The Top Quark Mass at Future Lepton Colliders:

Measurements using Radiative Events with a Matched Top Quark Pair Production Cross Section

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DESY Zeuthen Theory Seminar
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Outline

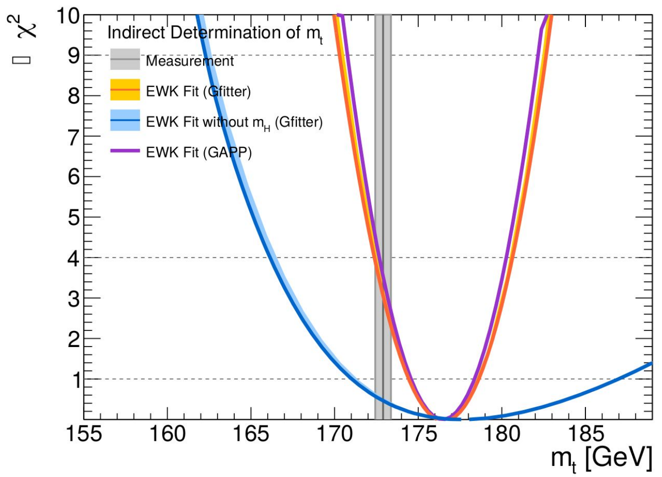
- Introduction - the Top Quark Mass
- Top Quark Mass Determination at Lepton Colliders
  - Overview
  - Threshold Scan
  - Radiative Events [Boronat, Fullana, Fuster, Gomis, Hoang, Mateu, Vos, AW]
- Top Quark Pair Production Cross Section from Threshold to Continuum
  - Threshold Region
  - Continuum Region
  - Mass Schemes at and above Threshold
  - Matching at $\mathrm{NNLL}_{\mathrm{threshold}}$ + $\mathrm{NNNLO}_{\mathrm{continuum}}$ [Dehnadi, Hoang, Mateu, Stahlhofen, AW]
- Measurement of the MSR Mass Running
  [Boronat, Fullana, Fuster, Gomis, Hoang, Mateu, Vos, AW]
- Conclusions

Introduction

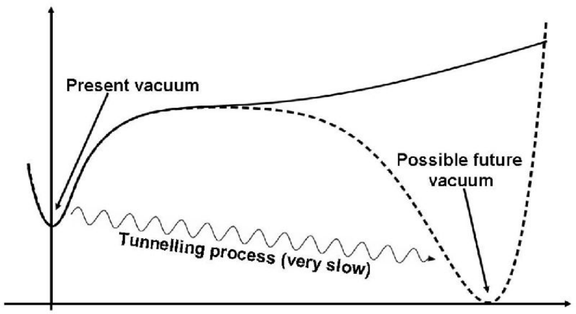
Why measure the Top Quark Mass?

- input parameter of the Standard Model
- input for global electroweak fits
- determination of electroweak vacuum stability

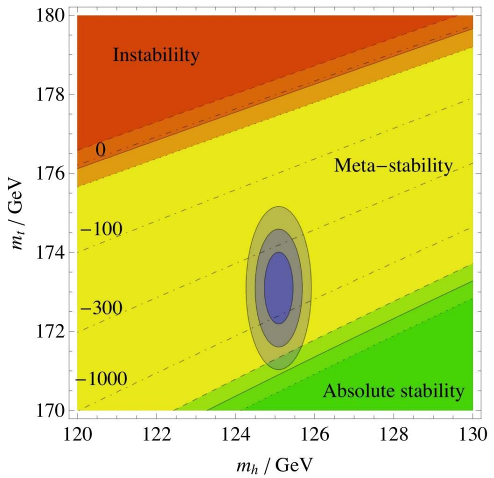
[J. Erler, M. Schott 2019]



top mass from EW fit vs. direct measurement



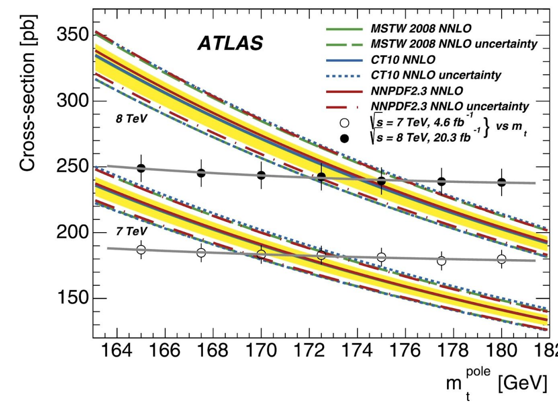
[S. Abel et al. 2008]



[S. Chigusa, T. Moroi, Y. Shoji 2018]

How to Measure the Top Quark Mass?

[ATLAS collaboration 2016, arXiv 1406.5375]

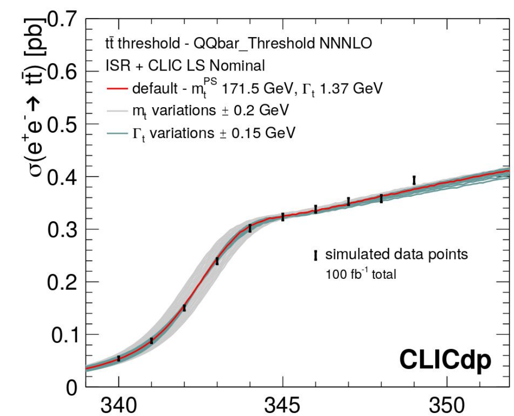


Hadron Colliders [PDG averages]

- direct reconstruction: 172.9 GeV ± 400 MeV (Monte Carlo mass)
- cross-section measurements: 173.1 GeV ± 900 MeV (pole mass)
- cross-section measurements: 160.0 GeV $^{+4800\text{ MeV}}_{-4300\text{ MeV}}$ (MS mass)

Future Lepton Colliders [Abramowicz et al. 2019]

- direct reconstruction: ± (50 - 100 MeV) (Monte Carlo mass)
- cross-section measurement: (threshold) ± (< 75 MeV) (short-distance mass)
- cross-section measurement: (radiative events) ± (105 - 150 MeV) (short-distance mass)



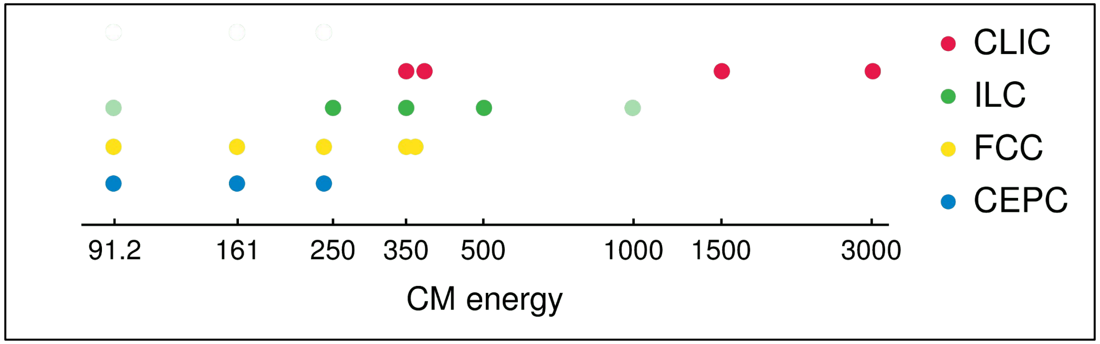
[Abramowicz et al. 2019]

Top Quark Mass Determination at Lepton Colliders

Overview

Lepton Colliders

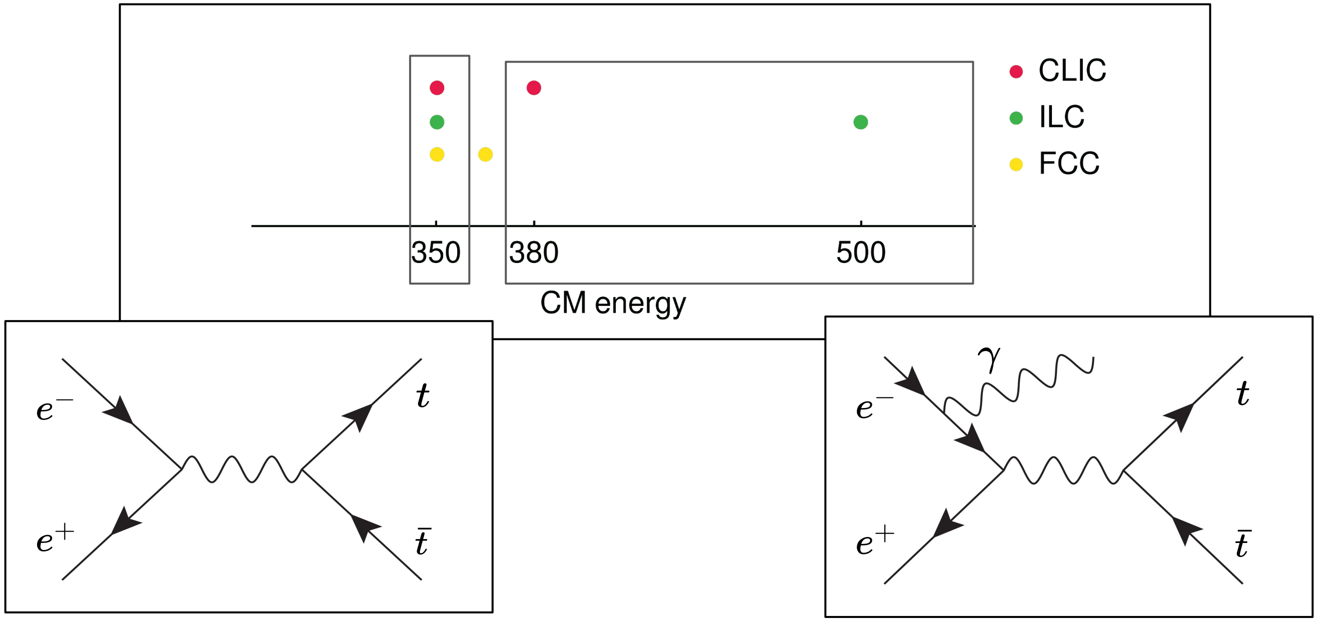
CLIC (CERN)	350, 380, 1500, 3000 GeV	[CLIC collaboration 2016]
ILC (Japan)	250, 350, 500 GeV (+ 91.2, 1000 GeV)	[LCC Sept. 2019]
FCC-ee (CERN)	91.2, 161, 240, 350, 365 GeV	[FCC collaboration 2019]
CEPC (China)	91.2, 161, 240 GeV	[CEPC study group 2018]



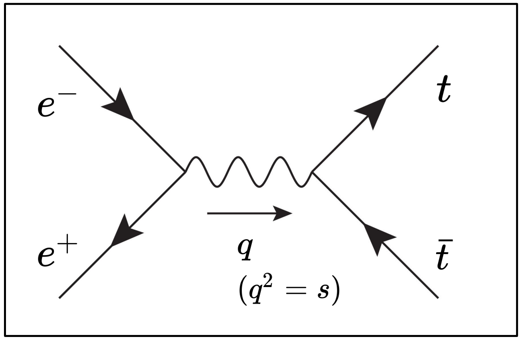
Overview

top quark mass measurements (main methods)

threshold scan	350 GeV	$\sigma(e^+e^- \to t\bar{t})$	< 75 MeV precision [Simon 2019]
radiative events	380, 500 GeV	$\sigma(e^+e^- \to t\bar{t}\gamma)$	~ 105 - 150 MeV precision [Boronat et al. 2019 - in preparation]
direct reconstruction	380, 500 GeV	invariant mass	~ 50 - 100 MeV precision [Abramowicz et al. 2019], [Seidel et al. 2013]



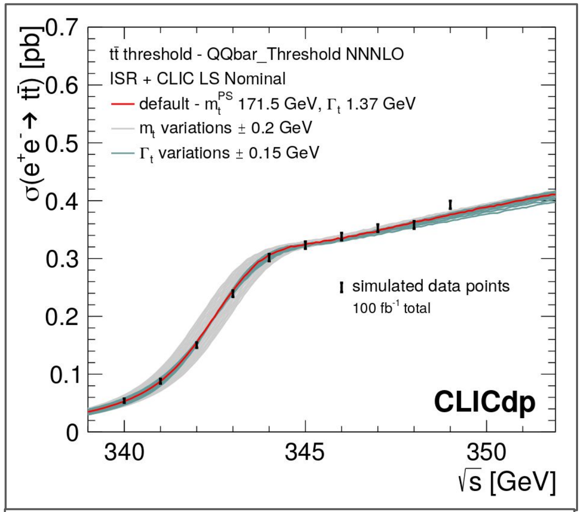
Threshold Scan



- top mass: precision < 75 MeV [Simon 2019]
  (now: $m_t^{\mathrm{MC}} = 172.9 \pm 0.4$ GeV [PDG])
- top width: precision < 100 MeV [Simon 2019]
  (now: $\Gamma_t = 1.42^{+0.19}_{-0.15}$ GeV [PDG])
- threshold also sensitive to top Yukawa coupling, strong coupling constant

uncertainties for top quark mass determination	
QCD scale variation	~ 40 MeV
parametric	~ 30 MeV (for $\Delta\alpha_s = 0.001$)
statistical	~ 20 MeV
systematic (experimental)	~ 25 - 50 MeV

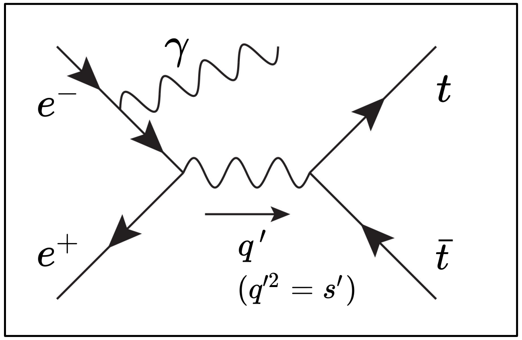
[Abramowicz et al. 2019]



threshold scan points

Radiative Events (380 GeV, 500 GeV)

[Boronat, Fullana, Fuster, Gomis, Hoang, Mateu, Vos, AW 2019 - to appear soon]



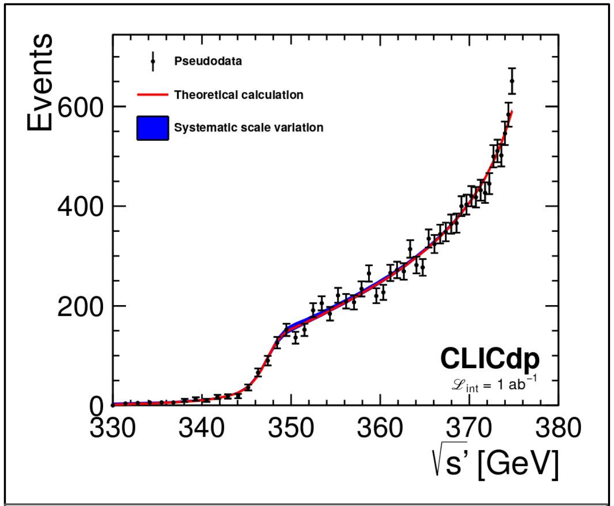
invariant mass of top quark pair: $$(q')^2 = s' = s\left(1 - \frac{2E_\gamma}{\sqrt{s}}\right)$$

factorization (in ISR approximation): $$\frac{d\sigma_{t\bar{t}\gamma}}{d\sqrt{s'}} = f(E_\gamma)\,\sigma_{t\bar{t}}(s')$$

→ E_γ can be measured with high precision

→ measurements of different E_γ give a scan over the pair production cross section

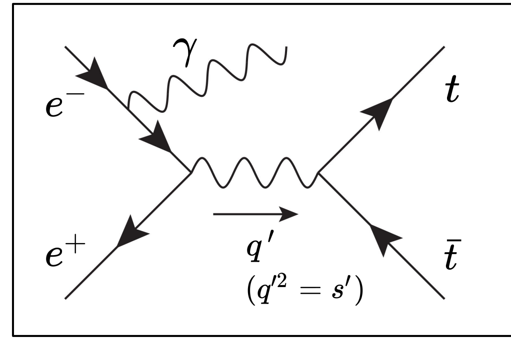
→ radiative return to threshold gives high mass sensitivity



measurement points

Radiative Events (380 GeV, 500 GeV)

[Boronat, Fullana, Fuster, Gomis, Hoang, Mateu, Vos, AW 2019 - to appear soon]



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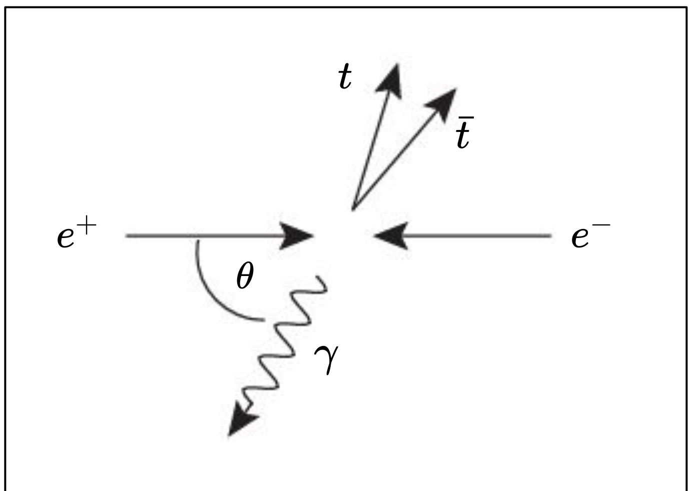
factorization (in ISR approximation):

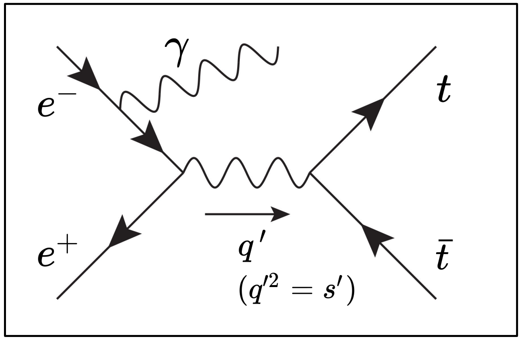
$$\frac{\mathrm{d}\sigma_{t\bar{t}\gamma}}{\mathrm{d}\cos\theta\,\mathrm{d}\sqrt{s'}} = \frac{\alpha_{\mathrm{em}}}{\pi\sqrt{s}}\, g(x,\theta)\,\sigma_{t\bar{t}}(s') + \mathcal{O}(\alpha_{\mathrm{em}}^2)\,, \quad g(x,\theta) = \frac{2\sqrt{(1-2x)}}{x\sin^2\theta}\left[1 - 2x + (1+\cos^2\theta)x^2\right], \quad x = \frac{E_\gamma}{\sqrt{s}}$$



- large photon energy $E_\gamma > 5$ GeV
- θ integrated from 8° to 172°
- highest mass sensitivity for collinear top quarks
  - $s' \sim 4\,m_t^2$
  - radiative return to threshold

Radiative Events (380 GeV, 500 GeV)

[Boronat, Fullana, Fuster, Gomis, Hoang, Mateu, Vos, AW 2019 - to appear soon]



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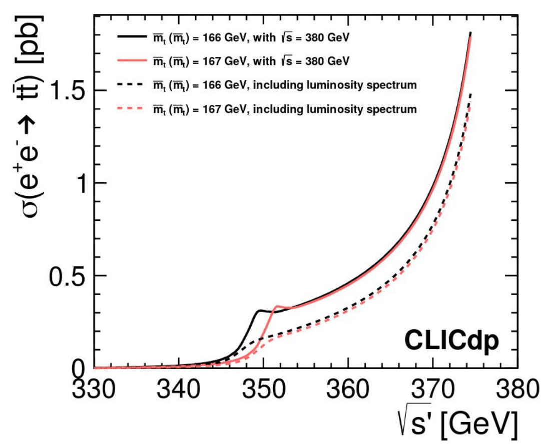

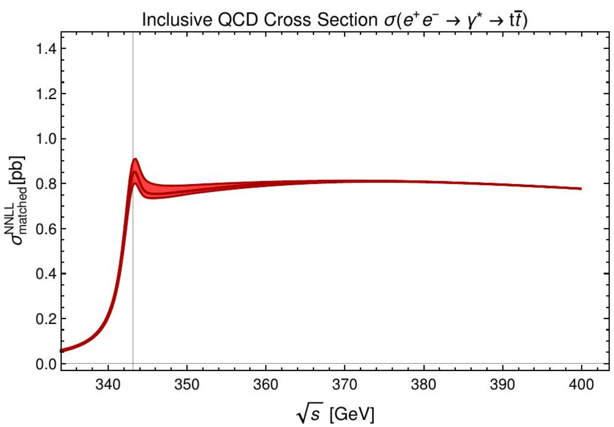

Radiative Events (380 GeV, 500 GeV)

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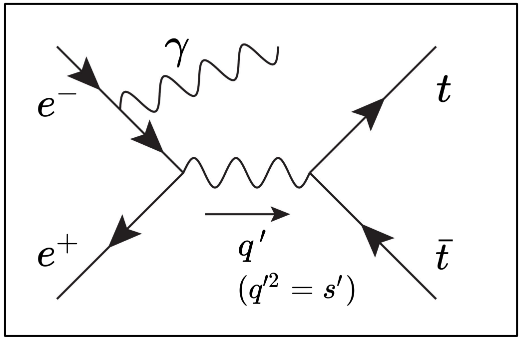

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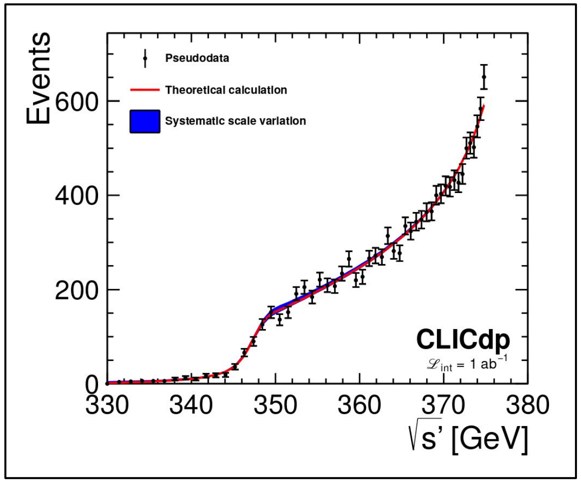
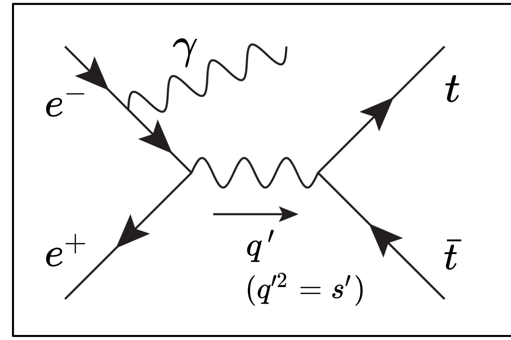


cms energy	CLIC, $\sqrt{s} = 380$ GeV		ILC, $\sqrt{s} = 500$ GeV	
luminosity [fb^{-1}]	500	1000	500	4000
statistical	140 MeV	90 MeV	350 MeV	110 MeV
theory	46 MeV		55 MeV	
lum. spectrum	20 MeV		20 MeV	
photon response	16 MeV		85 MeV	
total	150 MeV	110 MeV	360 MeV	150 MeV

uncertainties for top quark mass

Radiative Events (380 GeV, 500 GeV)

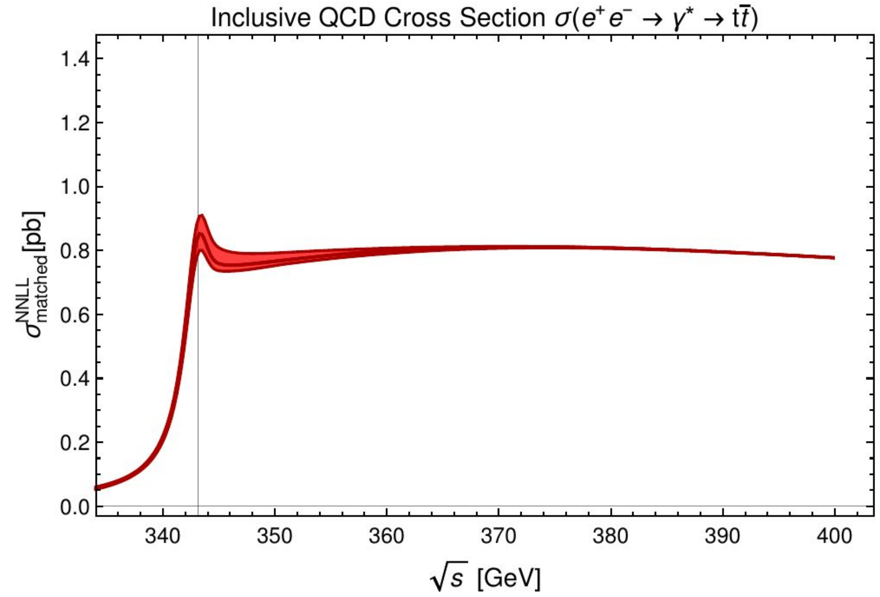
[Boronat, Fullana, Fuster, Gomis, Hoang, Mateu, Vos, AW 2019 - to appear soon]



currently missing / possible improvements:

- parametric uncertainty from α_s
- final state radiation
- including forward calorimeters (higher statistics)
- $t\bar{t}$ cross section: axial vector current
- $t\bar{t}$ cross section: higher order electroweak effects

Inclusive Top Quark Pair Production Cross Section from Threshold to Continuum



Inclusive Cross Section - Theory Overview

<u>non-relativistic QCD</u>
(NRQCD)

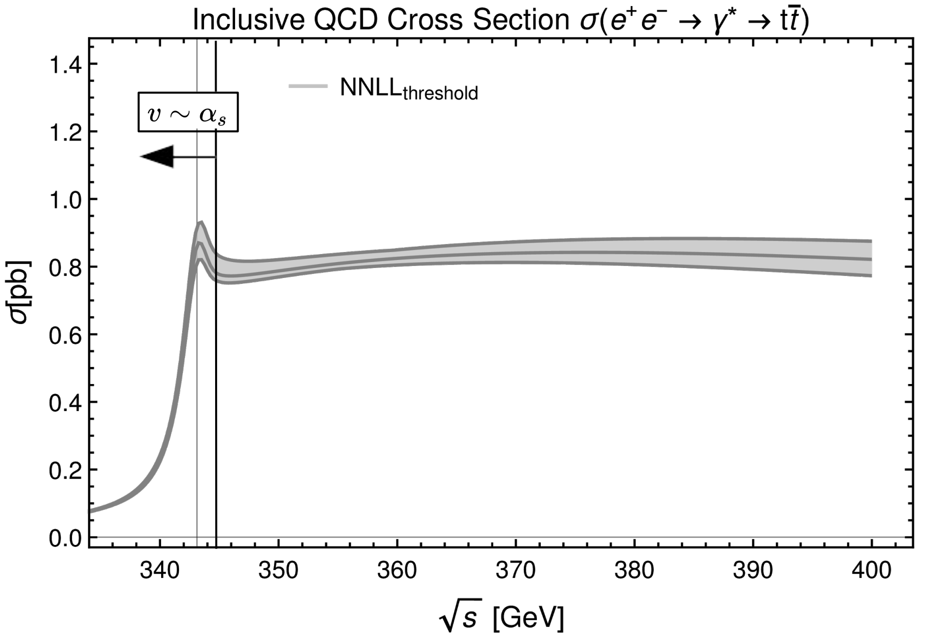
$\text{NNLL}_{\text{threshold}}$
[Hoang, Stahlhofen 2013]

- QCD + LO electroweak (double-resonant)
- vNRQCD
- resummation of velocity-logarithms
- complete for all practical purposes
- 1S mass

$\text{NNNLO}_{\text{threshold}}$
[Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser 2015]
[Beneke, Kiyo, Maier, Piclum 2016]

- QCD + EW + Higgs
- pNRQCD
- PS mass

threshold



Inclusive Cross Section - Theory Overview

<u>non-relativistic QCD</u> (NRQCD)

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NNNLO$_{\text{threshold}}$

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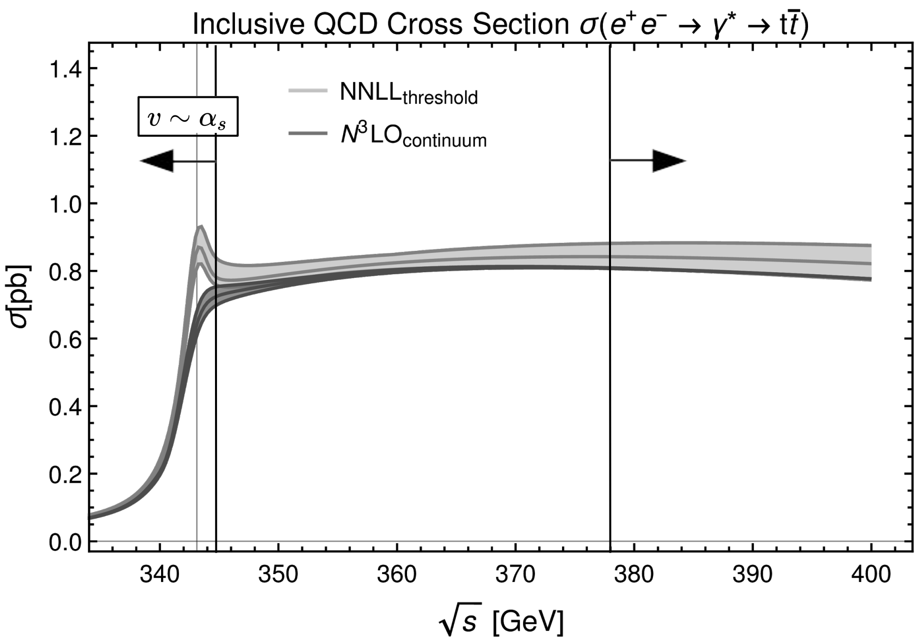
[Beneke, Kiyo, Maier, Piclum 2016]

- QCD + EW + Higgs
- pNRQCD
- PS mass

threshold

continuum



<u>full QCD</u>

NNNLO$_{\text{continuum}}$

[Hoang, Mateu, Zebarjad 2009]
[Kiyo, Maier, Maierhofer, Marquard 2009]

- QCD corrections

Inclusive Cross Section - Theory Overview

non-relativistic QCD (NRQCD)

NNLL$_{threshold}$

[Hoang, Stahlhofen 2013]

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[Beneke, Kiyo, Marquard, Penin, Piclum, Steinhauser 2015]
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- QCD + EW + Higgs
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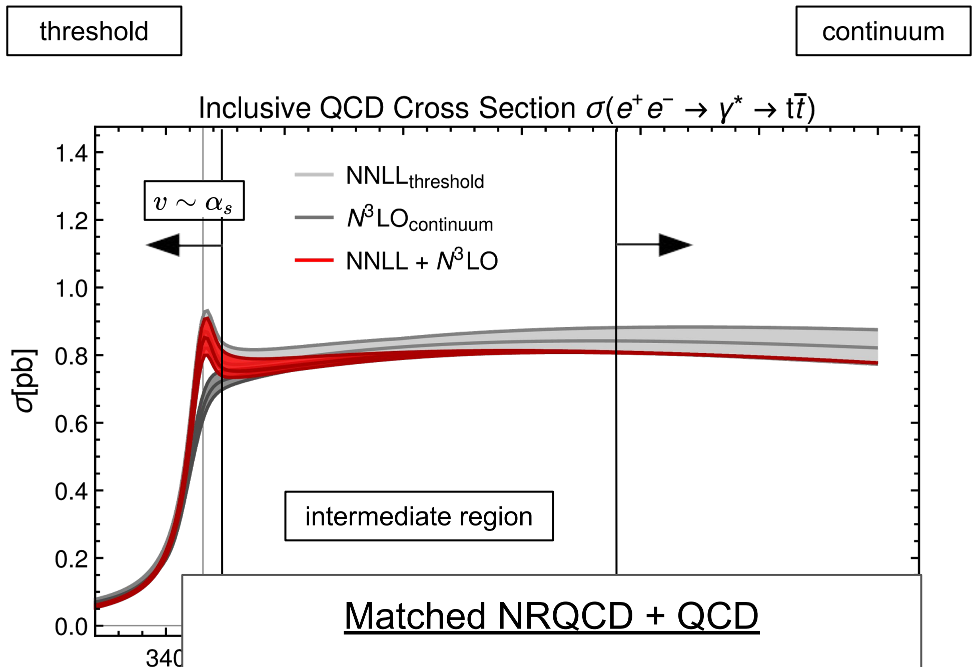


Matched NRQCD + QCD

LL$_{threshold}$ + NLO$_{continuum}$ (differential cross section)

[Bach et al. 2017]

NNLL$_{threshold}$ + NNNLO$_{continuum}$ this talk

- photon-induced cross section
- QCD + LO EW at threshold
- 1S + MSR mass scheme

full QCD

NNNLO$_{continuum}$

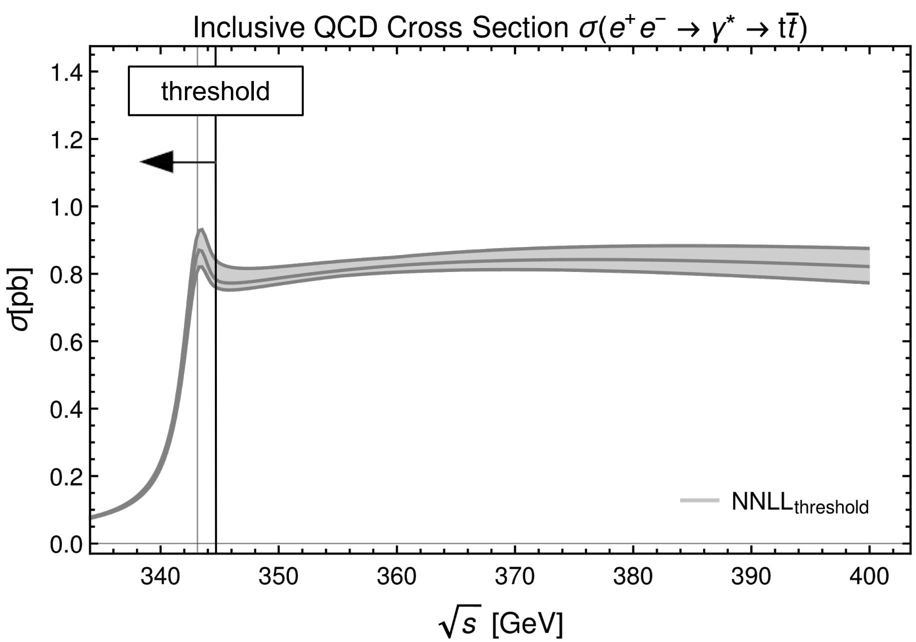
[Hoang, Mateu, Zebarjad 2009]

[Kiyo, Maier, Maierhofer, Marquard 2009]

- QCD corrections

Inclusive Cross Section -

Threshold



Threshold - Coulomb Resummation

At threshold: $v \sim \alpha_s,\, \alpha_s \log(v) \sim 1$

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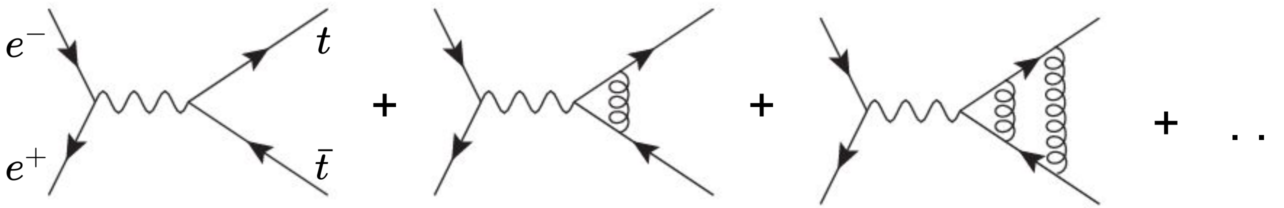
At threshold: $\boxed{v \sim \alpha_s,}\, \alpha_s \log(v) \sim 1$

→ ladder diagrams are enhanced



1 $\qquad \dfrac{\alpha_s}{v} \qquad \left(\dfrac{\alpha_s}{v}\right)^2$

→ resummation of ladder diagrams with Schrödinger equation

→ numerical solution with Toppik [Hoang, Teubner 1999]

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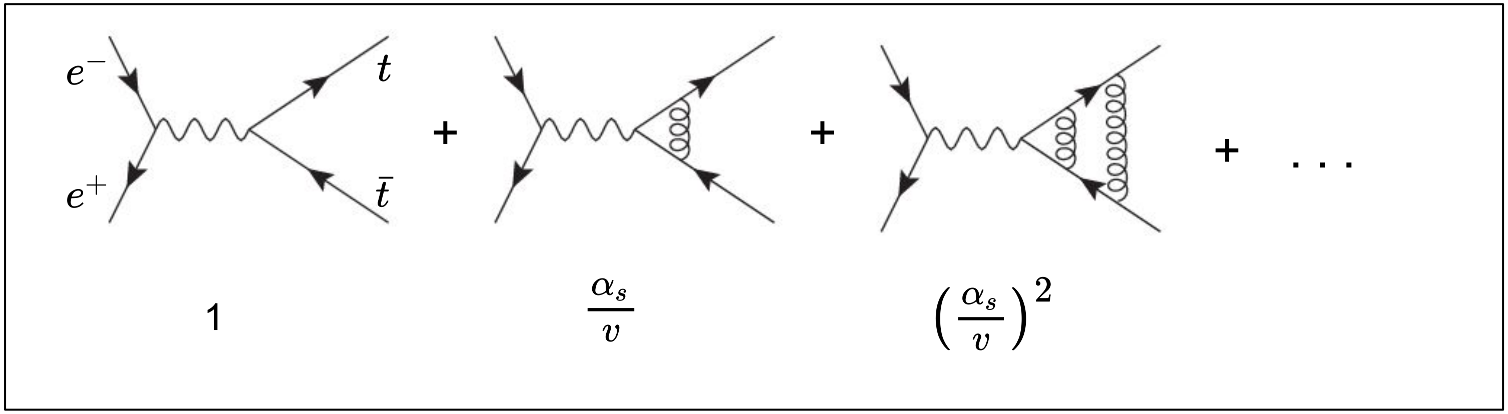
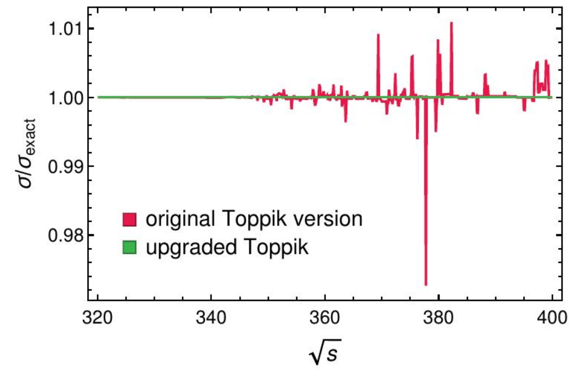


→ resummation of ladder diagrams with Schrödinger equation

→ numerical solution with Toppik [Hoang, Teubner 1999]

→ upgraded version of Toppik

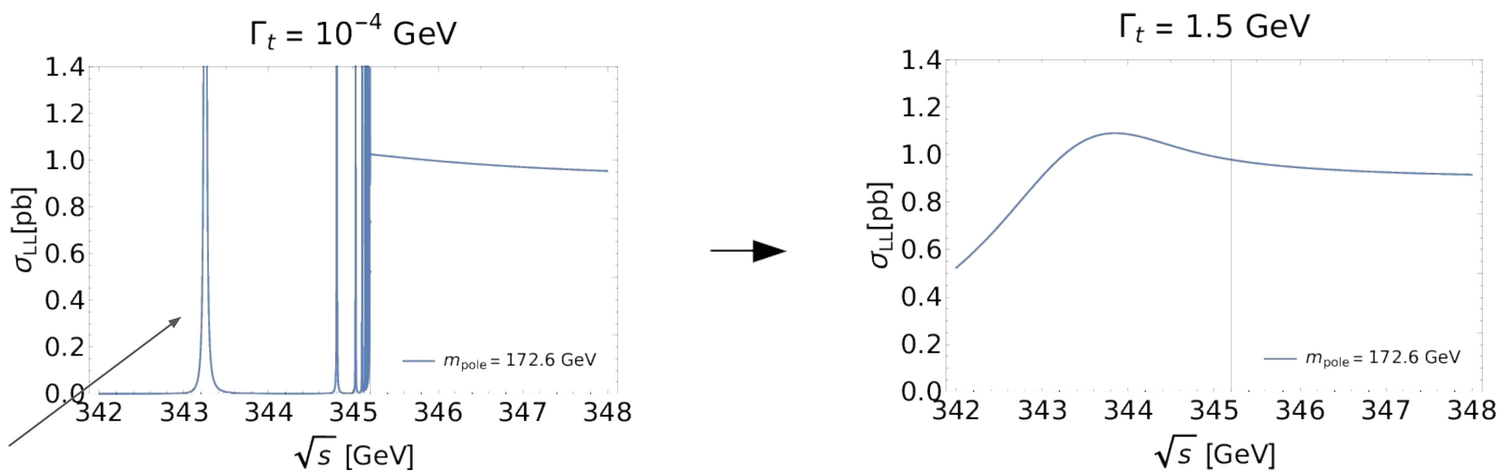
- precision now 10^{-4}
- 10 - 50 times faster than original version



Threshold - Coulomb Resummation

At threshold: $\boxed{v \sim \alpha_s,}\,\alpha_s \log(v) \sim 1$

- resummation of ladder diagrams gives toponium resonances
- large top quark width smears out the top quark resonances



1S resonance

→ inclusion of width by the replacement $\sqrt{s} + i\epsilon \to \sqrt{s} + i\,\Gamma_t$ (gives LO electroweak contributions at threshold) [Fadin, Khoze 1987]

Threshold - Large Logarithms

At threshold: $v \sim \alpha_s, \boxed{\alpha_s \log(v) \sim 1}$

→ resummation with vNRQCD (velocity non-relativistic QCD) [Hoang, Stahlhofen 2013]

Contributions to the cross section at threshold :

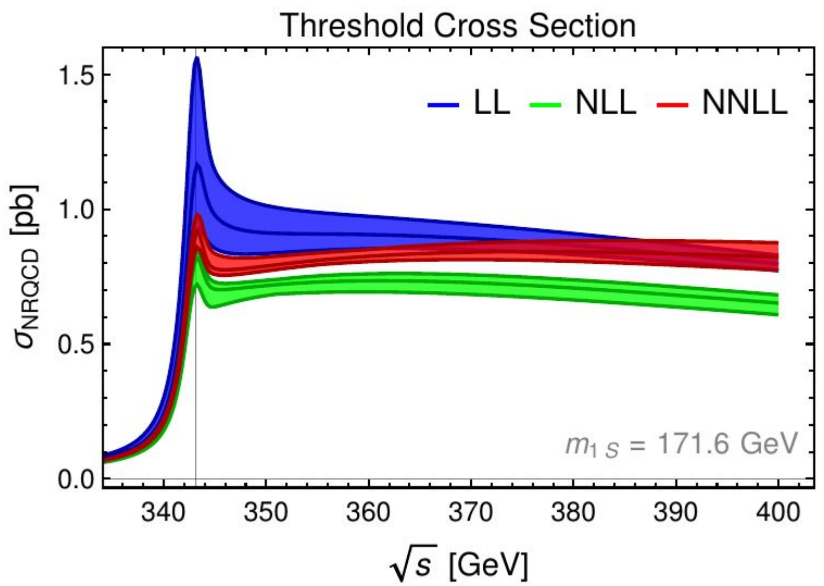
$$\begin{aligned} \sigma_{\mathrm{NRQCD}}^{\mathrm{NNLL}} = v &\sum_{n,m} \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \log v)^m && \mathrm{LL} \\ + v^2 &\sum_{n,m} \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \log v)^m && \mathrm{NLL} \\ + v^3 &\sum_{n,m} \left(\frac{\alpha_s}{v}\right)^n (\alpha_s \log v)^m && \mathrm{NNLL} \end{aligned}$$

Threshold - Large Logarithms

At threshold: $v \sim \alpha_s, \boxed{\alpha_s \log(v) \sim 1}$

→ resummation with vNRQCD (velocity non-relativistic QCD) [Hoang, Stahlhofen 2013]

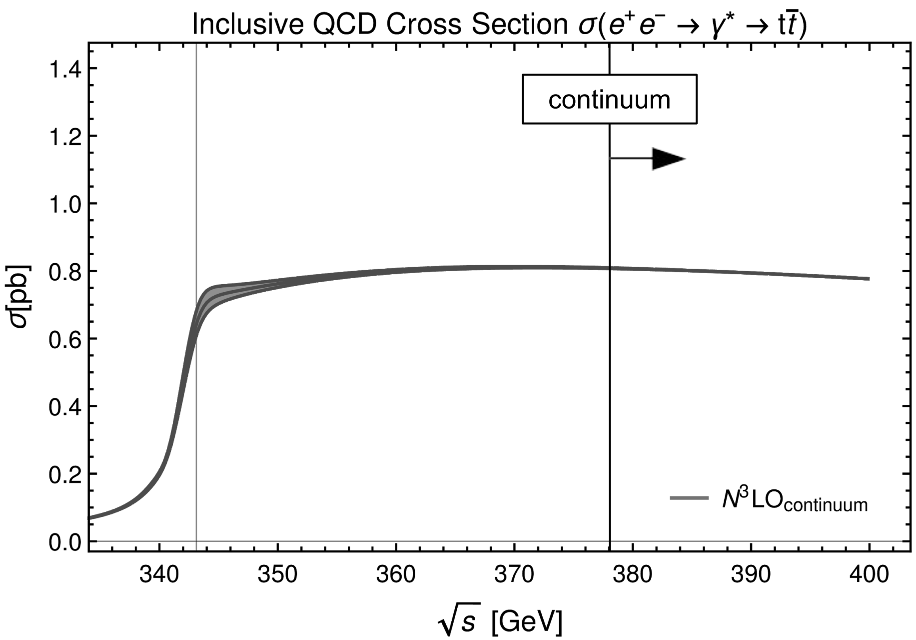
Contributions to the cross section at threshold :



(error bands from variation of renormalization scales)

Inclusive Cross Section -

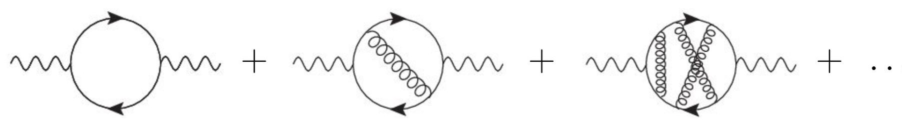
Continuum



Continuum Cross Section

The inclusive cross section is related to the vacuum polarization by the optical theorem:

$$\sigma_{t\bar{t}} = \frac{(4\pi\alpha)^2}{s} Q_t^2 \,\mathrm{Im}\Big[$$



$$\Big]$$

$$= \frac{(4\pi\alpha)^2}{s} Q_t^2 \,\mathrm{Im}\left[\Pi(\sqrt{s} + i\Gamma_t)\right]$$

In the continuum:

$$\sigma_{\mathrm{QCD}}^{\mathrm{N^3LO}} = \frac{(4\pi\alpha)^2}{s} Q_t^2 \cdot \mathrm{Im}\left[\Pi^{(0)} + \alpha_s\,\Pi^{(1)} + \alpha_s^2\,\Pi^{(2)} + \alpha_s^3\,\Pi^{(3)}\right]$$

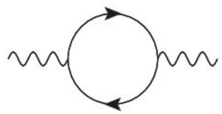
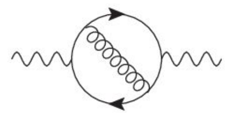
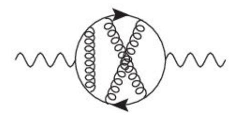
$\Pi^{(0)}$, $\Pi^{(1)}$ ... known analytically

$\Pi^{(2)}$, $\Pi^{(3)}$ … reconstructed with Padé approximations [Hoang, Mateu, Zebarjad 2009] [Kiyo, Maier, Maierhofer, Marquard 2009]

(validity of Padé approximations for $\Pi^{(2)}$ shown by comparison to exact numerical result in [Maier, Marquard 2017])

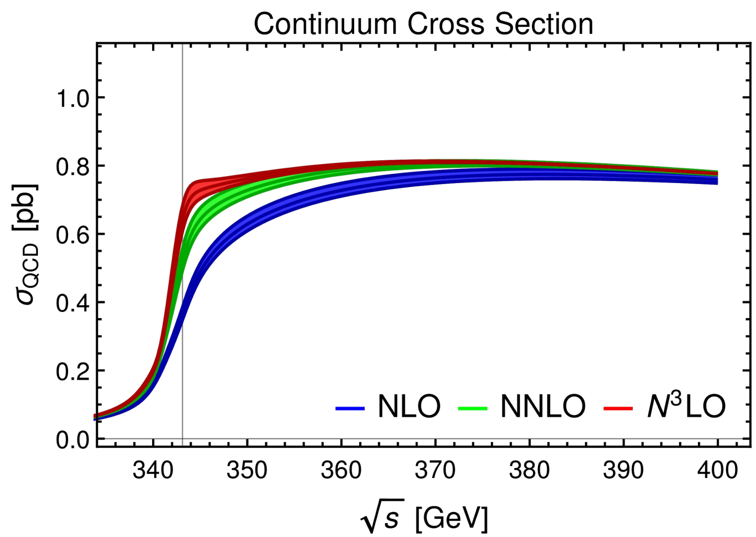
Continuum Cross Section

The inclusive cross section is related to the vacuum polarization by the optical theorem:

$$\sigma_{t\bar{t}} = \frac{(4\pi\alpha)^2}{s} Q_t^2 \, \mathrm{Im}\Big[$$  $$+$$  $$+$$  $$+ \; \ldots \Big]$$

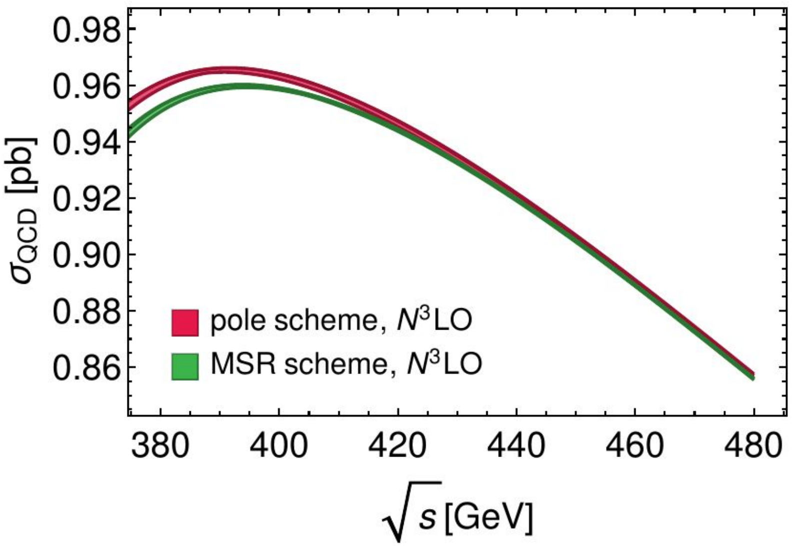
$$= \frac{(4\pi\alpha)^2}{s} Q_t^2 \, \mathrm{Im}\left[\Pi(\sqrt{s} + i\Gamma_t)\right]$$

In the continuum:

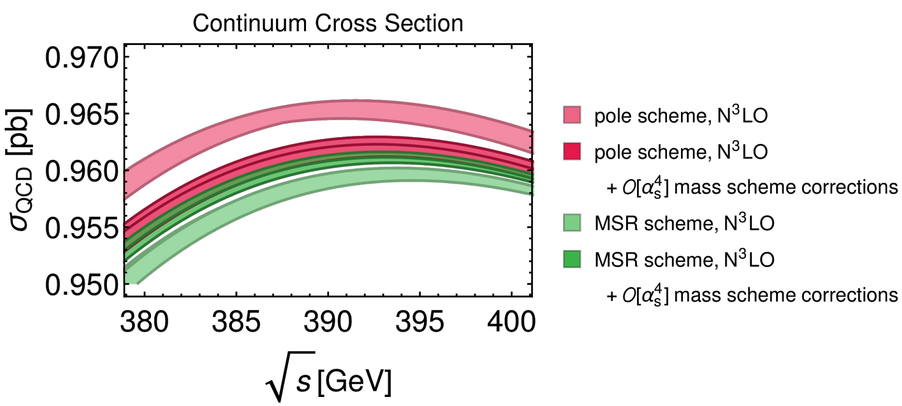


Theory Error for NNNLO$_{\text{continuum}}$

the cross section at NNNLO$_{\text{continuum}}$ shows a difference between the pole scheme and the MSR scheme:



- cross section in the pole scheme and the MSR scheme are incompatible (error bands do not overlap)
- scale variation seems to underestimate the error
- difference corresponds to 1 GeV difference in the top quark mass



- (known) $\mathcal{O}(\alpha_s^4)$ mass scheme corrections seem to favor MSR mass scheme

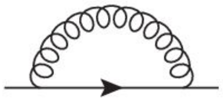
Inclusive Cross Section -

Mass Schemes

- pole mass scheme - renormalon
- 1S mass scheme - for the threshold
- $\overline{\text{MS}}$ mass scheme - for the continuum
- MSR mass scheme - for all regions

Mass Schemes - Pole Mass

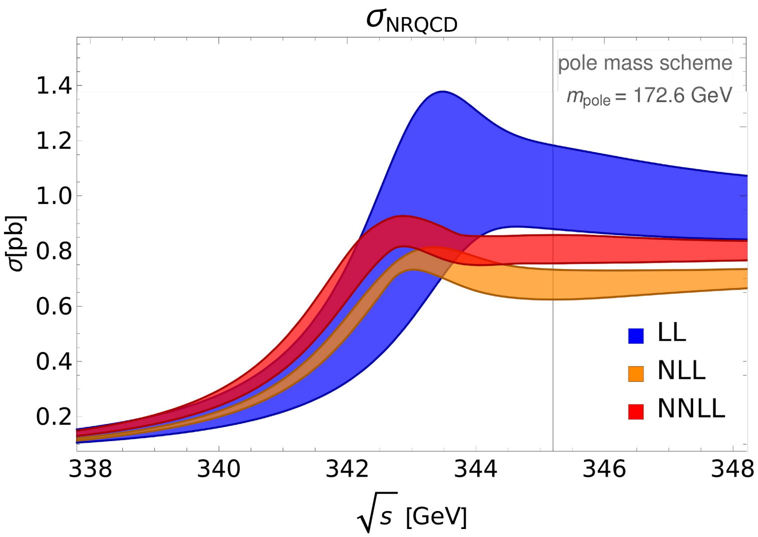
Full propagator: $$S_F^0 = \frac{i}{\not{p} - m_0 + \Sigma(\not{p}, m_0)} \, ,$$

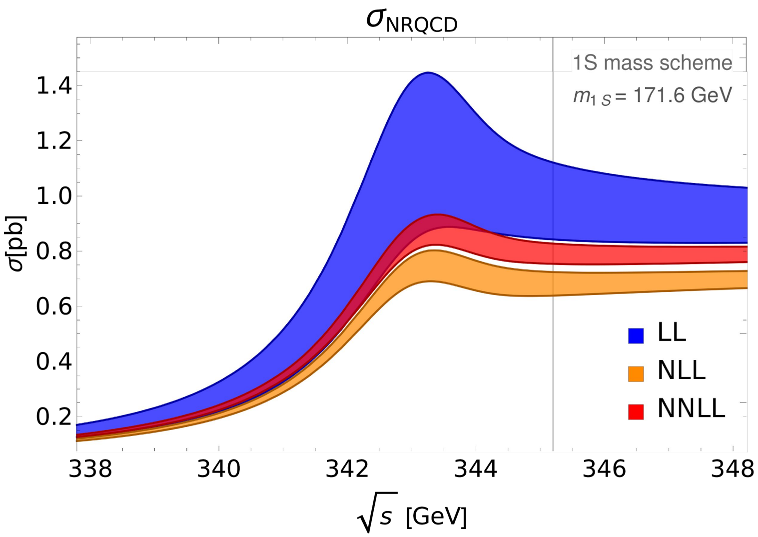
$$\Sigma(\not{p}, m_0) =$$  $$+ \ldots$$

Pole mass: $$\not{p} - m_0 + \Sigma(\not{p}, m_0)\big|_{p^2 = m_{\text{pole}}^2} = 0$$

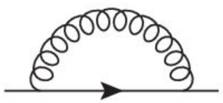
→ pole mass renormalon leads to bad convergence of the cross section already at lower orders

Renormalon at threshold:





Mass Schemes - $\overline{\text{MS}}$ Mass

Full propagator: $$S_F = \frac{i}{\not{p} - \overline{m}(\mu) + \Sigma_{\text{finite}}(\not{p}, \overline{m}(\mu))}\,, \qquad \Sigma(\not{p}, m_0) =$$  $+ \ldots$

Conversion:

$$m_{pole} = \overline{m} + \overline{m} \sum_{n=1}^{\infty} a_n(n_l, n_h)\, \alpha_s(\overline{m})^n$$

$$= \overline{m} + \overline{m}\, \alpha_s\, a_1 + \ldots \qquad \left(\overline{m} = \overline{m}^{(n_l+1)}(\overline{m}^{(n_l+1)}) \right)$$

$\sim mv$

→ works only in the continuum

Breaking of non-relativistic power counting in the MS scheme:

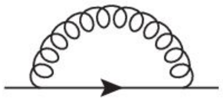
$$v_{\text{pole}} = \sqrt{\frac{\sqrt{s} - 2\, m_{\text{pole}}}{m_{\text{pole}}}}$$

$$= \sqrt{\frac{\sqrt{s} - 2\,(\overline{m} + \overline{m}\, a_1\, \alpha_s)}{\overline{m} + \overline{m}\, a_1\, \alpha_s}}$$

$$= v_{\overline{\text{MS}}} - a_1 \left(\frac{\alpha_s}{v_{\overline{\text{MS}}}}\right) \left(1 + \frac{1}{2} v_{\overline{\text{MS}}}^2\right) + a_1^2 \left(\frac{\alpha_s^2}{v_{\overline{\text{MS}}}^3}\right) \left(-\frac{1}{2} + \frac{1}{2} v_{\overline{\text{MS}}}^2 + \frac{3}{8}\, v_{\overline{\text{MS}}}^4\right) + \mathcal{O}(\alpha_s^3)$$

At threshold: $\sim \alpha_s$ $\quad \sim \alpha_s^0$ $\quad \sim \alpha_s^{-1}$

Mass Schemes - $\overline{\text{MS}}$ Mass

Full propagator: $$S_F = \frac{i}{\not{p} - \overline{m}(\mu) + \Sigma_{\text{finite}}(\not{p},\, \overline{m}(\mu))}\,, \qquad \Sigma(\not{p},\, m_0) =$$  $+\ \dots$

Conversion:
$$m_{pole} = \overline{m} + \overline{m}\sum_{n=1}^{\infty} a_n(n_l, n_h)\,\alpha_s(\overline{m})^n$$
$$= \overline{m} + \underbrace{\overline{m}\,\alpha_s}_{\sim\, mv}\,a_1 + \dots \qquad \left(\,\overline{m} = \overline{m}^{(n_l+1)}(\overline{m}^{(n_l+1)})\,\right)$$

➞ works only in the continuum

Breaking of non-relativistic power counting in the MS scheme:

$$v_{\text{pole}} = \sqrt{\frac{\sqrt{s} - 2\,m_{\text{pole}}}{m_{\text{pole}}}}$$
$$= \sqrt{\frac{\sqrt{s} - 2\,(\overline{m} + \overline{m}\,a_1\,\alpha_s)}{\overline{m} + \overline{m}\,a_1\,\alpha_s}}$$
$$= v_{\overline{\text{MS}}}\left(\left(\frac{\alpha_s}{v_{\overline{\text{MS}}}^2}\right)^0 - a_1\left(\frac{\alpha_s}{v_{\overline{\text{MS}}}^2}\right)^1\left(1 + \frac{1}{2}v_{\overline{\text{MS}}}^2\right) + a_1^2\left(\frac{\alpha_s}{v_{\overline{\text{MS}}}^2}\right)^2\left(-\frac{1}{2} + \frac{1}{2}v_{\overline{\text{MS}}}^2 + \frac{3}{8}\,v_{\overline{\text{MS}}}^4\right) + \mathcal{O}\left(\left(\alpha_s/v_{\overline{\text{MS}}}^2\right)^3\right)\right)$$

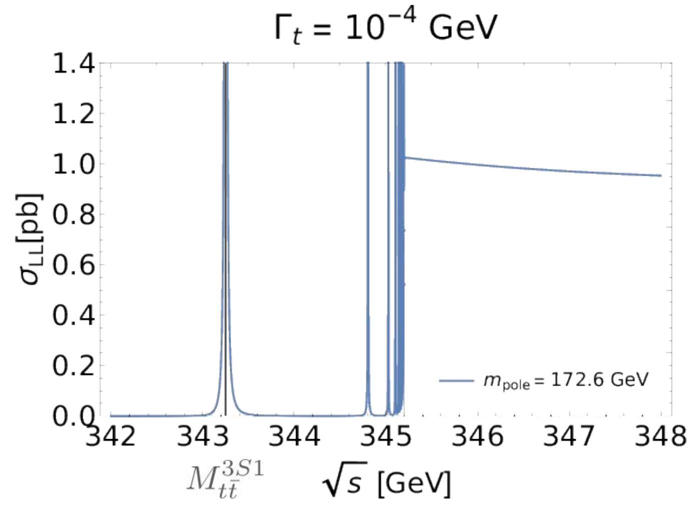
In the intermediate region: power counting breaking for $v_{\overline{\text{MS}}}^2 \lesssim \alpha_s$

Mass Schemes - 1S Mass

[Hoang, Ligeti, Manohar 1998]

Mass of 1S resonance: $M_{t\bar{t}}^{3S1} = E_{\rm bin} + 2\,m_{\rm pole}$

1S mass: $$m_{\rm 1S} = \frac{1}{2} M_{tt}^{3S1} = m_{\rm pole} + \frac{1}{2} E_{\rm bin}$$



other low-scale short-distance mass schemes:
PS mass [Beneke 1998] , RS mass [Pineda 2001] , kinetic mass [Czarnecki, Melnikov, Uraltsev 1998]

Mass Schemes - 1S Mass

[Hoang, Ligeti, Manohar 1998]

Mass of 1S resonance: $M_{t\bar{t}}^{3S1} = E_{\text{bin}} + 2\, m_{\text{pole}}$

1S mass: $$m_{1\text{S}} = \frac{1}{2} M_{tt}^{3S1} = m_{\text{pole}} + \frac{1}{2} E_{\text{bin}}$$

Conversion: $$\begin{aligned} m_{1S} &= m_{pole} + (C_F\, \alpha_s(\mu) m_{pole}) \sum_{n=1}^{\infty} \sum_{k=0}^{n-1} c_{n,k} \alpha_s(\mu)^n \log\left(\frac{\mu}{C_F\, \alpha_s(\mu) m_{pole}}\right) \\ &= m_{pole} - \underbrace{\frac{2}{9} \alpha_s^2\, m_{pole}}_{\sim\, mv^2} + \ldots \end{aligned}$$

no breaking of the non-relativistic power counting at threshold:

$$\begin{aligned} v_{\text{pole}} &= \sqrt{\frac{\sqrt{s} - 2\, m_{\text{pole}}}{m_{\text{pole}}}} \\ &= \sqrt{\frac{\sqrt{s} - 2\,(m_{1\text{S}} - m_{1\text{S}}\, c_{1,0}\, \alpha_s^2)}{m_{1\text{S}} - m_{1\text{S}}\, c_{1,0}\, \alpha_s^2}} \\ &= \underbrace{v_{1\text{S}}}_{\sim\, \alpha_s} + c_{1,0} \underbrace{\left(\frac{\alpha_s^2}{v_{1\text{S}}}\right)}_{\sim\, \alpha_s} \left(1 + \frac{v_{1\text{S}}^2}{2}\right) + c_{1,0}^2 \underbrace{\left(\frac{\alpha_s^4}{v_{1\text{S}}^3}\right)}_{\sim\, \alpha_s} \left(-\frac{1}{2} + \frac{1}{2} v_{1\text{S}}^2 + \frac{3}{8} v_{1\text{S}}^4\right) + \mathcal{O}(\alpha_s^6) \end{aligned}$$

Mass Schemes - MSR Mass

[Hoang, Jain, Scimemi, Stewart 2008], [Hoang, Jain, Lepenik, Mateu, Preisser, Scimemi, Stewart 2017]

Conversion:
$$m_{pole} = \overline{m} + \overline{m} \sum_{n=1}^{\infty} a_n \, \alpha_s(\overline{m})^n = \overline{m} + \overline{m}\, \alpha_s \, a_1 + \ldots$$
$$m_{pole} = m_{MSR}(R) + R \sum_{n=1}^{\infty} a_n \, \alpha_s(R)^n = m_{MSR}(R) + R\, \alpha_s \, a_1 + \ldots$$

→ no breaking of the non-relativistic power counting at threshold

$$\begin{aligned} v_{\text{pole}} &= \sqrt{\frac{\sqrt{s} - 2\, m_{\text{pole}}}{m_{\text{pole}}}} \\ &= \sqrt{\frac{\sqrt{s} - 2\,(m_{\text{MSR}} + R\, a_1\, \alpha_s)}{m_{\text{MSR}} + R\, a_1\, \alpha_s}} \\ &= \underbrace{v_{\text{MSR}}}_{\sim \alpha_s} - a_1 \underbrace{\alpha_s}_{\sim \alpha_s} \left(\frac{R}{m_{\text{MSR}} v_{\text{MSR}}}\right) \left(1 + \frac{v_{\text{MSR}}^2}{2}\right) + a_1^2 \underbrace{\frac{\alpha_s^2}{v_{\text{MSR}}}}_{\sim \alpha_s} \left(\frac{R}{m_{\text{MSR}} v_{\text{MSR}}}\right)^2 \left(-\frac{1}{2} + \frac{1}{2} v_{\text{MSR}}^2 + \frac{3}{8} v_{\text{MSR}}^4\right) + \mathcal{O}(\alpha_s^3) \end{aligned}$$

Mass Schemes - MSR Mass

[Hoang, Jain, Scimemi, Stewart 2008], [Hoang, Jain, Lepenik, Mateu, Preisser, Scimemi, Stewart 2017]

Conversion: $$m_{pole} = \overline{m} + \overline{m} \sum_{n=1}^{\infty} a_n \ \alpha_s(\overline{m})^n = \overline{m} + \overline{m} \ \alpha_s \, a_1 + \ldots$$

$$m_{pole} = m_{MSR}(R) + R \sum_{n=1}^{\infty} a_n \ \alpha_s(R)^n = m_{MSR}(R) + R \ \alpha_s \, a_1 + \ldots$$

➞ no breaking of the non-relativistic power counting at threshold

$$\begin{aligned} v_{\rm pole} &= \sqrt{\frac{\sqrt{s} - 2\, m_{\rm pole}}{m_{\rm pole}}} \\ &= \sqrt{\frac{\sqrt{s} - 2\,(m_{\rm MSR} + R\, a_1\, \alpha_s)}{m_{\rm MSR} + R\, a_1\, \alpha_s}} \\ &= v_{\rm MSR} - a_1\, \alpha_s \left(\frac{R}{m_{\rm MSR} v_{\rm MSR}}\right) \left(1 + \frac{v_{\rm MSR}^2}{2}\right) + a_1^2 \, \frac{\alpha_s^2}{v_{\rm MSR}} \left(\frac{R}{m_{\rm MSR} v_{\rm MSR}}\right)^2 \left(-\frac{1}{2} + \frac{1}{2} v_{\rm MSR}^2 + \frac{3}{8} v_{\rm MSR}^4\right) + \mathcal{O}(\alpha_s^3) \end{aligned}$$

➞ no power counting breaking for $R \sim mv$

Mass Schemes - MSR Mass

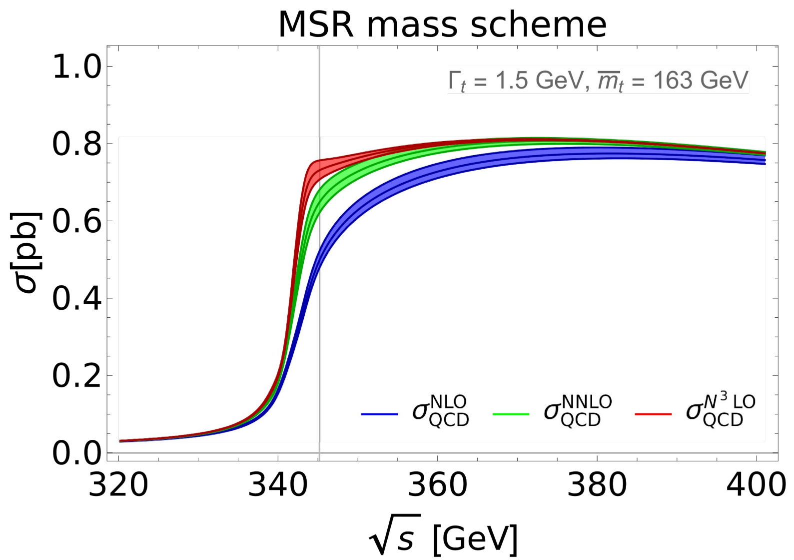
[Hoang, Jain, Scimemi, Stewart 2008], [Hoang, Jain, Lepenik, Mateu, Preisser, Scimemi, Stewart 2017]

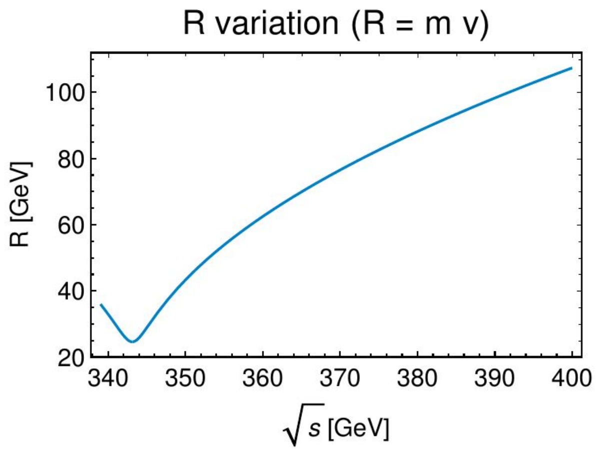
Conversion: $$m_{pole} = \overline{m} + \overline{m} \sum_{n=1}^{\infty} a_n \, \alpha_s(\overline{m})^n = \overline{m} + \overline{m} \, \alpha_s \, a_1 + \ldots$$

$$m_{pole} = m_{MSR}(R) + R \sum_{n=1}^{\infty} a_n \, \alpha_s(R)^n = m_{MSR}(R) + R \, \alpha_s \, a_1 + \ldots$$

➞ no breaking of the non-relativistic power counting at threshold

➞ improves convergence of the continuum cross section in the intermediate region:





Mass Schemes - MSR Mass

[Hoang, Jain, Scimemi, Stewart 2008], [Hoang, Jain, Lepenik, Mateu, Preisser, Scimemi, Stewart 2017]

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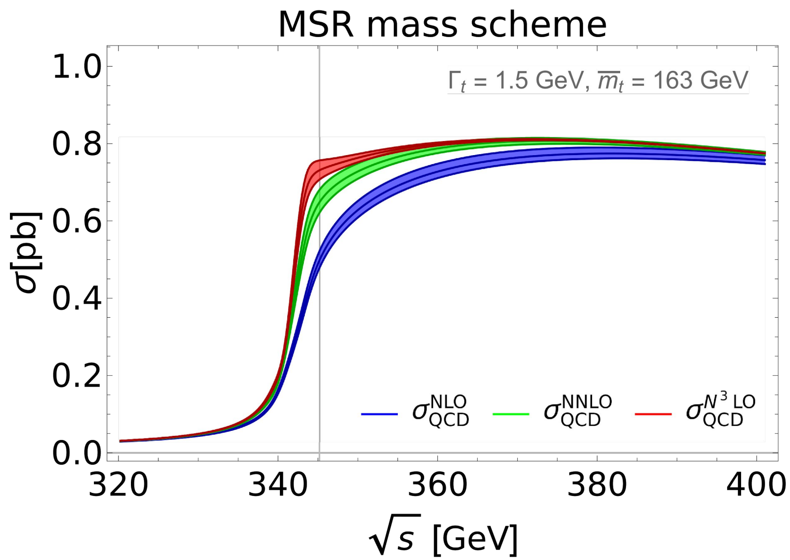
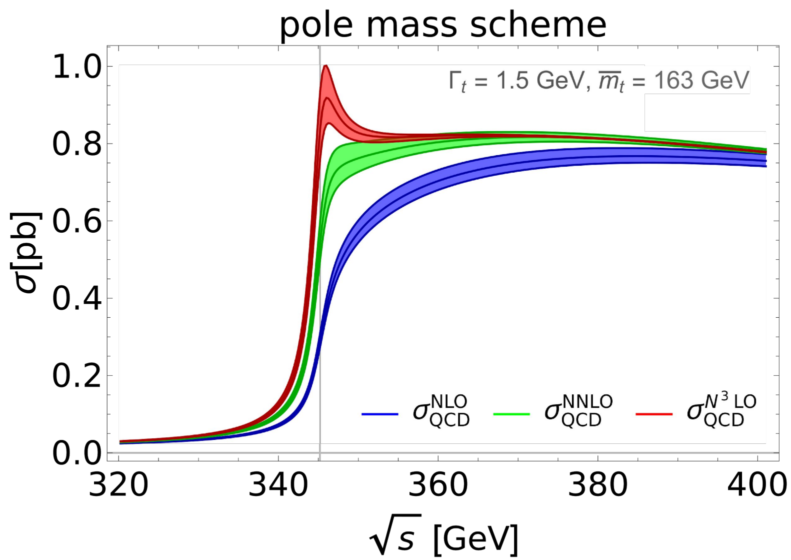
➞ no breaking of the non-relativistic power counting at threshold

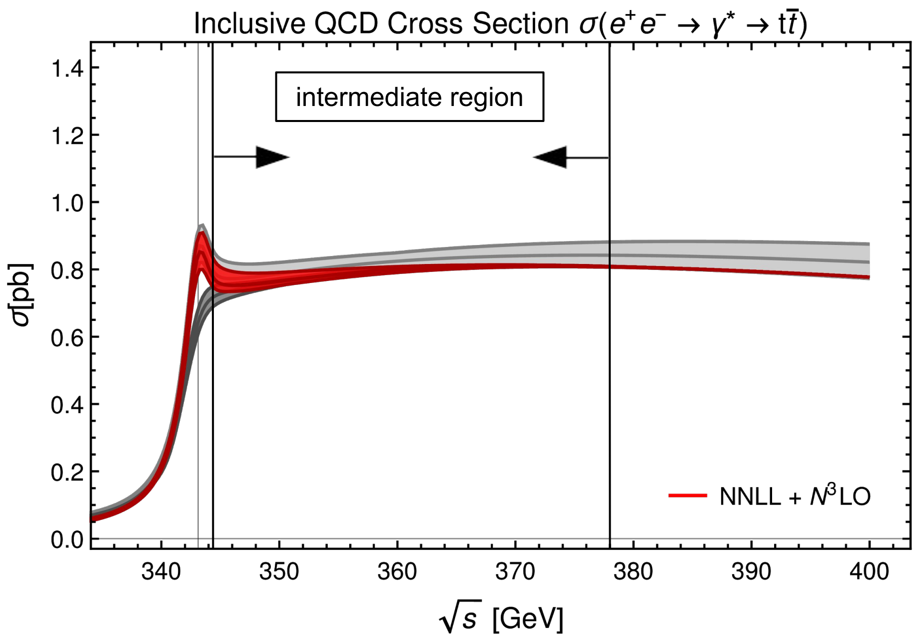
➞ improves convergence of the continuum cross section in the intermediate region:





Inclusive Cross Section -

Matching



Matching

$$\boxed{\sigma_{\text{matched}} = \sigma_{\text{QCD}} + \left(\sigma_{\text{vNRQCD}} - \sigma_{\text{double-counted}}\right) \cdot f_s}$$

Matching

$$\boxed{\sigma_{\text{matched}} = \sigma_{\text{QCD}} + \left(\sigma_{\text{vNRQCD}} - \sigma_{\text{double-counted}}\right) \cdot f_s}$$

$$\begin{aligned}
\sigma_{\text{vNRQCD}}^{\text{NNLO}} = \quad & v \;+\; \alpha_s \;+\; \alpha_s^2/v + \alpha_s^3/v^2 + \alpha_s^4/v^3 + \ldots \\
& + v^2 + \alpha_s\, v + \alpha_s^2 + \alpha_s^3/v + \alpha_s^4/v^2 + \ldots \\
& + v^3 + \alpha_s\, v^2 + \alpha_s^2\, v + \alpha_s^3 + \alpha_s^4/v + \ldots
\end{aligned}$$

$$\begin{aligned}
\sigma_{\text{QCD}}^{\text{N}^3\text{LO}} = \quad & v + v^2 + v^3 + v^4 + \ldots \\
& + \alpha_s + \alpha_s\, v + \alpha_s\, v^2 + \alpha_s\, v^3 + \ldots \\
& + \alpha_s^2/v + \alpha_s^2 + \alpha_s^2\, v + \alpha_s^2\, v^2 + \ldots \\
& + \alpha_s^3/v^2 + \alpha_s^3/v + \alpha_s^3 + \alpha_s^3 + \ldots
\end{aligned}$$

Matching

$$\boxed{\sigma_{\text{matched}} = \sigma_{\text{QCD}} + \left(\sigma_{\text{vNRQCD}} - \sigma_{\text{double-counted}}\right) \cdot f_s}$$

$$\sigma^{\text{NNLO}}_{\text{vNRQCD}} = \begin{array}{lllll} v \; + & \alpha_s \; + & \alpha_s^2/v \; + & \alpha_s^3/v^2 & + \; \alpha_s^4/v^3 \; + \; \ldots \\ + \; v^2 \; + & \alpha_s\, v \; + & \alpha_s^2 \; + & \alpha_s^3/v & + \; \alpha_s^4/v^2 \; + \; \ldots \\ + \; v^3 \; + & \alpha_s\, v^2 \; + & \alpha_s^2\, v \; + & \alpha_s^3 & + \; \alpha_s^4/v \; + \; \ldots \end{array}$$

$$\sigma^{\text{N}^3\text{LO}}_{\text{QCD}} = \begin{array}{llll} v & + \; v^2 & + \; v^3 & + \; v^4 \; + \; \ldots \\ + \; \alpha_s & + \; \alpha_s\, v & + \; \alpha_s\, v^2 & + \; \alpha_s\, v^3 \; + \; \ldots \\ + \; \alpha_s^2/v & + \; \alpha_s^2 & + \; \alpha_s^2\, v & + \; \alpha_s^2\, v^2 \; + \; \ldots \\ + \; \alpha_s^3/v^2 & + \; \alpha_s^3/v & + \; \alpha_s^3 & + \; \alpha_s^3 \; + \; \ldots \end{array}$$

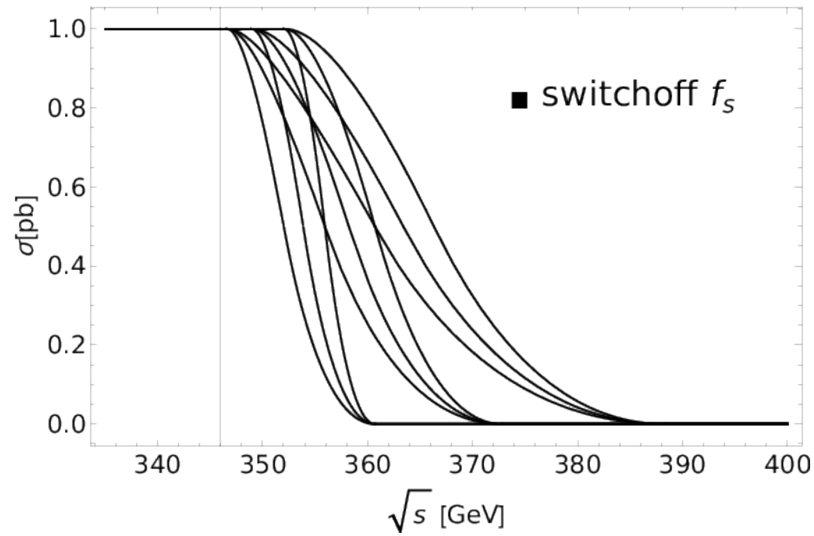
$\sigma_{\text{double-counted}}$

Matching

$$\boxed{\sigma_{\mathrm{matched}} = \sigma_{\mathrm{QCD}} + \left(\sigma_{\mathrm{vNRQCD}} - \sigma_{\mathrm{double\text{-}counted}}\right) \cdot f_s}$$

switch-off function:

- variation gives an error estimate of the matching
- introduces scheme dependence
- do we get convergence when going to higher orders?

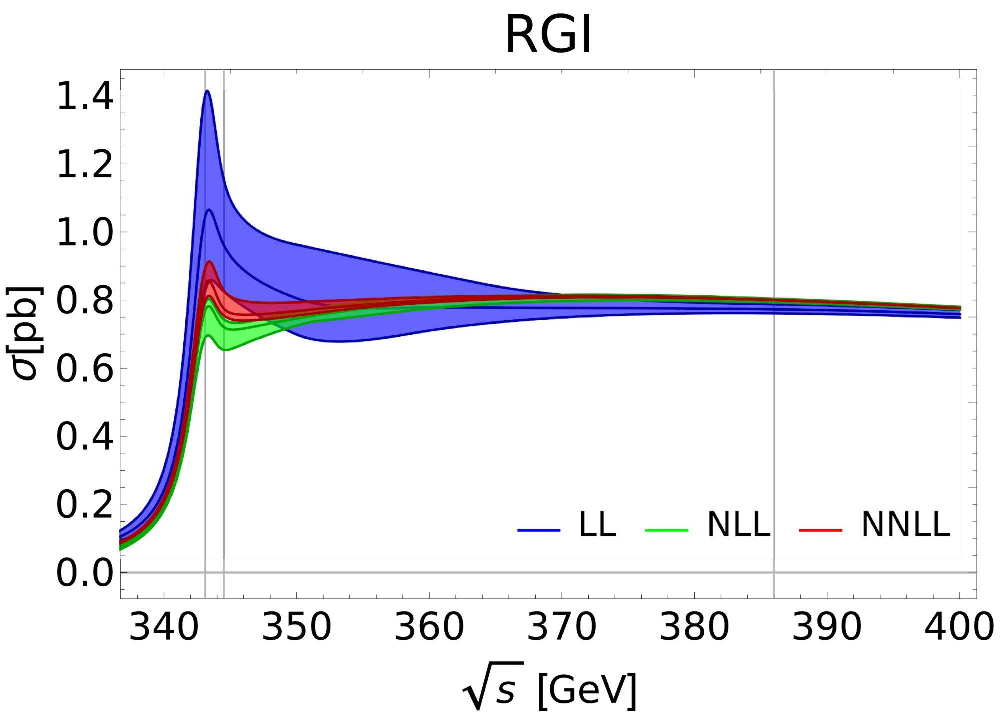


mass schemes:

σ_{vNRQCD}	1S mass scheme
σ_{QCD}	MSR mass scheme
$\sigma_{\mathrm{double\text{-}counted}}$	MSR mass scheme

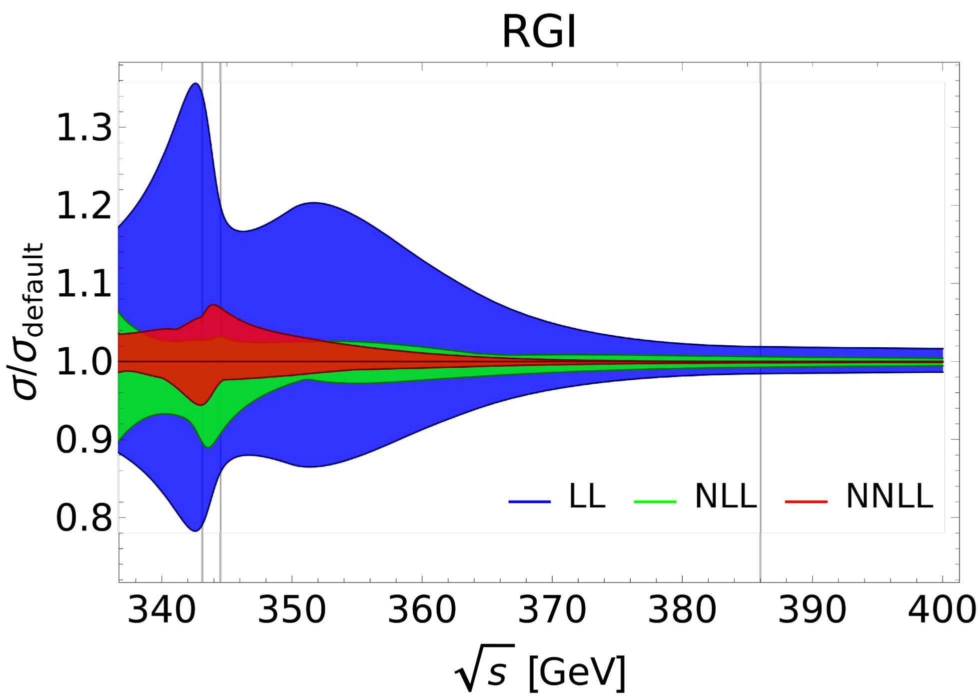
Matching

matched cross section from lowest to highest order:

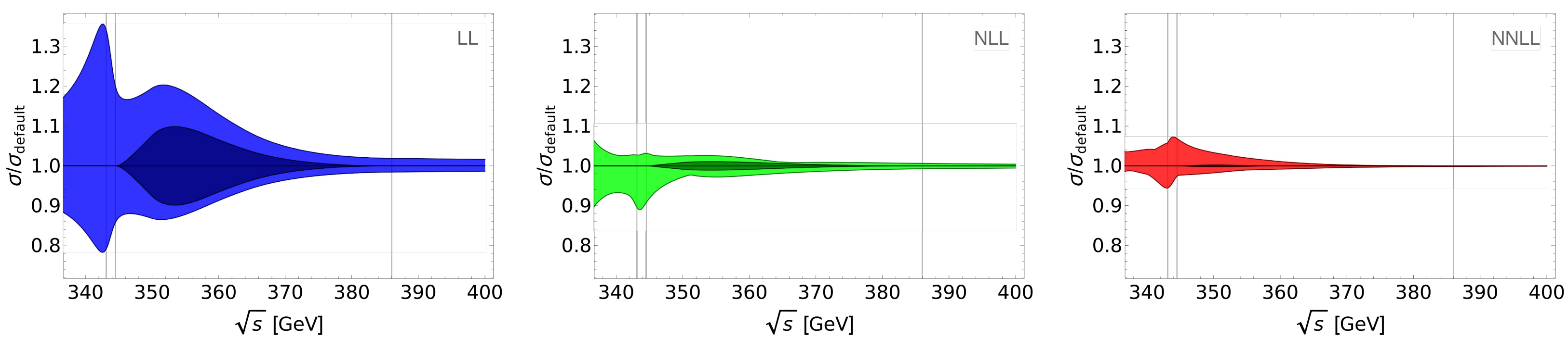


- error from variation of renormalization scales and the switch off function
- matching smoothly connects threshold with continuum
- overall error reduces from order to order

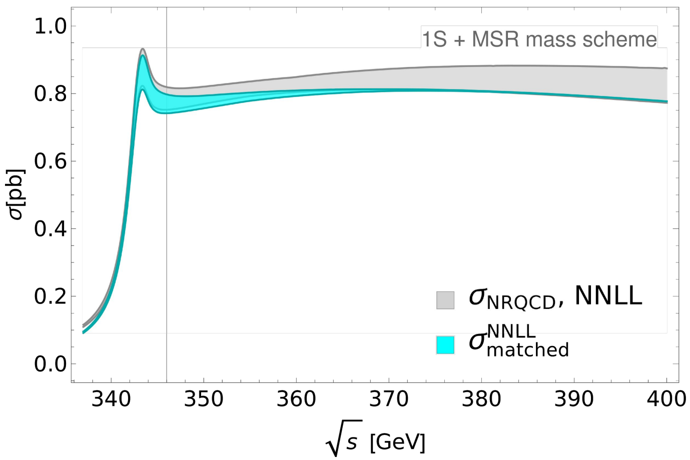
Matching



- good convergence from order to order
- matching error smaller than variation of renormalization scales
- matching error reduces from order to order



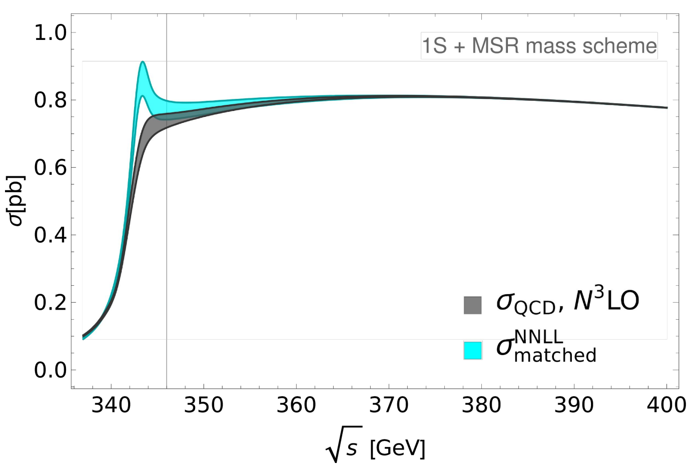
Matching



threshold cross section vs. matched cross section

→ matched cross section starts to differ from the threshold cross section immediately above the peak region

→ higher order corrections from continuum cross section give small shift at threshold

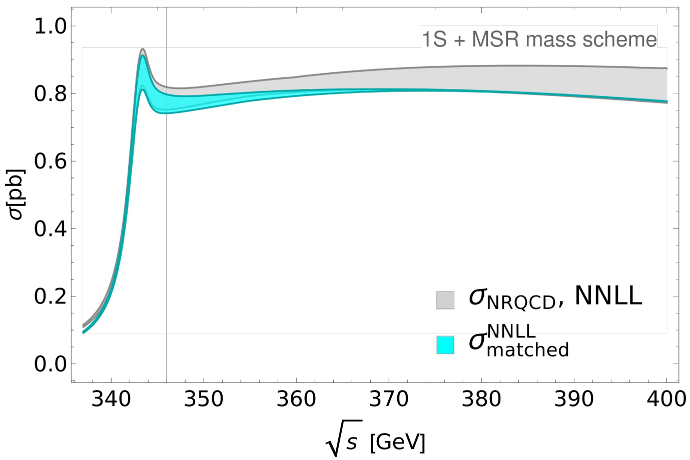


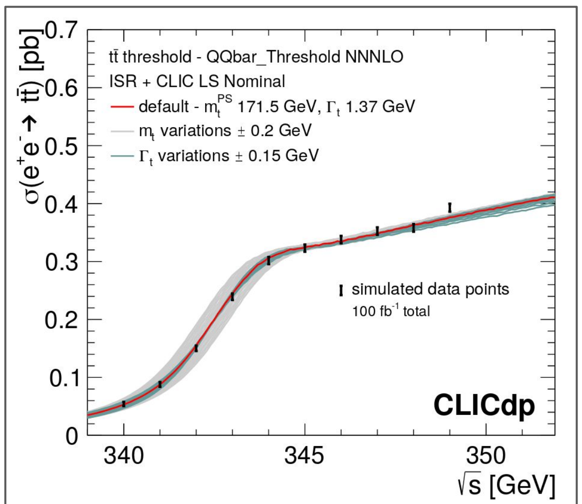
continuum cross section vs. matched cross section

→ matched cross section and continuum MSR cross section overlap above 365 GeV

→ MSR mass scheme valid down to smaller center-of-mass energies than pole mass scheme and MS mass scheme

Matching





threshold scan points

threshold cross section vs. matched cross section

→ matched cross section starts to differ from the threshold cross section immediately above the peak region

→ higher order corrections from continuum cross section give small shift at threshold

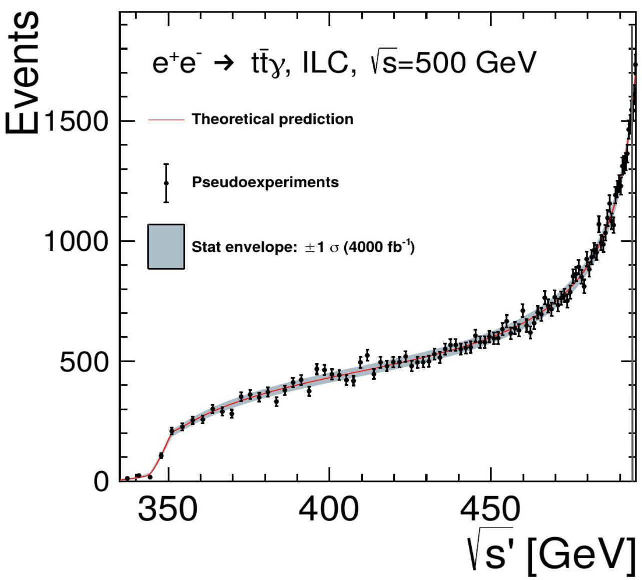
→ a part of the threshold scan points is in the intermediate region

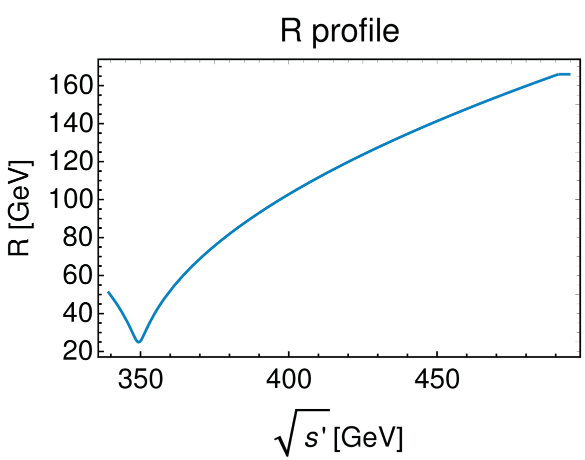
Measurement of the MSR Mass Running

Measurement of MSR Mass Running

[Boronat, Fullana, Fuster, Gomis, Hoang, Mateu, Vos, AW 2019 - to appear soon]

radiative events for ILC at 500 GeV:



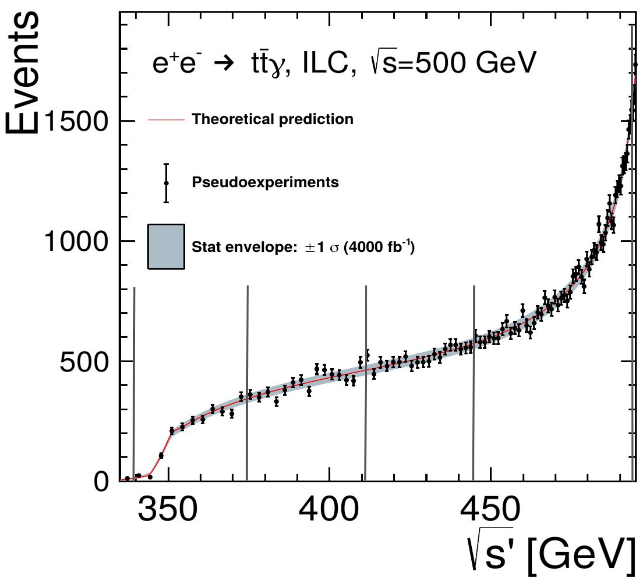


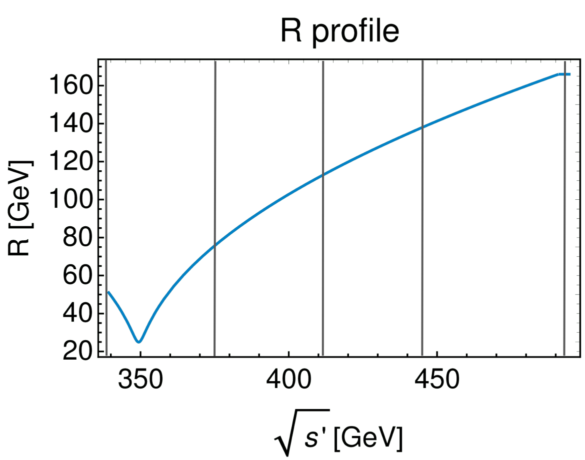
Measurement of MSR Mass Running

[Boronat, Fullana, Fuster, Gomis, Hoang, Mateu, Vos, AW 2019 - to appear soon]

radiative events for ILC at 500 GeV:

→ extraction of MSR mass $m_t^{\mathrm{MSR}}(R)$ from 4 different bins





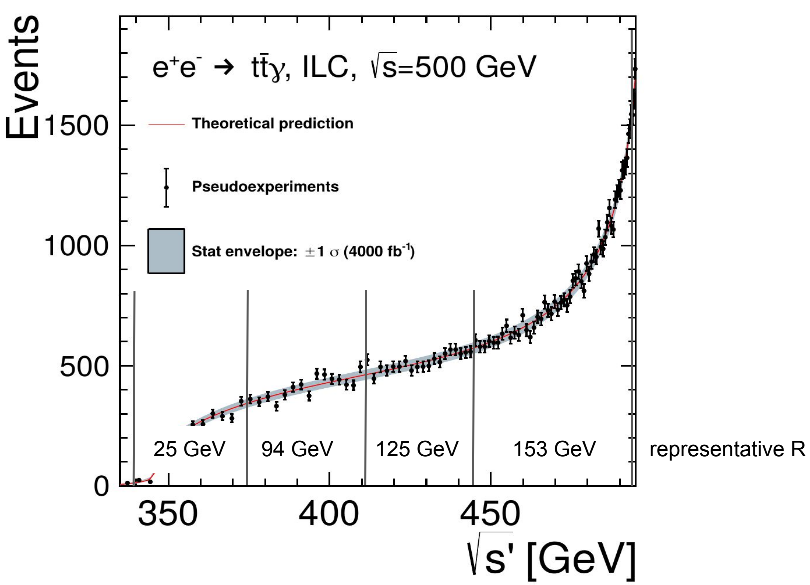
Measurement of MSR Mass Running

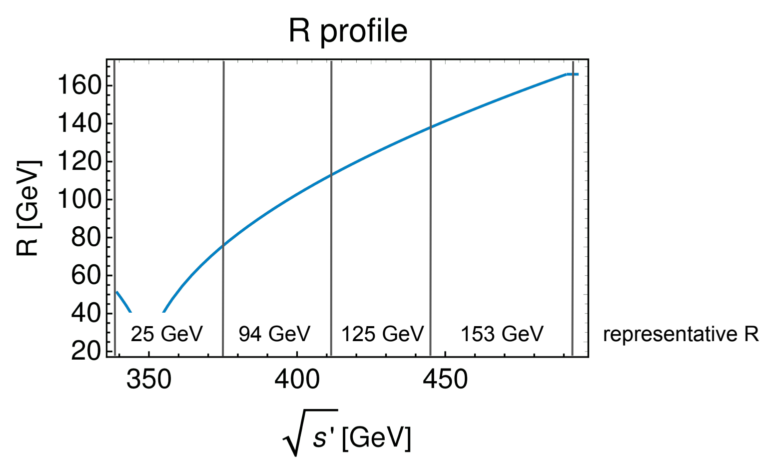
[Boronat, Fullana, Fuster, Gomis, Hoang, Mateu, Vos, AW 2019 - to appear soon]

radiative events for ILC at 500 GeV:

→ extraction of MSR mass $m_t^{\mathrm{MSR}}(R)$ from 4 different bins

→ mass determined at representative *R* scale of each bin

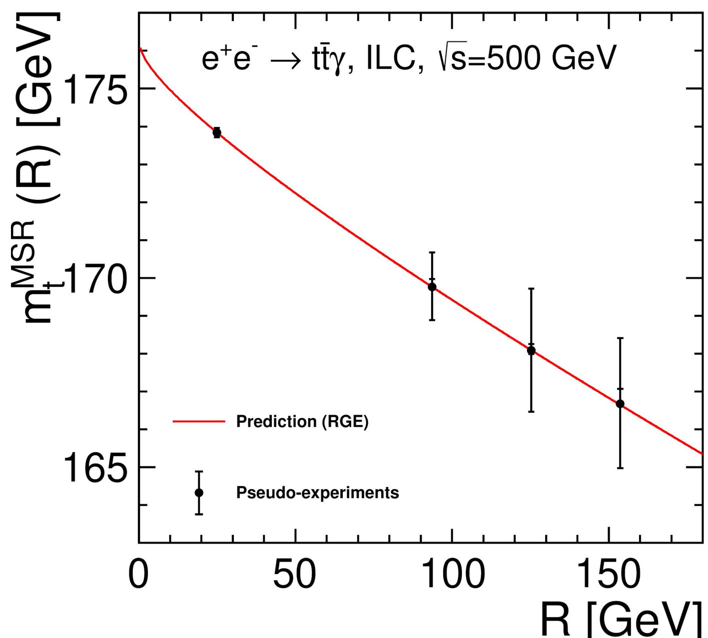




Measurement of MSR Mass Running

[Boronat, Fullana, Fuster, Gomis, Hoang, Mateu, Vos, AW 2019 - to appear soon]

- radiative events make extraction of MSR mass $m_t^{\mathrm{MSR}}(R)$ at different energies, and therefore at different scales R, possible
- provides a consistency check of QCD with running MSR mass
- ILC at 500 GeV can test running of MSR mass with over 5σ significance



[preliminary]

Conclusions

- Measurements using radiative events can determine the top quark mass with a precision of 110 MeV for CLIC at 380 GeV and 150 MeV for ILC at 500 GeV

- Consistent matching of the cross section at QCD $\mathrm{NNLL}_{\mathrm{threshold}}$ + $\mathrm{NNNLO}_{\mathrm{continuum}}$ with LO electroweak corrections at threshold has been implemented.

- The MSR mass provides a consistent mass scheme in all regions from threshold to the continuum.

- Extraction of the MSR mass at different energy scales provides a consistency check of QCD with the running MSR mass.

- Outlook:
    - differential matched cross section at $\mathrm{NLL}_{\mathrm{threshold}}$ + $\mathrm{NLO}_{\mathrm{continuum}}$

Conclusions

- Measurements using radiative events can determine the top quark mass with a precision of 110 MeV for CLIC at 380 GeV and 150 MeV for ILC at 500 GeV

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- <u>Outlook</u>:
    - differential matched cross section at $\mathrm{NLL}_{\mathrm{threshold}}$ + $\mathrm{NLO}_{\mathrm{continuum}}$

Thank you for your attention!

Backup

Matching - a few Subtleties

- Full matching for $\mathrm{NNLL}_{\text{threshold}}$ + $\mathrm{NNNLO}_{\text{continuum}}$ involves logarithms

  (Expanded terms on slide 25 shown for $\mathrm{NNLO}_{\text{threshold}}$ + $\mathrm{NNNLO}_{\text{continuum}}$ for illustration)

$$
\begin{aligned}
\sigma^{\text{NNLL}}_{\text{NRQCD}} = \quad & v + \alpha_s + \alpha_s^2/v + \alpha_s^3/v^2 + \ldots \\
& + v^2 + \alpha_s\, v + \alpha_s^2 + \alpha_s^3/v + \ldots \\
& + v^3 + \alpha_s\, v^2 + \alpha_s^2\, v + \alpha_s^3 + \ldots \\
& + \alpha_s\, v^3\, L + \alpha_s^2\, v^2\, L + \alpha_s^3\, v\, L + \ldots \\
& + \alpha_s^2\, v^3\, L^2 + \alpha_s^3\, v^2\, L^2 + \ldots \\
& + \alpha_s^3\, v^3\, L^3 + \ldots \qquad (L = \log v)
\end{aligned}
$$

- logarithms of vNRQCD cross section depend on renormalization scale, logarithms of QCD cross section depend on velocity

- coefficients of logarithms from QCD cross section and from vNRQCD cross section can be different, depending on the Padé approximation used for the QCD cross section

Matching - a few Subtleties

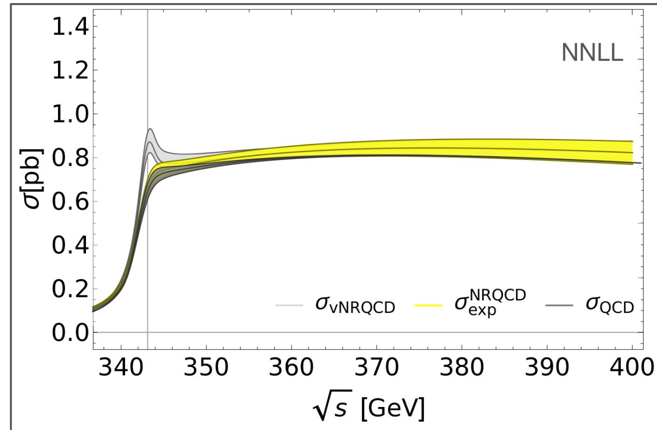
- difference from velocity vs. renormalization scale:

  small at threshold, large in the continuum

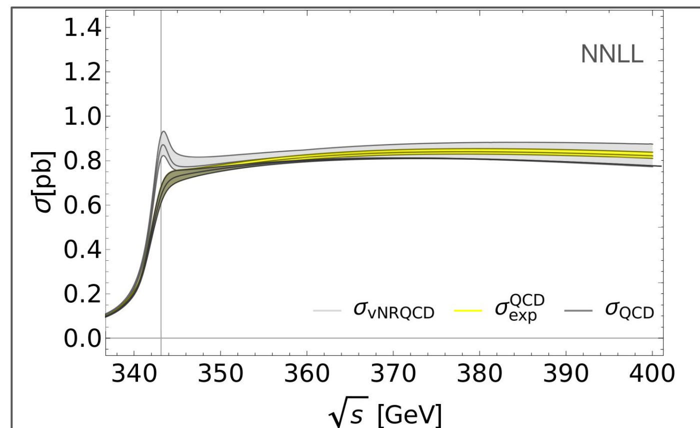
- different coefficients in the expansion (for a Padé approximation which does not include the exact logarithms):

  significant also at threshold

→ we use expansions from vNRQCD cross section for consistent matching



double-counted terms from vNRQCD cross section



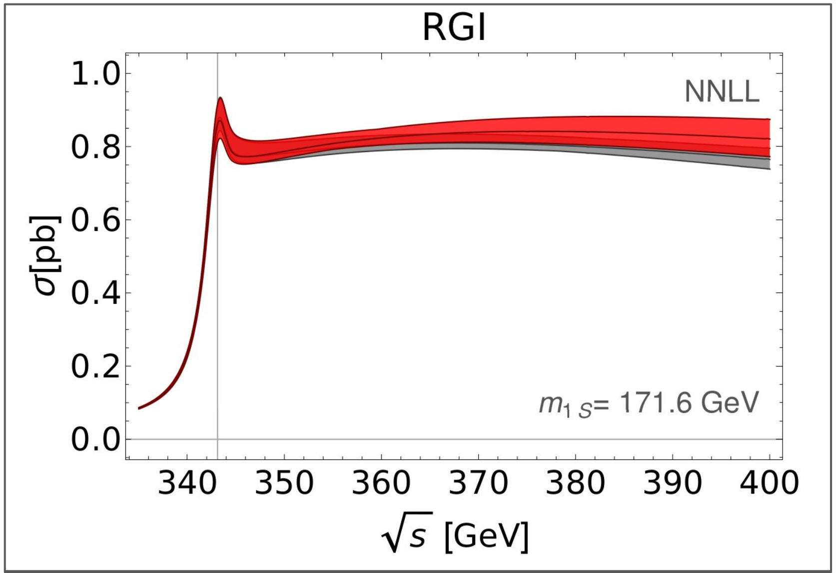
double-counted terms from QCD cross section

Matching - a few Subtleties

- Wilson coefficients can be used in expanded or unexpanded form, e.g.:

$$\left(c_1^{\rm NLL}\right)^2 = \left(1 - \frac{2\,C_F}{\pi}\,\alpha_s(\mu_h)\right)^2 \exp\left[2\,\xi^{\rm NLL}(h,\nu)\right]$$
$$= \left(1 - \frac{4\,C_F}{\pi}\,\alpha_s(\mu_h) + \mathcal{O}(\alpha_s^2)\right) \exp\left[2\,\xi^{\rm NLL}(h,\nu)\right]$$

- unexpanded form includes higher order corrections
- higher order corrections small at threshold, but large in the continuum
- only expanded form gives consistent matching



NNLL cross section with expanded (red) and unexpanded (gray) squared Wilson coefficients