Critical behavior of the QED_3-Gross-Neveu-Yukawa model at four loops

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DESY Zeuthen 2019

[arXiv:1808.00549]

Overview

Motivation

- Obtain reliable & precise predictions
- In HEP the SM (QFT with $\mathcal{L}_{\text{SM}}$) provides collider observables
- New physics $\mathcal{L}_{\text{“real”}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_X$?
- New physics includes realization of $SUSY$? (MSSM, nMSSM, ...)
- Origin of $\mathcal{L}_{\text{SM}}$?
- Origin of QFTs?

How to proceed?

- Build and run LHC to find new physics
- Calculate observables at higher order in PT
- Investigate on various QFTs (maybe no HEP relevancy)

Work in collaboration with...

- *University of Alberta (Edmonton)*
  Joseph Maciejko
  Rufus Boyack
- *DESY Zeuthen*
  Peter Marquard

The chiral Ising QED$_3$-Gross-Neveu-Yukawa Model

- chiral Ising Gross-Neveu-model:

$$\mathcal{L}_{\mathsf{GN}} = \bar{\Psi} \cdot \partial\!\!\!/ \Psi + \tfrac{1}{2} g^2 (\bar{\Psi} \cdot \Psi)^2 \,.$$

  - Ψ: Fermion with N flavours (closed fermion loops $\sim N$)
  - 4-Fermion interaction of Ising type (real singlet)
  - Lorenzian Symmetry
  - Renormalizable in $d = 2$

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- chiral Ising Gross-Neveu-Yukawa-model:

$$\begin{aligned}\mathcal{L}_{\text{GNY}} =& \bar{\Psi} \cdot \partial\!\!\!/ \Psi + g\phi \bar{\Psi} \cdot \Psi \\ &+ \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 + \frac{1}{4!} \lambda^2 \phi^4 \,.\end{aligned}$$

  - Lorenzian Symmetry
  - additional real scalar field ϕ
  - UV complete in $d = 4$ (renormalizable)

The chiral Ising QED_3-Gross-Neveu-Yukawa Model

- QED_3:

$$\mathcal{L}_{\text{QED}} = \bar{\Psi} \cdot \gamma_\mu (\partial_\mu - ieA_\mu)\Psi + \frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A_\mu)^2 \,.$$

The chiral Ising QED$_3$-Gross-Neveu-Yukawa Model

- The chiral Ising QED$_3$-Gross-Neveu-Yukawa model:

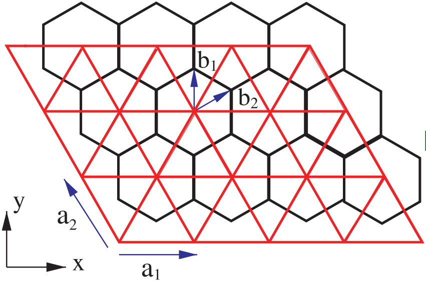
$$\begin{aligned}\mathcal{L}_{\text{QED}_3\text{GNY}} =& \bar{\Psi} \cdot \not{D} \Psi + g\phi\bar{\Psi} \cdot \Psi \\ &+ \tfrac{1}{2}(\partial_\mu \phi)^2 + \tfrac{1}{2} m^2 \phi^2 + \tfrac{1}{4!}\lambda^2 \phi^4 \\ &+ \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A_\mu)^2 \,.\end{aligned}$$

  - Ψ: four-component fermion with N flavours
  - UV complete in $d = 4$ (renormalizable)
  - discrete, chiral Z_2 symmetry:

$$\Psi \to \gamma_5 \Psi, \qquad \bar{\Psi} = \Psi^\dagger \gamma_0 \to -\bar{\Psi}\gamma_5, \qquad \phi \to -\phi \,.$$

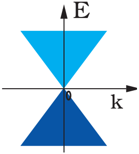
Where to find such a QFT?

- In a condensed matter systems like in layers of graphene (d=2+1)



[S.-S.Lee'07]

- Appearance as LowEnergyTheory in a mean field approximation
- Lorenzian Symmetry emergent at IR fixed point
- Band structure yields a linear fermionic dispersion relation at the Dirac-point:



The relevant physical Observables

- Critical exponents and Anomalous Dimenions at the non-trivial QuantumCricticalPoint in $d = 2 + 1$:

$$
\begin{aligned}
\eta_\phi &\equiv \gamma_\phi(e_*^2, g_*^2, \lambda_*^2), \\
\eta_{\phi^2} &\equiv \gamma_{\phi^2}(e_*^2, g_*^2, \lambda_*^2), \\
\eta_A &\equiv \gamma_A(e_*^2, g_*^2, \lambda_*^2), \\
\omega &\equiv \dots \\
\nu^{-1} &\equiv \dots \\
&\dots
\end{aligned}
$$

where $e_*, g_*, \lambda_* \neq 0$ holds

- In $d = 2 + 1$: Strong dynamics

Methods

- FunctionalRenormalizationGroup
- MonteCarlo
- conformal Boot Strap
- Large N approximation
- Weak coupling expansion (PT) around $d = 4 - \epsilon$

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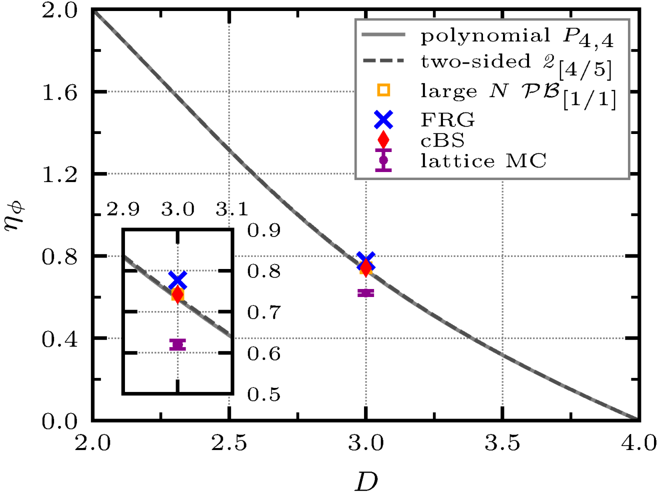
Example: chiral Ising Gross-Neveu UC

- η_ϕ for $N = 8$ [Ihrig,Mihaila,Scherer'18]:



Renormalization of χ_I-QED$_3$-GNY

- $$\begin{aligned}\mathcal{L}_{\mathrm{QED_3GNY}} =& \bar\Psi \cdot \not{D}\Psi + g\phi\bar\Psi\cdot\Psi \\ &+ \tfrac{1}{2}(\partial_\mu\phi)^2 + \tfrac{1}{2}m^2\phi^2 + \tfrac{1}{4!}\lambda^2\phi^4 \\ &+ \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A_\mu)^2\,.\end{aligned}$$

Renormalization of χ_I-QED$_3$-GNY

$$\mathcal{L}^0_{\mathrm{QED_3GNY}} = \bar{\Psi}^0 \cdot \not{D}^0 \Psi^0 + g^0 \phi^0 \bar{\Psi}^0 \cdot \Psi^0 + \tfrac{1}{2}(\partial_\mu \phi^0)^2 + \tfrac{1}{2}(m^2)^0 (\phi^0)^2 + \tfrac{1}{4!}(\lambda^0)^2 (\phi^0)^4 + \frac{1}{4} F^0_{\mu\nu} (F^0)^{\mu\nu} + \frac{1}{2\xi^0} (\partial_\mu A^0_\mu)^2 \,.$$

Renormalization of χ_I-QED$_3$-GNY

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- Field and coupling redefinitions (using $\overline{\text{MS}}$ and RS$=\mu$):

$$\phi^0 = \sqrt{Z_\phi}\phi\,, \qquad \Psi^0 = \sqrt{Z_\Psi}\Psi\,, \qquad A^0 = \sqrt{Z_A}A\,,$$
$$\lambda^0 = \mu^{\epsilon/2} Z_\lambda \lambda\,, \qquad g^0 = \mu^{\epsilon/2} Z_g g\,, \qquad e^0 = \mu^{\epsilon/2} Z_e e\,,$$
$$(m^2)^0 = Z_{m^2} m^2 \mu^2\,, \qquad \xi^0 = Z_\xi \xi\,.$$

Renormalization of χ_I-QED$_3$-GNY

$$\begin{aligned}\mathcal{L}^0_{\mathrm{QED_3GNY}} =& \bar\Psi^0 \cdot \not{D}^0 \Psi^0 + g^0 \phi^0 \bar\Psi^0 \cdot \Psi^0 \\ & + \tfrac{1}{2}(\partial_\mu \phi^0)^2 + \tfrac{1}{2}(m^2)^0 (\phi^0)^2 + \tfrac{1}{4!}(\lambda^0)^2 (\phi^0)^4 \\ & + \frac{1}{4} F^0_{\mu\nu} (F^0)^{\mu\nu} + \frac{1}{2\xi^0} (\partial_\mu A^0_\mu)^2 \,.\end{aligned}$$

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- Use short hands:

$$Z_{\phi^4} = Z_\lambda^2 Z_\phi^2\,, \quad Z_{\bar\Psi\Psi\phi} = Z_g Z_\Psi \sqrt{Z_\phi}\,, \quad Z_{\bar\Psi\Psi A} = Z_e Z_\Psi \sqrt{Z_A}\,, \quad Z_{\phi^2} = Z_{m^2} Z_\phi\,.$$

Renormalization of χ_I-QED$_3$-GNY

- $$\begin{aligned}\mathcal{L}^0_{\text{QED}_3\text{GNY}} =& Z_\Psi \bar\Psi \cdot \partial\!\!\!/ \Psi + i\mu^{\epsilon/2} Z_{\bar\Psi\Psi A} e \bar\Psi \cdot A\!\!\!/ \Psi + \mu^{\epsilon/2} Z_{\bar\Psi\Psi\phi} g \phi \bar\Psi \cdot \Psi \\ &+ Z_\phi \tfrac{1}{2}(\partial_\mu \phi)^2 + Z_{\phi^2} \tfrac{1}{2} m^2 \mu^2 \phi^2 + \mu^\epsilon Z_{\phi^4} \tfrac{1}{4!} \lambda^2 \phi^4 \\ &+ Z_A \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A_\mu)^2 \,.\end{aligned}$$

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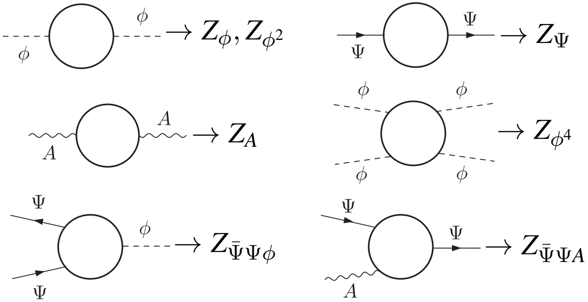
How to obtain Zs?

- Using DREG get poles in small $\epsilon = 4 - d$ of 1-PI n-point functions at $L = 1, 2, 3, 4$ loops for “any” kinematics



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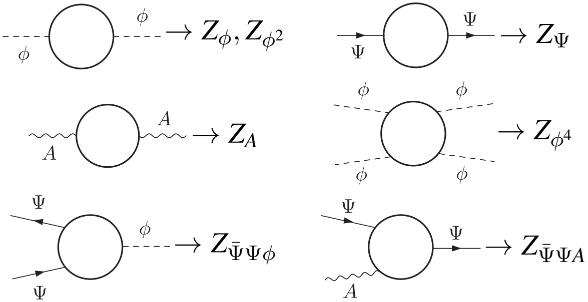
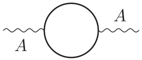
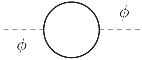
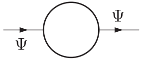
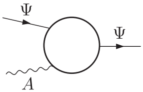
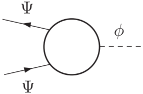
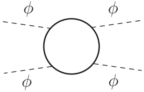


- Further we obtain $Z_{m_{\Psi,1}}$ and $Z_{m_{\Psi,A}}$
  flavour singlet & adjoint “mass” term:

$$\mathcal{L}^0_{m_1} = \mu Z_{\bar\Psi 1 \Psi} m_1 \bar\Psi \cdot 1 \cdot \Psi\,, \qquad \mathcal{L}^0_{m_{T_A}} = \mu Z_{\bar\Psi T_A \Psi} m_{T_A} \bar\Psi \cdot T_A \cdot \Psi\,.$$

How to obtain Zs?

- Number of Feynman diagrams:

Loops	1	2	3	4
	1	6	83	1610
	2	9	99	1808
	2	13	177	3387
	2	37	844	22818
	2	38	876	23767
	9	153	4248	138849

How to obtain Zs? (technics)

- Chosing “massiv tadpole kinematics”
  - Use single IR regulator mass M in every denominator
  - Expand in small external momenta $|p_i|/M \ll 1$
  - All integrals are 1-scale tadpole integrals
  - Use infrared rearrangement [Chetyrkin,Misiak,Munz] to consistently subtract artificial terms $\sim M^2$

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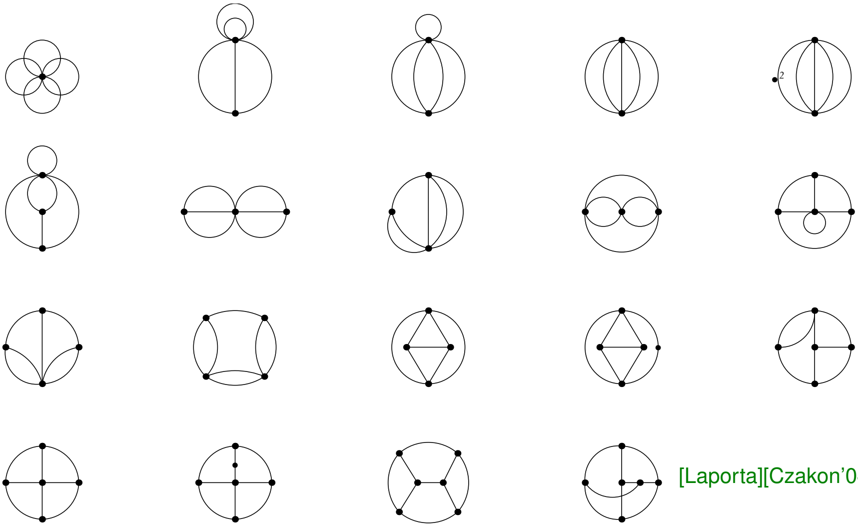
- Fully automatized setup
  - *QGRAF* [Nogueira]
  - *q2e* [Seidensticker]
  - *exp* [Seidensticker]
  - *FORM* [Vermaseren & Co]
  - *CRUSHER* [Marquard,Seidel]

19 Fully Massive Master Integrals



[Laporta][Czakon'04]

β- & γ- Functions

- Bare parameters do not depend on RS μ

$\Rightarrow$ $\beta_{g^2} = \frac{d\, g^2}{d \ln \mu} \quad \beta_{\lambda^2} = \frac{d\lambda^2}{d \ln \mu} \quad \beta_{e^2} = \frac{de^2}{d \ln \mu}$

$$\begin{aligned}
\beta_{g^2} &= -\,\epsilon g^2 + \beta_{g^2}^{(1L)} + \beta_{g^2}^{(2L)} + \beta_{g^2}^{(3L)} + \beta_{g^2}^{(4L)}\,, \\
\beta_{\lambda^2} &= -\,\epsilon \lambda^2 + \beta_{\lambda}^{(1L)} + \beta_{\lambda}^{(2L)} + \beta_{\lambda}^{(3L)} + \beta_{\lambda}^{(4L)}\,, \\
\beta_{e^2} &= -\,\epsilon e^2 + \beta_{e^2}^{(1L)} + \beta_{e^2}^{(2L)} + \beta_{e^2}^{(3L)} + \beta_{e^2}^{(4L)}\,,
\end{aligned}$$

- $\gamma_x = d \ln Z_x / d \ln \mu \qquad \forall x \in \{A, \phi, \phi^2, \dots\}$

$$\begin{aligned}
\gamma_A &= \gamma_A^{(1L)} + \gamma_A^{(2L)} + \gamma_A^{(3L)} + \gamma_A^{(4L)}\,, \\
\gamma_\phi &= \gamma_\phi^{(1L)} + \gamma_\phi^{(2L)} + \gamma_\phi^{(3L)} + \gamma_\phi^{(4L)}\,, \\
\gamma_{\phi^2} &= \gamma_{\phi^2}^{(1L)} + \gamma_{\phi^2}^{(2L)} + \gamma_{\phi^2}^{(3L)} + \gamma_{\phi^2}^{(4L)}\,.
\end{aligned}$$

- QED Ward-Identity: $\beta_{e^2} = (-\epsilon + \gamma_A) e^2\,.$

Showcasing β_s- & γ_s

$$
\begin{aligned}
\beta_{e^2}^{(1\mathrm{L})} &= \frac{8N}{3} e^4 \,, \\
\beta_{g^2}^{(1\mathrm{L})} &= -12 e^2 g^2 + 2(2N+3) g^4 \,, \\
\beta_{\lambda^2}^{(1\mathrm{L})} &= -2N g^4 + 8N g^2 \lambda^2 + 72 \lambda^4 \,, \\
\gamma_{\phi}^{(1\mathrm{L})} &= 4N g^2, \\
\gamma_{\phi^2}^{(1\mathrm{L})} &= -24 \lambda^2 .
\end{aligned}
$$

Showcasing β_s- & γ_s

$$
\begin{aligned}
\beta^{(2\mathrm{L})}_{e^2} &= 8Ne^6 - 4Ne^4g^2\,,\\
\beta^{(2\mathrm{L})}_{g^2} &= -\left(24N + \frac{9}{2}\right)g^6 + \left(\frac{40N}{3} - 6\right)e^4g^2\\
&\quad + 4(5N+12)e^2g^4 - 96g^4\lambda^2 + 96g^2\lambda^4,\\
\beta^{(2\mathrm{L})}_{\lambda^2} &= 16Ng^6 + 28Ng^4\lambda^2 - 288Ng^2\lambda^4 - 3264\lambda^6\\
&\quad - 8Ne^2g^4 + 40Ne^2g^2\lambda^2,\\
\gamma^{(2\mathrm{L})}_{\phi} &= 20Ne^2g^2 - 10Ng^4 + 96\lambda^4,\\
\gamma^{(2\mathrm{L})}_{\phi^2} &= -8Ng^4 + 96Ng^2\lambda^2 + 576\lambda^4.
\end{aligned}
$$

Showcasing β_s- & γ_s

$$
\begin{aligned}
\beta_{e^2}^{(3\mathrm{L})} &= -6Ne^6g^2 + 2N(7N+6)e^4g^4 - \frac{4N}{9}(22N+9)e^8\,,\\
\beta_{\lambda^2}^{(3\mathrm{L})} &= -\frac{N}{4}(628N - 5 + 384\zeta_3)g^8\\
&\quad + \frac{N}{2}(1736N - 4395 - 1872\zeta_3)g^6\lambda^2\\
&\quad + 12N(-72N + 361 + 648\zeta_3)g^4\lambda^4\\
&\quad + 12384Ng^2\lambda^6 + 1728(145+96\zeta_3)\lambda^8\\
&\quad + N(116N + 131 - 96\zeta_3)e^4g^4\\
&\quad - 2N(32N + 119 - 144\zeta_3)e^4g^2\lambda^2\\
&\quad + 2N(-11 + 96\zeta_3)e^2g^6 + 6N(217 - 304\zeta_3)e^2g^4\lambda^2\\
&\quad + 216N(-17 + 16\zeta_3)e^2g^2\lambda^4\,,\\
\beta_{g^2}^{(3\mathrm{L})} &= \left[-32N^2 + N(49 - 432\zeta_3) + \frac{327}{2} - 504\zeta_3\right]e^4g^4\\
&\quad + \left[\frac{560N^2}{27} + 8N(23 - 24\zeta_3) - 258\right]e^6g^2\\
&\quad + \left[28N^2 + N\left(\frac{67}{4} + 108\zeta_3\right) - \frac{697}{8} + 114\zeta_3\right]g^8\\
&\quad + 144(5N+7)g^6\lambda^2 + 24(-30N+91)g^4\lambda^4\\
&\quad - 1728g^2\lambda^6 + [2N(-79+48\zeta_3) - 348 + 72\zeta_3]e^2g^6\\
&\quad + 96e^2g^4\lambda^2\,.
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{\phi}^{(3\mathrm{L})} &= -3N(7+16\zeta_3)e^2g^4 + \frac{N}{4}(200N + 21 + 48\zeta_3)g^6\\
&\quad + N(-32N - 119 + 144\zeta_3)e^4g^2 + 240Ng^4\lambda^2\\
&\quad - 720Ng^2\lambda^4 - 1728\lambda^6.
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{\phi^2}^{(3\mathrm{L})} &= -32N(4N - 9 + 3\zeta_3)g^6\\
&\quad + 12N(24N - 11 - 120\zeta_3)g^4\lambda^2 - 2304Ng^2\lambda^4\\
&\quad - 50112\lambda^6 + 32N(-7 + 9\zeta_3)e^2g^4\\
&\quad + 72N(17 - 16\zeta_3)e^2g^2\lambda^2.
\end{aligned}
$$

Showcasing β_s- & γ_s

$$
\begin{aligned}
\beta_{e^2}^{(4L)} &= \frac{2N}{3}[N(267-432\zeta_3)+43-336\zeta_3]e^6g^4 + \frac{2N}{9}(322N-27+648\zeta_3)e^8g^2 - 240Ne^4g^4\lambda^2 + 576Ne^4g^2\lambda^4 \\
&\quad - \frac{4N}{243}[616N^2+36N(-95+312\zeta_3)+5589]e^{10} - \frac{4N}{9}[27N^2+N(299-24\zeta_3)+105+9\zeta_3]e^4g^6\,,\\
\beta_{\lambda^2}^{(4L)} &= \frac{N}{24}\Big[64N^2(-193+252\zeta_3)-32N(-1289+540\zeta_3-288\zeta_4-1920\zeta_5)+3(-4473+8864\zeta_3-3696\zeta_4+10400\zeta_5)\Big] \\
&\quad \times g^{10} - \frac{N}{3}\Big[4N^2(-1685+2736\zeta_3)+N(-69220-49872\zeta_3+32400\zeta_4+40320\zeta_5)-3(9745+18708\zeta_3-984\zeta_4 \\
&\quad +12560\zeta_5)\Big]g^8\lambda^2 + N[2304N^2(-1+2\zeta_3)-8N(15649+8784\zeta_3-3888\zeta_4)+211565+7296\zeta_3+40176\zeta_4 \\
&\quad +167040\zeta_5]g^6\lambda^4 + 64N[6N(263+72\zeta_3)-14521-7452\zeta_3-5184\zeta_4-17280\zeta_5]g^4\lambda^6 \\
&\quad -576N(1355+3456\zeta_3-1728\zeta_4)g^2\lambda^8 - 6912(3499+3744\zeta_3-864\zeta_4+5760\zeta_5)\lambda^{10} \\
&\quad -\frac{4N}{243}[16N^2(-125+324\zeta_3)-81N(185+16\zeta_3-240\zeta_4)-81(-1471+72\zeta_3+432\zeta_4+240\zeta_5)]e^6g^4 \\
&\quad +N\Big[\frac{32N^2}{243}(-1625+1296\zeta_3)+8N(-125+16\zeta_3+96\zeta_4)+\frac{9302}{3}+1856\zeta_3-864\zeta_4-2880\zeta_5\Big]e^6g^2\lambda^2 \\
&\quad -\frac{N}{6}[8N(-275+680\zeta_3+336\zeta_4+1920\zeta_5)+3(1747-8928\zeta_3+5040\zeta_4+3680\zeta_5)]e^4g^6 \\
&\quad +\frac{2N}{9}[2N(-26873+18912\zeta_3-144\zeta_4+5760\zeta_5)-9(7459+15272\zeta_3-7920\zeta_4-12000\zeta_5)]e^4g^4\lambda^2 \\
&\quad -12N[4N(-155+64\zeta_3-48\zeta_4)+3(-479-1056\zeta_3+432\zeta_4+960\zeta_5)]e^4g^2\lambda^4 \\
&\quad -\frac{N}{3}[3N(335+2464\zeta_3-1200\zeta_4)+4(845+3246\zeta_3-1566\zeta_4+2220\zeta_5)]e^2g^8 \\
&\quad +\frac{N}{6}[16N(2725+5376\zeta_3-2736\zeta_4)+30419+138720\zeta_3-24624\zeta_4-30240\zeta_5]e^2g^6\lambda^2 \\
&\quad -8N[9N(199+288\zeta_3-144\zeta_4)+19661-27216\zeta_3+1080\zeta_4+6480\zeta_5]e^2g^4\lambda^4 \\
&\quad +288N(1109-1104\zeta_3)e^2g^2\lambda^6.\,,\\
\beta_{g^2}^{(4L)} &= \Big[\frac{88N^3}{3}-\frac{2N^2}{3}(899+1500\zeta_3-324\zeta_4)+N\left(\frac{9907}{4}-2648\zeta_3+552\zeta_4-1680\zeta_5\right)+\frac{30529}{32}+10\zeta_3+342\zeta_4 \\
&\quad -1720\zeta_5\Big]g^{10} + \Big[\frac{32N^3}{81}(83-144\zeta_3)+\frac{64N^2}{27}(-19+270\zeta_3-162\zeta_4)+N\left(\frac{352}{3}-288\zeta_3+1920\zeta_5\right)+\frac{1261}{2} \\
&\quad +1344\zeta_3\Big]e^8g^2 + \Big[\frac{16N^3}{243}(-1625+1296\zeta_3)+\frac{4N^2}{81}(-27739+35856\zeta_3-7776\zeta_4) \\
&\quad +N\left(\frac{35}{3}-\frac{9856\zeta_3}{3}+912\zeta_4+10080\zeta_5\right)+\frac{9899}{2}-7464\zeta_3+1512\zeta_4+10800\zeta_5\Big]e^6g^4 \\
&\quad +2[96N^2-8N(683+648\zeta_3)-3(943+1008\zeta_3)]g^8\lambda^2 - 8[24N^2+4N(635-324\zeta_3)+135(33+40\zeta_3)]g^6\lambda^4 \\
&\quad +576(8N-455+144\zeta_3)g^4\lambda^6+224640g^2\lambda^8+\Big[N^2\left(-\frac{1022}{9}+928\zeta_3+64\zeta_4-640\zeta_5\right) \\
&\quad +N\left(-\frac{38065}{9}+\frac{22264\zeta_3}{3}-528\zeta_4+1920\zeta_5\right)-\frac{16949}{4}+6024\zeta_3-432\zeta_4+2640\zeta_5\Big]e^4g^6 \\
&\quad +\left(\frac{14944N}{3}-456+5760\zeta_3\right)e^4g^4\lambda^2+\Big[N^2(216-640\zeta_3+96\zeta_4)+N\left(\frac{27133}{12}+2056\zeta_3-756\zeta_4-1400\zeta_5\right) \\
&\quad +\frac{19659}{8}+3386\zeta_3-918\zeta_4-1460\zeta_5\Big]e^2g^8+16[N(250-432\zeta_3)+291+360\zeta_3]e^2g^6\lambda^2 \\
&\quad +24[6N(-67+48\zeta_3)-815]e^2g^4\lambda^4.
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{\phi}^{(4L)} &= \frac{N}{243}[16N^2(-1625+1296\zeta_3)+972N(-125+16\zeta_3+96\zeta_4)+81(4651+2784\zeta_3-1296\zeta_4-4320\zeta_5)]e^6g^2 \\
&\quad -\frac{2N}{3}[N^2(-101+144\zeta_3)+N(-211+636\zeta_3+108\zeta_4)-435+369\zeta_3+90\zeta_4+120\zeta_5]g^8 \\
&\quad +\frac{N}{9}[N(-4570+3264\zeta_3+4896\zeta_4-5760\zeta_5)-9(673+1064\zeta_3-1008\zeta_4+480\zeta_5)]e^4g^4 \\
&\quad -16N(76N+249-48\zeta_3)g^6\lambda^2-64N(3N+182-162\zeta_3)g^4\lambda^4+4608Ng^2\lambda^6+224640\lambda^8 \\
&\quad +\frac{N}{12}[32N(161+96\zeta_3-72\zeta_4)+11363+2784\zeta_3-3888\zeta_4-1440\zeta_5]e^2g^6-944Ne^2g^4\lambda^2 \\
&\quad +144N(-67+48\zeta_3)e^2g^2\lambda^4\,,
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{\phi^2}^{(4L)} &= \frac{N}{2}[64N^2(-11+18\zeta_3)+8N(-651+40\zeta_3+108\zeta_4+560\zeta_5)-1423-2688\zeta_3-2016\zeta_4+5040\zeta_5]g^8 \\
&\quad -3N[256N^2(-1+2\zeta_3)-8N(809+336\zeta_3-240\zeta_4)+12989-5120\zeta_3+3312\zeta_4-5760\zeta_5]g^6\lambda^2 \\
&\quad -96N[16N(11+3\zeta_3)-949-1440\zeta_3-216\zeta_4]g^4\lambda^4+576N(313+96\zeta_3)g^2\lambda^6+27648(187+18\zeta_3+36\zeta_4)\lambda^8 \\
&\quad -\frac{2N}{3}[4N(-683+480\zeta_3-72\zeta_4+240\zeta_5)-3393-7104\zeta_3+3456\zeta_4+6240\zeta_5]e^4g^4 \\
&\quad +4N[4N(-155+64\zeta_3-48\zeta_4)+3(-479-1056\zeta_3+432\zeta_4+960\zeta_5)]e^4g^2\lambda^2 \\
&\quad -4N[3N(89+192\zeta_3-96\zeta_4)-177+1808\zeta_3-720\zeta_4-200\zeta_5]e^2g^6 \\
&\quad +8N[N(597+864\zeta_3-432\zeta_4)+3019-2928\zeta_3-504\zeta_4+1200\zeta_5]e^2g^4\lambda^2+1152N(-49+48\zeta_3)e^2g^2\lambda^4.
\end{aligned}
$$

Determination of e_*, g_*, λ_*

IR fixed point is reached for $\mu \downarrow$:

$$\{\beta_{e^2}, \beta_{g^2}, \beta_{\lambda^2}\}^T = \vec{0}\,.$$

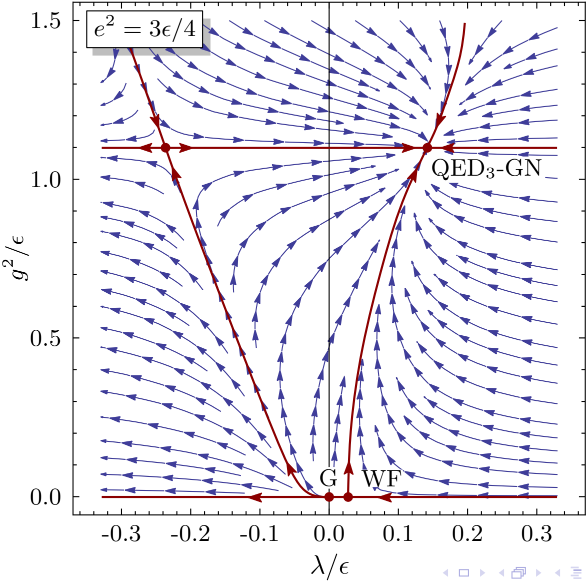
Solve for zeros in $\{e, g, \lambda\}$ loop by loop in dep. of ϵ

χ_I-QED$_3$-GNY fixed point (1-Loop):

$$e_*^2 = \frac{3}{8N}\epsilon + \mathcal{O}(\epsilon^2),$$
$$g_*^2 = \frac{2N+9}{4N(2N+3)}\epsilon + \mathcal{O}(\epsilon^2),$$
$$\lambda_*^2 = \frac{-2N-15+X}{144N(2N+3)}\epsilon + \mathcal{O}(\epsilon^2)\,,$$
$$X \equiv \sqrt{4N^4 + 204N^3 + 1521N^2 + 2916N}\,.$$

Determination of e_*, g_*, λ_*

$N = 1$: [Janssen,He'17]



Critical Exponents

- inverse correlation length exponent ν^{-1}: RG eigenvalue of (relevant) scalar mass term:

$$\nu^{-1} = -\left.\frac{d\ln(m^2)}{d\ln\mu}\right|_{(e_*^2,g_*^2,\lambda_*^2)} = 2 + \eta_{\phi^2} - \eta_\phi \,.$$

- stability exponent ω:

$$\omega = \min(\mathrm{EV}[\mathcal{J}])\,,$$

$$\mathcal{J} = \left.\begin{pmatrix} \frac{\partial\beta_{e^2}}{\partial e^2} & \frac{\partial\beta_{e^2}}{\partial g^2} & \frac{\partial\beta_{e^2}}{\partial\lambda^2} \\ \frac{\partial\beta_{g^2}}{\partial e^2} & \frac{\partial\beta_{g^2}}{\partial g^2} & \frac{\partial\beta_{g^2}}{\partial\lambda^2} \\ \frac{\partial\beta_{\lambda^2}}{\partial e^2} & \frac{\partial\beta_{\lambda^2}}{\partial g^2} & \frac{\partial\beta_{\lambda^2}}{\partial\lambda^2} \end{pmatrix}\right|_{(e_*^2,g_*^2,\lambda_*^2)} \,.$$

➡ stable FP/CP: $\omega > 0$.
unstable FP/CP: $\omega < 0$.

Quantum Phasetransitions

Phenomenology of Quantum Phasetransitions

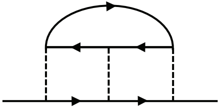
$x = (X - X_c)/X_c$

$$\xi \sim |x|^{-\nu}(1 + C|x|^{\omega} + ...)$$

- correlation length around Quantum Critical Point (QCP) ξ
- correlation length exponent ν
- subleading/stability exponent ω

Excursion: FS Fermion Bilinear

- During the calculation of fermion mass renormalization:



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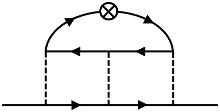

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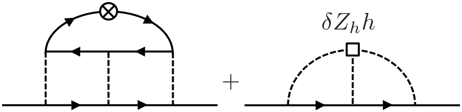

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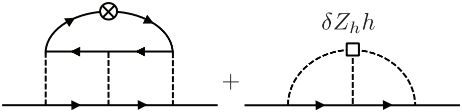

- $\mathcal{L}_{m_1}$ mixes (in $d = 4$) with: $\mathcal{L}_{\phi^3} = \frac{1}{3!} h\phi^3$

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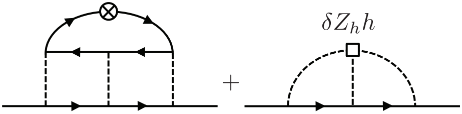

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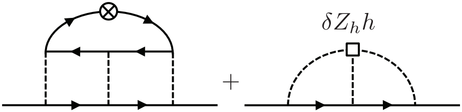

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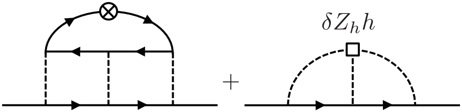

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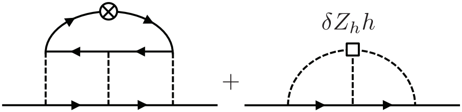

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- Coupled system for β_h and β_{m_1}
- Its Eigenvalues at fixed point reproduce corresponding large N results expanded up to $\mathcal{O}(\epsilon^4)$
- $\mathcal{L}_{m_A}$ does not mix ($\mathrm{tr}[T_A] = 0$)

Fermion Bilinear Scaling

- β-function for both masses and ϕ^3 coupling: $x \in \{1, T_A\}$

$$\beta_{m_x} = \frac{d\, m_x}{d \ln \mu} = (-1 - \gamma_{\bar{\Psi} x \Psi} + \gamma_\Psi) m_x \,,$$

$$\beta_h = \frac{d\, h}{d \ln \mu} = (-1 - \tfrac{1}{2}\epsilon - \gamma_{\phi^3} + \tfrac{3}{2}\gamma_\phi) h \,.$$

- FS operator mixing yields:

$$(\beta_{m_1}, \beta_h)^T = M \cdot (m_1, h)^T \,,$$

$$\Lambda_\pm = \tfrac{1}{2}(M_{11} + M_{22}) \pm \sqrt{\tfrac{1}{4}(M_{11} - M_{22})^2 + M_{12} M_{21}} \,.$$

- Scaling dimension Δ of $\bar{\Psi} x \Psi$ and ϕ^3 at fixed point:

$$\Delta_{\bar{\Psi} T_A \Psi} = d - 1 - \eta_{\bar{\Psi} T_A \Psi} + \eta_\Psi \,,$$

$$\Delta_{\bar{\Psi} 1 \Psi} = d + \Lambda_- \big|_{(e_*^2, g_*^2, \lambda_*^2)} \,,$$

$$\Delta_{\phi^3} = d + \Lambda_+ \big|_{(e_*^2, g_*^2, \lambda_*^2)} \,.$$

Checks

- Full agreement with indep. calc. at 1-Loop [Janssen,He'17] & 3-Loop [Ihrig,Mihaila,Scherer'18]
- $e = 0, g = 0 \to \phi^4$ Ising Universality Class [Vladimirov,Kazakov,Tarasov'79]
- $g = 0, \lambda = 0 \to$ QED [Gorishny,Kataev,Larin,Surguladze'91]
- $e = 0 \to \chi_I$ GNY [Zerf,Mihaila,Marquard,Herbut,Scherer,'17]
- large N calculations [Manashov,Strohmaier,18][Gracey'92'18]
- ξ dependence in γ_Ψ only linear and only in $\gamma_\Psi^{(1L)}$ [Gracey,'7][Kißler,Kreimer'17]

Results

➡ At 1-Loop: [Janssen,He'17]

$$\eta_\phi = \frac{2N+9}{2N+3}\epsilon + \mathcal{O}(\epsilon^2),$$

$$\nu^{-1} = 2 - \frac{10N^2+39N+X}{6N(2N+3)}\epsilon + \mathcal{O}(\epsilon^2),$$

$$\omega = \epsilon + \mathcal{O}(\epsilon^2),$$

$$\Delta_{\bar{\Psi}x\Psi} = 3 - \left(\frac{2N+6}{2N+3}\right)\epsilon + \mathcal{O}(\epsilon^2)\,.$$

Results

➡ At 4-Loop ($N = 1$ numeric rep.):

$$\begin{aligned}
\eta_\phi &\approx 2.2\epsilon - 0.2227\epsilon^2 + 16.88\epsilon^3 - 205.1\epsilon^4 + \mathcal{O}(\epsilon^5),\\
\nu^{-1} &\approx 2 - 3.905\epsilon + 7.471\epsilon^2 - 90.6\epsilon^3 + 1154\epsilon^4 + \mathcal{O}(\epsilon^5),\\
\omega &\approx \epsilon + 0.3\epsilon^2 + 4.294\epsilon^3 - 119.1\epsilon^4 + \mathcal{O}(\epsilon^5),\\
\Delta_{\bar\Psi 1 \Psi} &\approx 3 - 1.6\epsilon + 0.1114\epsilon^2 - 8.442\epsilon^3 + 102.5\epsilon^4 + \mathcal{O}(\epsilon^5),\\
\Delta_{\bar\Psi T_A \Psi} &\approx 3 - 1.6\epsilon + 1.987\epsilon^2 - 17.46\epsilon^3 + 215.7\epsilon^4 + \mathcal{O}(\epsilon^5).
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- naive extrapolation to $d = 3$ means $\epsilon = 1$!
- Use Padé / Padé-Borel approximants for extrapolation

Application

Conjectured duality in $d = 3$ at QCP [Wang,Nahum,Metlitski,Xu,Senthil'17]:

$$\chi_I\text{-QED}_3\text{-GNY}(N=1) \Leftrightarrow SU(2)\text{NCCP}^1 \,.$$

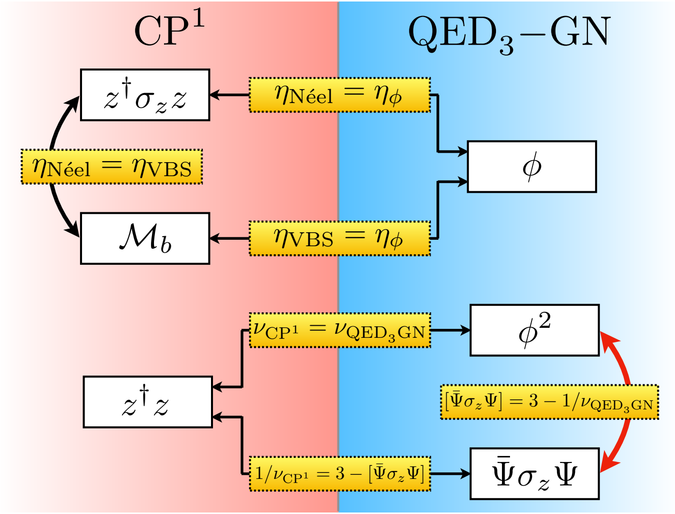
- $SU(2)\text{NCCP}^1$:

$$\mathcal{L}_{\text{CP}^1} = \sum_{\alpha=1,2} |D_b z_\alpha|^2 - (|z_1|^2 + |z_2|^2)^2 \,.$$

  - z_α: complex bosonic "spinon" field with $SU(2)$ flavour
  - D_b: covariant derivative containing NC $U(1)$ gauge field b^μ
  - EFT for Néel $\leftrightarrow$ VBS phase transitions by spin 1/2 "magnets" on square lattice

Emergent $SO(5)$ symmetry at the QCP
(seen in num. calc. [Nahum,Serna,Chalker,.'15])

Web of Dualities



[Ihrig,Mihaila,Scherer'18]

Web of Dualities

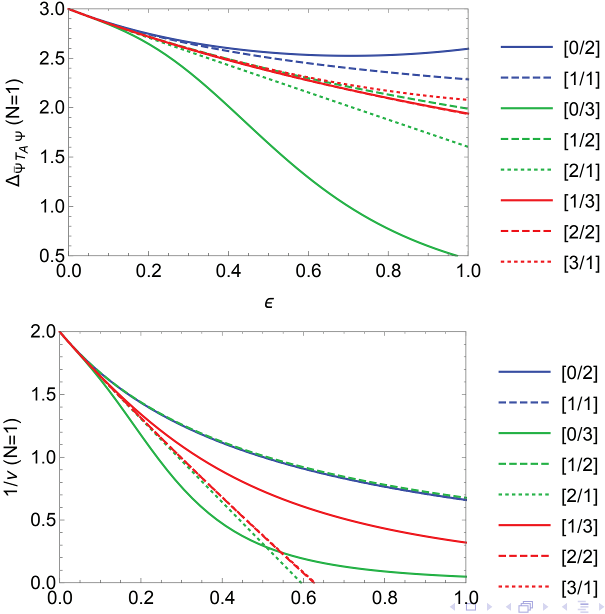
- Duality/$SO(5)$ symmetry implies relation within χ_I-QED$_3$-GNY$(N=1)$:

$$\Delta_{\bar{\Psi}T_A\Psi} = 3 - \nu^{-1}\,.$$

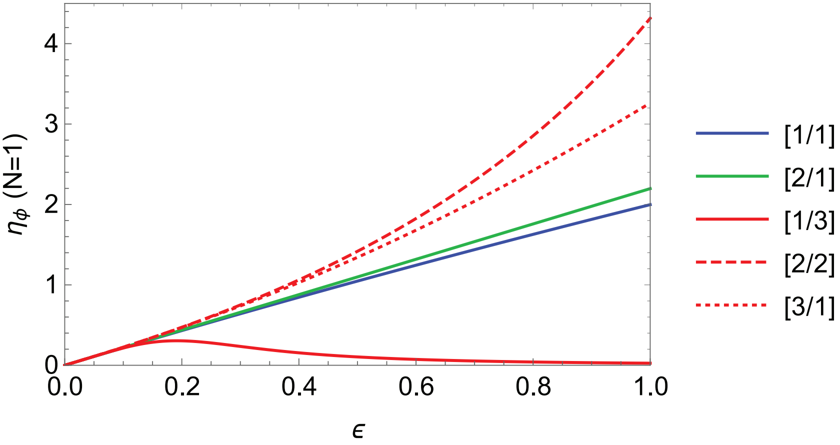
- Results for η_ϕ can be directly compared:

$$\eta_\phi = \eta_{\text{Néel}} = \eta_{\text{VBS}}\,.$$

Padée Extrapolation ($\Delta_{\bar{\Psi}T_A\Psi} = 3 - \nu^{-1}$.)



Padée Extrapolation η_ϕ



Comparing with Literature

$N=1$	$1/\nu$	$3-1/\nu$	η_ϕ	$\Delta_{\bar\Psi T_A \Psi}$
$[0/2]$	0.660	2.34	$\times$	2.60
$[0/2]_{\text{PB}}$	0.748	2.252	$\times$	2.20
$[1/1]$	0.660	2.34	2.00	2.29
$[1/1]_{\text{PB}}$	0.387	2.613	2.01	2.19
$[0/3]$	0.0486	2.9514	$\times$	0.467
$[0/3]_{\text{PB}}$	0.597	2.403	$\times$	1.69
$[1/2]$	0.677	2.323	$\times$	1.99
$[1/2]_{\text{PB}}$	$\times$	$\times$	$\times$	2.14
$[2/1]$	$\times$	$\times$	2.20	1.60
$[2/1]_{\text{PB}}$	$\times$	$\times$	2.20	1.67
$[0/4]$	$\times$	$\times$	$\times$	$\times$
$[0/4]_{\text{PB}}$	0.584	2.416	$\times$	$\times$
$[1/3]$	0.320	2.68	0.0259	1.94
$[1/3]_{\text{PB}}$	0.580	2.42	0.494	1.97
$[2/2]$	$\times$	$\times$	4.32	1.94
$[2/2]_{\text{PB}}$	$\times$	$\times$	$\times$	1.74
$[3/1]$	$\times$	$\times$	3.26	2.08
$[3/1]_{\text{PB}}$	$\times$	$\times$	3.59	1.75

NCCP[1]	χ_I-QED$_3$-GNY $(N=1)$
$\eta_{\text{Néel}} \approx 0.26(3)$ [1]	$\eta_\phi \approx 2.1(1)$ [2]
$\approx 0.35(3)$ [3]	$\approx 1.3(9)$ [4]
$\approx 0.30(5)$ [5]	
≈ 0.22 [6]	
$\approx 0.259(6)$ [7]	
$\eta_{\text{VBS}} \approx 0.28(8)$ [5]	
$\approx 0.25(3)$ [7]	
$3-1/\nu \approx 1.72(5)$ [1]	$3-1/\nu \approx 2.33(1)$ [2]
$\approx 1.53(9)$ [3]	$\approx 2.7(4)$ [4]
$\approx 1.15(19)$ [5]	
≈ 1.21 [6]	$\Delta_{\bar\Psi T_A \Psi} \approx 2.12(50)$ [2]
≈ 1 [7]	$\approx 1.8(5)$ [4]
$\approx 0.76(4)$ [8]	

[1]·[Sandvik'07]
[2]·[Ihrig,Janssen,Mihaila,Scherer'18]
[3]·[Melko,Kaul'08]
[4]·[Janssen,He'17]
[5]·[Pujari,Damle,Alet'13]
[6]·[Bartosch'13]
[7]·[Nahum,Serna,Chalker,.'15]
[8]·[Shao,Guo,Sandvik'16]

Summary

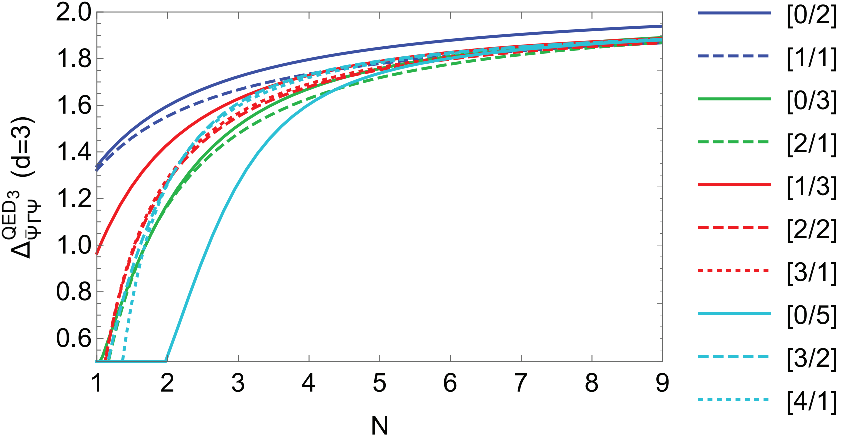
- ✓ Renormalization of χ_I-QED$_3$-GNY around $d = 4$ performed for general N at the four loop level
- ✓ Fixed point analysis of $\eta_\phi, \nu^{-1}, \omega, \Delta_{\bar{\Psi}T_A\Psi}, \Delta_{\bar{\Psi}1\Psi}$ via ϵ expansion
- ✓ Padé extrapolation to $d = 3$
- ☹ Inconclusive numerical results concerning the duality for $N = 1$

$$\chi_I\text{-QED}_3\text{-GNY}(N = 1) \Leftrightarrow SU(2)\text{NCCP}^1 \, .$$

- ? Five loop analysis?

To backup slides $\rightarrow$

Pure QED$_3$ at 5-Loop



Padé Approximants

- Padé:

$$[m/n](\epsilon) \equiv \frac{\sum_{i=0}^{m} a_i \epsilon^i}{1 + \sum_{j=1}^{n} b_j \epsilon^j},$$

- Borel:

$$\Delta(\epsilon) = \sum_k \Delta_k \epsilon^k ,$$

$$B_\Delta(\epsilon) \equiv \sum_k \frac{\Delta_k}{k!} \epsilon^k ,$$

$$\Delta(\epsilon) = \int_0^\infty dt\, e^{-t} B_\Delta(\epsilon t) .$$

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