Theory for muon-electron scattering @ 10ppm

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HU-DESY Zeuthen Seminar

July 11th 2019

The Muon g-2: experimental status

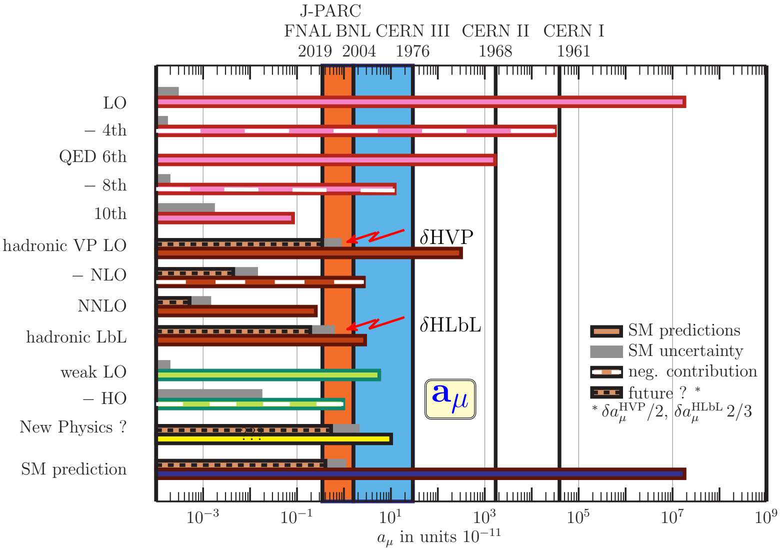
$$a_\mu^{\rm exp} = (16\,592\,091 \pm 54 \pm 33\,[63]) \times 10^{-11} \quad [540 \mathrm{ppb}]$$

Muon g-2 coll., PRD 73 (2006) 072003

New experiments at

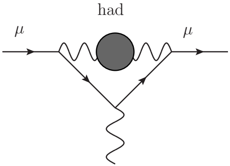
- Fermilab
  - aim at 140 ppb.
  - Data unblinded in May. Analysis in progress . . .
  - First result this year with a precision comparable to BNL E821.
- J-PARC proposal: phase-1 start with 0.46 ppm.

Is a_μ^{SM} ready for this precision?



F. Jegerlehner, Acta Phys.Polon. B49 (2018) 1157

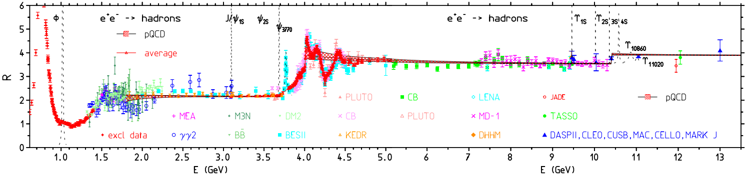
Hadronic LO contribution



$$a_\mu^{\mathrm{HLO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} R(s)\hat{K}(s)$$

with $R(s) = \sigma(e^+e^- \to \mathrm{had.}) / \frac{4\pi|\alpha(s)|^2}{3s}$

Durand, Phys. Rev. 128 (1962) 441; Gourdin, De Rafael, NPB 10 (1969) 667.



F. Jegerlehner, hep-ph/1804.07409

$$a_\mu^{\rm HLO} = 6932.7\,(24.6) \times 10^{-11}$$

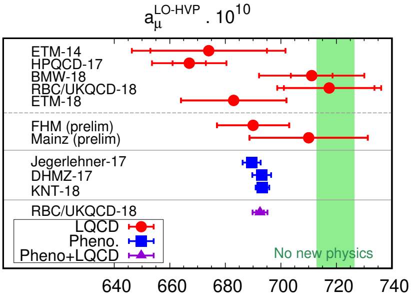
Keshavarzi, Nomura, Teubner, PRD 97 (2018) 114025

$$= 6894.6\,(32.5) \times 10^{-11}$$

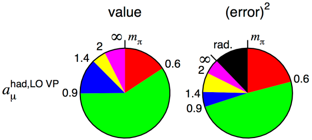
F. Jegerlehner, arXiv:1711.06089

$$= 6931\,(34) \times 10^{-11}$$

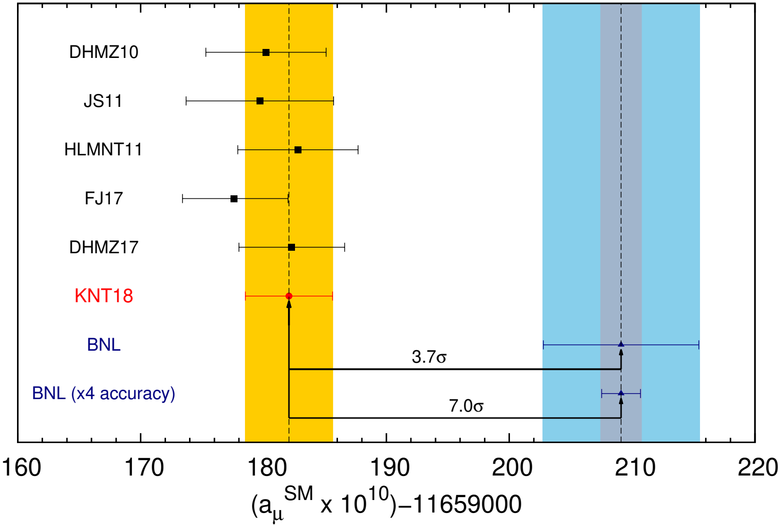
Davier, Hoecker, Malaescu, Zhang, EPJ C77 (2017) 827



Miura, hep-lat/1901.09052

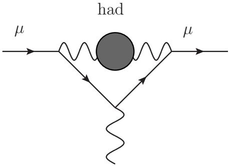


Keshavarzi, Nomura, Teubner, PRD 97 (2018) 114025



Keshavarzi, Nomura, Teubner, PRD 97 (2018) 114025

The MUonE Project



$$a_\mu^{\mathrm{HLO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} R(s)\hat{K}(s)$$

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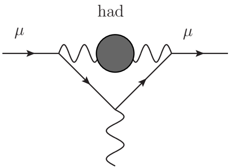
Durand, Phys. Rev. 128 (1962) 441; Gourdin, De Rafael, NPB 10 (1969) 667.

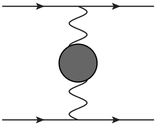


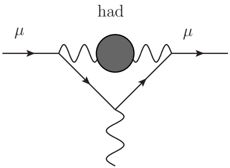
$$a_\mu^{\mathrm{HLO}} = -\frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha^{\mathrm{had}}(t(x))$$

with $t(x) = m_\mu^2 x^2/(x-1) < 0.$

Lautrup, Peterman, De Rafael, Phys. Rept. 3 (1972) 193

- $\Delta\alpha^{\mathrm{had}}(t)$: the hadronic contribution to the running of α_{em} in the space-like region.
- Can it be extracted from scattering data?





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Lautrup, Peterman, De Rafael, Phys. Rept. 3 (1972) 193

- $\Delta\alpha^{\rm had}(t)$: the hadronic contribution to the running of $\alpha_{\rm em}$ in the space-like region.
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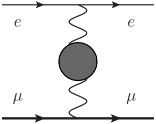


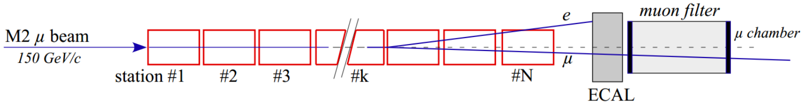
- The MUonE proposal: measure $\Delta\alpha^{\rm had}(t)$ via the elastic scattering $\mu e \to \mu e$.
- Use the 150 GeV muon beam, available at CERNs North Area.
- Fixed electron target (Beryllium).

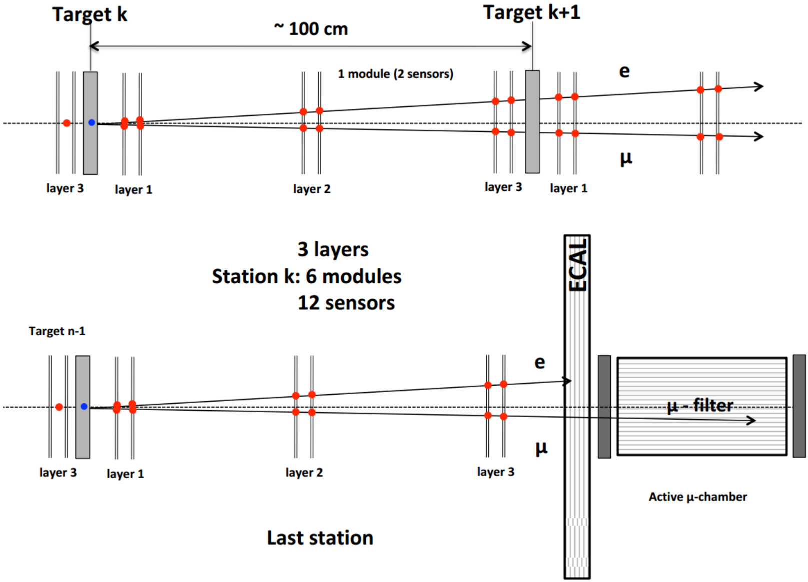
$$\frac{d\sigma}{dt} = \left|\frac{\alpha(t)}{\alpha(0)}\right|^2 \frac{d\sigma_0}{dt}$$

$$\text{with } \alpha(t) = \frac{\alpha(0)}{1-\Delta\alpha(t)}$$

Carloni Calame, Passera, Trentadue, Venanzoni, PLB 2015;

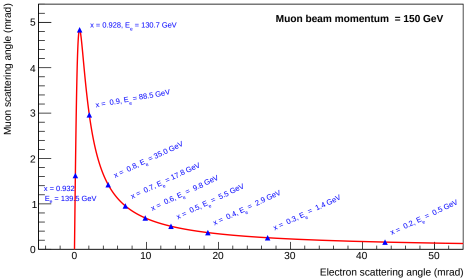
Abbiendi et al. EPJ C77 (2017) 139

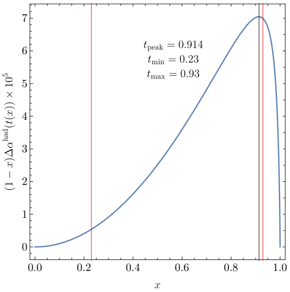




MUonE, LoI, 2019

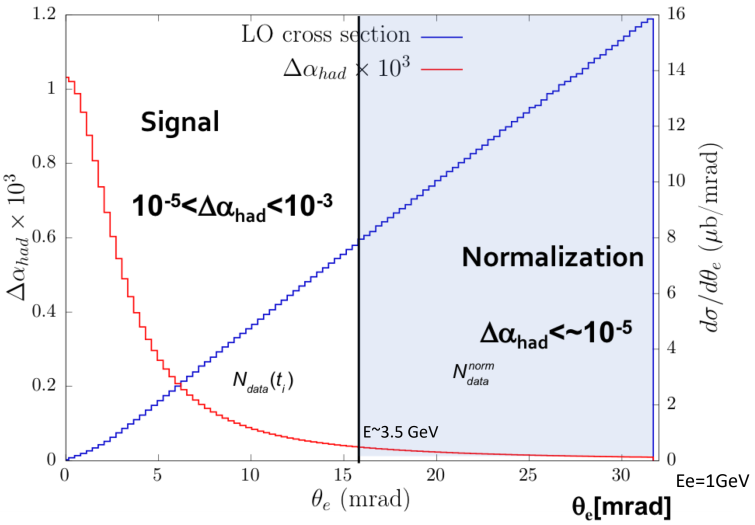
Measurement Strategy





$$10^{-3}\ \mathrm{GeV}^2 < |t| < 0.143\ \mathrm{GeV}^2 \quad \rightarrow \quad 0.23 < x < 0.93$$

$$\int_{0.23}^{0.93} dx\,(1-x)\,\Delta\alpha^{\mathrm{had}}(t) = 586.84 \pm 4.28 = 85\% \times a_\mu^{\mathrm{HLO}}$$



LO cross section
$\Delta\alpha_{had} \times 10^3$
Signal
$10^{-5}<\Delta\alpha_{had}<10^{-3}$
Normalization
$\Delta\alpha_{had}<\sim10^{-5}$
$N_{data}(t_i)$
N^{norm}_{data}
E~3.5 GeV
Ee=1GeV
θ_e (mrad)
θ_e[mrad]
$\Delta\alpha_{had} \times 10^3$
$d\sigma/d\theta_e$ (μb/mrad)

Extraction of $\Delta\alpha^{\mathrm{had}}$

At LO:

$$\left(1 - \frac{\Delta\alpha^{\mathrm{had}}(t)}{1 - \Delta\alpha^{\mathrm{lep}}(t)}\right)^{-2} = \frac{N_{\mathrm{signal}}(t_i)}{N_{\mathrm{norm}}} \times \frac{\sigma^{\mathrm{MC}}_{\mathrm{norm}}}{\sigma^{\mathrm{MC}}_0(t_i)}$$

At N^{x}LO:

- The ratio becomes a complex expression.
- Evaluated by Monte Carlo simulation.
- Observables: $d\sigma/d\theta_e$, $d\sigma/d\theta_\mu$ and $d^2\sigma/(d\theta_e\, d\theta_\mu)$.
- The extraction is carried out by template fit method.

$\Delta\alpha^{\mathrm{had}}(t)$ ansatz

- Polynomial:

$$\Delta\alpha^{\mathrm{had}}(t) = c_1 t + c_2 t^2 + c_3 t^3$$

- Padé approximant:

$$\Delta\alpha^{\mathrm{had}}(t) = at\frac{1+bt}{1+ct}$$

- Fermion-like function:

$$\Delta\alpha^{\mathrm{had}}(t) =$$

$$K\left[-\frac{5}{9}-\frac{4M}{t}\left(\frac{4M^2}{3t^2}+\frac{M}{3t}-\frac{1}{6}\right)\frac{2}{\sqrt{1-4M/t}}\log\left(\frac{1-\sqrt{1-4M/t}}{1+\sqrt{1-4M/t}}\right)\right]$$

To reach a precision on a_μ^{HLO} of 20×10^{-11}:

Theory

- Cross section ratio in signal and normalization regions must be know < 10 ppm.
- Cross section at NNLO
- Resummation

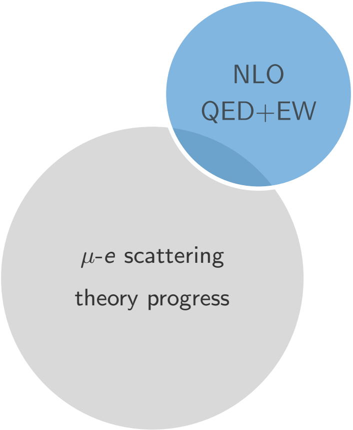
Experiment

- statistical sensitivity in 2 years of data taking, muon beam intensity of $1.3 \times 10^7\, s^{-1}$.
- 60 cm of low-Z material segmented in 10mm thin layers.
- Keep efficiency highly uniform over the entire t range.
- Control of the alignment of the tracker with high precision.
- Describe multiple scattering with an accuracy below 1%.

Theory for μ-e scattering at 10 ppm

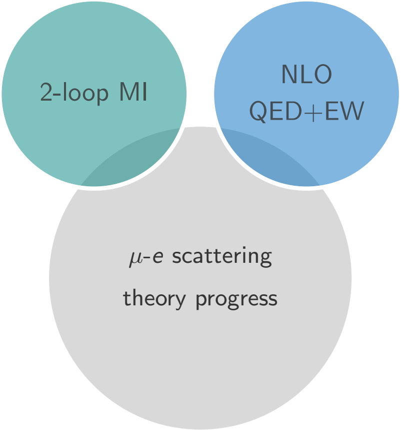
μ-e scattering

theory progress



M. Alacevich,
C. Carloni Calame,
M. Chiesa,
G. Montagna,
O. Nicrosini,
F. Piccinini.

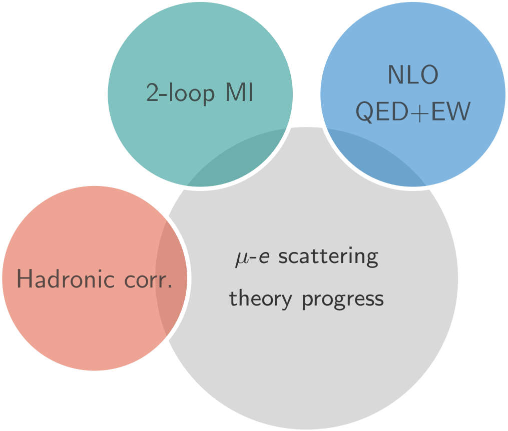
JHEP 1902 (2019) 155



S. Di Vita,
S. Laporta,
P. Mastrolia,
M. Passera, A. Primo,
U. Schubert,
W.J. Torres
Bobadilla.

JHEP 1809 (2018) 016

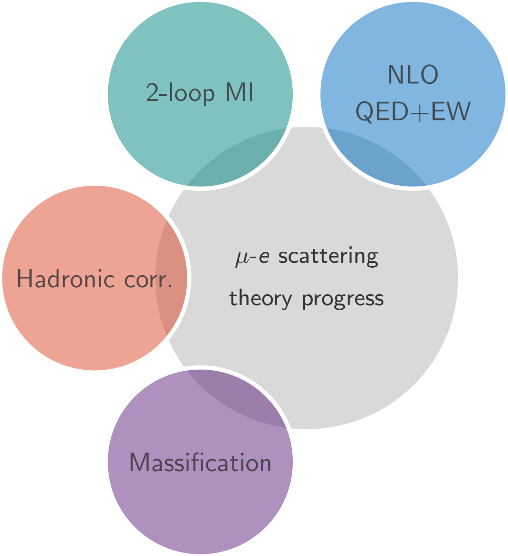
JHEP 1711 (2017) 198



MF, M. Passera,

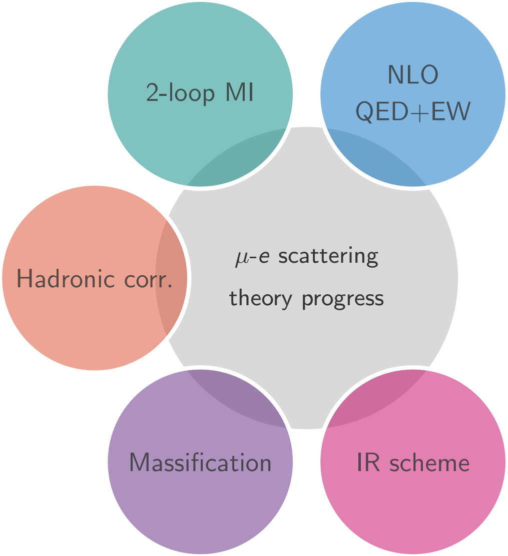
PRL 122 (2019) 192001,

JHEP 1902 (2019) 027



P. Banerjee T. Engel,
C. Gnendiger,
A. Signer, Y. Ulrich.

JHEP 1902 (2019) 118



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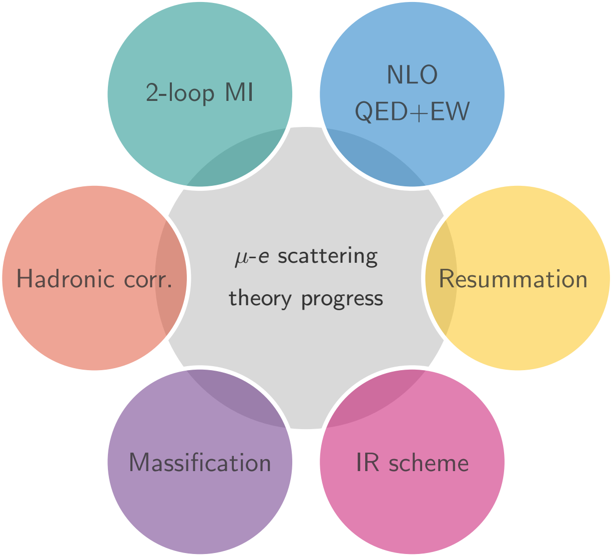


P. Banerjee
T. Becher,
A. Broggio, T. Engel,
C. Gnendiger,
M. Passera,
A. Signer, Y. Ulrich.

NLO Corrections

Old calculations

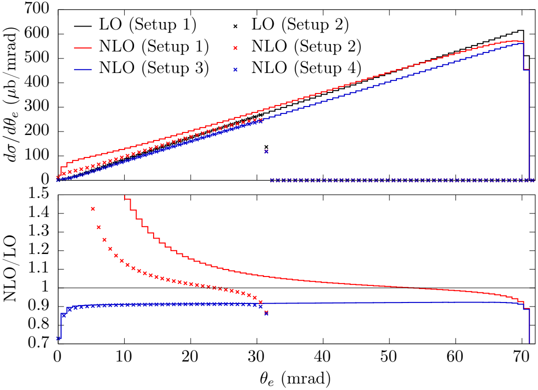
Nikishov, Sov. Phys. JETP 12, (1961) 529; Eriksson, Nuovo Cimento 19 (1961) 1029; Eriksson, Larsson, Rinander, Nuovo Cimento 30 (1963) 1434; Van Nieuwenhuizen, NPB 28 (1971) 429; Kaiser, J. Phys.G 37 (2010) 115005.

Fully differential MC generator

Alacevich, Chiesa, Montagna, Nicrosini, Piccinini, Carloni Calame, JHEP 1902 (2019) 155

- NLO (QED & EW) corrections to $\mu^{\pm} e^{-} \to \mu^{\pm} e^{-}$.
- EW corrections are small:
  - $\gamma - Z$ interference is $O(10^{-5})$.
  - NLO EW are well below 10^{-5}.
- Study of 4 cut scenarios.
- Study of finite m_e effects
- Sensitivity on $\Delta\alpha^{\mathrm{had}}$ with RC

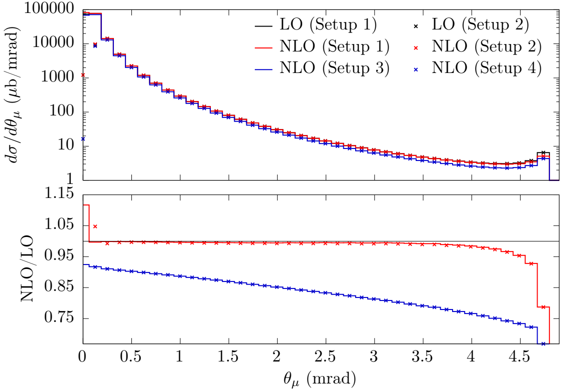
$\mu^+ e^- \to \mu^+ e^-$



Alacevich, Chiesa, Montagna, Nicrosini, Piccinini, Carloni Calame, JHEP 1902 (2019) 155

- Setup 1: $E_e > 0.2\text{GeV}$, $\theta_e, \theta_\mu < 100$ mrad
- Setup 2: $E_e > 1\text{GeV}$, $\theta_e, \theta_\mu < 100$ mrad
- Setup 3: setup 1 + acoplanarity cut $|\pi - (\varphi_e - \varphi_\mu)| < 3.5$ mrad
- Setup 4: setup 2 + acoplanarity cut $|\pi - (\varphi_e - \varphi_\mu)| < 3.5$ mrad

$\mu^+ e^- \to \mu^+ e^-$



Alacevich, Chiesa, Montagna, Nicrosini, Piccinini, Carloni Calame, JHEP 1902 (2019) 155

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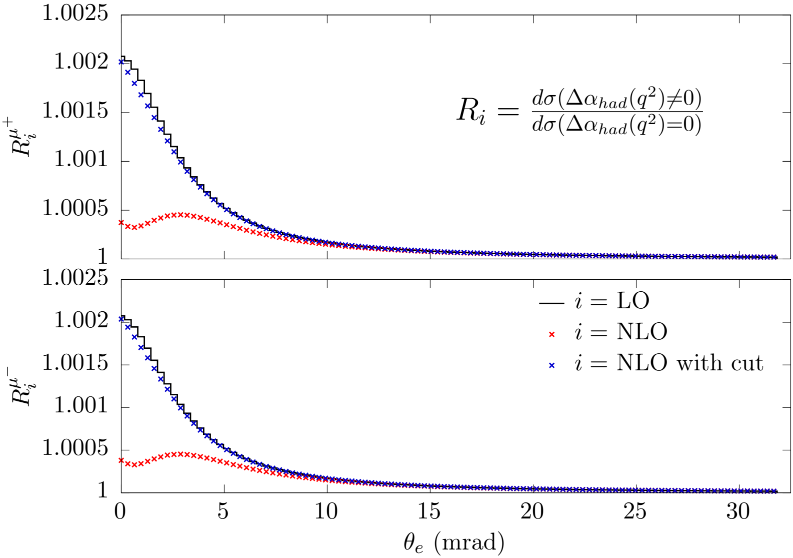
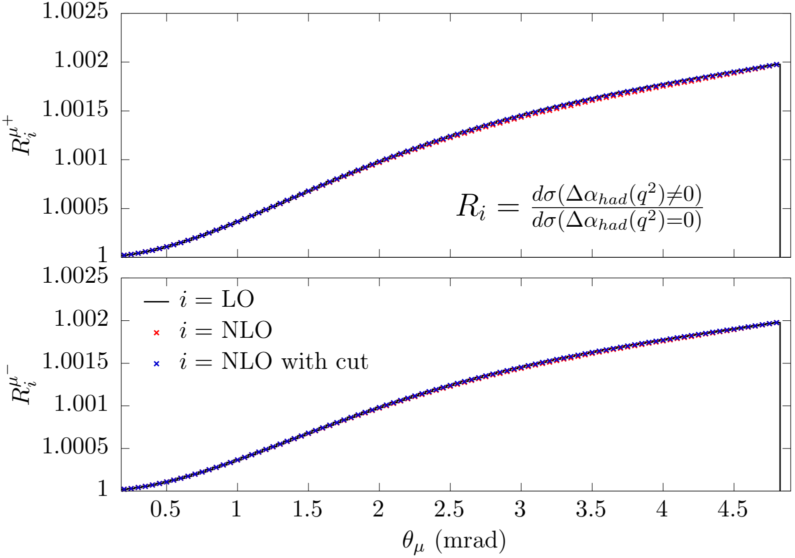
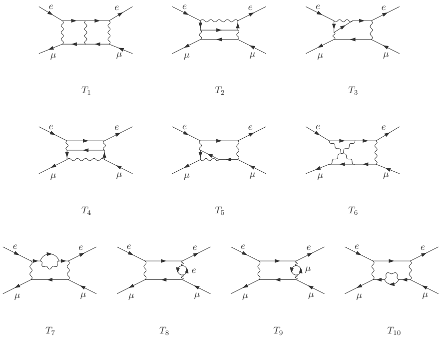


$R_i = \frac{d\sigma(\Delta\alpha_{had}(q^2)\neq 0)}{d\sigma(\Delta\alpha_{had}(q^2)=0)}$
$R_i^{\mu^+}$
$R_i^{\mu^-}$
i = LO
i = NLO
i = NLO with cut
θ_e (mrad)


1.0025
1.002
1.0015
1.001
1.0005
1
$R_i^{\mu^+}$
$R_i = \frac{d\sigma(\Delta\alpha_{had}(q^2)\neq 0)}{d\sigma(\Delta\alpha_{had}(q^2)=0)}$
i = LO
i = NLO
i = NLO with cut
$R_i^{\mu^-}$
0.5
1
1.5
2
2.5
3
3.5
4
4.5
θ_μ (mrad)

QED two-loop master integrals

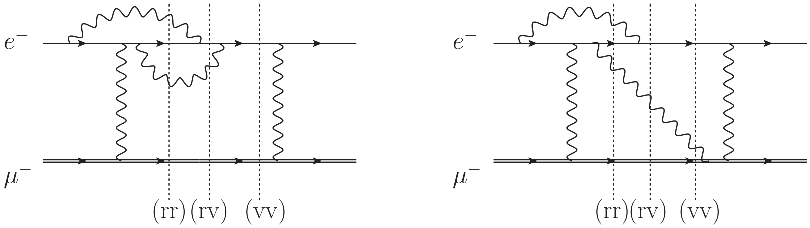


Di Vita, Laporta, Mastrolia, Primo, Schubert, JHEP 1809 (2018) 016;
Mastrolia, Passera, Primo, Schubert, JHEP 1711 (2017) 198.

- Two-loop planar and non planar master integrals for box diagrams computed.
- Full M_μ dependence. Massless electron
- Method of differential equations and the Magnus exponential series.
- Relevant also for $e^+e^- \to \ell^+\ell^-$ and $t\bar{t}$ production.

Massification:

how to recover the leading m_e-dependence, i.e. the logarithmic corrections $\log(m_e^2/M_\mu^2)$ and $\log(m^2/s)$, from massless two-loop amplitudes.



- Previous works for Bhabha scattering:

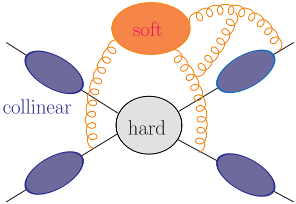
  Penin, NPB 734 (2006) 185; Mitov, Moch, JHEP 05 (2007) 001; Becher, Melnikov, JHEP 06 (2007) 084.

- New features in μ-e scattering: $m_e \ll M_\mu \sim s$.

- no large external mass [Mitov, Moch; Becher, Melnikov]

$$\mathcal{M}^{(n)}(s,t,m) = \prod_{i=1,4} \sqrt{Z_i'(m)} \times \mathcal{S}' \times \mathcal{M}^{(n)}(s,t,0)$$

  - hard: $\mathcal{M}^{(n)}(s,t,0)$, massless amplitude
  - soft: $\mathcal{S}'$, process dependent but 'easy'
  - collinear: $\sqrt{Z_i'(m)}$, process independent !
  - ultrasoft: $\to 0$, cancellation is check !

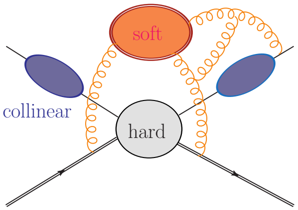


from A. Signer's talk at CERN 25.3.19

- with external mass [Engel, Gnendiger, AS, Ulrich 18]

$$\mathcal{M}^{(n)}(s,t,M,m) = \prod_{i=1,2} \sqrt{Z_i(m)} \times \mathcal{S} \times \mathcal{M}^{(n)}(s,t,M,0)$$

  - hard: $\mathcal{M}^{(n)}(s,t,M,0)$, massless amplitude
  - soft: $\mathcal{S}$, process dependent but pretty 'easy'
  - collinear: $\sqrt{Z_i(m)}$, process independent !
  - ultrasoft: $\to 0$, cancellation is check !



from A. Signer's talk at CERN 25.3.19

Engel, Gnendiger, Signer, Ulrich, JHEP 1902 (2019) 118

FKS^2: double-soft extension of FKS scheme

- FKS scheme very efficient when there are only soft sing.

$$\underbrace{d\varphi_{n+1}}_{\xi^{1-2\varepsilon}}\underbrace{\mathcal{M}^{(0)}_{n+1}} _{\xi^{-2}}\sim d\xi\left(\underbrace{-\frac{\xi_{\text{cut}}^{-2\varepsilon}}{2\varepsilon}\delta(\xi)}_{(s)}+\underbrace{\left(\xi^{-1-2\varepsilon}\right)_{\xi_{\text{cut}}}}_{(h)}\right)\left(\xi^2\mathcal{M}^{(0)}_{n+1}\right)$$

FKS extension at NNLO: Engel, Signer, Ulrich 2019 (to appear)

- real × virtual: $\mathcal{M}^{(1)}_{n+1}=\mathcal{M}^{(1)f}_{n+1}-\hat{\mathcal{E}}(\xi^2_{\text{cut}})\mathcal{M}^{(0)}_{n+1}$
- real × real: do FKS twice with two ξ_{cut}.

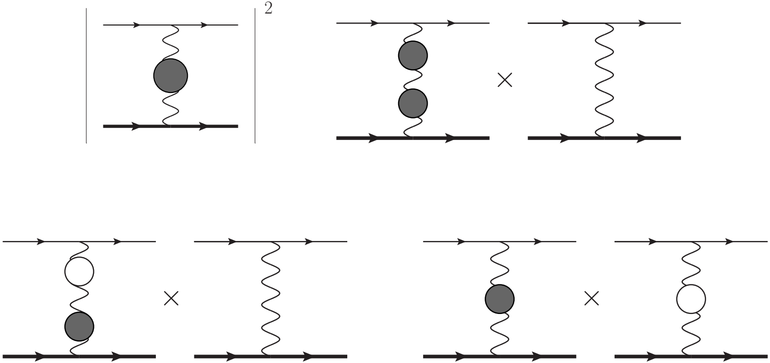
$$d\sigma_{n+2}=d\sigma^{(hh)}+d\sigma^{(ss)}+d\sigma^{(hs)}+d\sigma^{(sh)}$$

Hadronic Corrections to NNLO Cross Section

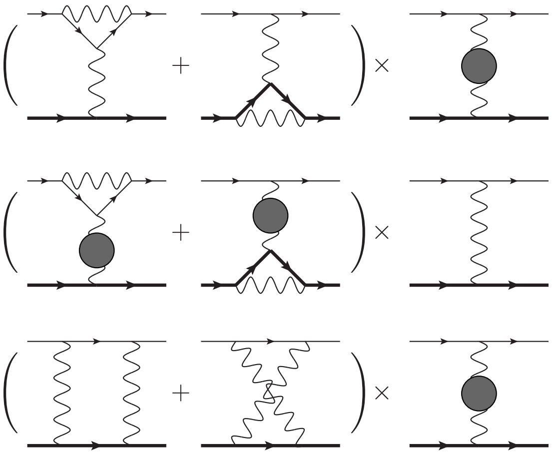
e
had.
μ

$$\Delta\alpha_{\mathrm{had}}(s > 0)?$$

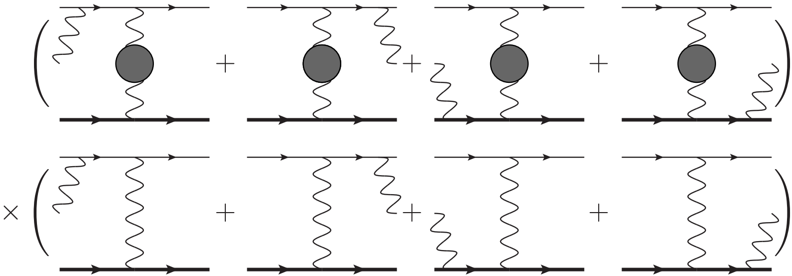
Class I



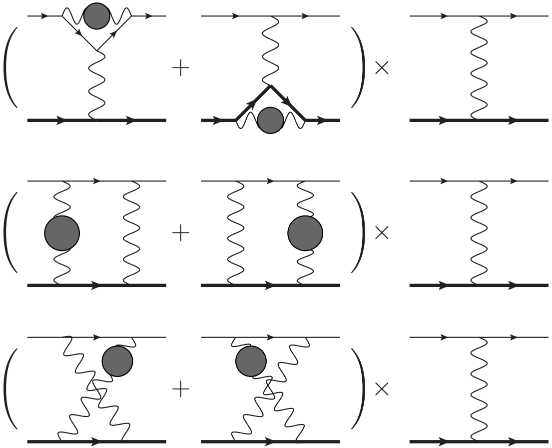
Class II

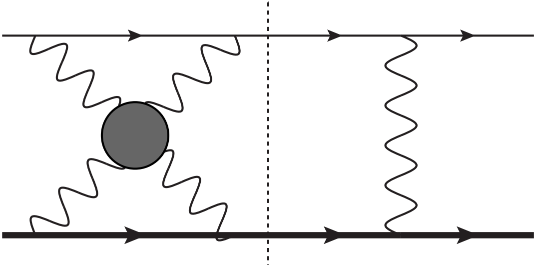


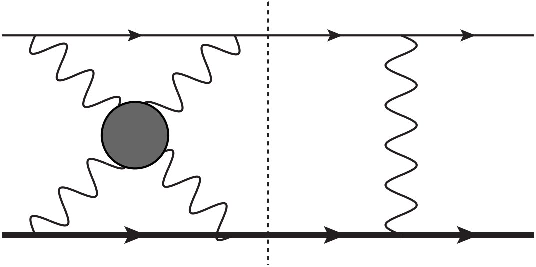
Class III



Class IV







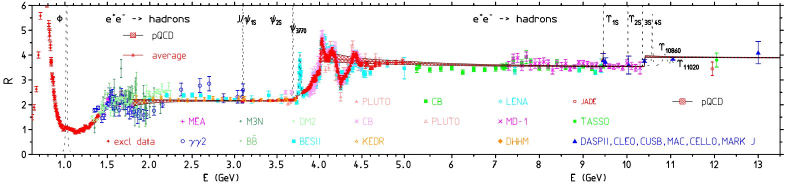
is of $O(\alpha^5)$.

Hadron production

μ-e scattering with hadrons in the final state:

- Phase space very small: $\sqrt{s} = 405$ MeV.
- $\mu e \to \mu e \pi\pi$, $\sqrt{s} - m_\mu - 2m_\pi = 22$ MeV
- $\mu e \to \mu e \pi^0$, $\sqrt{s} - m_\mu - m_\pi = 164$ MeV

- To $R(S)$
  - Method: dispersion relation.
  - Based on $\mathrm{Im}\,\Pi^{\mathrm{had}}(s > 0)$.
  - input: $e^+e^- \to$ had data.



F. Jegerlehner, hep-ph/1804.07409

Two Roads:

- To $R(S)$
  - Method: dispersion relation.
  - Based on $\mathrm{Im}\,\Pi^{\mathrm{had}}(s > 0)$.
  - input: $e^+e^- \to$ had data.

- Not to $R(s)$:
  - Method: hyperspherical integration.
  - Based on $\Pi^{\mathrm{had}}(t < 0)$.
  - input: MUonE's $e\mu \to e\mu$ data.

To $R(s)$

the dispersive method

Dispersion Relation + Optical Theorem

$$\Pi_{\text{had}}(q^2) = -\frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty} \frac{dz}{z}\frac{\text{Im}\Pi_{\text{had}}(z)}{q^2 - z + i\varepsilon}$$

Dispersion Relation + Optical Theorem

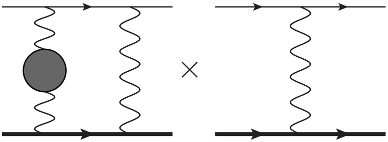
$$\Pi_{\mathrm{had}}(q^2) = -\frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dz}{z}\frac{\mathrm{Im}\Pi_{\mathrm{had}}(z)}{q^2 - z + i\varepsilon}$$

$$\frac{-ig^{\mu\nu}}{q^2} \to \frac{-ig^{\mu\nu}}{q^2}\Pi_{\mathrm{had}}(q^2)$$

Dispersion Relation + Optical Theorem

$$\Pi_{\mathrm{had}}(q^2) = -\frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dz}{z}\frac{\mathrm{Im}\Pi_{\mathrm{had}}(z)}{q^2 - z + i\varepsilon}$$

$$\frac{-ig^{\mu\nu}}{q^2} \to \frac{-ig^{\mu\nu}}{q^2}\Pi_{\mathrm{had}}(q^2) \to -\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dz}{z}\,\mathrm{Im}\Pi_{\mathrm{had}}(z)\left[\frac{-ig^{\mu\nu}}{q^2 - z}\right]$$



Hadronic NNLO Bhabha scattering:

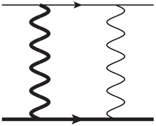
Actis, Czakon, Gluza, Riemann, PRL 100 (2008) 131602; Actis, Gluza, Riemann, Nucl. Phys. Proc. Suppl. 183 (2008) 174; Kühn, Uccirati, NPB 806 (2009) 300; Carloni Calame et al. JHEP 1107 (2011) 126.

Dispersion Relation + Optical Theorem

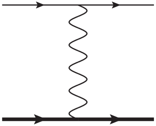
$$\Pi_{\rm had}(q^2) = -\frac{q^2}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dz}{z}\frac{{\rm Im}\Pi_{\rm had}(z)}{q^2 - z + i\varepsilon}$$

$$\frac{-ig^{\mu\nu}}{q^2} \to \frac{-ig^{\mu\nu}}{q^2}\Pi_{\rm had}(q^2) \to -\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{dz}{z}{\rm Im}\Pi_{\rm had}(z)\left[\frac{-ig^{\mu\nu}}{q^2 - z}\right]$$

$$-\frac{\alpha}{3\pi}\int_{4m_\pi^2}^{\infty}\frac{dz}{z}R^{\rm had}(z)$$

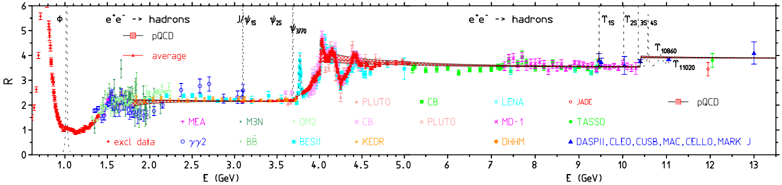


$\times$



Hadronic NNLO Bhabha scattering:

Actis, Czakon, Gluza, Riemann, PRL 100 (2008) 131602; Actis, Gluza, Riemann, Nucl. Phys. Proc. Suppl. 183 (2008) 174; Kühn, Uccirati, NPB 806 (2009) 300; Carloni Calame et al. JHEP 1107 (2011) 126.



F. Jegerlehner, hep-ph/1804.07409

HNNLO: Fortran Implementation

With the FeynArts + FormCalc framework:

- FeynArts generates Feynman diagrams in QED+extra massive photon.
- FormCalc calculates tree-level and one-loop diagrams.
- FormCalc exports $|\mathcal{M}|^2$ as Fortran code.

T. Hahn, Comput. Phys. Commun. 140 (2001) 418;
T. Hahn, S. Passehr and C. Schappacher, J. Phys. Conf. Ser. 762 (2016) 012065

HNNLO: Fortran Implementation

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T. Hahn, Comput. Phys. Commun. 140 (2001) 418;
T. Hahn, S. Passehr and C. Schappacher, J. Phys. Conf. Ser. 762 (2016) 012065

The Montecarlo code:

- Full dependence on m_e and m_μ.
- `Collier` evaluates one-loop functions.

  Denner, Dittmaier, Hofer, Comput. Phys. Commun. **212** (2017) 220
- Soft singularities with FKS subtraction.
  Frixione, Kunszt, Signer, NPB 467 (1996) 399
  Frederix, Frixione, Maltoni, Stelzer, JHEP 0910 (2009) 003

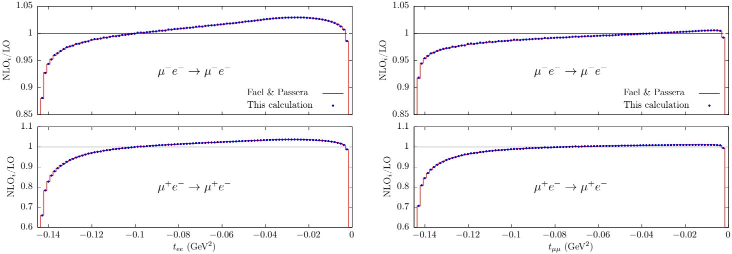
HNNLO: Mathematica Implementation

- One-loop functions `Package-X`

  Patel, Comput.Phys.Commun. 197 (2015) 276
- Checked with `LoopTools-quad`

  Hahn, Perez-Victoria, Comput.Phys.Commun. 118 (1999) 153
- Use of MATHEMATICAs arbitrary-precision numbers.
- Slicing method for IR.

Our QED NLO in good agreement with JHEP 1902 (2019) 155

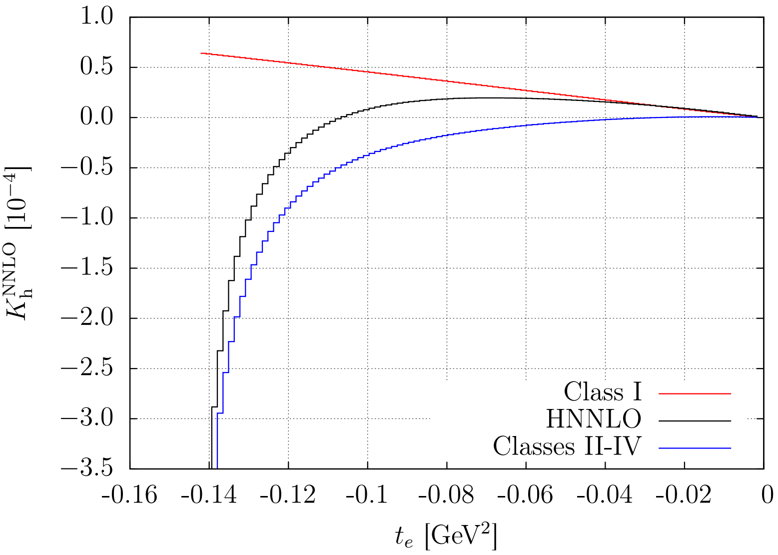


To $R(s)$

Both $\Pi_{\rm had}(t < 0)$ and $R_{\rm had}(z)$ provided by:

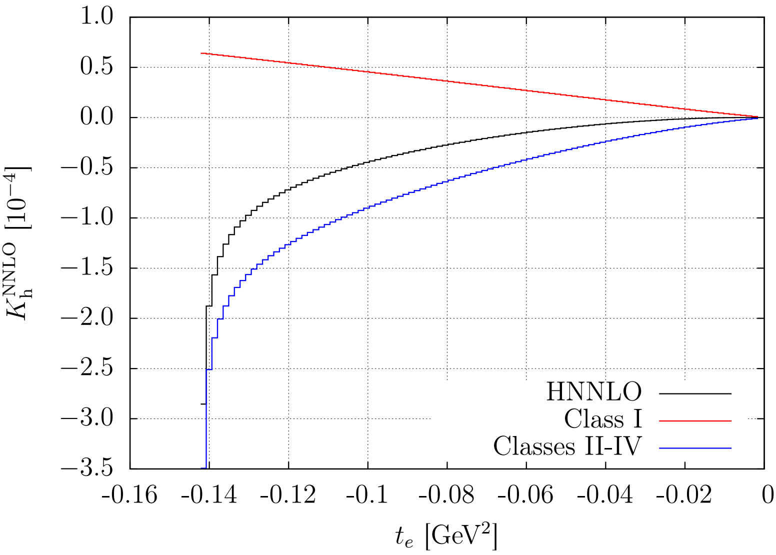
- Jegerlehner's package `alphaQED`:
  hep-ph/0105283,
  `www-com.physik.hu-berlin.de/~fjeger/alphaQEDc17.tar.gz`
- A Keshavarzi, D Nomura, T Teubner: `VP_KNT_v3_0`
  A Keshavarzi et al., PRD 97 (2018) 114025
- Top quark analytic contribution included (but negligible!).

$e^- \mu^+ \to e^- \mu^+$



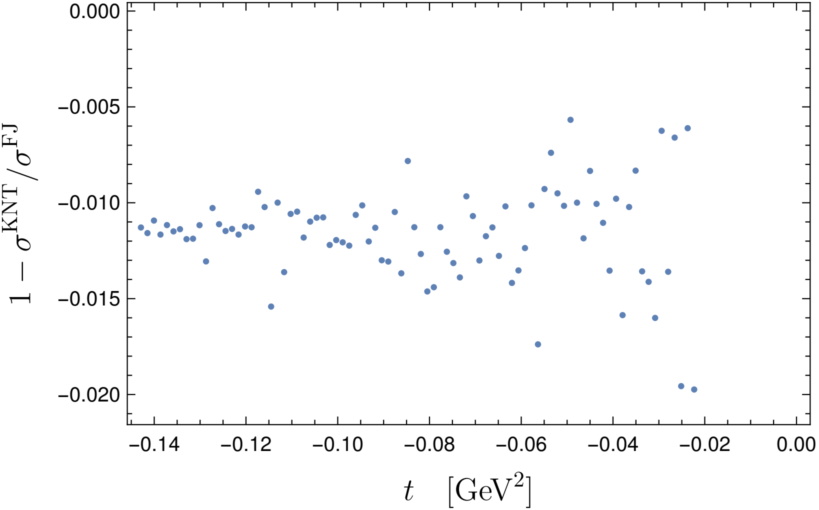
MF, M. Passera, hep-ph/1901.03106

$e^- \mu^- \to e^- \mu^-$



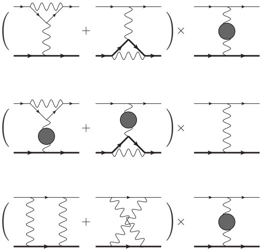
MF, M. Passera, hep-ph/1901.03106

Cross Section Uncertainty



A Potential Pitfall: IR cancellation

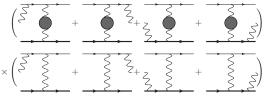
Class II



$\downarrow$

$\Pi_{\mathrm{had}}(t < 0)$

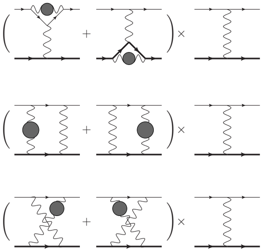
Class III



$\downarrow$

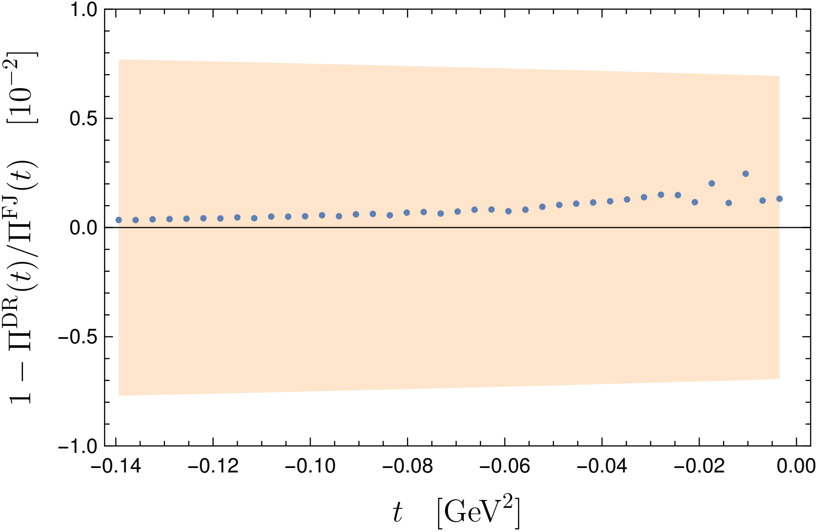
$\Pi_{\mathrm{had}}(t < 0)$

Class IV



$\downarrow$

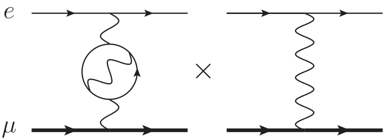
$\mathrm{Im}\Pi_{\mathrm{had}}(s > 0)$



$1 - \Pi^{\mathrm{DR}}(t)/\Pi^{\mathrm{FJ}}(t)$ $[10^{-2}]$
1.0
0.5
0.0
−0.5
−1.0
−0.14
−0.12
−0.10
−0.08
−0.06
−0.04
−0.02
0.00
t $[\mathrm{GeV}^2]$

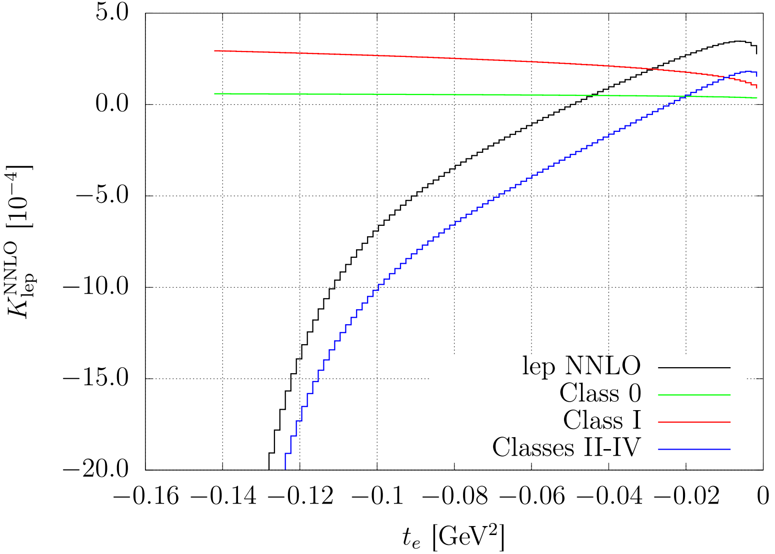
Leptonic contribution at NNLO

- $\Pi_{\rm had}(q^2) \to \Pi_{\rm lep}(q^2)$
- Classes I-IV as before
- Class 0:

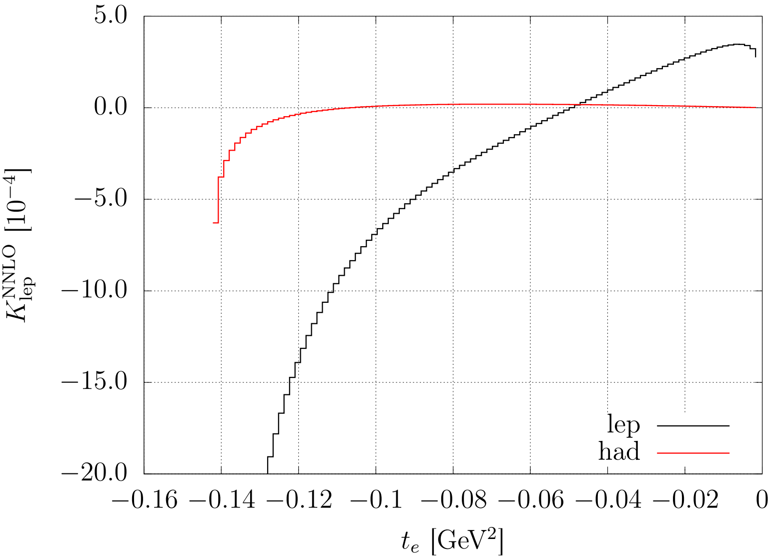


- $\Pi^{(2l)}_{\rm lep}(q^2)$ from Broadhurst, Fleischer, Tarasov, Z.Phys. C60 (1993) 287
- Pair production $\mu^\pm e^- \to \mu^\pm e^- e^+ e^-$ and $\mu^\pm e^- \to \mu^\pm e^- \mu^+ \mu^-$ under study

$e^- \mu^+ \to e^- \mu^+$


5.0
0.0
−5.0
−10.0
−15.0
−20.0
−0.16
−0.14
−0.12
−0.1
−0.08
−0.06
−0.04
−0.02
0
t_e [GeV^2]
$K_{\rm lep}^{\rm NNLO}$ [10^{-4}]
lep NNLO
Class 0
Class I
Classes II-IV

$e^- \mu^+ \to e^- \mu^+$

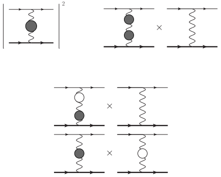


5.0
0.0
−5.0
−10.0
−15.0
−20.0
$K_{\text{lep}}^{\text{NNLO}}$ [10^{-4}]
−0.16
−0.14
−0.12
−0.1
−0.08
−0.06
−0.04
−0.02
0
t_e [GeV^2]
lep
had

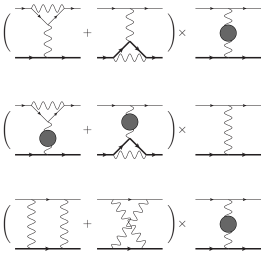
Or not to $R(s)$?

the hypersphérical method

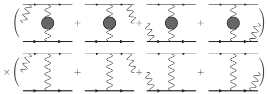
Class I



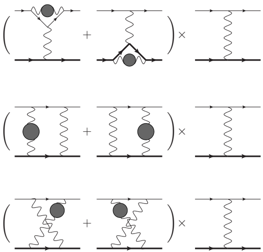
Class II



Class III



Class IV



$\Delta\alpha^{\text{had}}(t)$ ansatz

- Polynomial:

$$\Delta\alpha^{\text{had}}(t) = c_1 t + c_2 t^2 + c_3 t^3$$

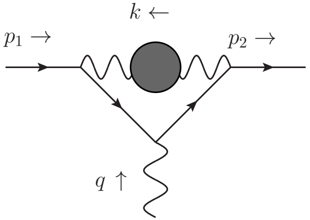
- Padé approximant:

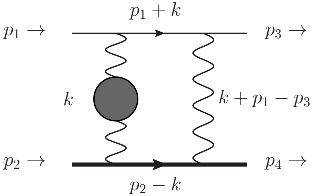
$$\Delta\alpha^{\text{had}}(t) = at\frac{1+bt}{1+ct}$$

- Fermion-like function:

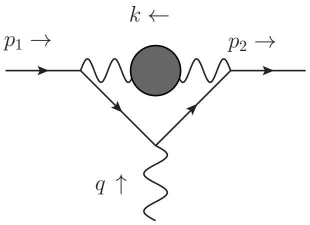
$$\Delta\alpha^{\text{had}}(t) =$$

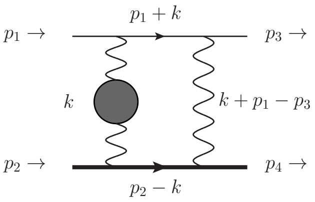
$$K\left[-\frac{5}{9} - \frac{4M}{t}\left(\frac{4M^2}{3t^2} + \frac{M}{3t} - \frac{1}{6}\right)\frac{2}{\sqrt{1-4M/t}}\log\left(\frac{1-\sqrt{1-4M/t}}{1+\sqrt{1-4M/t}}\right)\right]$$





$$I(p_1, \ldots, p_n) = \int d^4q \frac{\Pi_{\rm had}(q^2)}{q^2 + i\varepsilon} \frac{\mathcal{N}(q, p_1, \ldots, p_n)}{\mathcal{D}_1 \cdots \mathcal{D}_n}$$



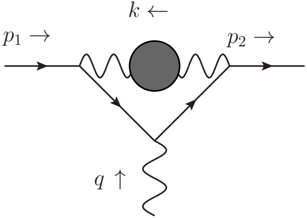


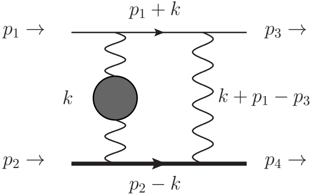
- Continue external momenta to Euclidean region:

$$p_i^2 = -P_i^2 < 0$$

- Wick rotation.

$$I(P_1, \ldots, P_n) = \int d^4Q \frac{\Pi_{\rm had}(-Q^2)}{Q^2} \frac{\mathcal{N}(Q, P_1, \ldots, P_n)}{\mathcal{D}_1 \cdots \mathcal{D}_n}$$





- Introduce spherical coordinates:

$$I(P_1, \ldots, P_n) = \int_0^\infty \frac{dQ^2}{2} \Pi_{\text{had}}(-Q^2) \int d\Omega_q \frac{\mathcal{N}(Q, P_1, \ldots, P_n)}{\mathcal{D}_1 \cdots \mathcal{D}_n}$$

The Angular Integrals

- **Euclidean** denominators expanded in Gegenbauer polynomials:

$$\frac{1}{(Q-P)^2+m^2} = \frac{Z_{QP}}{|Q||P|}\sum_{n=0}^{\infty} Z_{QP}^n \, C_n^{(1)}(\hat{Q}\cdot\hat{P})$$

with

$$Z_{QP} = \frac{Q^2+P^2+m^2-\lambda^{1/2}(Q^2,P^2,-m^2)}{2|Q||P|}$$

The Angular Integrals

- Angular integrals:

$$\int \frac{d\Omega_Q}{2\pi^2} \, C_n^{(1)}(\hat{Q} \cdot \hat{P}_i) \, C_m^{(1)}(\hat{Q} \cdot \hat{P}_j) = \frac{\delta_{nm}}{n+1} C_n^{(1)}(\hat{P}_i \cdot \hat{P}_j)$$

- Continue the result back to the physical region:

$$p_i^2 = -P^2 \to m_i^2$$

The Angular Integrals

- Angular integrals:

$$\int \frac{d\Omega_Q}{2\pi^2}\, C_n^{(1)}(\hat{Q}\cdot\hat{P}_i)\, C_m^{(1)}(\hat{Q}\cdot\hat{P}_j) = \frac{\delta_{nm}}{n+1} C_n^{(1)}(\hat{P}_i\cdot\hat{P}_j)$$

- Continue the result back to the physical region:

$$p_i^2 = -P^2 \to m_i^2$$

- For the box diagrams:

$$\int \frac{d\Omega_Q}{2\pi^2}\, C_l^{(1)}(\hat{Q}\cdot\hat{P}_i)\, C_m^{(1)}(\hat{Q}\cdot\hat{P}_j) C_n^{(1)}(\hat{Q}\cdot\hat{P}_k) = ?$$

HEP	1 records found

1. **Hyperspherical integration and the triple cross vertex graphs**

   S. Laporta (Bologna U. & INFN, Bologna). Feb 1994. 13 pp.

   Published in **Nuovo Cim. A107 (1994) 1729-1738**

   DFUB-94-01

   DOI: 10.1007/BF02780705

   e-Print: **hep-ph/9404203 |** PDF

   References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
   ADS Abstract Service

   Detailed record - Cited by 3 records

Brute-force integration:

$$\int d\Omega_Q = \int_0^{\pi} \sin^2\theta_1 d\theta_1 \int_0^{\pi} \sin\theta_2 d\theta_2 \int_0^{2\pi} d\theta_3$$

- General two propagator case:

$$\int \frac{d\Omega_Q}{[(Q-P_1)^2+m_1^2]\,[(Q-P_2)^2+m_2^2]}$$

- General three propagator case:

$$\int \frac{d\Omega_Q}{[(Q-P_1)^2+m_1^2]\,[(Q-P_2)^2+m_2^2]\,[(Q-P_3)^2+m_3^2]}$$

Sixth order contributions to the electron g-2:

- Levine and Roskies, Phys. Rev. Lett. **30** (1973) 772.
- Levine and Roskies, Phys. Rev. D **9** (1974) 421.
- Levine, Remiddi and Roskies, Phys. Rev. D **20** (1979) 2068.
- Roskies, Levine and Remiddi, Adv. Ser. Direct. High Energy Phys. **7** (1990) 162.
- Laporta and Remiddi, Phys. Lett. B **265** (1991) 182.
- Laporta and Remiddi, Phys. Lett. B **301** (1993) 440.

Pion pole contribution to a_μ^{HLBL}:

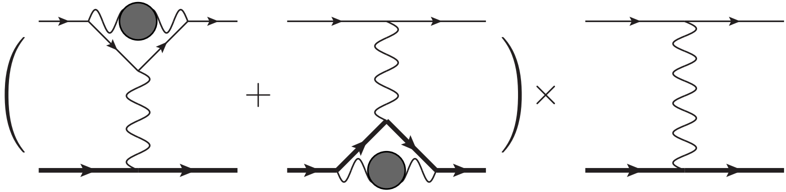
- Knecht and Nyffeler, Phys. Rev. D **65** (2002) 073034.
- Nyffeler and Jegerlehner, Phys.Rept. 477 (2009) 1

Dispersive approach to a_μ^{HLBL}:

- Colangelo, Hoferichter, Procura and Stoffer, JHEP **1409**, 091 (2014).
- Colangelo, Hoferichter, Kubis, Procura and Stoffer, Phys. Lett. B **738** (2014) 6.
- Colangelo, Hoferichter, Procura and Stoffer, JHEP **1509** (2015) 074.

Integrand decomposition of scattering amplitudes:

- Mastrolia, Peraro, Primo, JHEP 1608 (2016) 164.



Vertex Correction with Hyperspherical Approach

$$\Gamma^{\mu}(k) = \gamma^{\mu} F_1(k^2) + i\frac{\sigma^{\mu\nu} q_{\nu}}{2m_{\ell}} F_2(k^2).$$

Vertex Correction with Hyperspherical Approach

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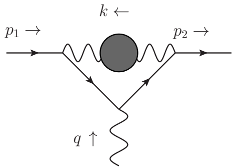
Hadronic v.p. contribution to QED form factors:



$$F_{1,2}^{\mathrm{had}}(q^2) = \int \frac{d^4q}{(2\pi)^4}\left[-\Pi_{\mathrm{had}}(q^2)\right]\frac{\text{some numerator}}{\mathcal{D}_0\mathcal{D}_1\mathcal{D}_2}$$

with

- $\mathcal{D}_0 = q^2$;
- $\mathcal{D}_{1,2} = (p_{1,2} + q)^2 - m_\ell^2$.

Hadronic Contribution to the Form Factors

$$F_{1,2}^{\mathrm{had}}(t) = -\frac{\alpha}{\pi}\int_0^1 dx\,\Pi_{\mathrm{had}}\left(\frac{m_\ell^2 x^2}{x-1}\right)\; f_{1,2}\left(x, \frac{t}{m_\ell^2}\right)$$

with $Q^2 = \frac{m_\ell^2 x^2}{1-x}$.

Hadronic Contribution to the Form Factors

$$
\begin{aligned}
f_1(x,y) =& \frac{3x^3-4x^2+4}{4(1-x)x} + \frac{2-x}{1-x}\Bigg\{ \frac{6x^2}{(4-y)^2(x-1)} + \frac{x^2-6x+4}{2(4-y)(x-1)} \\
&+ \left[\frac{\left(x^2+8x-8\right)x}{(4-y)(1-x)^2} - \frac{12x^3}{(4-y)^2(1-x)^2} + \frac{4-y}{x} + \frac{2\left(x^2+x-1\right)}{(1-x)x} \right] \\
&\times \frac{1}{\sqrt{y(y-4)}} \operatorname{arctanh}\left(\frac{(1-x)\sqrt{y\,(y-4)}}{2x+y-4-yx} \right) \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
f_2(x,y) =& \frac{2-x}{1-x}\Bigg\{ \frac{6x^2}{(4-y)^2(1-x)} + \frac{2-x}{4-y} + \left[\frac{2x}{(4-y)(1-x)} + \frac{3x^3}{(4-y)^2(1-x)^2} \right] \\
&\times \frac{4}{\sqrt{y(y-4)}} \operatorname{arctanh}\left(\frac{(1-x)\sqrt{y\,(y-4)}}{2x+y-4-yx} \right) \Bigg\}
\end{aligned}
$$

MF, JHEP 02 (2019) 027

Check

- a_μ^{HLO} for $t = 0$: $f_2(x, 0) = 1 - x$.

  T. Blum, PRL 91 (2003) 052001
- Setting $\Pi_{\rm had}(q^2) = -1$:

$$F_2^{(1l)}(q^2) = \frac{\alpha}{\pi}\frac{\xi \log \xi}{\xi^2 - 1}, \quad \text{with } q^2/m^2 = -(1-\xi)^2/\xi.$$

Check

- a_μ^{HLO} for $t = 0$: $f_2(x, 0) = 1 - x$.

  T. Blum, PRL 91 (2003) 052001

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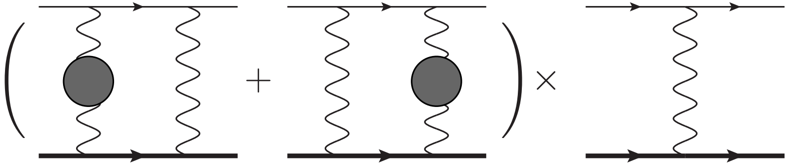
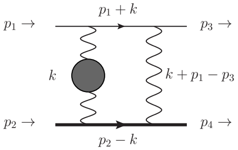
- Setting $\Pi_{\rm had}(q^2) = -q^2/(q^2 - \lambda^2)$,
  keeping non-vanishing terms in the limit $\lambda \to 0$:

$$F_1^{(1l)}(q^2)\left(\frac{\alpha}{\pi}\right)^{-1} = \log\left(\frac{\lambda}{m}\right)\left[\frac{\xi^2+1}{\xi^2-1}\log(\xi) - 1\right] + \frac{3\xi^2 + 2\xi + 3}{4\left(\xi^2 - 1\right)}\log(\xi) - 1$$
$$+ \frac{1+\xi^2}{1-\xi^2}\left[\mathrm{Li}_2(-\xi) - \frac{\log^2(\xi)}{4} + \frac{\pi^2}{12} + \log(\xi)\log(\xi+1)\right].$$

Barbieri, Mignaco and Remiddi, Nuovo Cim. A **11** (1972) 824.



Boxes with the Hyperspherical Method



$$\text{box}(s,t) = \int \frac{d^4 q}{(2\pi)^4} \Pi_{\rm had}(q^2) \frac{\text{some numerator}}{\mathcal{D}_0 \mathcal{D}_1 \mathcal{D}_2 \mathcal{D}_3}$$

$$\mathcal{D}_0 = q^2, \qquad \mathcal{D}_1 = (q + p_1)^2 - m_e^2,$$
$$\mathcal{D}_2 = (q + p_1 - p_3)^2, \quad \mathcal{D}_3 = (q - p_2)^2 - m_\mu^2.$$

- 14 "Radial" Integrations to be performed numerically:

$$I_\alpha = \frac{1}{i\pi^2}\int d^4q\,\Pi_{\rm had}(q^2)\,\cdots = \int_0^{+\infty} dQ^2\,Q^2\,\Pi_{\rm had}(-Q^2)\left\langle\,\ldots\,\right\rangle$$

- Angular integrals

$$\left\langle\,\ldots\,\right\rangle = \int \frac{d\Omega_Q}{2\pi^2}\cdots\,\Bigg|_{p_i^2\to m_i^2}$$

$$I_{0ij} = \int dQ^2\, Q^2\, \Pi_{\mathrm{had}}(-Q^2) \left\langle \frac{1}{\mathcal{D}_0 \mathcal{D}_i \mathcal{D}_j} \right\rangle$$

$$I^i_{jk} = \int dQ^2\, Q^2\, \Pi^{\mathrm{had}}(-Q^2) \left\langle \frac{q \cdot p_i}{\mathcal{D}_0 \mathcal{D}_j \mathcal{D}_k} \right\rangle$$

$$I_{ijk} = \int dQ^2\, Q^2\, \Pi^{\mathrm{had}}(-Q^2) \left\langle \frac{1}{\mathcal{D}_i \mathcal{D}_j \mathcal{D}_k} \right\rangle$$

$$I_{0ijk} = \int dQ^2\, Q^2\, \Pi^{\mathrm{had}}(-Q^2) \left\langle \frac{1}{\mathcal{D}_0 \mathcal{D}_i \mathcal{D}_j \mathcal{D}_k} \right\rangle$$

$$I_{\Delta 0ik} = \int dQ^2\, Q^2\, \Pi^{\rm had}(-Q^2) \left\langle \frac{1}{\mathcal{D}_0 \mathcal{D}_i} - \frac{\mathcal{D}_0}{\mathcal{D}_1 \mathcal{D}_2 \mathcal{D}_k} \right\rangle$$

$$I_{\Delta ijk} = \int dQ^2\, Q^2\, \Pi^{\rm had}(-Q^2) \left\langle \frac{1}{\mathcal{D}_i \mathcal{D}_j} - \frac{\mathcal{D}_0}{\mathcal{D}_1 \mathcal{D}_2 \mathcal{D}_k} \right\rangle$$

An Example:

$$I_{013} = \frac{1}{i\pi^2}\int d^4q\,\frac{\Pi^{\rm had}(q^2)}{q^2[(q+p_1)^2-m^2][(q-p_2)^2-M^2]}$$

$$= \quad \int dQ^2\,Q^2\,\Pi^{\rm had}(-Q^2)\left\langle\frac{1}{\mathcal{D}_0\mathcal{D}_1\mathcal{D}_3}\right\rangle$$

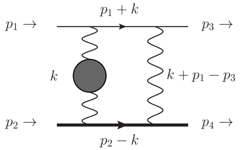
with

$$\left\langle\frac{1}{\mathcal{D}_0\mathcal{D}_1\mathcal{D}_3}\right\rangle = \frac{-2}{Q^2\lambda^{1/2}(s,M^2,m^2)}$$

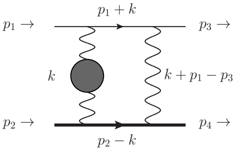
$$\times{\rm Arctanh}\left(\frac{\lambda^{1/2}(s,M^2,m^2)}{s-M^2-m^2-8M^2m^2\Big/\left[Q^2\left(1-\sqrt{1+\frac{4m^2}{Q^2}}\right)\left(1-\sqrt{1+\frac{4M^2}{Q^2}}\right)\right]}\right)$$

IR poles

$$I_{01234} = \int dQ^2\, Q^2\, \Pi^{\rm had}(-Q^2) \left\langle \frac{1}{\mathcal{D}_0 \mathcal{D}_1 \mathcal{D}_2 \mathcal{D}_3} \right\rangle$$



$$I_{01234} = \int dQ^2\, Q^2\, \Pi^{\rm had}(-Q^2) \left\langle \frac{1}{\mathcal{D}_0 \mathcal{D}_1 \mathcal{D}_2 \mathcal{D}_3} \right\rangle$$



IR singularity for $Q^2 \to |t|$

IR poles

Subtraction Method:

$$I_{ijk} = \int dQ^2 \, Q^2 \, \Pi^{\text{had}}(-Q^2) \left\langle \frac{1}{\mathcal{D}_i \mathcal{D}_j \mathcal{D}_k} \right\rangle$$

$$I_{0ijk} = \int dQ^2 \, Q^2 \, \Pi^{\text{had}}(-Q^2) \left\langle \frac{1}{\mathcal{D}_0 \mathcal{D}_i \mathcal{D}_j \mathcal{D}_k} \right\rangle$$

IR poles

Subtraction Method:

$$I_{ijk} = \int dQ^2\, Q^2 \left[\Pi^{\rm had}(-Q^2) - \Pi^{\rm had}(t) \right] \left\langle \frac{1}{\mathcal{D}_i \mathcal{D}_j \mathcal{D}_k} \right\rangle + \Pi^{\rm had}(t) \int \frac{d^D q}{i\pi^2} \frac{1}{\mathcal{D}_i \mathcal{D}_j \mathcal{D}_k}\,,$$

$$I_{0ijk} = \int dQ^2\, Q^2 \left[\Pi^{\rm had}(-Q^2) - \frac{2Q^2}{Q^2+|t|} \Pi^{\rm had}(t) \right] \left\langle \frac{1}{\mathcal{D}_0 \mathcal{D}_i \mathcal{D}_j \mathcal{D}_k} \right\rangle + 2\,\Pi^{\rm had}(t) \int \frac{d^D q}{i\pi^2} \frac{1}{(q^2-|t|)\mathcal{D}_i \mathcal{D}_j \mathcal{D}_k}\,,$$

Conclusions

- MUonE aim at measuring $\Delta\alpha(t)$ in μ-e scattering and independently determine a_μ^{HLO}.
- LoI submitted to CERN SPS on June 5th.
- SM Theory prediction must be under control at 10 ppm. Many developments already ...
- We computed hadronic NNLO corrections making use of R ratio. $K \simeq 10^{-4} - 10^{-5} \to$ relevant for MUonE.
- We showed an alternative computation via the hyperspherical method.
- It allows to employ (almost) any ansatz also at NNLO, without making use of e^+e^- data!

To Do & Theory Open Issues

- QED double-virtual matrix elements
- QED double real radiation & real-virtual corrections
- inelastic processes with leptonic and hadronic final states
- $\Delta\alpha^{\mathrm{had}}$ ansatz and NNLO hadronic corrections with spacelike data. Study model dependence.
- Extend the MC to fixed order NNLO
- Match the NNLO calculations with resummation of the log contributions
- How to implement experimental elasticity cuts for resummation.
- Reliable evaluate theory uncertainty due to missing higher order corr.
- Bound electron effects.
- New physics sensitivity of the MUonE experiment
- ...

Muon-electron scattering: Theory Kickoff Workshop

Padova, Sept. 4 - 5 2017

MITP T(r)opical Workshop

MITP Mainz, Feb. 19 - 23 2018

Organizers:

C. Carloni Calame, M. Passera, L. Trentadue, G. Venanzoni

2. Workstop/Thinkstart on Theory for μ-e scattering

Zurich, Feb. 4 - 7 2019

Organizers:
A. Signer and Y. Ulrich

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Mainz Institute for Theoretical Physics

The Evaluation of the Leading Hadronic Contribution to the Muon g-2: Toward the MUonE Experiment

30 March 2020 to 3 April 2020
Mainz Institute for Theoretical Physics, Johannes Gutenberg University
Europe/Berlin timezone

Contact @ MITP : NN

mitp@uni-mainz.de

The aim of the topical workshop is to assess the state of the art of the theoretical determination of the muon anomalous magnetic moment, a_μ , in view of the anticipated new measurement of the E989 Muon g-2 experiment in Fermilab, which is expected to improve the present precision by a factor of four, and the new approach to measure a_μ under development by the E34 collaboration at J-PARC. The workshop will focus on a_μ^{HLO} , the leading hadronic contribution to the muon g-2, both analyzing the most recent experimental and theoretical results for its determination, as well as taking into consideration the most recent dedicated lattice calculations. Particular attention will be devoted to MUonE, a recently proposed experiment at CERN which aims at measuring the running of the electromagnetic coupling constant in the spacelike region by scattering high-energy muons on atomic electrons of a low-Z target through the process $\mu e \to \mu e$. The differential cross section of this process provides the direct measurement of a_μ^{HLO} and, therefore, an independent determination of the theoretical prediction for a_μ competitive with the timelike dispersive approach.

Starts 30 Mar 2020, 08:00
Ends 3 Apr 2020, 18:00
Europe/Berlin

Mainz Institute for Theoretical Physics, Johannes Gutenberg University
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