TWO-LOOP FIVE-POINT AMPLITUDES FROM TWO-LOOP NUMERICAL UNITARITY

DESY ZEUTHEN 2019

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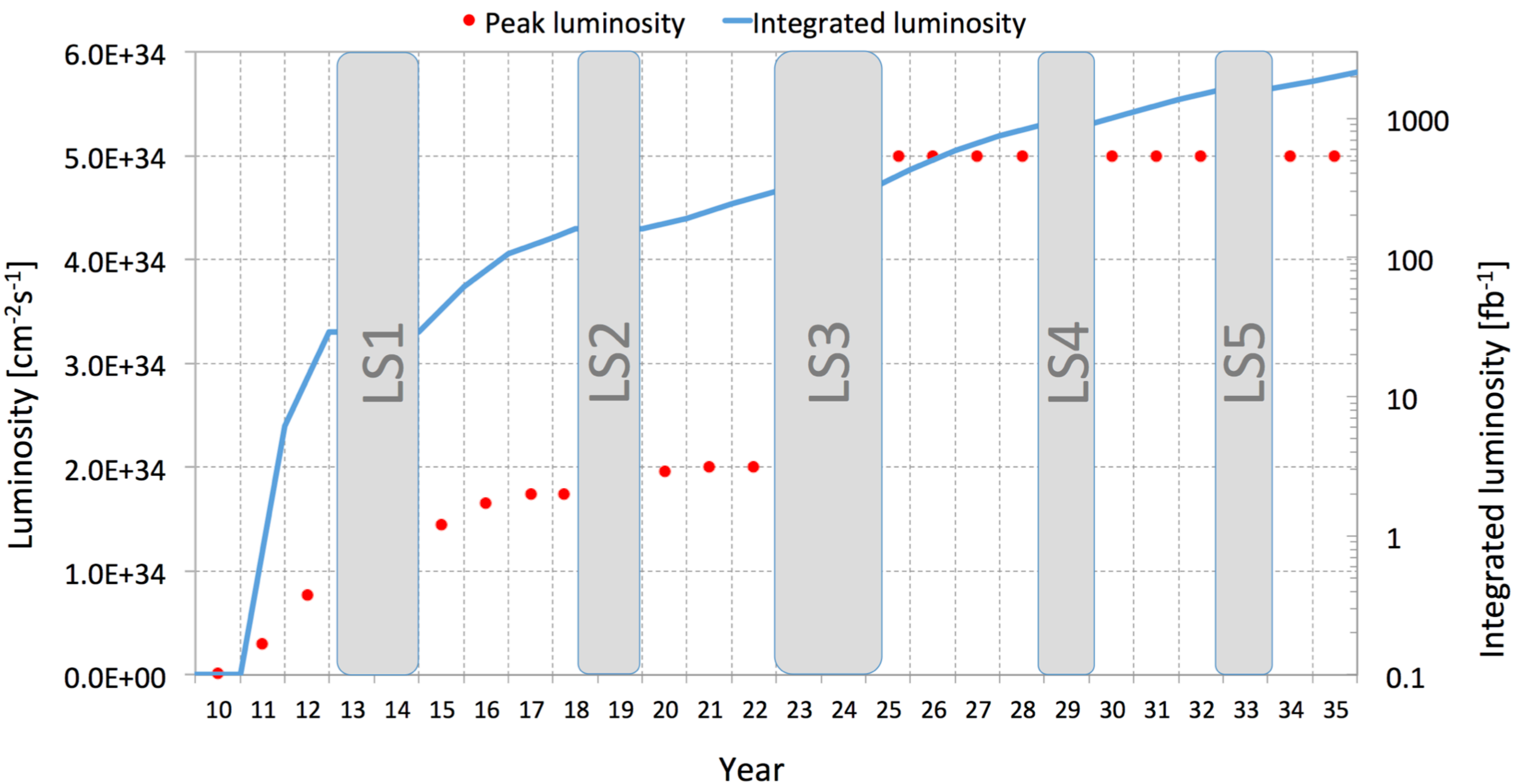
OUTLINE

1. Motivation

2. Two-loop numerical unitarity

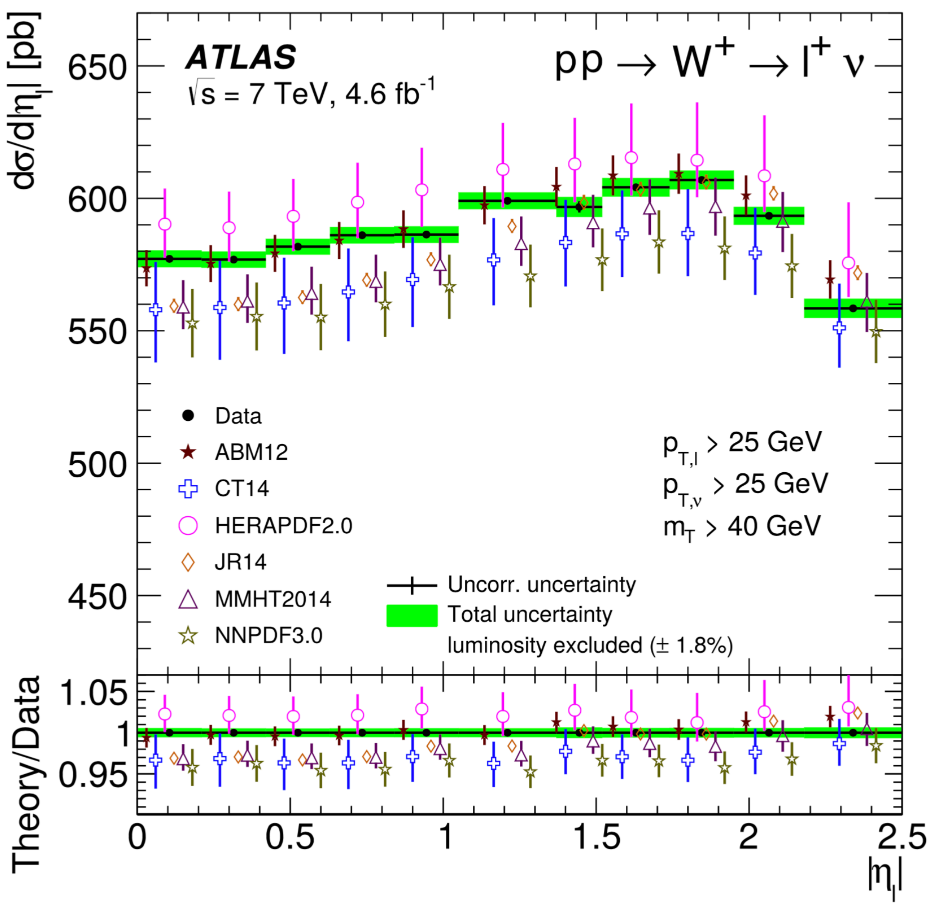
3. Analytic five-patron amplitudes

MOTIVATION



The LHC will still be running for many years, probing the Standard Model at the percent level for many observables: theory must keep up with experiments

MOTIVATION



[Eur.Phys.J.C 77, no.6, 367 (2017)]

Les Houches whishlist 2017 [1803.07977]

process	known	desired
$pp \to 2\,\text{jets}$	$\text{N}^2\text{LO}_\text{QCD}$ $\text{NLO}_\text{QCD}+\text{NLO}_\text{EW}$	
$pp \to 3\,\text{jets}$	NLO_QCD	$\text{N}^2\text{LO}_\text{QCD}$

Table I.2: Precision wish list: jet final states.

α_s determination

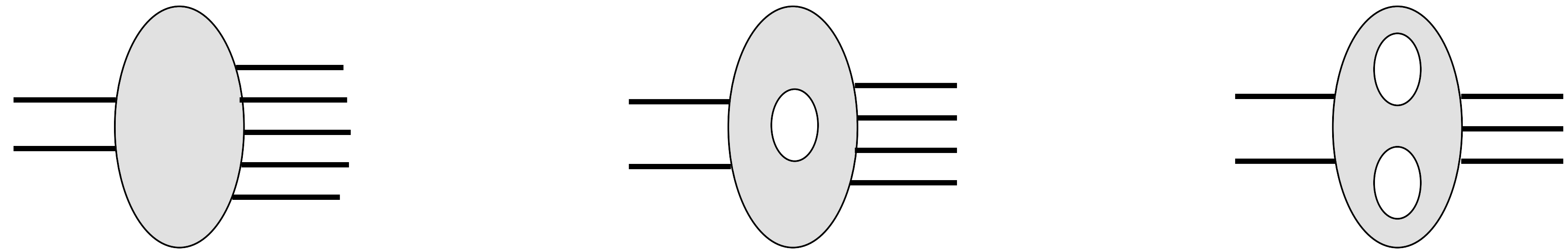
- Percent-level precision for many observables: requires NNLO QCD.
- Many 2 to 2 processes are known at NNLO
- Can we probe kinematic dependence of final states? Add recoiling jet?
- Can we add mass effects?

Two key building blocks:

- Handling of IR divergences at NNLO.
- Calculation of two-loop (multi-leg/scale) amplitudes.

TWO-LOOP FIVE-PARTON QCD AMPLITUDES

Cross-section for
3jet production
at NNLO



$$A = \sum_i c_i(\vec{x}, \epsilon)\, m_i(\vec{x}, \epsilon)$$

Master coefficients: specific to each process.

Rational (algebraic) functions of the external kinematics and the dimensional regulator

Master integrals: depend only on the kinematics of the process

Special functions (polylogarithms, elliptic, …) of the external kinematics and the dimensional regulator

- Important contribution to NNLO corrections
- Laboratory for complex multi-scale calculation
- Analytic results: faster, stable and flexible
- Study interesting mathematical properties

RECENT 5-POINT INTEGRALS/AMPLITUDE COMPUTATIONS

Analytic massless 5-point integrals

- Planar [Papadopoulos, Tommasini, Wever 15] [Gehrmann, Henn, lo Presti 15, 18]
- Non-planar: symbol results [Abreu, Dixon, Herrman, Page, Zeng 18] [Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia, 18]

IBP tables

- Planar [Boels, Jin, Luo 18] [Chawdhry, Lim, Mitov 18]
- Non-planar: partial [Böhm, Georgoudis, Larsen, Schönemann, Zhang 18]

Recent progress in 5-point QCD amplitudes

- Numerics 5-parton: [Badger, Brønnum-Hansen, Bayu, Gehrmann, Hartanto, Henn, lo Presti, Peraro 17, 18] [Abreu, Febres Cordero, Ita, Page, Sotnikov, Zeng 17, 18]
- Analytic 5g -++++: [Badger, Brønnum-Hansen, Bayu, Hartanto, Peraro 18]
- Analytic 5-parton: [Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov 18, 19]
- Analytic 5g +++++: [Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zoia, 19]

TWO-LOOP AMPLITUDE — STANDARD APPROACH

Feynman diagrams

- Tensor reduction [Passarino, Veltman 79]
- IBPs [Tkachov, Chetyrkin 81, Laporta 01]

Master integral decomposition

$$\mathcal{A} = \sum_{\Gamma} \sum_{i \in M_\Gamma} c_{\Gamma,i}(D) I_{\Gamma,i}(D)$$

- Diff. eq. [Kotikov 91, Gehrmann, Remiddi 01; Henn 13]
- Direct integration [..., HyperInt]
- Numeric integration [..., SecDec, Fiesta]

Integrated expression

(very simple compared to size of intermediate expressions)

Completely general approach, but:

- large intermediate expressions
- large IBP system
- parallelisation is non-trivial

Bad scaling with number of legs and masses

Numerical unitarity method

1. Numerical approach: suitable for multi-scale processes
2. Reduction & coefficient evaluation done simultaneously
3. Use numerical data to reconstruct analytics

TWO-LOOP NUMERICAL UNITARITY

TWO-LOOP UNITARITY AT 2-LOOPS

[S. Abreu, F. Febres Cordero, H. Ita, M. Jaquier, B. Page, M. Zeng, 17]

$$\mathcal{A}(\ell_l) = \sum_{\Gamma} \sum_{i=1}^{\dim(\Gamma)} c_{\Gamma,i} \frac{\tilde{m}_{\Gamma,i}(\ell_l)}{\prod_{j\in P_\Gamma} \rho_j}$$

Choosing good variables:
Polynomial and **unitarity compatible** parametrisation of the integrand

Change of integrand basis:
Construct **surface terms** and **master integrands**

$$\mathcal{A}(\ell_l) = \sum_{\Gamma} \sum_{i\in M_\Gamma \cup S_\Gamma} c_{\Gamma,i}(D) \frac{m_{\Gamma,i}(\ell_l, D)}{\prod_{j\in P_\Gamma} \rho_j}$$

Obtain integrated amplitude:
Determine **coefficients with (generalised) unitarity** and insert expression for **master integrals**

$$\mathcal{A} = \sum_{\Gamma} \sum_{i\in M_\Gamma} c_{\Gamma,i}(D) I_{\Gamma,i}(D)$$

Generalisation of one-loop: Ossola, Papadopoulos, Pittau 07; Ellis, Giele Kunszt 07; Giele Kunszt, Melnikov 08; Berger, Bern, Dixon, Febres Cordero, Ita, Kosower, Maitre 08

Related two-loop approaches: Badger, Frellesvig, Zhang 12, 13; Mastrolia, Mirabella, Ossola, Peraro 12; Mastrolia, Peraro, Primo 16; Badger, Brønnum-Hansen, Hartanto, Peraro 17; Boels, Jin, Luo 18; Chawdhry, Lim, Mitov 18

INTEGRAND PARAMETRISATION & SURFACE TERMS

[H. Ita 15; S. Abreu, F. Febres Cordero, H. Ita, M. Jaquier, B. Page, M. Zeng 17]

Make decomposition in terms of master integrals explicit at the integrand level:

$$\mathcal{A} = \sum_{\Gamma} \sum_{i \in M_\Gamma} c_{\Gamma,i}(D) I_{\Gamma,i}(D) \longrightarrow \mathcal{A}(\ell_l) = \sum_{\Gamma} \sum_{i \in M_\Gamma \cup S_\Gamma} c_{\Gamma,i}(D) \frac{m_{\Gamma,i}(\ell_l, D)}{\prod_{j \in P_\Gamma} \rho_j}$$

Integrand numerator is polynomial in components of loop momentum:

$$m_{\Gamma,i}(\ell_l) \longrightarrow m_{\Gamma,i}(\rho, \lambda, \alpha) \longrightarrow m_{\Gamma,i}(\lambda, \alpha)$$

Monomials that depend on transverse variables: [Passarino-Veltman reduction]

- Monomials with odd powers are surface terms
- Construct surface terms for even powers

$$\alpha_k^i \alpha_k^j \to \left(\alpha_k^i \alpha_k^j - \frac{\mu_{ij}}{D-4} \right)$$

Polynomial in propagators and ISPs

SURFACE TERMS & IBPS

[H. Ita 15; S. Abreu, F. Febres Cordero, H. Ita, M. Jaquier, B. Page, M. Zeng 17]

Surface terms from specific IBP relations: control propagator power

$$\int d^D\ell_l \sum_k \frac{\partial}{\partial \ell_k^\nu}\left[\frac{u_k^\nu}{\prod_{j\in P_\Gamma}\rho_j}\right] = 0 \qquad\qquad u_k^\nu \frac{\partial}{\partial \ell_k^\nu}\rho_j = f_j \rho_j$$

[Gluza, Kajda, Kosower 10; Schabinger 11]

Construct IBP-generating vectors as polynomial solution to,

$$\left(u_{ka}^{\text{loop}}\ell_a^\nu + u_{kb}^{\text{ext}} p_b^\nu\right)\frac{\partial}{\partial \ell_k^\nu}\begin{pmatrix}\rho_1\\ \rho_2\\ \vdots\\ \rho_{|\Gamma|}\end{pmatrix} - \begin{pmatrix}f_1\rho_1\\ f_2\rho_2\\ \vdots\\ f_{|\Gamma|}\rho_{|\Gamma|}\end{pmatrix} = \begin{pmatrix}0\\ 0\\ \vdots\\ 0\end{pmatrix}$$

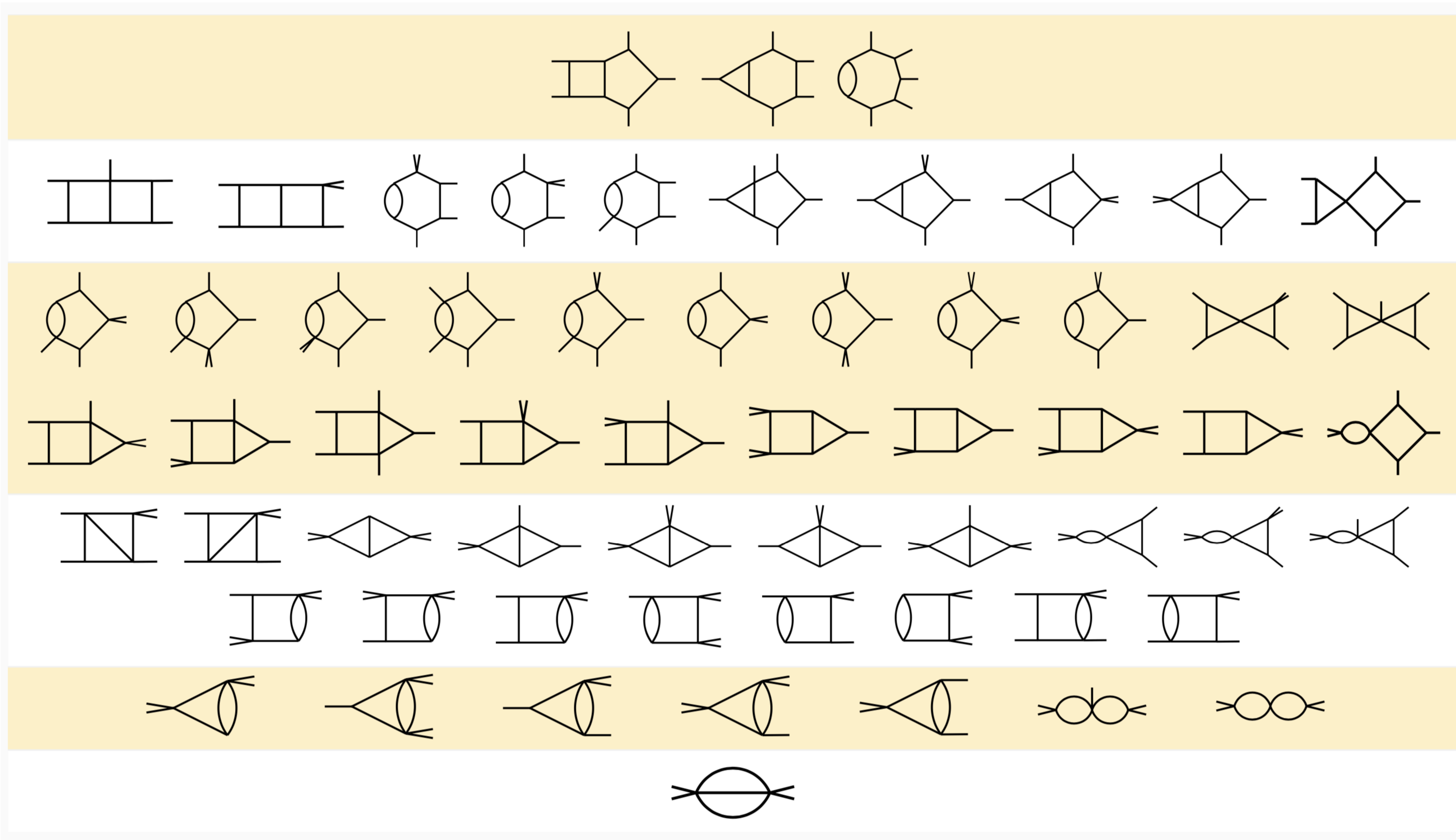
[generating set from Singular]

[related work J. Boehm, A. Georgoudis, K. J. Larsen, M. Schulze, Y. Zhang 16 - 18]

Fill remaining integrand function space with master integrands

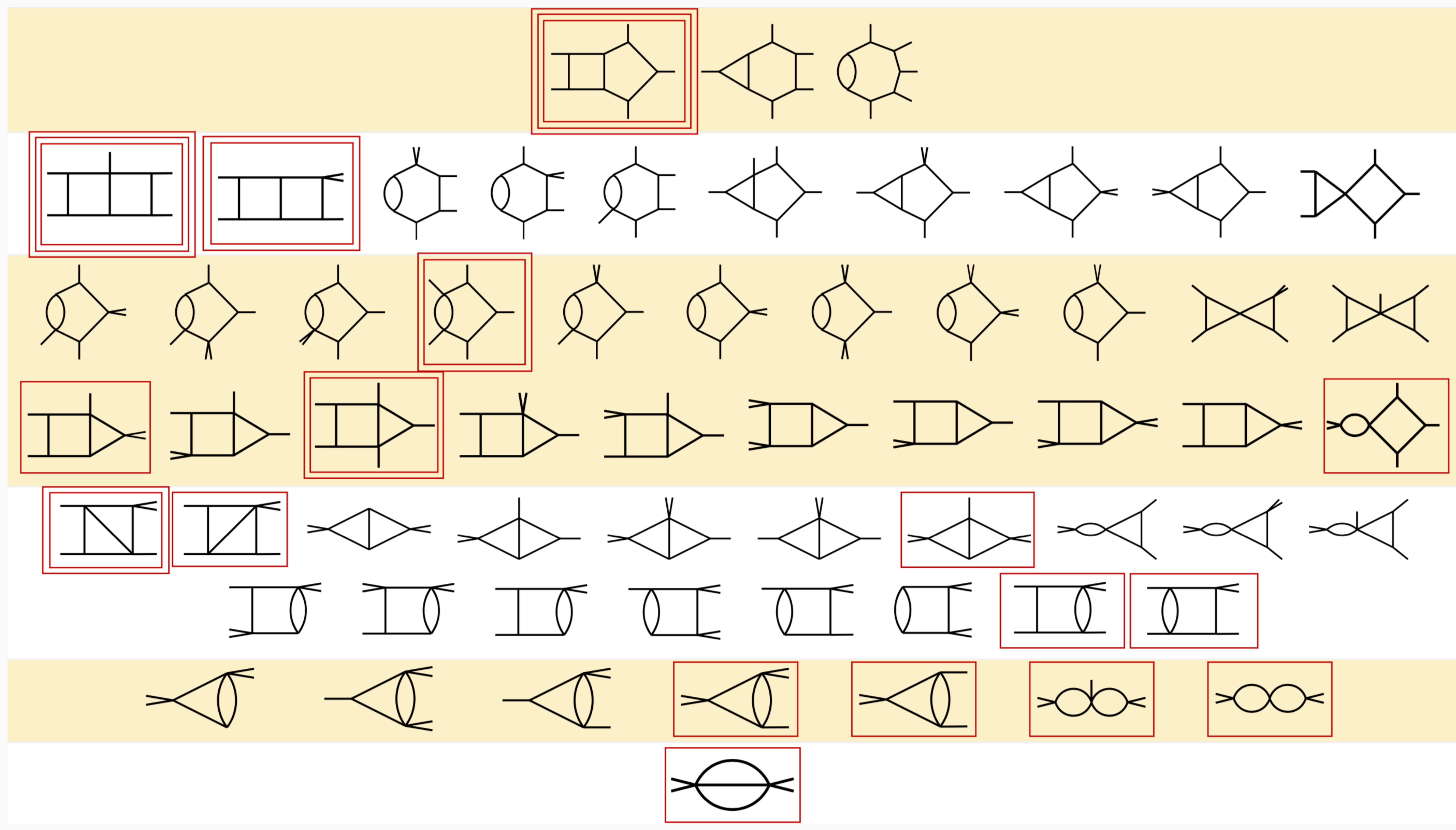
$$\mathcal{A}(\ell_l) = \sum_\Gamma \sum_{i\in M_\Gamma\cup S_\Gamma} c_{\Gamma,i}(D)\frac{m_{\Gamma,i}(\ell_l, D)}{\prod_{j\in P_\Gamma}\rho_j}$$

INTEGRAND FOR TWO-LOOP FIVE-POINT MASSLESS



All propagator structures

INTEGRAND FOR TWO-LOOP FIVE-POINT MASSLESS



Topologies with master integrals

DETERMINING THE COEFFICIENTS WITH UNITARITY

$$\mathcal{A}(\ell_l) = \sum_{\Gamma} \sum_{i \in M_\Gamma \cup S_\Gamma} c_{\Gamma,i}(D) \frac{m_{\Gamma,i}(\ell_l, D)}{\prod_{j \in P_\Gamma} \rho_j}$$

Determine the coefficients from on-shell information [Bern, Dixon, Kosower, Dunbar 94,95]

$$\lim_{\ell_l \to \ell_l^\Gamma} \mathcal{A}(\ell_l) = \frac{\prod \mathcal{A}^{\text{tree}}(\ell_l^\Gamma)}{\prod_{j \in P_\Gamma} \rho_j(\ell_l^\Gamma)} + \mathcal{O}(\rho_j)$$

ℓ^Γ: loop momenta evaluated at $\rho_j = 0\,, \qquad \forall j \in P_\Gamma$

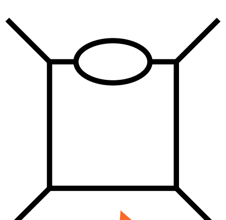
Requires fast evaluation of tree amplitudes: Berends-Giele recursion

Construct and solve linear system for the coefficients

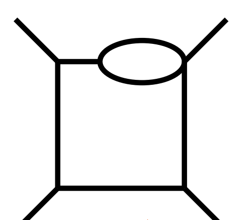
Final results: decomposition in terms of master integrals:

$$\mathcal{A} = \sum_{\Gamma} \sum_{i \in M_\Gamma} c_{\Gamma,i}(D) I_{\Gamma,i}(D)$$

Starting at two-loops, need to worry about subleading poles



Leading pole: standard procedure



Subleading pole: need to be more careful

Construct both numerators together

Same on-shell phase space

[S. Abreu, F. Febres Cordero, H. Ita, M. Jaquier, B. Page 17]

TWO-LOOP NUMERICAL UNITARITY AT 2-LOOPS

$$\mathcal{A}(\ell_l) = \sum_{\Gamma} \sum_{i=1}^{\dim(\Gamma)} c_{\Gamma,i} \frac{m_{\Gamma,i}(\ell_l)}{\prod_{j\in P_\Gamma} \rho_j}$$

Parametrisation of integrand

$$\mathcal{A}(\ell_l) = \sum_{\Gamma} \sum_{i\in M_\Gamma \cup S_\Gamma} c_{\Gamma,i}(D) \frac{m_{\Gamma,i}(\ell_l, D)}{\prod_{j\in P_\Gamma} \rho_j}$$

Master/surface term decomposition

$$\mathcal{A} = \sum_{\Gamma} \sum_{i\in M_\Gamma} c_{\Gamma,i}(D) I_{\Gamma,i}(D)$$

Evaluation of coefficients

Analytic expressions

Numerics

Floating point arithmetic:

- good behaviour with increase in number of scales
- can compute at arbitrary (even non-rational) PS point
- need to worry about loss of precision

Used in 4-gluon 2-loop calculation [S. Abreu, F. Febres Cordero, H. Ita, M. Jaquier, B. Page, M. Zeng, 17]

Finite-field arithmetic:

[Schabinger, von Manteuffel 14], [Peraro 16]

- exact calculation, no loss of precision
- opens the door to possible analytic reconstruction
- only rational PS points
- \`complicated' PS points take longer (more FF needed)
- update algorithm to avoid non-rational numbers at intermediate steps

Used in 5-gluon 2-loop calculation [S. Abreu, F. Febres Cordero, H. Ita, B. Page, M. Zeng, 17]

FINITE FIELDS AND NUMERICAL UNITARITY

[Peraro 16] [S. Abreu, F. Febres Cordero, H. Ita, B. Page, M. Zeng 17]

- Work in a finite field: $\mathbb{F}_p = \{0, \ldots, p-1\}$
  - Integers modulo *p*
  - Operations modulo *p*, define inverse through Euclidean algorithm
  - Algorithms exist for mapping from $\mathbb{F}_p$ to $\mathbb{Q}$. Might require several finite fields.

- Requires:
  - Rational external kinematics (e.g. use mom. twistors for 5 points)
  - Avoid complex numbers (use +-+- metric)
  - Rational on-shell momenta (use orthogonal vectors)
  - Avoid constructing polarisation states in BG recursion

TWO-LOOP NUMERICAL UNITARITY

[H. Ita 15; S. Abreu, F. Febres Cordero, H. Ita, M. Jaquier, B. Page, M. Zeng 17]

Step 1: construct analytic ansatz for the integrand

Step 2: numerically fix the ansatz at each phase-space point

- Generalised unitarity, factorisation in on-shell limits

Step 3: solve numerical linear system for master integral coefficient

Step 4: insert expressions for master integrals

Improvements: numerical calculations in finite field, get exact coefficients

[Schabinger, von Manteuffel 14], [Peraro 16], [S. Abreu, F. Febres Cordero, H. Ita, B. Page, M. Zeng, 17]

Computed all planar two-loop five-parton amplitude (full Nf dependence)

JHEP 1811 (2018) 116 [S. Abreu, H. Ita, F. Febres Cordero, B. Page, V. Sotnikov]

RESULTS — 5-PARTON AMPLITUDES

[S. Abreu, F. Febres Cordero, H. Ita, B. Page, M. Zeng 17]

- All helicities, no quark-loops

$\mathcal{A}^{(2)[N_f^0]}/\mathcal{A}^{(\mathrm{norm})}$	ϵ^{-4}	ϵ^{-3}	ϵ^{-2}	ϵ^{-1}	ϵ^{0}
$(1_g^+, 2_g^+, 3_g^+, 4_g^+, 5_g^+)$	0	0	−5.000000000	−29.38541207	−62.68413553
$(1_g^-, 2_g^+, 3_g^+, 4_g^+, 5_g^+)$	0	0	−5.000000000	−42.33840431	−159.9778589
$(1_g^-, 2_g^-, 3_g^+, 4_g^+, 5_g^+)$	12.50000000	84.83123596	243.4660216	301.9565843	−152.0528809
$(1_g^-, 2_g^+, 3_g^-, 4_g^+, 5_g^+)$	12.50000000	84.83123596	269.4635002	551.6251881	984.0882231
$(1_q^+, 2_{\bar{q}}^-, 3_g^+, 4_g^+, 5_g^+)$	0	0	−4.000000000	−33.66432052	−117.5792214
$(1_q^+, 2_{\bar{q}}^-, 3_g^+, 4_g^+, 5_g^-)$	8.000000000	51.38308777	127.3357346	55.24748112	−511.9128286
$(1_q^+, 2_{\bar{q}}^-, 3_g^+, 4_g^-, 5_g^+)$	8.000000000	51.38308777	137.2047686	143.1002284	−154.2224796
$(1_q^+, 2_{\bar{q}}^-, 3_g^-, 4_g^+, 5_g^+)$	8.000000000	51.38308777	133.2453937	110.9941406	−263.9507190
$(1_q^+, 2_{\bar{q}}^-, 3_Q^+, 4_{\bar{Q}}^-, 5_g^+)$	4.500000000	23.78050411	33.01035431	−76.65528489	−305.7123751
$(1_q^+, 2_{\bar{q}}^-, 3_Q^-, 4_{\bar{Q}}^+, 5_g^+)$	4.500000000	23.78050411	25.33119767	−122.8050519	−400.0885233
$(1_q^+, 2_{\bar{q}}^-, 3_Q^+, 4_{\bar{Q}}^-, 5_g^-)$	4.500000000	23.78050411	25.00917906	16.91995611	579.1225796
$(1_q^+, 2_{\bar{q}}^-, 3_Q^-, 4_{\bar{Q}}^+, 5_g^-)$	4.500000000	23.78050411	−1009.208812	−4797.768367	4827.790534

Exact numeric results: precision depends on number of digits of GiNaC output when evaluating master integrals

ANALYTIC 5 PARTON AMPLITUDES

WHAT IS A QUARK HELICITY AMPLITUDE IN DIM REG?

[JHEP **0409** (2004) 039, Bern, de Freitas]
[JHEP **0404** (2004) 021, Glover]

Extend (Clifford algebra) to D dimensions

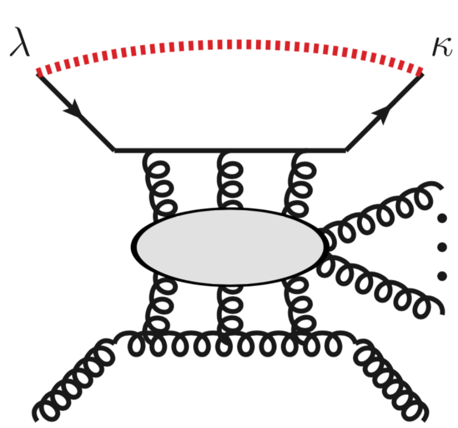
$$(\gamma^{\mu}_{[D_s]})^{b\lambda}_{a\kappa} = \begin{cases} \left(\gamma^{\mu}_{[4]}\right)_a^{\ b} \delta^{\lambda}_{\kappa}\,, & 0 \le \mu \le 3\,, \\ \left(\tilde{\gamma}_{[4]}\right)_a^{\ b} \left(\gamma^{(\mu-4)}_{[D_s-4]}\right)_{\kappa}^{\ \lambda}\,, & \mu > 3\,, \end{cases}$$

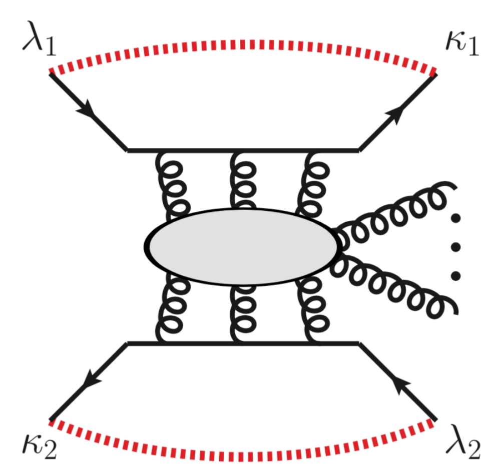
Quark amplitudes are tensors:

$$M^{(k)} = \sum_n v_n M_n^{(k)}$$

$$\left(M^{(0)}\right)^{\lambda_1,\lambda_2}_{\kappa_1,\kappa_2} = \begin{cases} M_0^{(0)}\,\delta^{\lambda_1}_{\kappa_1}\delta^{\lambda_2}_{\kappa_2} & \text{HV} \\ M_0^{(0)}\,\delta^{\lambda_1}_{\kappa_1}\delta^{\lambda_2}_{\kappa_2} + M_1^{(0)}\left(\gamma^{\mu}_{[D_s-4]}\right)^{\lambda_1}_{\kappa_1}\left(\gamma_{[D_s-4]\mu}\right)^{\lambda_2}_{\kappa_2} & \text{CDR} \end{cases}$$

Interference with tree gives tracing prescription





Note: Not trivial to implement in numerical calculation, efficiently handled with extra scalar particle

[arXiv:1803.11127, Anger, Sotnikov]
[Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov, 19]

NUMERICAL UNITARITY & ANALYTIC RECONSTRUCTION

Two-loop numerical unitarity:

- generic approach, scales well with the number of variables 😀
- possible numerical instabilities over phase space, longer calculation times 🙁

Use exact two-loop numerical unitarity results to determine the analytic expressions

[JHEP **1901** (2019) 186, Badger, Brønnum-Hansen, Hartanto, Peraro]
[PRL **122** (2019) 082002, Abreu, Dormans, Ita, Febres Cordero, Page]

$$A = \sum_i c_i(\vec{x}, \epsilon)\, m_i(\vec{x}, \epsilon)$$

The c_i are rational functions, can be determined from enough numerical data [Peraro 16]

- fast and exact data from two-loop numerical unitarity in finite field

The c_i are simpler than what intermediate steps in standard approach suggest

- directly target analytics of simpler expressions

Analytic reconstruction programs are now available [FireFly, 2019, J. Klappert, F. Lange] [FiniteFlow, 2019, Peraro]

ANALYTIC RECONSTRUCTION

GOAL: reconstruct the simplest objects you can think of

[Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov 18, 19]

Finite two-loop remainder: subtract known contributions from lower loops

$$R^{(2)} = A^{(2)} - \mathbf{I}^{(1)} A^{(1)} - \mathbf{I}^{(2)} A^{(0)}$$

see e.g. [Gehrmann, Henn, lo Presti 15] in the context of 2-loop 5-pt amplitudes

`Pentagon functions' decomposition: remove relations between master integrals

[D. Chicherin, J. Henn, V. Mitev 17; T. Gehrmann, J. Henn, N. Lo Presti 18]

$$R^{(2)} = \sum_i c_i(\vec{x}) h_i(\vec{x})$$

Master integrals from IBPs are linearly independent before expanding in epsilon, but many relations after expansion

VARIABLES

[Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov 18, 19]

$$R^{(2)} = \sum_i c_i(\vec{x}) h_i(\vec{x})$$

$$\vec{s} = \{1, s_{23}, s_{34}, s_{45}, s_{15}\} \qquad \mathrm{tr}_5 = 4i\varepsilon(p_1, p_2, p_3, p_4)$$

need to rationalise square-root to use finite fields!

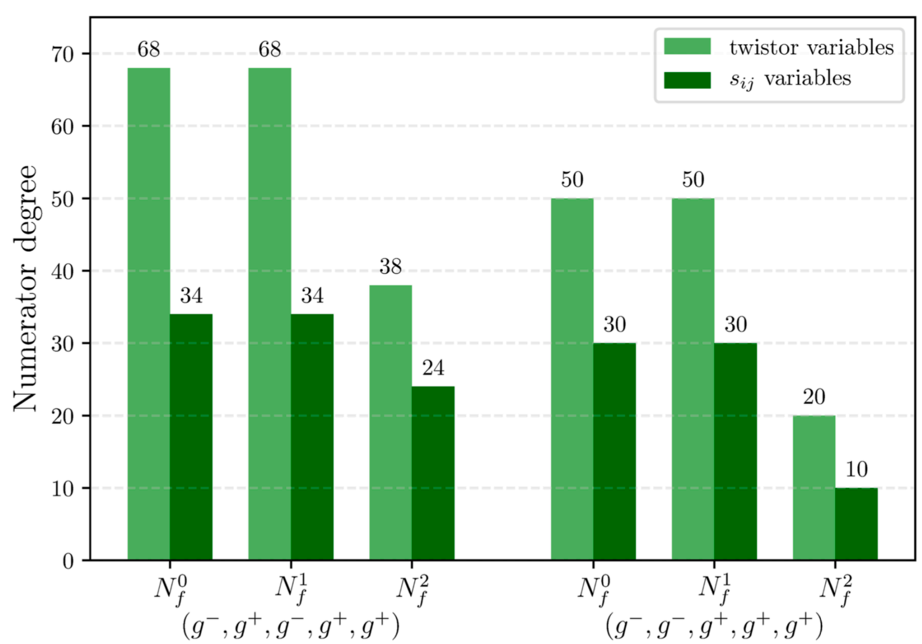
Finite fields requires rational parametrisation of 5-point massless phase-space

Solution 1: twistor variables [JHEP **1305** (2013) 135, Hodges]

Solution 2: keep square root explicit, use Mandelstams

$$N = N^+(\vec{s}) + \mathrm{tr}_5\, N^-(\vec{s})$$

Compute at parity conjugate points to extract even and odd coefficients!



DENOMINATOR GUESSING

$$R^{(2)} = \sum_i c_i(\vec{x}) h_i(\vec{x})$$

Pentagon functions: linear combinations of iterated integrals with logarithmic singularities

Position of logarithmic singularities defines an alphabet A

Conjecture: denominators are products of *symbol letters*

$$c_i^{\pm}(\vec{s}) = \frac{n_i^{\pm}(\vec{s})}{\prod_A W^{\vec{q}_i}(\vec{s})}$$

1. Reconstruct amplitude on a univariate slice to determine powers of each letter
2. Verify correctness on different univariate slice

E.g.: $x(t) = c_0, \quad s_{23}(t) = c_1 + d_1 t, \quad s_{45}(t) = c_2 + d_2 t, \quad s_{51}(t) = c_3\big(x(t) - s_{23}(t) + s_{45}(t)\big)$

Completely determine denominators of the rational functions

$\Longrightarrow$only need to reconstruct multivariate polynomial!

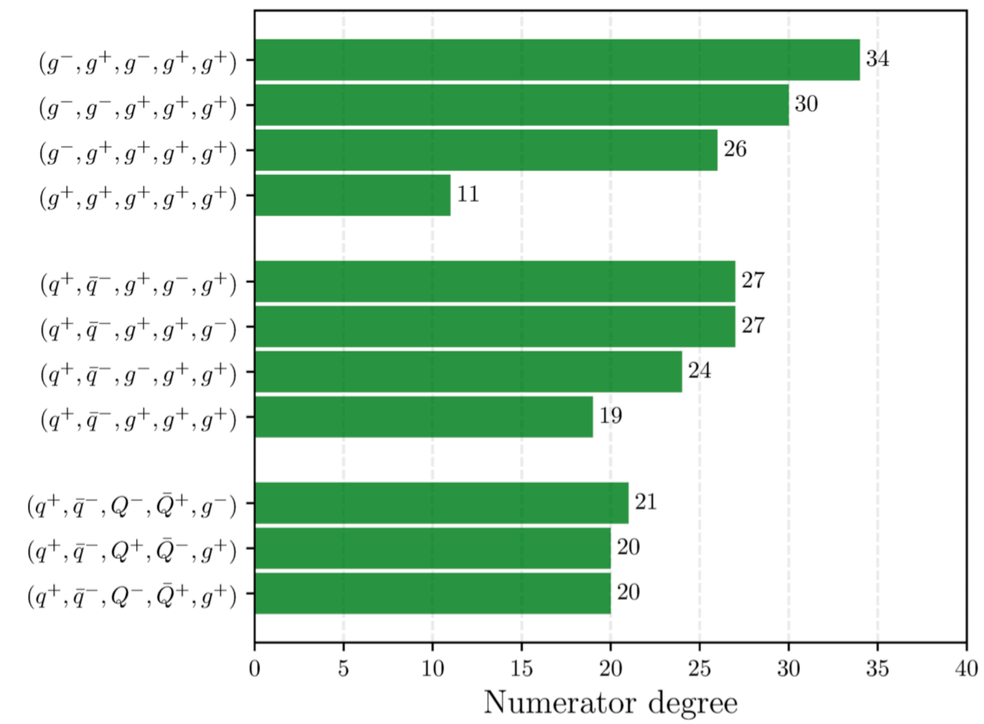
[Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov 18, 19]

NUMERATOR RECONSTRUCTION

$$c_i^{\pm}(\vec{s}) = \frac{n_i^{\pm}(\vec{s})}{\prod_A W^{\vec{q}_i}(\vec{s})}$$

Numerators are polynomials in the Mandelstam variables

$$n_i^{\pm}(\vec{s}) = \sum_{|\vec{\alpha}|\leq N_{\max}} a_{\vec{\alpha}} \vec{s}^{\vec{\alpha}}$$



Use univariate reconstruction to block-diagonalise system and recurse [Peraro 16]

$$n_i^{\pm}(\vec{s}) = \sum_{j=N_{\min}}^{N_{\max}} n_{i,j}^{\pm}(s_{23}, s_{45}, s_{15}) s_{34}^j$$

Use Newton interpolation formula for univariate reconstruction

Cluster capable: can predetermine evaluation points

RESULTS FOR FIVE-PARTON QCD AMPLITUDES

[Abreu, Dormans, Febres Cordero, Ita, Page, Sotnikov 19]

Caravel

Analytic reconstruction of most complex amplitude (5g, -+-++):

- 95000 phase-space points, ~4.5min / point: easy calculation on a cluster

Multi-variate partial fractioning to simplify final results:

- Only requires the evaluation in a single finite field

Analytic results for basis of 1-loop amplitudes and 2-loop remainders (HV-scheme):

- ggggg: (+,+,+,+,+), (-,+,+,+,+), (-,-,+,+,+), (-,+,-,+,+)
- qqggg: (+,-,+,+,+), (+,-,+,+,-), (+,-,+,-,+), (+,-,-,+,+)
- qqQQg: (+,-,-,+,+), (+,-,-,+,-), (+,-,+,-,+)

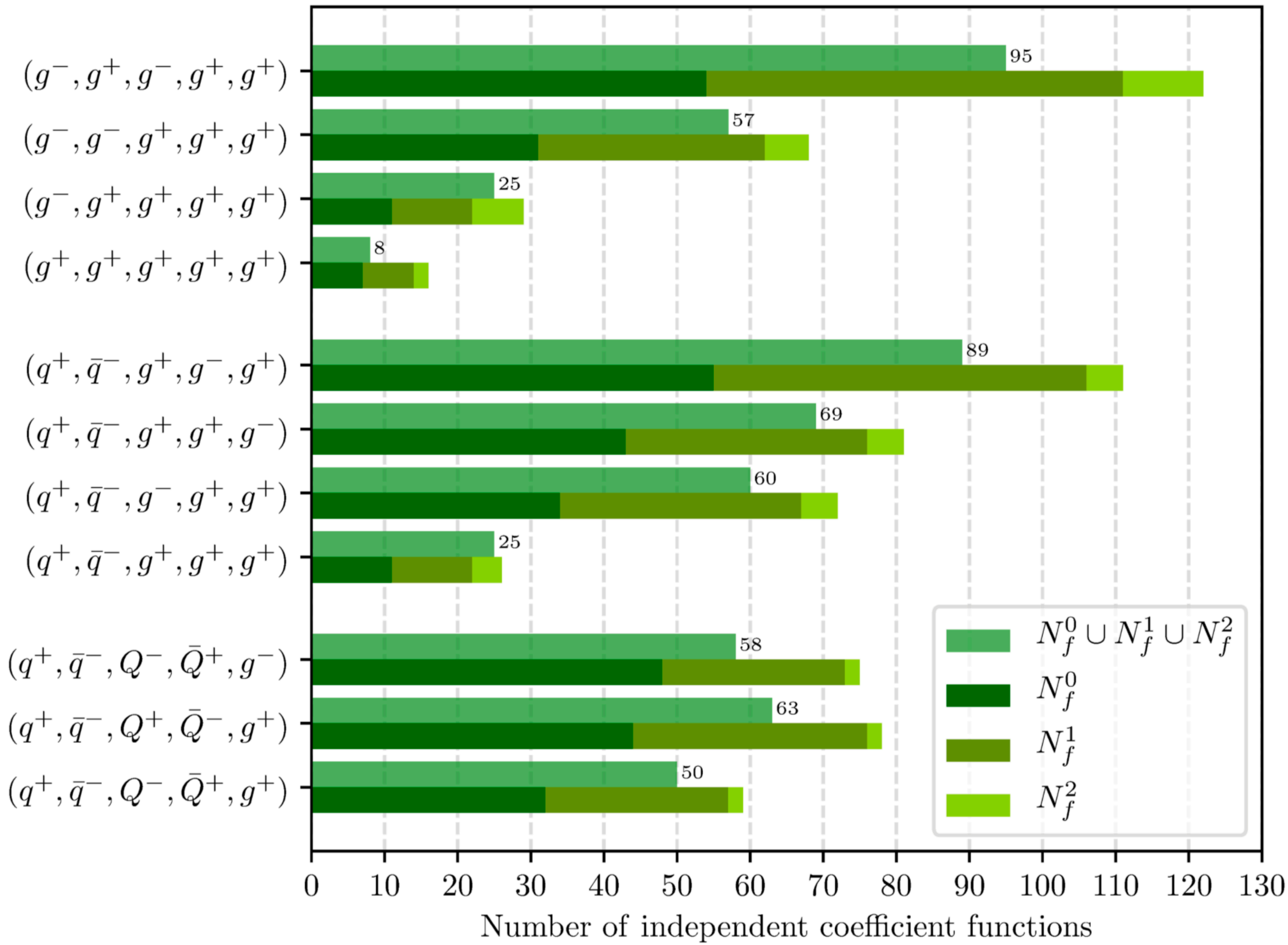
Reproduce all numerical targets

Total size for all amplitudes ~10MB (uncompressed)

⇒ Fast (~1s in Mathematica) and stable evaluation of coefficients, suitable for phenomenology studies

SIMPLICITY OF FIVE-PARTON QCD AMPLITUDES

There are ~ 400 pentagon functions, but final answer is much simpler



CONCLUSION AND OUTLOOK

New approach to amplitude evaluation: two-loop numerical unitarity & analytic reconstruction

- Bypass large intermediate analytic expressions, target simpler final result
- Preserve the advantages of an analytic result

Planar 5pt massless QCD amplitudes, a bottleneck for many years, become a simple calculation

- Opens the door to phenomenology of 3-jet production at the LHC

$pp \to V + 2j$	$\mathrm{NLO_{QCD}}{+}\mathrm{NLO_{EW}}$ $\mathrm{NLO_{EW}}$	$\mathrm{N^2LO_{QCD}}$
$pp \to V + b\bar{b}$	$\mathrm{NLO_{QCD}}$	$\mathrm{N^2LO_{QCD}} + \mathrm{NLO_{EW}}$

$pp \to H + 2j$	$\mathrm{NLO_{HEFT}}\otimes\mathrm{LO_{QCD}}$ $\mathrm{N^3LO_{QCD}^{(VBF^*)}}$ (incl.) $\mathrm{N^2LO_{QCD}^{(VBF^*)}}$ $\mathrm{NLO_{EW}^{(VBF)}}$	$\mathrm{N^2LO_{HEFT}}\otimes\mathrm{NLO_{QCD}}{+}\mathrm{NLO_{EW}}$ $\mathrm{N^2LO_{QCD}}{+}\mathrm{NLO_{EW}^{(VBF)}}$

Les Houches whishlist 2017 [1803.07977]

Efficiency of the approach opens the door to other previously impossible calculations:

- Non-planar massless QCD: related work with similar approach in simpler theories

[Abreu, Dixon, Herrmann, Page, Zeng 18]

With different approach [Chicherin, Gehrmann, Henn, Wasser, Zhang, Zoia 18]

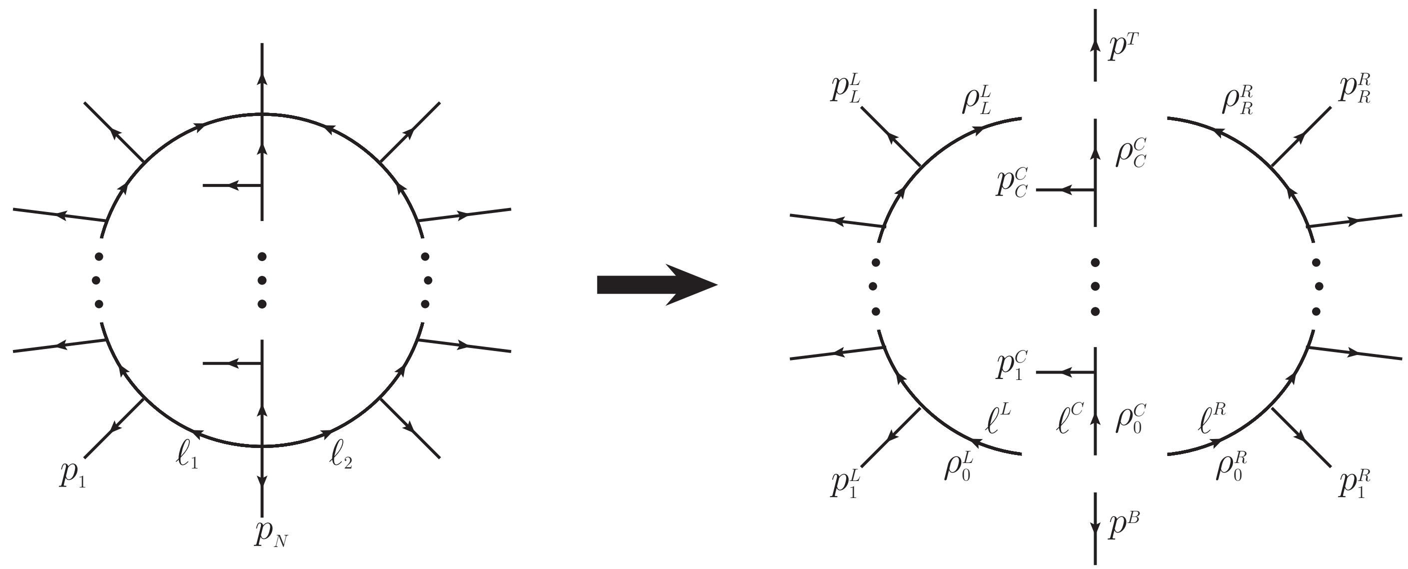
[Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zoia, 19]

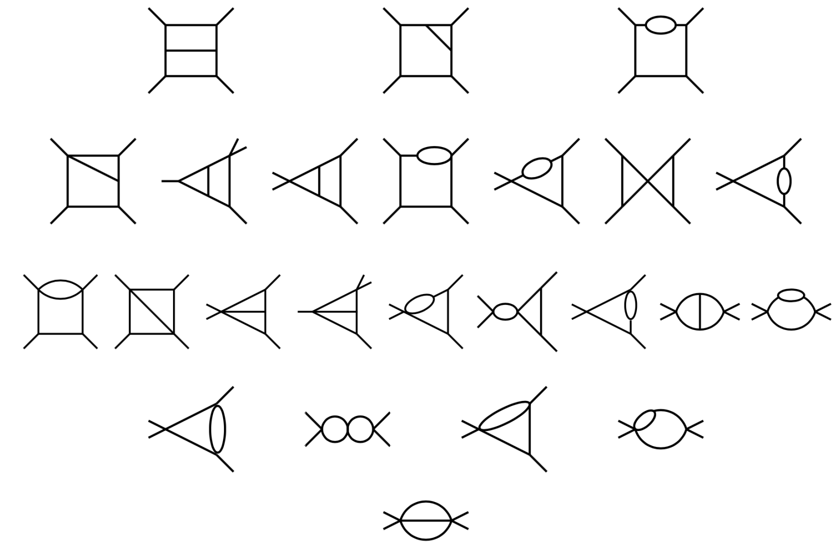
- Massive 2-loop 5pt amplitudes: H+2 jets, Z/W+2 jets production at the LHC

THANK YOU!

UNITARITY COMPATIBLE VARIABLES

[H. Ita 15; S. Abreu, F. Febres Cordero, H. Ita, M. Jaquier, B. Page, M. Zeng 17]





- Rung by rung approach to determine suitable variables:

$$\ell_L = \sum_{j\in D_\Gamma^L} v_L^j r_L^j + \sum_{j\in \bar{D}_\Gamma^L} v_L^j \lambda_L^j + \sum_{j\in \bar{D}_\Gamma^4} n_\Gamma^j \alpha_L^j + \sum_{j\in D^\epsilon} n_\Gamma^j \mu_L^j$$

$$p_i \cdot v_L^j = \delta_{ij}$$
$$p_i \cdot n_\Gamma^j = 0$$
$$n_\Gamma^i \cdot n_\Gamma^j = \delta_{ij}$$

Propagators | Irreducible scalar products (ISPs) | Transverse variables | Transverse variables beyond 4d

- Remove redundancy:
  - in each rung: $\rho_{L,0} + m_{L,0}^2 = \ell_L^2 = c_L(r_L, \lambda_L) + \vec{\mu}_L \cdot \vec{\mu}_L$
  - overall: use momentum conservation at bottom vertex $\alpha_C^j = -\alpha_1^j - \alpha_2^j$

INTEGRAND

$$\mathcal{A}(\ell_l) = \sum_{\Gamma} \sum_{i=1}^{\dim(\Gamma)} c_{\Gamma,i} \frac{m_{\Gamma,i}(\ell_l)}{\prod_{j \in P_\Gamma} \rho_j}$$

- Independent variables

$\lambda_1^j \quad \lambda_2^j$ $\qquad$ $\alpha_1^j \quad \alpha_2^j$ $\qquad$ $\cancel{\rho_j}$

On-shell: associated with another propagator structure

- Parametrisation of the **on-shell integrand**

Polynomial in $\ell_i \cdot \ell_j \quad \ell_i \cdot p_j \quad \ell_i \cdot n_j$ $\longrightarrow$ Polynomial in $\alpha_i^j \quad \lambda_i^j$

$m_{\Gamma,k}(\ell_l) \sim$ monomials in the independent variables

- Dimension easy to determine: number of independent monomials allowed by power counting.

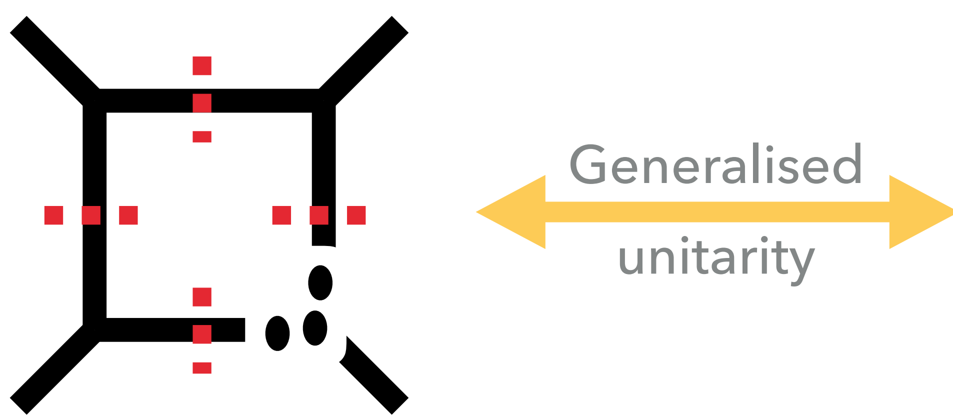
DETERMINING THE COEFFICIENTS — UNITARITY

- Integrand as a product of trees

$$\lim_{\ell \to \ell^\Gamma} \mathcal{A}(l, p_i) = \frac{\prod \mathcal{A}^{\text{tree}}(\ell^\Gamma, p_i)}{\prod_{k\in\Gamma} \rho^k(\ell^\Gamma)} = \frac{R(\Gamma, \ell^\Gamma)}{\prod_{k\in\Gamma} \rho^k(\ell^\Gamma)}$$

ℓ^Γ: loop momenta evaluated at $\rho^k = 0\,, \quad \forall\, k \in \Gamma$ ⟵ On-shell phase-space associated with Γ

- Product of trees implemented with a Berends-Giele recursion



$$R(\Gamma, \ell^\Gamma)$$

Bern, Dixon, Kosower, Dunbar 94; Bern, Dixon, Dunbar, Perelstein, Rozowsky 98; Bern, Dixon, Kosower 00; Britto, Cachazo, Feng 04

NUMERATORS FROM PRODUCT OF TREES

- Top down approach, solve numerically for the coefficients

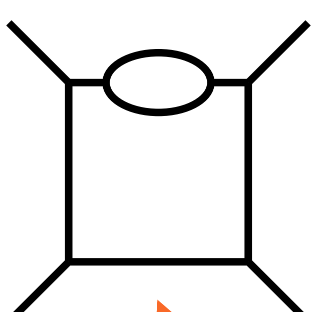
Already known from previous step

$$N(\Gamma',\ell^{\Gamma'}) = R(\Gamma',\ell^{\Gamma'}) - \sum_{\Gamma>\Gamma'} \frac{N(\Gamma,\ell^{\Gamma'})}{\prod_{k\in\Gamma}\rho^k(\ell^{\Gamma})}$$

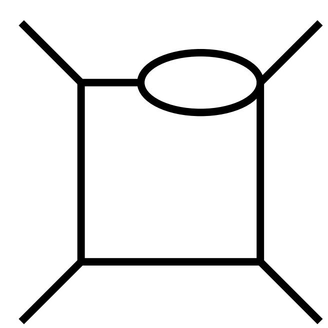
Product of trees

Propagator structures with more propagators

- Assumes each propagators structure corresponds to a different on-shell phase-space, problematic at two-loops and beyond:



Leading pole: standard procedure



Subleading pole: need to be more careful

Same on-shell phase space