Higher order corrections to spin correlations in top quark pair production at the LHC

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Outline

Introduction

Technical aspects

Results

Conclusions
The top quark

- Heaviest known elementary particle
- Discovered in 1995 at the Tevatron
- Abundantly produced at the LHC
- Very active field of research (theory & experiment)
- Interesting in its own right, but also as a probe of BSM physics
- Both aspects require precise understanding of its properties (mass, width, spin, couplings, ...)

[ATLлас+CMS Preliminary m_{top} summary, \( \sqrt{s} = 7-13 \text{ TeV} \), November 2018

\[ m_{top} \text{ [GeV]} \]

\[ \begin{array}{cccc}
\text{7 TeV} & 165 & 170 & 175 \\
\text{ATLAS, l+jets} & 172.2 \pm 1.7 & 173.3 \pm 1.7 & 174.4 \pm 1.7 \\
\text{CMS, dilepton} & 172.4 \pm 0.9 & 173.4 \pm 1.0 & 174.5 \pm 1.1 \\
\text{CMS, all jets} & 170.8 \pm 1.6 & 171.8 \pm 1.6 & 172.8 \pm 1.6 \\
\text{CMS, single top} & 168.8 \pm 1.6 & 170.8 \pm 1.6 & 172.8 \pm 1.6 \\
\text{CMS, l+jets} & 173.3 \pm 1.7 & 174.4 \pm 1.7 & 175.5 \pm 1.7 \\
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\end{array} \]

\[ \text{13 TeV} & 180 & 185 \\
\text{ATLAS, l+jets} & 173.3 \pm 1.7 & 174.4 \pm 1.7 & 175.5 \pm 1.7 \\
\text{CMS, dilepton} & 172.4 \pm 0.9 & 173.4 \pm 1.0 & 174.5 \pm 1.1 \\
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\[ \text{LHCtopWG '18] CC-BY 4.0} \]
Top pair production

- $t\bar{t}$ production at hadron colliders: mostly QCD
  - Tevatron: $q\bar{q}$ dominates
  - LHC: $gg$ dominates

- Known to NNLO QCD, NLO EW and with NNLL resummation for stable top quarks

- SM predictions for differential distribution agree well with data
Top quarks decay almost exclusively via $t \rightarrow Wb$, i.e. $pp \rightarrow t\bar{t} \rightarrow W^+W^-b\bar{b}$

- Classify reactions according to $W$ decays
  - all jets: $t\bar{t} \rightarrow b\bar{b} + 4j$
  - lepton+jets: $t\bar{t} \rightarrow bb + \ell\nu + 2j$
  - dilepton: $t\bar{t} \rightarrow bb + \ell\nu + \ell'\nu'$

- This talk: dilepton channel with $e^\pm\mu^\mp$
  - Cleanest signature, but also smallest rate
  - Incomplete kinematic information due to invisible neutrinos
  - $e^+e^-$ and $\mu^+\mu^-$ have larger Drell-Yan backgrounds
Top quark spin

Spin properties of top quarks

- Top quark is a fermion $\Rightarrow$ Couplings are spin-dependent
- Direct measurement of top quark spin not possible $\Rightarrow$ measurement through decay products
- Unique situation: Top quarks decay before decorrelation

Spin-dependence of top pair production

- Individual top quarks from $pp \rightarrow t\bar{t}$ are not polarised
- But: Spin correlation between $t$ and $\bar{t}$
- Can be measured in differential distributions
Spin correlations: Strategies and observables

Dilepton channel is very well suited for measuring spin correlations:

- Very clean signal
- Lepton momenta can be measured well

Direct measurement

- Decompose cross section into basis in spin space
- Use lepton momenta to probe coefficients

\[
\frac{1}{\sigma} \frac{d\sigma}{d \cos \theta_1^i \, d \cos \theta_2^i} = \frac{1}{4} \left( 1 + B_1^i \cos \theta_1^i + B_2^i \cos \theta_2^i - C_{ij} \cos \theta_1^i \cos \theta_2^j \right)
\]

where \( \theta_{1(2)}^{i(j)} \) are lepton angles of \( t(\bar{t}) \) wrt. axis \( i(j) \) in \( t(\bar{t}) \) rest frames

- Measure differential distributions in angles and extract coefficients
- Pro: Full spin information
- Contra: Requires reconstruction of \( t \) & \( \bar{t} \) rest frames
- Has actually been pursued, e.g., by CMS [CMS-PAS-TOP-18-006]
Spin correlations: Strategies and observables

Dilepton channel is very well suited for measuring spin correlations:

- Very clean signal
- Lepton momenta can be measured well

**Indirect measurement**

- Measure differential distributions of lepton geometry in *lab frame*
  - Lepton azimuthal opening angle $\Delta \phi(\ell, \bar{\ell})$
  - Lepton rapidity difference $|\Delta \eta(\ell, \bar{\ell})|$

- Boost of top quarks enhances anti-parallel leptons
- Spin correlations counteract and enhance parallel leptons
Spin correlations: Experimental situation

- Compare measurement to predictions with and without spin correlations
- Bin-wise linear fit
  \[ x_i = f_{SM} x_{\text{spin},i} + (1 - f_{SM}) x_{\text{nospin},i} \]
- SM expectation: \( f_{SM} = 1 \)
- At ICHEP2018 ATLAS reported measurement of \( \Delta \phi(\ell, \bar{\ell}) \) distributions and
  \[ f_{SM} = 1.25 \pm 0.08 \]
  \( \Rightarrow \) 3.2\( \sigma \) deviation from SM

Largest deviation from the SM in the top sector at that time

Question: Could this be due to missing NNLO corrections?
Ingredients of a NNLO fixed-order calculation

Matrix elements

- 2-loop
- 1-loop + 1 real
- tree-level + 2 real

Collinear factorisation

UV renormalisation

PDFs & parameters

Scheme to deal with IR divergences
Narrow width approximation

\[ pp \rightarrow t\bar{t} \rightarrow b\bar{b}\ell^+\ell'^-\nu\bar{\nu}' \] at NNLO

- In general needs 2-loop 8-point amplitudes \( \Rightarrow \) too difficult
- However: \( \Gamma_t \ll m_t \Rightarrow \) Use narrow width approximation

Narrow width approximation (NWA)

\[ \frac{1}{(p^2 - m_t^2)^2 + m_t^2 \Gamma_t^2} \xrightarrow{\Gamma_t/m_t \to 0} \frac{\pi \delta(p^2 - m_t^2)}{m_t \Gamma_t} \]

- On-shell tops \( \Rightarrow \) Revert spin sum for propagator numerators
  \[ p + m_t = \sum_\lambda u_\lambda(p) \bar{u}_\lambda(p) \]
- Factorizes production and decay
- Requires polarised matrix elements for production and decay
Matrix elements

$pp \rightarrow t\bar{t}$ with polarised top quarks

[Chen, Czakon, Poncelet '17]

- Projection onto spin- and colour structures
- IBP reduction onto master integrals
  - 422 master integrals
  - Involve also elliptic structures
  - Not yet completely known analytically
- Numerical calculation of master integrals
  - Solve differential equations numerically from boundary conditions in high-energy limit
  - New: Partially canonicalised DEQ system $\rightarrow$ CANONICA [Meyer '18]
Infrared divergences

- Real and virtual corrections contain IR divergences
- IR safe observables: Divergences cancel between real and virtual contributions
- Dimensional regularisation \((d = 4 - 2\epsilon)\):
  Real and virtual contain poles in \(1/\epsilon\)
- Arise from integration over massless degrees of freedom
  \[
  \frac{1}{(p + k)^2} = \frac{1}{2p \cdot k} = \frac{1}{2E_p E_k (1 - \cos \theta)}
  \]
  - Requires performing the loop and phase space integrals first
  - Phase space integrals are usually done numerically
  - Can’t naively expand integrand in \(\epsilon\) before integration
- Devise scheme to extract and cancel divergences
  - Slicing methods
  - Subtraction methods
Sector-improved residue subtraction scheme

Problem: Many singular limits; overlapping singularities

Idea:

- Subdivide phase space into sectors (similar to FKS scheme)
- Use sector decomposition to extract the singularities
  - Parametrise phasespace such that soft and collinear singularities are mapped to integration boundary of just one variable
  - Use plus distributions to generate subtraction terms
  - Express subtraction terms through QCD factorisation formulae
- Regulated terms can be expanded in $\epsilon$ at integrand level
- Integrate coefficients of truncated Laurent series numerically
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\[ 1 = \sum_{i,j} \left[ \sum_{k} S_{ij,k} + \sum_{k,l} S_{i,k;j,l} \right] \]

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    \[
    \hat{\eta}_1 = \eta_1 \quad \hat{\eta}_2 = \eta_1 \left(1 - \frac{\eta_2}{2}\right)
    \]
- Use plus distributions to generate subtraction terms
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\[
\int_0^1 dx \frac{f(x)}{x^{1+b\epsilon}} = -\frac{f(0)}{b\epsilon} + \int_0^1 dx \frac{f(x) - f(0)}{x^{1+b\epsilon}}
\]

- Express subtraction terms through QCD factorisation formulae
- Regulated terms can be expanded in $\epsilon$ at integrand level
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\[
\langle M_{n+1}^{(0)} | M_{n+1}^{(0)} \rangle \rightarrow 0 \approx -g_s^2 \sum_{i,j} S_{ij}(q) \langle M_n^{(0)} | T_i \cdot T_j | M_n^{(0)} \rangle
\]

• Regulated terms can be expanded in \( \epsilon \) at integrand level
• Integrate coefficients of truncated Laurent series numerically
Sector-improved residue subtraction: New developments

- New phasespace construction
  - Minimises the number of resolved kinematics for subtraction terms
  - Reduces the problem of mis-binning
  - Improved stability of invariant mass distribution
- Rederivation of the four-dimensional formulation of the scheme
  - Treat resolved momenta and spins as four-dimensional
  - Non-trivial since poles are calculated numerically
  - Crucial for high-multiplicity final states
  - New phasespace construction works in lab frame
    ⇒ rederivation of four-dimensional formulation necessary
  - Allows to check numerical pole cancellation for individual phase space points
Subtraction for top pair production and decay

- Extended implementation of subtraction scheme to include decays
- NNLO \(\times\) LO and LO \(\times\) NNLO done with sector-improved residue subtraction
- NLO \(\times\) NLO used Catani-Seymour for NLO decays
Setup of the calculation

ATLAS published data for two selection cuts, which we try to reproduce: Fiducial and inclusive

Fiducial setup

- Exactly 2 opposite sign leptons with $p_T > 27\,(25)$ GeV for the harder (softer) lepton and $|\eta| < 2.5$
- At least 2 jets with $p_T > 25$ GeV and $|\eta| < 2.5$ at least one of them $b$-flavoured
- Jets defined with anti-$k_T$ algorithm with $R = 0.4$

Inclusive setup

- No selection cuts
- ATLAS extrapolates via Monte Carlo simulation
Agreement between data and theory improves at NNLO

NNLO corrections are larger in fiducial than in inclusive phase space

Inclusive: NNLO predictions still somewhat disagree with data

⇒ Issue with extrapolation due to NLO Monte Carlo?
Differential distributions for $\Delta \phi_{\ell\bar{\ell}}$ at NNLO

- Agreement between data and theory improves at NNLO
- NNLO corrections are larger in fiducial than in inclusive phasespace
- Inclusive: NNLO predictions still somewhat disagree with data
  ⇒ Issue with extrapolation due to NLO Monte Carlo?
Differential distributions for $\Delta \eta_{\ell\bar{\ell}}$ at NNLO

- NNLO corrections improve agreement with data
- Slightly larger corrections in inclusive case
- Overall good agreement with available data
Differential distributions for $\Delta\eta_{\ell\bar{\ell}}$ at NNLO

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Anatomy of the higher-order $\Delta \phi_{\ell \bar{\ell}}$ corrections

Size of the NNLO correction

- **Fiducial:** For $\mu_{F,R} = H_T/4$ NNLO corrections are at most $\sim 5%$
  - Consistent with NLO scale uncertainty
  - NLO/LO K-factor up to $\sim 15%$
- **Inclusive:** Smaller K-factor and less scale variation
- Consistent with good perturbative convergence
- NNLO corrections are important
  - Reduce scale uncertainty by more than factor 2
  - Modify shape of distribution in same direction as spin correlations
  - Improve agreement with data
Anatomy of the higher-order $\Delta \phi_{\ell\bar{\ell}}$ corrections

Scale choices

- We tried three scale choices: $\mu_{F,R} = H_T/4$, $m_t/2$ and $m_t$
  with $H_T = \sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2}$
- $H_T/4$ and $m_t/2$ show similar behaviour
- $m_t$ shows slower perturbative convergence
  $\rightarrow$ non-negligible corrections beyond NNLO?
Anatomy of the higher-order $\Delta \phi_{\ell\bar{\ell}}$ corrections

Quantifying spin correlations

- Compare correlated and uncorrelated results at each order
- Correlations are an important effect (up to $\sim 25\%$)
- Change little at higher orders
Anatomy of the higher-order $\Delta \phi_{\ell\bar{\ell}}$ corrections

Quantifying spin correlations

- Disentangle kinematics and spin effects
- Higher order corrections are mostly a kinematic effect
Conclusions

Summary

• Calculated NNLO QCD corrections to top pair production with decays at the LHC in the NWA
• Allows for consistent treatment of production and decay
• Fiducial cross sections become available at NNLO
• Reduced scale uncertainty
• NNLO corrections to $\Delta \phi(\ell, \bar{\ell})$ and $\Delta \eta(\ell, \bar{\ell})$ distributions improve agreement with data from ATLAS collaboration

Outlook

• $m_t$ dependence of differential distributions
  $\Rightarrow$ extraction of $m_t$ from fiducial cross sections
• Predictions for lepton charge asymmetry