

N3LO QCD PRECISION FOR HIGGS BOSON Phenomenology at the LHC

with Falko Dulat, Achilleas Lazopoulos, Andrea Pelloni

- 4th of July 2012: The begin of the precision physics age of Higgs boson phenomenology
- Immediately after the discovery of the Higgs boson we started to ask questions about it's nature:
 $m_h = 125.09 {
 m GeV} \pm 0.2\%$ Couplings, spin, parity, mass, cross sections ...

- The basis for testing our understanding of nature is on the one side precise measurements that are sensitive to the Higgs boson properties.
- LHC provides the input!
 Run 2: Data, data, data

Now: 2 % of future LHC data analysed.



DEMAND FOR PRECISION ON THEORY SIDE

- Testing our understanding of nature: Compare experiment and theory!
- The key to theoretical predictions at the LHC:



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- Testing our understanding of nature: Compare experiment and theory!
- The key to theoretical predictions at the LHC:

$$\sigma \sim \int dx dy f(x) f(y) \hat{\sigma} + \mathcal{O}\left(\frac{\Lambda}{Q}\right)$$
$$\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_S^1 \hat{\sigma}^{(0)} + \alpha_S^2 \hat{\sigma}^{(2)} + \alpha_S^3 \hat{\sigma}^{(3)} \dots$$

- Compute perturbativ corrections from first principle QFT: Standard Model
- Allows for % level predictions for experimental precision will reach comparable levels!



produced?

4 WAYS TO PRODUCE A HIGGS



PERTURBATIVE CORRECTIONS TO HIGGS BOSON PRODUCTION



- N3LO corrections stabilise perturbative expansion.
- Significant reduction in residual perturbative uncertainty estimates.
- High orders are required!

GLUON FUSION

INGREDIENT NR1: GLUONS.

Probability to get two gluons out of the proton as a function of the partonic center of mass energy:



COMPUTING N3LO CROSS SECTIONS IS CHALLENGING

Simplifications:



- Removes one loop!
- > Excellent approximation: Captures dominant QCD effects. $\delta_t^{\rm LO}\sim 7\%$

$$\delta_t^{\rm NLO} \sim 0.7\%$$

Supplement with mass corrections, EWK corrections etc.

COMPUTING N3LO CROSS SECTIONS IS CHALLENGING

Simplifications:

Perform expansion around kinematic limit: Production Threshold



- Simplifies the analytic functions: Only numbers!
- Expand to sufficiently high order to ensure stable results.



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Simplifications:



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NEW: EXACT SOLUTION FOR GLUON FUSION @ N3L0

- Major analytic effort: 912 master integrals!
- Challenge of new analytic functions: Elliptic integrals!
- Elliptic integrals:

We are at the beginning of understanding these functions! Analytic continuation, numerical evaluation, functional identities, ...



EXPANDED VS. EXACT



INDIVIDUAL INITIAL STATE CHANNELS



INDIVIDUAL INITIAL STATE CHANNELS



PHENOMENOLOGY: IT'S COMPLICATED



Many contributions to be take into account: QCD, EWK,

 m_t, m_b, m_c

Many sources of uncertainty to estimate! Perturbative truncation, PDF, α_S



HIGGS BOSON MEASUREMENTS

ENTER THE AGE OF PRECISION HIGGS PHYSICS



- Incredible agreement of data and theory
- Triumph of SM predictions
- Higgs production
 ~10 sigma observed

IHIXS 2 – NEW CODE!

ihixs 2 Inclusive Higgs XS at N3LO

ihixs 2

by F.Dulat, A. Lazopoulos, B. Mistlberger

A program for the inclusive Higgs boson cross-section at hadron colliders. It incorporates QCD corrections through N3LO, electroweak corrections, mixed QCDelectroweak corrections, light quark-mass effects through NLO in QCD, top mass effects at NNLO, and a reliable estimate of theoretical uncertainties due to various sources, including PDFs and the strong coupling.

Download

https://people.phys.ethz.ch/~pheno/ihixs/



Differential Cross Sections

- Inclusive cross sections are idealised objects
 Important test of QFT, extraction of coupling constants, etc.
- Real life observables: Fiducial cross sections for realistic final states!
- Avoid extrapolation:
 - Predict as close to experimental outcome as possible

CHALLENGES OF DIFFERENTIAL PREDICTIONS

- Analytic complexity of high order perturbative computation
 - Complicated mathematical structures: Elliptic / multiple polylogarithms, couple differential equations, algebraic complexity, ...
- Numerical integration over complicated and "divergent" final state configurations:
 - Infrared subtraction at 2-loops and beyond.
 - Main challenge of the last couple of years.
 - Many methods available now.

- Sector decomposition
- Non-Linear Mappings

qT
FKS+
N-Jettiness H+J
Antenna
Colourful

. . .

Projection-To-Born VB

CHALLENGES OF DIFFERENTIAL PREDICTIONS

- Analytic complexity of high order perturbative computation
 - Complicated mathematical structures: Elliptic / multiple polylogarithms, couple differential equations, algebraic complexity, ...
- Numerical integration over complicated and "divergent" final state configurations:

- Introduce a framework that allows to compute differential cross sections at N3LO.
- Circumvent problems of NNLO infrared subtraction.
- Applicable for real life observables at the LHC.

Specifically: Differential Higgs Production in QCD



$$P \ P \to H + X \to \gamma \gamma + X$$

$$P P \to H + X \to 4l + X$$

Today: Recent Progress, NNLO, Obstacles, Method

Focus on the degrees of freedom of the Higgs boson:

 $p_h \equiv (E, p_x, p_y, p_z) = \left(\sqrt{p_T^2 + m_h^2} \cosh Y, \ p_T \cos \phi, \ p_T \sin \phi, \ \sqrt{p_T^2 + m_h^2} \sinh Y\right)$

Entirely described in terms of pT and Y (and a trivial azimuthal angle).

$$Y = \frac{1}{2} \log \left(\frac{E + p_z}{E - p_z} \right)$$

$$p_T = \sqrt{E^2 - p_z^2 - m_h^2}.$$

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$$\sigma_{PP \to H+X} \left[\mathcal{O} \right] = \sum_{i,j} \int_{-\infty}^{+\infty} dY \int_{0}^{\infty} dp_T^2 \int_{0}^{2\pi} \frac{d\phi}{2\pi} \int_{0}^{1} dx_1 dx_2 f_i(x_1) f_j(x_2)$$

$$\times \frac{d^2 \hat{\sigma}_{ij}}{dY dp_T^2} (S, x_1, x_2, m_h^2, Y, p_T^2) \mathcal{J}_{\mathcal{O}(Y, p_T^2, \phi, m_h^2)}$$

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- Entirely described in terms of pT and Y (and a trivial azimuthal angle).
- Higgs-differential cross section:

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Integrate over
Higgs degrees of freedom

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Partonic Higgs-differential
$$\times \underbrace{\frac{d^{2} \hat{\sigma}_{ij}}{dY dp_{T}^{2}}}_{\text{cross section}} \left(S, x_{1}, x_{2}, m_{h}^{2}, Y, p_{T}^{2} \right) \mathcal{J}_{\mathcal{O}}(Y, p_{T}^{2}, \phi, m_{h}^{2})$$

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PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS

$$\frac{d^2 \hat{\sigma}_{ij}}{dY dp_T^2} \sim \sum_X \int d\Phi_n \left| \mathcal{M}_{ij \to H+X} \right|^2$$

PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS

$$\frac{d^2 \hat{\sigma}_{ij}}{dY dp_T^2} \sim \sum_X \int d\Phi_n \left| \mathcal{M}_{ij \to H+X} \right|^2$$

Compute all required matrix elements of different final states X to a given order in perturbation theory.



PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS



- Phase space integral over partonic final state phase space momenta
- Integrate over as many partons as there are in X.
- Integration over fixed multiplicity matrix elements is divergent! (KLN).

PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS



- Perform integration over parton phase space analytically
- Rely on tools to perform analytic computation learned from inclusive N3LO
- Make singularities of final state parton integrations manifest using dimensional regularisation.

$$d = 4 - 2\epsilon$$

PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS

REVERSE UNITARITY FRAMEWORK:

Replace on-shell constraints with cut propagators

$$\delta_+(p_i^2) \sim \lim_{\delta \to 0} \left[\frac{1}{p_i^2 + i\delta} - \frac{1}{p_i^2 - i\delta} \right] = \left[\frac{1}{p_i^2} \right]_c$$

REVERSE UNITARITY FRAMEWORK:



Opens the door to large variety of loop integral technology!

IBPs + Differential equations

Key observation: Cut propagators can be differentiated similar to usual propagators.

REVERSE UNITARITY FRAMEWORK:

$$\frac{d^2 \hat{\sigma}_{ij}}{dY dp_T^2} \sim \sum_X \int d\Phi_n \left| \mathcal{M}_{ij \to H+X} \right|^2$$
$$= \sum_X \sum_i c_{X,i} F_{X,i}(S, p_T, Y, m_h^2)$$

- Coefficient: Rational function of remaining kinematic variables.
- Master Integral: Integrated Feynman integrals: Polylogarithms, rational functions of remaining kinematic variables.
- Explicit Laurent series in dimensional regulator.

HIGGS - DIFFERENTIAL CROSS SECTIONS: RAPIDITY

- Inclusive rapidity distribution
- Large K-factors



HIGGS - DIFFERENTIAL CROSS SECTIONS: FIDUCIAL XS (ATLAS)

 $P \ P \to H + X \to \gamma \gamma + X$

- Fiducial rapidity distribution.
- Non-trivial features due to selection criteria.
- Relatively flat K-factors
- Similar perturbative behaviour as inclusive distribution



HIGGS – DIFFERENTIAL CROSS SECTIONS: FIDUCIAL XS

 $P \ P \to H + X \to \gamma \gamma + X$

- Distributions of the photon momenta:
 - Leading Photon pT
 - Pseudo rapidity difference



 $\Delta \eta$



$$\Delta \eta = |\eta_{\gamma_1} - \eta_{\gamma_2}|$$

BEYOND NNLO



WHAT DID WE LEARN FROM NNLO

- Higgs-differential cross sections: fast and stable framework for fiducial cross sections.
- Analytic computation at NNLO comparably simple.

MAIN CHALLENGES FOR N3LO

- Rapid growth in analytic complexity: Many more integrals to compute, large rational expressions as a result
- Numerical stability vs. speed in evaluation of analytic coefficients.

FIXED ORDER MATRIX ELEMENTS

Rapid growth in complexity



1000 @ NNLO



1000 @ NNLO



Missing matrix elements with 2 or 3 final state partons.

- Same strategy as for NNLO: Analytic computation using reverse unitarity, master integrals and differential equations.
- Number of master integrals required: 100 x NNLO.
- Solving differential equations for master integrals:
 Need boundary conditions = Master integrals evaluated at one single point.

THRESHOLD EXPANSION

THRESHOLD EXPANSION FOR DIFFERENTIAL CROSS SECTIONS ???



Excellent approximation for inclusive cross section.

Reason Nr.1:

Crucial analytic information a full calculation relies on - boundary conditions. + checks, testing ground for technology, etc.

Reason Nr. 2: Can we use it for phenomenology?

THRESHOLD EXPANSION

THRESHOLD EXPANSION @ NNLO DIFFERENTIAL



THRESHOLD EXPANSION @ NNLO DIFFERENTIAL

Rapidity distribution normalised to true value.



Bulk of XS is described well with a couple of terms

Systematic improvement possible

THRESHOLD EXPANSION @ NNLO DIFFERENTIAL

PT distribution



THRESHOLD EXPANSION @ NNLO DIFFERENTIAL

PT distribution normalised to true value.



THRESHOLD EXPANSION

- Systematically improvable approximation.
- Soft expansion gives the opportunity to study differential distribution
- Doing phenomenology in this approximation requires careful case by case analysis to see if the approximation is valid!



IVI

THE ROAD TO N3LO VIA THRESHOLD EXPANSIONS

- Extend analytic techniques for automatic soft amplitude expansions.
- Apply reverse unitarity, differential equations, Multiple PolyLog, IBPs, Symbol tools,
- Compute 110 new double differential soft master integrals.
- Compute the first terms (Soft-Virtual SV) at N3LO
- Put into code and look at the N3LO corrections to the rapidity distribution and ...

SV @ N3L0



Rapidity y

BE CAREFUL WHEN YOU DO SOMETHING NEW

$$\sigma \sim \int dz \mathcal{L}_{gg}(z) \left[\frac{log^5(1-z)}{1-z} \right]_+$$
LHAPDF

- LHAPDF: Grid of points for PDFs in x and Q
- Interpolation between points with certain precision
- Not meant to be precise enough for N3LO plus distributions yet
- Improvements required: New interpolator, evolve from smooth PDF ?

N3LO CORRECTIONS TO THE RAPIDITY DISTRIBUTION



N3LO CORRECTIONS TO THE RAPIDITY DISTRIBUTION



FIRST RESULTS

- So far: We computed the first two terms in the threshold expansion.
- Important step: All required analytical boundary information for full computation obtained.
- Remember: 2 terms in the threshold expansion are not enough!
- However

FIRST RESULTS: N3LO CORRECTIONS ON RAPIDITY DISTRIBUTION



FIRST RESULTS: FIRST COMBINATION WITH LOWER ORDERS



- Exact computation of N3LO inclusive Higgs production cross section.
- Higgs-differential cross sections: Promising framework for realistic final state observables.
- Threshold expansions provide a key ingredient for analytic computation.
- Threshold expansion can be used at the differential level to approximate differential cross section predictions.
- Many interesting things to be encountered when going to higher order.

Thank you!

UV RENORMALISATION AND IR FACTORISATION

- > To derive UV counter terms and IR subtraction terms we require NNLO cross sections computed beyond the finite term in ϵ
- Allow to derive complete N3LO scale variation from DGLAP

$$\hat{\sigma}^{(3)} = \hat{\sigma}_0^{(3)} + \hat{\sigma}_1^{(3)} \log\left(\frac{m_h^2}{\mu^2}\right) + \hat{\sigma}_2^{(3)} \log^2\left(\frac{m_h^2}{\mu^2}\right) + \hat{\sigma}_3^{(3)} \log^3\left(\frac{m_h^2}{\mu^2}\right)$$

