Czakon/Mitov/Sterman 09/Cacciari et al. 11, ⇒ TOP++)

- SCET/NRQCD resummation of threshold logarithms and Coulomb corrections α_s/β (Beneke/Falgari/(Klein)/CS 09/11; ⇒ TOPIXS)

$$\sigma_{t\bar{t}}^{\text{NNLO}}(13\text{TeV}) = 802.83^{+28.12(3.5\%)}_{-44.97(5.6\%)}\text{pb} \Rightarrow \begin{cases} \text{NNLL(top++)}: & 821.37^{+20.28(2.5\%)}_{-29.60(3.6\%)}\text{pb} \\ \text{NNLL(topixs)}: & 807.13^{+24.72(3.2\%)}_{-39.03(5.0\%)}\text{pb} \end{cases}$$

(NNLL' terms in top++; resummation uncertainty $\approx \pm 2\%$ in topixs

higher-order Coulomb effects small; repulsive 8-channel)

Prospects of N³LL resummation (Piclum/CS 18)

- Input to N³LL resummation formula:
  - two-loop hard function (Bärnreuther/Czakon/Fiedler 13)
  - two-loop soft functions (Belitzky 98; Becher et al. 07; Czakon/Fiedler 13)
  - N³LO potential corrections

    (colour singlet: Beneke et al. 15; incomplete colour octet)
  - anomalous dimensions (4-loop γ_{cusp} (Moch et al. 17/18);

    3-loop collinear anomalous dimensions (Moch/Vermaseren/Vogt 04/05)

    3-loop massive soft anomalous dimension
    (Brüser/Liu/Stahlhofen 19; extracted from Grozin et al. 15 in Hoang et al. 15))
- Interplay of Coulomb and power-suppressed corrections
  - "next-to-eikonal' corrections (Krämer et al. 98; Laenen et al. 10)

$$\frac{\alpha_s^2}{\beta^2} \times \alpha_s \beta^2 \log \beta \sim \alpha_s^3 \log \beta$$

  - P-wave production channels: $\beta^2 \times \{\frac{\alpha_s^3}{\beta^3}, \frac{\alpha_s^2}{\beta^2} \times \alpha_s \ln^{2,1} \beta\}$

General form of potential ($R = 1, 8,\ S = 1, 3$)

$$V^{R,S}(\mathbf{p}, \mathbf{p}') = \frac{4\pi\alpha_s D_R}{\mathbf{q}^2}\left[\mathcal{V}_C^R - \mathcal{V}_{1/m}^R\,\frac{\pi^2\,|\mathbf{q}|}{m_t} + \mathcal{V}_{1/m^2}^{R,S}\,\frac{\mathbf{q}^2}{m_t^2} + \mathcal{V}_p^R\,\frac{\mathbf{p}^2 + \mathbf{p}'^{\,2}}{2m_t^2}\right] + \frac{\pi\alpha_s}{m_t^2}\nu_{\text{ann}}^{R,S},$$

Known for colour singlet and octet:

- Coulomb potential (two-loop singlet: Schröder 98; octet: Kniehl et al. 04)
- One-loop spin-dependent $\mathcal{V}_{1/m^2}^{R,S}$

  (Wüster 03; colour-singlet: Beneke/Kiyo/Schuller 13, colour octet: Piclum/CS 18)
- One-loop annihilation contributions $\nu_{\text{ann}}^{R,S}$ (Pineda/Soto 98)

Unknown for octet

- Two-loop $\mathcal{V}_{1/m}^R$ (singlet: Kniehl et al. 01)
- (ultra)-soft corrections (singlet: Beneke/Kiyo 08)

  with chromoelectric vertex $\psi^\dagger \vec{x} \cdot \vec{E}_{us} \psi'^{\,\dagger}$

Unknown contributions at $\mathcal{O}(\alpha_s^3)$: $\alpha_s^3(\delta c^{(2)} \ln^2\beta + \delta c^{(1)} \ln\beta + \ldots)$

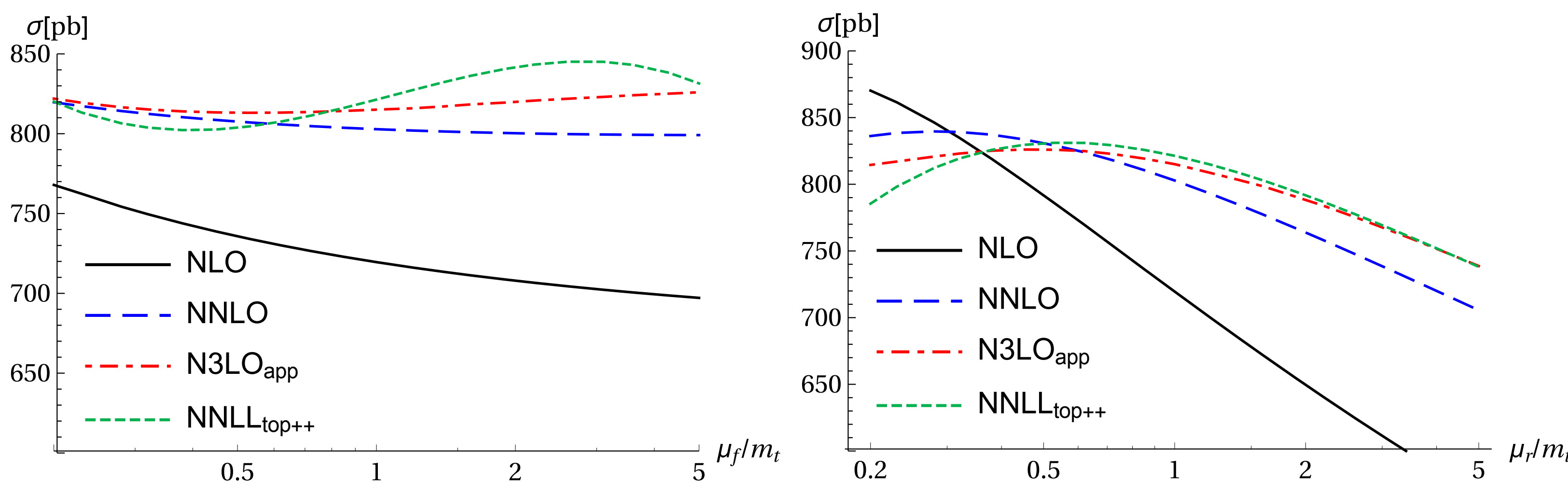
Estimate for octet: naive replacement $C_F \to (C_F - C_A/2)$

Expansion to N^{3}LO

Moderate correction $+1.6\%$ relative to NNLO (Piclum/CS 18)

$$\sigma_{t\bar{t}}^{\mathrm{N^3LO_{app}}}(13\mathrm{TeV}) = 815.70^{+19.88(2.4\%)}_{-27.12(3.3\%)}(\mathrm{scale})^{+9.49(1.2\%)}_{-6.27(0.8\%)}(\mathrm{approx})\mathrm{pb},$$

- Reduction of scale uncertainty to $\sim 3\%$
- Estimate of systematic uncertainty of approx:
$$\Delta\sigma_{t\bar{t}}^{\mathrm{N^3LO_{app}}}(\mathrm{approx.}) = \underbrace{{}^{+7.87}_{-6.24}}_{C^{(3)}} \underbrace{{}^{+5.3}_{-0.0}}_{\text{kin.ambiguity}} \pm \underbrace{0.11}_{3-\text{loop soft-an.dim}} \pm \underbrace{0.60}_{\text{Coulomb octet}} \mathrm{pb},$$
- Available in latest version of `topixs`



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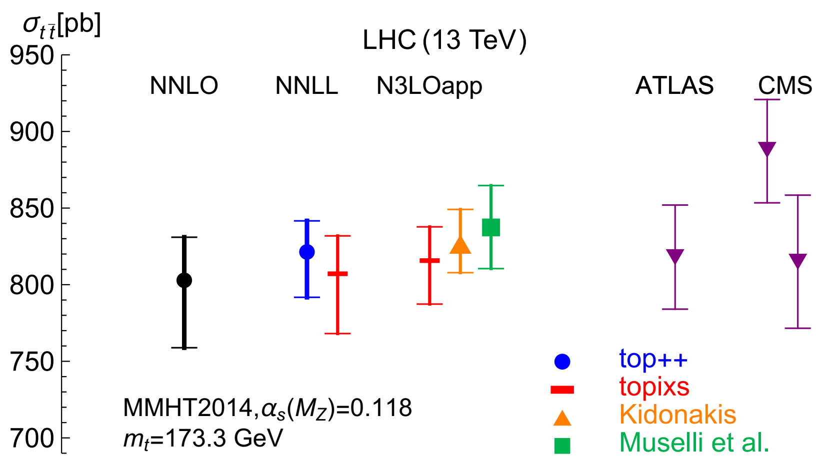
Other N^3LO_{approx} results:

- NNLL in one-particle inclusive kinematics: (Kidonakis 14)
- Including subleading collinear; $\beta \to 1$ terms (Muselli et al. 15)

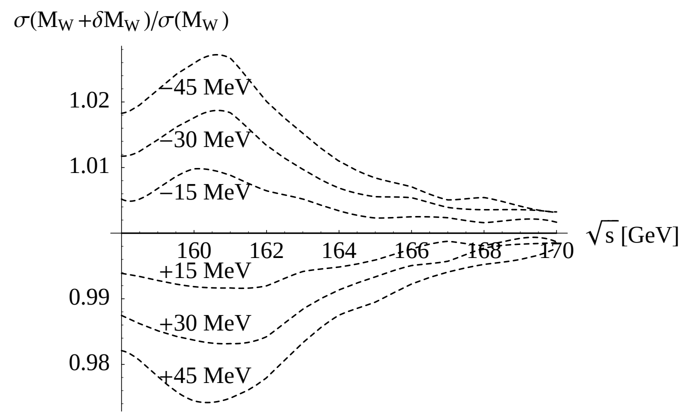


$e^- e^+ \to W^- W^+ \to 4f$ **threshold scan**

- Sensitivity to M_W

$$\Delta\sigma \sim 1\% \Leftrightarrow \Delta M_W \sim 15\,\text{MeV}$$

- Precision goal
  of future $e^- e^+$ colliders:



$$\Delta M_W \sim \begin{cases} \text{ILC}: & 4\,\text{MeV}\,(100\,\text{fb}^{-1}) \quad 2\,\text{MeV}\,(500\,\text{fb}^{-1}) \\ \text{FCC}-\text{ee}/\text{CEPC}: & < 1\,\text{MeV} \end{cases}$$

Current theory status:

- Full NLO calculation for $e^+ e^- \to 4f$ (Denner, Dittmaier, Roth, Wieders 05)
  remaining uncertainty few $\times 0.1\%$
- Leading NNLO corrections $\Delta\sigma \lesssim 0.5\%$ (Actis et al. 08)

FCC-ee goal $\Delta\sigma \lesssim 0.01\%$: full NNLO $e^- e^+ \to 4f$ required?

Threshold expansion of $\sigma(e^-e^+ \to 4f)$ in $\delta \sim \alpha \sim \beta^2$

- resonant contribution from soft/Coulomb factorization

  (soft $\log\beta$ corrections not resummed)

- finite-width effects; non-resonant contributions important
- Collinear photon ISR: mass logarithms $\ln\left(\frac{m_e}{M_W}\right)$ absorbed in electron structure functions

  (finite m_e: Skrzypek 92; $\overline{\text{MS}}$: Blümlein et al. 11; Bertone et al. 19)

- Flavour-specific decay corrections

NLO calculation of total cross section near threshold

(Beneke/Falgari/CS/Signer/Zanderighi 07)

$$\underbrace{\alpha/\beta}_{\text{1st Coulomb; non-res Born}} \sim \delta^{1/2}, \quad \underbrace{\beta^2,}_{\text{Born}} \quad \underbrace{\alpha,}_{\text{one-loop soft/hard}} \quad \underbrace{\alpha^2/\beta^2}_{\text{2nd Coulomb}} \sim \delta$$

Difference to NLO$_{4f}$ result: $\sigma^{4f}_{\text{NLO}}(s) - \sigma^{(1)}_{\text{EFT}}(s) = \sigma^{4f}_{\text{Born}}(s) \times (0.9 - 1.2)\%$

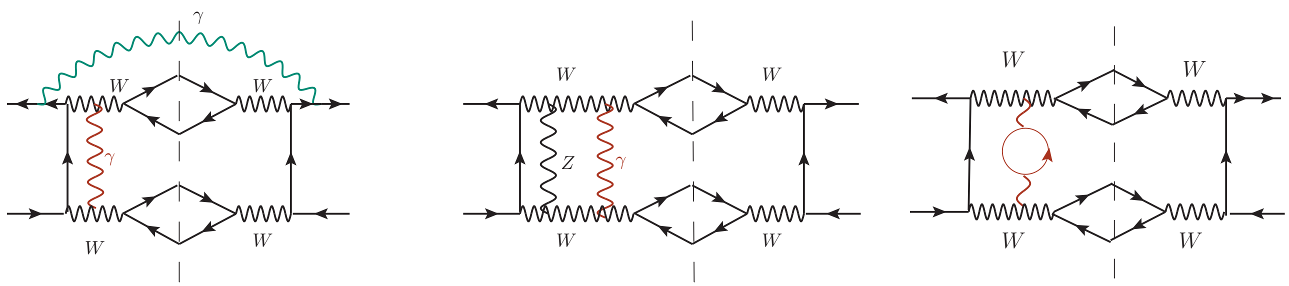
(Missing in EFT: $\mathcal{O}(\alpha)$ non-res. corrections; $\mathcal{O}(\alpha, \alpha/\beta)$ corrections to $\sim \beta^2$ suppressed Born)

Leading NNLO corrections

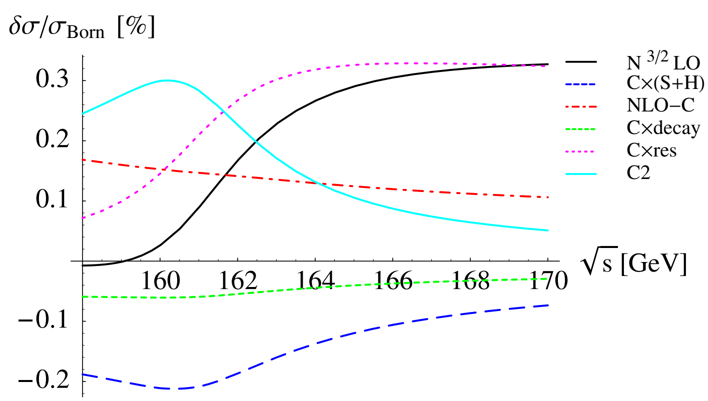
(Actis et al. 08)

$$\mathcal{O}(\delta^{3/2}): \quad \underbrace{\alpha\beta}_{\text{included in NLO}_{4f}} \quad \underbrace{\alpha \times \alpha/\beta}_{\text{Coulomb-enhanced NNLO}} \quad \underbrace{\alpha^3/\beta^3}_{\text{3rd Coulomb}}$$

$\mathcal{O}(\alpha^2/\beta)$ corrections:



- combination of 1st Coulomb with NLO hard/soft
- NLO renormalization of V_{Coul}
- Numerical effect: $< 0.5\%$; $[\delta M_W] \lesssim 3$ MeV



What is required for FCC-ee accuracy $\Delta\sigma \lesssim 0.01\%$? (CS 19)

$$\mathcal{O}(\delta^2): \underbrace{\beta^4}_{\text{Born}} \quad \underbrace{\alpha\beta^2}_{\text{included in NLO}_{4f}} \quad \underbrace{\alpha^2}_{\text{two-loop soft/hard}} \quad \underbrace{\alpha \times \alpha^2/\beta^2}_{\text{Coulomb-enhanced N}^3\text{LO}} \quad \underbrace{\alpha^4/\beta^4}_{\text{4th Coulomb}}$$

$\mathcal{O}(\alpha^2)$ corrections in EFT from soft/Coulomb factorization:

- Two-loop soft/collinear: ingredients known

  (Soft function: Belitzky 98; Collinear: Blümlein et al. 11)

- Hard corrections from two-loop $\mathcal{A}(e^-e^+ \to WW)|_{s=4M_W^2}$; estimate $C^{(2)} \approx (C^{(1)})^2 \Rightarrow \Delta\sigma \approx 0.06\%$; mandatory for FCC-ee/CEPC

- W-decay at $\mathcal{O}(\alpha^2, \alpha\alpha_s^2, \alpha_s^4)$ ($\mathcal{O}(\alpha^2)$ for Z decay: Dubovyk et al. 18)

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Is a full NNLO $e^- e^+ \to 4f$ calculation required?

Naive estimate:

$$\sigma^{\mathrm{4f}}_{\mathrm{NNLO}}(s) - \sigma^{(2)}_{\mathrm{EFT}}(s) \approx \frac{\alpha}{s_w^2}\left(\sigma^{\mathrm{4f}}_{\mathrm{NLO}}(s) - \sigma^{(1)}_{\mathrm{EFT}}(s)\right) = \sigma^{\mathrm{4f}}_{\mathrm{Born}}(s) \times (0.03 - 0.04)\%$$

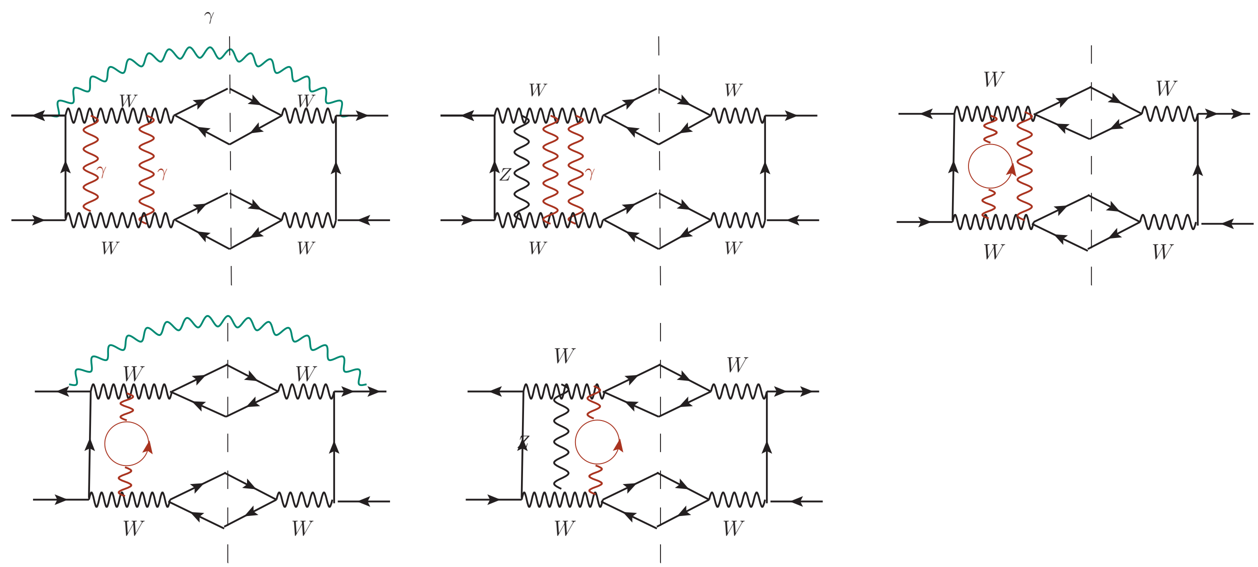
$\Rightarrow$ effects need to be controlled!

Leading $\mathcal{O}(\alpha^3)$ corrections

- $\delta^2 \sim \alpha \times \alpha^2/\beta^2$ corrections in NNLO$^{\text{EFT}}$:
  Combination of second Coulomb with NLO soft/hard; $V_{\text{Coul}}^{\text{NLO}}$
  (Most contributions follow from soft/Coulomb factorization; extension of Actis et al. 08)
- $\delta^{5/2} \sim \alpha^2 \times \alpha/\beta$ corrections:
  Combination of first Coulomb with NNLO soft/hard;
  $V_{\text{Coul}}^{\text{NLO}}$ with NLO soft/hard; $V_{\text{Coul}}^{\text{NNLO}}$
  (Can be computed once NNLO$^{\text{EFT}}$ is known; similar to N^{3}LO$_{\text{app}}$ for $pp \to t\bar{t}$)

α^3/v^2

α^3/v



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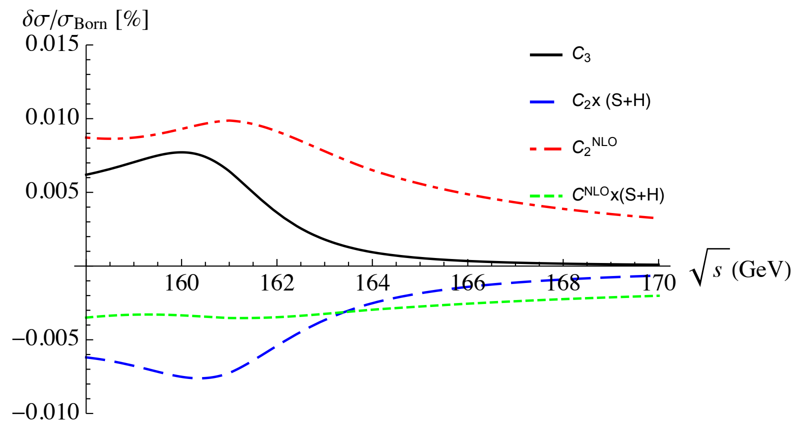
  $V_{\text{Coul}}^{\text{NLO}}$ with NLO soft/hard; $V_{\text{Coul}}^{\text{NNLO}}$
  (Can be computed once NNLO$^{\text{EFT}}$ is known; similar to N^{3}LO$_{\text{app}}$ for $pp \to t\bar{t}$)
- Known subset of corrections:

$$\Delta\sigma_{\alpha^3} \sim 0.01\%$$

$\Rightarrow$ relevant for FCC-ee



- **Threshold production** of heavy-particles
  - precision measurements of m_t, M_W at future e^-e^+ colliders
  - universal structure (soft/Coulomb corrections)
    all-order resummations possible
  - hadron colliders: threshold enhanced by parton luminosities
- **Predictions for $t\bar{t}$ hadroproduction**
  - NNLL resummation moderate effect;
    reduction of scale ambiguity to 3%
  - N^3LO expansion of partial N^3LL complementary to NNLL
    resummation (includes input beyond NNLL)
- **Prospects for WW threshold scan** at ILC/FCC-ee/CEPC
  - FCC-ee goal $\Delta\sigma_{WW} < 0.01\%$ requires
    NNLO corrections in threshold expansion; non-res NNLO;
    Coulomb-enhanced 3-loop corrections
  - Precision $\Delta M_W = (0.15 - 0.60)\text{MeV}$ appears feasible