Regge Limits of Scattering Amplitudes

...and their use in predictions for processes at the LHC

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Overview of Talk

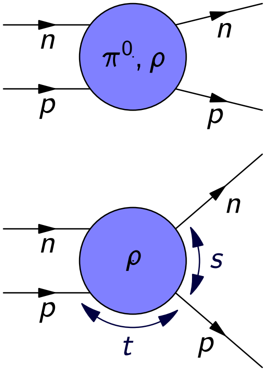
1. Simple ideas from Regge Theory
2. Scaling of QCD Amplitudes in the Multi-Regge-Kinematic limit
3. Their use in Approximating All-Order Corrections
4. Comparison to Data. Perturbative Stability for Large s_{jj} (VBF-cuts)

Regge theory

Regge theory describes scattering from a **central potential** in terms of the projections on Legendre polynomial and states of **definite orbital angular momentum**

The analysis of **scattering amplitudes** in terms of Regge Theory:

$$\mathcal{M} = \sum_i \Gamma_i(t)\ (s)^{j_i}$$



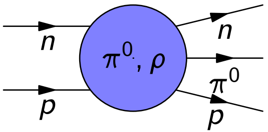
At **large energies** s, the contribution from particle of **highest spin *j* dominates**

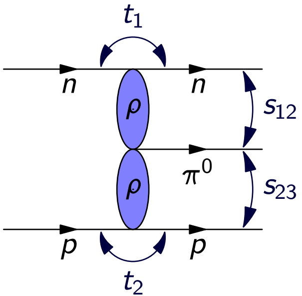
$$\mathcal{M} \to \Gamma(t)\ (s)^{j}$$

Regge limit: $s \gg -t$ or $s \gg p_t^2$

Multi-Regge theory

Large s of course leads to the possibility of **multi-particle production**





Multi-Regge limit:

$s_{12}, s_{23} \gg p_{t_i}^2, |t_i|, \quad |t_i| \sim |t_j|, \quad |p_{t_i}| \sim |p_{t_j}|$

$$\mathcal{M} = s_{12}^j s_{23}^j \, \Gamma\left(t_1, t_2, s/(s_{12}s_{23})\right)$$

This framework obtains **good descriptions of hadronic processes.**
Based primarily on **analyticity of scattering matrix.**

Multi-Regge Limits and QCD

What does **Regge analysis** predict for amplitudes in **perturbative QCD**?

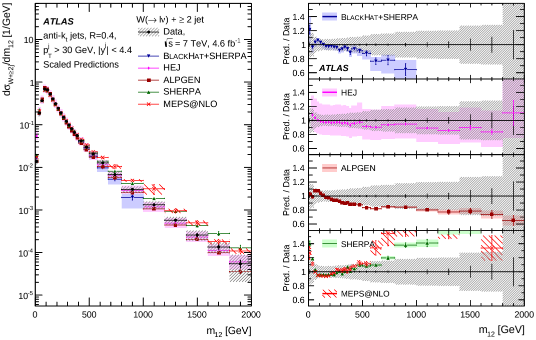
- How does one define the flavour (and **spin**) **of the exchanged particle** for a gauge theory, where particles of many types can contribute, and the **contribution from each exchange is gauge-dependent**?
- How does one even **define the** t**-channel with identical particles**?

High Energy Jets:

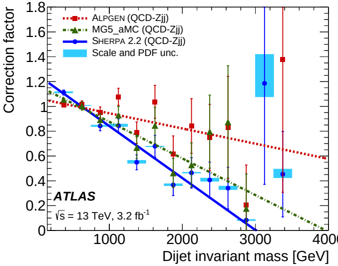
- **Factorisation of matrix elements** using **currents**
- systematic **power expansion of QCD amplitudes**
- all-order **leading and sub-leading logarithmic corrections**
- matching, **results**. . .

Why We Should Develop a Better Understanding

- At LHC energies, $(n+1)$-jet rates are not necessarily small compared to n-jet rates
- For some analyses, the "**difficult regions**" of phase space where NLO-calculations fail are exactly the regions of interest **VBF Higgs** production, **Vector Boson Scattering**,...
- Powers of α_s accompanied by $\log(\hat{s}/p_\perp^2) \propto \Delta y$



Huge variations in the perturbative description of regions of large $\hat{s}$ arXiv:1703.04362, 1709.10264



HEJ (High Energy Jets)

Goal (inspired by the great Fadin & Lipatov)

Sufficiently **simple** model for hard radiative corrections that the all-order sum can be evaluated explicitly (completely exclusive)

but. . .

Sufficiently **accurate** that the description is relevant

Comparison of 3-jet hard scattering matrix elements

Universal behaviour of the hard scattering matrix element in the High energy (Multi-Regge Kinematic) limit (Fadin, Lipatov, Kuraeev):

$$\forall i \in \{2,\ldots,n-1\} : y_{i-1} \gg y_i \gg y_{i+1}$$
$$\forall i,j : |p_{i\perp}| \approx |p_{j\perp}|$$

$$\left|\overline{\mathcal{M}}_{gg\to g\cdots g}\right|^2 \longrightarrow \frac{4\ \hat{s}^2}{N_C^2-1}\ \frac{g^2\ C_A}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\ g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\ C_A}{|p_{n\perp}|^2}.$$

$$\left|\overline{\mathcal{M}}_{qg\to qg\cdots g}\right|^2 \longrightarrow \frac{4\ \hat{s}^2}{N_C^2-1}\ \frac{g^2\ C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\ g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\ C_A}{|p_{n\perp}|^2},$$

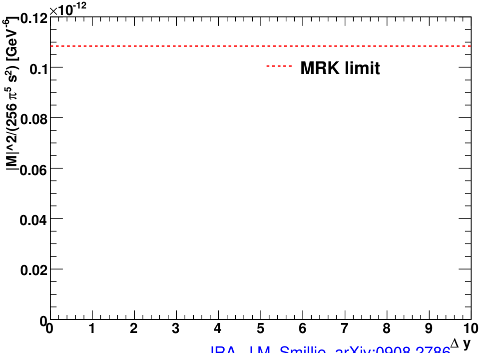
$$\left|\overline{\mathcal{M}}_{qQ\to qg\cdots Q}\right|^2 \longrightarrow \frac{4\ \hat{s}^2}{N_C^2-1}\ \frac{g^2\ C_F}{|p_{1\perp}|^2}\left(\prod_{i=2}^{n-1}\frac{4\ g^2 C_A}{|p_{i\perp}|^2}\right)\frac{g^2\ C_F}{|p_{n\perp}|^2},$$

Allow for analytic resummation (BFKL equation) capturing $(\alpha_s \Delta y)^n$
However, how well does this actually approximate the amplitude?

Comparison of 3-jet hard scattering matrix elements

Study just a slice in phase space, and compare full tree-level with α_s^3-approximation from resummation:

40GeV jets in Mercedes star (transverse) configuration. Rapidities at $-\Delta y, 0, \Delta y$.



JRA, J.M. Smillie, arXiv:0908.2786

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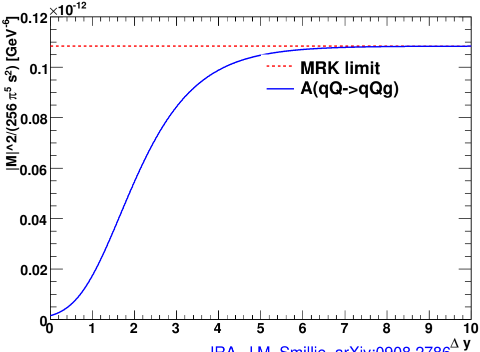

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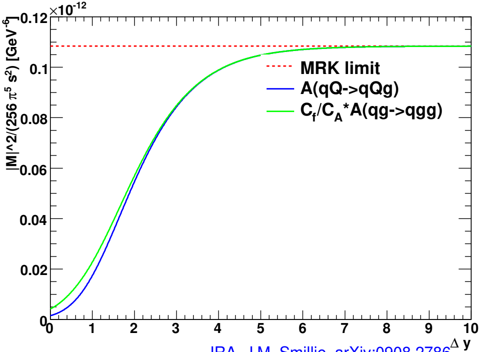

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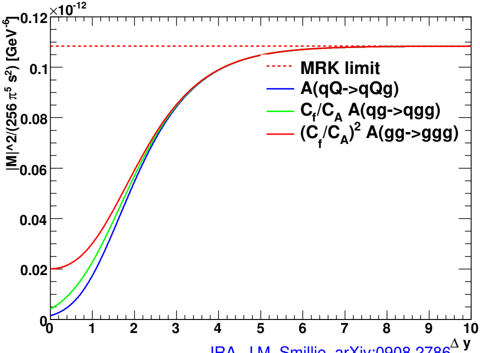

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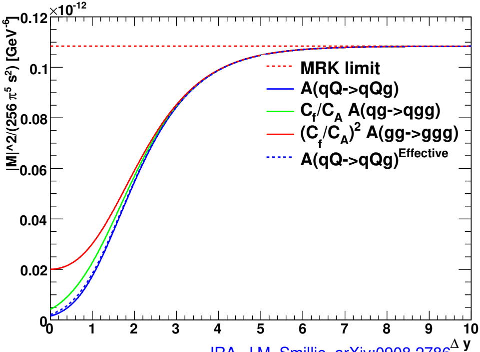

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High Energy Jets (HEJ):

1) Inspiration from Fadin&Lipatov: dominance by *t*-channel colour octet exchange
2) No kinematic approximations in invariants
3) Accurate definition of currents (coupling through *t*-channel exchange)
4) Full gauge invariance.

Factorisation of QCD Matrix Elements

It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:
Collinear limit $\rightarrow$ enters many resummation formalisms, parton showers....

Like all good limits, the collinear approximation is applied **outside its strict region of validity.**

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

Factorisation only **becomes exact** in a region **outside** the reach of any collider...

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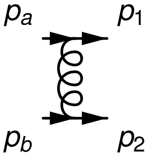
Scattering of qQ-Helicity States

Start by describing quark scattering. Simple matrix element for $q(a)Q(b) \to q(1)Q(2)$: $\quad j_\mu^{1a} = \bar{\psi}_1 \gamma_\mu \psi_a, \quad j_\nu^{2b} = \bar{\psi}_2 \gamma_\nu \psi_b$

$$M_{q^- Q^- \to q^- Q^-} = j_\mu^{1a} \quad \frac{g^{\mu\nu}}{t} \quad j_\nu^{2b}$$



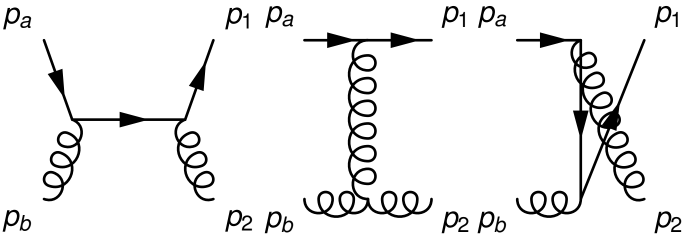
***t*-channel factorised**: Contraction of (local) currents across t-channel pole. $\left\| S_{qQ\to qQ} \right\|^2 = \hat{s}^2 + \hat{u}^2$

$$\left| \overline{\mathcal{M}}^t_{qQ\to qQ} \right|^2 = \frac{1}{4\,(N_C^2 - 1)} \left\| S_{qQ\to qQ} \right\|^2 \cdot \left(g^2\ C_F\ \frac{1}{t_1} \right) \cdot \left(g^2\ C_F\ \frac{1}{t_2} \right).$$

$|\mathcal{M}| \propto \hat{s}^1$ for a gluon exchange ! J.M.Smillie and JRA: arXiv:0908.2786

Quark-Gluon Scattering

"What happens in $2 \to 2$-processes with gluons? Surely the t-channel factorisation is spoiled!"



Direct calculation ($q^- g^- \to q^- g^-$):

$$\mathcal{M} = \frac{g^2}{\hat{t}} \times \frac{p^*_{2\perp}}{|p_{2\perp}|} \left(t^2_{ae} t^b_{e1} \sqrt{\frac{p_b^-}{p_2^-}} - t^b_{ae} t^2_{e1} \sqrt{\frac{p_2^-}{p_b^-}} \right) j^\mu_{b2}\, j^{1a}_\mu$$

J.M.Smillie and JRA

Complete t-channel factorisation! Same scaling as qQ scattering

Quark-Gluon Scattering

The t-channel current generated by a gluon in qg scattering is that generated by a quark, but with a colour factor

$$\frac{1}{2}\left(C_A - \frac{1}{C_A}\right)\left(\frac{p_b^-}{p_2^-} + \frac{p_2^-}{p_b^-}\right) + \frac{1}{C_A}$$

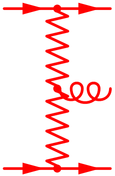
instead of C_F. Tends to C_A in the MRK limit.

Similar results for e.g. $g^+g^- \to g^+g^-$ (well-defined t-channel):
Exact, complete t-channel **factorisation.**

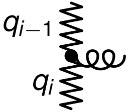
By using the formalism of **current-current scattering**, we get a better description of the t-channel pole than by using just the MRK kinematic limit of BFKL.

Building Blocks for an Amplitude

Identification of the **dominant contributions** to the **perturbative series** in the limit of well-separated particles

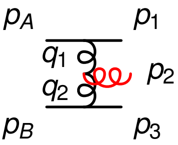


$$\frac{1}{q^2}\exp\left(\hat{\alpha}(q)\Delta y\right)$$

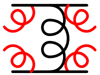


$$V^\mu(q_{i-1}, q_i)$$

$$j^\nu = \overline{\psi}\gamma^\nu\psi$$



$$\begin{aligned} V^\rho(q_1, q_2) = &-(q_1+q_2)^\rho \\ &+\frac{p_A^\rho}{2}\left(\frac{q_1^2}{p_2\cdot p_A}+\frac{p_2\cdot p_B}{p_A\cdot p_B}+\frac{p_2\cdot p_n}{p_A\cdot p_n}\right)+p_A\leftrightarrow p_1 \\ &-\frac{p_B^\rho}{2}\left(\frac{q_2^2}{p_2\cdot p_B}+\frac{p_2\cdot p_A}{p_B\cdot p_A}+\frac{p_2\cdot p_1}{p_A\cdot p_1}\right)-p_B\leftrightarrow p_3. \end{aligned}$$



Performing the Explicit Resummation

Analytic subtraction of soft divergence from real radiation:

$$\left|\mathcal{M}_t^{p_a p_b \to p_0 p_1 p_2 p_3}\right|^2 \overset{\mathbf{p}_1^2 \to 0}{\longrightarrow} \left(\frac{4 g_s^2 C_A}{\mathbf{p}_1^2}\right) \left|\mathcal{M}_t^{p_a p_b \to p_0 p_2 p_3}\right|^2$$

Integrate over the soft part $\mathbf{p}_1^2 < \lambda^2$ of phase space in $D = 4 + 2\varepsilon$ dimensions

$$\int_0^\lambda \frac{\mathrm{d}^{2+2\varepsilon}\mathbf{p}\ \mathrm{d}y_1}{(2\pi)^{2+2\varepsilon}\ 4\pi} \left(\frac{4 g_s^2 C_A}{\mathbf{p}^2}\right) \mu^{-2\varepsilon}$$
$$= \frac{4 g_s^2 C_A}{(2\pi)^{2+2\varepsilon} 4\pi}\ \Delta y_{02}\ \frac{\pi^{1+\varepsilon}}{\Gamma(1+\varepsilon)} \frac{1}{\varepsilon} (\lambda^2/\mu^2)^\varepsilon$$

Pole in ε cancels with that from the **virtual corrections**

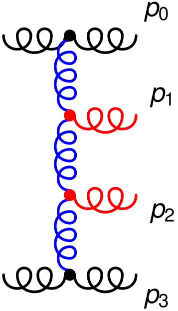
$$\frac{1}{t_1} \to \frac{1}{t_1} \exp\left(\hat{\alpha}(t) \Delta y_{02}\right) \qquad \hat{\alpha}(t) = -\frac{g_s^2 C_A \Gamma(1-\varepsilon)}{(4\pi)^{2+\varepsilon}} \frac{2}{\varepsilon} \left(\mathbf{q}^2/\mu^2\right)^\varepsilon.$$

Expression for the Regularised Amplitude

$$\overline{\left|\mathcal{M}_{\mathrm{HEJ}}^{\mathrm{reg}}(\{p_i\})\right|}^2 = \frac{1}{4\,(N_C^2-1)}\ \left\|S_{f_1 f_2\to f_1 f_2}\right\|^2\cdot\ \left(g^2\ K_{f_1}\ \frac{1}{t_1}\right)\cdot\ \left(g^2\ K_{f_2}\ \frac{1}{t_{n-1}}\right)$$

$$\cdot\prod_{i=1}^{n-2}\left(g^2 C_A\ \left(\frac{-1}{t_i t_{i+1}} V^\mu(q_i,q_{i+1})V_\mu(q_i,q_{i+1}) - \frac{4}{\mathbf{p}_i^2}\ \theta\left(\mathbf{p}_i^2<\lambda^2\right)\right)\right)$$

$$\cdot\prod_{j=1}^{n-1}\exp\left[\omega^0(q_j,\lambda)(y_{j-1}-y_j)\right],\qquad \omega^0(q_j,\lambda) = -\frac{\alpha_s N_C}{\pi}\log\frac{\mathbf{q}_j^2}{\lambda^2}.$$



All-Order Summed (and Matched) Cross Section

The cross section is calculated as the sum over the phase space integrals of the explicit *n*-body phase space

$$\sigma_{2j}^{\text{sum,match}} = \sum_{n=2}^{\infty} \sum_{f_1,f_2} \prod_{i=1}^{n} \left(\int \frac{\mathrm{d}^2\mathbf{p}_{i\perp}}{(2\pi)^3} \int \frac{\mathrm{d}y_i}{2} \right) \frac{\overline{\left|\mathcal{M}_{\text{HEJ}}^{f_1 f_2 \to f_1 g \cdots g f_2}(\{p_i\})\right|}^2}{\hat{s}^2}$$

$$\times \quad \mathcal{O}_{2j}(\{p_i\}) \times \sum_m \mathcal{O}_{mj}^e(\{p_i\})\ w_{m-\text{jet}}$$

$$\times \quad x_a f_{A,f_1}(x_a, Q_a)\ x_2 f_{B,f_2}(x_b, Q_b)\ (2\pi)^4\ \delta^2\left(\sum_{i=1}^{n} \mathbf{p}_{i\perp}\right).$$

Matching to fixed order (tree-level so far) is obtained by clustering the *n*-parton phase space point into *m*-jet momenta and multiply by the ratio of full to approximate matrix element:

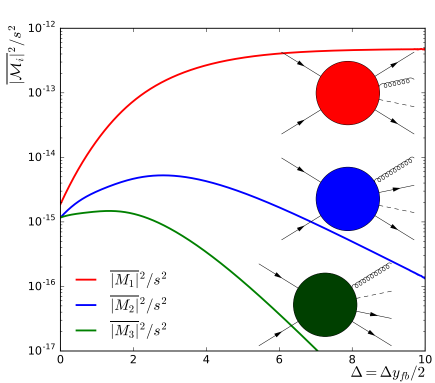
$$w_{m-\text{jet}} \equiv \frac{\overline{\left|\mathcal{M}^{f_1 f_2 \to f_1 g \cdots g f_2}\left(\{p_{\mathcal{J}_l}(\{p_i\})\}\right)\right|}^2}{\overline{\left|\mathcal{M}^{t, f_1 f_2 \to f_1 g \cdots g f_2}\left(\{p_{\mathcal{J}_l}(\{p_i\})\}\right)\right|}^2}.$$

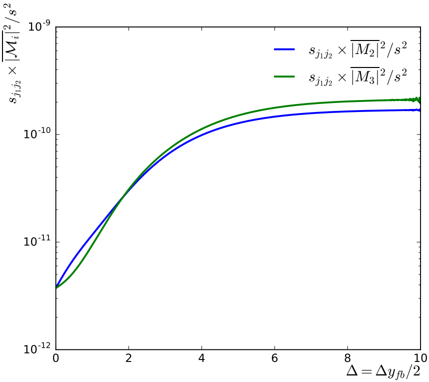
Summary: All-Orders, Regularisation, etc.

- Have prescription for $2 \to n$ matrix element, including virtual corrections: Lipatov Ansatz $1/t \to 1/t \exp(-\omega(t)\Delta y_{ij})$
- Organisation of cancellation of IR (soft) divergences is easy
- Can **calculate** the **sum over the *n*-particle phase space** explicitly ($n \sim 30$) to get the **all-order corrections** (just as if one had provided all the $N^{30}LO$ matrix elements and a regularisation procedure)
- **Merge** *n*-jet tree-level MEs (by merging *m*-parton momenta to *n* hard jet-momenta) where these can be evaluated in reasonable time
- HEJ merged with **parton shower** (Pythia) JRA, Brooks, Lönnblad, arXiv:1712.00178

Factorisation For H+$\geq$2jets Too

The factorisation extends to dijet processes involving Higgs bosons, W, Z and photon production too.





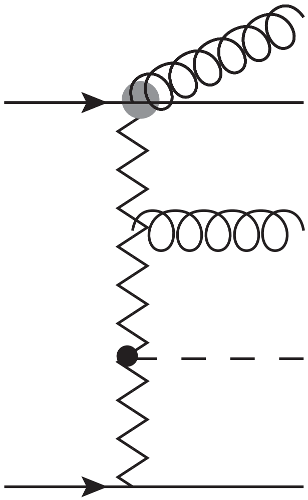
The **scaling** for different kinematic evaluations of the same amplitude is exactly as predicted by Regge theory applied to the **planar graph** connecting the rapidity-ordered configuration. JRA, A. Maier et al., arXiv:1706.01002

Subleading Corrections and Quark Mass Dependence

The first **subleading correction** can be included by **relaxing** the constraint between two emissions: **Quasi** Multi Regge Kinematics

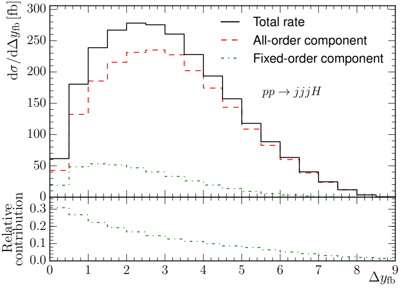
Effective currents can be extracted from the full amplitude, now dependent on more momenta. These currents obey **crossing** ($q \to qgg^*$ related to $g \to q\bar{q}g^*$)

Simplicity of the amplitudes allow for calculation of **full quark mass dependence** of Higgs coupling (requires just triangles and boxes).

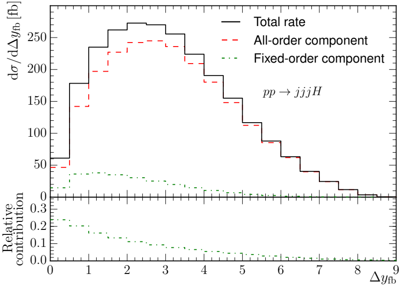


JRA, A. Maier et al., arXiv:1706.01002,1812.08072

Impact of Subleading Corrections



Leading logs only + matching

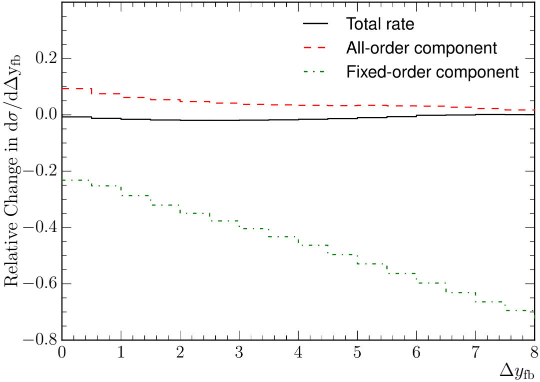


LL, one unordered emission + matching

The **all-order component** of the cross section **increases**, the impact from **matching is decreased**, but the **total result** changes only **little.**

JRA, A. Maier et al., arXiv:1706.01002

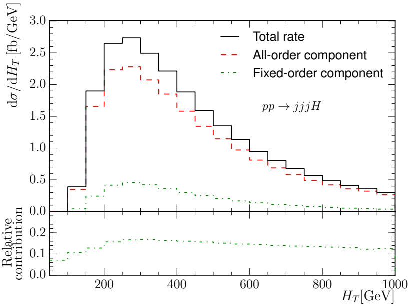
Subleading Corrections as Expected



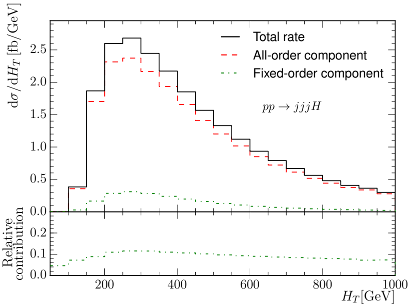
The fixed-order component of the cross section is **systematically reduced** by a component growing with Δy_{fb} (as expected, of course).

JRA, A. Maier et al., arXiv:1706.01002

Impact of Subleading Corrections for H_T



Leading logs only + matching



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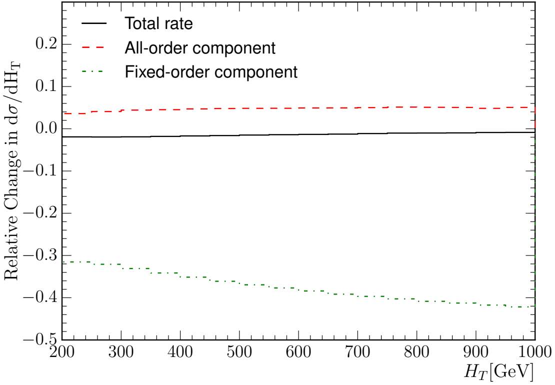
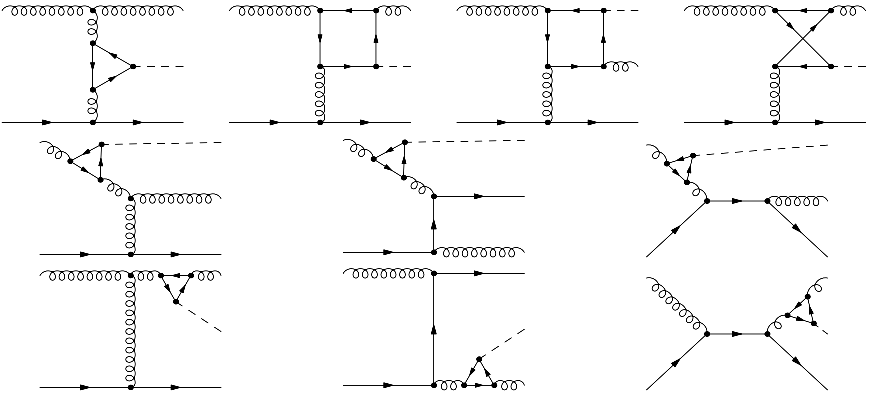
Subleading Corrections for H_T



The all-order and fixed-order component of the cross section is **changed uniformly**, but their **sum changes only little.**
Conclusion: The **all-order perturbative treatment** is very **stable.**

JRA, A. Maier et al., arXiv:1706.01002

Finite quark mass effects

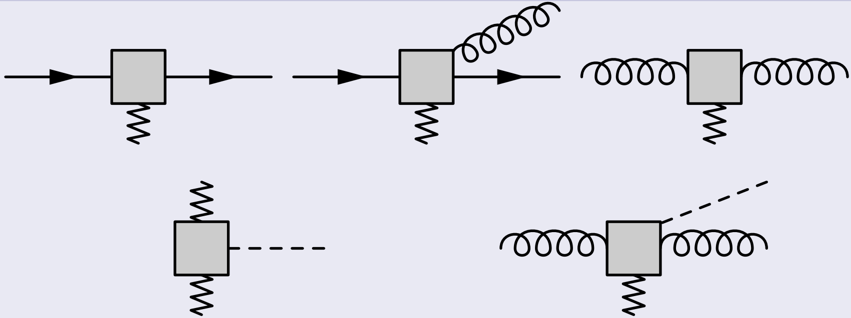


Del Duca, Kilgore, Oleari, Schmidt, Zeppenfeld, hep-ph/0105129

Effective HEJ-currents with full mass dependence can be extracted similarly to simple *qg*-scattering.

Finite quark mass effects

Building pieces in HEJ (currents) arxiv:1812.08072



Del Duca, Kilgore, Oleari, Schmidt, Zeppenfeld, hep-ph/0105129

Effective HEJ-currents with full mass dependence can be extracted similarly to simple qg-scattering.

Matching & merging with Fixed Order

$$
\begin{aligned}
\sigma_{2j}^{\text{resum,match}} = & \sum_{f_1,f_2} \sum_m \prod_{j=1}^m \left(\int_{p_{j\perp}^B=0}^{p_{j\perp}^B=\infty} \frac{\mathrm{d}^2\mathbf{p}_{j\perp}^B}{(2\pi)^3} \int \frac{\mathrm{d}y_j^B}{2} \right) (2\pi)^4 \, \delta^{(2)}\left(\sum_{k=1}^m \mathbf{p}_{k\perp}^B \right) \\
& \cdot \; x_a^B \; f_a(x_a^B, Q_a^B) \; x_b^B \; f_b(x_b^B, Q_b^B) \; \frac{\overline{|\mathcal{M}^B|}^2}{(\hat{s}^B)^2} \\
& \cdot \; \overline{|\mathcal{M}_{\text{HEJ}}^{tree}|}^{-2} \; (2\pi)^{-4+3m} \; 2^m \; \frac{(\hat{s}^B)^2}{x_a^B \; f_{a,f_1}(x_a^B, Q_a^B) \; x_b^B \; f_{b,f_2}(x_b^B, Q_b^B)} \\
& \cdot \; \sum_{n=2}^{\infty} \int_{p_{1\perp}=.9p_{j,\perp}}^{p_{1\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{1\perp}}{(2\pi)^3} \int_{p_{n\perp}=.9p_{j,\perp}}^{p_{n\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{n\perp}}{(2\pi)^3} \prod_{i=2}^{n-1} \int_{p_{i\perp}=\lambda}^{p_{i\perp}=\infty} \frac{\mathrm{d}^2\mathbf{p}_{i\perp}}{(2\pi)^3} (2\pi)^4 \, \delta^{(2)}\left(\sum_{k=1}^n \mathbf{p}_{k\perp} \right) \\
& \cdot \; \mathbf{T}_y \prod_{i=1}^n \left(\int \frac{\mathrm{d}y_i}{2} \right) \mathcal{O}_{mj}^e \left(\prod_{l=1}^{m-1} \delta^{(2)}(\mathbf{p}_{\mathcal{J}_l\perp}^B - \mathbf{j}_{l\perp}) \right) \left(\prod_{l=1}^m \delta(y_{\mathcal{J}_l}^B - y_{\mathcal{J}_l}) \right) \mathcal{O}_{2j}(\{p_i\}) \\
& \cdot \; x_a f_{a,f_1}(x_a, Q_a) \, x_b f_{b,f_2}(x_b, Q_b) \; \frac{\overline{|\mathcal{M}_{\text{HEJ}}^{f_1 f_2 \to f_1 g \cdots g f_2}(\{p_i\})|}^2}{\hat{s}^2} .
\end{aligned}
$$

- Generate Fixed Order, explore all of higher-order phase space

$\Rightarrow$ computationally efficient, merging limited by Fixed Order ($\sim$ 5 jets)

JRA, A. Maier et al., arXiv:1805.04446

Matching & merging with Fixed Order

$$\sigma_{2j}^{\text{resum,match}} = \sum_{f_1,f_2} \sum_m \prod_{j=1}^{m} \left(\int_{p_{j\perp}^B=0}^{p_{j\perp}^B=\infty} \frac{d^2\mathbf{p}_{j\perp}^B}{(2\pi)^3} \int \frac{dy_j^B}{2} \right) (2\pi)^4 \delta^{(2)}\left(\sum_{k=1}^{m} \mathbf{p}_{k\perp}^B \right)$$

$$\cdot\ x_a^B f_a(x_a^B, Q_a^B)\, x_b^B f_b(x_b^B, Q_b^B) \frac{\overline{|\mathcal{M}^B|}^2}{(\hat{s}^B)^2}$$

Fixed Order

$$\cdot\ \overline{|\mathcal{M}_{\text{HEJ}}^{tree}|}^{-2} (2\pi)^{-4+3m}\, 2^m \frac{(\hat{s}^B)^2}{x_a^B f_{a,f_1}(x_a^B, Q_a^B)\, x_b^B f_{b,f_2}(x_b^B, Q_b^B)}$$

Matching

All order

$$\cdot\ \sum_{n=2}^{\infty} \int_{p_{1\perp}=.9p_{j,\perp}}^{p_{1\perp}=\infty} \frac{d^2\mathbf{p}_{1\perp}}{(2\pi)^3} \int_{p_{n\perp}=.9p_{j,\perp}}^{p_{n\perp}=\infty} \frac{d^2\mathbf{p}_{n\perp}}{(2\pi)^3} \prod_{i=2}^{n-1} \int_{p_{i\perp}=\lambda}^{p_{i\perp}=\infty} \frac{d^2\mathbf{p}_{i\perp}}{(2\pi)^3} (2\pi)^4 \delta^{(2)}\left(\sum_{k=1}^{n} \mathbf{p}_{k\perp} \right)$$

$$\cdot\ \mathbf{T}_y \prod_{i=1}^{n} \left(\int \frac{dy_i}{2} \right) \mathcal{O}_{mj}^e \left(\prod_{l=1}^{m-1} \delta^{(2)}(\mathbf{p}_{\mathcal{J}_l\perp}^B - \mathbf{j}_{l\perp}) \right) \left(\prod_{l=1}^{m} \delta(y_{\mathcal{J}_l}^B - y_{\mathcal{J}_l}) \right) \mathcal{O}_{2j}(\{p_i\})$$

$$\cdot\ x_a f_{a,f_1}(x_a, Q_a)\, x_b f_{b,f_2}(x_b, Q_b) \frac{\overline{|\mathcal{M}_{\text{HEJ}}^{f_1 f_2 \to f_1 g \cdots g f_2}(\{p_i\})|}^2}{\hat{s}^2}.$$

- Generate Fixed Order, explore all of higher-order phase space

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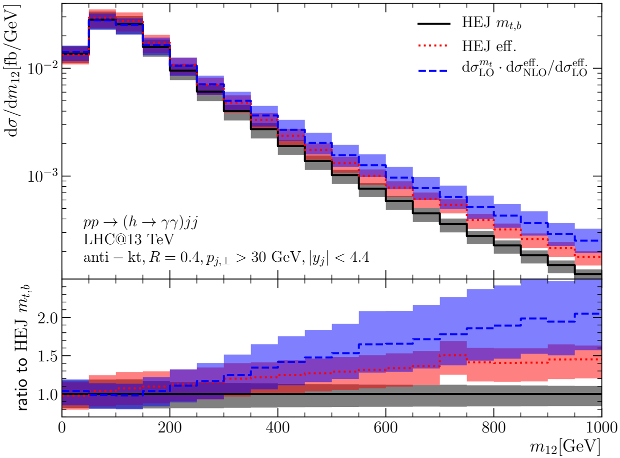
Spectrum in Invariant dijet mass



⇒ After VBF-cuts: $\sigma^{m_t \to \infty}_{\mathrm{HEJ}} \approx 1.1 \cdot \sigma_{\mathrm{HEJ}} \approx 0.5 \cdot \sigma_{\mathrm{NLO}}$

⇒ HEJ suppresses the region of large m_{jj} compared to fixed order.

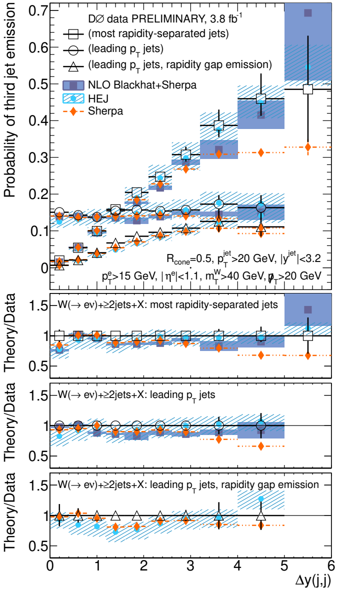
Comparison to Data

D0: W+Jets Phys. Rev. D 88, 092001

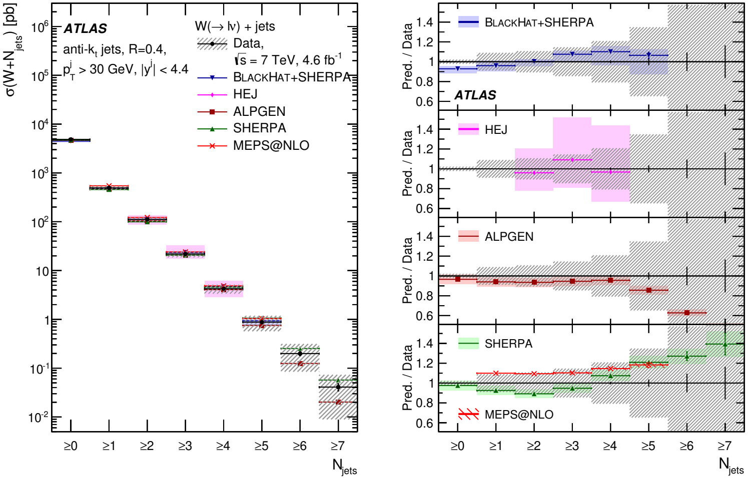
D0 measurement of the probability of at least one additional jet when requiring just a *W* in association with two jets. Probability measured vs. rapidity separation of

1. the two most rapidity separated jets
2. the two hardest (in pt) jets
3. the two hardest (in pt) jets, counting additional jets only in the rapidity interval between the two hardest jets

Good agreement between data and HEJ for all observables - effects are even more pronounced at the LHC.



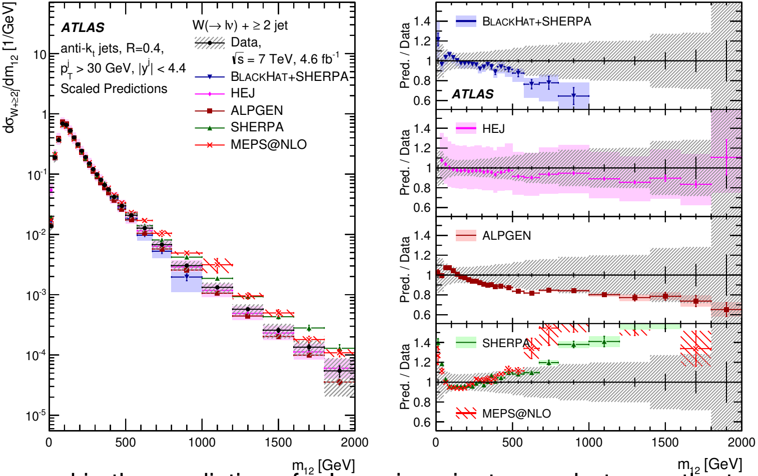
ATLAS: W+DiJets ($\alpha_s\alpha$)



arXiv:1409.8639

Good agreement between all predictions and data - on inclusive quantities.

ATLAS: W+DiJets ($\alpha_s \alpha$)



Large spread in the predictions for large invariant mass between the two hardest jets. Here, the terms systematically dealt with in HEJ are important, and HEJ gives a good description.

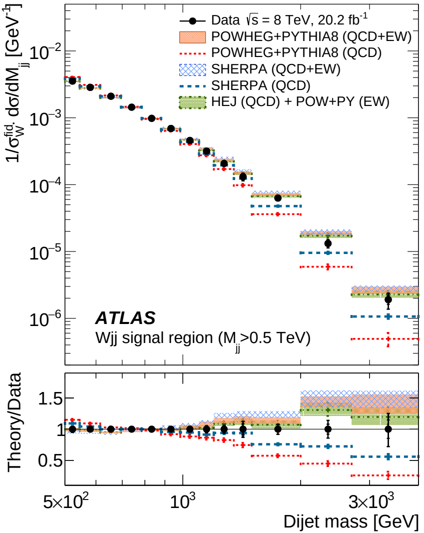
Note: hjj interesting for $m_{jj} > 400 - 600$GeV.

arXiv:1409.8639

Electroweak Wjj-production

Electroweak Wjj production.
Background from $\alpha_s^2\alpha$-component.
Compare NLO+PS (Powheg+Pythia, Sherpa) and HEJ+EW(Powheg)

- QCD contribution decreases at large dijet mass, but remains significant
- NLO+PS slightly overshoot



arXiv:1703.04362

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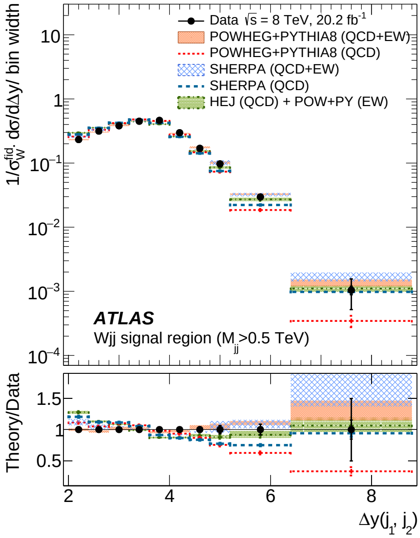
Similar conclusions for $\Delta y_{j_1 j_2}$

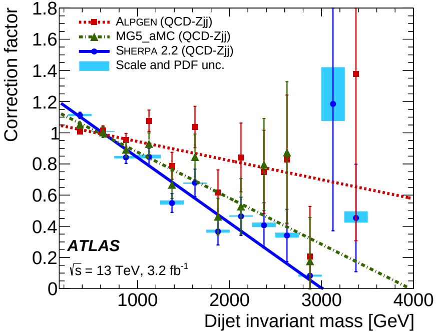


arXiv:1703.04362

Electroweak Zjj-production

Electroweak Zjj production. Background from $\alpha_s^2\alpha$-component. Compare Alpgen, MG5_aMC, SHERPA in control region.

Bin-by-bin corrections factors very different from unity are necessary in order to “describe” the control regions, and are then used to subtract the QCD contributions in order to analyse the EW process.



arXiv: 1709.10264

Conclusions

- The phase space explored by the LHC can enhance higher-order terms, which hinders the convergence of the fixed-order expansion
- The effect is present in LHC data (also Tevatron)
- Sizeable effects and implications for analyses of Higgs VBF and Vector-Boson-Scattering
- HEJ systematically includes the logarithmic corrections to all orders. Next-to-leading logarithmic corrections are being included.
- Public code, documentation, sample analyses, . . .

http://hej.web.cern.ch