B_d^0 and B_s^0 mixing with hadronic matrix elements from sum rules

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Matthew Kirk, Alexander Lenz, TR, arXiv:1711.02100
JHEP 1712 (2017) 068

Daniel King, Alexander Lenz, TR, arXiv:1904.00940

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FOR FUNDAMENTAL PHYSICS

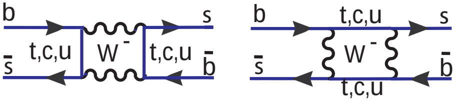
Outline

- Introduction
- The sum rule
- Three-point correlator at three loops
- Results and phenomenology
- Summary & outlook

B_s mixing in the Standard Model

Eigenstates

Flavor eigenstates mix:



$$i\frac{d}{dt}\begin{pmatrix} |B_s^0(t)\rangle \\ |\bar{B}_s^0(t)\rangle \end{pmatrix} = \left(\hat{M}^s - \frac{i}{2}\hat{\Gamma}^s\right)\begin{pmatrix} |B_s^0(t)\rangle \\ |\bar{B}_s^0(t)\rangle \end{pmatrix}$$

$$= \begin{pmatrix} M^s - \frac{i\Gamma^s}{2} & M_{12}^s - \frac{i\Gamma_{12}^s}{2} \\ M_{12}^{s*} - \frac{i\Gamma_{12}^{s*}}{2} & M^s - \frac{i\Gamma^s}{2} \end{pmatrix}\begin{pmatrix} |B_s^0(t)\rangle \\ |\bar{B}_s^0(t)\rangle \end{pmatrix}$$

Diagonalization gives mass eigenstates $|B_{s,L/H}(t)\rangle$ with

$$\Delta M_s \equiv M_H^s - M_L^s = 2|M_{12}^s| + \mathcal{O}\left(\frac{|\Gamma_{12}^s|^2}{|M_{12}^s|^2}\right),$$

$$\Delta\Gamma_s \equiv \Gamma_L^s - \Gamma_H^s = 2|\Gamma_{12}^s|\cos\phi_{12}^s + \mathcal{O}\left(\frac{|\Gamma_{12}^s|^2}{|M_{12}^s|^2}\right).$$

B_s mixing in the Standard Model

Mass and decay rate difference in the SM

$$M_{12}^s = \frac{G_F^2 M_W^2}{16\pi^2}\,\lambda_t^2 S_0(x_t)\hat{\eta}_B\,\frac{\langle \bar{B}_s^0|Q_1|B_s^0\rangle}{2M_{B_s^0}}\,,$$

$$\Gamma_{12}^s = \frac{-G_F^2 m_b^2}{24\pi M_{B_s^0}}\sum_{x=u,c}\sum_{y=u,c}\left[G_1^{s,xy}\langle \bar{B}_s^0|Q_1|B_s^0\rangle - G_2^{s,xy}\langle \bar{B}_s^0|Q_2|B_s^0\rangle\right] + \mathcal{O}\left(\frac{1}{m_b}\right).$$

Hadronic physics are contained in matrix elements of local operators:

$$\begin{aligned}
Q_1 &= \bar{b}_i\gamma_\mu(1-\gamma^5)s_i\,\bar{b}_j\gamma^\mu(1-\gamma^5)s_j,\\
Q_2 &= \bar{b}_i(1-\gamma^5)s_i\,\bar{b}_j(1-\gamma^5)s_j, \qquad Q_3 = \bar{b}_i(1-\gamma^5)s_j\,\bar{b}_j(1-\gamma^5)s_i,\\
Q_4 &= \bar{b}_i(1-\gamma^5)s_i\,\bar{b}_j(1+\gamma^5)s_j, \qquad Q_5 = \bar{b}_i(1-\gamma^5)s_j\,\bar{b}_j(1+\gamma^5)s_i.
\end{aligned}$$

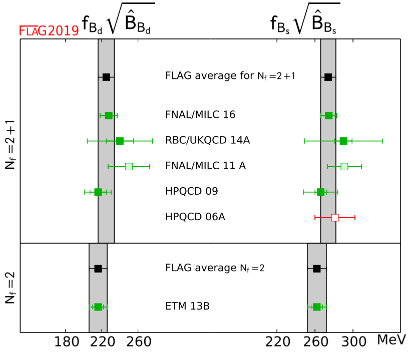
Bag parameters (Vacuum saturation approximation: $B_Q^s = 1$)

$$\langle Q(\mu)\rangle = A_Q\, f_{B_s}^2 M_{B_s}^2\, B_Q^s(\mu) = \overline{A}_Q(\mu)\, f_{B_s}^2 M_{B_s}^2\, \overline{B}_Q^s(\mu)$$

B_s mixing in the Standard Model

Mass and decay rate difference in the SM

Hadronic matrix elements can be determined with lattice QCD:



$$\Delta M_s^{\text{Lat.}} = (20.3^{+1.3}_{-1.7})\,\text{ps}^{-1}\,,$$

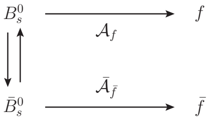
$$\Delta\Gamma_s^{\text{Lat.}} = (0.102^{+0.023}_{-0.032})\,\text{ps}^{-1}$$

B_s mixing in the Standard Model

CP violation

- In mixing: consider flavor-specific decays, e.g. $B_s^0 \to D_s^{(*)-}\mu^+\nu$

$$a_{fs}^s = \frac{\Gamma(\bar{B}_s^0(t) \to f) - \Gamma(B_s^0(t) \to \bar{f})}{\Gamma(\bar{B}_s^0(t) \to f) + \Gamma(B_s^0(t) \to \bar{f})}$$



B_s mixing in the Standard Model

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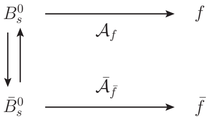
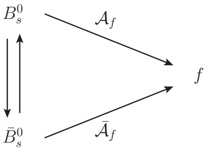


- In interference between mixing and decay:

$$A_{CP,f}(t) = \frac{\Gamma(\bar{B}_s^0(t) \to f) - \Gamma(B_s^0(t) \to f)}{\Gamma(\bar{B}_s^0(t) \to f) + \Gamma(B_s^0(t) \to f)}$$
$$= -\frac{\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}}\cos(\Delta M_s t) + \mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}}\sin(\Delta M_s t)}{\cosh(\Delta\Gamma_s t/2) + \mathcal{A}_{\Delta\Gamma}\sinh(\Delta\Gamma_s t/2)}$$



Golden modes (e.g. $B_s^0 \to J/\psi\phi$):

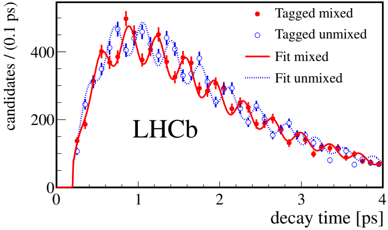
$$\mathcal{A}_{\mathrm{CP}}^{\mathrm{dir}} = 0,\ \mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}} = \sin(\phi_s),\ \mathcal{A}_{\Delta\Gamma} = -\cos(\phi_s)$$

B_s mixing in the Standard Model

CP violation

B_s mixing

Experiment



mixed: different flavor at decay and production

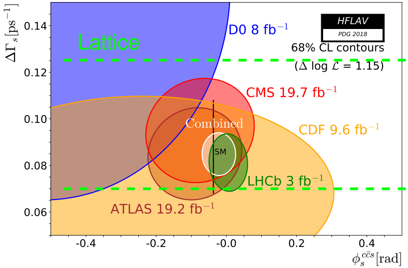
unmixed: same flavor at decay and production

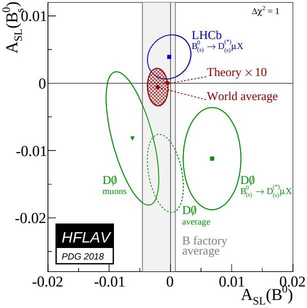
$$\Delta M_s^{\text{Lat.}} = (20.3^{+1.3}_{-1.7})\,\text{ps}^{-1}\,,$$

$$\Delta M_s^{\text{HFLAV}} = (17.757 \pm 0.021)\text{ps}^{-1}$$

$$\Delta\Gamma_s^{\text{Lat.}} = (0.102^{+0.023}_{-0.032})\,\text{ps}^{-1}$$

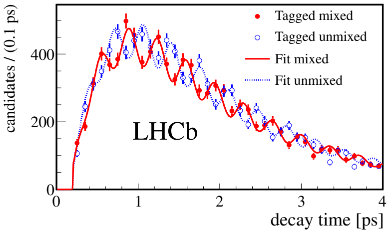
$$\Delta\Gamma_s^{\text{HFLAV}} = (0.088 \pm 0.006)\text{ps}^{-1}$$

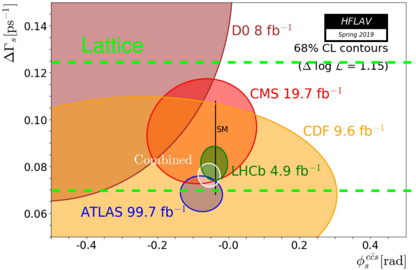




B_s mixing

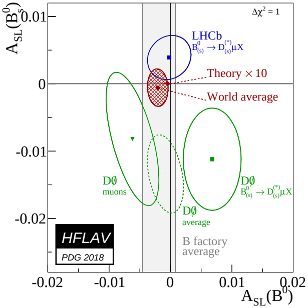
Experiment: New combination after Moriond 2019





mixed: different flavor at decay and production

unmixed: same flavor at decay and production



$$\Delta M_s^{\text{Lat.}} = (20.3^{+1.3}_{-1.7})\,\text{ps}^{-1}\,,$$

$$\Delta M_s^{\text{HFLAV}} = (17.757 \pm 0.021)\text{ps}^{-1}$$

$$\Delta\Gamma_s^{\text{Lat.}} = (0.102^{+0.023}_{-0.032})\,\text{ps}^{-1}$$

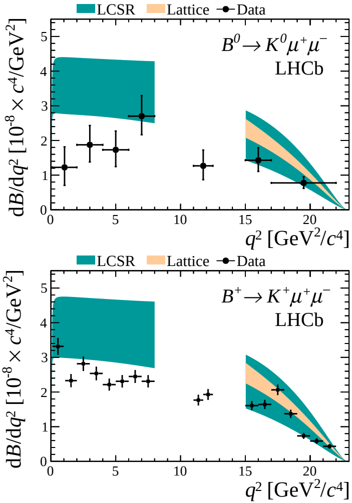
$$\Delta\Gamma_s^{\text{HFLAV}} = (0.0762 \pm 0.0033)\text{ps}^{-1}$$

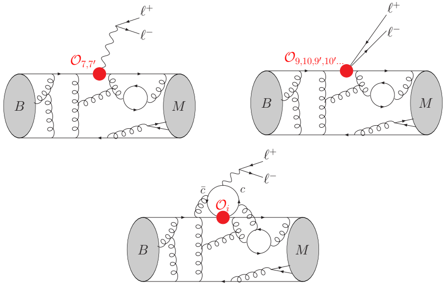
$$\phi_s^{\text{HFLAV}} = -0.0544 \pm 0.0205$$

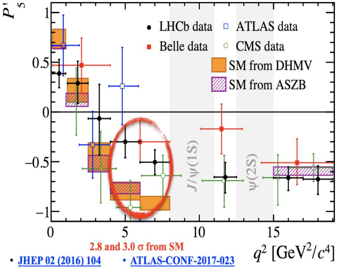
Flavour anomalies

Status before Moriond 2019

- $B^0 \to K^{(*)0}\mu^+\mu^-$







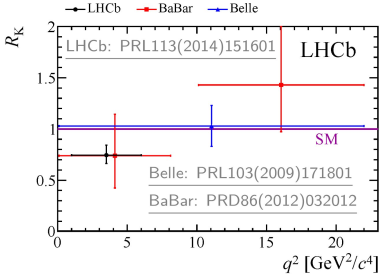
- JHEP 02 (2016) 104
- PRL 118 (2017)
- ATLAS-CONF-2017-023
- CMS-PAS-BPH-15-008

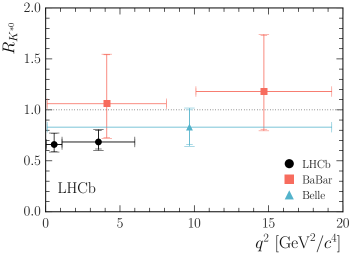
Flavour anomalies

Status before Moriond 2019

- $B^0 \to K^{(*)0}\mu^+\mu^-$
- Lepton-flavor universality: $B^0 \to K^{(*)0} l^+ l^-$

$$R_{K^{(*)}} = \frac{\mathrm{Br}(B^0 \to K^{(*)0}\mu^+\mu^-)}{\mathrm{Br}(B^0 \to K^{(*)0}e^+e^-)} \overset{\mathrm{SM}}{\simeq} 1$$



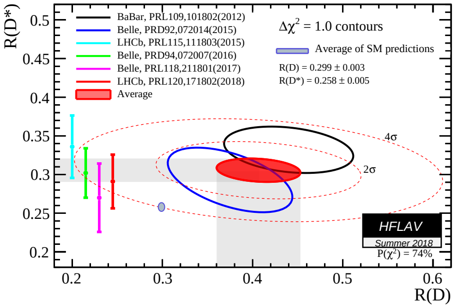


Flavour anomalies

Status before Moriond 2019

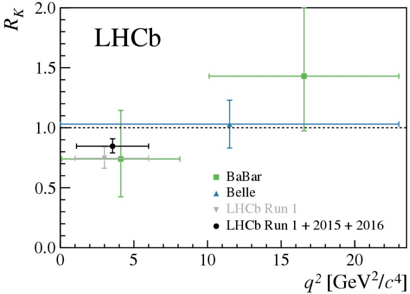
- $B^0 \to K^{(*)0}\mu^+\mu^-$
- Lepton-flavor universality: $B^0 \to K^{(*)0}l^+l^-$
- Lepton-flavor universality: $B^0 \to D^{(*)-}l^+\nu$

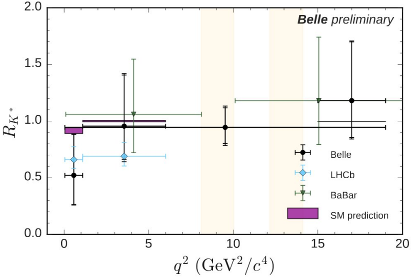
$$R(D^{(*)}) = \frac{\mathrm{Br}(B^0 \to D^{(*)-}\tau^+\nu)}{\mathrm{Br}(B^0 \to D^{(*)-}l^+\nu)}$$

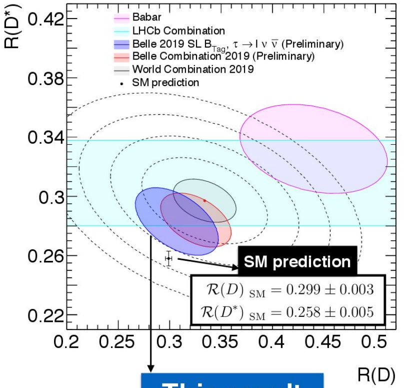


Flavour anomalies

Status after Moriond 2019 [M. Prim (Belle), T. Humair (LHCb), G. Caria (Belle)]







This result

$$\mathcal{R}(D) = 0.307 \pm 0.037 \pm 0.016$$

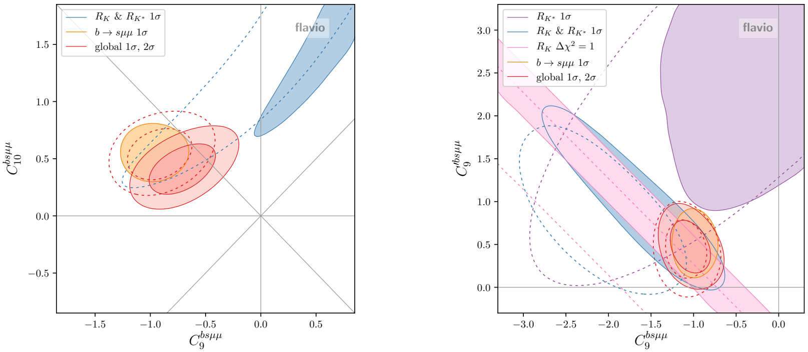
$$\mathcal{R}(D^*) = 0.283 \pm 0.018 \pm 0.014$$

Flavour anomalies

Fit to Wilson coefficients [Aebischer et al. 2019]

$$O_9^{bsll} = (\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu l)\,, \qquad O_9'^{bsll} = (\bar{s}\gamma_\mu P_R b)(\bar{l}\gamma^\mu l)\,,$$

$$O_{10}^{bsll} = (\bar{s}\gamma_\mu P_L b)(\bar{l}\gamma^\mu \gamma_5 l)\,, \qquad O_{10}'^{bsll} = (\bar{s}\gamma_\mu P_R b)(\bar{l}\gamma^\mu \gamma_5 l)\,,$$

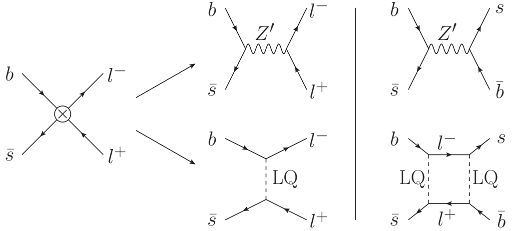


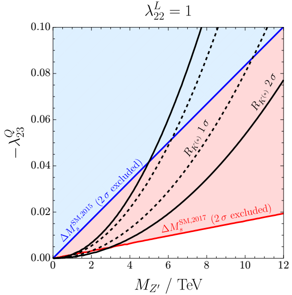
dashed: before Moriond 2019, solid: after Moriond 2019

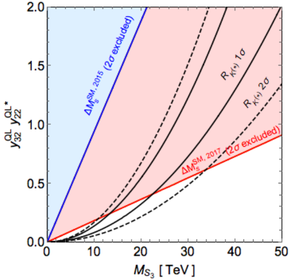
Flavour anomalies

Interplay with mixing [Di Luzio, Kirk, Lenz 2019]

Moving from EFT fits to models shows effects in mixing:







The sum rule

Alternative to lattice simulations

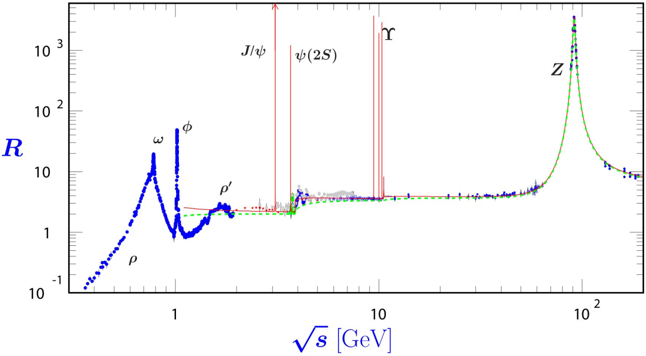
[Peskin, Schroeder 1995]

The sum rule

Basics: sum rule for $e^+e^- \to$ hadrons [Shifman, Vainshtein, Zakharov 1979]

Based on:

- Quark-hadron duality [Poggio, Quinn, Weinberg 1976]



red line: 3-loop perturbative QCD prediction [Chetyrkin, Harlander, Kühn 2000]

[PDG 2018]

The sum rule

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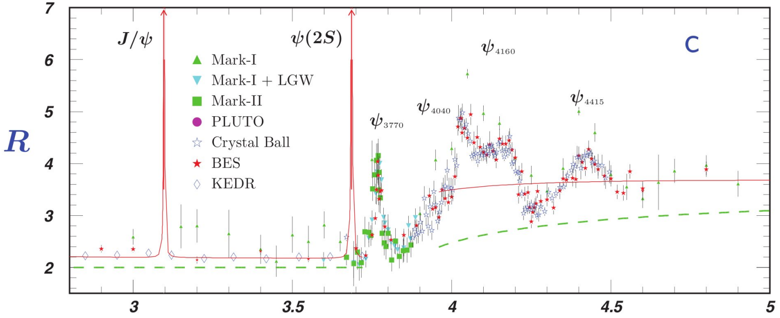

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The sum rule

Basics: sum rule for $e^+e^- \to$ hadrons [Shifman, Vainshtein, Zakharov 1979]

Based on:

- Quark-hadron duality [Poggio, Quinn, Weinberg 1976]
  Consider the smeared cross section:

$$\bar{R}(s,\Delta) = \frac{\Delta}{\pi}\int_0^\infty ds' \frac{R(s')}{(s'-s)^2+\Delta^2}$$

  Related to forward scattering amplitude Π at off-shell momenta:

$$\bar{R}(s,\Delta) = \frac{\Pi(s-i\Delta)-\Pi(s+i\Delta)}{2i}$$

  Off-shell partons do not hadronize. Thus, for suffficiently large Δ

$$\bar{R}_{\text{pert}}(s,\Delta) \simeq \bar{R}_{\text{exp}}(s,\Delta)$$

The sum rule

Basics: sum rule for $e^+e^- \to$ hadrons [Shifman, Vainshtein, Zakharov 1979]

Based on:

- Quark-hadron duality [Poggio, Quinn, Weinberg 1976]
- Analyticity of correlation functions

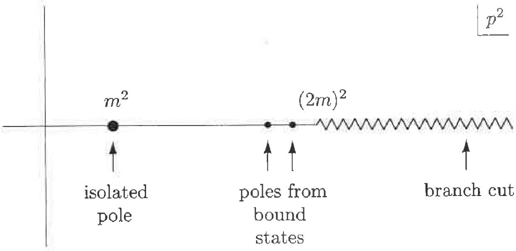

Figure 7.3. Analytic structure in the complex p^2-plane of the Fourier transform of the two-point function for a typical theory. The one-particle states contribute an isolated pole at the square of the particle mass. States of two or more free particles give a branch cut, while bound states give additional poles.

[Peskin, Schroeder 1995]

The sum rule

Basics: sum rule for $e^+e^- \to$ hadrons [Shifman, Vainshtein, Zakharov 1979]

Based on:

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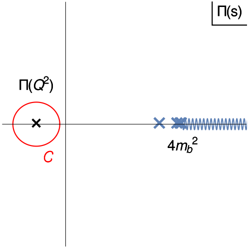
Use Cauchy's theorem



$$\Pi(Q^2) = \frac{1}{2\pi i}\oint_{\mathcal{C}} ds \frac{\Pi(s')}{s - Q^2}$$

The sum rule

Basics: sum rule for $e^+e^- \to$ hadrons [Shifman, Vainshtein, Zakharov 1979]

Based on:

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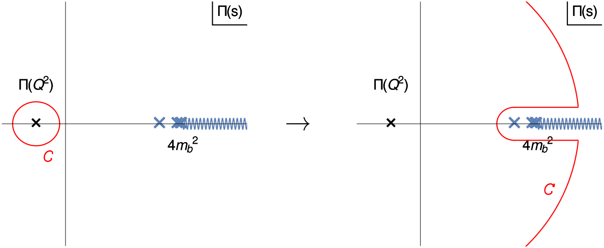
Use Cauchy's theorem and deform the contour



$$\Pi(Q^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\left[\Pi(s)\right]}{s-Q^2} + \frac{1}{2\pi i}\oint_{|s|=\infty} ds \frac{\Pi(s)}{s-Q^2}$$

The sum rule

Basics: sum rule for $e^+e^- \to$ hadrons [Shifman, Vainshtein, Zakharov 1979]

Based on:

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Taking derivatives at $Q^2 = 0$ yields the sum rule

$$\mathcal{M}_n^{\mathrm{exp}} \equiv \int_0^\infty ds \frac{R^{\mathrm{exp}}(s)}{s^{n+1}} \overset{\mathrm{QHD}}{=} \frac{12\pi^2}{n!}\left(\frac{d}{dQ^2}\right)^n \Pi^{\mathrm{OPE}}(Q^2)\Big|_{Q^2=0} \equiv \mathcal{M}_n^{\mathrm{th}}$$

The sum rule

Decay constant in HQET [Broadhurst, Grozin 1992; Bagan, Ball, Braun, Dosch 1992; Neubert 1992]

Heavy-quark effective theory (HQET) [Eichten, Hill 1990; Georgi 1990]:

- Describes IR dynamics inside heavy-light mesons
- Small momentum fluctuations around heavy quark at rest: $p_Q = m_Q v + k$ with $k \sim \Lambda_{\rm QCD} \ll m_Q$
- Expansion in inverse HQ mass m_Q

$$\mathcal{L}_{\rm HQET} = \bar{h}_v \, i v \cdot D \, h_v + \mathcal{O}(1/m_Q)$$

$$M_B = m_b + \bar{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2}{2 m_b} + \dots$$

HQET decay constant:

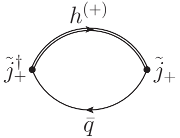
$$\langle 0 | \bar{h}^{(-)} \gamma^\mu \gamma^5 s | \mathbf{B}(v) \rangle = -i F(\mu) v^\mu$$

The sum rule

Decay constant in HQET [Broadhurst, Grozin 1992; Bagan, Ball, Braun, Dosch 1992; Neubert 1992]

Consider the two-point correlator

$$\Pi(\omega) = i \int d^d x e^{ipx} \langle 0 | \mathrm{T} \left[\tilde{j}_+^\dagger(0) \tilde{j}_+(x) \right] | 0 \rangle \,,$$



with $\omega = p \cdot v$ and $\tilde{j}_+ = \bar{q} \gamma^5 h^{(+)}$.

Using a Borel transform (instead of derivatives) gives the sum rule

$$\int_0^\infty d\omega \, e^{-\frac{\omega}{t}} \rho_\Pi^{\text{had}}(\omega) = \int_0^\infty d\omega \, e^{-\frac{\omega}{t}} \rho_\Pi^{\text{OPE}}(\omega)$$

with the discontinuity

$$\rho_\Pi(\omega) \equiv \frac{\Pi(\omega + i0) - \Pi(\omega - i0)}{2\pi i} = F^2(\mu) \delta(\omega - \overline{\Lambda}) + \rho_\Pi^{\text{cont}}(\omega) \,.$$

Sum rule for the decay constant with cutoff ω_c:

$$\boxed{F^2(\mu_\rho) e^{-\frac{\overline{\Lambda}}{t}} = \int_0^{\omega_c} d\omega \, e^{-\frac{\omega}{t}} \rho_\Pi(\omega)}$$

The sum rule

Decay constant in HQET [Broadhurst, Grozin 1992; Bagan, Ball, Braun, Dosch 1992; Neubert 1992]

Good agreement with lattice but with larger uncertainties

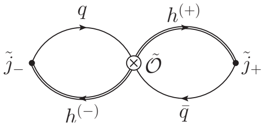
Reference	Method	N_f	f_{B^+}(MeV)	f_{B_s}(MeV)	f_{B_s}/f_{B^+}
ETM 13 [85] *,†	LQCD	2+1+1	196(9)	235(9)	1.201(25)
HPQCD 13 [86]	LQCD	2+1+1	184(4)	224(5)	1.217(8)
Average	LQCD	2+1+1	184(4)	224(5)	1.217(8)
Aoki 14 [87] *,‡	LQCD	2+1	218.8(6.5)(30.8)	263.5(4.8)(36.7)	1.193(20)(44)
RBC/UKQCD 14 [88]	LQCD	2+1	195.6(6.4)(13.3)	235.4(5.2)(11.1)	1.223(14)(70)
HPQCD 12 [89] *	LQCD	2+1	191(1)(8)	228(3)(10)	1.188(12)(13)
HPQCD 12 [89] *	LQCD	2+1	189(3)(3)*	–	–
HPQCD 11 [90]	LQCD	2+1	–	225(3)(3)	–
Fermilab/MILC 11 [69]	LQCD	2+1	196.9(5.5)(7.0)	242.0(5.1)(8.0)	1.229(13)(23)
Average	LQCD	2+1	189.9(4.2)	228.6(3.8)	1.210(15)
Our average	LQCD	Both	187.1(4.2)	227.2(3.4)	1.215(7)
Wang 15 [71] §	QCD SR		194(15)	231(16)	1.19(10)
Baker 13 [91]	QCD SR		186(14)	222 (12)	1.19(4)
Lucha 13 [92]	QCD SR		192.0(14.6)	228.0(19.8)	1.184(24)
Gelhausen 13 [72]	QCD SR		$207\binom{+17}{-9}$	$242\binom{+17}{-12}$	$1.17\binom{+3}{-4}$
Narison 12 [73]	QCD SR		206(7)	234(5)	1.14(3)
Hwang 09 [75]	LFQM		–	270.0(42.8)¶	1.32(8)

[PDG 2018]

The sum rule

Sum rule for Bag parameters [Chetyrkin et al. 1986; Körner et al. 2003; Mannel et al. 2011; Grozin et al. 2016; Kirk, Lenz, TR 2017; King, Lenz, TR 2019]

Consider the three-point correlator

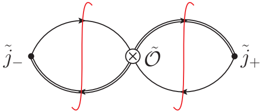


$$K_{\tilde{Q}}(\omega_1,\omega_2) = \int d^d x_1 d^d x_2 e^{ip_1\cdot x_1 - ip_2\cdot x_2} \langle 0|\mathrm{T}\left[\tilde{j}_+(x_2)\tilde{Q}(0)\tilde{j}_-(x_1)\right]|0\rangle$$

Going through the same steps one obtains the sum rule:

$$F^2(\mu)\langle\tilde{Q}(\mu)\rangle e^{-\frac{\overline{\Lambda}}{t_1}-\frac{\overline{\Lambda}}{t_2}} = \int_0^{\omega_c} d\omega_1 d\omega_2 e^{-\frac{\omega_1}{t_1}-\frac{\omega_2}{t_2}} \rho^{\mathrm{OPE}}_{\tilde{Q}}(\omega_1,\omega_2).$$

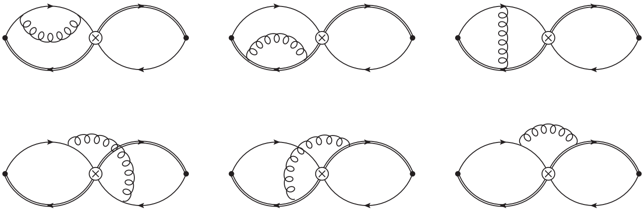
with the double discontinuity $\rho_{\tilde{Q}}$



The sum rule

Three-point correlator

NLO accuracy in the perturbative part requires a three-loop calculation:



$$\rho_{\tilde{Q}_i}(\omega_1,\omega_2) = A_{\tilde{Q}_i}\rho_\Pi(\omega_1)\rho_\Pi(\omega_2) + \Delta\rho_{\tilde{Q}_i}(\omega_1,\omega_2),$$

Factorizable contribution, reproduces the vacuum saturation approximation $B = 1$ (VSA)

Non-factorizable contribution:

$$\Delta\rho^{\text{pert}}_{\tilde{Q}_i} \equiv \frac{\omega_1^2\omega_2^2}{\pi^4}\frac{\alpha_s}{4\pi} r_{\tilde{Q}}\left(\frac{\omega_2}{\omega_1}, L_\omega\right)$$

The sum rule

Deviation from VSA

Formulate sum rule for deviation $\Delta B_{\tilde{Q}} = B_{\tilde{Q}} - 1$

$$\Delta B_{\tilde{Q}_i} = \frac{1}{A_{\tilde{Q}_i} F(\mu)^4} \int_0^{\omega_c} d\omega_1 d\omega_2 e^{\frac{\overline{\Lambda}-\omega_1}{t_1} + \frac{\overline{\Lambda}-\omega_2}{t_2}} \Delta\rho_{\tilde{Q}_i}(\omega_1, \omega_2)$$

Dispersion relation is not violated by arbitrary analytical weight function (Note of caution: Duality breaks down for pathological choices)

$$F^4(\mu) e^{-\frac{\overline{\Lambda}}{t_1} - \frac{\overline{\Lambda}}{t_2}} w(\overline{\Lambda}, \overline{\Lambda}) = \int_0^{\omega_c} d\omega_1 d\omega_2 e^{-\frac{\omega_1}{t_1} - \frac{\omega_2}{t_2}} w(\omega_1, \omega_2) \rho_\Pi(\omega_1) \rho_\Pi(\omega_2) + \ldots .$$

With the choice $w_{\tilde{Q}_i}(\omega_1, \omega_2) = \frac{\Delta\rho^{\text{pert}}_{\tilde{Q}_i}(\omega_1, \omega_2)}{\rho^{\text{pert}}_\Pi(\omega_1)\rho^{\text{pert}}_\Pi(\omega_2)} = \frac{C_F}{N_c} \frac{\alpha_s}{4\pi} r_{\tilde{Q}_i}(x, L_\omega)$ we obtain an analytic result for the pert contribution:

$$\boxed{\Delta B^{\text{pert}}_{\tilde{Q}_i}(\mu_\rho) = \frac{C_F}{N_c A_{\tilde{Q}_i}} \frac{\alpha_s(\mu_\rho)}{4\pi} r_{\tilde{Q}_i} \left(1, \log \frac{\mu_\rho^2}{4\overline{\Lambda}^2}\right).}$$

The sum rule

SU(3) breaking effects

For the B_s^0 system we employ an expansion in $m_s/\overline{\Lambda} \sim 0.2$

$$\Delta B_{\tilde{Q}_i}^{s,\text{pert}}(\mu_\rho) = \frac{w_{\tilde{Q}_i}(\overline{\Lambda}+m_s,\overline{\Lambda}+m_s)}{A_{\tilde{Q}_i}} =$$

$$\frac{C_F}{N_c A_{\tilde{Q}_i}} \frac{\alpha_s(\mu_\rho)}{4\pi} \Bigg\{ r_{\tilde{Q}_i}^{(0)}\left(1, L_{\overline{\Lambda}+m_s}\right) + \frac{2m_s}{\overline{\Lambda}+m_s}\left[r_{\tilde{Q}_i}^{(1)}\left(1, L_{\overline{\Lambda}+m_s}\right) - r_{\tilde{Q}_i}^{(0)}\left(1, L_{\overline{\Lambda}+m_s}\right)\right]$$

$$+ \frac{2m_s^2}{(\overline{\Lambda}+m_s)^2}\left[r_{\tilde{Q}_i}^{(2)}\left(1, L_{\overline{\Lambda}+m_s}\right) - 2r_{\tilde{Q}_i}^{(1)}\left(1, L_{\overline{\Lambda}+m_s}\right) + 2r_{\tilde{Q}_i}^{(0)}\left(1, L_{\overline{\Lambda}+m_s}\right)\right] + \ldots \Bigg\},$$

with

$$\Delta\rho_{\tilde{Q}_i}^{\text{pert}}(\omega_1,\omega_2) \equiv \frac{N_c C_F}{4} \frac{\omega_1^2\omega_2^2}{\pi^4} \frac{\alpha_s}{4\pi}\Bigg[r_{\tilde{Q}_i}^{(0)}(x, L_\omega) + \left(\frac{m_s}{\omega_1} + \frac{m_s}{\omega_2}\right) r_{\tilde{Q}_i}^{(1)}(x, L_\omega)$$

$$+ \left(\frac{m_s^2}{\omega_1^2} + \frac{m_s^2}{\omega_2^2}\right) r_{\tilde{Q}_i}^{(2)}(x, L_\omega) + \ldots \Bigg] \theta(\omega_1 - m_s)\theta(\omega_2 - m_s).$$

The sum rule

Calculation of the three-point correlator

- Generation of diagrams with `QGRAF` [Nogueira 1991]
- Dirac algebra with private implementation or `TRACER` [Jamin, Lautenbacher 1991]
- IBP reduction with `FIRE5` [Smirnov 2014]
- Master integrals to all orders in ϵ [Grozin, Lee 2008]
- Expansion of master integrals with `HypExp` [Huber, Maitre 2007]
- Take the double discontinuity of the correlator

The sum rule

Calculation of the three-point correlator

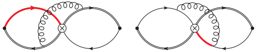


Figure 3: Examples for soft corrections to the non-factorizable part of the three-point correlator (2.22). The red, thick light-quark line carries momentum of the order of $m_s \ll \omega \sim \overline{\Lambda}$.

to the required order in ϵ using HypExp [35]. For completeness we state the results $r^{(0)}_{\tilde{Q}_i} = r^{(0)}_{\tilde{Q}_i}(x, L_\omega)$ for $m_s = 0$ previously presented in [15]

$$\begin{aligned}
r^{(0)}_{\tilde{Q}_1} &= 8 - \frac{a_2}{2} - \frac{8\pi^2}{3}, \\
r^{(0)}_{\tilde{Q}_2} &= 25 + \frac{a_1}{2} - \frac{4\pi^2}{3} + 6L_\omega + \phi(x), \\
r^{(0)}_{\tilde{Q}_4} &= 16 - \frac{a_3}{4} - \frac{4\pi^2}{3} + 3L_\omega + \frac{\phi(x)}{2}, \\
r^{(0)}_{\tilde{Q}_5} &= 29 - \frac{a_3}{2} - \frac{8\pi^2}{3} + 6L_\omega + \phi(x),
\end{aligned} \tag{2.30}$$

with

$$\phi(x) = \begin{cases} x^2 - 8x + 6\ln(x), & x \le 1, \\ \frac{1}{x^2} - \frac{8}{x} - 6\ln(x), & x > 1. \end{cases} \tag{2.31}$$

For the linear terms $r^{(1)}_{\tilde{Q}_i} = r^{(1)}_{\tilde{Q}_i}(x, L_\omega)$ we obtain

$$\begin{aligned}
r^{(1)}_{\tilde{Q}_1} &= -\frac{a_2}{2} - \frac{8\pi^2}{3} - 2\psi(x) + \begin{cases} \frac{2(18-63x+23x^2)}{9(1+x)} + \left(2 - \frac{2(3+x^3)}{3x(1+x)}\right)\ln(x), & x \le 1, \\ \frac{2(23-63x+18x^2)}{9x(1+x)} - \left(2 - \frac{2(1+3x^3)}{3x(1+x)}\right)\ln(x), & x > 1, \end{cases} \\
r^{(1)}_{\tilde{Q}_2} &= \frac{a_1}{2} - \frac{4\pi^2}{3} + 6L_\omega + \psi(x) + \begin{cases} \frac{243+162x-41x^2}{9(1+x)} + \left(5 + \frac{3+x^3}{3x(1+x)}\right)\ln(x), & x \le 1, \\ \frac{243x^2+162x-41}{9x(1+x)} - \left(5 + \frac{1+3x^3}{3x(1+x)}\right)\ln(x), & x > 1, \end{cases} \\
r^{(1)}_{\tilde{Q}_4} &= -\frac{a_3}{4} - \frac{4\pi^2}{3} + 3L_\omega + \begin{cases} \frac{4(36+9x+x^2)}{9(1+x)} + \left(3 - \frac{2x^2}{3(1+x)}\right)\ln(x), & x \le 1, \\ \frac{4(1+9x+36x^2)}{9x(1+x)} - \left(3 - \frac{2}{3x(1+x)}\right)\ln(x), & x > 1, \end{cases}
\end{aligned}$$

$$r^{(1)}_{\tilde{Q}_5} = -\frac{a_3}{2} - \frac{8\pi^2}{3} + 6L_\omega + \begin{cases} \frac{29+11x-2x^2}{1+x} + 6\ln(x), & x \le 1, \\ \frac{29x^2+11x-2}{x(1+x)} - 6\ln(x), & x > 1, \end{cases} \tag{2.32}$$

with

$$\psi(x) = \begin{cases} \frac{(1-x)^2}{x}\left[2\ln(1-x) - \ln(x)\right], & x \le 1, \\ \frac{(1-x)^2}{x}\left[2\ln(x-1) - \ln(x)\right], & x > 1. \end{cases} \tag{2.33}$$

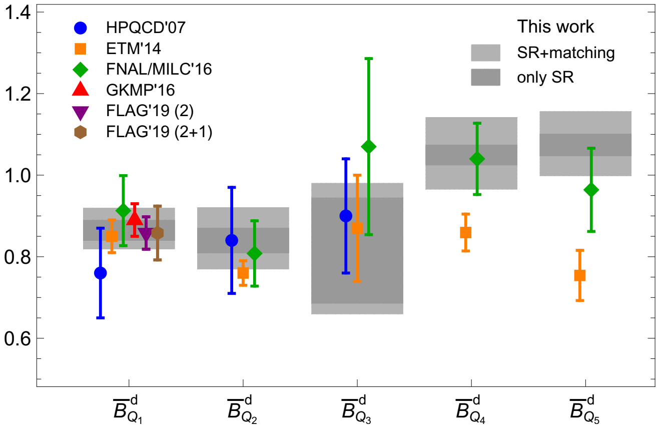
Last but not least, our results for the quadratic terms $r^{(2)}_{\tilde{Q}_i} = r^{(2)}_{\tilde{Q}_i}(x, L_\omega)$ are

$$\begin{aligned}
r^{(2)}_{\tilde{Q}_1} &= \frac{1}{1+x^2}\left[\frac{(1-x)^2 a_2}{4} + \frac{2\pi^2(1-4x+x^2)}{3} + 2x\psi(x)\left(2 + \frac{1+x}{1-x}\ln(x)\right)\right. \\
&\quad \left. + \begin{cases} -\frac{2(6+6x-x^2+2x^3)}{3} + 2(2-4x+x^2)\ln(x) - 4(1-x^2)\mathrm{Li}_2(1-1/x), & x \le 1, \\ -\frac{2(2-x+6x^2+6x^3)}{3x} - 2(1-4x+2x^2)\ln(x) + 4(1-x^2)\mathrm{Li}_2(1-x), & x > 1, \end{cases}\right], \\
r^{(2)}_{\tilde{Q}_2} &= \frac{1}{1+x^2}\left[\frac{-(1-x)^2 a_1}{4} - 3(1-x)^2 L_\omega + \frac{\pi^2(1-4x+x^2)}{3} + \frac{x(1+x)}{1-x}\ln(x)\psi(x)\right. \\
&\quad \left. + \begin{cases} -\frac{75-198x+89x^2-4x^3}{6} - (3-6x+2x^2)\ln(x) - 2(1-x^2)\mathrm{Li}_2(1-1/x), & x \le 1, \\ +\frac{4-89x+198x^2-75x^3}{6x} + (2-6x+3x^2)\ln(x) + 2(1-x^2)\mathrm{Li}_2(1-x), & x > 1, \end{cases}\right], \\
r^{(2)}_{\tilde{Q}_4} &= \frac{1}{1+x^2}\left[\frac{(1-x)^2 a_3}{8} - \frac{3(1-x)^2}{2}L_\omega + \frac{x\psi(x)}{2}\left(1 + \frac{3(1+x)}{1-x}\ln(x)\right)\right. \\
&\quad \left. + \begin{cases} -(1+8x-5x^2)\frac{\pi^2}{6} - \frac{24-48x+16x^2+x^3}{3} - (1+x^2)\ln(x) \\ -(1-x^2)\ln^2(x) - 5(1-x^2)\mathrm{Li}_2(1-1/x), & x \le 1, \\ +(5-8x-x^2)\frac{\pi^2}{6} - \frac{1+16x-48x^2+24x^3}{3x} + (1+x^2)\ln(x) \\ +(1-x^2)\ln^2(x) + 5(1-x^2)\mathrm{Li}_2(1-x), & x > 1, \end{cases}\right], \\
r^{(2)}_{\tilde{Q}_5} &= \frac{1}{1+x^2}\left[\frac{(1-x)^2 a_3}{4} - 3(1-x)^2 L_\omega + \frac{2\pi^2(1-4x+x^2)}{3}\right. \\
&\quad +2x\psi(x)\left(1 + \frac{1+x}{1-x}\ln(x)\right) - \frac{29-62x+29x^2}{2} \\
&\quad \left. + \begin{cases} -(1-x)^2\ln(x) - 4(1-x^2)\mathrm{Li}_2(1-1/x), & x \le 1, \\ +(1-x)^2\ln(x) + 4(1-x^2)\mathrm{Li}_2(1-x), & x > 1, \end{cases}\right].
\end{aligned} \tag{2.34}$$

Results and phenomenology

Bag parameters

- Determine HQET Bag parameters at low scale μ_ρ from sum rule
- Run up to $\mu_m \sim m_b$ and match to QCD Bag parameters at NLO



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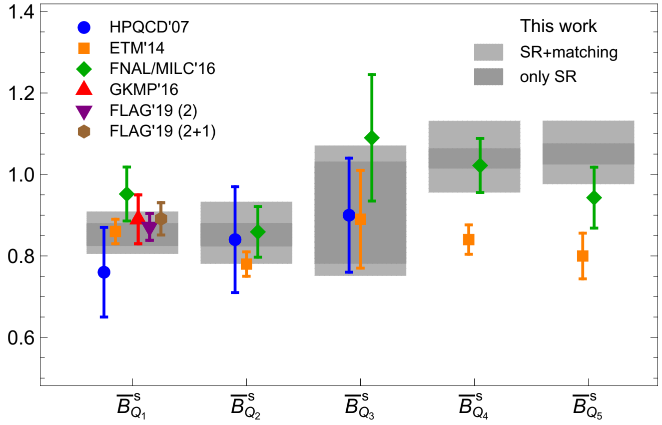
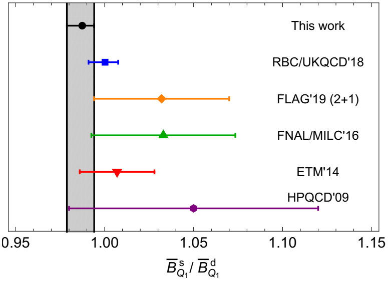


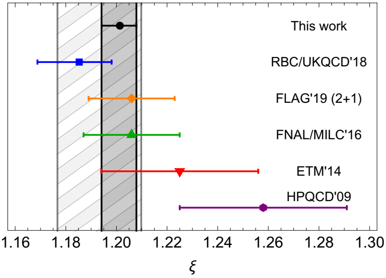
Results and phenomenology

SU(3) breaking ratios

- Small SU(3) breaking effects $m_s/\overline{\Lambda} \times \alpha_s(\mu_\rho)/\pi \sim 0.02$
- Using FLAG 2+1+1 (hatched band with 2+1) average for f_{B_s}/f_B we obtain the most precise result for

$$\xi \equiv \frac{f_{B_s}}{f_B}\sqrt{\overline{B}^{s/d}_{Q_1}} = 1.2014^{+0.0065}_{-0.0072} = 1.2014 \pm 0.0050 \left(\frac{f_{B_s}}{f_B}\right) {}^{+0.0043}_{-0.0053} \left(\overline{B}^{s/d}_{Q_1}\right)$$



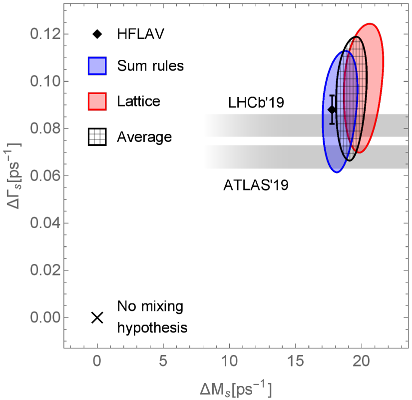


Results and phenomenology

B_s mixing observables

$$\Delta M_s^{\text{exp}} = (17.757 \pm 0.021)\,\text{ps}^{-1},$$
$$\Delta M_s^{\text{SR}} = (18.5^{+1.2}_{-1.5})\,\text{ps}^{-1},$$
$$\Delta M_s^{\text{Lat.}} = (20.3^{+1.3}_{-1.7})\,\text{ps}^{-1},$$
$$\Delta M_s^{\text{Av.}} = (19.4^{+1.0}_{-1.4})\,\text{ps}^{-1}.$$

$$\Delta \Gamma_s^{\text{exp}} = (0.088 \pm 0.006)\,\text{ps}^{-1},$$
$$\Delta \Gamma_s^{\text{SR}} = (0.091^{+0.022}_{-0.030})\,\text{ps}^{-1},$$
$$\Delta \Gamma_s^{\text{Lat.}} = (0.102^{+0.023}_{-0.032})\,\text{ps}^{-1},$$
$$\Delta \Gamma_s^{\text{Av.}} = (0.097^{+0.022}_{-0.031})\,\text{ps}^{-1}.$$

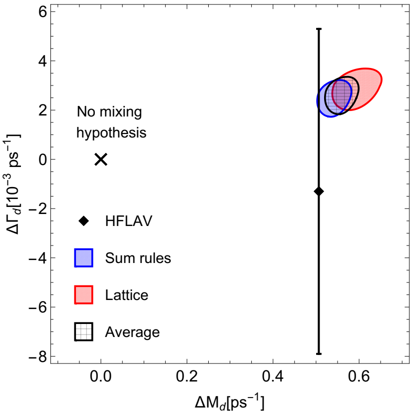


Results and phenomenology

B_d mixing observables

$$\Delta M_d^{\text{exp}} = (0.5064 \pm 0.0019)\,\text{ps}^{-1},$$
$$\Delta M_d^{\text{SR}} = (0.547^{+0.035}_{-0.046})\,\text{ps}^{-1},$$
$$\Delta M_d^{\text{Lat.}} = (0.596^{+0.054}_{-0.063})\,\text{ps}^{-1},$$
$$\Delta M_d^{\text{Av.}} = (0.565^{+0.034}_{-0.046})\,\text{ps}^{-1}.$$

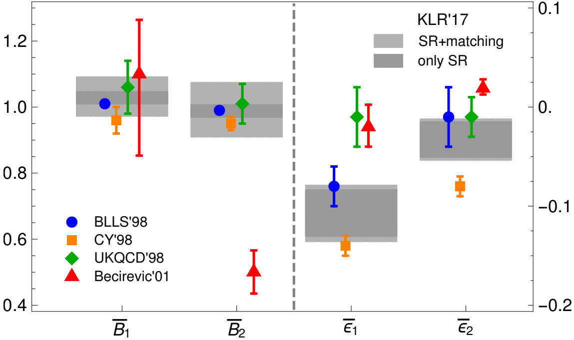
$$\Delta\Gamma_d^{\text{exp}} = (-1.3 \pm 6.6)\cdot 10^{-3}\,\text{ps}^{-1},$$
$$\Delta\Gamma_d^{\text{SR}} = (2.6^{+0.6}_{-0.9})\cdot 10^{-3}\,\text{ps}^{-1},$$
$$\Delta\Gamma_d^{\text{Lat.}} = (3.0^{+0.7}_{-1.0})\cdot 10^{-3}\,\text{ps}^{-1},$$
$$\Delta\Gamma_d^{\text{Av.}} = (2.7^{+0.6}_{-0.9})\cdot 10^{-3}\,\text{ps}^{-1}.$$



Results and phenomenology

B meson lifetimes

First state-of-the art results for $\Delta B = 0$ Bag parameters



$$\left.\frac{\tau(B^+)}{\tau(B^0)}\right|_{\text{exp}} = 1.076 \pm 0.004, \qquad \left.\frac{\tau(B_s^0)}{\tau(B^0)}\right|_{\text{exp}} = 0.994 \pm 0.004,$$

$$\left.\frac{\tau(B^+)}{\tau(B^0)}\right|_{\text{SR}} = 1.082^{+0.022}_{-0.026}. \qquad \left.\frac{\tau(B_s^0)}{\tau(B^0)}\right|_{\text{SR}} = 0.9994 \pm 0.0025.$$

Results and phenomenology

Heavy quark expansion in charm?

B-physics: HQE is well established approach, $\Lambda/m_b \sim 0.2 \ll 1$

D-physics: HQE commonly dismissed, $\Lambda/m_c \sim 0.2 m_b/m_c \sim 0.7 \approx 1$,

Results and phenomenology

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D-physics: HQE commonly dismissed, $\Lambda/m_c \sim 0.2 m_b/m_c \sim 0.7 \approx 1$,

But: HQE is really an expansion in Λ/momentum release

- $\Delta\Gamma_s$ dominated by $D_s^{(*)+} D_s^{(*)-}$ final state, mom. release ~3.5 GeV
- D decays dominated by $K\pi^{1-3}$ final state, mom. release ~1.7 GeV
- expected expansion parameter is of the order 0.4

Small enough for convergence?

Results and phenomenology

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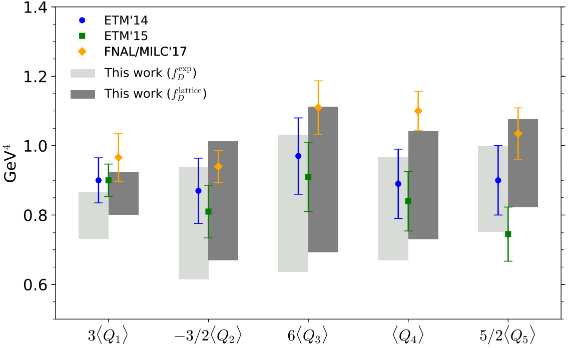
- expected expansion parameter is of the order 0.4

Small enough for convergence?

Shut up and calculate!

Results and phenomenology

Comparison of matrix elements for D mixing



- Good agreement with lattice (using lattice results for the decay constant)
- Larger uncertainties due to lower matching scale compared to B system

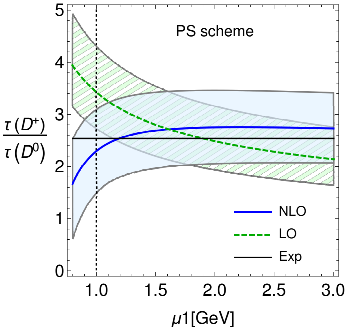
Results and phenomenology

D meson lifetimes as test of the HQE

Good description of lifetimes in charm sector: [Lenz, TR 2013; Kirk, Lenz, TR 2017]

$$\left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\text{exp}} = 2.536 \pm 0.019, \qquad \left.\frac{\bar\tau(D_s^+)}{\tau(D^0)}\right|_{\text{exp}} = 1.292 \pm 0.019,$$

$$\left.\frac{\tau(D^+)}{\tau(D^0)}\right|_{\text{SR}} = 2.70^{+0.74}_{-0.82}. \qquad \left.\frac{\bar\tau(D_s^+)}{\tau(D^0)}\right|_{\text{SR}} = 1.19 \pm 0.13.$$



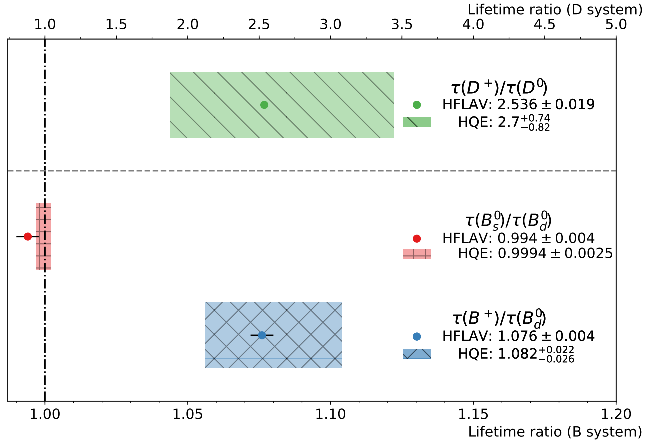
Good convergence:
NLO QCD +28%, $1/m_c$ -34%.

$$\frac{\tau(D^+)}{\tau(D^0)} = 1 + 16\pi^2(0.25)^3[1 - 0.34]$$

Good behaviour under scale variation above about 1 GeV.

Results and phenomenology

Lifetime overview



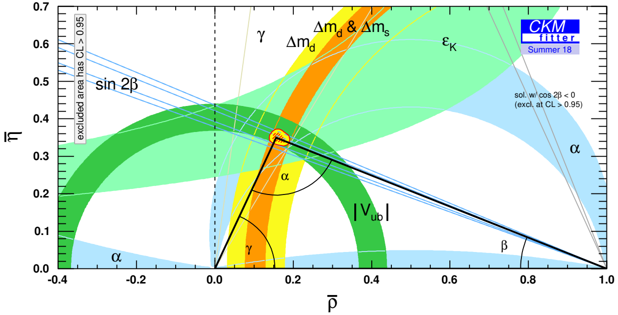
Results and phenomenology

Impact on CKM picture

Unitarity triangle (UT): Display the rescaled unitarity relation

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{cd}V_{cb}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

Overconstrain the UT to check the SM (e.g. 4th generation)

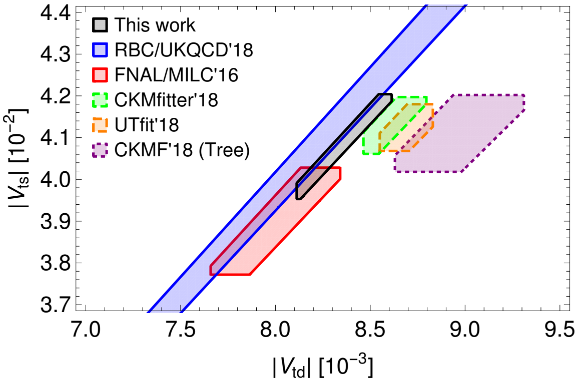


Results and phenomenology

Determination of CKM elements

Assuming the validity of the SM we get a precise determination of $|V_{td}|$ and $|V_{ts}|$ and in particular the ratio from

$$\frac{\Delta M_d}{\Delta M_s} = \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{1}{\xi^2} \frac{M_{B_d}}{M_{B_s}} .$$

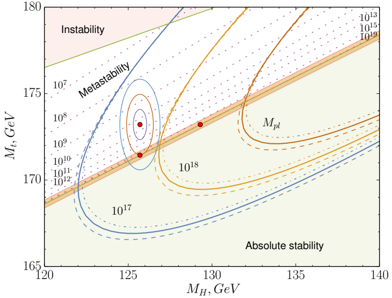


Results and phenomenology

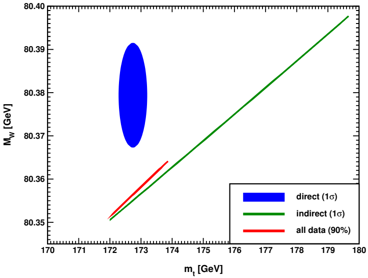
Determination of the top-quark mass

Top-quark mass is an important input parameter, e.g. for

- Stability of the EW vacuum
- Electroweak precision observables



[Bednyakov, Kniehl, Pikelner, Veretin 2015]



[PDG 2018]

Results and phenomenology

Determination of the top-quark mass

Top-quark mass is an important input parameter, e.g. for

- Stability of the EW vacuum
- Electroweak precision observables

Direct measurement $m_t^{\mathrm{MC}} = (173.0 \pm 0.4)\,\mathrm{GeV}$ by reconstruction from decay products corresponds to Monte-Carlo mass. Uncertainty in scheme conversion difficult to assess [Hoang, Plätzer, Samitz 2018]

Determination of $\overline{\mathrm{MS}}$ mass from mixing

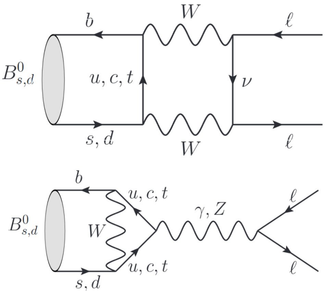
$$\overline{m}_t(\overline{m}_t) = (157^{+8}_{-6})\,\mathrm{GeV} = (157^{+7}_{-6}\,(\mathrm{had.})^{+0}_{-1}\,(\mu)^{+4}_{-1}\,(\mathrm{param.}))\,\mathrm{GeV},$$

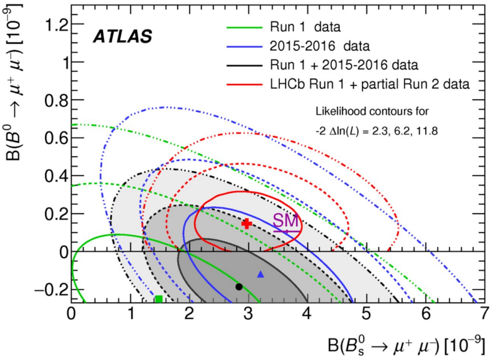
compatible with result from cross section measurements [PDG 2018]

$$\overline{m}_t(\overline{m}_t) = (160^{+5}_{-4})\,\mathrm{GeV}.$$

Results and phenomenology

Predictions for $B_{s/d} \to \mu^+\mu^-$





$$\mathrm{Br}(B_q \to l^+ l^-) = \frac{G_F^4 M_W^4 M_{B_q} f_{B_q}^2}{2\pi^5 \Gamma_H^q} |V_{tb}^* V_{tq}|^2 m_l^2 \sqrt{1 - \frac{4 m_l^2}{M_{B_q}^2}} |C_A(\mu)|^2 + \mathcal{O}(\alpha_{\mathrm{em}})$$

with C_A known at NNLO QCD plus NLO EW [Bobeth et al. 2013]

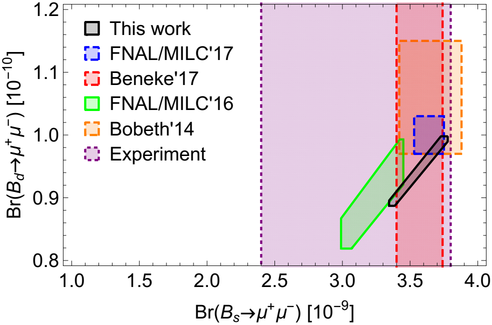
Results and phenomenology

Predictions for $B_{s/d} \to \mu^+\mu^-$

Dominant uncertainties from CKM elements and decay constants cancel in the ratio

$$\frac{\mathrm{Br}(B_q \to l^+ l^-)}{\Delta M_q} = \frac{3 G_F^2 M_W^2 m_l^2 \tau_{B_q^H}}{\pi^3} \sqrt{1 - \frac{4 m_l^2}{M_{B_q}^2}} \frac{|C_A(\mu)|^2}{S_0(x_t)\hat{\eta}_B \overline{B}^q_{Q_1}(\mu)} .$$

Gives alternative prediction with $\overline{B}^q_{Q_1}(\mu)$ as only relevant uncertainty.



Summary and outlook

- Sum rules provide highly competitive alternative to lattice simulations for the matrix elements of 4-quark operators and truly independent comparisons.
- The HQE is in terrific shape. Lifetimes even look promising in the charm sector.
- Mixing gives strong constraints on models that are frequently invoked to explain the current ’anomalies’.
- First state-of-the-art results for $\Delta F = 0$ matrix elements. Confirmation from lattice desirable.

Summary and outlook

- Sum rules provide highly competitive alternative to lattice simulations for the matrix elements of 4-quark operators and truly independent comparisons.
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- Mixing gives strong constraints on models that are frequently invoked to explain the current 'anomalies'.
- First state-of-the-art results for $\Delta F = 0$ matrix elements. Confirmation from lattice desirable.
- NNLO QCD-HQET matching calculations can significantly decrease uncertainties. [Q1: Grozin, Mannel, Pivovarev 2017/18]
- Uncertainties in decay rate difference and lifetimes can be reduced considerably by a sum rule determination of the dimension seven matrix elements.

Backup: B_s mixing in the Standard Model

Determination of mixing matrix

$$2M_{B_s}\left(M_{12}^s - \frac{i\Gamma_{12}^s}{2}\right)$$

$$= \left\langle \bar{B}_s^0 \middle| H^{|\Delta B|=2} \middle| B_s^0 \right\rangle + \sum_n \frac{\left\langle \bar{B}_s^0 \middle| H^{|\Delta B|=1} \middle| n \right\rangle \left\langle n \middle| \bar{B}_s^0 \middle| H^{|\Delta B|=1} \middle| B_s^0 \right\rangle}{M_{B_s} - E_n + i0}$$

↓ Short-distance part: only contributes to M_{12}^s

↓ Long-distance part: contributes to both ($M_{12}^{s,\text{LD}} \ll M_{12}^{s,\text{SD}}$)

With $\lambda_q = V_{qs}^* V_{qb}$ and the unitarity relation $\lambda_u + \lambda_c + \lambda_t = 0$ we get

$$\begin{aligned} M_{12}^s \propto & \sum_{q,q'=u,c,t} \lambda_q \lambda_{q'} F(q,q') \\ = & \lambda_c^2 [F(c,c) - 2F(u,c) + F(u,u)] \\ & + 2\lambda_c\lambda_t [F(c,t) - F(u,t) - F(u,c) + F(u,u)] \\ & + \lambda_t^2 [F(t,t) - 2F(u,t) + F(u,u)] \end{aligned}$$

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$$M_{12}^s \propto \sum_{q,q'=u,c,t} \lambda_q \lambda_{q'} F(q,q')$$

$$\approx \lambda_t^2 [F(t,t) - 2F(u,t) + F(u,u)] \propto \lambda_t^2 S_0\left(\frac{m_t^2}{M_W^2}\right)$$

where $F(q,q')$ is the contribution with quarks q and q' in the loop and we approximate $m_u^2/M_W^2 \approx 0$ and $m_c^2/M_W^2 \approx 0$.

Backup: B_s mixing in the Standard Model

Heavy-quark expansions for lifetime (differences)

Use the optical theorem:

$$\Gamma(B_s^0) = \frac{1}{2M_{B_s^0}} \left\langle B_s^0 \middle| \mathrm{Im}\left(i \int d^4x T\left[\mathcal{H}^{|\Delta B|=1}(x)\mathcal{H}^{|\Delta B|=1}(0)\right]\right) \middle| B_s^0 \right\rangle$$

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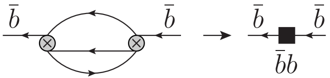
and apply an OPE for small x, i.e. large momentum release

$$\Gamma(B_q \to f) = \frac{G_F^2 m_b^5}{192\pi^3} \frac{|V_{\mathrm{CKM}}|^2}{2M_B} \left[c_3^f \left\langle B_q \left| \bar{b}b \right| B_q \right\rangle \right.$$



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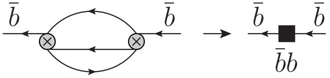

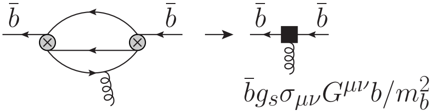

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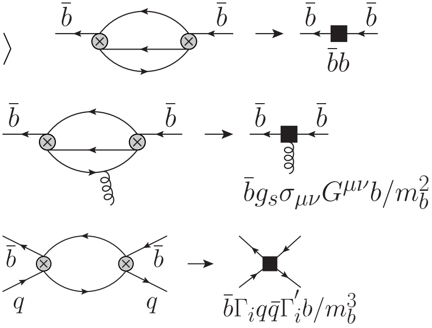
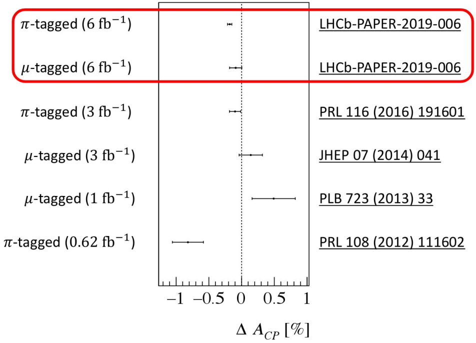


Backup

More flavor news from Moriond 2019: CP violation in charm [F. Betti for LHCb]

Partial cancellation of experimental uncertainties in difference

$$\Delta A_{\mathrm{CP}} \equiv A_{\mathrm{CP}}(D^0 \to K^- K^+) - A_{\mathrm{CP}}(D^0 \to \pi^- \pi^+)$$



$$\Delta A_{\mathrm{CP}} = (-15.4 \pm 2.9) \times 10^{-4}$$

Discovery at 5.3σ!

SM prediction is very challenging: SM explanation requires enhancement of penguin effects by an order of magnitude compared to naive PT expectation

Backup

Finite m_s effects in the sum rule for the decay constant

Exact m_s dependence at LO:

$$\rho_{\Pi}^{\text{pert}}(\omega) = \frac{N_c}{2\pi^2}\left[(\omega+m_s)\sqrt{\omega^2-m_s^2}\,\theta(\omega-m_s)+\mathcal{O}(\alpha_s)\right].$$

The finite-energy (FESR) version of the sum rule with $t\to\infty$ gives

$$\begin{aligned} F_s^2(\mu_\rho)|_{\text{FESR}} &= \frac{N_c}{6\pi^2}\Bigg[\left(\omega_c-\frac{m_s}{2}\right)(\omega_c+2m_s)\sqrt{\omega_c^2-m_s^2} \\ &\quad +\frac{3m_s^3}{2}\ln\left(\frac{m_s}{\omega_c+\sqrt{\omega_c^2-m_s^2}}\right)+\mathcal{O}(\alpha_s)+[\text{condensates}]\Bigg] \\ &= \frac{N_c\omega_c^3}{6\pi^2}\left[1+\frac{3m_s}{2\omega_c}-\frac{3m_s^2}{2\omega_c^2}-\frac{3m_s^3}{4\omega_c^3}\left(1-\ln\frac{m_s^2}{4\omega_c^2}\right)+\dots\right]. \end{aligned}$$

Backup

Finite m_s effects in the sum rule for the decay constant

Split the integration at an arbitrary scale ν with $m_s \ll \nu \ll \omega_c$

$$\mathcal{T}_{\frac{m_s}{\omega_c}}[F_s^2(\mu_\rho)]e^{-\frac{\bar{\Lambda}+m_s}{t}} = \mathcal{T}_{\{\frac{m_s}{\omega_c},\frac{m_s}{\nu},\frac{\nu}{\omega_c}\}}\left[\int_{m_s}^{\nu} d\omega\, e^{-\frac{\omega}{t}}\rho_\Pi(\omega) + \int_{\nu}^{\omega_c} d\omega\, e^{-\frac{\omega}{t}}\mathcal{T}_{\frac{m_s}{\omega}}[\rho_\Pi(\omega)]\right]$$

Taking the limit $\nu \to m_s$ *after* the expansion the first term is polynomial in m_s starting at m_s^3. Thus, knowledge of expanded discontinuity is sufficient to obtain result up to m_s^2

$$\mathcal{T}_{\frac{m_s}{\omega_c}}\left[\int_{m_s}^{\omega_c} d\omega\, \mathcal{T}_{\frac{m_s}{\omega}}[\rho_\Pi(\omega)]\right] = \frac{N_c\omega_c^3}{6\pi^2}\left[1 + \frac{3m_s}{2\omega_c} - \frac{3m_s^2}{2\omega_c^2} - \frac{m_s^3}{\omega_c^3}\left(1 - \frac{3}{4}\ln\frac{m_s^2}{\omega_c^2}\right) + \ldots\right].$$

Results for CKM elements

$$\begin{aligned} |V_{ts}|_{\text{SR}} &= (40.74^{+1.30}_{-1.21}) \cdot 10^{-3} \\ &= (40.74^{+1.29}_{-1.20}\,(\text{had.})\,^{+0.09}_{-0.14}\,(\mu) \pm 0.05\,(\text{param.})) \cdot 10^{-3}\,, \\ |V_{td}|_{\text{SR}} &= (8.36^{+0.26}_{-0.24}) \cdot 10^{-3} \\ &= (8.36^{+0.26}_{-0.24}\,(\text{had.})\,^{+0.02}_{-0.03}\,(\mu) \pm 0.02\,(\text{param.})) \cdot 10^{-3}\,. \end{aligned}$$

$$\begin{aligned} |V_{ts}|_{\text{CKMfitter}} &= (41.69^{+0.28}_{-1.08}) \cdot 10^{-3} \\ |V_{td}|_{\text{CKMfitter}} &= (8.710^{+0.086}_{-0.246}) \cdot 10^{-3}\,. \end{aligned}$$

$$\begin{aligned} |V_{ts}|_{\text{CKMfitter, tree}} &= (41.63^{+0.39}_{-1.45}) \cdot 10^{-3} \\ |V_{td}|_{\text{CKMfitter, tree}} &= (9.08^{+0.23}_{-0.45}) \cdot 10^{-3}\,. \end{aligned}$$

$$|V_{td}/V_{ts}|_{\text{SR}} = 0.2045^{+0.0012}_{-0.0013} = 0.2045^{+0.0011}_{-0.0012}\,(\text{had.}) \pm 0.0004\,(\text{exp.})\,,$$

$$|V_{td}/V_{ts}| = 0.2052 \pm 0.0033 \quad [\text{FNAL/MILC'16}]\,,$$
$$|V_{td}/V_{ts}| = 0.2018^{+0.0020}_{-0.0027} \quad [\text{RBC-UKQCD'18}]\,.$$

$$|V_{td}/V_{ts}| = 0.2088^{+0.0016}_{-0.0030} \quad [\text{CKMfitter}]\,,$$
$$|V_{td}/V_{ts}| = 0.211 \pm 0.003 \quad [\text{UTfit}]\,,$$

Individual errors for the Bag parameters of the $\Delta B = 2$ matrix elements

$\Delta B = 2$	$\overline{\Lambda}$	intrinsic SR	condensates	μ_ρ	$1/m_b$	μ_m	a_i
$\overline{B}_{Q_1}$	$^{+0.001}_{-0.002}$	±0.018	±0.004	$^{+0.011}_{-0.022}$	±0.010	$^{+0.045}_{-0.039}$	$^{+0.007}_{-0.007}$
$\overline{B}_{Q_2}$	$^{+0.014}_{-0.017}$	∓0.020	±0.004	$^{+0.012}_{-0.019}$	±0.010	$^{+0.071}_{-0.062}$	$^{+0.015}_{-0.015}$
$\overline{B}_{Q_3}$	$^{+0.060}_{-0.074}$	±0.107	±0.023	$^{+0.016}_{-0.008}$	±0.010	$^{+0.086}_{-0.069}$	$^{+0.053}_{-0.052}$
$\overline{B}_{Q_4}$	$^{+0.007}_{-0.006}$	±0.021	±0.011	$^{+0.003}_{-0.003}$	±0.010	$^{+0.088}_{-0.079}$	$^{+0.005}_{-0.006}$
$\overline{B}_{Q_5}$	$^{+0.019}_{-0.015}$	±0.018	±0.009	$^{+0.004}_{-0.006}$	±0.010	$^{+0.077}_{-0.068}$	$^{+0.012}_{-0.012}$

Individual errors for the Bag parameters of the $\Delta B = 0$ matrix elements

$\Delta B = 0$	$\overline{\Lambda}$	intrinsic SR	condensates	μ_ρ	$1/m_b$	μ_m	a_i
$\overline{B}_1$	$^{+0.003}_{-0.002}$	±0.019	±0.002	$^{+0.002}_{-0.002}$	±0.010	$^{+0.060}_{-0.052}$	$^{+0.002}_{-0.003}$
$\overline{B}_2$	$^{+0.001}_{-0.001}$	∓0.020	±0.002	$^{+0.000}_{-0.001}$	±0.010	$^{+0.084}_{-0.076}$	$^{+0.001}_{-0.002}$
$\overline{\epsilon}_1$	$^{+0.006}_{-0.007}$	±0.022	±0.003	$^{+0.003}_{-0.003}$	±0.010	$^{+0.010}_{-0.012}$	$^{+0.006}_{-0.007}$
$\overline{\epsilon}_2$	$^{+0.005}_{-0.006}$	±0.017	±0.003	$^{+0.002}_{-0.001}$	±0.010	$^{+0.001}_{-0.002}$	$^{+0.003}_{-0.004}$

Backup

	$\Delta M_s^{\rm SM}$ [ps^{-1}]	$\Delta\Gamma_s^{\rm PS}$ [ps^{-1}]	$\Delta M_d^{\rm SM}$ [ps^{-1}]	$\Delta\Gamma_d^{\rm SM}$ [10^{-3}ps^{-1}]
$\overline{B}^q_{Q_1}$	±1.1	±0.005	±0.031	$^{+0.16}_{-0.15}$
$\overline{B}^q_{Q_3}$	±0.0	$^{+0.006}_{-0.005}$	±0.000	$^{+0.17}_{-0.16}$
$\overline{B}^q_{R_0}$	±0.0	±0.004	±0.000	±0.10
$\overline{B}^q_{R_1}$	±0.0	±0.000	±0.000	±0.01
$\overline{B}^q_{R'_1}$	±0.0	±0.000	±0.000	±0.01
$\overline{B}^q_{R_2}$	±0.0	±0.018	±0.000	±0.53
$\overline{B}^q_{R_3}$	±0.0	±0.000	±0.000	±0.00
$\overline{B}^q_{R'_3}$	±0.0	±0.000	±0.000	±0.01
f_{B_q}	±0.2	±0.001	$^{+0.008}_{-0.007}$	±0.04
μ_1	±0.0	$^{+0.008}_{-0.021}$	±0.000	$^{+0.24}_{-0.60}$
μ_2	±0.1	$^{+0.000}_{-0.003}$	$^{+0.004}_{-0.002}$	$^{+0.00}_{-0.08}$
m_b	±0.0	$^{+0.000}_{-0.001}$	±0.000	$^{+0.01}_{-0.04}$
m_c	±0.0	±0.001	±0.000	±0.02
α_s	±0.0	±0.000	±0.001	±0.01
CKM	$^{+0.3}_{-1.0}$	$^{+0.001}_{-0.005}$	$^{+0.011}_{-0.032}$	$^{+0.06}_{-0.15}$

Individual errors for the ratio $\tau(B^+)/\tau(B^0)$ in the PS mass scheme

$\overline{B}_1$	$\overline{B}_2$	$\bar{\epsilon}_1$	$\bar{\epsilon}_2$	ρ_3	ρ_4	σ_3	σ_4
± 0.002	± 0.000	$^{+0.016}_{-0.015}$	± 0.004	± 0.001	± 0.000	± 0.013	± 0.000

f_B	μ_1	μ_0	m_b	m_c	α_s	CKM
$^{+0.004}_{-0.003}$	$^{+0.000}_{-0.013}$	$^{+0.000}_{-0.006}$	$^{+0.000}_{-0.001}$	± 0.000	± 0.002	± 0.006

Individual errors for the ratio $\tau(D^+)/\tau(D^0)$ in the PS mass scheme

$\overline{B}_1$	$\overline{B}_2$	$\bar{\epsilon}_1$	$\bar{\epsilon}_2$	ρ_3	ρ_4	σ_3	σ_4
$^{+0.07}_{-0.05}$	± 0.00	$^{+0.52}_{-0.47}$	± 0.017	± 0.05	± 0.00	± 0.46	± 0.00

f_B	μ_1	μ_0	m_c	m_s	α_s	CKM
± 0.08	$^{+0.07}_{-0.40}$	$^{+0.08}_{-0.21}$	± 0.08	± 0.00	$^{+0.07}_{0.06}$	± 0.00