Leading-logarithmic threshold resummation at next-to-leading power

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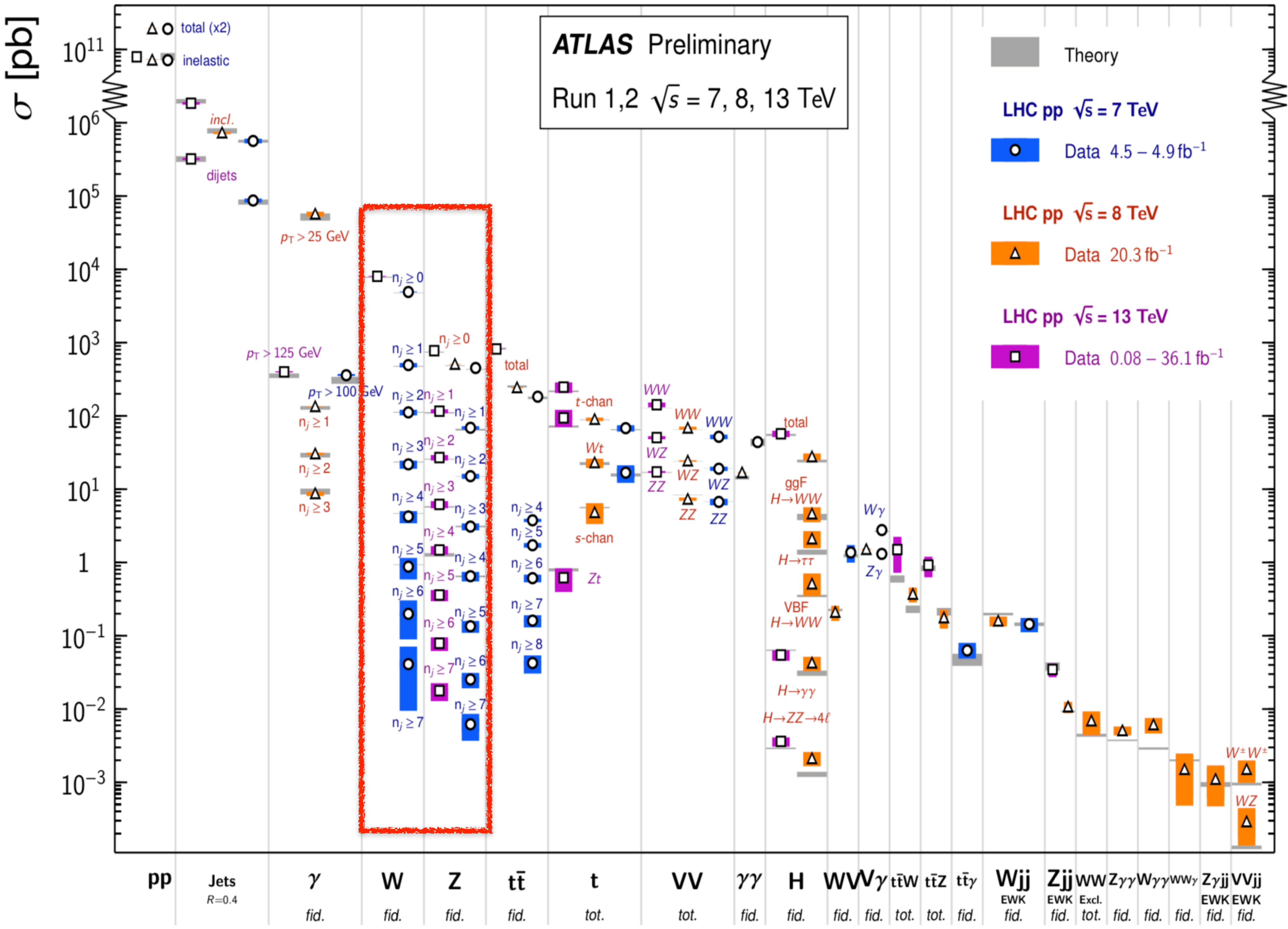
arXiv:1809.10631

DESY Zeuthen, 1/11/2018

Standard Model Production Cross Section Measurements

Status: July 2017



DY process

NNLO x-section: Hamberg, van Neerven, Matsuura 1991

Near threshold z=Q²/s->1

$$\hat{\sigma}_{ab}(z) = \sum_{n=0}^{\infty} \alpha_s^n \left[\underbrace{c_n \delta(1-z)}_{\text{LP}} + \sum_{m=0}^{2n-1} \left(\underbrace{c_{nm} \left[\frac{\ln^m(1-z)}{1-z} \right]_+}_{\text{LP}} + \underbrace{d_{nm} \ln^m(1-z)}_{\text{NLP}} \right) + \dots \right]$$

Can we know the coefficients c's and d's for any n?

Factorization and Resummation

LP results

$$\hat{\sigma}(z) = H(Q^2)\, Q S_{\rm DY}(Q(1-z))$$

P. A. Baikov, K. G. Chetyrkin, A. V. Smirnov, V. A. Smirnov and M. Steinhauser, R. N. Lee, '09

T. Gehrmann, E. W. N. Glover, T. Huber, N. Ikizlerli and C. Studerus, '10

$$S_{\rm DY}(\Omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2}\, \frac{1}{N_c}\, {\rm Tr}\, \langle 0|\bar{\mathbf{T}}(Y_+^\dagger(x^0)Y_-(x^0))\, \mathbf{T}(Y_-^\dagger(0)Y_+(0))|0\rangle$$

C. Anastasiou, C. Duhr, F. Dulat, E. Furlan, T. Gehrmann, F. Herzog and B. Mistlberger, '13

Y. Li, A. von Manteuffel, R. M. Schabinger and H. X. Zhu, '13

In contrast, much less is understood at NLP.

Structure of NLP logarithms

- the method of region approach, Bonocore et al 2014, Anastasiou et al 2014, Bahjat-Abbas et al 2018
- diagrammatic factorization techniques, Bonocore et al 2015, Bonocore et al 2016, Del Duca et al:2017

Low-Burnett-Kroll theorem 1958,1968

$$-g_s \sum_{i=1}^{N} \mathbf{T}_i \left(\frac{p_i \cdot \epsilon(k)}{p_i \cdot k} + \frac{\epsilon_\mu(k) k_\nu J_i^{\mu\nu}}{p_i \cdot k} \right) A_0(\{p_i\})$$

$$J_i^{\mu\nu} = p_i^\mu \frac{\partial}{\partial p_{i\nu}} - p_i^\nu \frac{\partial}{\partial p_{i\mu}} + \Sigma_i^{\mu\nu}$$

LP factorization

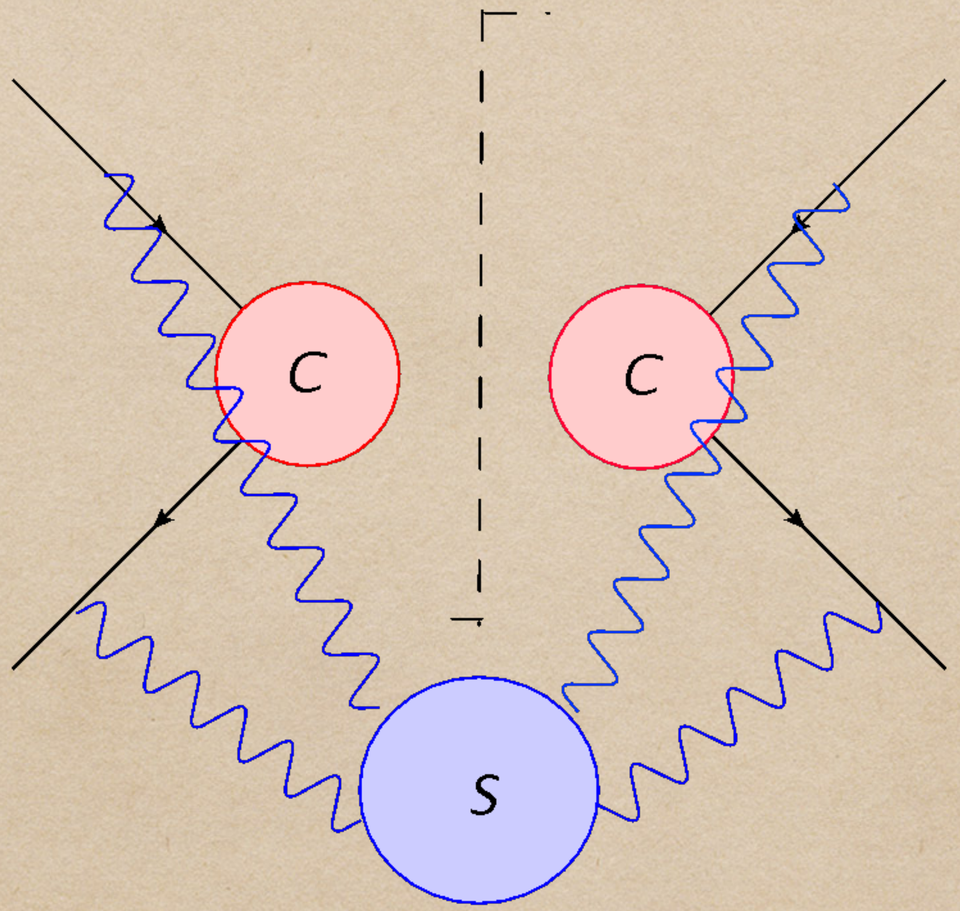
$$\hat{\sigma}(z) = H(Q^2)\, Q S_{\mathrm{DY}}(Q(1-z))$$

$$(n_+p, n_-p, p_\perp)$$

$\mu_h \sim Q$

$\mu_s \sim Q(1-z)$

$\mu_c \sim Q\sqrt{1-z}$



$Q(1,1,1)$

$Q(1,\lambda^2,\lambda)$

$Q(\lambda^2,\lambda^2,\lambda^2)$

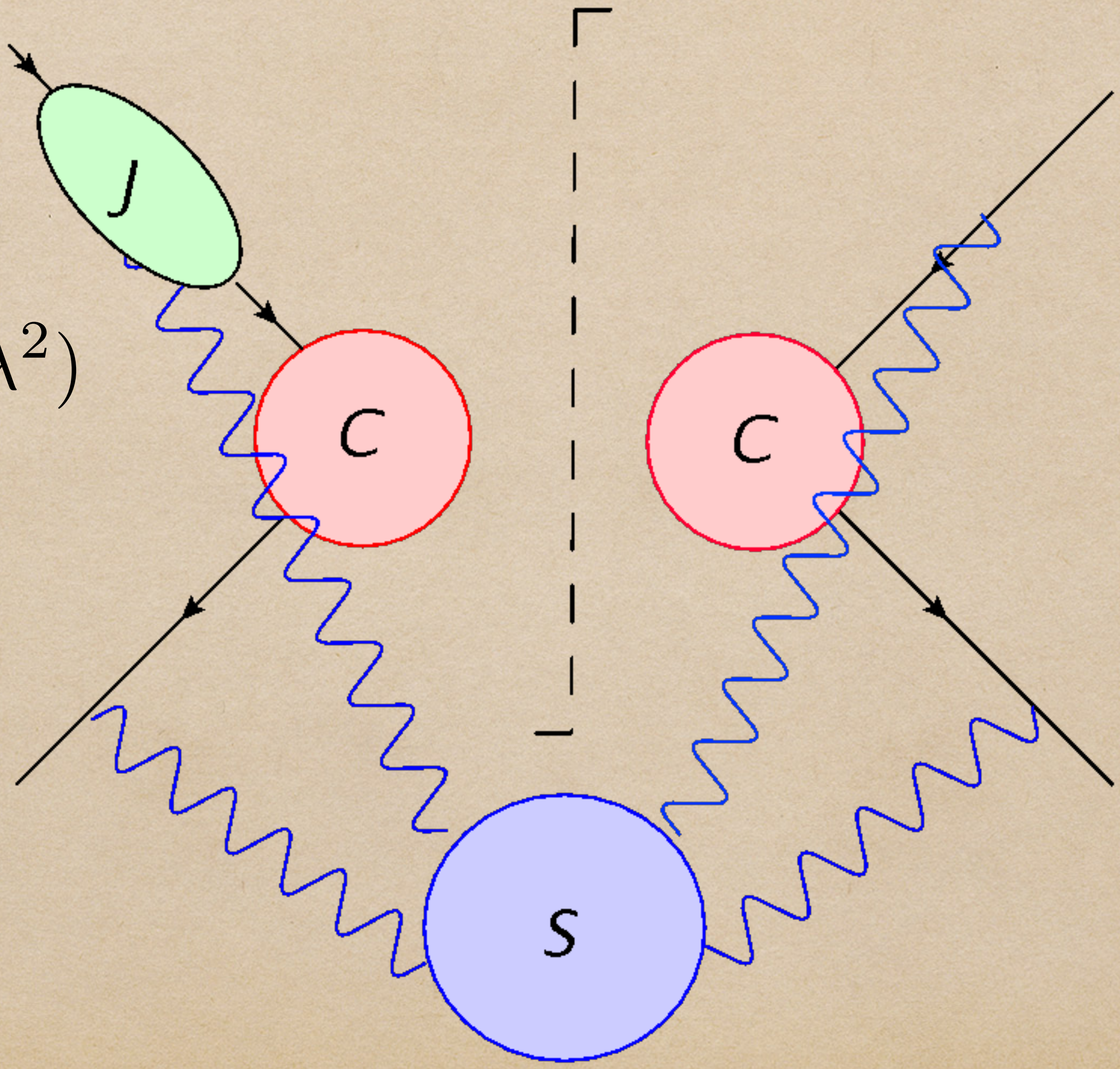
$\lambda = \sqrt{1-z}$

Tricky point: no collinear function at LP

NLP factorization

$Q(1, \boxed{\lambda^2}, \boxed{\lambda})$

$Q(\lambda^2, \lambda^2, \lambda^2)$

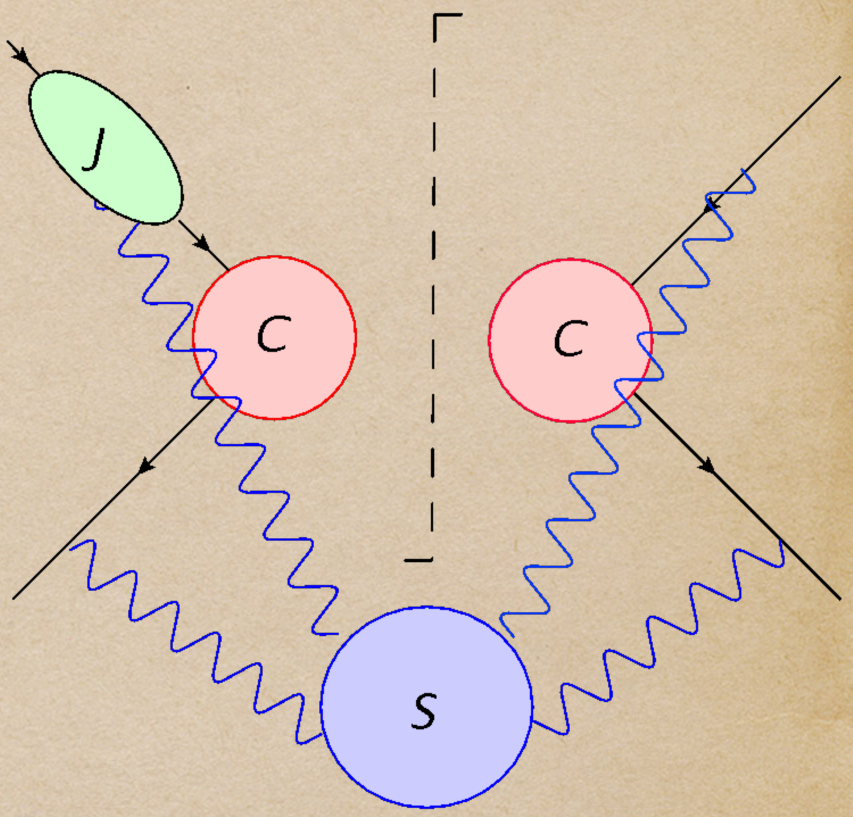


NLP factorization

$$\hat{\sigma}(z) = \sum_i \int d\omega_i d\bar{\omega}_i d\omega_i' d\bar{\omega}_i' \, D(-\hat{s};\omega_i,\bar{\omega}_i) D^*(-\hat{s};\omega_i',\bar{\omega}_i')$$

$$\times Q^2 \int \frac{d^3\vec{q}}{(2\pi)^3 \, 2\sqrt{Q^2+\vec{q}^{\,2}}} \, \frac{1}{2\pi}$$

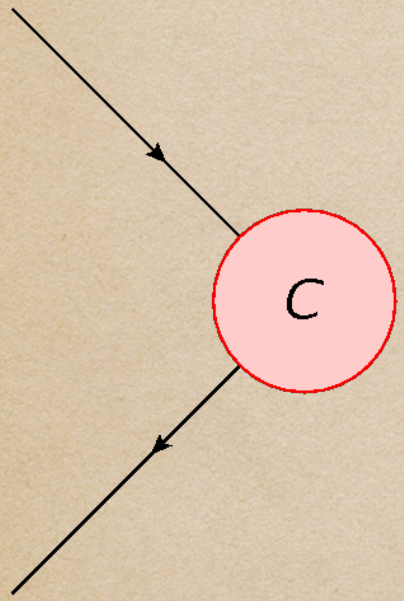
$$\int d^4x \, e^{i(x_a p_A + x_b p_B - q)\cdot x} \, \widetilde{S}(x;\omega_i,\bar{\omega}_i,\omega_i',\bar{\omega}_i')$$



$$D(-\hat{s};\omega_i,\bar{\omega}_i) = \int d(n_+p_i) d(n_-\bar{p}_i) \, C(n_+p_i, n_-\bar{p}_i)$$

$$\times J(n_+p_i, x_a n_+ p_A;\omega_i) \, \bar{J}(n_-\bar{p}_i, -x_b n_- p_B;\bar{\omega}_i)$$

Hard function



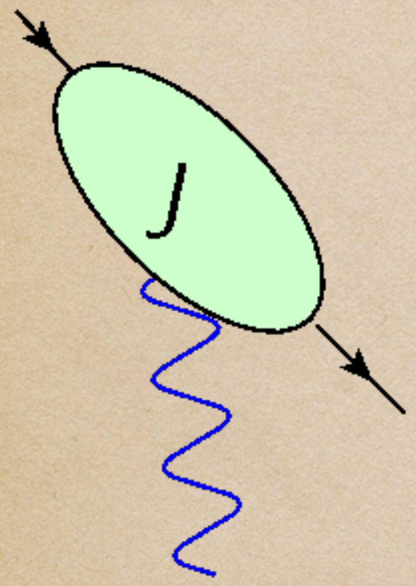
$$\bar{\psi}\gamma_\mu\psi(0) = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}\,)\, J_\mu^{A0}(t,\bar{t}\,)$$

$$C^{A0}(n_+p, n_-\bar{p}) = \int dt\, d\bar{t}\, e^{-itn_+p - i\bar{t}n_-\bar{p}}\, \widetilde{C}^{A0}(t,\bar{t}\,)$$

$$J_\mu^{A0}(t,\bar{t}\,) = \bar{\chi}_{\bar{c}}(\bar{t}n_-)\gamma_{\perp\mu}\chi_c(tn_+)$$

$$\hat{\sigma}(z) = H(Q^2)\, Q S_{\mathrm{DY}}(Q(1-z))$$

NLP jet function



$$i\int d^4z\, e^{i\omega(n_+z)/2}\,\mathbf{T}\left[\chi_{c,\alpha a}(tn_+)\bar{\chi}_{c,d}(z)\frac{\not{n}_+}{2}\chi_{c,e}(z)\right]$$

$$= 2\pi\int du\,\widetilde{J}_{\alpha\beta,abde}(t,u;\omega)\,\chi^{\mathrm{PDF}}_{c,\beta b}(un_+)$$

Field definition of radiative jet function

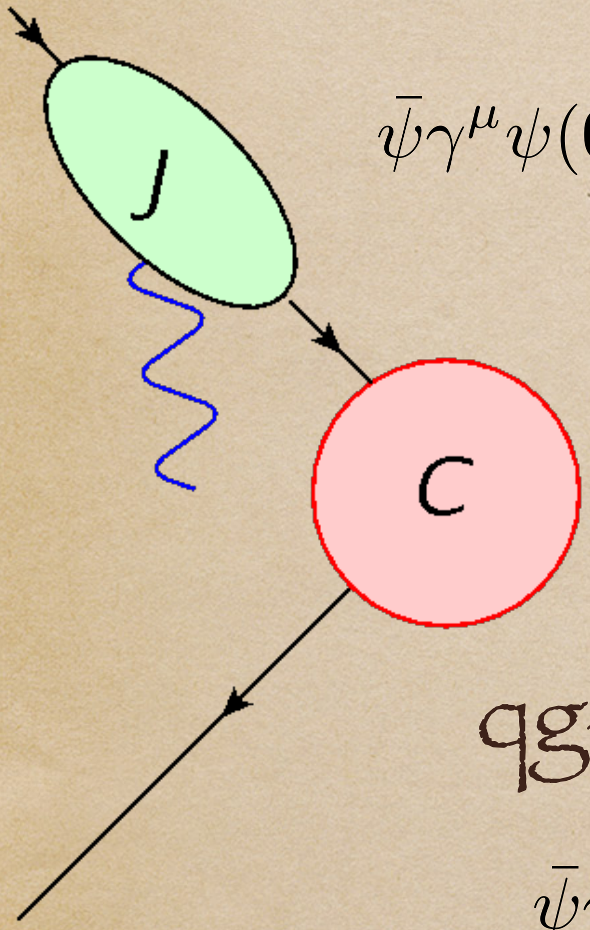
del Duca 1990,
Bonocore et al '15,'16

NLP quark-gluon interaction: Beneke et al 2002

$$\mathcal{L}^{(2)}_{2\xi} = \frac{1}{2}\bar{\chi}_c x^\mu_\perp x^\nu_\perp\left[i\partial_\nu in_-\partial\mathcal{B}^+_\mu\right]\frac{\not{n}_+}{2}\chi_c \qquad \mathcal{B}^\mu_\pm = Y^\dagger_\pm\left[iD^\mu_s Y_\pm\right]$$

LO: $$J^{\mu\rho}_{2\xi;\alpha\beta,abde}(n_+p,n_+p';\omega) = -\frac{g^{\mu\rho}_\perp}{n_+p}\delta(n_+p-n_+p')\delta_{\alpha\beta}\delta_{ad}\delta_{eb}$$

NLP factorization



$$\bar{\psi}\gamma^\mu\psi(0) = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}\,) \left[J^\mu_{A0}(t,\bar{t}\,) + \left(J^{T2}_{A0,2\xi}(t,\bar{t}\,)\right)^\mu + \ \bar{c}\text{-term}\right]$$

$$\left(J^{T2}_{A0,2\xi}(s,t)\right)^\mu = i\int d^4x\, \mathbf{T}\left[J^\mu_{A0}(s,t)\, \mathcal{L}^{(2)}_{2\xi}(x)\right]$$

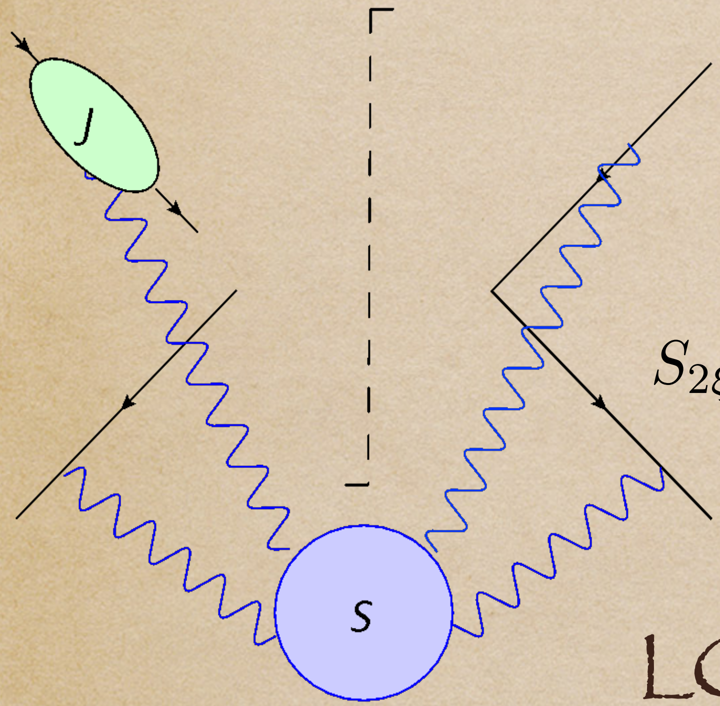
qg-channel:

$$\bar{\psi}\gamma^\mu\psi(0) = \int dt\, d\bar{t}\, \widetilde{C}^{A0}(t,\bar{t}\,) \left[\left(J^{T1}_{A0,\xi q}(t,\bar{t}\,)\right)^\mu + \ \bar{c}\text{-term}\right]$$

$$\left(J^{T1}_{A0,\xi q}(s,t)\right)^\mu = i\int d^4x\, \mathbf{T}\left[J^\mu_{A0}(s,t)\, \mathcal{L}^{(1)}_{\xi q}(x)\right]$$

$$\mathcal{L}^{(1)}_{\xi q} = \bar{q}_+ \mathcal{A}_{c\perp}\chi_c + \text{h.c.}$$

Soft function at NLP



$$\widetilde{S}_{2\xi}\,(x,z_-) = \bar{\mathbf{T}}\left[Y_+^\dagger(x)Y_-(x)\right]\mathbf{T}\left[Y_-^\dagger(0)Y_+(0)\frac{i\partial_\perp^\nu}{in_-\partial}\mathcal{B}_{\perp\nu}^+(z_-)\right]$$

$$S_{2\xi}(\Omega,\omega) = \int\frac{dx^0}{4\pi}\int\frac{d(n_+z)}{4\pi}\,e^{ix^0\Omega/2-i\omega(n_+z)/2}\,\frac{1}{N_c}\,\mathrm{Tr}\,\langle 0|\widetilde{S}_{2\xi}(x^0,z_-)|0\rangle$$

LO:

$$S_{2\xi}(\Omega,\omega) = \frac{\alpha_s C_F}{2\pi}\left\{\theta(\Omega)\delta(\omega)\left(-\frac{1}{\epsilon}+\ln\frac{\Omega^2}{\mu^2}\right)+\left[\frac{1}{\omega}\right]_+\theta(\omega)\theta(\Omega-\omega)\right\}$$

A puzzle: divergence at LO

RG condition

$$S_{2\xi}(\Omega,\omega)_{|\mathrm{ren}} = \int d\Omega' \int d\omega'\, Z_{2\xi,2\xi}(\Omega,\omega;\Omega',\omega')\, S_{2\xi}(\Omega',\omega')_{|\mathrm{bare}}$$
$$+ \int d\Omega'\, Z_{2\xi,x_0}(\Omega,\omega;\Omega')\, S_{x_0}(\Omega')_{|\mathrm{bare}}$$

$$Z_{2\xi,2\xi}(\Omega,\omega;\Omega,\omega') = \delta(\Omega-\Omega')\delta(\omega-\omega') + \mathcal{O}(\alpha_s)\,,$$
$$Z_{2\xi,x_0}(\Omega,\omega;\Omega') = \frac{\alpha_s C_F}{2\pi}\frac{1}{\epsilon}\delta(\Omega-\Omega')\delta(\omega) + \mathcal{O}(\alpha_s^2)$$

The mixing term subtracts the divergent part of the first term on the right-hand side, resulting in a finite, renormalized soft function

auxiliary soft function

$$S_{x_0}(\Omega) = \theta(\Omega)$$

We propose

$$S_{x_0}(\Omega) = \int \frac{dx^0}{4\pi}\, e^{ix^0\Omega/2}\, \frac{-2i}{x^0 - i\varepsilon}\, \frac{1}{N_c}\, \mathrm{Tr}\, \langle 0|\bar{\mathbf{T}}\left[Y_+^\dagger(x^0)Y_-(x^0)\right]\mathbf{T}\left[Y_-^\dagger(0)Y_+(0)\right]|0\rangle$$

"Theta-soft function" in NLP thrust distribution,

Moult, Stewart,Vita, Zhu '18

We check this form by requiring the poles cancel at two loop

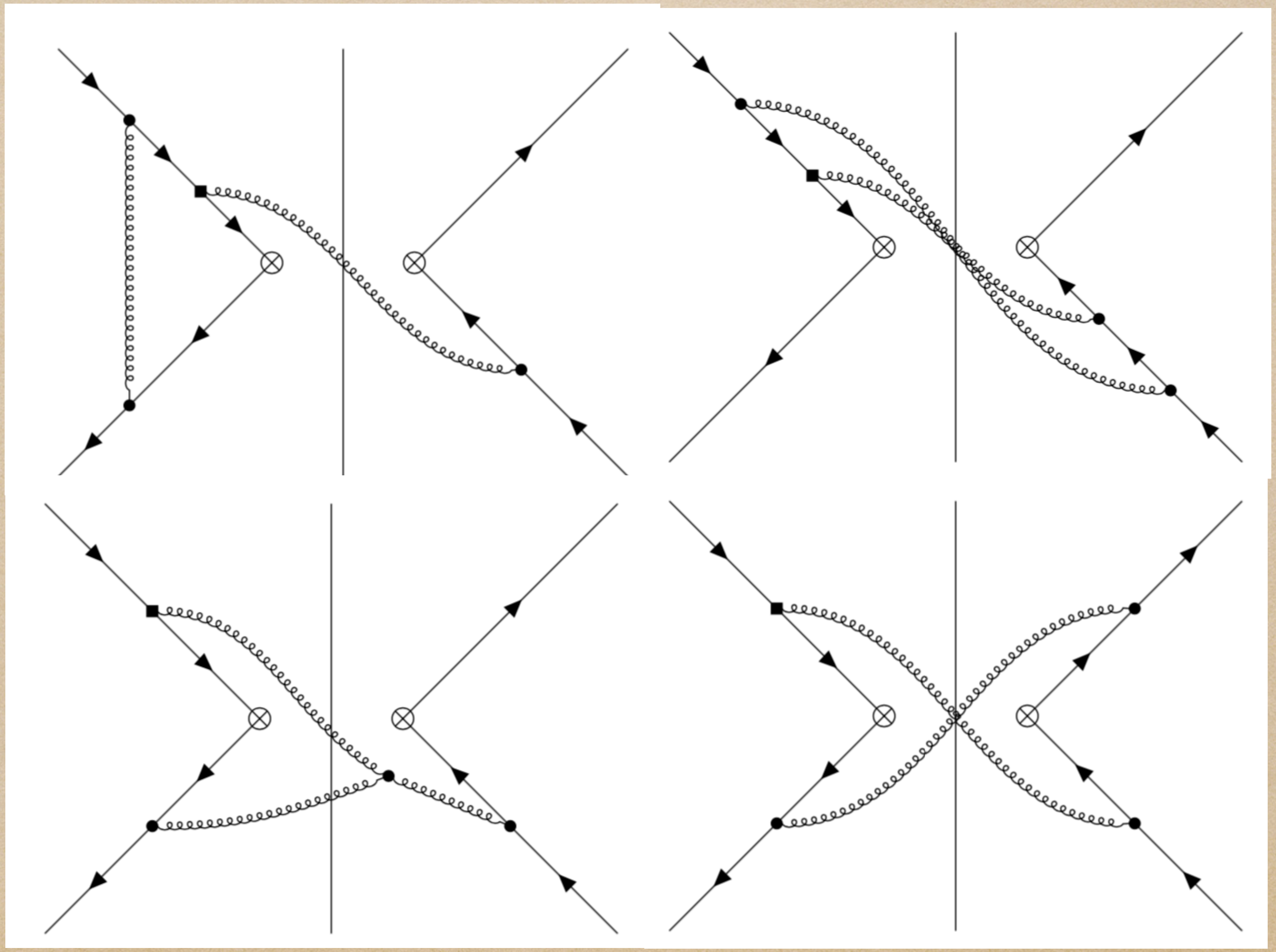
Check

$$S^{(2)}_{2\xi} + Z^{(1)}_{2\xi\, x_0} S^{(1)}_{x_0} + Z^{(2)}_{2\xi\, x_0} S^{(0)}_{x_0} + Z^{(1)}_{2\xi\, 2\xi} S^{(1)}_{2\xi} = \text{finite}$$

Under assumption that the off-diag has only subleading pole

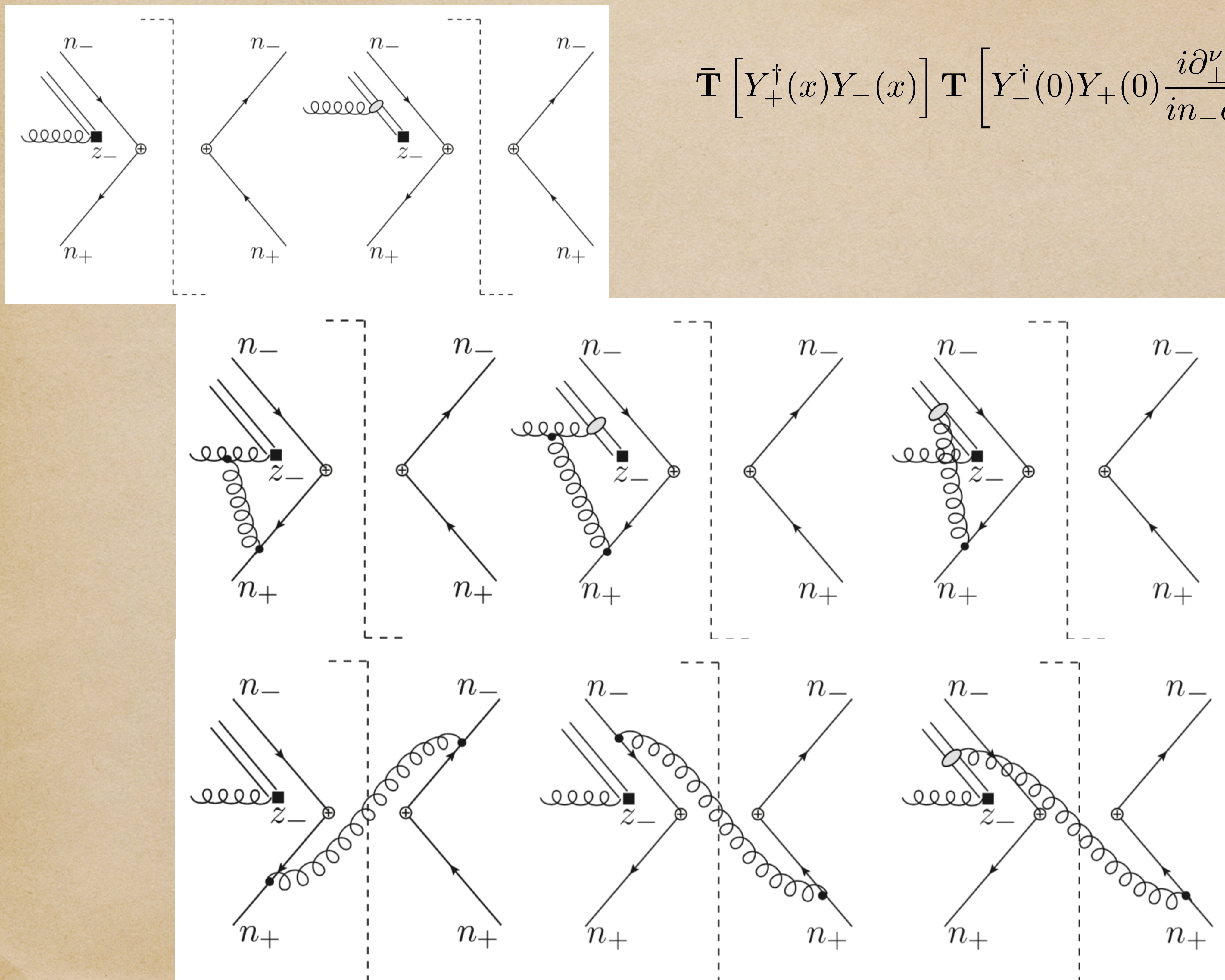
$$S^{(2)}_{2\xi} - \frac{1}{4} \underbrace{Z^{(1)}_{2\xi\, x_0}}_{\text{Known from 1-loop}} \left(3 Z^{(1)}_{2\xi\, 2\xi} + \underbrace{Z^{(1)}_{x_0\, x_0}}_{\text{Same as LP soft fun.}} \right) S^{(0)}_{x_0} = \mathcal{O}\left(\frac{1}{\epsilon^2}\right)$$

$$S^{(2)}_{2\xi}$$



$Z^{(1)}_{2\xi\ 2\xi}$

$$\bar{\mathbf{T}}\left[Y_+^\dagger(x)Y_-(x)\right]\mathbf{T}\left[Y_-^\dagger(0)Y_+(0)\frac{i\partial_\perp^\nu}{in_-\partial}\mathcal{B}^+_{\perp\nu}(z_-)\right]$$



RG eq. of soft fun.

$$\frac{d}{d\ln\mu}\begin{pmatrix} S_{2\xi}\left(\Omega,\omega\right) \\ S_{x_0}\left(\Omega\right)\end{pmatrix} = \frac{\alpha_s}{\pi}\begin{pmatrix} 4C_F\ln\frac{\mu}{\mu_s} & -C_F\delta(\omega) \\ 0 & 4C_F\ln\frac{\mu}{\mu_s}\end{pmatrix}\begin{pmatrix} S_{2\xi}\left(\Omega,\omega\right) \\ S_{x_0}\left(\Omega\right)\end{pmatrix}$$

$$S_{2\xi}^{\mathrm{LL}}(\Omega,\omega,\mu) = \underbrace{\frac{2C_F}{\beta_0}\ln\frac{\alpha_s(\mu)}{\alpha_s(\mu_s)}}_{\alpha_s\ln\frac{\mu}{\mu_s}}\exp\left[\underbrace{-4S^{\mathrm{LL}}(\mu_s,\mu)}_{\alpha_s\ln^2\frac{\mu}{\mu_s}}\right]\theta(\Omega)\delta(\omega)$$

Similarly, for the hard function

$$H(Q^2,\mu) = \exp\left[\underbrace{4S(\mu_h,\mu)}_{\alpha_s\ln^2\frac{\mu}{\mu_h}}\right]$$

Kinematic corrections

In the partonic c.o.m frame, the energy of the soft hadronic final state is expanded as

$$[x_1 p_1 + x_2 p_2 - q]^0 = p^0_{X_s} = \sqrt{\hat{s}} - \sqrt{Q^2 + \vec{q}^{\,2}} = \frac{\Omega_*}{2} - \frac{\vec{q}^{\,2}}{2Q} + O(\lambda^6)$$

$$\Omega_* = 2Q\frac{1-\sqrt{z}}{\sqrt{z}} = Q(1-z) + \frac{3}{4}Q(1-z)^2 + O(\lambda^6)$$

The soft function expands

$$S_{\mathrm{DY}}(Q(1-z)) + \frac{1}{Q}S_{K1}(Q(1-z)) + \frac{1}{Q}S_{K2}(Q(1-z)) + \mathcal{O}(\lambda^4)$$

$$\Delta_{ab}(z) = \frac{\hat{\sigma}_{ab}(z)}{z}$$

$$S_{K1}(\Omega) = \frac{\partial}{\partial\Omega}\partial^2_{\vec{x}} S_0\left(\Omega, \vec{x}\right)_{|\vec{x}=0}\ ,$$

$$S_{K2}(\Omega) = \frac{3}{4}\,\Omega^2\frac{\partial}{\partial\Omega}S_0(\Omega,\vec{x})_{|\vec{x}=0}$$

$$S_{K3}(\Omega) = \Omega\, S_0(\Omega,\vec{x})_{|\vec{x}=0}$$

$$\boxed{\sum_{i=1}^{3} S_{Ki}(\Omega) = 2\,\frac{\alpha_s C_F}{\pi}}$$

No kine.cor.

Final results

$$\mu_h \sim Q$$
$$\mu_s \sim Q(1-z)$$
$$\mu_c \sim Q\sqrt{1-z}$$

$$\Delta^{\rm LL}_{\rm NLP}(z) = -\exp\left[4S^{\rm LL}(\mu_h,\mu_c) - 4S^{\rm LL}(\mu_s,\mu_c)\right] \times \frac{8C_F}{\beta_0}\ln\frac{\alpha_s(\mu_c)}{\alpha_s(\mu_s)}\,\theta(1-z)$$

Why we evolve the hard/soft function to the jet scale?

We use the LO jet function. Recover
the general scale dependence by the AP splitting kernels

$$\frac{d}{d\ln\mu}\hat{\sigma}_{ab}(z,\mu) = -\sum_c \int_z^1 dx\left(P_{ca}(x)\hat{\sigma}_{cb}\left(\frac{z}{x},\mu\right) + P_{cb}(x)\hat{\sigma}_{ac}\left(\frac{z}{x},\mu\right)\right)$$

$$P^{\rm LP}_{ab}(x) = \left(2\Gamma_{\rm cusp}(\alpha_s)\frac{1}{[1-x]_+} + 2\gamma^{\phi}(\alpha_s)\delta(1-x)\right)\delta_{ab}$$

$$P^{\rm NLP}_{ab} = \gamma^{\rm NLP}_{ab}(\alpha_s)$$

Final results

$$\frac{d}{d\ln\mu}\Delta_{\mathrm{NLP}}(z,\mu)$$

$$= -4\left[\Gamma_{\mathrm{cusp}}(\alpha_s)\left(\underline{\ln(1-z)} - \gamma_E - \psi\left(1+\frac{d}{d\ln(1-z)}\right)\right) + \gamma^{\phi}(\alpha_s)\right]\Delta_{\mathrm{NLP}}(z,\mu)$$

$$+\ K(z,\mu)$$

$$K(z,\mu) = \underline{-2\gamma_{qq}^{\mathrm{NLP}}(\alpha_s)\int_z^1 dy\,\Delta_{\mathrm{LP}}(y,\mu)} - 4\,\Gamma_{\mathrm{cusp}}(\alpha_s)(1-z)\Delta_{\mathrm{LP}}(z,\mu)$$

LL

$$\Delta_{\mathrm{NLP}}^{\mathrm{LL}}(z,\mu) = \quad \exp\left[4S^{\mathrm{LL}}(\mu_h,\mu) - 4S^{\mathrm{LL}}(\mu_s,\mu)\right] \times \frac{-8C_F}{\beta_0}\ln\frac{\alpha_s(\mu)}{\alpha_s(\mu_s)}\,\theta(1-z)$$

Expansion

$$\Delta^{\rm LL}_{\rm NLP}(z,\mu) = \exp\left[-2\frac{\alpha_s C_F}{\pi}\ln^2\frac{\mu}{\mu_h}\right]\exp\left[+2\frac{\alpha_s C_F}{\pi}\ln^2\frac{\mu}{\mu_s}\right]$$
$$\times\,(-4)\frac{\alpha_s C_F}{\pi}\ln\frac{\mu_s}{\mu}\,\theta(1-z)$$

$$\begin{aligned}
\Delta^{\rm LL}_{\rm NLP}(z,\mu) = &-\theta(1-z)\Big\{4C_F\frac{\alpha_s}{\pi}\Big[\ln(1-z)-L_\mu\Big] \\
&+8C_F^2\left(\frac{\alpha_s}{\pi}\right)^2\Big[\ln^3(1-z)-3L_\mu\ln^2(1-z)+2L_\mu^2\ln(1-z)\Big] \\
&+8C_F^3\left(\frac{\alpha_s}{\pi}\right)^3\Big[\ln^5(1-z)-5L_\mu\ln^4(1-z)+8L_\mu^2\ln^3(1-z)-4L_\mu^3\ln^2(1-z)\Big] \\
&+\frac{16}{3}C_F^4\left(\frac{\alpha_s}{\pi}\right)^4\Big[\ln^7(1-z)-7L_\mu\ln^6(1-z)+18L_\mu^2\ln^5(1-z)-20L_\mu^3\ln^4(1-z) \\
&\qquad+8L_\mu^4\ln^3(1-z)\Big] \\
&+\frac{8}{3}C_F^5\left(\frac{\alpha_s}{\pi}\right)^5\Big[\ln^9(1-z)-9L_\mu\ln^8(1-z)+32L_\mu^2\ln^7(1-z)-56L_\mu^3\ln^6(1-z) \\
&\qquad+48L_\mu^4\ln^5(1-z)-16L_\mu^5\ln^4(1-z)\Big]\Big\}+\mathcal{O}(\alpha_s^6\times(\log)^{11})
\end{aligned}$$

Hamberg, van Neervan, Matsuura, 1991

Florian, Mazzitelli, Moch, Vogt, '14

Kramer, Laenen, Spira, '96, Kidonakis '07

Summary and outlook

- The LP threshold resummation was developed in 1987/89, extended to higher accuracy later.
- We provide an NLP resummation of the leading logs in the soft-collinear effective theory.
- The LO divergences in the soft function are cancelled by an auxiliary soft function.
- There is no kinematic power correction.
- The resumed result has no leading log at the jet scale.
- At a general scale, we reproduce the first few orders.

Summary and outlook

- Extension to NLL is interesting and will reveal the full difficulty and complexity of NLP resummation, which can be seen from the anomalous dimension of NLP operators. Beneke, Garny, Robert, JW ’18

Thank you for your attention!