Mixed QCD-EW corrections to Higgs boson gluon fusion

Marco Bonetti

Karlsruher Institut für Technologie
Institut für Theoretische Teilchenphysik

January 25, 2018

Karlsruher Institut für Technologie

[arXiv:1610.05497] & [arXiv:1711.11113] & [arXiv:XXXX.XXXXX]
with
K. Melnikov (TTP, KIT) & L. Tancredi (CERN)

Topics

Looking for new physics

- Existence of physics beyond the Standard Model
- Lack of direct evidences of new physics (at the LHC)

Looking for new physics

- Existence of physics beyond the Standard Model
- Lack of direct evidences of new physics (at the LHC)

Game of precision

New physics from investigating known processes at higher precision

Higgs boson: good candidate

- Couples to particles proportionally to their masses
- Is sensible to new physics: $\delta g/g \sim 5\%$ at $\Lambda = 1\,\text{TeV}$

Looking for new physics

- Existence of physics beyond the Standard Model
- Lack of direct evidences of new physics (at the LHC)

Game of precision

New physics from investigating known processes at higher precision

Higgs boson: good candidate

- Couples to particles proportionally to their masses
- Is sensible to new physics: $\delta g/g \sim 5\%$ at $\Lambda = 1\,\text{TeV}$

Goal: provide theoretical predictions for Higgs physics with error lower than $O(1\%)$

Higgs boson at the LHC

[Anastasiou. . . ,2009;Anastasiou. . . ,2016]

Gluon fusion

Dominant channel for H production at the LHC: $\sim$ 90% of σ_{tot} at 13 TeV

- 95% pure QCD
- 5% mixed QCD-EW

Higgs boson at the LHC

[Anastasiou. . . ,2009;Anastasiou. . . ,2016]

Gluon fusion

Dominant channel for H production at the LHC: $\sim 90\%$ of σ_{tot} at 13 TeV

- 95% pure QCD
- 5% mixed QCD-EW

Theoretical uncertainties

PDF + α_S	δ_{scale}	δ_{trunc}	$\delta_{\text{PDF-TH}}$	$\delta_{\text{QCD-EW}}$	$\delta_{t,\ b,\ c}$	δ_{1/m_t}
$\sim 4\%$	$\sim 2\%$	0.37%	1.16%	1%	0.83%	1%

Higgs boson at the LHC

[Anastasiou. . . ,2009;Anastasiou. . . ,2016]

Gluon fusion

Dominant channel for H production at the LHC: $\sim$ 90% of σ_{tot} at 13 TeV

- 95% pure QCD
- 5% mixed QCD-EW

Theoretical uncertainties

PDF + α_S	δ_{scale}	δ_{trunc}	$\delta_{\text{PDF-TH}}$	$\delta_{\text{QCD-EW}}$	$\delta_{t,\ b,\ c}$	δ_{1/m_t}
$\sim$ 4%	$\sim$ 2%	0.37%	1.16%	1%	0.83%	1%

$$\sigma^{\text{QCD-EW}} = \underbrace{\sigma_{\text{LO}}}_{\text{Exact}} + \underbrace{\sigma_{\text{NLO}}}_{m_{W,Z} \gg m_H}$$

Higgs boson at the LHC

[Anastasiou. . . ,2009;Anastasiou. . . ,2016]

Gluon fusion

Dominant channel for H production at the LHC: $\sim$ 90% of σ_{tot} at 13 TeV

- 95% pure QCD
- 5% mixed QCD-EW

Theoretical uncertainties

PDF $+ \alpha_S$	δ_{scale}	δ_{trunc}	$\delta_{\text{PDF-TH}}$	$\delta_{\text{QCD-EW}}$	$\delta_{t,\ b,\ c}$	δ_{1/m_t}
$\sim$ 4%	$\sim$ 2%	0.37%	1.16%	1%	0.83%	1%

$$\sigma^{\text{QCD-EW}} = \underbrace{\sigma_{\text{LO}}}_{\text{Exact}} + \underbrace{\sigma_{\text{NLO}}}_{m_{W,Z} \gg m_H}$$

Goal: compute $\sigma^{\text{QCD-EW}}_{\text{NLO}}$ for physical values of $m_{W,Z}$ and m_H

NLO building blocks

Hadronic cross-section

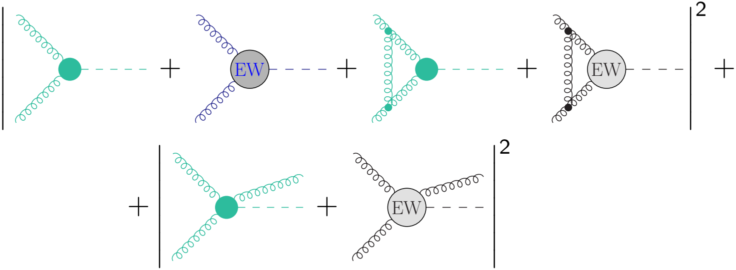
$$\sigma_{\text{NLO}}^{\text{QCD-EW}} = \int_0^1 \int_0^1 f(x_1,\mu) f(x_2,\mu) \sigma_{\text{part}} \, \mathrm{d}x_2 \mathrm{d}x_1$$

NLO building blocks

Hadronic cross-section

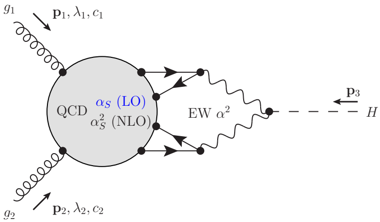
$$\sigma_{\text{NLO}}^{\text{QCD-EW}} = \int_0^1 \int_0^1 f(x_1,\mu) f(x_2,\mu) \sigma_{\text{part}} \, \mathrm{d}x_2 \mathrm{d}x_1$$

$\sigma_{\text{part}} \propto$



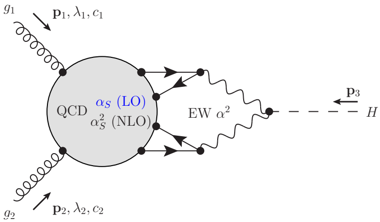
- Pure QCD: known up to N^{3}LO in $m_t \to +\infty$
- QCD-EW: known in a physical region at LO

$gg \to H$ QCD-EW amplitude



$$\mathcal{M}^{c_1 c_2}_{\lambda_1 \lambda_2} = \mathcal{F}(s, m_W, m_Z)\delta^{c_1 c_2}\epsilon_{\lambda_1}(\mathbf{p}_1) \cdot \epsilon_{\lambda_2}(\mathbf{p}_2)$$

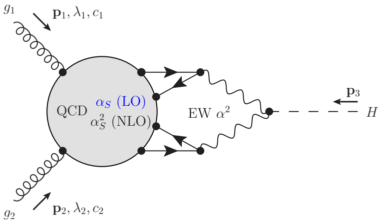
$gg \to H$ QCD-EW amplitude



$$\mathcal{M}^{c_1 c_2}_{\lambda_1 \lambda_2} = \mathcal{F}(s, m_W, m_Z)\delta^{c_1 c_2}\epsilon_{\lambda_1}(\mathbf{p}_1) \cdot \epsilon_{\lambda_2}(\mathbf{p}_2)$$

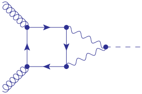
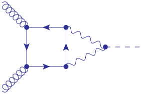
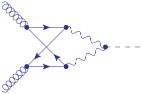
Higgs couples with W or Z bosons: top quark suppressed, $m_q = 0$.

$gg \to H$ QCD-EW amplitude

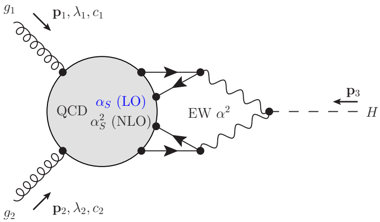


$$\mathcal{M}^{c_1 c_2}_{\lambda_1 \lambda_2} = \mathcal{F}(s, m_W, m_Z)\delta^{c_1 c_2}\epsilon_{\lambda_1}(\mathbf{p}_1) \cdot \epsilon_{\lambda_2}(\mathbf{p}_2)$$

Higgs couples with W or Z bosons: top quark suppressed, $m_q = 0$.

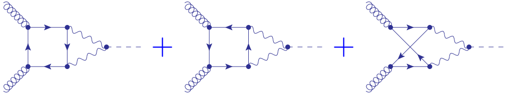
$\mathcal{M}(\alpha^2, \alpha_S) =$  $+$  $+$  .

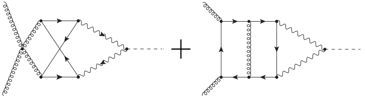
$gg \to H$ QCD-EW amplitude



$$\mathcal{M}^{c_1 c_2}_{\lambda_1 \lambda_2} = \mathcal{F}(s, m_W, m_Z)\delta^{c_1 c_2}\epsilon_{\lambda_1}(\mathbf{p}_1) \cdot \epsilon_{\lambda_2}(\mathbf{p}_2)$$

Higgs couples with W or Z bosons: top quark suppressed, $m_q = 0$.

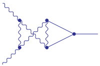
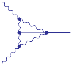
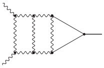
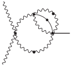
$\mathcal{M}(\alpha^2, \alpha_S) =$  .

$\mathcal{M}(\alpha^2, \alpha_S^2) =$  + [45 FDs].

Feynman Integrals

Tensorial decomposition

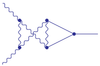
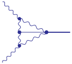
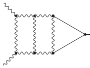
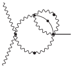
$$\mathcal{F}(s, m_W, m_Z) = \frac{\epsilon^{*\lambda_1}(\mathbf{p}_1) \cdot \epsilon^{*\lambda_2}(\mathbf{p}_2)}{2(1-\epsilon)} \frac{\delta_{c_1 c_2}}{N_C^2 - 1} \mathcal{M}^{c_1 c_2}_{\lambda_1 \lambda_2} =$$

$= \Big($  $+$  $(p_1 \cdot k_2) + \ldots \Big) + \Big($  $+$  $+ \ldots \Big)$

Feynman Integrals

Tensorial decomposition

$$\mathcal{F}(s, m_W, m_Z) = \frac{\epsilon^{*\lambda_1}(\mathbf{p}_1) \cdot \epsilon^{*\lambda_2}(\mathbf{p}_2)}{2(1-\epsilon)} \frac{\delta_{c_1 c_2}}{N_C^2 - 1} \mathcal{M}^{c_1 c_2}_{\lambda_1 \lambda_2} =$$

$= \Big($  $+$  $(p_1 \cdot k_2) + \dots \Big) + \Big($  $+$  $+ \dots \Big)$

Feynman Integral

$$\mathcal{I}(\{s\}, \{m\}) = \int \prod_{j=1}^{J} \frac{1}{\mathcal{D}_j^{a_j}} \prod_{i=1}^{L} \frac{\mathrm{d}^{4-2\epsilon} k_i}{\mathrm{i}\pi^{2-\epsilon}\Gamma(1+\epsilon)}$$

- $\mathcal{D}_j = \left(\alpha_{j1} p_1^\mu + \dots + \alpha_{jE} p_E^\mu + \beta_{j1} k_1^\mu + \dots + \beta_{jL} k_L^\mu\right)^2 - m_j^2$
- $a_j \in \mathbb{Z}$ (scalar products equivalent to $a_j < 0$)

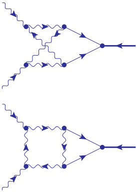
Topologies

- FIs are classified according to their topology, i.e. their set of denominators.
- Parent topologies: maximal topologies, other topologies are subsets of them.

Topologies

- FIs are classified according to their topology, i.e. their set of denominators.
- Parent topologies: maximal topologies, other topologies are subsets of them.

2 loops

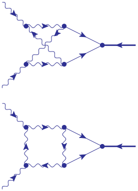


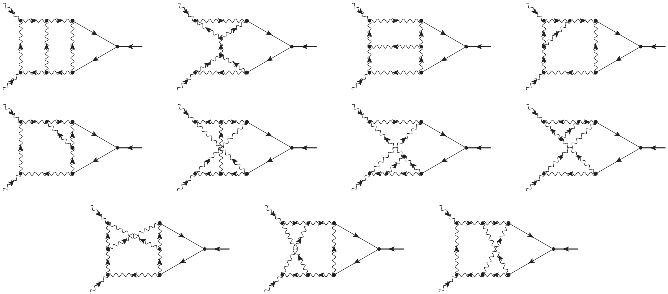
Topologies

- FIs are classified according to their topology, i.e. their set of denominators.
- Parent topologies: maximal topologies, other topologies are subsets of them.

3 loops

2 loops





Properties of Feynman Integrals [Chetyrkin. . . ,1981;Gehrmann. . . ,2000]

3 loop FIs

Feynman diagrams		Feynman Integrals
10^2	$\Rightarrow$	10^5
Tensorial		Scalar

Properties of Feynman Integrals [Chetyrkin. . . ,1981;Gehrmann. . . ,2000]

3 loop FIs

Feynman diagrams		Feynman Integrals
10^2	$\Rightarrow$	10^5
Tensorial		Scalar

Not all FIs are independent.

Properties of Feynman Integrals [Chetyrkin...,1981;Gehrmann...,2000]

3 loop FIs

Feynman diagrams		Feynman Integrals
10^2	$\Rightarrow$	10^5
Tensorial		Scalar

Not all FIs are independent.

Integration-by-Parts identities

$$\int \frac{\partial}{\partial k^\mu}\left[q^\mu \prod_{j=1}^{J}\frac{1}{\mathcal{D}_j^{a_j}}\right] \mathrm{d}^{4-2\epsilon}k_1 \ldots \mathrm{d}^{4-2\epsilon}k_L = 0, \qquad q^\mu = k_l^\mu, p_e^\mu.$$

- IBPs provide a set of relations among FIs at fixed number of loops.
- Other relations: Lorentz invariance, dim. reg., symmetries.

Master Integrals

[Laporta,2000]

Master Integrals

All FIs are linear combinations of the Master Integrals: basis of MIs.

Master Integrals

[Laporta,2000]

Master Integrals

All FIs are linear combinations of the Master Integrals: basis of MIs.

- Automated procedure to find a basis of MIs: Laporta's algorithm; implemented in many codes (`Reduze`, `Kira`...).
- The basis of MIs is not unique: finding the optimal basis still an open problem.
- Not rarely topologies require more than a single MI.

Master Integrals

[Laporta,2000]

Master Integrals

All FIs are linear combinations of the Master Integrals: basis of MIs.

- Automated procedure to find a basis of MIs: Laporta's algorithm; implemented in many codes (`Reduze`, `Kira`...).
- The basis of MIs is not unique: finding the optimal basis still an open problem.
- Not rarely topologies require more than a single MI.

2 loop	12 MIs	from 3 to 6 denominators
3 loop	95 MIs	from 4 to 9 denominators

Master Integrals

[Laporta,2000]

Master Integrals

All FIs are linear combinations of the Master Integrals: basis of MIs.

- Automated procedure to find a basis of MIs: Laporta's algorithm; implemented in many codes (`Reduze`, `Kira`. . .).
- The basis of MIs is not unique: finding the optimal basis still an open problem.
- Not rarely topologies require more than a single MI.

2 loop	12 MIs	from 3 to 6 denominators
3 loop	95 MIs	from 4 to 9 denominators

Feynman diagrams		Feynman Integrals		Master Integrals
10^2	$\Rightarrow$	10^5	$\Rightarrow$	10^2
Tensorial		Scalar		Scalar

Master Integrals Evaluation [Kotikov,1991;Remiddi,1997;Gehrmann...,2000]

Master Integrals Evaluation [Kotikov,1991;Remiddi,1997;Gehrmann...,2000]

Differential Equations for MIs

Master Integrals Evaluation [Kotikov,1991;Remiddi,1997;Gehrmann...,2000]

Differential Equations for MIs

1. Differentiate MIs w.r.t. masses or scalar kinematic invariants

$$\frac{\partial}{\partial\,(m^2)}$$  $$= -2$$ 

Master Integrals Evaluation [Kotikov,1991;Remiddi,1997;Gehrmann...,2000]

Differential Equations for MIs

1. Differentiate MIs w.r.t. masses or scalar kinematic invariants

$$\frac{\partial}{\partial\left(m^2\right)}$$  $$= -2$$ 

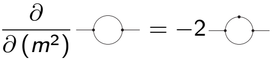
2. Apply IBPs to recover the MIs

 $$= -\frac{1-2\epsilon}{4m^2+s}$$  $$- \frac{1-\epsilon}{m^2\left(4m^2+s\right)}$$ 

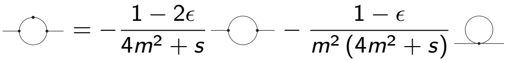
Master Integrals Evaluation [Kotikov,1991;Remiddi,1997;Gehrmann. . . ,2000]

Differential Equations for MIs

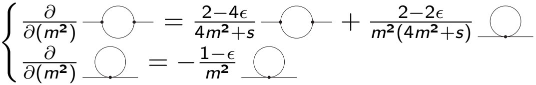
1. Differentiate MIs w.r.t. masses or scalar kinematic invariants



2. Apply IBPs to recover the MIs



3. Include DEs for the subtopologies



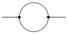
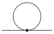
Master Integrals Evaluation [Kotikov,1991;Remiddi,1997;Gehrmann. . . ,2000]

Differential Equations for MIs

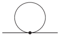
1. Differentiate MIs w.r.t. masses or scalar kinematic invariants

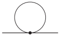
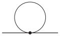
$\frac{\partial}{\partial\left(m^2\right)}$  $= -2$ 

2. Apply IBPs to recover the MIs

 $= -\frac{1-2\epsilon}{4m^2+s}$  $- \frac{1-\epsilon}{m^2\left(4m^2+s\right)}$ 

3. Include DEs for the subtopologies

$\frac{\partial}{\partial(m^2)}$  $= \frac{2-4\epsilon}{4m^2+s}$  $+ \frac{2-2\epsilon}{m^2(4m^2+s)}$ 

$\frac{\partial}{\partial(m^2)}$  $= -\frac{1-\epsilon}{m^2}$ 

System of linear Partial Differential Equations

$$\frac{\partial \mathbf{I}(\mathbf{x},\epsilon)}{\partial x_i} = A_i(\mathbf{x},\epsilon)\mathbf{I}(\mathbf{x},\epsilon)$$

Preliminary Analysis

- MIs depend on s and m^2 (Z and W never in the same diagram).

Preliminary Analysis

- MIs depend on s and m^2 (Z and W never in the same diagram).

New variables

$$y = \frac{\sqrt{1-4m^2/s}-1}{\sqrt{1-4m^2/s}+1} \qquad \& \qquad s$$

s factorised out by dimensional analysis, only y-dependence not trivial

$$\mathbf{I}_n(\epsilon, y, s) = (-s)^{-a_n - L\epsilon}\mathbf{J}_n(\epsilon, y)$$

Preliminary Analysis

- MIs depend on s and m^2 (Z and W never in the same diagram).

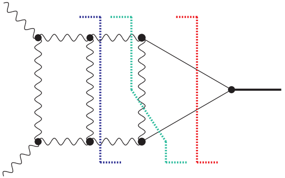
New variables

$$y = \frac{\sqrt{1-4m^2/s}-1}{\sqrt{1-4m^2/s}+1} \qquad \& \qquad s$$

s factorised out by dimensional analysis, only y-dependence not trivial

$$\mathbf{I}_n(\epsilon, y, s) = (-s)^{-a_n - L\epsilon}\mathbf{J}_n(\epsilon, y)$$

- Cuts: hints on physical singularities of the MIs.



Possible singularities at both 2 & 3 loops

s	0	m^2	$4m^2$	(∞)
y	$+1$	$e^{i\pi/3}$	-1	(0)

A Special System

[Henn,2013;Mastrolia. . . ,2014;Papadopoulos,2014]

The basis of functions is not unique:

- Different choices for the MIs
- Algebraic transformations

A Special System

[Henn,2013;Mastrolia. . . ,2014;Papadopoulos,2014]

The basis of functions is not unique:

- Different choices for the MIs
- Algebraic transformations

Choice of basis to simplify the DEs:

A Special System

[Henn,2013;Mastrolia. . . ,2014;Papadopoulos,2014]

The basis of functions is not unique:

- Different choices for the MIs
- Algebraic transformations

Choice of basis to simplify the DEs:

Canonical system $A(\epsilon, y)$ is homogeneous in ϵ

A Special System

[Henn,2013;Mastrolia. . . ,2014;Papadopoulos,2014]

The basis of functions is not unique:

- Different choices for the MIs
- Algebraic transformations

Choice of basis to simplify the DEs:

Canonical system $A(\epsilon, y)$ is homogeneous in ϵ

Fuchsian equations $A(\epsilon, y)$ contains only simple poles in y (or $\mathrm{d}\log$ form)

A Special System

[Henn,2013;Mastrolia. . . ,2014;Papadopoulos,2014]

The basis of functions is not unique:

- Different choices for the MIs
- Algebraic transformations

Choice of basis to simplify the DEs:

Canonical system	$A(\epsilon, y)$ is homogeneous in ϵ
Fuchsian equations	$A(\epsilon, y)$ contains only simple poles in y (or $\mathrm{d}\log$ form)
ϵ-finite solution	$\lim_{\epsilon\to 0} \mathbf{J}(\epsilon, y)$ finite

A Special System

[Henn,2013;Mastrolia. . . ,2014;Papadopoulos,2014]

The basis of functions is not unique:

- Different choices for the MIs
- Algebraic transformations

Choice of basis to simplify the DEs:

Canonical system	$A(\epsilon, y)$ is homogeneous in ϵ
Fuchsian equations	$A(\epsilon, y)$ contains only simple poles in y (or $\mathrm{d}\log$ form)
ϵ-finite solution	$\lim_{\epsilon \to 0} \mathbf{J}(\epsilon, y)$ finite

Canonical fuchsian system

$$\frac{\mathrm{d}\mathbf{J}(\epsilon, y)}{\mathrm{d}y} = \epsilon \sum_{k=0}^{K} \frac{B_k}{y - y_k} \mathbf{J}(\epsilon, y)$$

Form of the solution [Goncharov,1994;Remiddi. . . ,2000;Henn,2004;Lee,2015]

Form of the solution [Goncharov,1994;Remiddi...,2000;Henn,2004;Lee,2015]

Canonical system & ϵ-finite solution: Dyson Series in ϵ

$$\mathbf{J}(\epsilon, y) = \mathcal{P}_y \mathrm{e}^{\epsilon \int A(\xi)\,\mathrm{d}\xi} \mathbf{J}_0(\epsilon) = \mathbf{J}_0^{(0)} + \epsilon \left(\int_y A(\xi_1) \mathbf{J}_0^{(0)}\,\mathrm{d}\xi_1 + \mathbf{J}_0^{(1)} \right) +$$

$$+ \epsilon^2 \left(\int_y A(\xi_1) \int_{\xi_1} A(\xi_2) \mathbf{J}_0^{(0)}\,\mathrm{d}\xi_2 \mathrm{d}\xi_1 + \int_y A(\xi_1) \mathbf{J}_0^{(1)}\,\mathrm{d}\xi_1 + \mathbf{J}_0^{(2)} \right) + O\left(\epsilon^3\right)$$

ϵ^n-term contains at most n nested integrations.

Form of the solution [Goncharov,1994;Remiddi...,2000;Henn,2004;Lee,2015]

Canonical system & ϵ-finite solution: Dyson Series in ϵ

$$\mathbf{J}(\epsilon, y) = \mathcal{P}_y \mathrm{e}^{\epsilon \int A(\xi)\,\mathrm{d}\xi} \mathbf{J}_0(\epsilon) = \mathbf{J}_0^{(0)} + \epsilon \left(\int_y A(\xi_1) \mathbf{J}_0^{(0)}\,\mathrm{d}\xi_1 + \mathbf{J}_0^{(1)} \right) +$$

$$+ \epsilon^2 \left(\int_y A(\xi_1) \int_{\xi_1} A(\xi_2) \mathbf{J}_0^{(0)}\,\mathrm{d}\xi_2 \mathrm{d}\xi_1 + \int_y A(\xi_1) \mathbf{J}_0^{(1)}\,\mathrm{d}\xi_1 + \mathbf{J}_0^{(2)} \right) + O\left(\epsilon^3\right)$$

ϵ^n-term contains at most n nested integrations.

Fuchsian equations: Goncharov's polylogarithms

$$G(\mathbf{m}_w; y) := \begin{cases} \frac{1}{w!} \log^w y & \text{if } \mathbf{m} = (0, \ldots, 0) \\ \int_0^y \frac{1}{\xi - m_w} G(\mathbf{m}_{w-1}; \xi)\,\mathrm{d}\xi & \text{if } \mathbf{m} \neq (0, \ldots, 0) \end{cases}$$

Weight w: length of the vector $\mathbf{m}_w$.

The system of Differential Equations for $gg \to H$

$$\frac{\mathrm{d}\mathbf{J}(\epsilon, y)}{\mathrm{d}y} = A(\epsilon, y)\mathbf{J}(\epsilon, y)$$

In general neither canonical nor fuchsian.

The system of Differential Equations for $gg \to H$

$$\frac{\mathrm{d}\mathbf{J}(\epsilon, y)}{\mathrm{d}y} = A(\epsilon, y)\mathbf{J}(\epsilon, y)$$

In general neither canonical nor fuchsian.

- Block-triangular form: off-diagonal terms related to subtopologies

The system of Differential Equations for $gg \to H$

$$\frac{\mathrm{d}\mathbf{J}(\epsilon, y)}{\mathrm{d}y} = A(\epsilon, y)\mathbf{J}(\epsilon, y)$$

In general neither canonical nor fuchsian.

- Block-triangular form: off-diagonal terms related to subtopologies
  - MIs with fewer denominators have simpler equations
  - Blocks correspond to topologies requiring more MIs (up to 4)
  - Different topologies with same number of denominators are not related

$$\downarrow$$

The system of Differential Equations for $gg \to H$

$$\frac{\mathrm{d}\mathbf{J}(\epsilon, y)}{\mathrm{d}y} = A(\epsilon, y)\mathbf{J}(\epsilon, y)$$

In general neither canonical nor fuchsian.

- Block-triangular form: off-diagonal terms related to subtopologies
  - MIs with fewer denominators have simpler equations
  - Blocks correspond to topologies requiring more MIs (up to 4)
  - Different topologies with same number of denominators are not related

$$\downarrow$$

Bottom-up approach

From MIs with small number of denominators and off-diagonal terms, towards the largest and most coupled ones.

The system of Differential Equations for $gg \to H$

$$\frac{\mathrm{d}\mathbf{J}(\epsilon, y)}{\mathrm{d}y} = A(\epsilon, y)\mathbf{J}(\epsilon, y)$$

In general neither canonical nor fuchsian.

- Block-triangular form: off-diagonal terms related to subtopologies
  - MIs with fewer denominators have simpler equations
  - Blocks correspond to topologies requiring more MIs (up to 4)
  - Different topologies with same number of denominators are not related

$$\downarrow$$

Bottom-up approach

From MIs with small number of denominators and off-diagonal terms, towards the largest and most coupled ones.

- Lack of general and efficient algorithm: approach tuned case by case.

Few denominators: integrals [Gehrmann. . . ,2014;Mastrolia. . . ,2014]

MIs up to 6 denominators: no scalar products, at most 3×3 blocks.

Few denominators: integrals

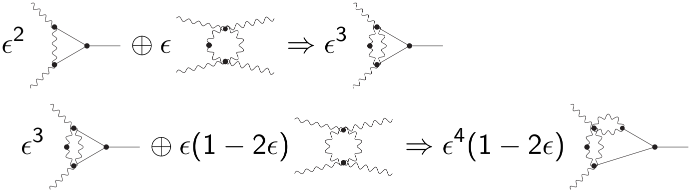
[Gehrmann. . . ,2014;Mastrolia. . . ,2014]

MIs up to 6 denominators: no scalar products, at most 3×3 blocks.

1. Adjusting powers of denominators in the MIs

Guiding principles

Building blocks canonical functions with right mass dimension



No UV divergencies adding dots for negative mass dimension

Few denominators: integrals [Gehrmann...,2014;Mastrolia...,2014]

MIs up to 6 denominators: no scalar products, at most 3×3 blocks.

1. Adjusting powers of denominators in the MIs
2. Multiplication of MIs by $\epsilon^{a_1}(c_1 + \epsilon c_2)^{a_2}$ to obtain an ϵ-linear system

$$\frac{\mathrm{d}\tilde{\mathbf{J}}(\epsilon, y)}{\mathrm{d}y} = [A_0(y) + \epsilon A_1(y)]\tilde{\mathbf{J}}(\epsilon, y)$$

Few denominators: integrals [Gehrmann...,2014;Mastrolia...,2014]

MIs up to 6 denominators: no scalar products, at most 3×3 blocks.

1. Adjusting powers of denominators in the MIs
2. Multiplication of MIs by $\epsilon^{a_1}(c_1 + \epsilon c_2)^{a_2}$ to obtain an ϵ-linear system
3. Integrating out $A_0(y)$

$$\frac{\mathrm{d}\mathbf{F}(\epsilon, y)}{\mathrm{d}y} = \epsilon \hat{S}_{A_0}^{-1}(y) A_1(y) \hat{S}_{A_0}(y) \mathbf{F}(\epsilon, y)$$

$$\mathbf{F}(\epsilon, y) = \hat{S}_{A_0}^{-1}(y) \tilde{\mathbf{J}}(\epsilon, y)$$

$$\hat{S}_{A_0}(y) = \sum_{k=0}^{+\infty} \int_{y_0}^{y} A_0(\xi_1) \dots \int_{y_0}^{\xi_{k-1}} A_0(\xi_k) A_0(\xi_k) \, \mathrm{d}\xi_k \dots \mathrm{d}\xi_1$$

- $\hat{S}_{A_0}(y)$ by Magnus Series or direct integration
- Fuchsianity not guaranteed: logarithms may arise from integration

Many denominators: algebra [Lee,2015;Gituliar,2017;Meyer,2017]

MIs from 7 to 9 denominators: scalar products, up to 4×4 blocks.

Many denominators: algebra

[Lee,2015;Gituliar,2017;Meyer,2017]

MIs from 7 to 9 denominators: scalar products, up to 4×4 blocks.

Balance Transformation

$$\mathcal{B}(y_1, y_2, \mathbb{P}|y) = (\mathbb{I} - \mathbb{P}) + \frac{y - y_2}{y - y_1}\mathbb{P}, \qquad \mathbb{P} = \mathbf{u}_{1i}\mathbf{v}_{2j}^{\dagger}$$

$$A_1 \mathbf{u}_{1i} = a_{1i}\mathbf{u}_{1i} \qquad \mathbf{v}_{2j}^{\dagger} A_2 = a_{2j}\mathbf{v}_{2j}^{\dagger}$$

$$\tilde{A} = \mathcal{B}^{-1} A \mathcal{B} - \mathcal{B}^{-1} \mathrm{d}_y \mathcal{B}$$

Many denominators: algebra

[Lee,2015;Gituliar,2017;Meyer,2017]

MIs from 7 to 9 denominators: scalar products, up to 4×4 blocks.

Balance Transformation

$$\mathcal{B}(y_1, y_2, \mathbb{P}|y) = (\mathbb{I} - \mathbb{P}) + \frac{y - y_2}{y - y_1}\mathbb{P}, \qquad \mathbb{P} = \mathbf{u}_{1i}\mathbf{v}^\dagger_{2j}$$

$$A_1\mathbf{u}_{1i} = a_{1i}\mathbf{u}_{1i} \qquad \mathbf{v}^\dagger_{2j}A_2 = a_{2j}\mathbf{v}^\dagger_{2j}$$

$$\tilde{A} = \mathcal{B}^{-1}A\mathcal{B} - \mathcal{B}^{-1}\mathrm{d}_y\mathcal{B}$$

Eigenvalues Deflation: `Fuchsia`

1. Fuchsification: reducing to 0 the rank of all non-fuchsian residues

Many denominators: algebra [Lee,2015;Gituliar,2017;Meyer,2017]

MIs from 7 to 9 denominators: scalar products, up to 4×4 blocks.

Balance Transformation

$$\mathcal{B}(y_1, y_2, \mathbb{P}|y) = (\mathbb{I} - \mathbb{P}) + \frac{y - y_2}{y - y_1}\mathbb{P}, \qquad \mathbb{P} = \mathbf{u}_{1i}\mathbf{v}^\dagger_{2j}$$

$$A_1\mathbf{u}_{1i} = a_{1i}\mathbf{u}_{1i} \qquad \mathbf{v}^\dagger_{2j}A_2 = a_{2j}\mathbf{v}^\dagger_{2j}$$

$$\tilde{A} = \mathcal{B}^{-1}A\mathcal{B} - \mathcal{B}^{-1}\mathrm{d}_y\mathcal{B}$$

Eigenvalues Deflation: `Fuchsia`

1. Fuchsification: reducing to 0 the rank of all non-fuchsian residues
2. ϵ-eigenvalues: moving units among eigenvalues of simple poles

Many denominators: algebra [Lee,2015;Gituliar,2017;Meyer,2017]

MIs from 7 to 9 denominators: scalar products, up to 4×4 blocks.

Balance Transformation

$$\mathcal{B}(y_1, y_2, \mathbb{P}|y) = (\mathbb{I} - \mathbb{P}) + \frac{y - y_2}{y - y_1}\mathbb{P}, \qquad \mathbb{P} = \mathbf{u}_{1i}\mathbf{v}_{2j}^{\dagger}$$

$$A_1 \mathbf{u}_{1i} = a_{1i}\mathbf{u}_{1i} \qquad \mathbf{v}_{2j}^{\dagger} A_2 = a_{2j}\mathbf{v}_{2j}^{\dagger}$$

$$\tilde{A} = \mathcal{B}^{-1} A \mathcal{B} - \mathcal{B}^{-1} \mathrm{d}_y \mathcal{B}$$

Eigenvalues Deflation: `Fuchsia`

1. Fuchsification: reducing to 0 the rank of all non-fuchsian residues
2. ϵ-eigenvalues: moving units among eigenvalues of simple poles
3. Similarity transformation: factorizing out ϵ

Optimizing Deflation

[Henn,2014;Primo...,2016;Frellesvig,2017]

- Can generate extremely complex solutions
- Can modify already canonical subtopologies
- Can spoil the integration constants

Optimizing Deflation

[Henn,2014;Primo. . . ,2016;Frellesvig,2017]

- Can generate extremely complex solutions
- Can modify already canonical subtopologies
- Can spoil the integration constants

Solution: tuning the DEs before applying eigenvalues deflation.

Optimizing Deflation

[Henn,2014;Primo. . . ,2016;Frellesvig,2017]

- Can generate extremely complex solutions
- Can modify already canonical subtopologies
- Can spoil the integration constants

Solution: tuning the DEs before applying eigenvalues deflation.

Unitarity Cuts

$$\frac{1}{q^2 - m^2} \qquad \rightarrow \qquad \delta(q^2 - m^2)$$

Optimizing Deflation

[Henn,2014;Primo. . . ,2016;Frellesvig,2017]

- Can generate extremely complex solutions
- Can modify already canonical subtopologies
- Can spoil the integration constants

Solution: tuning the DEs before applying eigenvalues deflation.

Unitarity Cuts

$$\frac{1}{q^2 - m^2} \quad \to \quad \delta(q^2 - m^2)$$

- Terms not containing the cut propagator become 0 in the DEs
- By using different cuts different coefficients can be inspected
- If all cuts are in $\mathrm{d}\log$ iterated form, the integral is a good candidate

In practice, only maximal cuts are investigated.

Final form of the system

At 3 loops: 95 MIs

$$\mathrm{d}\mathbf{F}(\epsilon, y) = \epsilon \ \Big[\ B_{+1} \, \mathrm{d}\log(1-y) + \\ + B_r \, \mathrm{d}\log(y^2 - y + 1) + \\ + B_{-1} \, \mathrm{d}\log(y+1) + \\ + B_0 \, \mathrm{d}\log y \ \Big] \mathbf{F}(\epsilon, y)$$

Alphabet of the GPLs (common to 2 & 3 loops):

$$\{+1, r, -1, 0\}$$

$$G(\ldots, r, \ldots; y) = G(\ldots, r_+, \ldots; y) + G(\ldots, r_-, \ldots; y)$$

$$r_+ = \mathrm{e}^{+\mathrm{i}\pi/3} \qquad r_- = \mathrm{e}^{-\mathrm{i}\pi/3}$$

Final form of the system

At 3 loops: 95 MIs

$$\mathrm{d}\mathbf{F}(\epsilon, y) = \epsilon \, \Big[\, B_{+1} \, \mathrm{d}\log(1-y) + B_r \, \mathrm{d}\log(y^2 - y + 1) + B_{-1} \, \mathrm{d}\log(y+1) + B_0 \, \mathrm{d}\log y \, \Big] \mathbf{F}(\epsilon, y)$$

Alphabet of the GPLs (common to 2 & 3 loops):

$$\{+1, r, -1, 0\}$$

$$G(\ldots, r, \ldots; y) = G(\ldots, r_+, \ldots; y) + G(\ldots, r_-, \ldots; y)$$

$$r_+ = \mathrm{e}^{+\mathrm{i}\pi/3} \qquad r_- = \mathrm{e}^{-\mathrm{i}\pi/3}$$

Evaluation of the Dyson series up to order ϵ^6 and $w = 6$.

Fixing the constants

[Smirnov,2000]

$$\mathsf{F}(\epsilon, y) = \mathcal{P}_y \mathrm{e}^{\epsilon \int B(\xi)\, \mathrm{d}\xi} \mathsf{F}_0(\epsilon)$$

Fixing the constants

[Smirnov,2000]

$$\mathbf{F}(\epsilon, y) = \mathcal{P}_y \mathrm{e}^{\epsilon \int B(\xi)\, \mathrm{d}\xi} \mathbf{F}_0(\epsilon)$$

Matching in y_0 between solution of the DEs and the same functions obtained with independent method

$$y_0 = 1 : \qquad \frac{m^2}{s} \to \infty$$

Easy limit for both GPLs evaluation and generation of Boudary Conditions

Fixing the constants

[Smirnov,2000]

Large-Mass Expansion

- MIs depending on
  - Energy of external gluons $p_1 \sim p_2 \sim \sqrt{s}$
  - Mass of the EW vector bosons m

Fixing the constants

[Smirnov,2000]

Large-Mass Expansion

- MIs depending on
  - Energy of external gluons $p_1 \sim p_2 \sim \sqrt{s}$
  - Mass of the EW vector bosons m
- Mathematical limit: $m^2 \gg s \quad \Leftrightarrow \quad y \to 1$

Fixing the constants

[Smirnov,2000]

Large-Mass Expansion

- MIs depending on
  - Energy of external gluons $p_1 \sim p_2 \sim \sqrt{s}$
  - Mass of the EW vector bosons m
- Mathematical limit: $m^2 \gg s \quad \Leftrightarrow \quad y \to 1$
- MIs can now receive contributions only from regions where

$$k_i \sim \sqrt{s} \quad \text{or} \quad k_i \sim m,$$

and k_i must satisfy

Large-momentum conservation law

Large momentum cannot be created, destroyed or provided by external legs: it must form at least one closed flow along the internal lines.

Fixing the constants [Smirnov,2000]

Large-Mass Expansion

- MIs depending on
  - Energy of external gluons $p_1 \sim p_2 \sim \sqrt{s}$
  - Mass of the EW vector bosons m
- Mathematical limit: $m^2 \gg s \quad \Leftrightarrow \quad y \to 1$
- MIs can now receive contributions only from regions where

  $$k_i \sim \sqrt{s} \quad \text{or} \quad k_i \sim m,$$

  and k_i must satisfy

  Large-momentum conservation law

  Large momentum cannot be created, destroyed or provided by external legs: it must form at least one closed flow along the internal lines.
- Taylor expansion in small momenta p_1, p_2, $k_i \sim \sqrt{s}$:
  loops with large momentum detach from the main graph

Fixing the constants

[Smirnov,2000]

Large-Mass Expansion

- MIs depending on
  - Energy of external gluons $p_1 \sim p_2 \sim \sqrt{s}$
  - Mass of the EW vector bosons m
- Mathematical limit: $m^2 \gg s \quad \Leftrightarrow \quad y \to 1$
- MIs can now receive contributions only from regions where

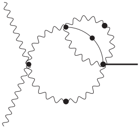
  $$k_i \sim \sqrt{s} \quad \text{or} \quad k_i \sim m,$$

  and k_i must satisfy

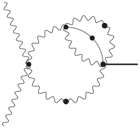
  Large-momentum conservation law

  Large momentum cannot be created, destroyed or provided by external legs: it must form at least one closed flow along the internal lines.
- Taylor expansion in small momenta p_1, p_2, $k_i \sim \sqrt{s}$:
  loops with large momentum detach from the main graph
- Evaluation of the resulting integrals and sum over all contributions

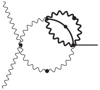
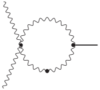
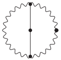
Large-Mass Expansion at 3 loops

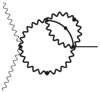
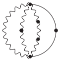
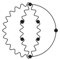


Large-Mass Expansion at 3 loops

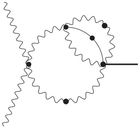


Non-vanishing configurations

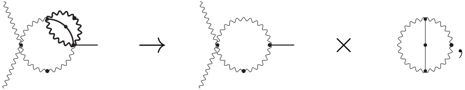
 $\rightarrow$  $\times$  ,

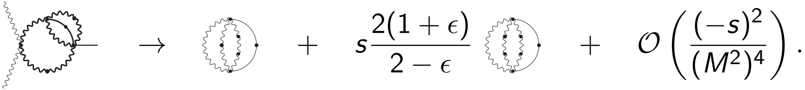
 $\rightarrow$  $+\; s\,\dfrac{2(1+\epsilon)}{2-\epsilon}$  $+\;\mathcal{O}\left(\dfrac{(-s)^2}{(M^2)^4}\right).$

Large-Mass Expansion at 3 loops



Non-vanishing configurations





Boundary terms consist of tadpoles and massless bubbles or triangles.

Evaluation of constants

- Match solution of the DEs with Large-Mass expansion in $y \to 1$

$$\lim_{y \to 1} [\mathbf{F}(\epsilon, y) - L(\epsilon, y)] = 0$$

Evaluation of constants

- Match solution of the DEs with Large-Mass expansion in $y \to 1$

$$\lim_{y \to 1} [\mathbf{F}(\epsilon, y) - L(\epsilon, y)] = 0$$

- r_+ and r_- in the alphabet: analytic expressions in $y = 1$ not known.

Evaluation of constants

- Match solution of the DEs with Large-Mass expansion in $y \to 1$

$$\lim_{y \to 1} [\mathbf{F}(\epsilon, y) - L(\epsilon, y)] = 0$$

- r_+ and r_- in the alphabet: analytic expressions in $y = 1$ not known.
- GLPs are functions of uniform weight w: their value at rational points is a *simple* $\mathbb{Q}$-linear combination of weight w constants.

w	0	1	2	3	4	5	6
$F_0^{(w)}$	1	$\emptyset$	π^2	$\zeta(3)$	π^4	$\pi^2\zeta(3)$ $\zeta(5)$	π^6 $\zeta^2(3)$

Evaluation of constants

- Match solution of the DEs with Large-Mass expansion in $y \to 1$

$$\lim_{y\to 1}[\mathbf{F}(\epsilon, y) - L(\epsilon, y)] = 0$$

- r_+ and r_- in the alphabet: analytic expressions in $y = 1$ not known.
- GLPs are functions of uniform weight w: their value at rational points is a *simple* $\mathbb{Q}$-linear combination of weight w constants.

w	0	1	2	3	4	5	6
$F_0^{(w)}$	1	$\emptyset$	π^2	$\zeta(3)$	π^4	$\pi^2\zeta(3)$ $\zeta(5)$	π^6 $\zeta^2(3)$

- If BCs for the simplest topologies have uniform weight w at order ϵ^w, higher BCs show the same pattern due to canonical fuchsian DEs.

Evaluation of constants

- Match solution of the DEs with Large-Mass expansion in $y \to 1$

$$\lim_{y \to 1} [\mathbf{F}(\epsilon, y) - L(\epsilon, y)] = 0$$

- r_+ and r_- in the alphabet: analytic expressions in $y = 1$ not known.
- GLPs are functions of uniform weight w: their value at rational points is a *simple* $\mathbb{Q}$-linear combination of weight w constants.

w	0	1	2	3	4	5	6
$F_0^{(w)}$	1	$\emptyset$	π^2	$\zeta(3)$	π^4	$\pi^2\zeta(3)$ $\zeta(5)$	π^6 $\zeta^2(3)$

- If BCs for the simplest topologies have uniform weight w at order ϵ^w, higher BCs show the same pattern due to canonical fuchsian DEs.

Numerical matching via PSLQ algorithm with > 750 digits.

The expression of the Form Factor

$$\mathcal{F} = -\mathrm{i}\frac{\alpha^2 \alpha_S(\mu) v}{64\pi \sin^4\theta_W} \sum_{i=W,Z} C_i A(m_i^2/s, \mu^2/s)$$

$$C_W = 4, \qquad C_Z = \frac{2}{\cos^4\theta_W}\left(\frac{5}{4} - \frac{7}{3}\sin^2\theta_W + \frac{22}{9}\sin^4\theta_W\right)$$

$$A(m^2/s, \mu^2/s) = A_{2\mathrm{L}} + \frac{\alpha_S(\mu)}{2\pi} A_{3\mathrm{L}} + \mathcal{O}(\alpha_S^2)$$

The expression of the Form Factor

$$\mathcal{F} = -\mathrm{i}\frac{\alpha^2\alpha_S(\mu)v}{64\pi\sin^4\theta_W}\sum_{i=W,Z} C_i A(m_i^2/s, \mu^2/s)$$

$$C_W = 4, \qquad C_Z = \frac{2}{\cos^4\theta_W}\left(\frac{5}{4} - \frac{7}{3}\sin^2\theta_W + \frac{22}{9}\sin^4\theta_W\right)$$

$$A(m^2/s, \mu^2/s) = A_{2\mathrm{L}} + \frac{\alpha_S(\mu)}{2\pi}A_{3\mathrm{L}} + \mathcal{O}(\alpha_S^2)$$

$A_{2\mathrm{L}}$ ϵ-finite, GPLs up to weight 3

$A_{3\mathrm{L}}$ $1/\epsilon^2$, GPLs up to weight 5

The expression of the Form Factor

$$\mathcal{F} = -\mathrm{i}\frac{\alpha^2 \alpha_S(\mu) v}{64\pi \sin^4\theta_W} \sum_{i=W,Z} C_i A(m_i^2/s, \mu^2/s)$$

$$C_W = 4, \qquad C_Z = \frac{2}{\cos^4\theta_W}\left(\frac{5}{4} - \frac{7}{3}\sin^2\theta_W + \frac{22}{9}\sin^4\theta_W\right)$$

$$A(m^2/s, \mu^2/s) = A_{2L} + \frac{\alpha_S(\mu)}{2\pi} A_{3L} + \mathcal{O}(\alpha_S^2)$$

A_{2L} ϵ-finite, GPLs up to weight 3

A_{3L} $1/\epsilon^2$, GPLs up to weight 5

$\mathcal{F}$ is both UV and IR divergent

- UV renormalization: α_S in $\overline{\text{MS}}$ scheme
- IR subtraction: Catani's prescription $\quad A_{3L} = \mathbf{I}^{(1)} A_{2L} + A_{3L}^{\text{fin}}$

Numerics for the Amplitude

$s = m_H = 125.09\,\text{GeV},\ m_W = 80.385\,\text{GeV},\ m_Z = 91.1876\,\text{GeV},\ N_C = 3,\ N_f = 5,\ \mu = m_H$

$$A_{2L}(m_Z^2/m_H^2, 1) = -6.880846 - i\,0.5784119$$
$$A_{2L}(m_W^2/m_H^2, 1) = -10.71693 - i\,2.302953$$
$$A_{3L}^{fin}(m_Z^2/m_H^2, 1) = -2.975801 - i\,41.19509$$
$$A_{3L}^{fin}(m_W^2/m_H^2, 1) = -11.31557 - i\,54.02989$$

Numerics for the Amplitude

$s = m_H = 125.09\,\text{GeV},\ m_W = 80.385\,\text{GeV},\ m_Z = 91.1876\,\text{GeV},\ N_C = 3,\ N_f = 5,\ \mu = m_H$

$$A_{2\text{L}}(m_Z^2/m_H^2, 1) = -6.880846 - \mathrm{i}\,0.5784119$$
$$A_{2\text{L}}(m_W^2/m_H^2, 1) = -10.71693 - \mathrm{i}\,2.302953$$
$$A_{3\text{L}}^{\text{fin}}(m_Z^2/m_H^2, 1) = -2.975801 - \mathrm{i}\,41.19509$$
$$A_{3\text{L}}^{\text{fin}}(m_W^2/m_H^2, 1) = -11.31557 - \mathrm{i}\,54.02989$$

$$|\Im A_{3\text{L}}^{\text{fin}} - \Im A_{2\text{L}}| \gg |\Re A_{3\text{L}}^{\text{fin}} - \Re A_{2\text{L}}|$$

Numerics for the Amplitude

$s = m_H = 125.09\,\text{GeV},\ m_W = 80.385\,\text{GeV},\ m_Z = 91.1876\,\text{GeV},\ N_C = 3,\ N_f = 5,\ \mu = m_H$

$$A_{2L}(m_Z^2/m_H^2, 1) = -6.880846 - \mathrm{i}\,0.5784119$$
$$A_{2L}(m_W^2/m_H^2, 1) = -10.71693 - \mathrm{i}\,2.302953$$
$$A_{3L}^{\text{fin}}(m_Z^2/m_H^2, 1) = -2.975801 - \mathrm{i}\,41.19509$$
$$A_{3L}^{\text{fin}}(m_W^2/m_H^2, 1) = -11.31557 - \mathrm{i}\,54.02989$$

$$|\Im A_{3L}^{\text{fin}} - \Im A_{2L}| \gg |\Re A_{3L}^{\text{fin}} - \Re A_{2L}|$$

- Diagram level: discontinuities at $s = 0$ & $s = m^2$ for 2 & 3 loops
- Amplitude level: discontinuity at $s = 0$ only for 3 loops

Numerics for the Amplitude

$s = m_H = 125.09\,\text{GeV},\ m_W = 80.385\,\text{GeV},\ m_Z = 91.1876\,\text{GeV},\ N_C = 3,\ N_f = 5,\ \mu = m_H$

$$A_{2L}(m_Z^2/m_H^2, 1) = -6.880846 - i\,0.5784119$$
$$A_{2L}(m_W^2/m_H^2, 1) = -10.71693 - i\,2.302953$$
$$A_{3L}^{fin}(m_Z^2/m_H^2, 1) = -2.975801 - i\,41.19509$$
$$A_{3L}^{fin}(m_W^2/m_H^2, 1) = -11.31557 - i\,54.02989$$

$$|\Im A_{3L}^{fin} - \Im A_{2L}| \gg |\Re A_{3L}^{fin} - \Re A_{2L}|$$

- Diagram level: discontinuities at $s = 0$ & $s = m^2$ for 2 & 3 loops
- Amplitude level: discontinuity at $s = 0$ only for 3 loops

$\Im A_{2L}$ for $0 < s < m^2$ obtained cutting through fermionic lines

$$gg \to q\bar{q} \quad | \quad q\bar{q} \to H$$

Higgs cannot couple to massless fermions: no contribution to $\Im A_{2L}$.

Cross-section at NLO

[de Florian,2013;Ball. . . ,2013]

- 2-loop QCD-EW $gg \to H$
- 3-loop QCD-EW $gg \to H$ (virtual corrections)
- 2-loop QCD-EW $gg \to Hg$ (real corrections) MISSING

Cross-section at NLO

[de Florian,2013;Ball...,2013]

- 2-loop QCD-EW $gg \to H$
- 3-loop QCD-EW $gg \to H$ (virtual corrections)
- 2-loop QCD-EW $gg \to Hg$ (real corrections) MISSING

Soft-gluon approximation

$$\sigma = \int_0^1 \int_0^1 f(x_1, \mu) f(x_2, \mu) \sigma_{\text{LO}} z G(z, \mu, \alpha_S) \, \mathrm{d}x_2 \mathrm{d}x_1$$

- $z := m_H^2 / (S_{\text{hadr}} x_1 x_2)$ $gg \to H$ energy
- $G = \delta(1-z) + \frac{\alpha_S}{2\pi} \left[8 C_A \mathcal{D}_1 + \left(\frac{2\pi^2}{3} C_A + \frac{\sigma_{\text{NLO}}^{\text{fin}}}{\sigma_{\text{LO}}} \right) \delta(1-z) \right]$
- $\mathcal{D}_1 = \left[\frac{\log(1-z)}{1-z} \right]_+ + (2 - 3z + 2z^2) \frac{\log[(1-z)/\sqrt{z}]}{1-z} - \frac{\log(1-z)}{1-z}$

σ_{LO}, $\sigma_{\text{NLO}}^{\text{fin}}$ are partonic cross-sections.

Numerics for the cross-section

QCD vs. QCD-EW

$\sigma_{\text{LO}}^{\text{QCD}} = 20.6\,\text{pb}$	$\sigma_{\text{LO}}^{\text{QCD-EW}} = 21.7\,\text{pb}$	$+5.3\%$
$\sigma_{\text{NLO}}^{\text{QCD}} = 37.0\,\text{pb}$	$\sigma_{\text{NLO}}^{\text{QCD-EW}} = 39.0\,\text{pb}$	$+5.5\%$

NNPDF30 for PDFs and running of $\alpha_S(\mu)$

Numerics for the cross-section

QCD vs. QCD-EW

$\sigma_{\text{LO}}^{\text{QCD}} = 20.6\,\text{pb}$	$\sigma_{\text{LO}}^{\text{QCD-EW}} = 21.7\,\text{pb}$	$+5.3\%$
$\sigma_{\text{NLO}}^{\text{QCD}} = 37.0\,\text{pb}$	$\sigma_{\text{NLO}}^{\text{QCD-EW}} = 39.0\,\text{pb}$	$+5.5\%$

NNPDF30 for PDFs and running of $\alpha_S(\mu)$

Confirmation of the estimate from evaluation in the limit $m_{W,Z} \gg m_H$.

- Large E_g suppressed by PDFs
- Standard and improved $\mathcal{D}_1$ give the same increase in σ_{NLO}
- Emission from internal lines not considered: suppressed by E_g^2

Outlook

- NLO virtual corrections to QCD-EW $gg \to H$ evaluated
- Soft-gluon limit for $\sigma_{\text{NLO}}^{\text{QCD-EW}}$ confirms the previous estimates

Outlook

- NLO virtual corrections to QCD-EW $gg \to H$ evaluated
- Soft-gluon limit for $\sigma_{\text{NLO}}^{\text{QCD-EW}}$ confirms the previous estimates

Exact real corrections

- Necessary ingredient for further improvements
- Still unknown

Outlook

- NLO virtual corrections to QCD-EW $gg \to H$ evaluated
- Soft-gluon limit for $\sigma_{\text{NLO}}^{\text{QCD-EW}}$ confirms the previous estimates

Exact real corrections

- Necessary ingredient for further improvements
- Still unknown
- Challenging problem: 2-loop, with more than one scale $(s,\ m^2,\ E_g)$
  - Larger number of MIs
  - Few techniques for DEs with more than one scale
  - GPLs may be not enough to express the solution of the system

Thank you for your attention

Karlsruher Institut für Technologie