AUTOMATING CALCULATIONS IN SOFT-COLLINEAR EFFECTIVE THEORY

[GUIDO BELL]

based on: GB, R. Rahn, J. Talbert, 1512.06100, 1801.04877, 1805.12414 + work in progress
GB, A. Hornig, C. Lee, J. Talbert, work in progress
GB, B. Dehnadi, T. Mohrmann, R. Rahn, work in progress

UNIVERSITÄT SIEGEN

OUTLINE

SOFT-COLLINEAR EFFECTIVE THEORY

SCALES, MODES, SCET-1 AND SCET-2

AUTOMATED CALCULATION OF SOFT FUNCTIONS

DIJET SOFT FUNCTIONS

SOFT SERVE

N-JET OBSERVABLES

ANGULARITIES

SCET-1 VS SCET-2

NNLL′ RESUMMATION

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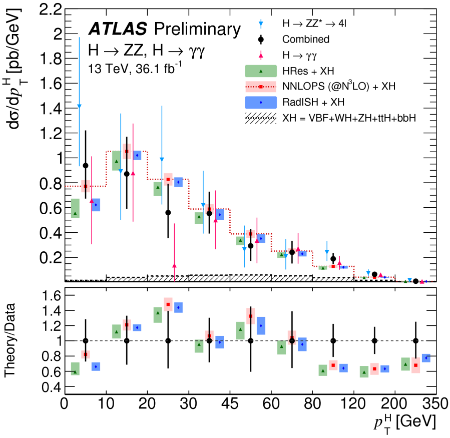
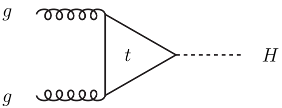
Momentum scales

Higgs p_T spectrum





$m_t \simeq 175$ GeV

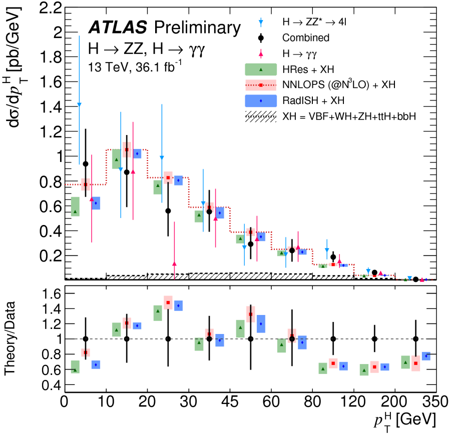
$m_H \simeq 125$ GeV

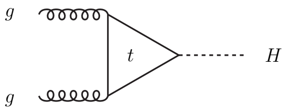
$p_T \simeq 0 - 200$ GeV

$\Lambda_{\rm QCD} \simeq 0.5$ GeV

Momentum scales

Higgs p_T spectrum





$m_t \simeq 175$ GeV

$m_H \simeq 125$ GeV

$p_T \simeq 10 - 20$ GeV

$\Lambda_{QCD} \simeq 0.5$ GeV

Scale separation

For $\Lambda_{\text{QCD}} \ll p_T, m_H, m_t$ the cross section factorises

$$d\sigma \simeq \sum_{i,j} f_{i/p}(\Lambda_{\text{QCD}}, \mu) \otimes f_{j/p}(\Lambda_{\text{QCD}}, \mu) \otimes d\hat{\sigma}_{ij \to HX}(p_T, m_H, m_t, \mu)$$

- universal parton-distribution functions $f_{i/p}$
- perturbative partonic cross section $d\hat{\sigma}_{ij \to HX}$

Factorisation scale μ separates short- and long-distance dynamics

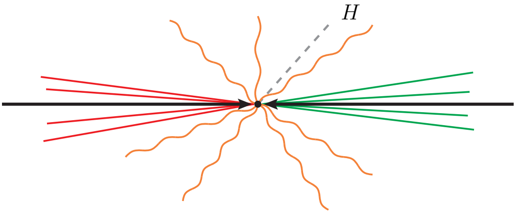
- single-logarithmic evolution controlled by DGLAP equations

$$\frac{df_{i/p}(\mu)}{d\ln\mu} = \sum_j P_{ij}(\alpha_s) \otimes f_{j/p}(\mu)$$

Small p_T

For $p_T \ll m_H, m_t$ the partonic cross section factorises further

$$\frac{d\hat{\sigma}}{dp_T} \simeq H(m_H, m_t, \mu)\; J_1(p_T, \mu) \otimes J_2(p_T, \mu) \otimes S(p_T, \mu)$$



- hard function H
- jet (beam) functions J_i
- soft function S

perturbative ($p_T \gg \Lambda_{\text{QCD}}$)

double-logarithmic RG evolution

$\Rightarrow$ Sudakov logarithms $\alpha_s^n \ln^{2n} \frac{m_H}{p_T}$

Soft-Collinear Effective Theory

[Bauer, Fleming, Pirjol, Stewart 00;
Beneke, Chapovsky, Diehl, Feldmann 02]

Effective field theory for energetic massless particles

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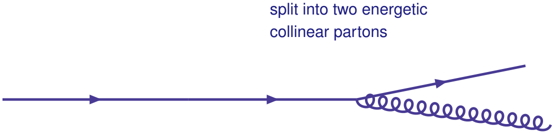

Soft-Collinear Effective Theory

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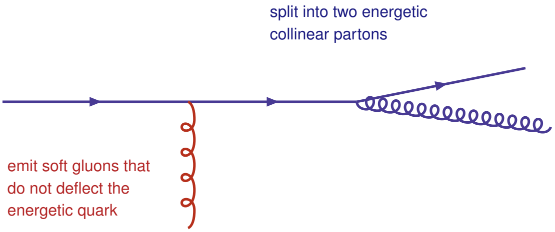
Effective field theory for energetic massless particles



Soft-Collinear Effective Theory

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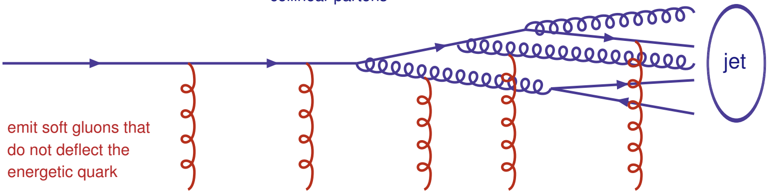
Effective field theory for energetic massless particles

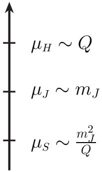


$\Rightarrow$ jet of collinear particles $\quad m_J^2 \ll E_J^2$

soft large-angle radiation $\quad E_s \ll E_J$

SCET-1

Three-scale problem: $\mu_S \ll \mu_J \ll \mu_H$



$$\boxed{d\hat{\sigma} \simeq H(Q,\mu)\; J(m_J,\mu) \otimes S(m_J^2/Q,\mu)}$$

$$\ln^2 \frac{Q^2}{m_J^2} = \frac{1}{2} \ln^2 \frac{Q^2}{\mu^2} - \ln^2 \frac{m_J^2}{\mu^2} + \frac{1}{2} \ln^2 \frac{m_J^4/Q^2}{\mu^2}$$

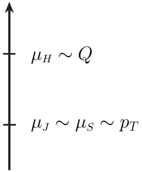
Sudakov resummation with standard RG techniques

$$\frac{dH(Q,\mu)}{d\ln\mu} = \left[2\,\Gamma_{\rm cusp}(\alpha_s)\, \ln\frac{Q^2}{\mu^2} + 4\gamma_H(\alpha_s)\right] H(Q,\mu)$$

- anomalous dimensions: $\Gamma_{\rm cusp}, \gamma_H, \gamma_J, \gamma_S$
- matching corrections: c_H, c_J, c_S

SCET-2

Two-scale problem: $\mu_S \sim \mu_J \ll \mu_H$



$$\boxed{d\hat{\sigma} \;\simeq\; H(Q,\mu)\;\, J(p_T,\mu) \,\otimes\, S(p_T,\mu)}$$

$$\ln^2 \frac{Q^2}{p_T^2} = \ln^2 \frac{Q^2}{\mu^2} - \ln^2 \frac{p_T^2}{\mu^2} + \quad ?$$

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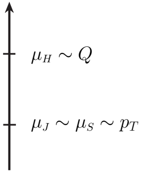

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$$\ln^2 \frac{Q^2}{p_T^2} = \ln^2 \frac{Q^2}{\mu^2} - \ln^2 \frac{p_T^2}{\mu^2} + \quad ?$$

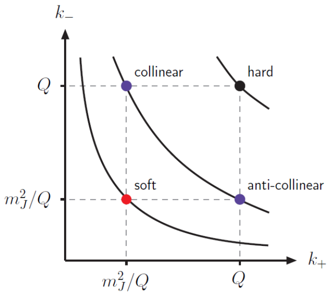
Jet and soft functions are ill-defined in dimensional regularisation

$$k^\mu = k_- \frac{n^\mu}{2} + k_+ \frac{\bar{n}^\mu}{2} + k_\perp^\mu \qquad \Rightarrow \qquad J \sim \int_0^Q \frac{dk_+}{k_+} \qquad S \sim \int_{p_T}^\infty \frac{dk_+}{k_+}$$

$\Rightarrow$ in light-cone coordinates DR is attached to the transverse space $d^{d-2}k_\perp$

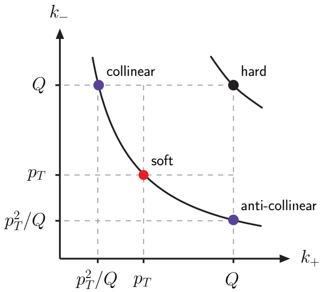
Momentum modes

SCET-1



$\mu_S \ll \mu_J$

SCET-2



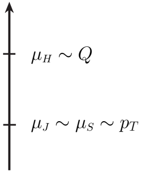
$\mu_S \sim \mu_J$

In SCET-2 one cannot distinguish soft from collinear modes when radiated into jet direction

$\Rightarrow$ need additional regulator that distinguishes modes by their rapidities

SCET-2

Two-scale problem: $\mu_S \sim \mu_J \ll \mu_H$



$$\boxed{d\hat{\sigma} \simeq H(Q,\mu)\; J(p_T,\mu) \otimes S(p_T,\mu)}$$

$$\ln^2 \frac{Q^2}{p_T^2} = \ln^2 \frac{Q^2}{\mu^2} - \ln^2 \frac{p_T^2}{\mu^2} + \quad ?$$

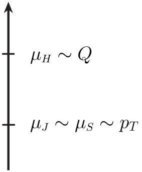
The regulator can be implemented on the level of phase-space integrals [Becher, GB 11]

$$\int d^4k\; \delta(k^2)\,\theta(k^0) \quad \Rightarrow \quad \int d^dk \left(\frac{\nu}{k_+}\right)^{\alpha} \delta(k^2)\,\theta(k^0)$$

$\Rightarrow$ induces rapidity logarithms that cannot be resummed with standard RG techniques

SCET-2

Two-scale problem: $\mu_S \sim \mu_J \ll \mu_H$



$$\boxed{d\hat{\sigma} \simeq H(Q,\mu)\;\; J(p_T,\mu,Q,\nu) \otimes S(p_T,\mu,p_T,\nu)}$$

$$\ln^2 \frac{Q^2}{p_T^2} = \ln^2 \frac{Q^2}{\mu^2} - \ln^2 \frac{p_T^2}{\mu^2} - 2\ln\frac{p_T^2}{\mu^2}\,\ln\frac{Q^2}{\nu^2} - 2\ln\frac{p_T^2}{\mu^2}\,\ln\frac{\nu^2}{p_T^2}$$

Rapidity logarithms exponentiate (in position space)

[Becher, Neubert 10;
Chiu, Jain, Neill, Rothstein 11]

$$\mathcal{J}(x_T,\mu,Q,\nu)\,\mathcal{S}(x_T,\mu,x_T,\nu) = \left(Q^2 x_T^2\right)^{-F(x_T,\mu)}\, W(x_T,\mu)$$

- anomalous dimensions: Γ_{cusp}, γ_H, F
- matching corrections: c_H, W

Applications

e^+e^- event-shape variables

- Thrust (N^{3}LL)
  [Becher, Schwartz 08; Abbate et al 10]
- Heavy jet mass (N^{3}LL)
  [Chien, Schwartz 10]
- C-parameter (N^{3}LL)
  [Hoang, Kolodrubetz, Mateu, Stewart 14]
- Jet broadening (NNLL)
  [Becher, GB 12]
- Angularities (NNLL)
  [GB, Hornig, Lee, Talbert, in progress]

hadron collider observables

- Threshold Drell-Yan (N^{3}LL)
  [Becher, Neubert, Xu 07]
- $W/Z/H$ at large p_T (N^{3}LL)
  [Becher, GB, Lorentzen, Marti 13,14]
- Higgs at small p_T (NNLL)
  [Becher, Neubert, Wilhelm 12]
- Jet veto (NNLL)
  [Becher et al 13; Stewart et al 13]
- N-jettiness (NNLL)
  [Berger et al 10; Jouttenus et al 13]

Can we automate these calculations?

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QCD-based resummation codes: `CAESAR` (NLL) [Banfi, Salam, Zanderighi 04]

`ARES` (NNLL, e^+e^-) [Banfi, McAslan, Monni, Zanderighi 14]

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NNLL′ RESUMMATION

Resummation ingredients

Accuracy	Γ_{cusp}	$\gamma_H, \begin{cases} \gamma_J, \gamma_S \\ F \end{cases}$	$c_H, \begin{cases} c_J, c_S \\ W \end{cases}$
LL	1-loop	–	–
NLL	2-loop	1-loop	tree
NNLL	3-loop	2-loop	1-loop
N^3LL	4-loop	3-loop	2-loop

SCET-1
SCET-2

Precision resummations require observable-dependent 2-loop ingredients

- so far analytic calculations on a case-by-case basis

⇒ develop generic method for automated computations

Dijet soft functions

Definition

$$S(\tau,\mu) = \frac{1}{N_c} \sum_{i\in X} \mathcal{M}(\tau;\{k_i\})\, \mathrm{Tr}\, \langle 0|S_{\bar{n}}^\dagger S_n|X\rangle\langle X|S_n^\dagger S_{\bar{n}}|0\rangle$$

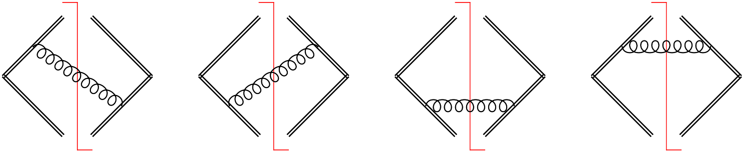
- soft Wilson lines $S_n, S_{\bar{n}}$
- generic measurement function $\mathcal{M}(\tau;\{k_i\})$
- SCET-1 and SCET-2 observables
- relevant for $e^+e^- \to 2$ jets, $e^-p \to 1$ jet, $pp \to 0$ jets

Structure of divergences is independent of the observable

$\Rightarrow$ isolate singularities with universal phase-space parametrisation

$\Rightarrow$ compute observable-dependent integrations numerically

NLO calculation



NLO calculation

One gluon emission

$$S^{(1)}(\tau,\mu) \sim \int d^d k \, \left(\frac{\nu}{k_+ + k_-}\right)^\alpha \delta(k^2)\,\theta(k^0)\ \mathcal{M}_1(\tau;k)\ \big|\mathcal{A}(k)\big|^2$$

- $n \leftrightarrow \bar{n}$ symmetrised version of phase-space regulator
- matrix element $\big|\mathcal{A}(k)\big|^2 \sim \dfrac{1}{k_+ k_-}$

Phase-space parametrisation

$$k_T = \sqrt{k_+ k_-} \qquad y = \frac{k_+}{k_-} \qquad t = \frac{1-\cos\theta}{2}$$

- k_T is only dimensionful variable
- measurement vector v^μ $\rightarrow$ one angle in transverse plane: $\theta \sphericalangle (\vec{k}_\perp, \vec{v}_\perp)$

Measurement function

Generic form

$$\mathcal{M}_1(\tau; k) = \exp\Big(-\tau\, k_T\, y^{n/2}\, f(y,t)\Big)$$

- assumes Laplace transform with $[\tau] = 1/\text{mass}$ $\rightarrow$ fixes k_T dependence
- parameter n is fixed by requirement that $f(y,t)$ is finite and non-zero for $y \rightarrow 0$

Measurement function

Generic form

$$\mathcal{M}_1(\tau; k) = \exp\Big(-\tau\, k_T\, y^{n/2}\, f(y,t)\Big)$$

Observable	n	$f(y,t)$
Thrust	1	1
Angularities	$1-A$	1
Recoil-free broadening	0	$1/2$
Threshold Drell-Yan	-1	$1+y$
W@large p_T	-1	$1+y-2\sqrt{y}\,\cos\theta$
e^+e^- transverse thrust	1	$\frac{1}{s\sqrt{y}}\left(\sqrt{\left(c\cos\theta+\left(\frac{1}{\sqrt{y}}-\sqrt{y}\right)\frac{s}{2}\right)^2+1-\cos^2\theta} - \left\lvert c\cos\theta+\left(\frac{1}{\sqrt{y}}-\sqrt{y}\right)\frac{s}{2}\right\rvert\right)$

$\cos\theta = 1-2t$

NLO master formula

After performing the observable-independent integrations one finds

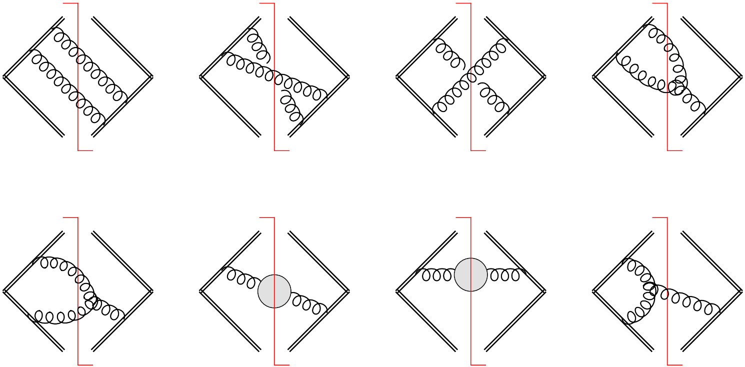
$$S^{(1)}(\tau,\mu) \sim \Gamma(-2\varepsilon-\alpha) \int_0^1 dy \, \frac{y^{-1+n\varepsilon+\alpha/2}}{(1+y)^\alpha} \int_0^1 dt \, \big(4t(1-t)\big)^{-1/2-\varepsilon} \, \big[f(y,t)\big]^{2\varepsilon+\alpha}$$

- singularities from $k_T \to 0$ and $y \to 0$ are factorised
- additional regulator is needed only for $n=0$ ($\to$ SCET-2)

Isolate singularities with standard subtraction techniques

$$\int_0^1 dx \; x^{-1+n\varepsilon} \, f(x) = \int_0^1 dx \; x^{-1+n\varepsilon} \, \Big[\underbrace{f(x)-f(0)}_{\text{finite}} + \underbrace{f(0)}_{1/\varepsilon} \Big]$$

NNLO calculation



NNLO calculation

Double real emission

$$S_{RR}^{(2)}(\tau,\mu) \sim \int d^dk \left(\frac{\nu}{k_+ + k_-}\right)^{\alpha} \delta(k^2)\,\theta(k^0) \int d^dl \left(\frac{\nu}{l_+ + l_-}\right)^{\alpha} \delta(l^2)\,\theta(l^0)\;\mathcal{M}_2(\tau;k,l)\;\left|\mathcal{A}(k,l)\right|^2$$

- higher dimensional phase-space integrations
- three colour structures: $\underbrace{C_F C_A,\; C_F T_F n_f,}_{\text{correlated}}\;\underbrace{C_F^2}_{\text{uncorrelated}}$

Non-trivial matrix element

$$\left|\mathcal{A}(k,l)\right|^2_{C_F T_F n_f} \sim \frac{2k\cdot l\,(k_- + l_-)\,(k_+ + l_+) - (k_- l_+ - k_+ l_-)^2}{(k_- + l_-)^2\,(k_+ + l_+)^2\,(2k\cdot l)^2}$$

$\Rightarrow$ complex singularity structure with overlapping divergences

Correlated emissions

Phase-space parametrisation

$$p_T = \sqrt{(k_+ + l_+)(k_- + l_-)} \qquad y = \frac{k_+ + l_+}{k_- + l_-} \qquad a = \sqrt{\frac{k_- l_+}{k_+ l_-}} \qquad b = \sqrt{\frac{k_- k_+}{l_- l_+}}$$

- p_T is only dimensionful variable
- three angles in transverse plane: $\theta_k \sphericalangle (\vec{k}_\perp, \vec{v}_\perp)$, $\theta_l \sphericalangle (\vec{l}_\perp, \vec{v}_\perp)$, $\theta_{kl} \sphericalangle (\vec{k}_\perp, \vec{l}_\perp)$

Measurement function

$$\mathcal{M}_2^{corr}(\tau; k, l) = \exp\Big(-\tau\, p_T\, y^{n/2}\, F(a, b, y, t_k, t_l, t_{kl})\Big)$$

- p_T dependence fixed on dimensional grounds
- $F(a, b, y, t_k, t_l, t_{kl})$ is finite and non-zero for $y \to 0$

Uncorrelated emissions

Phase-space parametrisation

$$y_k = \frac{k_+}{k_-} \qquad q_T = \sqrt{k_+ k_-} \left(\frac{\sqrt{l_+ l_-}}{l_- + l_+} \right)^{-n} + \sqrt{l_+ l_-} \left(\frac{\sqrt{k_+ k_-}}{k_- + k_+} \right)^{-n}$$

$$y_l = \frac{l_+}{l_-} \qquad b = \sqrt{\frac{k_+ k_-}{l_+ l_-}} \left(\frac{\sqrt{k_+ k_-}}{k_- + k_+} \right)^{n} \left(\frac{\sqrt{l_+ l_-}}{l_- + l_+} \right)^{-n}$$

- q_T is only dimensionful variable; again three angles θ_k, θ_l, θ_{kl}

Measurement function

$$\mathcal{M}_2^{unc}(\tau; k, l) = \exp\Big(- \tau\, q_T\, y_k^{n/2}\, y_l^{n/2}\, G(y_k, y_l, b, t_k, t_l, t_{kl}) \Big)$$

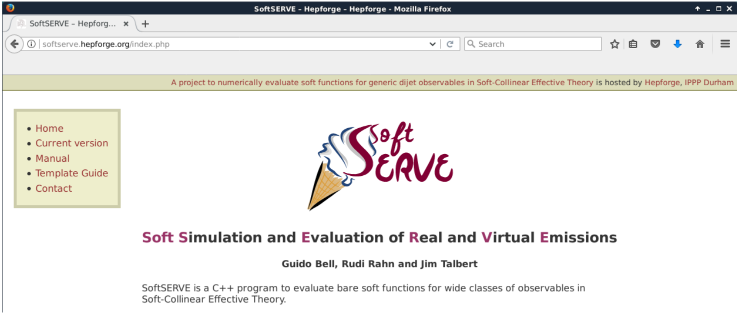
- q_T dependence fixed on dimensional grounds
- $G(y_k, y_l, b, t_k, t_l, t_{kl})$ is finite and non-zero for $y_k \to 0$ and $y_l \to 0$

SoftSERVE

[GB, Rahn, Talbert to appear]

C++ program for numerical evaluation of soft functions

- Divonne integrator from Cuba library
- phase-space remappings to improve numerical convergence
- option to work with multi-precision variables (`boost`, `GMP/MPFR`)
- bash scripts for renormalisation in Laplace and cumulant space



Available results

e^+e^- event-shape variables

- Thrust
  [Kelley et al 11; Monni et al 11]
- C-parameter
  [Hoang et al 14]
- Recoil-free broadening
  [Becher, GB 12]
- Angularities
  [—]
- Hemisphere masses
  [Kelley et al 11; Hornig al 11]

hadron collider observables

- Threshold Drell-Yan
  [Belitsky 98]
- W at large p_T
  [Becher et al 12]
- p_T resummation
  [Becher, Neubert 10; Echevarria et al 15]
- p_T jet veto
  [Banfi et al 12; Becher et al 13; Stewart et al 13]
- Rapidity dependent jet vetoes
  [Gangal et al 16]
- Soft-drop jet groomer
  [—]
- Transverse thrust
  [Becher at al 15]

Soft anomalous dimension

RG equation in Laplace space

$$\frac{d\,S(\tau,\mu)}{d\ln\mu} = -\frac{1}{n}\left[4\,\Gamma_{\rm cusp}(\alpha_s)\,\ln(\mu\bar\tau) - 2\gamma^S(\alpha_s)\right] S(\tau,\mu)$$

Two-loop solution with $L = \ln(\mu\bar\tau)$

$$S(\tau,\mu) = 1 + \left(\frac{\alpha_s}{4\pi}\right)\left\{-\frac{2\Gamma_0}{n}L^2 + \frac{2\gamma_0^S}{n}L + c_1^S\right\} + \left(\frac{\alpha_s}{4\pi}\right)^2\left\{\frac{2\Gamma_0^2}{n^2}L^4 - 4\Gamma_0\left(\frac{\gamma_0^S}{n^2}+\frac{\beta_0}{3n}\right)L^3\right.$$
$$\left. -2\left(\frac{\Gamma_1}{n} - \frac{(\gamma_0^S)^2}{n^2} - \frac{\beta_0\gamma_0^S}{n} + \frac{\Gamma_0 c_1^S}{n}\right)L^2 + 2\left(\frac{\gamma_1^S}{n} + \frac{\gamma_0^S c_1^S}{n} + \beta_0 c_1^S\right)L + c_2^S\right\}$$

Results will be presented in the form

$$\gamma_1^S = \gamma_1^{C_A}\,C_F C_A + \gamma_1^{n_f}\,C_F T_F n_f + \gamma_1^{C_F}\,C_F^2$$

$$c_2^S = c_2^{C_A}\,C_F C_A + c_2^{n_f}\,C_F T_F n_f + c_2^{C_F}\,C_F^2$$

Performance

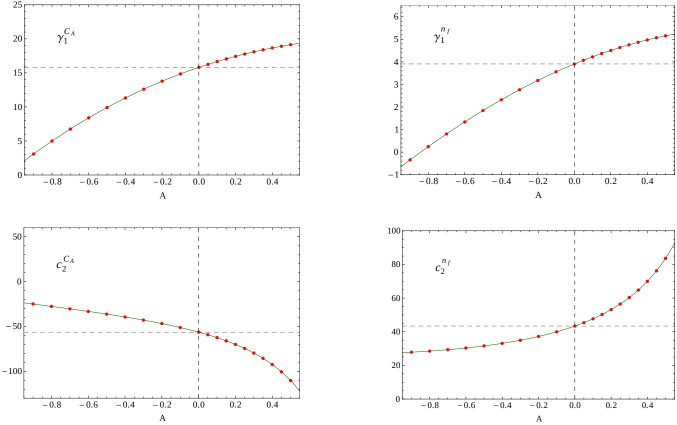
W at large p_T	$c_2^{C_A}$	$c_2^{n_f}$	runtime*
standard setting	-2.660 ± 0.075	-25.313 ± 0.009	30 sec
precision setting	-2.651 ± 0.005	-25.307 ± 0.001	9 h
analytic	-2.650	-25.307	[Becher et al 12]

C-parameter	$c_2^{C_A}$	$c_2^{n_f}$	runtime*
standard setting	-57.893 ± 0.039	43.817 ± 0.004	25 sec
precision setting	-57.973 ± 0.004	43.818 ± 0.001	20 min
EVENT2	-58.16 ± 0.26	43.74 ± 0.06	[Hoang et al 14]

* on a single 8-core machine

Angularities

e^+e^- event shape that interpolates between thrust ($A = 0$) and broadening ($A = 1$)



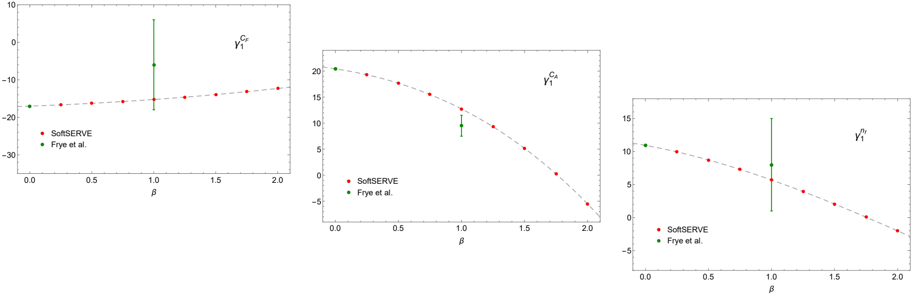
$\Rightarrow$ last missing ingredient for NNLL resummation

[GB, Hornig, Lee, Talbert to appear]

Soft-drop jet mass

Jet grooming removes soft radiation from jets [Frye, Larkoski, Schwartz, Yan 16]

- parameter β controls aggressiveness of groomer
- observable violates non-abelian exponentiation theorem $\Rightarrow$ non-trivial C_F^2 structure
- confirm and extend existing NNLO results



N-jet soft functions

Definition

$$S(\tau,\mu) = \sum_{i\in X} \mathcal{M}(\tau;\{k_i\})\, \langle 0|(S_{n_1} S_{n_2} S_{n_3} \ldots)^\dagger |X\rangle\langle X| S_{n_1} S_{n_2} S_{n_3} \ldots |0\rangle$$

- soft function is a matrix in colour space
- generic measurement function $\mathcal{M}(\tau;\{k_i\})$
- SCET-1 and SCET-2 observables
- assume non-abelian exponentiation in a first step

Motivation

- resummation for hadronic event shapes, boosted top observables, ...
- subtraction technique for NNLO calculations [Catani, Grazzini 07; Boughezal et al 15; Gaunt et al 15]

N-jet soft functions

Technical aspects

- 2-particle correlations are similar to dijet case

  $\Rightarrow$ generalise phase-space parametrisations to arbitrary $n_i \cdot n_j$

- 3-particle correlations arise only in real-virtual contribution ($\rightarrow$ NAE)
- no 4-particle correlations ($\rightarrow$ NAE)

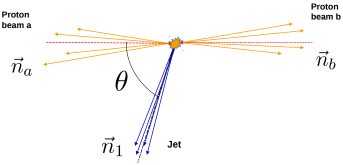
N-jettiness [Stewart, Tackmann, Waalewijn 10]

$$\mathcal{T}_N(\{k_i\}) = \sum_i \min_j (n_j \cdot k_i) \qquad j = \underbrace{a, b}_{\text{beams}}, \underbrace{1, \ldots, N}_{\text{jets}}$$

- 1-jettiness soft function known to NNLO [Boughezal, Liu, Petriello 15; Campbell, Ellis, Mondini, Williams 17]
- general case $N \geq 2$ known to NLO [Jouttenus, Stewart, Tackmann, Waalewijn 11]

1-jettiness

Kinematics

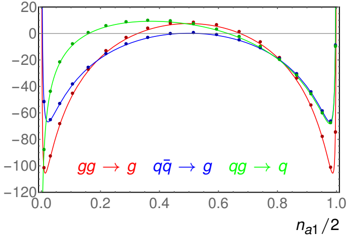


$$n_{ab} \equiv n_a \cdot n_b = 2$$

$$n_{a1} \equiv n_a \cdot n_1 = 1 - \cos\theta$$

$$n_{b1} \equiv n_b \cdot n_1 = 1 + \cos\theta$$

Two-loop finite terms for different partonic channels (preliminary)

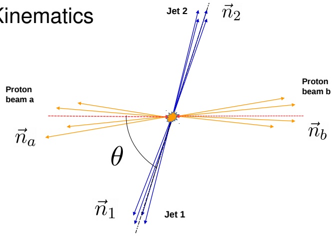


dots: [GB, Dehnadi, Mohrmann, Rahn to appear]

lines: [Campbell, Ellis, Mondini, Williams 17]

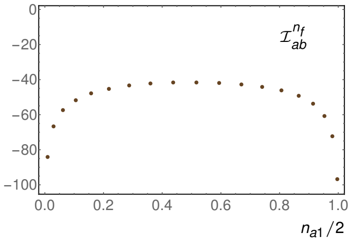
2-jettiness

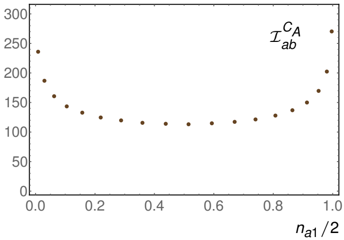
Kinematics



$$n_{ab} \equiv n_a \cdot n_b = n_1 \cdot n_2 = 2$$
$$n_{a1} \equiv n_a \cdot n_1 = n_b \cdot n_2 = 1 - \cos\theta$$
$$n_{b1} \equiv n_b \cdot n_1 = n_a \cdot n_2 = 1 + \cos\theta$$

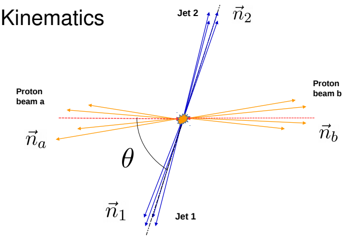
Two-loop finite terms: dipole contributions (preliminary)





2-jettiness

Kinematics



$$n_{ab} \equiv n_a \cdot n_b = n_1 \cdot n_2 = 2$$
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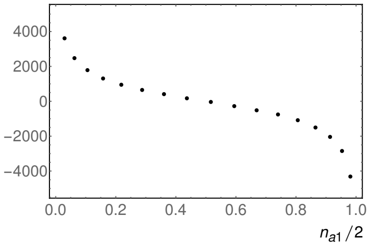
$$n_{b1} \equiv n_b \cdot n_1 = n_a \cdot n_2 = 1 + \cos\theta$$

Two-loop finite terms: sum of tripole contributions (preliminary)



dots: [GB, Dehnadi, Mohrmann, Rahn to appear]

OUTLINE

SOFT-COLLINEAR EFFECTIVE THEORY

SCALES, MODES, SCET-1 AND SCET-2

AUTOMATED CALCULATION OF SOFT FUNCTIONS

DIJET SOFT FUNCTIONS

SOFT SERVE

N-JET OBSERVABLES

ANGULARITIES

SCET-1 VS SCET-2

NNLL′ RESUMMATION

Angularities

e^+e^- event shape that depends on a continous parameter a [Berger, Kucs, Sterman 03]

$$e_a(\{k_i\}) = \sum_i |k_\perp^i|\, e^{-|\eta_i|\,(1-a)}$$

- interpolates between thrust ($a=0$) and total jet broadening ($a=1$)

Factorisation theorem for $e_a \to 0$

$$\frac{1}{\sigma_0}\frac{d\sigma}{de_a} \simeq H(Q,\mu) \int de_n\, de_{\bar{n}}\, de_s\;\, J_n(e_n,\mu)\, J_{\bar{n}}(e_{\bar{n}},\mu)\, S(e_s,\mu)\, \delta(e_a - e_n - e_{\bar{n}} - e_s)$$

- relevant scales: $\mu_H^2 \sim Q^2 \gg \mu_J^2 \sim Q^2 e_a^{\frac{2}{2-a}} \gg \mu_S^2 \sim Q^2 e_a^2$
- thrust: $\mu_H^2 \sim Q^2 \gg \mu_J^2 \sim Q^2 e_0 \gg \mu_S^2 \sim Q^2 e_0^2$ (SCET-1)
- broadening: $\mu_H^2 \sim Q^2 \gg \mu_J^2 \sim Q^2 e_1^2 \sim \mu_S^2 \sim Q^2 e_1^2$ (SCET-2)

SCET-1 vs SCET-2

Resummation in Laplace space

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau_a} \simeq H(Q,\mu)\, \widetilde{J}_n(\tau_a,\mu)\, \widetilde{J}_{\bar{n}}(\tau_a,\mu)\, \widetilde{S}(\tau_a,\mu)$$

Different formalisms

<u>SCET-1:</u>
$(a < 1)$

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_a} = e^{4S(\mu_H,\mu_J)-2A_H(\mu_H,\mu_J)+\frac{4}{1-a}S(\mu_S,\mu_J)+\frac{2}{1-a}A_S(\mu_J,\mu_S)} \left(\frac{Q^2}{\mu_H^2}\right)^{-2A_\Gamma(\mu_H,\mu_J)}$$
$$\times\, (\mu_s\bar{\tau}_a)^{-\frac{4}{1-a}A_\Gamma(\mu_J,\mu_S)}\; H(Q,\mu_H)\, \widetilde{J}_n(\tau_a,\mu_J)\, \widetilde{J}_{\bar{n}}(\tau_a,\mu_J)\, \widetilde{S}(\tau_a,\mu_S)$$

<u>SCET-2:</u>
$(a = 1)$

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\sigma}{\mathrm{d}\tau_a} = e^{4S(\mu_H,\mu_S)-2A_H(\mu_H,\mu_S)} \left(\frac{Q^2}{\mu_H^2}\right)^{-2A_\Gamma(\mu_H,\mu_J)} \left(\frac{\nu_J}{\nu_S}\right)^{-2F(\tau_a,\mu_S)}$$
$$\times\, H(Q,\mu_H)\, \widetilde{J}_n(\tau_a,\mu_J,\nu_J)\, \widetilde{J}_{\bar{n}}(\tau_a,\mu_J,\nu_J)\, \widetilde{S}(\tau_a,\mu_S,\nu_S)$$

Can one derive the SCET-2 expression directly from SCET-1? [Larkoski, Neill, Thaler 14]

Mapping rapidity onto RGE logs

Compare RG equations

$$\frac{d\ln\widetilde{S}(\tau_a,\mu_S)}{d\ln\mu_S} = -\tfrac{4}{1-a}\,\Gamma_{\rm cusp}\,\ln(\mu_S\bar{\tau}_a) + \ldots$$

$$\frac{d\ln\widetilde{S}(\tau_a,\mu_S,\nu_S)}{d\ln\mu_S} = 4\,\Gamma_{\rm cusp}\,\ln(\mu_S\bar{\tau}_a) - 4\,\Gamma_{\rm cusp}\,\ln(\nu_S\bar{\tau}_a) + \ldots$$

$$\Bigg\} \quad \mu_S = \nu_S^{\frac{1-a}{2-a}}\;\tau_a^{-\frac{1}{2-a}}$$

$$\frac{d\ln\widetilde{J}(\tau_a,\mu_J)}{d\ln\mu_J} = -2\,\tfrac{2-a}{1-a}\,\Gamma_{\rm cusp}\,\ln\left(\frac{Q^{(1-a)/(2-a)}}{\mu_J\,\bar{\tau}_a^{1/(2-a)}}\right) + \ldots$$

$$\frac{d\ln\widetilde{J}(\tau_a,\mu_J,\nu_J)}{d\ln\mu_J} = 2\,\Gamma_{\rm cusp}\,\ln\left(\frac{\nu_J}{Q}\right) + \ldots$$

$$\Bigg\} \quad \mu_J = \nu_J^{\frac{1-a}{2-a}}\;\tau_a^{-\frac{1}{2-a}}$$

$$\Rightarrow \quad \boxed{\frac{\mu_J}{\mu_S} = 1 + (1-a)\,\ln\frac{\nu_J}{\nu_S} + \mathcal{O}(1-a)^2}$$

SCET-2 limit

Smooth limit for RG kernels

$$\frac{1}{1-a}\ S(\mu_S,\mu_J)\ \xrightarrow{a\to 1}\ \mathcal{O}(1-a)$$

$$\frac{1}{1-a}\ A_S(\mu_J,\mu_S)\ \xrightarrow{a\to 1}\ \gamma^S\Big(\alpha_s(1/\bar{\tau})\Big)\ \ln\frac{\nu_J}{\nu_S}\ +\ \mathcal{O}(1-a)$$

Matching corrections are divergent in the limit $a \to 1$

$$c_1^J\ \xrightarrow{a\to 1}\ \frac{d_1'}{2(1-a)}\ +\ \hat{c}_1^J\ +\ \mathcal{O}(1-a)$$

$$c_1^S\ \xrightarrow{a\to 1}\ -\frac{d_1'}{(1-a)}\ +\ \hat{c}_1^S\ +\ \mathcal{O}(1-a)$$

but the product of jet and soft functions is well-defined!

SCET-2 limit

Matching corrections yield additional contribution to anomaly exponent

$$\frac{\alpha_s(\mu_J)}{4\pi}\left\{2c_1^J\right\} + \frac{\alpha_s(\mu_S)}{4\pi}\left\{c_1^S\right\} \xrightarrow{a\to 1} -2\left(\frac{\alpha_s(1/\bar{\tau})}{4\pi}\right)^2 \beta_0\, d_1' \ln\frac{\nu_J}{\nu_S} + \mathcal{O}(1-a)$$

Relation between collinear anomaly and soft anomalous dimension

$$\begin{aligned} d_1 &= -\gamma_0^S \\ d_2 &= -\gamma_1^S + \beta_0\, d_1' \end{aligned}$$

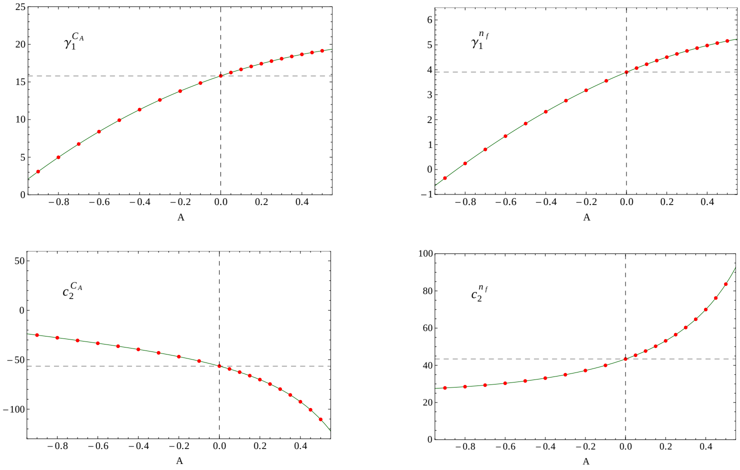
where d_1' is the ε-coefficient of the one-loop anomaly exponent

$$F(\tau,\mu=1/\bar{\tau}) = \frac{\alpha_s}{4\pi}\left\{d_1 + d_1'\,\varepsilon\right\} + \left(\frac{\alpha_s}{4\pi}\right)^2\left\{d_2 + d_2{}'\,\varepsilon\right\} + \dots$$

Resembles connection between threshold and p_T resummation [Li,Zhu 16; Vladimirov 16]

NNLO soft function

Two-loop soft anomalous dimension and matching correction from `SoftSERVE`



State of the art

Accuracy	Γ_{cusp}	$\gamma_H, \gamma_J, \gamma_S$	c_H, c_J, c_S
NLL	2-loop	1-loop	tree
NLL′	2-loop	1-loop	1-loop
NNLL	3-loop	2-loop	1-loop
NNLL′	3-loop	2-loop	2-loop

[Hornig, Lee, Ovanesyan 09]

- γ_H and c_H are known to 2-loop
- 2-loop γ_S and c_S from SoftSERVE
- $\gamma_J = \frac{1}{2}\,\gamma_H + \frac{1}{2(1-a)}\,\gamma_S$ fixed by RG invariance
- extract 2-loop c_J from EVENT2-fit

⇒ extend resummation to NNLL′ accuracy

Theory input

Further refinements

- matching to fixed-order α_s^2 calculation $\Rightarrow$ NNLL' + NLO accuracy
- angularity-dependent profile scales
- non-perturbative shape function

$$\frac{d\sigma}{de_a}(e_a) \xrightarrow{\text{tail region}} \frac{d\sigma}{de_a}\left(e_a - \frac{2}{1-a}\frac{\Omega_1}{Q}\right)$$

[Lee, Sterman 06]

same NP parameter Ω_1 that controls shift of thrust and C-parameter distributions

Theory input

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[Lee, Sterman 06]

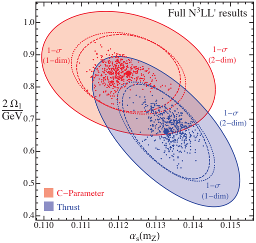
same NP parameter Ω_1 that controls shift of thrust and C-parameter distributions

Thrust:

[Abbate et al 10]

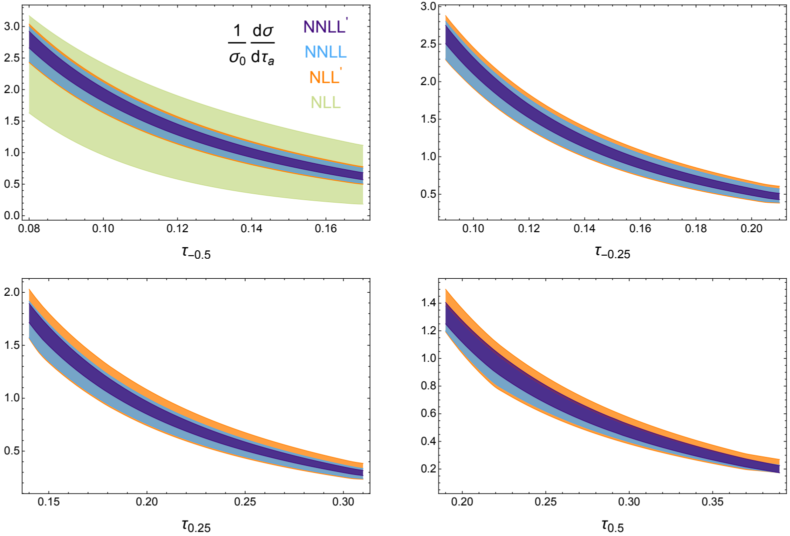
C-parameter:

[Hoang et al 15]



$\Rightarrow$ test implementation of non-perturbative effects with angularity distributions

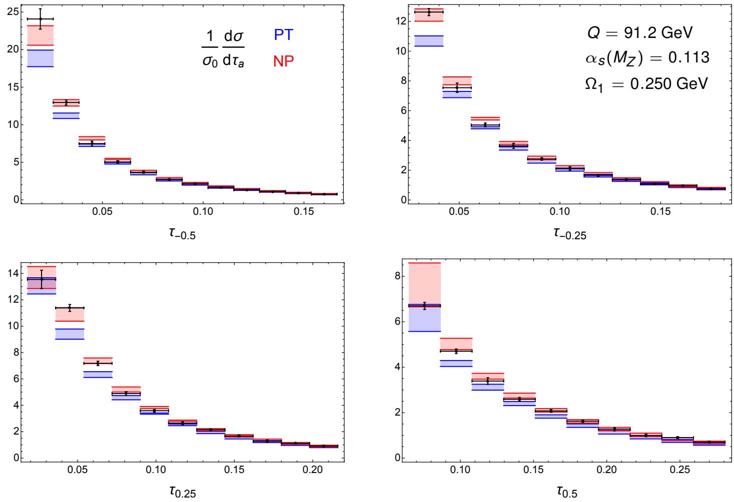
Convergence



Comparison to L3 data

[GB, Hornig, Lee, Talbert to appear]

Binned distributions without / with non-perturbative effects (preliminary)



Conclusions

Automated calculation of NNLO soft functions

- softserve.hepforge.org
- compact integral representation for anomalous dimensions [GB, Rahn, Talbert 18]
- explicit results for $\sim$ 15 dijet soft functions
- first results for N-jet observables

Angularities

- comparison of SCET-1 and SCET-2 resummations
- precision analysis at NNLL' + NLO accuracy
- α_s determination in progress

Backup slides

SCET-1 results

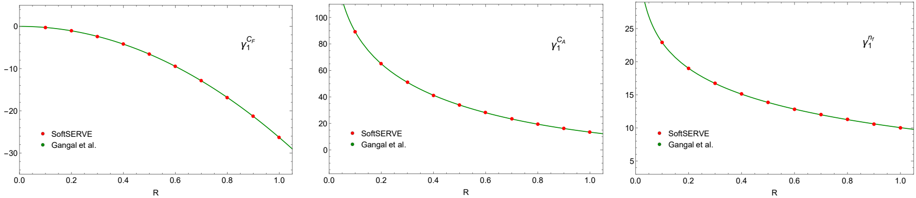
Observable	$\gamma_1^{C_A}$	$\gamma_1^{n_f}$	$c_2^{C_A}$	$c_2^{n_f}$
Thrust [Kelley et al, Monni et al 11]	15.7945 (15.7945)	3.90981 (3.90981)	−56.4992 (−56.4990)	43.3902 (43.3905)
C-parameter [Hoang et al 14]	15.7947 (15.7945)	3.90980 (3.90981)	−57.9754 (-58.16 ± 0.26)	43.8179 (43.74 ± 0.06)
Threshold Drell-Yan [Belitsky 98]	15.7946 (15.7945)	3.90982 (3.90981)	6.81281 (6.81287)	−10.6857 (−10.6857)
W@large p_T [Becher et al 12]	15.7947 (15.7945)	3.90981 (3.90981)	−2.65034 (−2.65010)	−25.3073 (−25.3073)
Transverse thrust [Becher, Garcia 15]	−158.278 (-148^{+20}_{-30})	19.3955 (18^{+2}_{-3})	—	—

- upper numbers: `SoftSERVE` / `SecDec` in a few hours on a single machine
- lower numbers: analytic (black) or fit to fixed-order code (gray)

Rapidity-dependent jet veto

Non-standard jet veto in Higgs production [Gangal, Stahlhofen, Tackmann 14]

- clustering effects violate non-abelian exponentiation
- RGE in momentum space ($\rightarrow$ inverse Laplace transform)
- confirm existing NNLO results [Gangal, Gaunt, Stahlhofen, Tackmann 16]



SCET-2

Anomaly exponent obeys simple RG equation

$$\frac{d\,F(\tau,\mu)}{d\ln\mu} = 2\,\Gamma_{\rm cusp}(\alpha_s)$$

Two-loop solution with $L = \ln(\mu\bar{\tau})$

$$F(\tau,\mu) = \left(\frac{\alpha_s}{4\pi}\right)\left\{2\Gamma_0 L + d_1\right\} + \left(\frac{\alpha_s}{4\pi}\right)^2\left\{2\Gamma_0\beta_0 L^2 + 2\,(\Gamma_1 + \beta_0 d_1)\,L + d_2\right\}$$

- two-loop anomaly coefficient $\quad d_2 = d_2^{C_A}\,C_F C_A + d_2^{n_f}\,C_F T_F n_f + d_2^{C_F}\,C_F^2$

SCET-2 results

Observable	$d_2^{C_A}$	$d_2^{n_f}$
Recoil-free broadening [Becher, GB 12]	7.03595 (7.03605)	-11.5393 (-11.5393)
p_T resummation [Becher, Neubert 10]	-3.73389 (-3.73167)	-8.29610 (-8.29630)
E_T resummation	15.9804 ($-$)	-18.7370 ($-$)
Transverse thrust [Becher et al 15]	208.098 (208.0 ± 0.1)	-37.1766 (-37.191 ± 0.006)

- do not confirm QCD result for E_T resummation

$$B_g^{(2)} = \frac{1}{16}\left(d_2 + 2\gamma_1^g + \beta_0\, e_1^g\right) = \begin{cases} 33.0081 & \texttt{SoftSERVE} \\ -5.1 \pm 1.6 & \text{[Grazzini et al 14]} \end{cases}$$

p_T jet veto

Standard jet veto based on transverse momenta

- SCET-2 observable with clustering effects
- RGE in momentum space ($\rightarrow$ inverse Laplace transform)
- confirm existing NNLO results [Banfi et al 12; Becher et al 13; Stewart et al 13]