

# Back to the future with next lepton colliders: status and perspectives for the Standard Model calculations

Janusz Gluza

in collaboration with

Ievgen Dubovyk, Ayres Freitas, Tord Riemann, Johann Usovitsch  
and Staszek Jadach, and Alain Blondel, and Patrick Janot, ...

HU Berlin, 13 June 2019

# Outline

- 1 Motivation
- 2 Half a century of the SM
- 3 Future HE experiments
- 4 Challenges for theory
  - Experimental demands
  - Intersection between QED and EW
  - S-matrix Ansatz
  - Calculation of EWPOs
  - EWPOs fits
  - EWPOs: calculational methods and results
  - Needs for EWPOs beyond 2-loops
- 5 Backup slides: numerics, FCC physics

# Motivation

The recent open European Strategy meeting in Granada as well as the discussions, indicated that

- (i)* the next machine ought to be an  $e^+e^-$  collider;
- (ii)* Europe should proceed with a flagship collider programme at CERN;
- (iii)* a vigorous R&D programme must continue to pave the way towards the highest possible centre-of-mass energy with high luminosities.

# Is there a future for our field?

*"in this field, almost everything is  
already discovered, and all that remains  
is to fill a few unimportant holes"*



Philipp von Jolly  
(1809-1884)

advice to the young Max Planck  
not to go into physics, Munich 1878

- (i) Theory
- (ii) Experiment

## The Z-boson theory (1967) – 2 pages work

S. Weinberg

"A MODEL OF LEPTONS"

VOLUME 19, NUMBER 21

PHYSICAL REVIEW LETTERS

20 NOVEMBER 1967

and

$$\varphi_1 \equiv (\varphi^0 + \varphi^{0\dagger} - 2\lambda)/\sqrt{2} \quad \varphi_2 \equiv (\varphi^0 - \varphi^{0\dagger})/i\sqrt{2}. \quad (5)$$

The condition that  $\varphi_1$  have zero vacuum expectation value to all orders of perturbation theory tells us that  $\lambda^2 \cong M_1^2/2h$ , and therefore the field  $\varphi_1$  has mass  $M_1$ , while  $\varphi_2$  and  $\varphi^-$  have mass zero. But we can easily see that the Goldstone bosons represented by  $\varphi_2$  and  $\varphi^-$  have no physical coupling. The Lagrangian is gauge invariant, so we can perform a combined isospin and hypercharge gauge transformation which eliminates  $\varphi^-$  and  $\varphi_2$  everywhere<sup>6</sup> without changing anything else. We will see that  $G_e$  is very small, and in any case  $M_1$  might be very large,<sup>7</sup> so the  $\varphi_1$  couplings will also be disregarded in the following.

The effect of all this is just to replace  $\varphi$  everywhere by its vacuum expectation value

$$\langle \varphi \rangle = \lambda \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (6)$$

The first four terms in  $\mathcal{L}$  remain intact, while the rest of the Lagrangian becomes

$$-\frac{1}{2}\lambda^2 g^2 [(A_\mu^1)^2 + (A_\mu^2)^2] - \frac{1}{2}\lambda^2 (gA_\mu^3 + g'B_\mu)^2 - \lambda G_e \bar{e}e. \quad (7)$$

We see immediately that the electron mass is  $\lambda G_e$ . The charged spin-1 field is

$$W_\mu \equiv 2^{-1/2}(A_\mu^1 + iA_\mu^2) \quad (8)$$

and has mass

$$M_W = \frac{1}{2}\lambda g. \quad (9)$$

The neutral spin-1 fields of definite mass are

$$Z_\mu = (g^2 + g'^2)^{-1/2}(gA_\mu^3 + g'B_\mu), \quad (10)$$

$$A_\mu = (g^2 + g'^2)^{-1/2}(-g'A_\mu^3 + gB_\mu). \quad (11)$$

Their masses are

$$M_Z = \frac{1}{2}\lambda(g^2 + g'^2)^{1/2}, \quad (12)$$

$$M_A = 0, \quad (13)$$

so  $A_\mu$  is to be identified as the photon field. The interaction between leptons and spin-1 mesons is

$$\frac{ig}{2\sqrt{2}} \bar{e} \gamma^\mu (1 + \gamma_5) \nu W_\mu + \text{H.c.} + \frac{igg'}{(g^2 + g'^2)^{1/2}} \bar{e} \gamma^\mu e A_\mu + \frac{i(g^2 + g'^2)^{1/2}}{4} \left[ \left( \frac{3g'^2 - g^2}{g'^2 + g^2} \right) \bar{e} \gamma^\mu e - \bar{e} \gamma^\mu \gamma_5 e + \nu \gamma^\mu (1 + \gamma_5) \nu \right] Z_\mu. \quad (14)$$

## J.C. Ward, 1950, the shortest, great publication?

182

LETTERS TO THE EDITOR

studies and the latter is studying atmospheric ionisation at ground level. These increases in ionisation are considered to be due to radioactive matter brought down with the rain. Between 0915 and 1900 hr. GMT on November 29 at Ottawa precipitation was falling. The precipitation started as snow and changed to rain about 1400 hr. Compared with the results of Dean and Wait and McNish the 35 percent increase in the salt component registered at Ottawa by counters seems too high to be explained in the same way, unless there was an exceptionally high density of radioactive matter in the atmosphere at the time. An alternative, but not very likely explanation, might be that there was a burst of hard gamma-rays or some other radiation which would increase the number of soft shower particles without any appreciable effect on the hard component.

An interesting feature of the November 19 increase is the difference between the measurements at the various stations, particularly between Resolute and Godhavn (geomagnetic latitude 80°). These two stations are about 900 miles apart and the differences confirm previous indications that sudden increments in cosmic-ray intensity occur over a limited area. The lack of a sudden decrease after the increment is unusual, since a decrease has been reported on previous occasions.

The cooperation of the Department of Transport of the Government of Canada is appreciated for supplying facilities at Resolute and for weather information.

- † A. Davall, *Comptes Rendus* 229, 1096 (1949).  
 ‡ *Fortsch. Strahlungsb. und Schein. Bull. Am. Phys. Soc.* 25, No. 1, 15 (1950).  
 § I. L. Chakraborty and S. D. Chatterjee, *Ind. J. Phys.* 23, 525 (1949).  
 ¶ *Fortsch. Gel. und Vakuum. Phys. Med. Phys.* 21, 44 (1950).  
 \* E. L. Dyer, *Phys. Rev.* 40, 107 (1938).  
 †† E. L. Dyer and M. McNish, *Weather Rev.* 62, 1 (1934).

## An Identity in Quantum Electrodynamics

J. C. WARD  
 The Cavendish Laboratory, Oxford, England  
 February 27, 1950

IT has been recently proved by Dyson<sup>1</sup> that all divergencies in the S-matrix of electrodynamics may be removed by a renormalisation of mass and charge. Dyson defines certain fundamental divergent operators  $T_n$ ,  $S_n'$ ,  $D_n'$  and gives a procedure for the calculation of their finite parts  $T_n$ ,  $S_n'$ ,  $D_n'$  by a process of successive approximation. It is then shown that

$$T_n = Z_1^{-1} T_{n1}(e_1), \quad S_n' = Z_2 S_n'(e_1), \quad D_n' = Z_2 D_n'(e_1),$$

$$e_1 = Z_1^{-1} Z_2 e, \quad Z_1 = Z_2 Z_3,$$

where  $Z_1$ ,  $Z_2$ , and  $Z_3$  are certain infinite constants and  $e_1$  is the renormalized electronic charge. Dyson conjectured that  $Z_1 = Z_2$ , and it is proposed here to give a formal proof of this relation.

In the first place, with any proper electron self-energy part  $\Pi$ , may be associated a set of proper vertex parts  $\Gamma^A$  obtained by inserting a photon line in one of the electron lines of  $\Pi$ . Now consider the operators  $A_n(V^A, \beta, \beta)$  in which the two external electron momentum variables  $\beta$  have been set equal, and the external photon variable made to vanish. Then  $A_n(V^A, \beta, \beta)$  may be obtained from  $\Sigma(W, \beta)$  by replacing  $S_F$  by  $S_F \gamma_A S_F$  at one electron line of  $W$ . Because of the identity

$$-(1/2\pi) \delta S_F / \delta \beta_A = S_F \gamma_A S_F,$$

on summing  $A_n(V^A, \beta, \beta)$  over all vertex parts  $\Gamma^A$  associated with  $W$ , one finds

$$\Sigma_n A_n(V^A, \beta, \beta) = -(1/2\pi) (\delta \Sigma(W, \beta) / \delta \beta_A).$$

(One can verify that any closed loop in  $W$  gives zero total effect.) Finally summing over all proper electron self-energy parts  $\Pi$ , one finds

$$A_n(\beta, \beta) = -(1/2\pi) (\delta \Sigma^n(\beta) / \delta \beta_A).$$

Now substitute this identity into Eqs. (91) and (95) of reference 1. One finds

$$A_n = Z_1^{-1} [(1 - Z_1) \gamma_A + \lambda_{n0}], \quad \Sigma^n = Z_1^{-1} [(Z_1 - 1) S_F^{-1} + S_F^{-1} \Sigma^n / 2\pi].$$

We have

$$-(1/2\pi) Z_1^{-1} [(Z_1 - 1) 2\pi \gamma_A + \gamma_A S_F^{-1} + (\gamma_A \beta_A - iK_n) (\delta S_F / \delta \beta_A)] = Z_1^{-1} [(1 - Z_1) \gamma_A + \lambda_{n0}(\beta, \beta)].$$

Now put

$$\gamma_A \beta_A = iK_n, \quad (\beta_A)^2 = -K_A^2.$$

The convergent parts of these equations then vanish and there is left the relation

$$-(1/2\pi) Z_1^{-1} (Z_1 - 1) 2\pi \gamma_A = Z_1^{-1} (1 - Z_1) \gamma_A,$$

which reduces immediately to  $Z_1 = Z_2$ .

† P. J. Dwyer, *Phys. Rev.* 78, 1736 (1949).

The Partial Molal Entropy of Superfluid in Pure He<sup>4</sup> below the  $\lambda$ -Point

O. K. RICE  
 Department of Chemistry, University of North Carolina,  
 Chapel Hill, North Carolina  
 March 3, 1950

IN a recent article<sup>1</sup> (the notation of which is retained here, except  $\lambda$  that subscripts 4e and 4s refer to normal fluid and superfluid, respectively, in place of I and II), I have considered the thermodynamics of liquid helium on the two-fluid theory, taking account of the fact that if two "phases" or "components," the normal fluid and the superfluid, exist together they must be in equilibrium with each other. On this basis, using the assumed relations<sup>2</sup> which states that the total molal entropy  $S$  at any temperature<sup>3</sup> is the mole fraction  $x_n$  of normal fluid times the molal entropy  $S_N$  at the  $\lambda$ -point

$$S = x_n S_N = (1 - x_n) S_\lambda \quad (1)$$

using the empirical relation for  $S$  as a function of temperature

$$S = S_0(T/T_A)^r \quad (2)$$

(with  $r=5.0$ ), and assuming that the partial molal enthalpy of superfluid,  $\bar{H}_n$ , is independent of temperature (at essentially constant pressure), and independent of  $x_n$  (i.e., there is no heat of mixing), I derived the equation for the partial molal entropy of superfluid

$$\bar{S}_n = S_N x_n / (r+1). \quad (3)$$

However, as I remarked in reference 1, there are some approximations involved in this procedure. Equation (1) is based on the assumption that below  $T_\lambda$  the entropy is contributed solely by the normal fluid, whose molal entropy is always set equal to the constant  $S_N$ , thus neglecting any temperature dependence. Furthermore, there is an implied inconsistency, since Eq. (1) assumes no entropy of mixing while Eq. (3) implies that there is a mixing entropy. In fact, in the following letter we shall show that we may derive a somewhat different expression for  $S$  from Eq. (3). We shall, therefore, discard Eq. (1) and turn to a consideration of the enthalpies.

If  $\bar{H}_n$  is independent of  $x_n$ , then  $\bar{H}_n$  must be also, and we have  $\bar{H}_n = \bar{H}_n^0$ , where  $\bar{H}_n^0$  is the enthalpy of pure normal helium. We can write for the total molal enthalpy<sup>4</sup>

$$H = x_n \bar{H}_n^0 \quad (4)$$

We will now proceed to derive an expression for  $\bar{S}_n$  in a somewhat more direct way than in reference 1, using Eq. (4) in place of Eq. (1). Since  $F = H - TS$  and  $\mu_n = \bar{H}_n - T\bar{S}_n = -T\bar{S}_n$ , the condition for internal equilibrium,  $F = \mu_n$ , gives

$$\bar{S}_n = S - H/T. \quad (5)$$

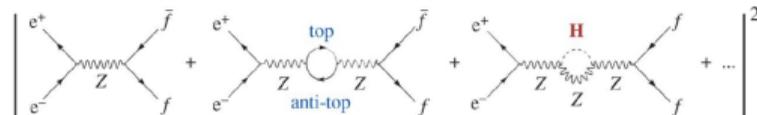
# And, 46 years of the Z-boson discovery (1973)



Gargamelle

# Dawn of the SM

- Is the SM renormalizable? In Weinberg's paper (1967):  
 "The model may be renormalizable"  
 "if this model is renormalizable, what happens when we extend it to include the couplings of  $\vec{A}_\mu$  and  $B_\mu$  to the hadrons?"
- $Z$ -boson needed? Glashow (1961):  
 found considerations on "partial symmetries" academic, "without decisive experimental consequence"
- Solution: Gargamelle, 1973. There is more photon (and  $O(3)$ , etc...)
- Direct detection  $p\bar{p}$ , UA1, UA2, CERN, 1983
- **SLC** (1980), LEP (1989) - 7-years, **17 millions of Z's**
- 1985 - 1-loop  $Z$ -decay EW correctios, global fits: top (Tevatron 1995) and Higgs indirect searches



- and LHC 2012, happened.

## F. Jegerlehner, in arXiv:1905.05078.

$$\sin^2 \Theta_i \cos^2 \Theta_i = \frac{\pi \alpha}{\sqrt{2} G_\mu M_Z^2} \frac{1}{1 - \Delta r_i} ; \quad \Delta r_i = \Delta r_i(\alpha, G_\mu, M_Z, m_H, m_{f \neq t}, m_t),$$

$$\sin^2 \Theta_W = 1 - \frac{M_W^2}{M_Z^2},$$

$$\sin^2 \Theta_g = e^2/g^2 = \frac{\pi \alpha}{\sqrt{2} G_\mu M_W^2},$$

$$\sin^2 \Theta_f = \frac{1}{4|Q_f|} \left( 1 - \frac{v_f}{a_f} \right), \quad f \neq \nu, \quad \mathbf{EXP \text{ at } M_Z!}$$

$$\Delta r_i = \Delta \alpha - f_i(\sin^2 \Theta_i) \Delta \rho + \Delta r_{i \text{ reminder}},$$

$\sim 30$  SD disagreement between some SM prediction and experiment without subleading SM corrections, and only with the leading corrections  $\Delta \alpha(M_Z^2)$  and  $\Delta \rho$ .

## F. Jegerlehner, in arXiv:1905.05078.

Example: the  $W$  and  $Z$  mass from  $\alpha(M_Z)$ ,  $G_\mu$  and  $\sin^2 \Theta_{\ell \text{ eff}}$ :

$$(i) \sin^2 \Theta_W = 1 - M_W^2/M_Z^2, \quad \sin^2 \theta_{\ell, \text{ eff}}(M_Z) = \left(1 + \frac{\cos^2 \Theta_W}{\sin^2 \Theta_W} \Delta\rho\right) \sin^2 \Theta_W,$$

$$\Delta\rho = \frac{3 M_t^2 \sqrt{2} G_\mu}{16 \pi^2}; \quad M_t = 173 \pm 0.4 \text{ GeV}$$

The iterative solution with input  $\sin^2 \theta_{\ell, \text{ eff}}(M_Z) \equiv (1 - v_\ell/a_\ell)/4 = 0.23148$  **EXP!** is  
 $\sin^2 \Theta_W = 0.22426$

$$(ii) M_W^{\text{exp}} = 80.379 \pm 0.012 \text{ GeV}; \quad M_Z^{\text{exp}} = 91.1876 \pm 0.0021 \text{ GeV}, \longrightarrow 1 - M_W^2/M_Z^2 = 0.22263$$

Predicting then the masses we have

$$M_W = \frac{A_0}{\sin^2 \Theta_W}; \quad A_0 = \sqrt{\frac{\pi\alpha}{\sqrt{2}G_\mu}}; \quad M_Z = \frac{M_W}{\cos \Theta_W}$$

where, including photon VP correction  $\alpha^{-1}(M_Z) = 128.953 \pm 0.016$ . For the  $W, Z$  mass we then get

$$M_W^{\text{the}} = 81.1636 \pm 0.0346 \text{ GeV}; \quad M_Z^{\text{the}} = 92.1484 \pm 0.0264 \text{ GeV}.$$

Deviations (errors added in quadrature):  $W$ :  $23\sigma$ ;  $Z$ :  $36\sigma$

## F. Jegerlehner, in arXiv:1905.05078.

Fred Jegerlehner (in report):

"The result is of course scheme-dependent, but illustrates well the sensitivity to taking into account the proper radiative corrections. Actually, including full one-loop and leading two-loop corrections reduces the disagreement below the  $2\sigma$  level."

Alain Blondel (e-mail):

"...to me this shows that one should *\*never, ever\** define  $\sin^2 \theta_W$  from the ratio of W and Z masses, and think of it as a mixing angle. This is an old mistake that has cost many many wrong conclusions up to this day, the meaning of both value *\*and error\** are different."

## 1-, some 2-, 3-, 4-loop contributions, from 1604.00406

Update?

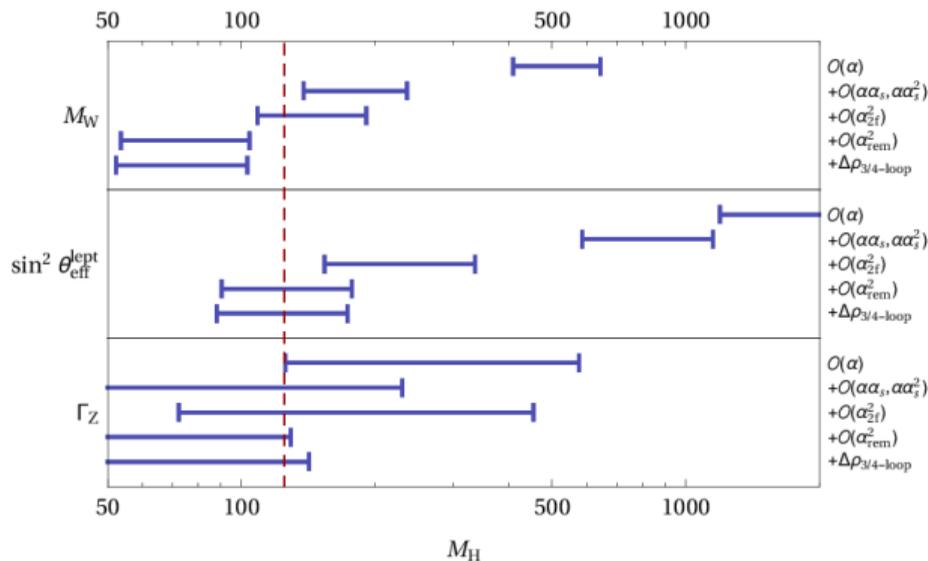


Figure 12: Impact of higher-order corrections on the indirect determination of  $M_H$  from  $M_W$  (predicted from  $G_F$ ),  $\sin^2 \theta_{\text{eff}}^{\text{lept}}$  and  $\Gamma_Z$ . Corrections of different order are cumulatively added as indicated on the right. Here  $O(\alpha^2_{2f})$  stands for electroweak two-loop contributions with two closed fermion loops,  $O(\alpha^2_{\text{rem}})$  denotes the remaining two-loop contributions (with one or no closed fermion loop), and  $\Delta\rho_{3/4\text{-loop}}$  are leading 3- and 4-loop corrections of  $O(\alpha^2\alpha_s)$ ,  $O(\alpha^3)$  and  $O(\alpha\alpha_s^3)$ . The error bars reflect the parametric uncertainty from the input parameters in Tab. B, but not the theory error from missing higher orders. The dashed lines indicates the value  $M_H = 125$  GeV from the direct measurement of the Higgs mass.

## Rich physics

Erler, Freitas, PDG'17

Presently:

Very good agreement

theory — experiment

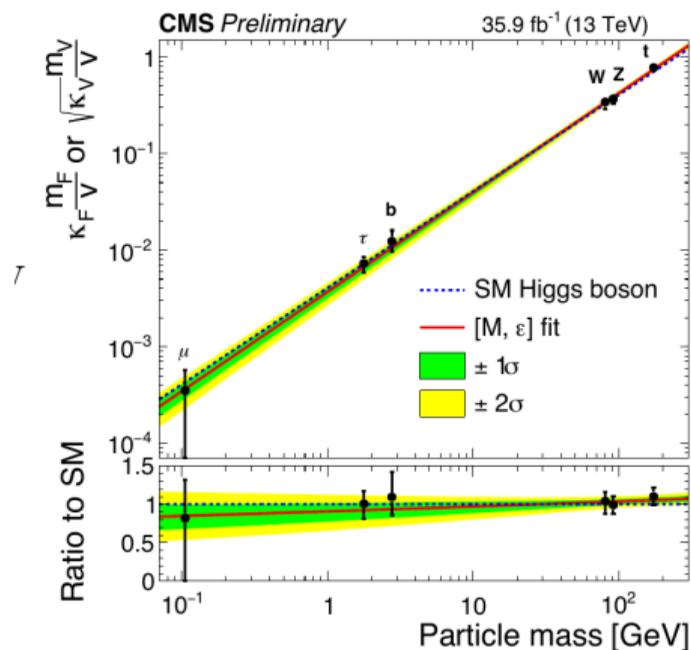
over large number of EWPOs

**Table 10.5:** Principal  $Z$  pole observables and their SM predictions (*cf.* Table 10.4). The first  $\tilde{s}_\ell^2$  is the effective weak mixing angle extracted from the hadronic charge asymmetry, the second is the combined value from the Tevatron [164–166], and the third from the LHC [170–172]. The values of  $A_e$  are (i) from  $A_{LR}$  for hadronic final states [159]; (ii) from  $A_{LR}$  for leptonic final states and from polarized Bhabha scattering [161]; and (iii) from the angular distribution of the  $\tau$  polarization at LEP 1. The  $A_\tau$  values are from SLD and the total  $\tau$  polarization, respectively.

Quantity	Value	Standard Model	Pull
$M_Z$ [GeV]	$91.1876 \pm 0.0021$	$91.1880 \pm 0.0020$	-0.2
$\Gamma_Z$ [GeV]	$2.4952 \pm 0.0023$	$2.4943 \pm 0.0008$	0.4
$\Gamma(\text{had})$ [GeV]	$1.7444 \pm 0.0020$	$1.7420 \pm 0.0008$	—
$\Gamma(\text{inv})$ [MeV]	$499.0 \pm 1.5$	$501.66 \pm 0.05$	—
$\Gamma(\ell^+\ell^-)$ [MeV]	$83.984 \pm 0.086$	$83.995 \pm 0.010$	—
$\sigma_{\text{had}}[\text{nb}]$	$41.541 \pm 0.037$	$41.484 \pm 0.008$	1.5
$R_e$	$20.804 \pm 0.050$	$20.734 \pm 0.010$	1.4
$R_\mu$	$20.785 \pm 0.033$	$20.734 \pm 0.010$	1.6
$R_\tau$	$20.764 \pm 0.045$	$20.779 \pm 0.010$	-0.3
$R_b$	$0.21629 \pm 0.00066$	$0.21579 \pm 0.00003$	0.8
$R_c$	$0.1721 \pm 0.0030$	$0.17221 \pm 0.00003$	0.0
$A_{FB}^{(0,e)}$	$0.0145 \pm 0.0025$	$0.01622 \pm 0.00009$	-3.7
$A_{FB}^{(0,\mu)}$	$0.0169 \pm 0.0013$		0.5
$A_{FB}^{(0,\tau)}$	$0.0188 \pm 0.0017$		1.5
$A_{FB}^{(0,b)}$	$0.0992 \pm 0.0016$	$0.1031 \pm 0.0003$	-2.4
$A_{FB}^{(0,c)}$	$0.0707 \pm 0.0035$	$0.0736 \pm 0.0002$	-0.8
$A_{FB}^{(0,s)}$	$0.0976 \pm 0.0114$	$0.1032 \pm 0.0003$	-0.5
$\tilde{s}_\ell^2$	$0.2324 \pm 0.0012$	$0.23152 \pm 0.00005$	0.7
	$0.23185 \pm 0.00035$		0.9
	$0.23105 \pm 0.00087$		-0.5
$A_e$	$0.15138 \pm 0.00216$	$0.1470 \pm 0.0004$	2.0
	$0.1544 \pm 0.0060$		1.2
	$0.1498 \pm 0.0049$		0.6
$A_\mu$	$0.142 \pm 0.015$		-0.3
$A_\tau$	$0.136 \pm 0.015$		-0.7
	$0.1439 \pm 0.0043$		-0.7
$A_b$	$0.923 \pm 0.020$	$0.9347$	-0.6
$A_c$	$0.670 \pm 0.027$	$0.6678 \pm 0.0002$	0.1
$A_s$	$0.895 \pm 0.091$	$0.9356$	-0.4

NOW: LHC

## LHC &amp; Higgs

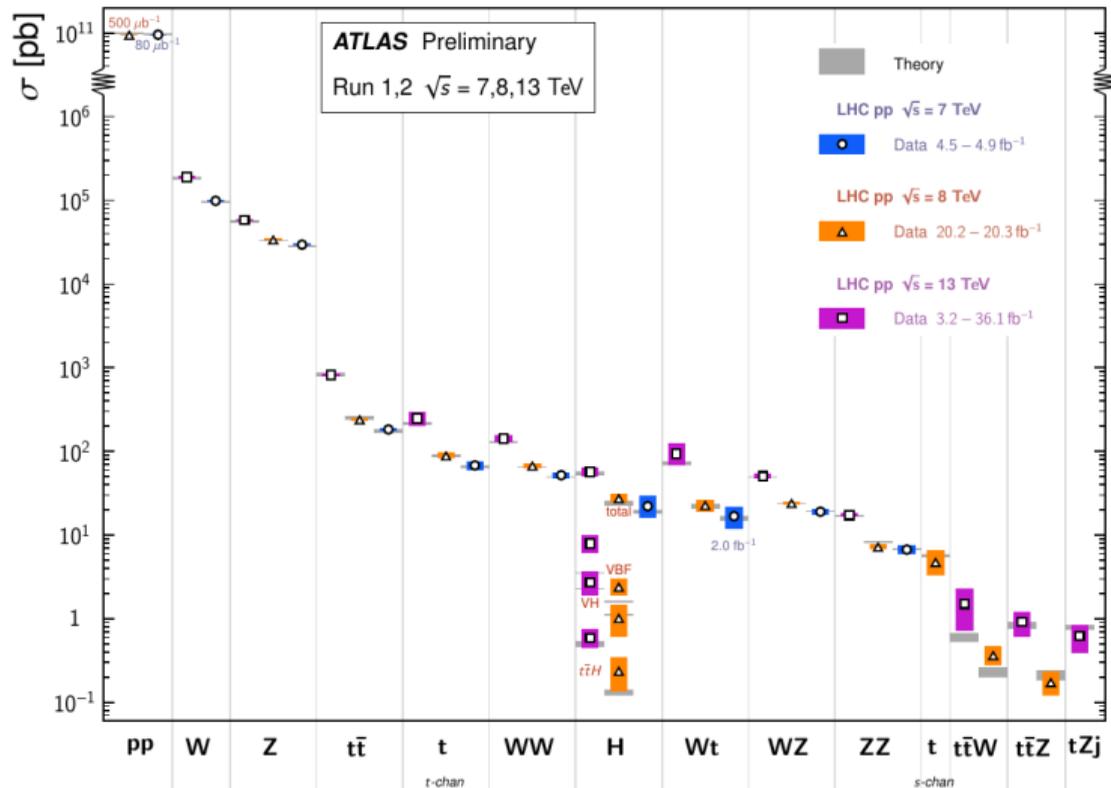


Known with 10-20% accuracy: in future  $\sim 1-5\%$  possible (HL-LHC)

## SM at the TeV scale

## Standard Model Total Production Cross Section Measurements

Status: March 2018

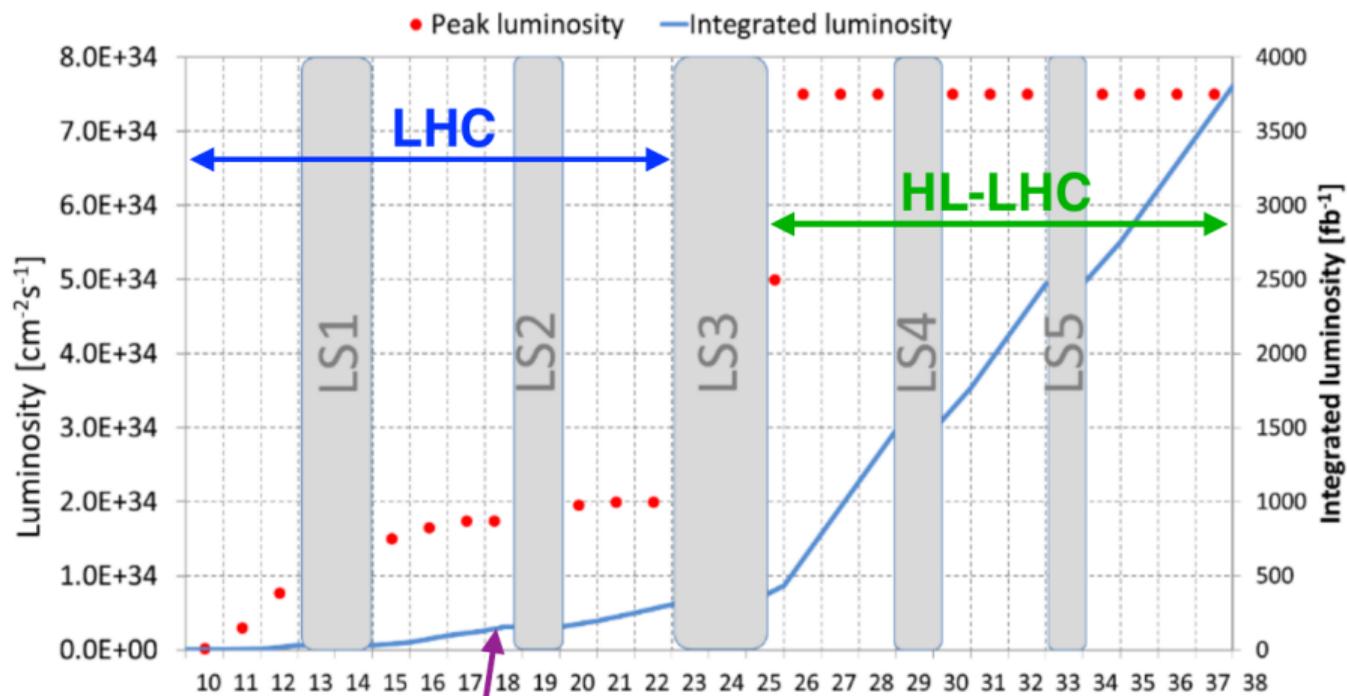


... and FUTURE?

Theory shaped by experiment

# HL-LHC/HE-LHC, FCC-hh, Ion, Ion-e, LHeC/FCC-eh, b/c/tau, muon, CEPC, SppC, FCC-ee, ILC, CLIC

## LHC: next steps



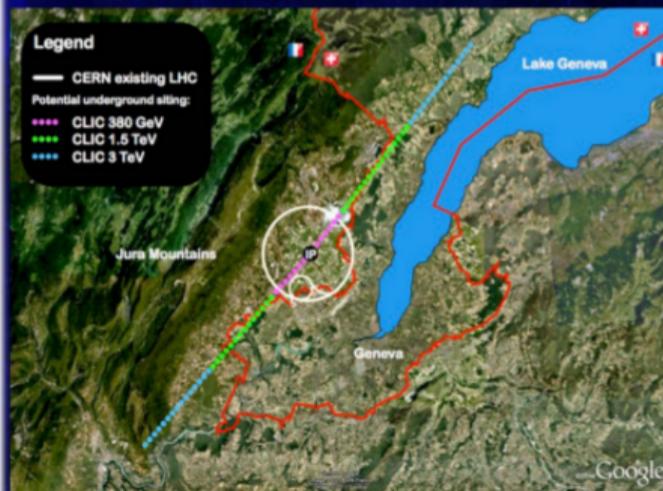


# Future Linear $e^+e^-$ Colliders



**ILC**

International Linear Collider,  
Kitakami, Japan

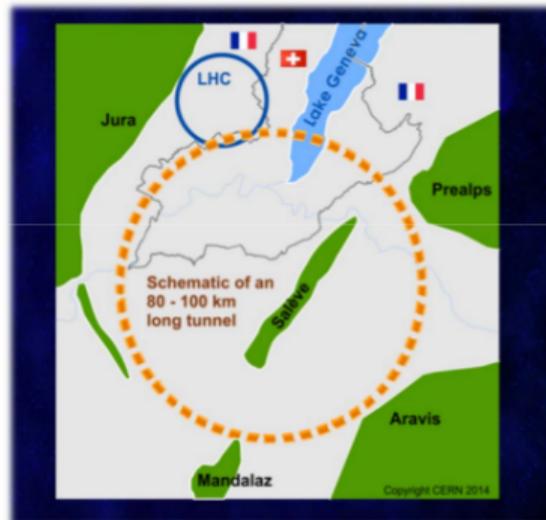


**CLIC**

Compact Linear Collider,  
CERN



# Future Circular $e^+e^-$ Colliders

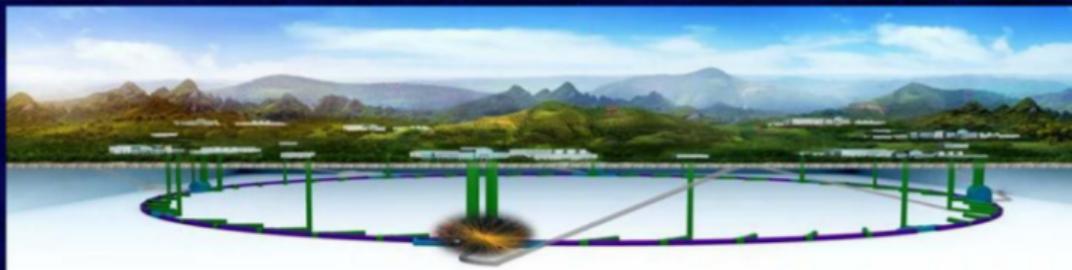


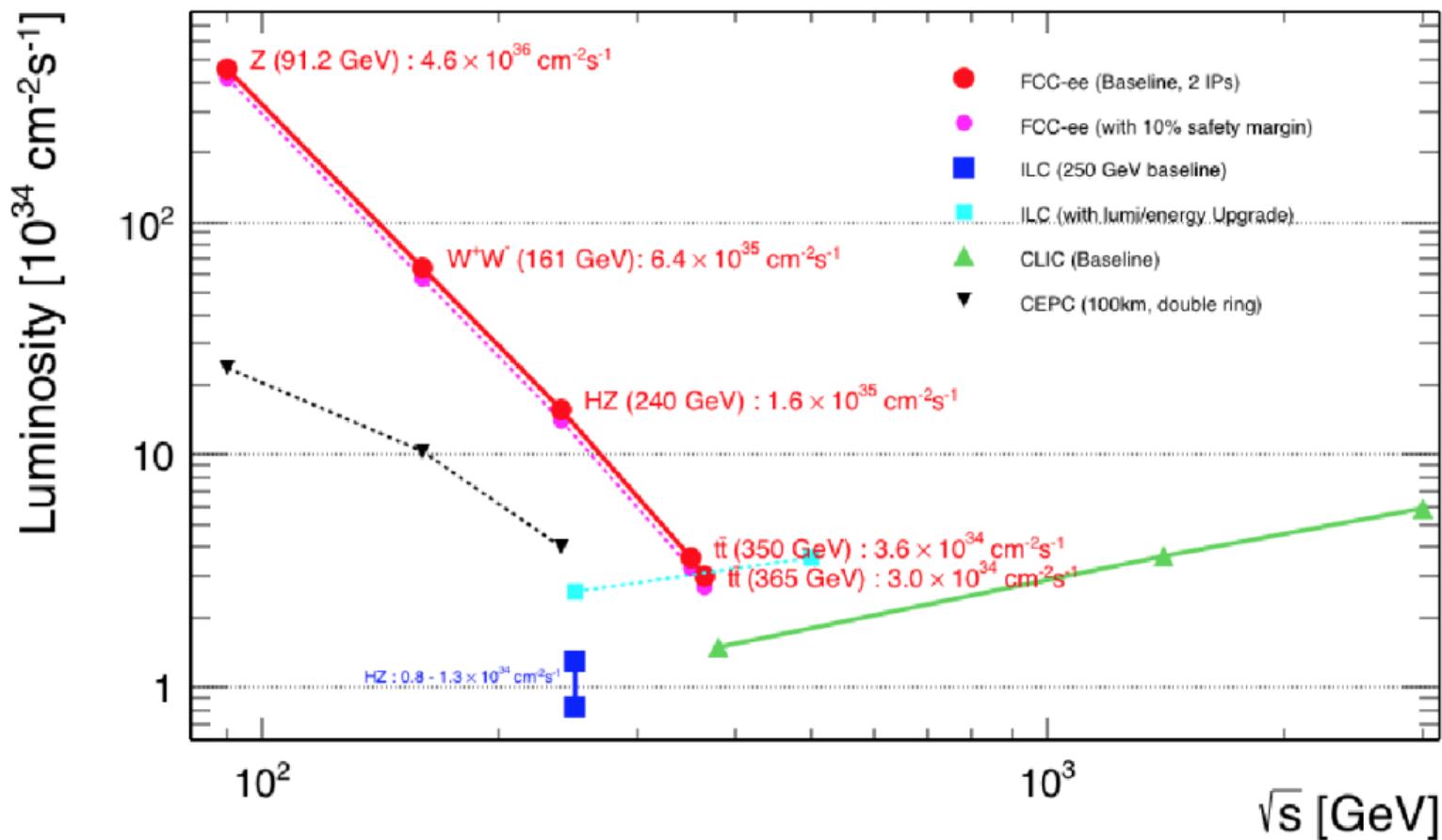
## FCC – $ee$

Future Circular Collider,  
CERN

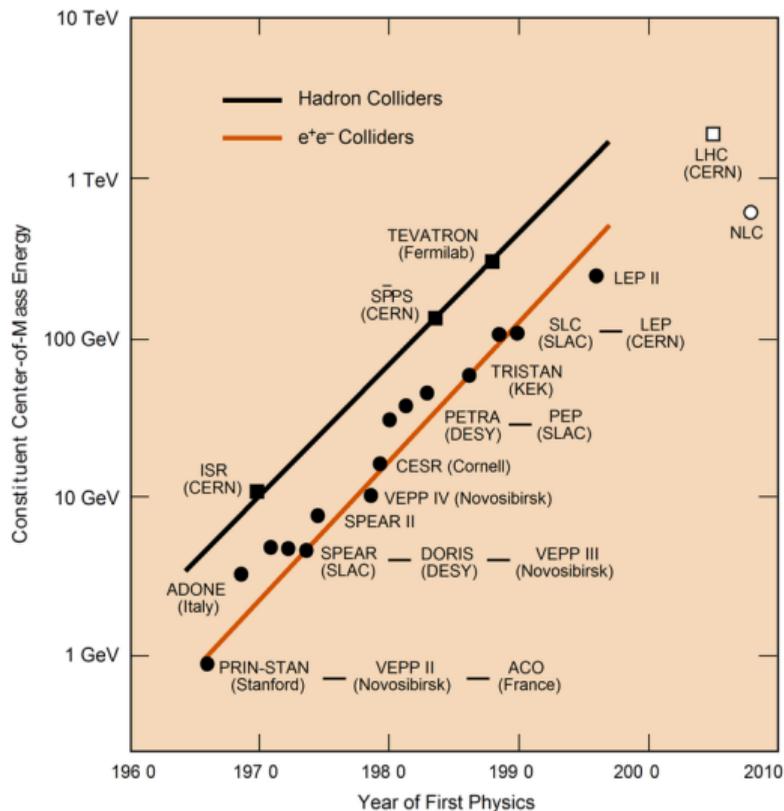
## CEPC

Circular Electron Positron Collider,  
China





# How many colliders so far?



## FCC-ee: Your Questions Answered

Contribution to the European Particle Physics Strategy Update 2018-2020

(See next page for the list of authors)

### Abstract

This document answers in simple terms many FAQs about FCC-ee, including comparisons with other colliders. It complements the FCC-ee CDR [1] and the FCC Physics CDR [2] by addressing many questions from non-experts and clarifying issues raised during the European Strategy symposium in Granada, with a view to informing discussions in the period between now and the final endorsement by the CERN Council in 2020 of the European Strategy Group recommendations. This document will be regularly updated as more questions<sup>1</sup> appear or new information becomes available.

arXiv:1906.02693v1 [hep-ph] 6 Jun 2019

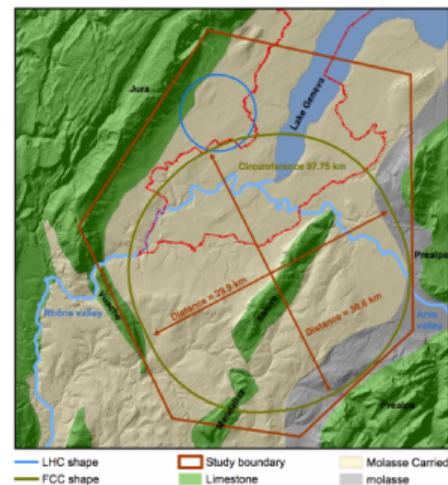


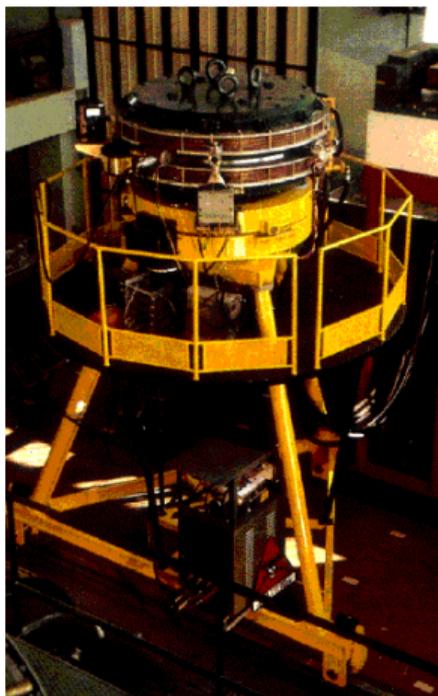
Figure 1: Baseline FCC tunnel layout with a perimeter of 97.5 km, and optimized placement in the Geneva basin, showing the main topographical and geological features.

<sup>1</sup>Send your questions to [patrick.jasot@cern.ch](mailto:patrick.jasot@cern.ch) and [alsin.blondel@cern.ch](mailto:alsin.blondel@cern.ch)

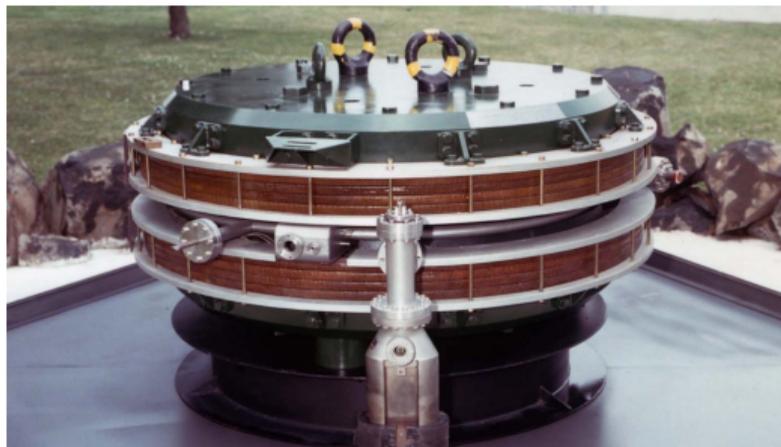
## Contents

<b>1</b>	<b>What is FCC-ee?</b>	<b>5</b>	
<b>2</b>	<b>Can I do Higgs physics in the first year of FCC-ee?</b>	<b>6</b>	
<b>3</b>	<b>How can the FCC-ee Machine Parameters reach such High Luminosities?</b>	<b>6</b>	
3.1	What is the basis for the FCC-ee machine parameters? . . . . .	7	
3.2	How do circular and linear $e^+e^-$ colliders compare in this respect? . . . . .	7	
3.2.1	<i>Historical record</i> . . . . .	7	
3.2.2	<i>Beam sizes</i> . . . . .	8	
3.2.3	<i>Positron source</i> . . . . .	8	
3.2.4	<i>Beam emittance</i> . . . . .	9	
3.3	Summary . . . . .	9	
<b>4</b>	<b>How will the FCC-ee Detectors deal with Beam Backgrounds?</b>	<b>9</b>	
<b>5</b>	<b>How good is the FCC-ee as a Higgs Factory?</b>	<b>9</b>	
<b>6</b>	<b>How Many Interaction Points at FCC-ee?</b>	<b>11</b>	
<b>7</b>	<b>Do we need an <math>e^+e^-</math> Energy of at least 500 GeV to Study the Higgs Boson Thoroughly?</b>	<b>12</b>	
<b>8</b>	<b>Why are the FCC-ee Beams not Polarized Longitudinally?</b>	<b>13</b>	
8.1	A choice: Longitudinal or Transverse Polarization? . . . . .	13	
8.2	Longitudinal GigaZ vs Transverse TeraZ . . . . .	14	
8.3	Longitudinal Polarization and Higgs Coupling Determination . . . . .	16	
<b>9</b>	<b>Will the Accuracy of FCC-ee Higgs Measurements be Affected by Experimental Uncertainties?</b>	<b>18</b>	
<b>10</b>	<b>How does a Muon Collider compare (as a Higgs Factory)?</b>	<b>19</b>	
<b>11</b>	<b>Can I do more than Higgs Physics at FCC-ee?</b>	<b>20</b>	
<b>12</b>	<b>Why do we need At Least <math>5 \times 10^{12}</math> Z Decays?</b>	<b>21</b>	
<b>13</b>	<b>Why is FCC-ee More Precise for Electroweak Measurements?</b>	<b>23</b>	
<b>14</b>	<b>Will Theory be Sufficiently Precise to Match this Experimental Precision?</b>	<b>24</b>	
<b>15</b>	<b>What can be discovered at FCC-ee?</b>	<b>25</b>	
<b>16</b>	<b>Is the FCC-ee Project "Ready to Go"?</b>	<b>26</b>	
			<b>17 What is the cost of the FCC-ee? . . . . . 26</b>
			17.1 What are the FCC-ee Construction Costs? . . . . . 26
			17.2 What are the Costs of Operating FCC-ee? . . . . . 27
			<b>18 Can FCC-ee be the First Stepping Stone for the Future of our Field? 27</b>
			<b>19 Can there be a Smooth Transition between HL-LHC and FCC-ee Experiments? 29</b>
			<b>20 Can Physics start at FCC-ee right after HL-LHC? 29</b>
			<b>21 Will FCC-ee delay FCC-hh? 30</b>
			<b>22 How long will the Shutdown between FCC-ee and FCC-hh be? 30</b>
			<b>23 Are there Better Ways to 100 TeV than FCC-ee? 31</b>
			23.1 Learning from History . . . . . 32
			23.2 Looking at the numbers . . . . . 33
			23.3 Should we by-pass FCC-ee and go directly for a 100 or 150 TeV Hadron Collider? . 33
			23.4 Should we by-pass FCC-ee and opt for a High-energy Upgrade of the LHC instead? 33
			23.5 Rather than starting with FCC-ee, should we build a Lower-Energy Hadron Collider in the FCC Tunnel? . . . . . 34
			23.6 Why not a Low-Energy Linear $e^+e^-$ Collider instead? . . . . . 35
			23.7 Should we leave FCC-ee to China? . . . . . 36
			<b>24 Why do we want the FCC in Europe? 36</b>

"The first accelerator dates back to prehistoric-historic times, when men built bows and arrows for hunting.", S.Y. Lee, "Accelerator Physics"

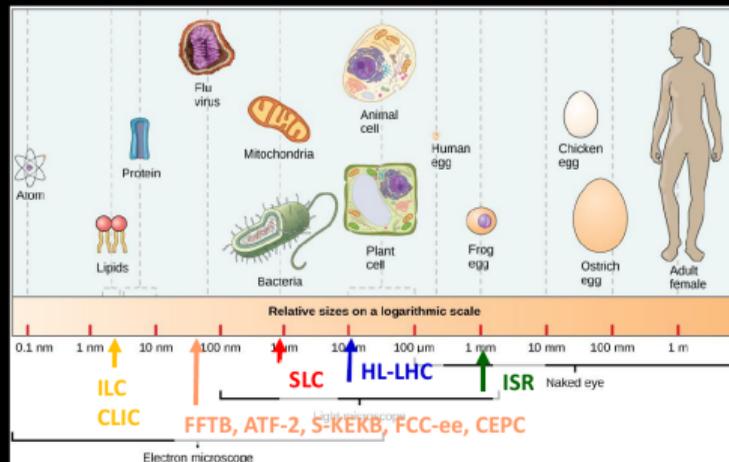


ADA/ADONE: The first [circular]  $e^+e^-$  collider  
**1969**-1993, Frascati,  $\sqrt{s} \leq 3$  GeV



# How do circular and linear $e^+e^-$ colliders compare?

## vertical spot size challenge

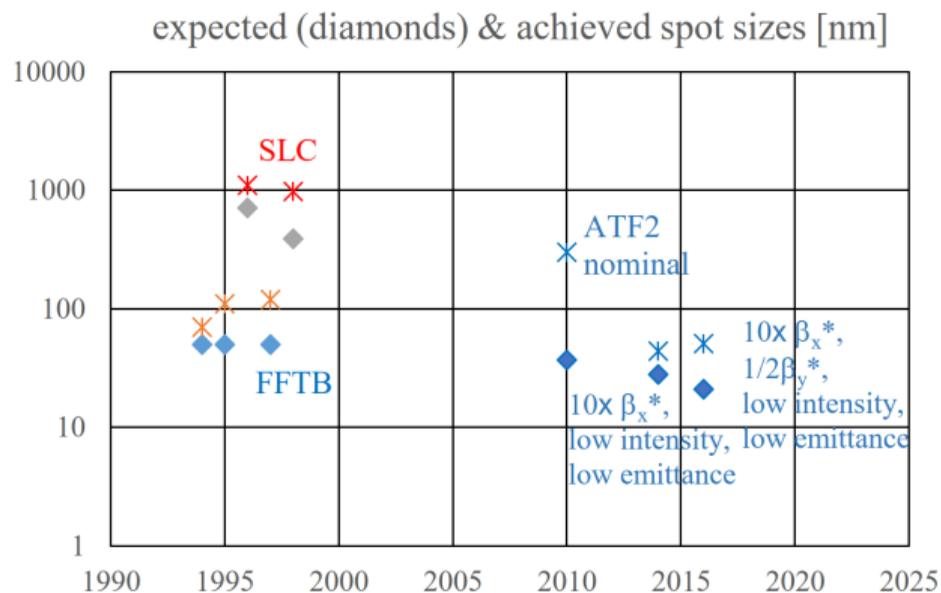


FCC-ee in the regime of FFTB, ATF-2, and especially SuperKEKB

F. Zimmermann, 11th FCC-ee Workshop, CERN, January 2019  
<https://indico.cern.ch/event/766859>

# How do circular and linear $e^+e^-$ colliders compare?

## (i) Beam sizes



More issues: (ii) Positron source (20 ÷ 40 times more needed than today's SLC record); (iii) Beam emittance;

# Costs, and who is greener?

- ① 4 GCHF for the FCC-ee collider and injector;
- ② 17 GCHF for the FCC-hh collider and injector (of which 9.4 GCHF for the magnets);
- ③ 7.6 GCHF for the common civil engineering and technical infrastructure.

ILC<sub>250</sub> - 6.5 GCHF, without the site construction;

First stage of CLIC (of the order of 6 GCHF).

CEPC - 6.5 GCHF.

FCC-ee: 1.7 TWh/year; 1.2 TWh/year LHC and the expected 1.4 TWh/year for HL-LHC.

CLIC<sub>380</sub>: 0.8 TWh/year

LEP2 energy consumption ranged between 0.9 and 1.1 TWh/year

**Table:** Operation costs of low-energy Higgs factories, expressed in euros per Higgs boson.

Collider	ILC <sub>250</sub>	CLIC <sub>380</sub>	FCC-ee <sub>240</sub>
Cost (Euros/Higgs)	7,000 to 12,000	2,000	255

# Energy: generation, consumption, storage. Magnets, RF, cooling, computing

## Orders of magnitude

generation	consumption	storage
1d cyclist „Tour de France“ (4h x 300W): <b>1.2 kWh</b>	1 run of cloth washing machine: <b>0.9 kWh</b>	Car battery (60 Ah): <b>0.72 kWh</b> BEV <sup>1</sup> Battery (Lithium-ion): <b>100 kWh</b>
1d Wind Power Station (avg): <b>12 MWh</b>	1d SwissLightSource 2.4 GeV, D.4 A: <b>82 MWh</b>	All LHC magnets @ 8.33 T: <b>3 MWh</b> ITER superconducting coil: <b>12.5 MWh</b>
1d nucl. Pow. Plant (e.g. Leibstadt, CH): <b>30 GWh</b>	1d CLIC Linear Collider @ 3 TeV c.m.: <b>14 GWh</b>	All German storage hydropower: <b>40 GWh</b>
1d Earth/Moon System E-loss: <b>77 TWh</b>	1d electrical consumption mankind: <b>53 TWh</b>	Energy stored in all BEVs <sup>1</sup> : <b>0(0.5 TWh)</b> World storage hydropower: <b>0(1 TWh)</b>
1d sunshine absorbed on Earth: <b>3,000,000 TWh = 3 EW (E<sub>16</sub> = 10<sup>16</sup>)</b>	1d total consumption humankind: <b>360 TWh</b>	Energy storage seems not to scale up!

<sup>1</sup> Battery Electric Vehicle

- Accelerators are in the range were they become relevant for society and public discussion.
- Desired turn to renewables is an enormous task; storage is the problem, not production!
- Fluctuations of energy availability, depending on time and weather, will be large!

14 May 2019 ESPFu Open Symposium, Granada E. Jensen, Energy Efficiency M. Seidel/PSI 7

## Energy not wasted is gained resource

International Energy Agency

In advanced economies, new sources of electricity demand growth such as digitalization and electrification of heat and mobility have been outpaced by savings from energy efficiency. In the absence of energy efficiency improvements, electricity demand in advanced economies would have grown at 1.6% per year since 2010, instead of 0.3%.

Electricity consumption in advanced economies, 2000-2017

Energy efficiency measures adopted since 2000 saved almost 1 800 TWh in 2017 or the equivalent of around 20% of overall current electricity use.

14 May 2019 ESPFu Open Symposium, Granada E. Jensen, Energy Efficiency 10

## Energy Management - example CERN: consumption

European Strategy Update

Multi-years cycles for LHC

LEP2

LHC

HL-LHC

Yearly Energy Consumption [GW.h]

Cycles: 9m+3m

Cycles: 3 or 4 x (10m+2m) + 1.5yr

1990 1995 2000 2005 2010 2015 2020 2025 2030

— Total (old) — Total Flyers — Forecast

V. Mertens/CERN

14 May 2019 ESPFu Open Symposium, Granada E. Jensen, Energy Efficiency 14

# Theory uncertainties for EWK physics

ILC and FCC-ee have great potential for high-precision Z, WW, and Higgs physics

Can theory provide the necessary precision?

↪ **Optimists:** "Yes. No show-stoppers seen, great progress can be anticipated."

**Sceptics:** "Enormous challenge! Conceptual progress difficult to extrapolate."

Some warnings:

- Produce solid and conservative uncertainty estimates!
- Always combine experimental and theoretical uncertainties!
- Employ different theoretical strategies and exp. analyses as much as possible!  
(e.g. for  $\alpha_s$ ,  $\Delta\alpha_{\text{had}}$ )

The greatest challenges: (+ many more very demanding tasks)

- **Z:**
  - ◊ full EW 2-loop calculation for off-shell  $e^+e^- \rightarrow f\bar{f}$   
+ theoretically sound concept of pseudo-observables
  - ◊ massive 3-loop calculations for  $1 \rightarrow 2$  decays and  $\mu$  decay
- **WW:**
  - ◊ NNLO threshold EFT calculation for  $e^+e^- \rightarrow WW$
- **Higgs:**
  - ◊ full EW 2-loop calculation for off-shell  $e^+e^- \rightarrow ZH$
  - ◊ massless 4-/5-loop QCD calculations for  $1 \rightarrow 2$  decays

↪ Certainly takes another generation of bright minds!

**Standard Model Theory for the FCC-ee: The Tera-Z**

A. Blondel (Geneva U.), J. Gluza (Silesia U.), S. Jadach (Cracow, INP), P. Janot (CERN), T. Riemann (Silesia U. & DESY, Zeuthen), A. Khudov (Istancia U. & Baku, Inst. Phys.), A. Arbuzov (Dubna, JINR), R. Boels (Hamburg U., Inst. Theor. Phys. IB), S. Bondarenko (Dubna, JINR), S. Borowka (CERN) et al. [View all 38 authors](#)

Sep 6, 2018 - 243 pages

Conference: [C19-01-12](#)

BU-HEPP-18-04, CERN-TH-2018-145, IFJ-PAN-IV-2018-09, KW-18-003, MITP-18-062, MPP-2018-143, SI-HEP-2018-21

e-Print: [arXiv:1809.01830](#) [hep-ph] | [PDF](#)

**Abstract** (arXiv)

The future 100-km circular collider FCC at CERN is planned to operate in one of its modes as an electron-positron FCC-ee machine. We give an overview of the theoretical status compared to the experimental demands of one of four foreseen FCC-ee operating stages, which is Z-boson resonance energy physics, FCC-ee Tera-Z, stage for short. The FCC-ee Tera-Z will deliver the highest integrated luminosities as well as very small systematic errors for a study the Standard Model (SM) with unprecedented precision. In fact, the FCC-ee Tera-Z will allow to study at least one more quantum field theoretical perturbative order compared to the LEP/SLC precision. The real problem is that the present precision of theoretical calculations of the various observables within the SM does not match that of the anticipated experimental measurements. The bottle-neck problems are specified. In particular, the issues of precise QED unfolding and of the correct calculation of SM pseudo-observables are critically reviewed. In an Executive Summary we specify which basic theoretical calculations are needed to meet the strong experimental expectations at the FCC-ee Tera-Z. Several methods, techniques and tools needed for higher order multi-loop calculations are presented. By inspection of the Z-boson partial and total decay widths analysis, arguments are given that at the beginning of operation of the FCC-ee Tera-Z, the theory predictions may be tuned to be precise enough not to limit the physics integration of the measurements. This statement is based on the anticipated progress in analytical and numerical calculations of multi-loop and multi-scale Feynman integrals and on the completion of two-loop electroweak radiative corrections to the SM pseudo-observables this year. However, the above statement is conditional as the theoretical issues demand a very dedicated and focused investment by the community.

**Note:** 243 pages, Report on the 1st Mini workshop: Precision EW and QCD calculations for the FCC studies: methods and tools, 12-13 January 2018, CERN, Geneva, Switzerland

**Keywords:** INSPIRE: ["Automatic, Keywords:"](#) | [electroweak interaction](#) | [radiative correction](#) | [resonance energy](#) | [Z0 resonance](#) | [decay width](#) | [FCC-ee](#) | [quantum electrodynamics](#) | [numerical calculations](#) | [field theory](#) | [electron](#)

**Theory report on the 11th FCC-ee workshop**

A. Blondel (ed.) (Geneva U.), J. Gluza (ed.) (Silesia U. & Hradec Králové U.), S. Jadach (ed.) (Cracow, INP), P. Janot (ed.) (CERN), T. Riemann (ed.) (Silesia U. & DESY, Zeuthen)

May 13, 2019 - 290 pages

Conference: [C19-01-08.1](#)

BU-HEPP-19-03, CERN-TH-2019-061, CP3-19-22, DESY-19-072, FR-PHENO-2019-005, IFIC/19-23, IFT-UAM-CSIC-19-058,

IPhT-19-050, IPPP/19/32, KW-19-003, MPP-2019-84, LTH 1203, ZU-TH-22-19, TUM-HEP-1200-19, TTP19-008, TTK-19-19

e-Print: [arXiv:1905.05078](#) [hep-ph] | [PDF](#)

Experiment: [CERN-FCC](#)

[Contributions](#)

**Abstract** (arXiv)

The FCC at CERN, a proposed 100-km circular facility with several colliders in succession, culminates with a 100 TeV proton-proton collider. It offers a vast new domain of exploration in particle physics, with orders of magnitude advances in terms of Precision, Sensitivity and Energy. The implementation plan foresees, as a first step, an Electroweak Factory electron-positron collider. This high luminosity facility, operating between 90 and 365 GeV centre-of-mass energy, will study the heavy particles of the Standard Model, Z, W, Higgs, and top with unprecedented accuracy. The Electroweak Factory  $e^+e^-$  collider constitutes a real challenge to the theory and to precision calculations, triggering the need for the development of new mathematical methods and software tools. A first workshop in 2018 had focused on the first FCC-ee stage, the Tera-Z, and confronted the theoretical status of precision Standard Model calculations on the Z-boson resonance to the experimental demands. The second workshop in January 2019, which is reported here, extended the scope to the next stages, with the production of W-bosons (FCC-ee-W), the Higgs boson (FCC-ee-H) and top quarks (FCC-ee-t). In particular, the theoretical precision in the determination of the crucial input parameters,  $\alpha_s$ ,  $\alpha_{\text{QED}}$ ,  $\alpha_s$ ,  $m_W$ ,  $m_t$  at the level of FCC-ee requirements is thoroughly discussed. The requirements on Standard Model theory calculations were spelled out, so as to meet the demanding accuracy of the FCC-ee experimental potential. The discussion of innovative methods and tools for multi-loop calculations was deepened. Furthermore, phenomenological analyses beyond the Standard Model were discussed, in particular the effective theory approaches.

**Note:** 290 pages, Report on the 11th FCC-ee workshop: Theory and Experiments, 8-11 January 2019, CERN, Geneva, Switzerland

Phase	Run duration (years)	Center-of-mass Energies ( GeV )	Integrated Luminosity ( $\text{ab}^{-1}$ )	Event Statistics
FCC-ee-Z	4	88-95	150	$3.10^{12}$ visible Z decays
FCC-ee-W	2	158-162	12	$10^8$ WW events
FCC-ee-H	3	240	5	$10^6$ ZH events
FCC-ee-tt	5	345-365	1.5	$10^6$ $t\bar{t}$ events

Table 3.1: Measurement of selected electroweak quantities at the FCC-ee, compared with the present precisions.

Observable	present value $\pm$ error	FCC-ee Stat.	FCC-ee Syst.	Comment and dominant exp. error
$m_Z$ (keV/c <sup>2</sup> )	91186700 $\pm$ 2200	5	100	From Z line shape scan Beam energy calibration
$\Gamma_Z$ (keV)	2495200 $\pm$ 2300	8	100	From Z line shape scan Beam energy calibration
$R_\ell^Z$ ( $\times 10^3$ )	20767 $\pm$ 25	0.06	0.2-1	ratio of hadrons to leptons acceptance for leptons
$\alpha_s(m_Z)$ ( $\times 10^4$ )	1196 $\pm$ 30	0.1	0.4-1.6	from $R_\ell^Z$ above [29]
$R_b$ ( $\times 10^6$ )	216290 $\pm$ 660	0.3	<60	ratio of $b\bar{b}$ to hadrons stat. extrapol. from SLD [30]
$\sigma_{\text{had}}^0$ ( $\times 10^3$ ) (nb)	41541 $\pm$ 37	0.1	4	peak hadronic cross-section luminosity measurement
$N_\nu$ ( $\times 10^3$ )	2991 $\pm$ 7	0.005	1	Z peak cross sections Luminosity measurement
$\sin^2 \theta_W^{\text{eff}}$ ( $\times 10^6$ )	231480 $\pm$ 160	3	2 - 5	from $A_{\text{FB}}^{\mu\mu}$ at Z peak Beam energy calibration
$1/\alpha_{\text{QED}}(m_Z)$ ( $\times 10^3$ )	128952 $\pm$ 14	4	small	from $A_{\text{FB}}^{\mu\mu}$ off peak [20]
$A_{\text{FB},0}^b$ ( $\times 10^4$ )	992 $\pm$ 16	0.02	1-3	b-quark asymmetry at Z pole from jet charge
$A_{\text{FB}}^{\text{pol},\tau}$ ( $\times 10^4$ )	1498 $\pm$ 49	0.15	<2	$\tau$ polarisation and charge asymmetry $\tau$ decay physics

## W and top

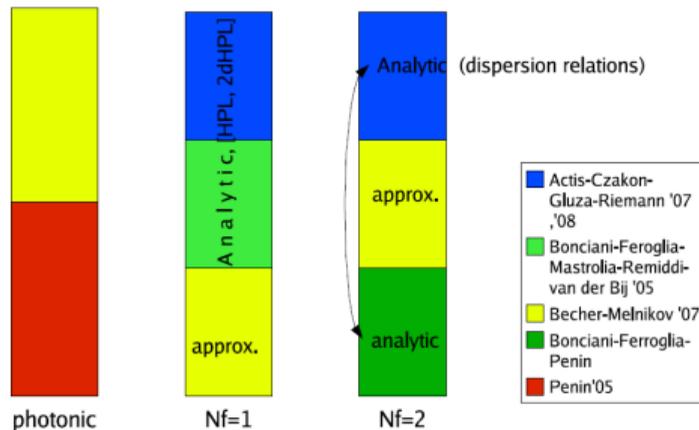
$m_W$ (keV/c <sup>2</sup> )	$803500 \pm 15000$	600	300	From WW threshold scan Beam energy calibration
$\Gamma_W$ (keV)	$208500 \pm 42000$	1500	300	From WW threshold scan Beam energy calibration
$\alpha_s(m_W)(\times 10^4)$	$1170 \pm 420$	3	small	from $R_\ell^W$ [31]
$N_\nu(\times 10^3)$	$2920 \pm 50$	0.8	small	ratio of invis. to leptonic in radiative Z returns
$m_{\text{top}}$ (MeV/c <sup>2</sup> )	$172740 \pm 500$	20	small	From $t\bar{t}$ threshold scan QCD errors dominate
$\Gamma_{\text{top}}$ (MeV/c <sup>2</sup> )	$1410 \pm 190$	40	small	From $t\bar{t}$ threshold scan QCD errors dominate
$\lambda_{\text{top}}/\lambda_{\text{top}}^{\text{SM}}$	$1.2 \pm 0.3$	0.08	small	From $t\bar{t}$ threshold scan QCD errors dominate
ttZ couplings	$\pm 30\%$	$<2\%$	small	From $E_{\text{CM}} = 365\text{GeV}$ run

## Several reasons to stay optimistic in "microscoping" higher order calculations

- ① **Steady** progress in numerical calculations, methods and tools;
- ② Lessons from the past (LEP, LHC,...) - anticipated SM predictions improved considerably - sometimes even several times after experiments took off;
- ③ Often problems can be attacked from different perspectives (it is needed for independent confirmations);

Present situation, virtual NNLO QED

### Bhabha scattering, 10 years ago



## We are not optimists: we are realists.

Many, many examples when needs (and competition!) triggered progress.

Now we have new arguments. And the table, with calculations behind. It is convincing me personally why we should aim at the most ambitious experimental programs.

A. Freitas: 1604.00406 (modified)

	Measurement error				Theory error	
	Present (LEP)	ILC	CEPC	FCC-ee	Current	Future <sup>†</sup>
$M_W$ [MeV]	15	3–4	3	1	4	1-1.5
$\Gamma_Z$ [MeV]	2.3	0.8	0.5	<b>0.1</b>	<b>0.5</b>	<b>0.2</b>
$R_b$ [ $10^{-5}$ ]	66	14	17	6	15	7
$\sin^2 \theta_{\text{eff}}^\ell$ [ $10^{-5}$ ]	16	1	2.3	<b>0.6</b>	4.5	<b>1.5</b>

**Table:** Projected experimental and theoretical uncertainties for some electroweak precision pseudo-observables.

<sup>†</sup> Based on estimations for:  $\mathcal{O}(\alpha_{bos}^2)$ ,  $\mathcal{O}(\alpha\alpha_s^2)$ ,  $\mathcal{O}(N_f\alpha^2\alpha_s)$ ,  $\mathcal{O}(N_f^2\alpha^3)$

## NLO Automations - many (complete) tools

Over the last decade or so modern methods of

- on-shell recursion relations (Britto, Cachazo, Feng, Witten,...)

and

- unitarity methods (Bern, Dixon, Kosower, ..., Ossola, Pittau, Papadopoulos, ..., Badger,.....)

overtaken to a large extent traditional Feynman diagrammatic approach, including one-loop calculations. Knowledge of (scalar) basis and their analytic structure allowed to focus and find

coefficients of reductions (OPP, Kosower, ..., Mastrolia,...), integrand reduction techniques (Ellis, Giele, Kunstz, Melnikov, Tramontano, Heinrich, Reiter,...)

1990-2000



**Altogether - a bunch of automatic packages:** FeynArts, BlackHat, Golem/Samurai, GoSam, Helac-NLO, MadGraph@NLO, Collier, PJFRY, ...

# "The two-loop explosion" - G. Zanderighi, CERN Courier Mar 17, 2017

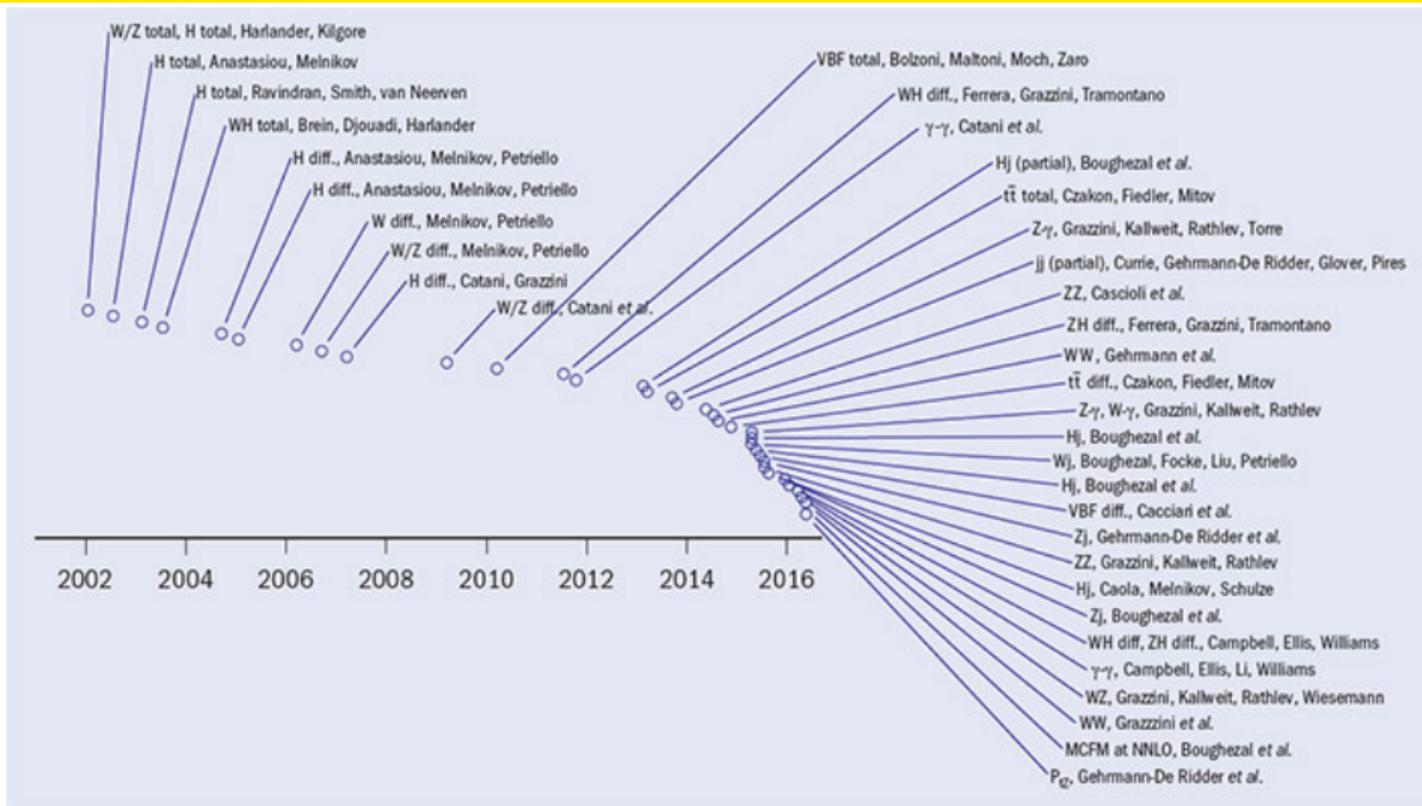


Fig. by G P Salam, 2016 LHCP conference

# Progress in analytical approaches

Annals of Mathematics, **141** (1995), 443-551



Pierre de Fermat

## Modular elliptic curves and Fermat's Last Theorem

By ANDREW JOHN WILES\*

*For Nada, Claire, Kate and Olivia*



Andrew John Wiles

*Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos, et generaliter nullam in infinitum ultra quadratum potestatum in duos ejusdem nominis fas est dividere: cujus rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.*

# Progress in analytical approaches

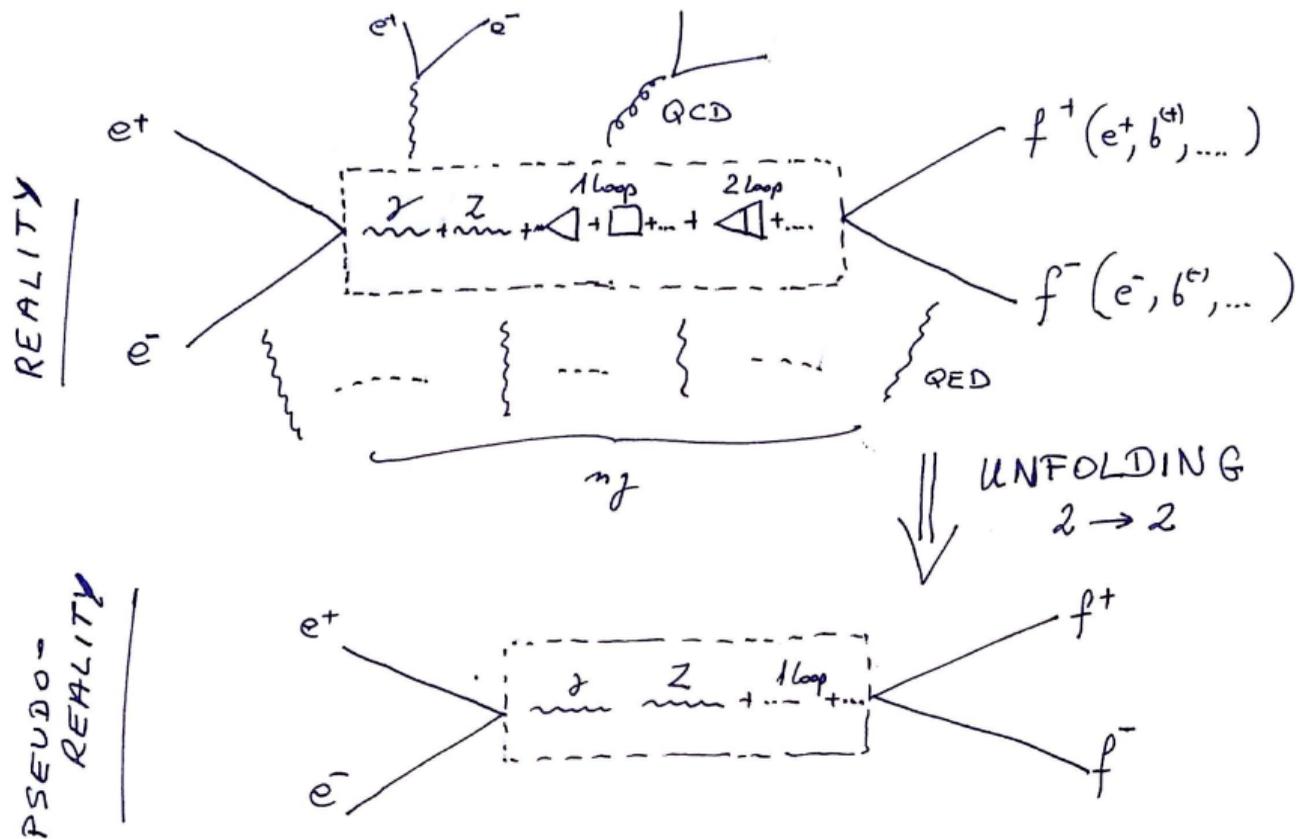
COMMUNICATIONS IN  
NUMBER THEORY AND PHYSICS  
Volume 12, Number 2, 193–251, 2018

## Feynman integrals and iterated integrals of modular forms

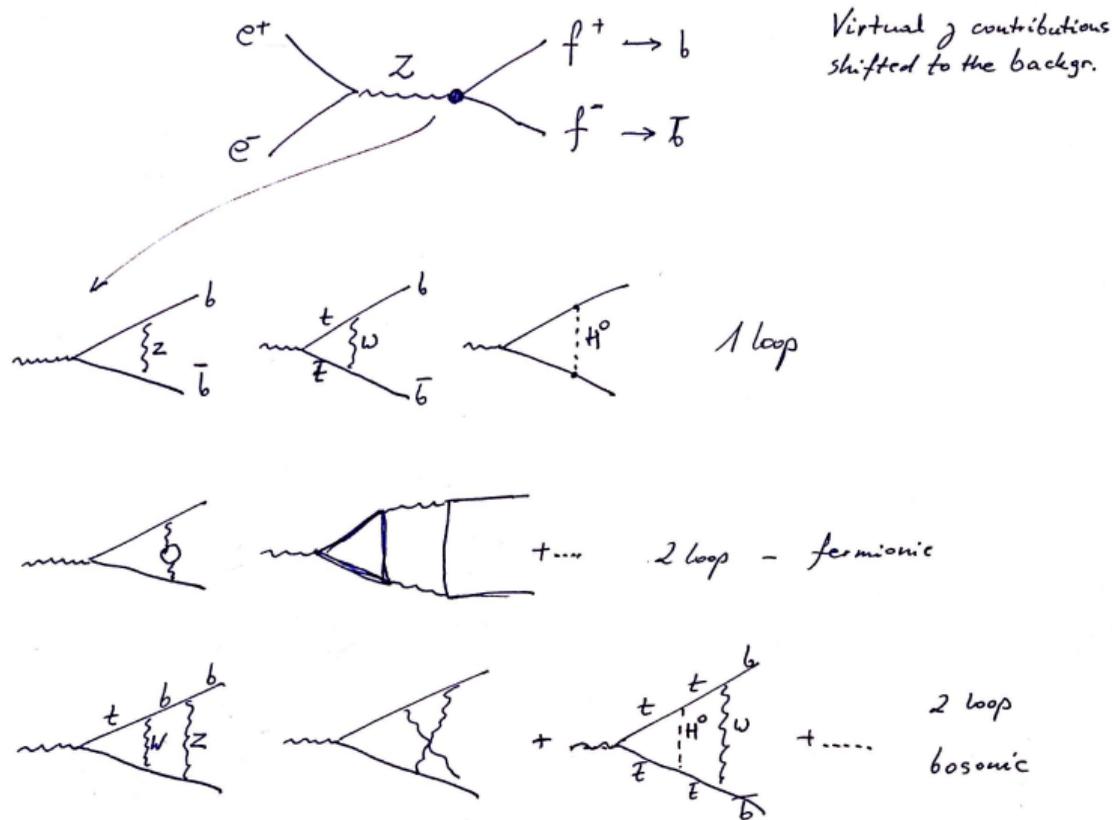
LUISE ADAMS AND STEFAN WEINZIERL

In this paper we show that certain Feynman integrals can be expressed as linear combinations of iterated integrals of modular forms to all orders in the dimensional regularisation parameter  $\varepsilon$ . We discuss explicitly the equal mass sunrise integral and the kite integral. For both cases we give the alphabet of letters occurring in the iterated integrals. For the sunrise integral we present a compact formula, expressing this integral to all orders in  $\varepsilon$  as iterated integrals of modular forms.

# Rough scheme for extracting the $Z\bar{f}f$ vertex and EW corrections (1)



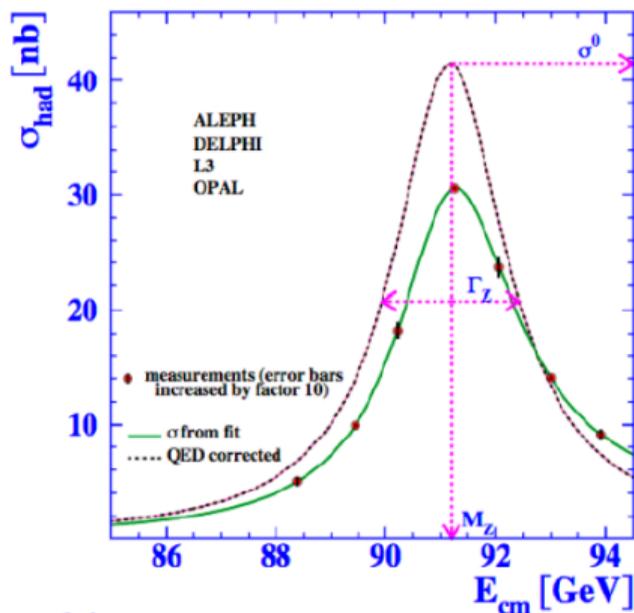
# Rough scheme for extracting the $Z\bar{f}f$ vertex and EW corrections (2)



# QED unfolding

Altogether  $17 \cdot 10^6$  Z-boson decays at LEP

## □ Cross section : Z mass and width



◆ ~30% QED corrections (ISR)

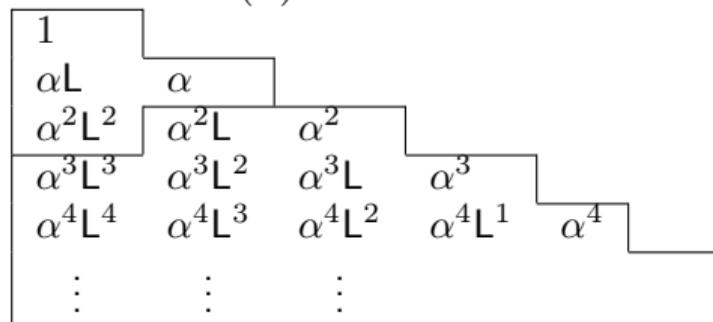
## What is needed in EWK HE studies: Basic issues

What we need:

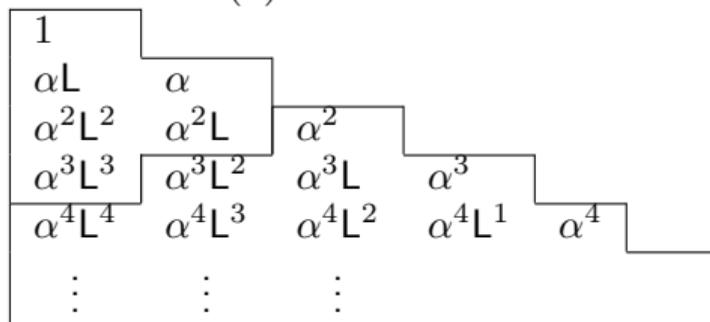
- Calculations at  $\sqrt{s} \doteq M_Z$  and around  
→ Line shape studies;
- Calculations for a clean setup of EWPOs at fixed order of virtual corrections  
Implementing higher order QED effects to MC, and resummations;

## QED perturbative leading and subleading corrections, 1903.09895

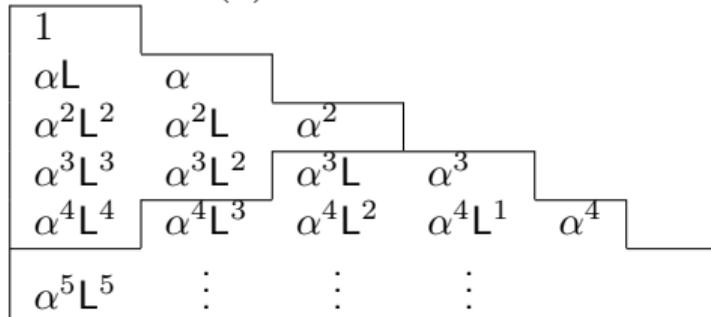
(a) 0.5%



(b) 0.02%



(c) 0.001%

ISR ( $e^\pm$ ) and FSR ( $\mu^\pm$ ) at the  $Z$  peak $\alpha \equiv \alpha_{QED}$  $L \equiv L_f = \ln(s/m_f^2)$ ,  $f = e, \mu$

EW SM theory at loops, an example ( $\Delta_{ef} \neq 0$ )

$$\left\{ \begin{array}{l} \Gamma_Z, \Gamma_{\text{partial}} \\ A_{FB, \text{peak}}^{\text{eff., Born}}, A_{LR, \text{peak}}^{\text{eff., Born}} \\ R_b, R_\ell, \dots \end{array} \right. \longrightarrow \left\{ \begin{array}{l} v_{\ell, \nu, u, d, b}^{\text{eff}} \\ a_{\ell, \nu, u, d, b}^{\text{eff}} \\ \sin^2 \theta_{\text{eff}}^b, \sin^2 \theta_{\text{eff}}^{\text{lept}} \end{array} \right.$$

e.g. : improvements needed for subtle corrections  $\Delta_{1,2}$  (e.g. boxes, **2L-boxes**)

$$A_{FB, \text{peak}}^{\text{eff., Born}} = \frac{2\Re \left[ \frac{v_e a_e^*}{|a_e|^2} \right] 2\Re \left[ \frac{v_f a_f^*}{|a_f|^2} \right]}{\left( 1 + \frac{|v_e|^2}{|a_e|^2} \right) \left( 1 + \frac{|v_f|^2}{|a_f|^2} \right)} + \Delta_1 - \Delta_2 \simeq \frac{3}{4} A_e A_f,$$

$$\Delta_1 = 2\Re [\Delta_{ef}], \quad \Delta_2 = |\Delta_{ef}|^2 + 2\Re \left[ \frac{v_e a_e^*}{|a_e|^2} \frac{v_f a_f^*}{|a_f|^2} \Delta_{ef}^* \right],$$

$$\Delta_{ef} = 16 |Q_e Q_f| s_W^4 (\kappa_{ef} - \kappa_e \kappa_f)$$

# How to unfold - rough scheme

We have to describe

$$e^+e^- \longrightarrow (\gamma, Z) \longrightarrow f^+ f^-(\gamma), \quad (1)$$

S-matrix Ansatz in the complex energy plane

$$\mathcal{A}^{e^+e^- \rightarrow b\bar{b}} = \frac{R_Z}{s - s_Z} + \underbrace{\frac{R_\gamma}{s} + S + (s - s_Z)S'}_{\text{Background}} + \dots,$$

$$s_Z = \underbrace{\hspace{10em}}_{\gamma\text{-}Z \text{ interference}} = \overline{M}_Z^2 - i\overline{M}_Z\overline{\Gamma}_Z$$

- $R, S, S', \dots$  are individually gauge-invariant and UV-finite - **unitarity and analyticity of the  $S$ -matrix**. IR-finite, when soft and collinear real photon emission is added.

[Willenbrock, Valencia,1991] [Sirlin,1991] [Stuart,1991] [Riemann, 1991, 1992] [H. Veltman,1994] [Passera, Sirlin, 1998] [Gambino, Grassi, 2000]

[Awramik, Czakon, Freitas, 2006].

# The term $R_\gamma(s)/s$ is part of the the background

- The poles of  $\mathcal{A}$  have complex residua  $R_Z$  and  $R_\gamma$ .
- There is only ONE pole in mathematics, while in physics we observe two of them: photon exchange at  $s = 0$ ,  $Z$  exchange at  $s_0 = s_Z$ . Mathematically, the appearance of the photon pole is result of summing of part of background around  $Z$  pole,  $s_0 = s_Z$

[T. Riemann, APPB 2015]

$$\begin{aligned}
 \frac{R_\gamma(s)}{s} &= \frac{\sum_{n=0}^{\infty} R_n(s - s_0)^n}{s} \\
 &= \frac{\sum_{n=0}^{\infty} R_n(s - s_0)^n}{s_0 - (s_0 - s)} \\
 &= \sum_{n=0}^{\infty} R_n(s - s_0)^n \frac{1}{s_0} \frac{1}{1 - \frac{s_0 - s}{s_0}} \\
 &= \sum_{n=0}^{\infty} R_n(s - s_0)^n \frac{1}{s_0} \left[ 1 + \frac{s_0 - s}{s_0} + \left( \frac{s_0 - s}{s_0} \right)^2 \dots \right];
 \end{aligned}$$

From Theory: The way to keep a proper track is the S-matrix approach

$$e^+e^- \longrightarrow (\gamma, Z) \longrightarrow f\bar{f}(\gamma),$$

$$A_0^{e^+e^- \rightarrow f\bar{f}} = \frac{R_Z}{s - M_Z^2 + i\Gamma_Z M_Z} + S_0 + (s - s_Z)S' + \dots,$$

$$\sigma_0 \simeq |A_0|^2 = \frac{sr + (s - M_Z^2)j_{int}^{\gamma-Z}}{|s - s_Z^2|^2} + corr.[background]$$

$$r = (v_e^2 + a_e^2)(v_f^2 + a_f^2) + \dots$$

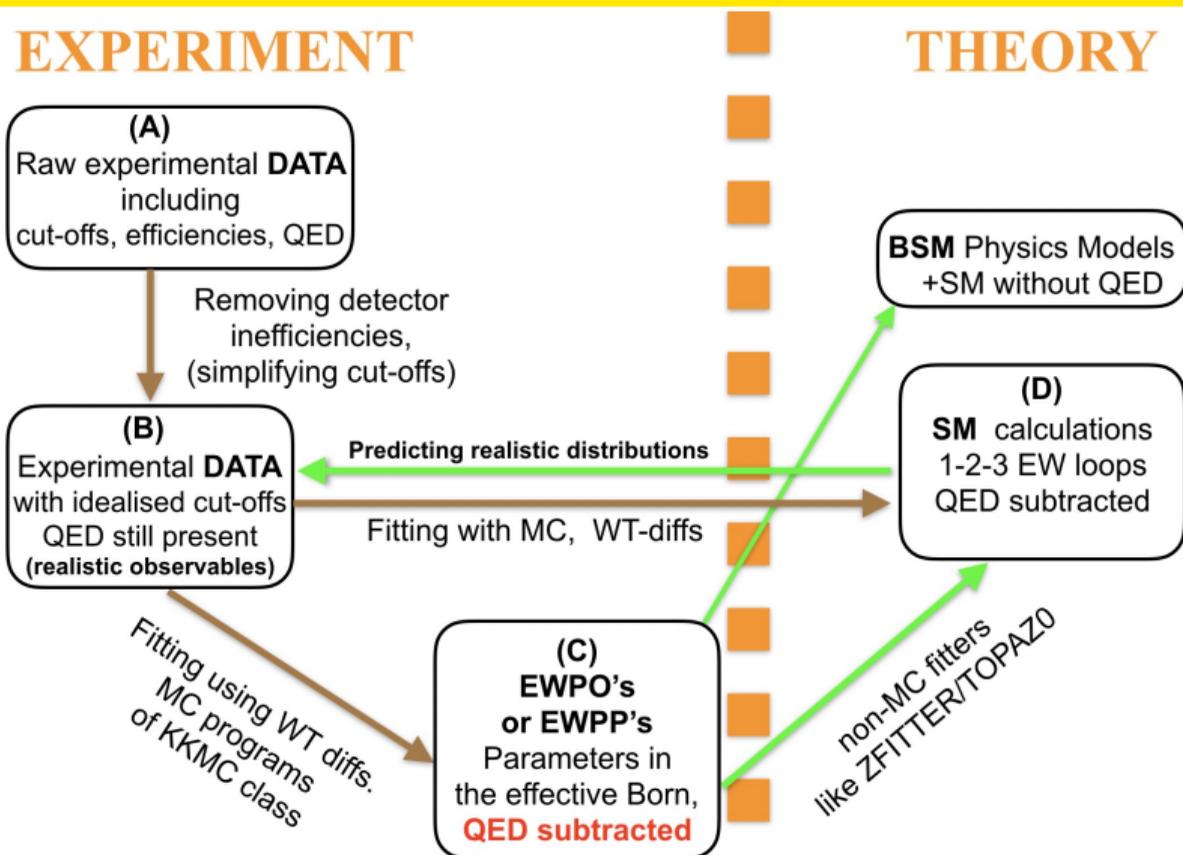
[Willenbrock, Valencia,1991] [Sirlin,1991] [Stuart,1991] [Riemann, 1991, 1992] [H. Veltman,1994]

[Passera, Sirlin, 1998] [Gambino, Grassi, 2000] [Awramik, Czakon, Freitas, 2006].

Solved issue: 2-loop vertex, present issue: 3-loop vertex, potential issue:  
2-loop boxes

# Scheme of construction and the use of EWPO/EWPP at post-LEP era

Report 1 and 1903.0985



EWPOs - refers to  $|M|^2$ ; EWPPs - refers to  $M$

Beyond Born level, one can write

$$\mathcal{M}_\gamma^{(0)}(e^-e^+ \rightarrow f^-f^+) = \frac{4\pi i \alpha_{em}(s)}{s} Q_e Q_f \gamma_\alpha \otimes \gamma^\alpha,$$

$$\begin{aligned} \mathcal{M}_Z^{(0)}(e^-e^+ \rightarrow f^-f^+) = & 4ie^2 \frac{\chi_Z(s)}{s} [M_{vv}^{ef} \gamma_\alpha \otimes \gamma^\alpha - M_{av}^{ef} \gamma_\alpha \gamma_5 \otimes \gamma^\alpha \\ & - M_{va}^{ef} \gamma_\alpha \times \gamma^\alpha \gamma_5 + M_{aa}^{ef} \gamma_\alpha \gamma_5 \otimes \gamma^\alpha \gamma_5]. \end{aligned}$$

In the pole scheme, where  $\bar{M}_Z$  is defined as the real part of the pole of the S matrix, one has

$$\chi_Z(s) = \frac{G_F M_Z^2}{\sqrt{2} 8\pi \alpha_{em}} K_Z(s) \simeq \frac{1}{1 + i \frac{\bar{\Gamma}_Z}{M_Z}} \frac{s}{s - \bar{M}_Z^2 + i \bar{M}_Z \bar{\Gamma}_Z} \simeq \frac{s}{s - M_Z^2 + i M_Z \Gamma_Z(s)},$$

$$\Gamma_Z(s) = \frac{s}{M_Z^2} \Gamma_Z$$

# EWPOs - refers to $|M|^2$ ; EWPPs - refers to $M$

Definitions are related:

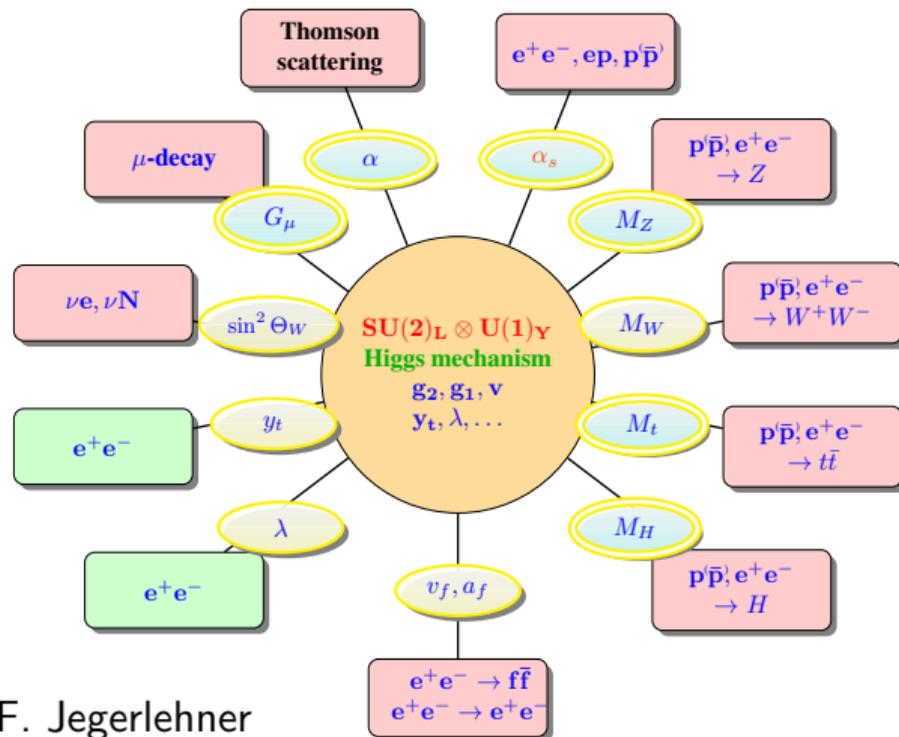
$$\bar{M}_Z \approx M_Z - \frac{1}{2} \frac{\Gamma_Z^2}{M_Z} \approx M_Z - 34 \text{ MeV},$$

$$\bar{\Gamma}_Z \approx \Gamma_Z - \frac{1}{2} \frac{\Gamma_Z^3}{M_Z^2} \approx \Gamma_Z - 0.9 \text{ MeV}.$$

- Known from LEP. One of examples why changing frameworks/assumptions/simplifications of calculations matter (!).
- However, at FCC-ee  $\delta\Gamma_Z \sim 0.1 \text{ MeV}$ . Non-factorization effects must be added properly beyond 1-loop.
- Is it necessary for FCC-ee accuracy to implement MC with radiative corrections calculated at the amplitudes level?
- At this precision it is important which parameters are taken as input parameters in schemes.

# Input and renormalization schemes

## 1 Input and calculated/measured parameters



$\alpha_{QED}$  in "Report 2", F. Jegerlehner

Let's assume we unfolded EW physics well.

And defined all contributions taking into account non-factorizable effects.

Then we can calculate comfortable SM corrections order by order in Z-boson physics.

# Currently most precise prediction for $\sin^2 \theta_{\text{eff}}^b$

$$\sin^2 \theta_{\text{eff}}^b = s_0 + d_1 L_H + d_2 L_H^2 + d_3 \Delta_\alpha + d_4 \Delta_t + d_5 \Delta_t^2 + d_6 \Delta_t L_H + d_7 \Delta_{\alpha_s} + d_8 \Delta_t \Delta_{\alpha_s} + d_9 \Delta_Z \quad (2)$$

$$L_H = \log \left( \frac{M_H}{125.7 \text{ GeV}} \right), \quad \Delta_t = \left( \frac{m_t}{173.2 \text{ GeV}} \right)^2 - 1, \quad \Delta_Z = \frac{M_Z}{91.1876 \text{ GeV}} - 1, \quad (3)$$

$$\Delta_\alpha = \frac{\Delta_\alpha}{0.0059} - 1, \quad \Delta_{\alpha_s} = \frac{\alpha_s}{0.1184} - 1.$$

$$\begin{aligned} s_0 = 0.232704, \quad d_1 = 4.723 \times 10^{-4}, \quad d_2 = 1.97 \times 10^{-4}, \quad d_3 = 2.07 \times 10^{-2}, \\ d_4 = -9.733 \times 10^{-4}, \quad d_5 = 3.93 \times 10^{-4}, \quad d_6 = -1.38 \times 10^{-4}, \\ d_7 = 2.42 \times 10^{-4}, \quad d_8 = -8.10 \times 10^{-4}, \quad d_9 = -0.664. \end{aligned} \quad (4)$$

- $M_W$  is calculated from the Fermi constant  $G_\mu$  [Awramik, et al., 2004]
- The deviations to the full calculation amount to average (maximal)  $2 \times 10^{-7}$  ( $1.3 \times 10^{-6}$ ), in the input parameter ranges.

## Goodnes of fits

Observable	max. dev.	EXP now	FCC-ee	CEPC	ILC
$\sin^2 \theta_{\text{eff}}^{\ell} \times 10^4$	0.056	1.6	0.06	0.23	0.1
$\sin^2 \theta_{\text{eff}}^{\text{b}} \times 10^4$	0.025	160	0.18	9	15

*Table: Influence of input parameters in the fitting formulas on the leptonic and bottom-quark effective weak mixing angles, compared with the envisaged precision of measurements (statistical errors) at the collider projects FCC-ee CDR, CEPC CDR and ILC. To our knowledge, there is no corresponding data available for EWPOs at CLIC. . The entry “EXP now” gives the present experimental precision, as known since LEP 1 .*

Not to be mixed up with theoretical errors due to missing higher order radiative corrections

# Published results on EWPOs in the SM @NNLO

Complete corrections  $\Delta r, \sin^2 \theta_{\text{eff}}^l$ :

Freitas, Hollik, Walter, Weiglein: '00  
 Awramik, Czakon: '02, Onishchenko, Veretin: '02  
 Awramik, Czakon, Freitas, Weiglein: '04  
 Awramik, Czakon, Freitas: '06  
 Hollik, Meier, Uccirati: '05, '07  
 Degrossi, Gambino, Giardino: '14

Fermionic corrections  $\sin^2 \theta_{\text{eff}}^b, a_f, v_f$ :

Awramik, Czakon, Freitas, Kniehl: '09  
 Czarnecki, Kühn: '96  
 Harlander, Seidensticker, Steinhauser: '98  
 Freitas: '13, '14

Bosonic corrections:  $\sin^2 \theta_{\text{eff}}^b$ :

Dubovyk, Freitas, JG, Riemann, Usovitsch '16

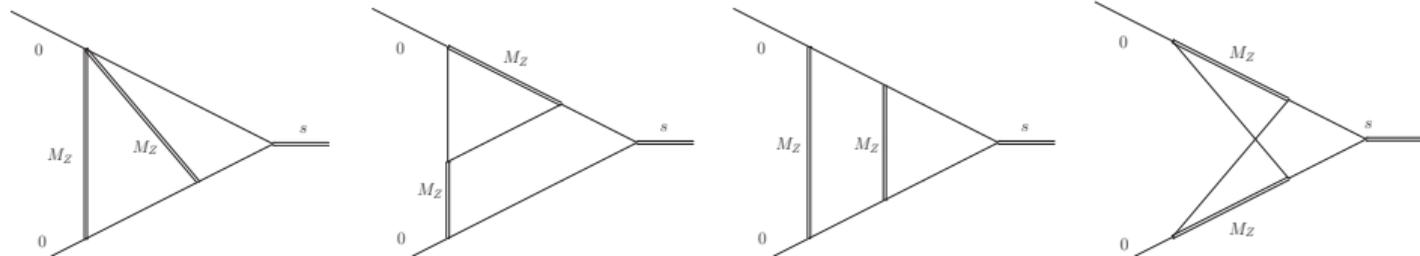
Bosonic corrections:  $\Gamma_Z, R_l, \dots$ :

Dubovyk, Freitas, JG, Riemann, Usovitsch '18

# Mellin-Barnes and Sector Decomposition methods are very much complementary

- MB works well for hard threshold, on-shell cases, not many internal masses (more IR);  
SD more useful for integrals with many internal masses
- talk by Johann Usovitsch, LL2018
- JG, Tord Riemann in PoS-LL2016 & DFGRU in PLB'16.

$10^{-8}$  accuracy achieved for **any** self-energy and vertex Feynman integral with one of the methods - in **Minkowskian region**.



Available for several years!

# NNLO results

Input parameters:

Parameter	Value	Parameter	Value
$M_Z$	91.1876 GeV	$m_b^{\overline{\text{MS}}}$	4.20 GeV
$\Gamma_Z$	2.4952 GeV	$m_c^{\overline{\text{MS}}}$	1.275 GeV
$M_W$	80.385 GeV	$m_\tau$	1.777 GeV
$\Gamma_W$	2.085 GeV	$\Delta\alpha$	0.05900
$M_H$	125.1 GeV	$\alpha_s(M_Z)$	0.1184
$m_t$	173.2 GeV	$G_\mu$	$1.16638 \times 10^{-5} \text{ GeV}^{-2}$

# The 2-loops EWPOs results\* for $\mathcal{O}(\alpha_{\text{bos}}^2)$ , [hep-ph/1804.10236](https://arxiv.org/abs/hep-ph/1804.10236)

$\Gamma_i$ [MeV]	$\Gamma_e, \Gamma_\mu, \Gamma_\tau$	$\Gamma_{\nu_e}, \Gamma_{\nu_\mu}, \Gamma_{\nu_\tau}$	$\Gamma_d, \Gamma_s$	$\Gamma_u, \Gamma_c$	$\Gamma_b$	$\Gamma_Z$
Born	81.142	160.096	371.141	292.445	369.56	2420.2
$\mathcal{O}(\alpha)$	2.273	6.174	9.717	5.799	3.857	60.22
$\mathcal{O}(\alpha\alpha_s)$	0.288	0.458	1.276	1.156	2.006	9.11
$\mathcal{O}(N_f^2\alpha^2)$	0.244	0.416	0.698	0.528	0.694	5.13
$\mathcal{O}(N_f\alpha^2)$	0.120	0.185	0.493	0.494	0.144	3.04
$\mathcal{O}(\alpha_{\text{bos}}^2)$	<b>0.017</b>	<b>0.019</b>	<b>0.058</b>	<b>0.057</b>	<b>0.167</b>	<b>0.505</b>
$\mathcal{O}(\alpha_t\alpha_s^2, \alpha_t\alpha_s^3, \alpha_t^2\alpha_s, \alpha_t^3)$	0.038	0.059	0.191	0.170	<b>0.190</b>	1.20

- ① Fun fact of the day: so far all contributions positive.
- ② 2016, estimation, bosonic NNLO  $\sim 0 \pm 0.1$  MeV  
**2018**, exact result: 0.505 MeV

\* Fixed values of  $M_W$

# The 2-loops EWPOs results for $\mathcal{O}(\alpha_{\text{bos}}^2)$ , [hep-ph/1804.10236](https://arxiv.org/abs/hep-ph/1804.10236)

	$\Gamma_Z$ [GeV]	$\sigma_{\text{had}}^0$ [nb]
Born	2.53601	41.6171
+ $\mathcal{O}(\alpha)$	2.49770	41.4687
+ $\mathcal{O}(\alpha\alpha_s)$	2.49649	41.4758
+ $\mathcal{O}(\alpha_t\alpha_s^2, \alpha_t\alpha_s^3, \alpha_t^2\alpha_s, \alpha_t^3)$	2.49560	41.4770
+ $\mathcal{O}(N_f^2\alpha^2, N_f\alpha^2)$	2.49441	41.4883
+ $\mathcal{O}(\alpha_{\text{bos}}^2)$	<b>[+0.34 MeV]=</b> 2.49475	<b>[+1.3 pb]=</b> 41.4896

Results for  $\Gamma_Z$  and  $\sigma_{\text{had}}^0$ , with  $M_W$  calculated from  $G_\mu$  using the same order of perturbation theory as indicated in each line.

# The 2-loops EWPOs results for $\mathcal{O}(\alpha_{\text{bos}}^2)$ , [hep-ph/1804.10236](https://arxiv.org/abs/hep-ph/1804.10236)

	$R_\ell$	$R_c$	$R_b$
Born	21.0272	0.17306	0.21733
+ $\mathcal{O}(\alpha)$	20.8031	0.17230	0.21558
+ $\mathcal{O}(\alpha\alpha_s)$	20.7963	0.17222	0.21593
+ $\mathcal{O}(\alpha_t\alpha_s^2, \alpha_t\alpha_s^3, \alpha_t^2\alpha_s, \alpha_t^3)$	20.7943	0.17222	0.21593
+ $\mathcal{O}(N_f^2\alpha^2, N_f\alpha^2)$	20.7512	0.17223	0.21580
+ $\mathcal{O}(\alpha_{\text{bos}}^2)$	20.7516	0.17222	0.21585

Results for the ratios  $R_\ell$ ,  $R_c$  and  $R_b$ , with  $M_W$  calculated from  $G_\mu$  to the same order as indicated in each line.

## Updates for error estimations

- Theory error estimate is not well defined, ideally  $\Delta_{\text{th}} \ll \Delta_{\text{exp}}$
- Common methods:
  - Count prefactors ( $\alpha, N_c, N_f, \dots$ )
  - Extrapolation of perturbative series
  - Renormalization scale dependence
  - Renormalization scheme dependence
- Also parametric error from external inputs ( $m_t, m_b, \alpha_s, \Delta\alpha_{\text{had}}, \dots$ )

see, Ayres Freitas: 1604.00406

E.g.: Theory error estimation for  $\Gamma_Z$ , 1804.10236 [1604.00406]

## ① Geometric series

$$\delta_1 : \mathcal{O}(\alpha^3) - \mathcal{O}(\alpha_t^3) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha^2) \sim 0.20 \text{ MeV} [0.26 \text{ MeV}]$$

$$\delta_2 : \mathcal{O}(\alpha^2 \alpha_s) - \mathcal{O}(\alpha_t^2 \alpha_s) \sim \frac{\mathcal{O}(\alpha^2) - \mathcal{O}(\alpha_t^2)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s) \sim 0.21 \text{ MeV} [0.3 \text{ MeV}]$$

$$\delta_3 : \mathcal{O}(\alpha \alpha_s^2) - \mathcal{O}(\alpha_t \alpha_s^2) \sim \frac{\mathcal{O}(\alpha \alpha_s) - \mathcal{O}(\alpha_t \alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s) \sim 0.23 \text{ MeV}$$

$$\delta_4 : \mathcal{O}(\alpha \alpha_s^3) - \mathcal{O}(\alpha_t \alpha_s^3) \sim \frac{\mathcal{O}(\alpha \alpha_s) - \mathcal{O}(\alpha_t \alpha_s)}{\mathcal{O}(\alpha)} \mathcal{O}(\alpha \alpha_s^2) \sim 0.035 \text{ MeV}$$

$$\delta_5 : \mathcal{O}(\alpha_{bos}^2) \sim \mathcal{O}(\alpha_{bos})^2 \sim \mathbf{0.1 \text{ MeV}} \text{ [Now we know it!]}$$

$$\text{Total: } \delta\Gamma_Z = \sqrt{\sum_{i=1}^5 \delta_i^2} \sim \mathbf{0.4 \text{ MeV}} \text{ [0.5 \text{ MeV}]}$$

## Summary: estimations for higher order EW and QCD corrections

$\delta_1 :$	$\delta_2 :$	$\delta_3 :$	$\delta_4 :$	$\delta_5 :$	$\delta\Gamma_Z$ [MeV]
$\mathcal{O}(\alpha^3)$	$\mathcal{O}(\alpha^2\alpha_s)$	$\mathcal{O}(\alpha\alpha_s^2)$	$\mathcal{O}(\alpha\alpha_s^3)$	$\mathcal{O}(\alpha_{bos}^2)$	$= \sqrt{\sum_{i=1}^5 \delta_i^2}$
TH1 (estimated error limits from <a href="#">geometric series of perturbation</a> )					
0.26	0.3	0.23	0.035	0.1	0.5
TH1-new (estimated error limits from <a href="#">geometric series of perturbation</a> )					
0.2	0.21	0.23	0.035	$< 10^{-4}$	0.4

$\delta'_1 :$	$\delta'_2 :$	$\delta'_3 :$	$\delta_4 :$		$\delta\Gamma_Z$ [MeV]
$\mathcal{O}(N_f^{\leq 1}\alpha^3)$	$\mathcal{O}(\alpha^3\alpha_s)$	$\mathcal{O}(\alpha^2\alpha_s^2)$	$\mathcal{O}(\alpha\alpha_s^3)$		$\sqrt{\delta_1'^2 + \delta_2'^2 + \delta_2'^3 + \delta_4^2}$
TH2 (extrapolation through <a href="#">prefactor scaling</a> )					
0.04	0.1	0.1	0.035	$10^{-4}$	0.15

## Crucial issue: accuracy of calculations

For 2-loops we maintained 4 digits for EWPOs.

A calculation of the radiative corrections  $\delta_1 \div \delta_4$  and  $\delta'_1 \div \delta'_3$  with a 10% accuracy (corresponding to two significant digits) should suffice to meet future experimental demands.

## Minimal precision of 3-loop EW calculations:

- ① Calculating  $N^3LO$  with 10% accuracy (two digits), we can replace theory error estimation  $\delta\Gamma_Z = \sqrt{\sum_{i=1}^5 \delta_i^2} \sim 0.4$  MeV by

$$\delta\Gamma_Z = \sqrt{\sum_{i=1}^5 (\delta_i/10)^2} \sim 0.04 \text{ MeV.}$$

- ① The requirement of FCC-ee<sup>exper. error</sup>( $\Gamma_Z$ )  $\sim 0.1$  MeV can be met and the condition

$$\delta[\text{FCCee}^{\text{theor.}}(\Gamma_Z)] \sim 0.04 \text{ MeV} < \delta[\text{FCCee}^{\text{exper.}}(\Gamma_Z)] \sim 0.1 \text{ MeV}$$

will be fulfilled.

# Estimations for total values of missing EWPOs

	$\delta\Gamma_Z$ [MeV]	$\delta R_l$ [ $10^{-4}$ ]	$\delta R_b$ [ $10^{-5}$ ]	$\sin^2 \theta_{\text{eff}}^l$ [ $10^{-5}$ ]	$\sin^2 \theta_{\text{eff}}^b$ [ $10^{-5}$ ]	$\sigma_{\text{had}}^0$ [pb]
EXP-FCCee	0.1	$2 \div 20$	$2 \div 6$	6	70	4
TH1*	0.4	60	10	4.5	5	6
TH2*	0.15	60	5	1.5	$1.5 \div 2$	6

TH1 - estimates from geometric series (3-loops)

TH2 - estimates from prefactor scaling (beyond 3-loops)

\* **10% knowledge (2 digits) of the error would decrease numbers by factor 10**

And this should be the goal for future  $\geq N^3 LO$  calculations

# BACKUP SLIDES

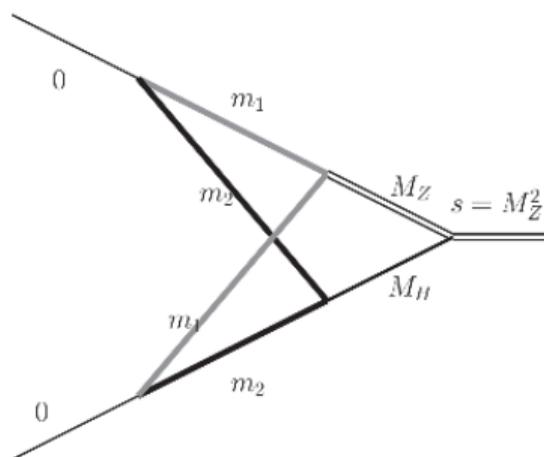
## Discussion of NNNLO accuracy

Two factors play role:

- Number of diagrams
- Their complexity

Goal: at least 2-digits accuracy for EWPOs.

We estimate it to be possible, even from present perspective.

2-loops  $\longrightarrow$  3-loops

$$m_1 = M_t, m_2 = M_W$$

The integrals contain up to three dimensionless parameters

$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{(M_Z + i\varepsilon)^2}{M_Z^2} \right\}$$

2-loops  $\longrightarrow$  3-loops

- The standard model prediction for the effective weak mixing angle can be written as

$$\sin^2 \theta_{\text{eff}}^{\text{b}} = \left( 1 - \frac{M_W^2}{M_Z^2} \right) (1 + \Delta\kappa_{\text{b}})$$

- The bosonic electroweak two-loop corrections amount to

$$\Delta\kappa_{\text{b}}^{(\alpha^2, \text{bos})} = -0.9855 \times 10^{-4}$$

DFGRU, Phys.Lett. B762 (2016) 184

Collection of radiative corrections: Full stabilization at  $10^{-4}$ ! $\pm 0.001 \xrightarrow{!}$ 

Order	Value [ $10^{-4}$ ]	Order	Value [ $10^{-4}$ ]
$\alpha$	468.945	$\alpha_t^2 \alpha_s$	1.362
$\alpha \alpha_s$	-42.655	$\alpha_t^3$	0.123
$\alpha_{\text{ferm}}^2$	3.866	$\alpha_t \alpha_s^2$	-7.074
$\alpha_{\text{bos}}^2$	<b>-0.9855</b>	$\alpha_t \alpha_s^3$	-1.196

Table: Comparison of different orders of radiative corrections to  $\Delta\kappa_b$ .

*Input Parameters:*  $M_Z, \Gamma_Z, M_W, \Gamma_W, M_H, m_t, \alpha_s$  and  $\Delta\alpha$

- one-loop contributions [Akhundov, Bardin, Riemann, 1986] [Beenakker, Hollik, 1988]
- two-loop fermionic contributions [Awramik, Czakon, Freitas, Kniehl, 2009]
- two-loop bosonic contributions [Dubovyk, Freitas, JG, Riemann, Usovitsch, 2016]

### Partial higher-order corrections

$\mathcal{O}(\alpha_t \alpha_s^2)$

Avdeev: 1994, Chetyrkin: 1995

$\mathcal{O}(\alpha_t \alpha_s^3)$

Schroder: 2005, Chetyrkin: 2006, Boughezal: 2006

$\mathcal{O}(\alpha^2 \alpha_t)$  and  $\mathcal{O}(\alpha_t^3)$

vanderBij: 2000, Faisst: 2003

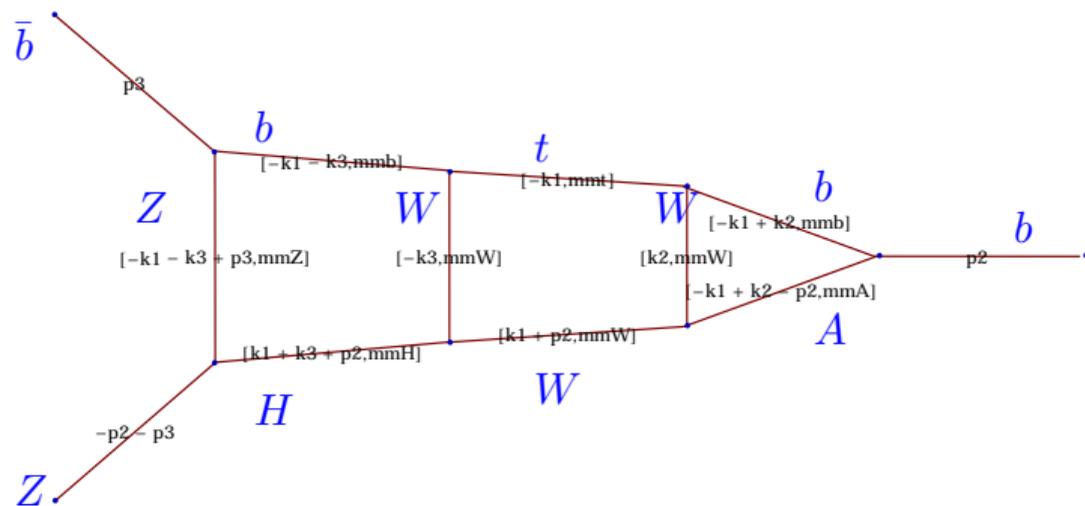
## 3-loops. Basic bookkeeping

$Z \rightarrow b\bar{b}$			
Number of topologies	1 loop	2 loops	3 loops
	1	$14 \xrightarrow{(A)}$ $7 \xrightarrow{(B)}$ <b>5</b>	$211 \xrightarrow{(A)}$ $84 \xrightarrow{(B)}$ <b>50</b>
Number of diagrams	15	$2383 \xrightarrow{(A,B)}$ <b>1114</b>	$490387 \xrightarrow{(A,B)}$ <b>120187</b>
<b>Fermionic loops</b>	0	150	<b>17580</b>
<b>Bosonic loops</b>	15	<b>964</b>	102607
Planar diagrams	1T/15D	4T/981D	35T/84059D
Non-planar diagrams	0	1T/133D	15T/36128D

**Table:** Some statistical overview for  $Z \rightarrow b\bar{b}$  multiloop studies. At 3 loops there are in total almost half a million of diagrams present. After basic refinements (A) and (B) about  $10^5$  genuine 3-loop vertex diagrams remain. In (A) tadpoles and products of lower loops are excluded, in (B) symmetries of topologies are taken into account.

A complete zoo of heavy particles  $m_t, m_W, m_Z, m_H$  @NNNLO level

MB:  $\epsilon^0$ [8-dim],  $1/\epsilon$ [7-dim]; SD:  $\epsilon^0$ [8-dim],  $1/\epsilon$ [7-dim];



At 2-loops up to three dimensionless parameters (all 4 at 3-loops):

$$\left\{ \frac{M_H^2}{M_Z^2}, \frac{M_W^2}{M_Z^2}, \frac{m_t^2}{M_Z^2}, \frac{(M_Z + i\epsilon)^2}{M_Z^2} \right\}$$

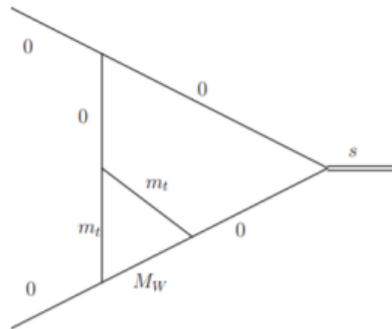
## Sector decomposition

FIESTA 3 [A.V.Smirnov, 2014], SecDec 3 [Borowka, et. al., 2015] and pySecDec [Borowka, et. al., 2017]

## Mellin-Barnes integral approach

- With AMBRE 2 [Gluza, et. al., 2011] (AMBRE 3 [Dubovyk, et. al., 2015]) we derive Mellin-Barnes representations for planar (non-planar) topologies. One may use PlanarityTest [Bielas, et. al, 2013] for automatic identification.
- Expansion in terms of  $\epsilon = (4 - D)/2$  is done with MB [Czakon, 2006], MBresolve [A. Smirnov, V. Smirnov, 2009], barnesroutines (D. Kosower).
- For the numerical treatment of massive Mellin-Barnes integrals with Minkowskian regions, the package MBnumerics is being developed since 2015.

soft7  $\epsilon^0$ : [MB - 3 dim] [SD - 5 dim],  $\epsilon^{-1}$ : [MB - 2 dim] [SD - 4 dim],  $\epsilon^{-2}$ : [MB - 1 dim] [SD - 3 dim]



MB	0.060266486557699 <b>9</b> $\epsilon^{-2}$	
SD - 90 Mio	0.0602664865 <b>5</b> $\epsilon^{-2}$	
MB	$(-0.03151248903$	$+0.18933275142i) \epsilon^{-1}$
SD - 90 Mio	$(-0.0315124816$	$+0.18933271696i) \epsilon^{-1}$
MB 1	$(-0.228231867511$	$-0.088247945691i) + \mathcal{O}(\epsilon)$
MB 2	$(-0.228231867551$	$-0.088247945739i) + \mathcal{O}(\epsilon)$
SD - 90 Mio	$(-0.22822653$	$-0.08824596i) + \mathcal{O}(\epsilon)$
SD - 15 Mio	$(-0.228162$	$-0.088209i) + \mathcal{O}(\epsilon)$

# Step 1

## Construction of MB integrals

<http://us.edu.pl/~gluza/ambre/>

# Mellin-Barnes representations in HEP - method

- "Om definita integraler", R. H. Mellin, Acta Soc. Sci. Fenn. 20(7), 1 (1895),  
 "The theory of the gamma function", E. W. Barnes Messenger Math. 29(2), 64 (1900).

$$\begin{aligned}
 \text{mathematics} &\longrightarrow \frac{1}{(A+B)^\lambda} = \frac{1}{\Gamma(\lambda)} \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dz \Gamma(\lambda+z) \Gamma(-z) \frac{B^z}{A^{\lambda+z}} \\
 \text{physics} &\longrightarrow \frac{1}{(p^2 - m^2)^a} = \frac{1}{\Gamma(a)} \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} dz \Gamma(a+z) \Gamma(-z) \frac{(m^2)^z}{(p^2)^{a+z}}
 \end{aligned}$$

It is recursive  $\implies$  multidimensional complex integrals.

$$\int_{-\frac{1}{3}-i\infty}^{-\frac{1}{3}+i\infty} dz_1 \int_{-\frac{2}{3}-i\infty}^{-\frac{2}{3}+i\infty} dz_2 \left( \frac{-s}{M_Z^2} \right)^{-z_1} \frac{\Gamma[-z_1]^3 \Gamma[1+z_1] \Gamma[z_1-z_2] \Gamma[-z_2]^3 \Gamma[1+z_2] \Gamma[1-z_1+z_2]}{s \Gamma[1-z_1]^2 \Gamma[-z_1-z_2] \Gamma[1+z_1-z_2]}$$

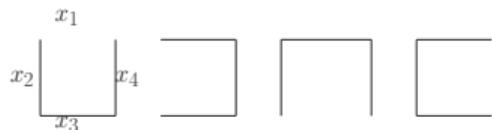
**Overlaped integrals**

# Multiloop Feynman diagrams, general MB integrals

$$\frac{1}{D_1^{n_1} D_2^{n_2} \dots D_N^{n_N}} \rightarrow \int \prod_{j=1}^N dx_j x_j^{n_j-1} \delta\left(1 - \sum_{i=1}^N x_i\right) \frac{U(\mathbf{x})^{N_\nu - d(L+1)/2}}{F(\mathbf{x})^{N_\nu - dL/2}}$$

$$N_\nu = n_1 + \dots + n_N$$

The functions  $U$  and  $F$  are called graph or Symanzik polynomials.



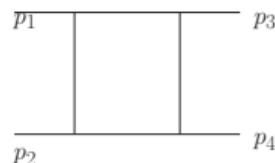
Trees contributing to the polynomial  $U$  for the square diagram

$$\mathbf{U} = \mathbf{x}_1 + \mathbf{x}_2 + \mathbf{x}_3 + \mathbf{x}_4 \quad ! \text{ 1-loop} \rightarrow 1$$



2 – trees contributing to the polynomial  $F$  for the square diagram

$$\mathbf{F} = \mathbf{t} \cdot \mathbf{x}_1 \mathbf{x}_3 + \mathbf{s} \cdot \mathbf{x}_2 \mathbf{x}_4$$



Cuts of internal lines such that:

- $U$ : (i) every vertex is still connected to every other vertex by a sequence of uncut lines; (ii) no further cuts without violating (i)
- $F$ : (iii) divide the graph into two disjoint parts such that within each part (i) and (ii) are obeyed and such that at least one external momentum line is connected to each part;

**Dimension of MB integrals depends on factorizations of  $F$  and  $U$ !**

## Step 2

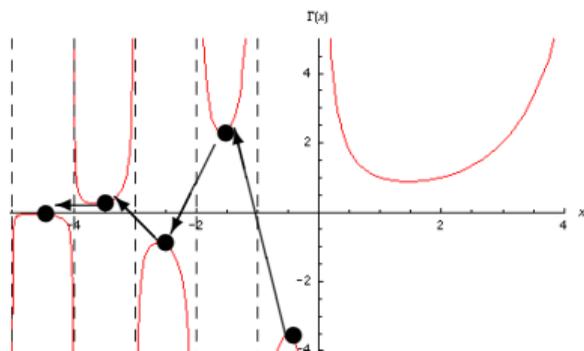
# Numerics of MB integrals

<http://mbtools.hepforge.org/>

## Two basic observations for shifting $z$ follows

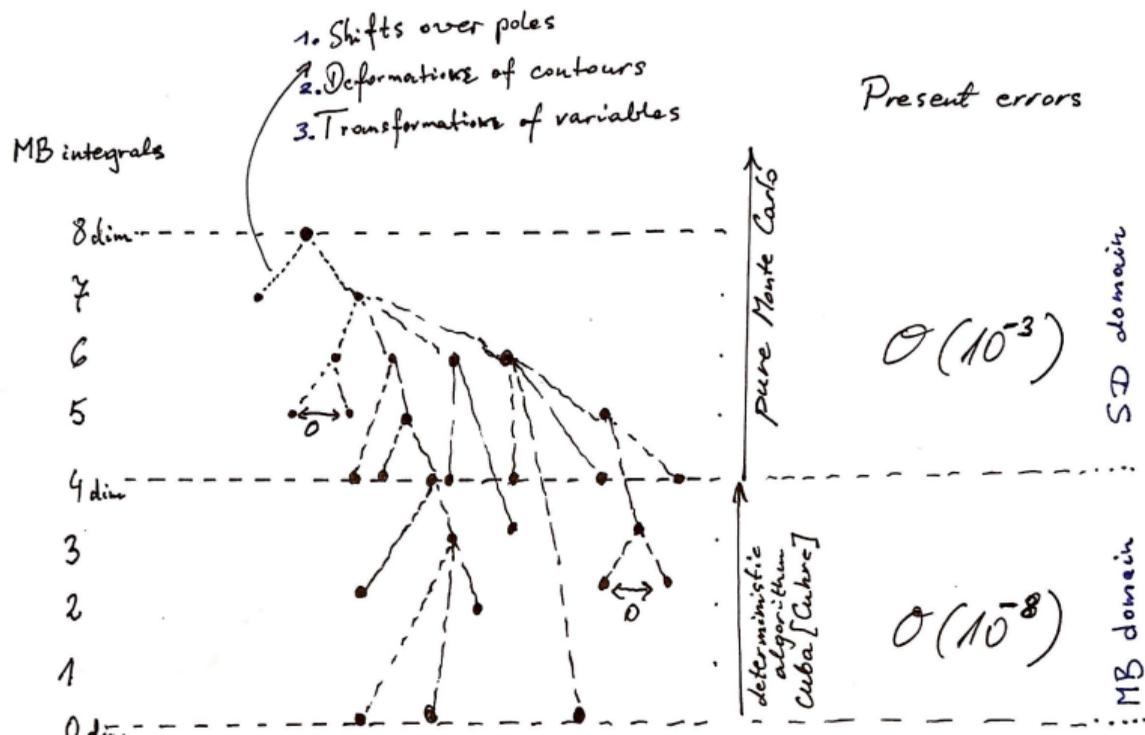
$$\begin{aligned}
 & \int dz_1 \dots dz_k \dots I(\dots, \text{Re}[z_k] + n + \text{Im}[z_k], \dots) && I_{orig} \\
 = & \text{Residue}[\int dz_1 \dots \cancel{dz_k} \dots I]_{\text{Re}[z_k] + n} && I_{Res} \\
 + & \int dz_1 \dots dz_k \dots I(\dots, \text{Re}[z_k] + (n + 1) + \text{Im}[z_k], \dots) && I_{new}
 \end{aligned}$$

- Residues **lower** dimensionality of original MB integrals.
- Integral after passing a pole (proper shifts) **can be made smaller**.



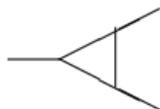
## Top-bottom approach to evaluation of multidimensional MB integrals

## MBnumerics.m - I. Dubovyk, J. Usovitsch, T. Riemann



# BASIC PROBLEMS in Minkowski kinematics

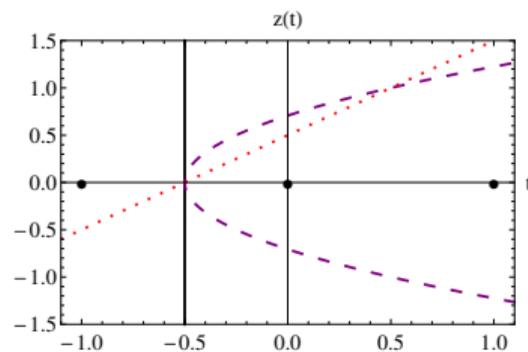
- I. Bad oscillatory behavior of integrands;
- II. Fragile stability for integrations over products and ratios of  $\Gamma$  functions.



$$\begin{aligned}
 V(s) &= \frac{e^{\epsilon\gamma_E}}{i\pi^{(4-2\epsilon)/2}} \int \frac{d^{(4-2\epsilon)}k}{[(k+p_1)^2 - m^2][k^2][(k-p_2)^2 - m^2]} \\
 &= \frac{V_{-1}(s)}{\epsilon} + V_0(s) + \dots,
 \end{aligned}$$

$$\begin{aligned}
 V_{-1}(s)|_{m=1} &= -\frac{1}{2s} \int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} \frac{dz}{2\pi i} \underbrace{(-s)^{-z}}_{\text{Problem I}} \overbrace{\frac{\Gamma^3(-z)\Gamma(1+z)}{\Gamma(-2z)}}^{\text{Problem II}} \\
 &= \frac{1}{2} \sum_{n=0}^{\infty} \frac{s^n}{\binom{2n}{n}(2n+1)} = \frac{2 \arcsin(\sqrt{s}/2)}{\sqrt{4-s}\sqrt{s}},
 \end{aligned}$$

# Contour deformations



$$z(t) = x_0 + it : V_{-1}^{C_1}(s) = \int_{-\infty}^{+\infty} (i) dt J[z(t)];$$

$$z(t) = x_0 + \theta t + it : V_{-1}^{C_2}(s) = \int_{-\infty}^{+\infty} (\theta + i) dt J[z(t)]$$

$$z(t) = x_0 + at^2 + it : V_{-1}^{C_3}(s) = \int_{-\infty}^{+\infty} (2at + i) dt J[z(t)]; .$$

$$s = 2, z(t) = \Re[-1/2] + i y, \quad y \in (-a, +a)$$

$$V_{-1}(2)|_{\text{analyt.}} = \mathbf{0.78539816339744830962} = \frac{\pi}{4}$$

$$V_{-1}(2)|_{\text{Pantis}}^{MB.m} = 0.7925 - \cancel{0.0225} i$$

$$V_{-1}(2)|_{C_1, a=15} = 0.7548660085063523 - \cancel{0.229985258820015} i$$

$$V_{-1}(2)|_{C_1, a=10^2} = 0.73479313088852537844 + \cancel{0.074901423602937676597} i$$

$$V_{-1}(2)|_{C_1, a=10^3} = 0.84718185073531076915 - \cancel{0.094865760649354977853} i$$

$$V_{-1}(2)|_{C_1, a=10^4} = 4.4574554985139977188 + \cancel{4.5139812364645122275} i$$

$$\checkmark V_{-1}(2)|_{C_2} = \mathbf{0.7853981633859819} - 5.420140575251864 \cdot 10^{-15} \checkmark i$$

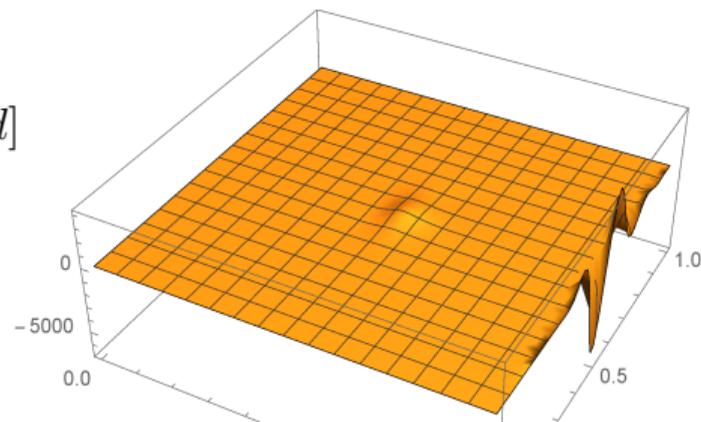
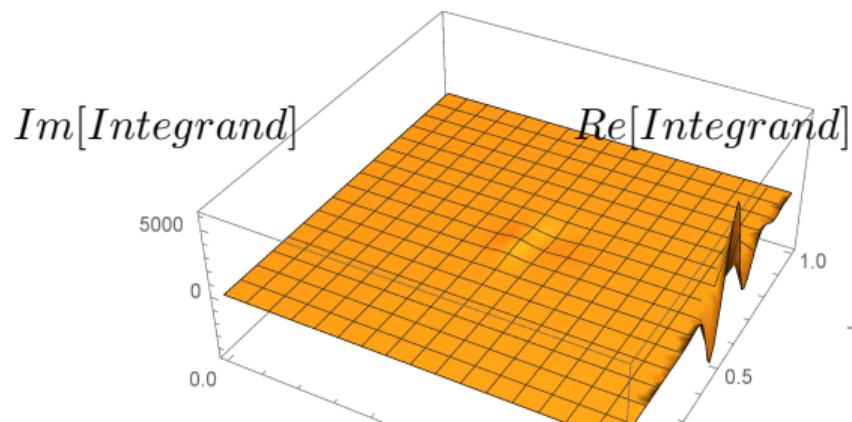
$$\checkmark V_{-1}(2)|_{C_3} = \mathbf{0.7853981632958756} + 2.435551760271437 \cdot 10^{-15} \checkmark i$$

# Transformations of integration variables (Mappings)

$$\int_{-\frac{1}{3}-i\infty}^{-\frac{1}{3}+i\infty} dz_1 \int_{-\frac{2}{3}-i\infty}^{-\frac{2}{3}+i\infty} dz_2 \left(\frac{-s}{M_Z^2}\right)^{-z_1} \frac{\Gamma[-z_1]^3 \Gamma[1+z_1] \Gamma[z_1-z_2] \Gamma[-z_2]^3 \Gamma[1+z_2] \Gamma[1-z_1+z_2]}{s \Gamma[1-z_1]^2 \Gamma[-z_1-z_2] \Gamma[1+z_1-z_2]}$$

Logarithmic (in MB.m, M. Czakon, CPC 2006):

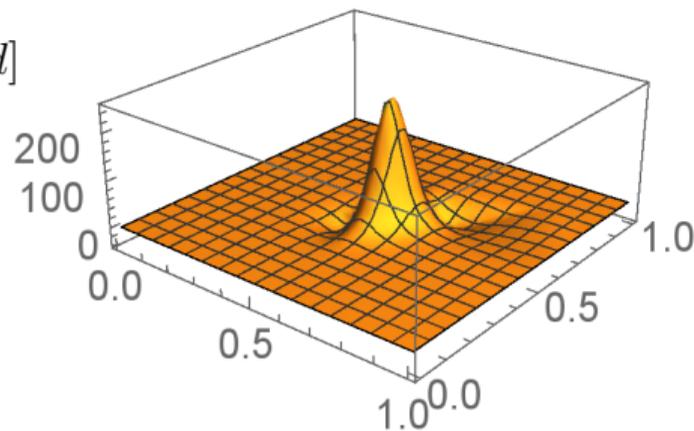
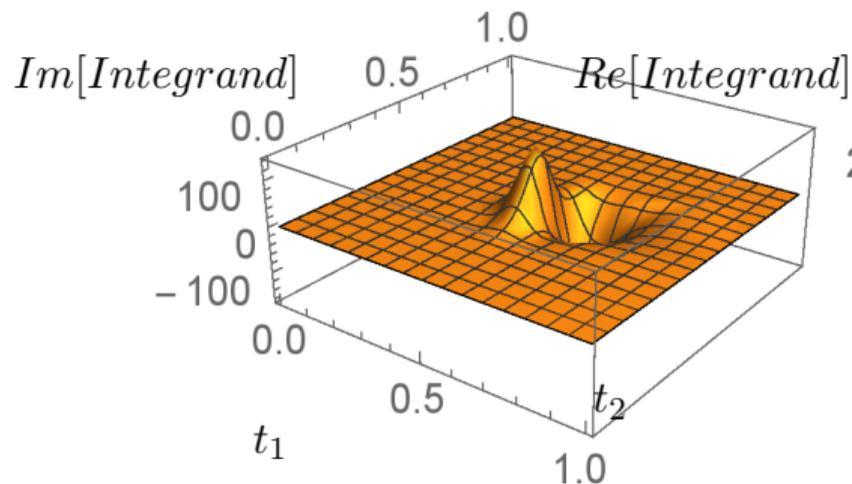
$$z_k = x_k + i \ln\left(\frac{t_k}{1-t_k}\right), \quad t_k \in (0, 1), \quad \text{the Jacobians: } J_k(t_k) = \frac{1}{t_k(1-t_k)}.$$



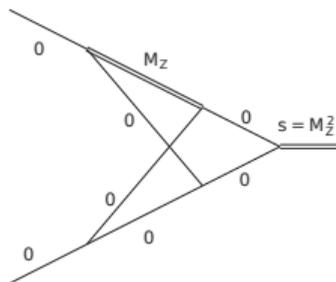
# Transformations of variables (Mappings)

Tangent (in MBnumerics.m, ID, JU, TR, 2016):

$$z_k = x_k + i \frac{1}{\tan(-\pi t_k)}, \quad t_k \in (0, 1), \quad \text{the Jacobians : } J_k = \frac{\pi}{\sin^2[(\pi t_k)]}.$$



## MB vs SD



Euclidean results (constant part):

Analytical :  $-0.4966198306057021$   
 MB(Vegas) :  $-0.4969417442183914$   
 MB(Cuhre) :  $-0.4966198313219404$   
 FIESTA :  $-0.4966184488196595$   
 SecDec :  $-0.4966192150541896$

Minkowskian results (constant part):

Analytical :  $-0.778599608979684 - 4.123512593396311 \cdot i$   
 MBnumerics :  $-0.778599608324769 - 4.123512600516016 \cdot i$   
 MB(Vegas) : big error  
 MB(Cuhre) : NaN  
 FIESTA : big error  
 SecDec : big error

# SM W-physics, more in the report for January 2019 CERN workshop



## Future improvements of theory predictions?

### Implementation of state-of-the art calculations in public tools?

- **NLO-EW**  $e^-e^+ \rightarrow 4f$  now possible with standard tools  
(RECOLA, OpenLoops, MadLoops + SHERPA, MadGraph, WHIZARD...)  
but not (yet) optimized for  $e^-e^+$  (ISR, Beamstrahlung)
- **Two-loop Coulomb-enhanced** corrections for differential observables doable; (related:  $t\bar{t}$  with Coulomb resummation in WHIZARD)  
(no guarantee of formal accuracy for general distributions)

### Full NNLO in EFT for total cross section

- Soft  $\log \beta$  terms can be adapted from QCD results
- NNLO  $\log(m_e/M_W)$  terms doable (c.f. Bhabha scattering)
- two-loop hard non-logarithmic corrections  
(from amplitudes for  $e^+e^- \rightarrow W^+W^-$  at threshold: border of current capabilities)  
resulting uncertainty from cross-section calculation

$$\Delta\sigma_{\text{hard}}^{(2)} = \left(\frac{\alpha}{2\pi}\right)^2 c^{(2)}\sigma^{(0)} \sim (1-2)\% \text{ for estimate } c^{(2)} = (c^{(1)})^2$$

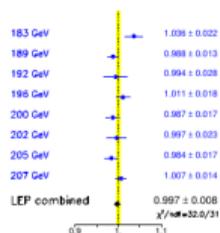
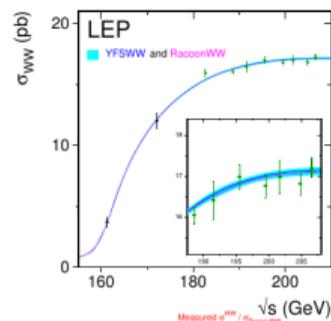
### Full NNLO for $e^+e^- \rightarrow 4f$ : completely new methods needed

# SM W-physics

## W-pair production

### Success story at LEP2:

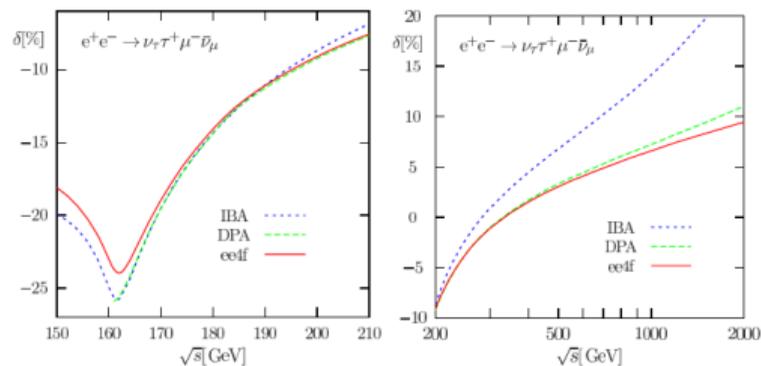
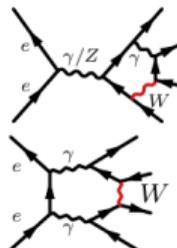
- $\sigma_{WW}$ : 1%-level agreement with NLO theory
- ⇒ test of EW-sector of SM at quantum level
- measurement of branching ratios (lepton universality)
- bounds on anomalous triple vector-boson couplings
- ⇒ test of non-abelian structure
- W-mass measurement from kinematic reconstruction (+  $\sigma_{WW}$  at threshold)



# SM W-physics

## Full NLO calculation for $e^+e^- \rightarrow 4f$ (Denner, Dittmaier, Roth, Wieders 05)

- More than 1000 1-loop diagrams, 5, 6-point loop integrals
- ⇒ pioneering methods for six-point diagrams  
now automated for LHC: RECOLA, OpenLoops, MadLoops
- [complex mass scheme](#) for  $W$  decay width
- fully differential calculation
- not easy to incorporate higher-order effects
- DPA not sufficient at threshold and for  $\sqrt{s} > 500$  GeV



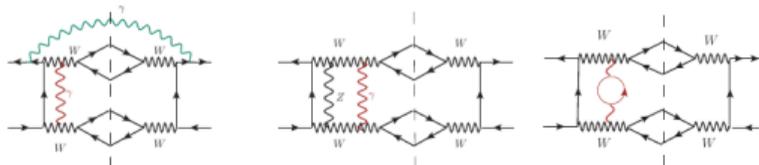
# SM W-physics

**EFT expansion** in  $\alpha \sim \frac{\Gamma_W}{M_W} \sim \beta^2$  (Beneke/Falgari/CS/Signer/Zanderighi 07)

- systematically possible to include higher-order corrections
- limited to total cross section near threshold

## Leading NNLO corrections

- 2nd Coulomb correction  $\sim \alpha^2/\beta^2 \sim \alpha$  (Fadin et al. 95)
- Coulomb-enhanced corrections  $\sim \alpha^2/\beta \sim \alpha^{3/2}$  (Actis et al. 08)



- Numerical effect:  $\Delta\sigma_{WW} \sim 5\text{‰}$ ;  $[\delta M_W] \lesssim 3 \text{ MeV}$

$\sqrt{s}$ [GeV]	$\sigma(e^-e^+ \rightarrow \mu^- \bar{\nu}_\mu u \bar{d})$ (fb)			
	NLO <sub>EFT</sub>	NLO <sub>ee4f</sub> [DDRW]	$\Delta_{\text{NNLO}}(\alpha^2/\beta^2)$	$\Delta_{\text{NNLO}}(\alpha^2/\beta)$
161	117.5	118.77	0.44 (3.7‰)	0.15 (1.3‰)
170	397.8	404.5	0.25 (0.6‰)	1.6 (3.9‰)

# SM precision parameters determination: $\alpha(M_Z^2)$

## 1. $\alpha(M_Z^2)$ in precision physics (precision physics limitations)

**Uncertainties of hadronic contributions to effective  $\alpha$  are a problem for electroweak precision physics:** besides top Yukawa  $y_t$  and Higgs self-coupling  $\lambda$

$\alpha$ ,  $G_\mu$ ,  $M_Z$  **most precise input parameters**  $\Rightarrow$  **precision predictions**  
 50% non-perturbative  $\sin^2 \Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \dots$   
 $\alpha(M_Z), G_\mu, M_Z$  **best effective input parameters for VB physics (Z,W) etc.**

$$\begin{aligned} \frac{\delta\alpha}{\alpha} &\sim 3.6 \times 10^{-9} \\ \frac{\delta G_\mu}{G_\mu} &\sim 8.6 \times 10^{-6} \\ \frac{\delta M_Z}{M_Z} &\sim 2.4 \times 10^{-5} \\ \frac{\delta\alpha(M_Z)}{\alpha(M_Z)} &\sim 0.9 \div 1.6 \times 10^{-4} \quad (\text{present : lost } 10^5 \text{ in precision!}) \\ \frac{\delta\alpha(M_Z)}{\alpha(M_Z)} &\sim 5.3 \times 10^{-5} \quad (\text{FCC - ee/ILC requirement}) \end{aligned}$$

**LEP/SLD:**  $\sin^2 \Theta_{\text{eff}} = (1 - v_l/a_l)/4 = 0.23148 \pm 0.00017$   
 $\delta\Delta\alpha(M_Z) = 0.00020 \Rightarrow \delta\sin^2 \Theta_{\text{eff}} = 0.00007$ ;  $\delta M_W/M_W \sim 4.3 \times 10^{-5}$

**affects most precision tests and new physics searches!!!**

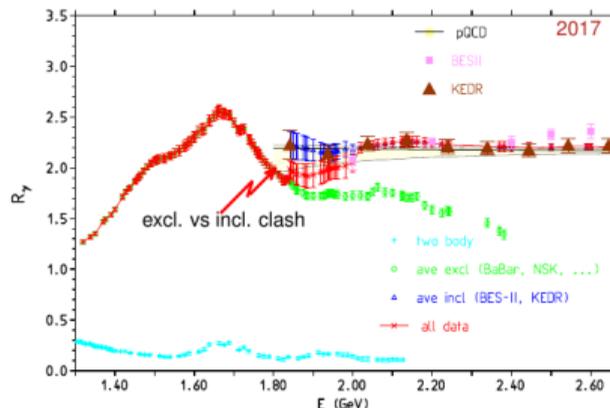
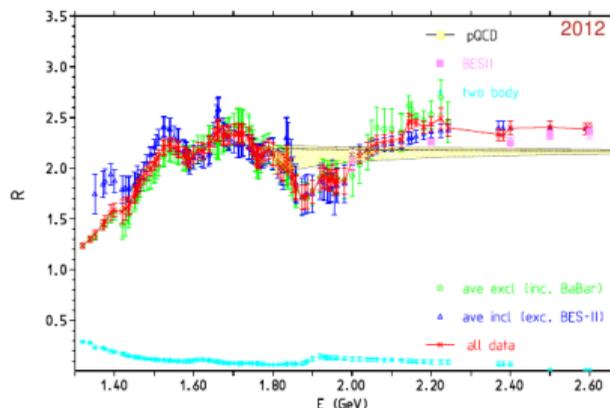
$$\frac{\delta M_W}{M_W} \sim 1.5 \times 10^{-4}, \quad \frac{\delta M_H}{M_H} \sim 1.3 \times 10^{-3}, \quad \frac{\delta M_t}{M_t} \sim 2.3 \times 10^{-3}$$

**For pQCD contributions very crucial: precise QCD parameters  $\alpha_s, m_c, m_b, m_t \Rightarrow$  Lattice-QCD**

# SM precision parameters determination: $\alpha(M_Z^2)$

## Still an issue in HVP

- region 1.2 to 2 GeV data; test-ground exclusive vs inclusive  $R$  measurements (more than 30 channels!) VEPP-2000 CMD-3, SND (NSK) scan, BaBar, BES III radiative return! still contributes 50% of uncertainty



- illustrating progress by BaBar and NSK exclusive channel data vs new inclusive data by KEDR. Why point at 1.84 GeV so high?

# Three approaches should be further explored for better error estimate

Note: **theory-driven** standard analyses ( $R(s)$  integral) using pQCD above 1.8 GeV cannot be improved by improved cross-section measurements above 2 GeV !!!

precision in $\alpha$ :	present	direct	$1.7 \times 10^{-4}$
		Adler	$1.2 \times 10^{-4}$
future	Adler QCD 0.2%	Adler QCD 0.1%	$5.4 \times 10^{-5}$
		Adler QCD 0.1%	$3.9 \times 10^{-5}$
future	via $A_{\text{FB}}^{\mu\mu}$ off Z		$3 \times 10^{-5}$

- Adler function method is competitive with **Patrick Janot's** direct near Z pole determination via forward backward asymmetry in  $e^+e^- \rightarrow \mu^+\mu^-$

$$A_{\text{FB}}^{\mu\mu} = A_{\text{FB},0}^{\mu\mu} + \frac{3a^2}{4v^2} \frac{I}{\mathcal{Z} + \mathcal{G}}$$

where

$\gamma - Z$  interference term  $I \propto \alpha(s) G_\mu$

Z alone  $\mathcal{Z} \propto G_\mu^2$

$\gamma$  only  $\mathcal{G} \propto \alpha^2(s)$

$v$  vector Z coupling also depends on  $\alpha(s \sim M_Z^2)$  and  $\sin^2 \Theta_f(s \sim M_Z^2)$

$a$  axial Z coupling sensitive to  $\rho$ -parameter (strong  $M_t$  dependence)

- using  $v, a$  as measured at Z-peak

$$e^+e^- \rightarrow \mu^+\mu^- \text{ and } \alpha^2(s)$$

$\sigma_{\mu\mu}$ :

- ① the photon-exchange term,  $\mathcal{G}$ , proportional to  $\alpha^2(s)$ ;
- ② the Z-exchange term,  $\mathcal{Z}$ , proportional to  $G_F^2$  (where  $G_F$  is the Fermi constant);
- ③ the Z-photon interference term,  $\mathcal{I}$ , proportional to  $\alpha(s) \times G_F$

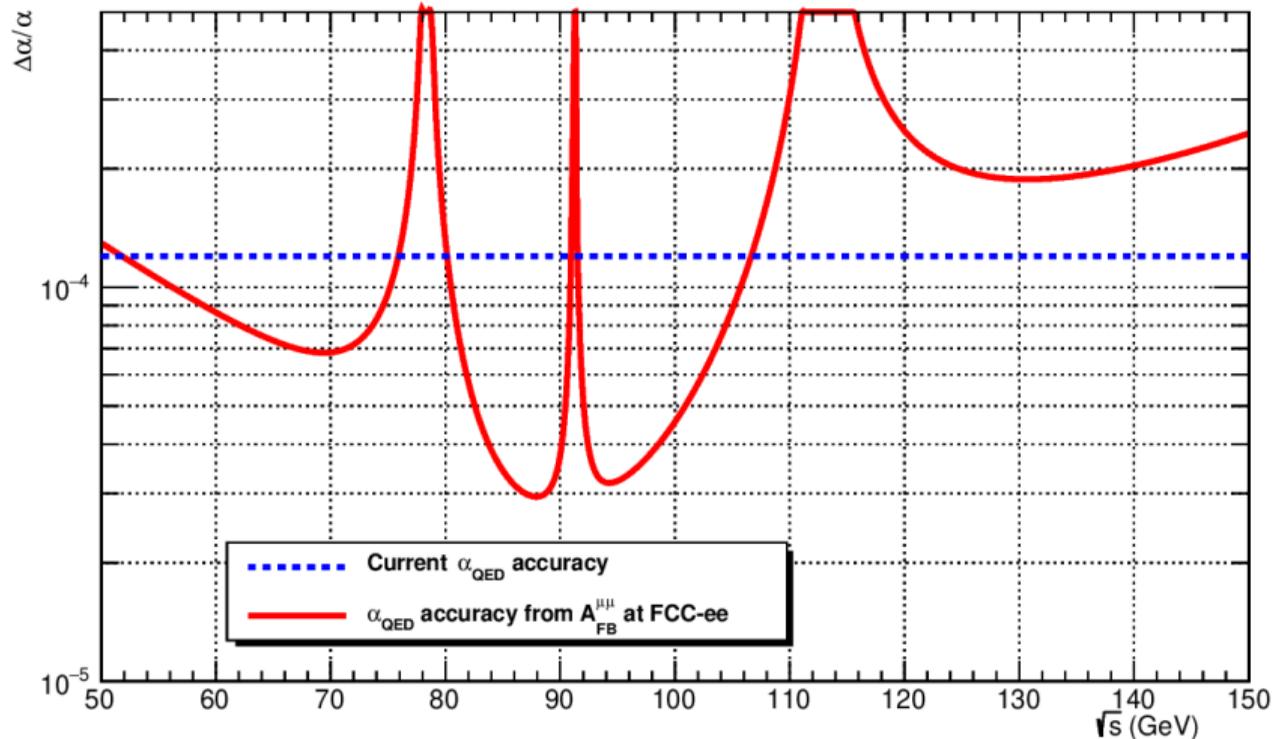
The muon forward-backward asymmetry,  $A_{\text{FB}}^{\mu\mu}$ , is maximally dependent on the interference term

$$A_{\text{FB}}^{\mu\mu} = A_{\text{FB},0}^{\mu\mu} + \frac{3^2}{4^2} \frac{\mathcal{I}}{\mathcal{G} + \mathcal{Z}},$$

varies with  $\alpha_{\text{QED}}(s)$  as follows:

$$\Delta A_{\text{FB}}^{\mu\mu} = \left( A_{\text{FB}}^{\mu\mu} - A_{\text{FB},0}^{\mu\mu} \right) \times \frac{\mathcal{Z} - \mathcal{G}}{\mathcal{Z} + \mathcal{G}} \times \frac{\Delta\alpha}{\alpha}.$$

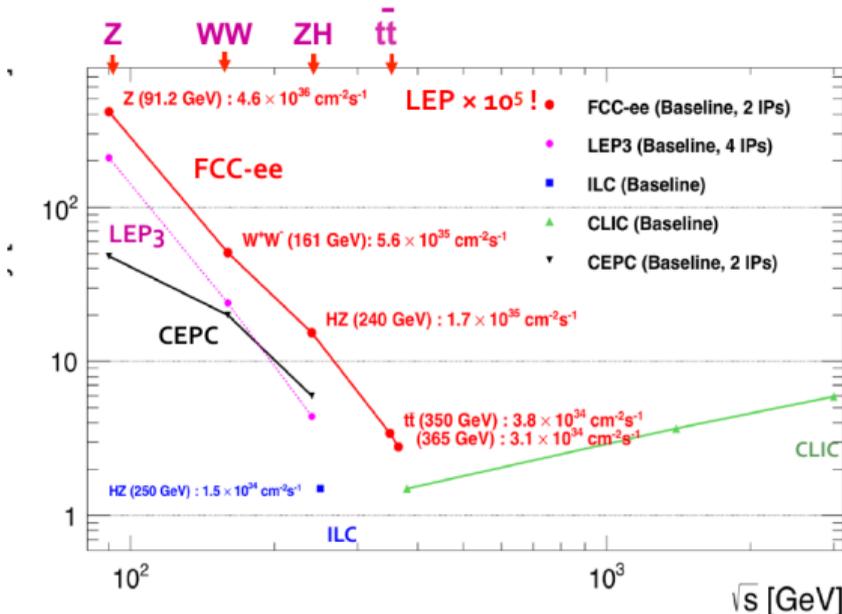
$$e^+e^- \rightarrow \mu^+\mu^- \text{ and } \alpha^2(s)$$



The best accuracy is obtained for one year of running either just below or just above the Z pole, at 87.9 and 94.3 GeV, respectively.

# EW factories : Energies and luminosities

- The FCC-ee offers the largest luminosities in the 88 → 365 GeV  $\sqrt{s}$  range



- Ultimate precision:

- ◆ 100 000 Z / second (!)
- 1 Z / second at LEP
- ◆ 10 000 W / hour
- 20 000 W at LEP
- ◆ 1 500 Higgs bosons / day
- 10 times ILC
- ◆ 1 500 top quarks / day
- in each detector

... in a clean environment:

- No pileup
- Beam backgrounds under control
- E,p constraints

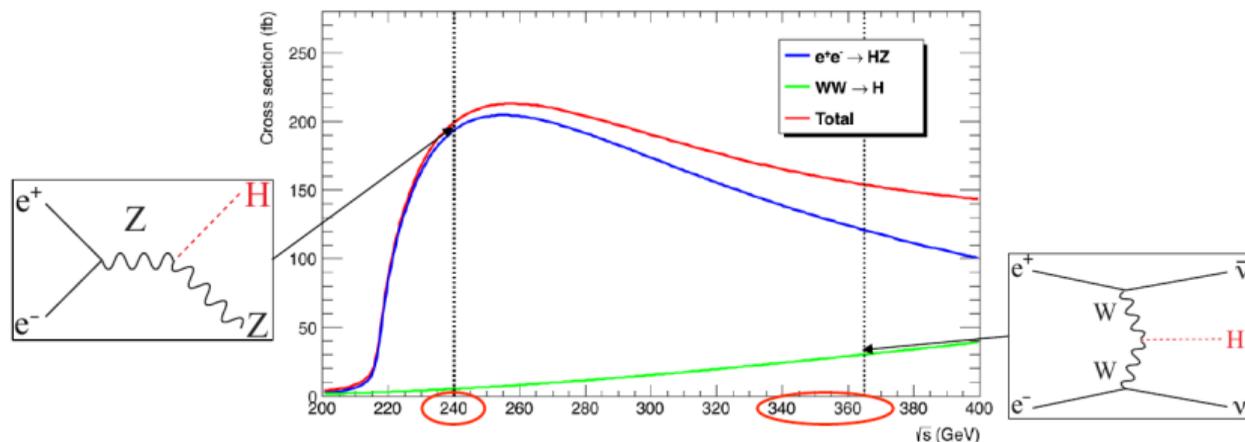
PRECISION and SENSITIVITY  
to rare or elusive phenomena

- ◆ The FCC-ee discovery potential at the precision frontier is multiplied by the presence of the four heaviest SM particles (Z, W, H, and top) in its energy range



# The FCC-ee as a Higgs factory

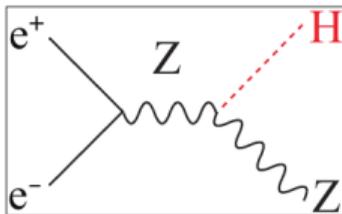
- Higgsstrahlung ( $e^+e^- \rightarrow ZH$ ) event rate largest at  $\sqrt{s} \sim 240$  GeV :  $\sigma \sim 200$  fb



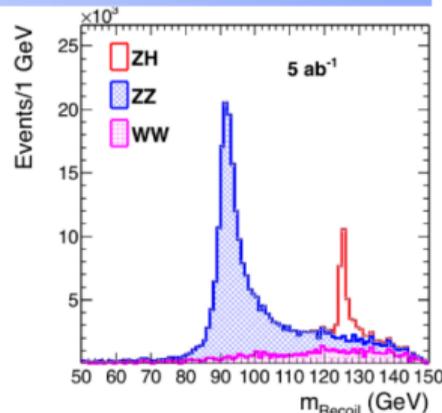
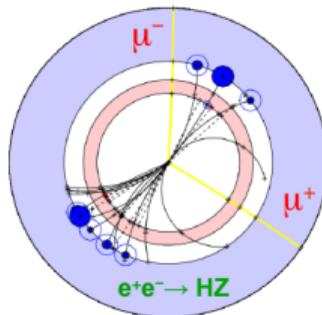
- ◆  $10^6$   $e^+e^- \rightarrow ZH$  events with  $5 \text{ ab}^{-1}$  – cross section predicted with great accuracy
  - Target : (few) per-mil precision, statistics-limited.
  - Complemented with 200k events at  $\sqrt{s} = 350 - 365$  GeV
    - Of which 30% in the WW fusion channel (useful for the  $\Gamma_H$  precision)

# Absolute coupling and width measurement

## □ Higgs tagged by a Z, Higgs mass from Z recoil



$$m_H^2 = s + m_Z^2 - 2\sqrt{s}(E_+ + E_-)$$



- ◆ Total rate  $\propto g_{\text{HZZ}}^2$   $\rightarrow$  measure  $g_{\text{HZZ}}$  to 0.2%
- ◆  $\text{ZH} \rightarrow \text{ZZZ}$  final state  $\propto g_{\text{HZZ}}^4 / \Gamma_H$   $\rightarrow$  measure  $\Gamma_H$  to a couple %
- ◆  $\text{ZH} \rightarrow \text{ZXX}$  final state  $\propto g_{\text{HXX}}^2 g_{\text{HZZ}}^2 / \Gamma_H$   $\rightarrow$  measure  $g_{\text{HXX}}$  to a few per-mil / per-cent
- ◆ Empty recoil = invisible Higgs width; Funny recoil = exotic Higgs decays

## □ Note: The HL-LHC is a great Higgs factory ( $10^9$ Higgs produced) but ...

- ◆  $\sigma_{i \rightarrow f}^{(\text{observed})} \propto \sigma_{\text{prod}} (g_{\text{Hi}})^2 (g_{\text{Hf}})^2 / \Gamma_H$ 
  - Difficult to extract the couplings:  $\sigma_{\text{prod}}$  is uncertain and  $\Gamma_H$  is largely unknown
  - ➔ Must do physics with ratios or with additional assumptions.

# Flavour physics

Table 7.1: Expected production yields of heavy-flavoured particles at Belle II ( $50 \text{ ab}^{-1}$ ) and FCC-ee.

Particle production ( $10^9$ )	$B^0 / \bar{B}^0$	$B^+ / B^-$	$B_s^0 / \bar{B}_s^0$	$\Lambda_b / \bar{\Lambda}_b$	$c\bar{c}$	$\tau^+\tau^-$
Belle II	27.5	27.5	n/a	n/a	65	45
FCC-ee	1000	1000	250	250	550	170

Table 7.2: Comparison of orders of magnitude for expected reconstructed yields of a selection of electroweak penguin and pure dileptonic decay modes in Belle II, LHCb upgrade and FCC-ee experiments. Standard model branching fractions are assumed. The yields for the electroweak penguin decay  $\bar{B}^0 \rightarrow K^{*0}(892)e^+e^-$  are given in the low  $q^2$  region.

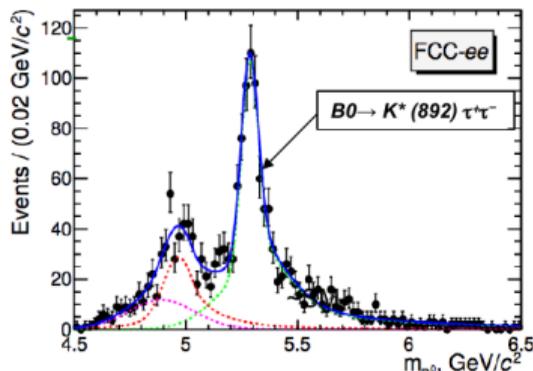
Decay mode	$B^0 \rightarrow K^*(892)e^+e^-$	$B^0 \rightarrow K^*(892)\tau^+\tau^-$	$B_s(B^0) \rightarrow \mu^+\mu^-$
Belle II	$\sim 2\,000$	$\sim 10$	n/a (5)
LHCb Run I	150	-	$\sim 15$ (-)
LHCb Upgrade	$\sim 5000$	-	$\sim 500$ (50)
FCC-ee	$\sim 200000$	$\sim 1000$	$\sim 1000$ (100)

# Flavours : B anomalies, $\tau$ physics, ...

- **Lepton flavour universality is challenged in  $b \rightarrow s \ell^+ \ell^-$  transitions @ LHCb**
  - ◆ This effect, if real, could be enhanced for  $\ell = \tau$ , in  $B \rightarrow K^{(*)} \tau^+ \tau^-$ 
    - Extremely challenging in hadron colliders
    - With  $10^{12} Z \rightarrow bb$ , FCC-ee is beyond any foreseeable competition
    - ◆ Decay can be fully reconstructed; full angular analysis possible

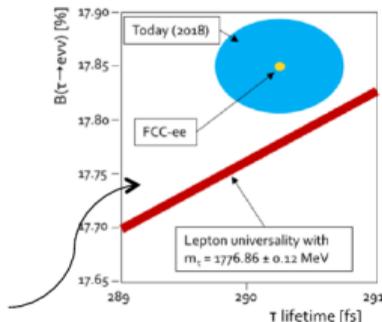
Talk from A. Bondar

J.F. Kamenik et al.  
[arXiv:1705.11106](https://arxiv.org/abs/1705.11106)



Also 100,000  $B_S \rightarrow \tau^+ \tau^-$  @ FCC-ee  
 Reconstruction efficiency under study

- **Not mentioning lepton-flavour-violating decays**
  - ◆  $BR(Z \rightarrow e\tau, \mu\tau)$  down to  $10^{-9}$  (improved by  $10^4$ )
  - ◆  $BR(\tau \rightarrow \mu\gamma, \mu\mu\mu)$  down to a few  $10^{-10}$
  - ◆  $\tau$  lifetime vs  $BR(\tau \rightarrow e\nu_e \nu_\tau, \mu\nu_\mu \nu_\tau)$  : lepton universality tests



# Is a $\sqrt{s} = 500$ GeV upgrade required/useful ?

## □ According to the white book of ESU 2013 :

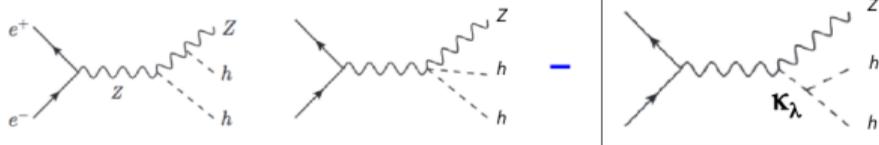
<https://cds.cern.ch/record/1567295/>

At energies of 500 GeV or higher, such a machine could explore the Higgs properties further, for example **the coupling to the top quark, the self-coupling, and the total width.**

- ◆ Responsible for the "... whose energy can be upgraded." in ESU update (CERN Council)
  - You will probably hear more of that during ESU 2020!
- ◆ So, should we foresee an upgrade of FCC-ee at  $\sqrt{s} = 500$  GeV ?
  - For the total width and the coupling to the top quark : the answer is NO (slide 18)
  - For the Higgs self-coupling ( $\kappa_\lambda$ ):

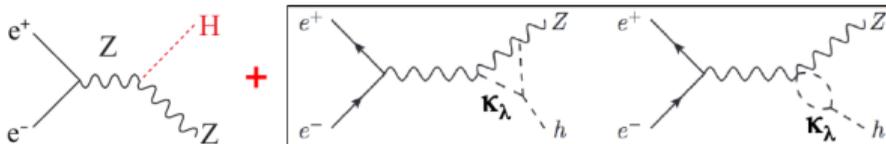
At  $\sqrt{s} = 500$  GeV

Di-Higgs production



At FCC-ee

$\sigma_{HZ}$

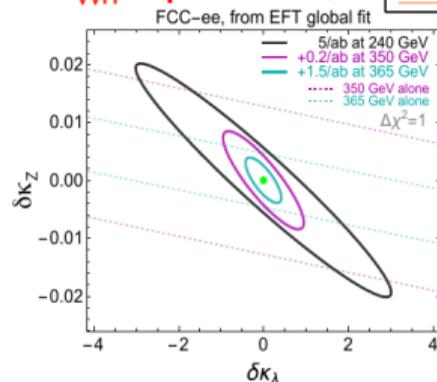
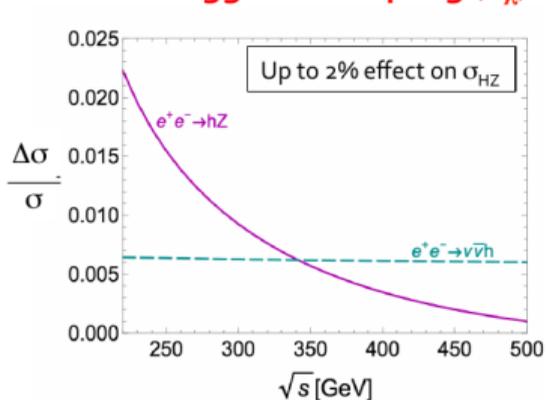


M. McCullough  
[arXiv:1812.0322](https://arxiv.org/abs/1812.0322)

# Higgs self-coupling at the FCC-ee

- Effect of Higgs self coupling ( $\kappa_\lambda$ ) on  $\sigma_{ZH}$  and  $\sigma_{\nu\nu h}$  depends on  $\sqrt{s}$

C. Grojean et al.  
arXiv:1711.03978



- ◆ Two energy points lift off the degeneracy between  $\delta\kappa_Z$  and  $\delta\kappa_H$ 
  - Precision on  $\kappa_\lambda$  with 2 IPs at the end of the FCC-ee (91+160+240+365 GeV)
    - Global EFT fit (model-independent) :  $\pm 35\%$  ; in the SM :  $\pm 24\%$  (3-4 $\sigma$ )
  - Precision on  $\kappa_\lambda$  with 4 IPs :  $\pm 23\%$  (EFT fit) ;  $\pm 16\%$  (SM fit)
    - 5 $\sigma$  discovery with 4 IPs instead of 2 – much less costly than 500 GeV upgrade (in time and funds, in view of FCC-hh)
- And, most importantly
  - ◆ Only FCC-hh, in combination with FCC-ee, can measure  $\kappa_{top}$  and  $\kappa_\lambda$  to 1% and 5%, resp.

A. Blondel, P. J.

arXiv:1809.10041

## FCC-ee and FCC-hh (FCC-eh)

- ① measure and tightly constrain a comprehensive set of electroweak and Higgs observables with high precision, 10-100 times better than so far;
- ② unveil small but significant deviations with respect to the standard model predictions,
- ③ observe rare new processes or particles, beyond the standard model expectations,

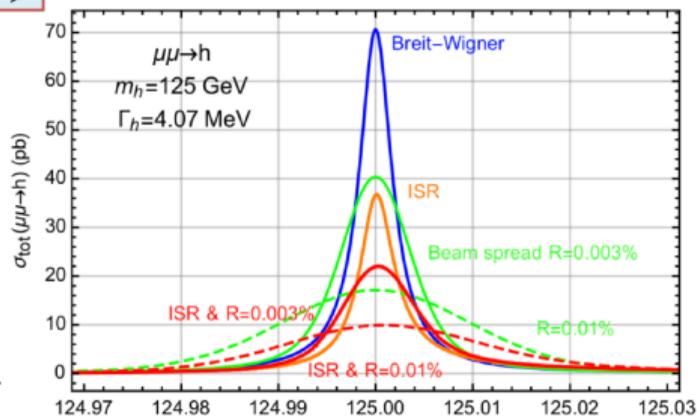
# Lepton Colliders: $\mu$ vs $e$ @ $\sqrt{s}=125$ GeV

Back on the envelope calculation:

$$\sigma(\mu^+\mu^- \rightarrow H) = \left(\frac{m_\mu}{m_e}\right)^2 \times \sigma(e^+e^- \rightarrow H) = \left(\frac{105.7\text{MeV}}{0.511\text{MeV}}\right)^2 \times \sigma(e^+e^- \rightarrow H)$$

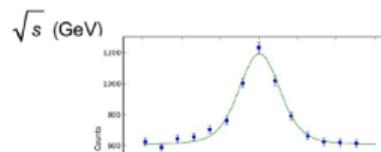
$$\sigma(\mu^+\mu^- \rightarrow H) = 4.3 \times 10^4 \times \sigma(e^+e^- \rightarrow H)$$

More precise determination  
by M. Greco et al. [arXiv:1607.03210v2](https://arxiv.org/abs/1607.03210v2)



R: percentage beam energy resolution, key parameter

$\sigma(\text{BW})$	ISR alone	R (%)	BES alone	BES+ISR
$\mu^+\mu^-$ : 71 pb	37	0.01	17	10
		0.003	41	22
$e^+e^-$ : 1.7 fb	0.50	0.04	0.12	0.048
		0.01	0.41	0.15



Higgs width 4.2 MeV  
Beam energy spread  $\sim 10^{-5}$

# Neutrino induced hazard

Neutrino radiation imposes major design and siting constraints on multi-TeV muon colliders or inventing smart solutions!

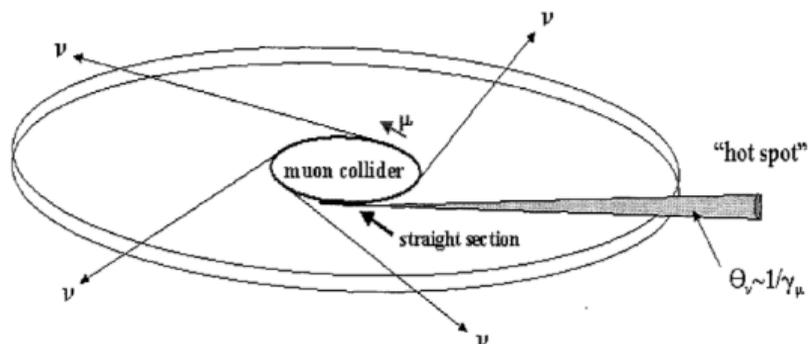


Table 4. Constraints on lattice designs to limit neutrino radiation.

$E$	$B(\text{min})$	$L(\text{max})$	$\mathcal{L}$
TeV	T	m	$10^{34} \text{ cm}^{-2} \text{ s}^{-1}$
1.5	0.25	2.4	0.008
3.0	1.5	0.28	0.6
6.0	1.5	0.28	12*

One concept

- Solution beyond 10 TeV unclear at present†

\* constrained by  $\nu$  radiation

Muon Collider '18, U Padova 7/1–3, 2018

† although cf. AIP Conf. Proc. 1507 (2012) 860



D. M. Kaplan

www.it.edu



12/17

New background generation with new neutrino cross sections planned with FLUKA

## Present situation

"The effect of a **concept-driven** revolution is to explain old things in new ways.

The effect of a **tool-driven revolution** is to discover new things that have to be explained"

"New directions in science are launched by new tools much more often than by new concepts ."

Freeman Dyson.

**No energy reference scale anymore!**

# Knowns and unknowns (from talk by Josh Ruderman, FCC Week 2018)

February 2002:

*“As we know there are **known knowns**;  
there are things we know we know.*



*We also know there are **known unknowns**;  
that is to say we know there are some things we do not know.*

*But there are also **unknown unknowns** –  
the ones we don't know we don't know.”*

Donald Rumsfeld,  
US Secretary of Defense  
1975-1977, 2001-2006

## Unknown Knowns

*“things you think you know that it turns  
out you did not.”*

- Donald Rumsfeld, 2004 memo

*“things that you possibly may know,  
that you don't know you know”*

- Donald Rumsfeld, 2013 interview



# Knowns and unknowns

(from talk by Josh Ruderman, FCC Week 2018)

## Rumsfeld's Matrix of Knowledge

<p><b>known knowns</b></p> <p><i>things we know we know</i></p>	<p><b>known unknowns</b></p> <p><i>things we know we don't know</i></p>
<p><b>unknown knowns</b></p> <p><i>things we think we know but we don't know</i></p>	<p><b>unknown unknowns</b></p> <p><i>things we don't know we don't know</i></p>

## Rumsfeld's Matrix of Particle Physics (from talk by JR, FCC Week 2018)

<b>known knowns</b>  Standard Model	<b>known unknowns</b>  "known" new physics
<b>unknown knowns</b>  new physics modifies known physics <hr/> and maybe we already measured it!	<b>unknown unknowns</b>  surprises