Threshold and jet radius joint resummation for single-inclusive jet production

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Based on work done in collaboration with:

- *Threshold and jet radius joint resummation for single-inclusive jet production*
  Xiaohui Liu, S. M. and Felix Ringer

- *Phenomenology of single-inclusive jet production with jet radius and threshold resummation*
  Xiaohui Liu, S. M. and Felix Ringer
QCD factorization
QCD factorization

\[ \sigma_{pp \rightarrow X} = \sum_{ij} f_i(\mu) \otimes f_j(\mu) \otimes \hat{\sigma}_{ij \rightarrow X} (\alpha_s(\mu), Q, \mu, m_X) \]

- Factorization at scale \( \mu \)
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section \( \hat{\sigma}_{ij \rightarrow X} \) calculable in perturbation theory
  - cross section \( \hat{\sigma}_{ij \rightarrow k} \) for parton types \( i, j \) and hadronic final state \( X \)
- Parton distribution functions \( f_i \), strong coupling \( \alpha_s \), particle masses \( m_X \)
  - known from global fits to exp. data, lattice computations, ...
Parton luminosity

• Long distance dynamics due to proton structure

\[ \sigma_{pp \rightarrow X} = \sum_{ij} f_i(\mu) \otimes f_j(\mu) \otimes \ldots \]

• Cross section depends on parton distributions \( f_i \)

• Parton distributions known from global fits to exp. data
  • available fits accurate to NNLO
  • information on proton structure depends on kinematic coverage
**Hard scattering cross section**

- Parton cross section $\hat{\sigma}_{ij \rightarrow k}$ calculable perturbatively in powers of $\alpha_s$
  - known to NLO, NNLO, ... ($\mathcal{O}(\text{few\%})$ theory uncertainty)

- Accuracy of perturbative predictions
  - LO (leading order) ($\mathcal{O}(50 - 100\%)$ unc.)
  - NLO (next-to-leading order) ($\mathcal{O}(10 - 30\%)$ unc.)
  - NNLO (next-to-next-to-leading order) ($\lesssim \mathcal{O}(10\%)$ unc.)
  - $N^3$LO (next-to-next-to-next-to-leading order)
  - ...
Perturbative QCD at higher orders (I)

Exact $N^3$LO Higgs cross section at LHC

- Apparent convergence of perturbative expansion
- Scale dependence of exact $N^3$LO prediction with residual uncertainty 3%
- Minimal sensitivity at scale $\mu = m_H / 2$

Anastasiou, Duhr, Dulat, Herzog, Mistlberger ‘15
Threshold logarithms provide approximate $N^4$LO corrections at two scales $\mu = m_H$ and $\mu = m_H/2$.

- $K$-factor $\approx 1\%$
  for $\mu = m_H/2$ with $\sqrt{s} = 13$ TeV.
Single-inclusive jet production

- Double differential cross section for $pp \to \text{jet} + X$ at $\sqrt{s} = 13$ TeV
  - transverse momentum $p_T$ and rapidity $y$ of signal-jet
- Comparison with NLO perturbative QCD predictions (NLOJET++ Nagy)
  - impressive agreement over several orders of magnitude
Uncertainties

- Relative QCD uncertainties in single-inclusive jet cross-sections at NLO
  - first and last $|y|$ bins for inclusive jet measurement
  - uncertainties due to renormalization and factorization scales, $\mu_R, \mu_F$, the strong coupling $\alpha_s$ with $\Delta\alpha_s = 0.0015$ and PDFs CT14

- Sizable uncertainties
  - $\mathcal{O}(10\%)$ for central rapidities $|y| < 0.5$
  - $\mathcal{O}(30 - 40\%)$ forward $2.5 \leq |y| < 3.0$ at large $p_T$
Uses of inclusive jet data

- Dermination of $\alpha_s(M_Z)$ and PDFs (gluon at medium to large $x$)
  - partonic cross sections $\hat{\sigma}_{ij\rightarrow\text{jet}} \propto \alpha_s^2(\mu)$
  - jet cross section $d\sigma_{pp\rightarrow\text{jet}} = \alpha_s^2(\mu) \sum_{ij} f_i(\mu) \otimes f_j(\mu) \otimes [\ldots]$

- $\alpha_s(M_Z)$ at NLO in QCD from inclusive jet cross section data
  Britzger, Rabbertz, Savoiu, Sieber ‘17

- correlations between PDFs and $\alpha_s(M_Z)$ are important
Single-inclusive jet production

Theory status at NNLO

- Fully differential kinematics to match experimental cuts and fiducial cross section
- QCD corrections at NNLO
  Gehrmann-De Ridder, Gehrmann, Glover, Pires ‘13; Currie, Glover, Pires ‘16
  - all partonic channels; leading color contributions only

Soft and collinear singularities

- Soft/collinear regions of phase space
- massless partons

\[
\frac{1}{(p + k)^2} = \frac{1}{2p \cdot k} = \frac{1}{2E_q E_g (1 - \cos \theta_{qg})}
\]

\[
\alpha_s \int \frac{1}{(p + k)^2} \rightarrow \alpha_s \int dE_g d\theta_{qg} \frac{1}{2E_q E_g (1 - \cos \theta_{qg})}
\]

\[
\rightarrow \alpha_s \frac{1}{\epsilon^2} \times (\ldots) \quad \text{in dim. reg.} \quad D = 4 - 2\epsilon
\]

- Regularization for real emissions required

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Threshold and jet radius joint resummation for single-inclusive jet production – p.12
Regularization

Subtraction

- removes all $\frac{1}{\epsilon^k}$ singularities analytically; $k \leq 4$ at NNLO
  - Antenna subtraction Gehrmann-De Ridder, Gehrmann, Glover ’05
  - Colourful subtraction Del Duca, Somogyi, Trocsanyi ’05
  - Sector subtraction Czakon ’10; Boughezal, Melnikov, Petriello ’11; Czakon, Heymes ’14
  - Projection to Born Cacciari, Dreyer, Karlberg, Salam, Zanderighi ’15

Slicing

- Cuts imposed in phase space; singularities $\ln^k$(some cut) need to cancel numerically; $k \leq 4$ at NNLO
  - $q_T$ subtraction Catani, Grazzini ’07
  - $N$-jettiness subtraction Boughezal, Focke, Liu, Petriello ’15; Gaunt, Stahlhofen, Tackmann, Walsh ’15

Computing challenges at NNLO

- NNLO differential computations require significant computational efforts
- Target for numerical accuracy is $\mathcal{O}(1 \text{ ppm})$
State-of-the-art (I): $e^+ e^- \rightarrow 3 \text{ jets}$

- Thrust $T = \max \left( \frac{\sum_i \vec{n} \cdot \vec{p}_i}{\sum_i |\vec{p}_i|} \right)$ for $e^+ e^- \rightarrow 3 \text{ jets}$ at NNLO

- Comparison for NNLO coefficient of weighted $\tau = 1 - T$ distribution
  - colourful subtraction Del Duca, Duhr, Kardos, Somogyi, Trocsanyi ‘16
  - SW Weinzierl ‘09; GGGH Gehrmann-De Ridder, Gehrmann, Glover, Heinrich ‘07
State-of-the-art (II): jet hadro-production

- Ratios of NLO and NNLO QCD predictions as function of jet $p_T$ for anti-$k_T$ jets with $R = 0.4$ to data at $\sqrt{s} = 13$ TeV ATLAS arXiv:1711.02692

- NNLO computation from standalone production run with $\sqrt{s}$, $\sqrt{R}$, PDFs fixed and three scale variations at CPU cost of $O(350000)$ h

- Scale choice $\mu_R = \mu_F = p_T^{max}$ (natural hard scale)
State-of-the-art (III): jet hadro-production

- Scale choice $\mu_R = \mu_F = p_T^{\text{jet}}$ with individual jet uses softer scale
  - generated events acquire different weight (softer jets with $\alpha_s(\mu_R)$)
- Ratios of theory to data display inverted hierarchy (NNLO corrections smaller than NLO)

ATLAS

$L = 81 \text{nb}^{-1} - 3.2 \text{ fb}^{-1}$

$\sqrt{s} = 13 \text{ TeV}$

anti-$k_t$, $R=0.4$

Data

NLO QCD

$\otimes k_{\text{EW}} \otimes k_{\text{NP}}$

NNLO QCD

$\otimes k_{\text{EW}} \otimes k_{\text{NP}}$

$\mu_R = \mu_F = p_T^{\text{jet}}$

NLO

MMHT 2014 NLO

NNLO

MMHT 2014 NNLO
QCD factorization

- Double differential cross section for $pp \rightarrow \text{jet} + X$
- Transverse momentum $p_T$ and rapidity $\eta$ of signal-jet

$$\frac{p_T^2 d^2 \sigma}{dp_T^2 d\eta} = \sum_{i_1 i_2} \int_0^{V(1-W)} dz \int_{\frac{1-V}{1-z}}^{1-\frac{1}{1-z}} dv \ x_1^2 f_{i_1}(x_1) x_2^2 f_{i_2}(x_2) \frac{d^2 \hat{\sigma}_{i_1 i_2}}{dv dz}(v, z, p_T, R)$$

- PDFs $f_i$ and variables $V = 1 - p_T e^{-\eta}/\sqrt{S}$, $V W = p_T e^{\eta}/\sqrt{S}$
- Partonic cross sections $\hat{\sigma}_{i_1 i_2}$ dependent on partonic kinematic variables on $s = x_1 x_2 S$, $v = u/(u + t)$ and partonic threshold $z = s_4/s \rightarrow 0$
- Mandelstam variables $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_2 - p_3)^2$ with kinematics constraint $s + t + u = s_4$
Threshold logarithms

- Soft and collinear regions of phase space
  - double logarithms from singular regions in Feynman diagrams
  - propagator vanishes for: \( E_g = 0 \), soft \( \theta_{qg} = 0 \) collinear

\[
\frac{1}{(p + k)^2} = \frac{1}{2p \cdot k} = \frac{1}{2E_qE_g(1 - \cos \theta_{qg})}
\]

\[
\alpha_s \int d^4k \frac{1}{(p + k)^2} \quad \rightarrow \quad \alpha_s \int dE_g d\sin \theta_{qg} \frac{1}{2E_qE_g(1 - \cos \theta_{qg})} \quad \rightarrow \quad \alpha_s \ln^2(\ldots)
\]

- Large double-logarithmic corrections \( \ln(\ldots) \gg 1 \) near threshold
- Single-inclusive jet production with threshold logarithms
  \( \alpha_s^n \frac{\ln^{2n-1}(z)}{z} \) for \( z = s_4/s \to 0 \)
  - positive corrections enhance partonic cross sections \( \hat{\sigma}_{i_1i_2} \)
  - long history of resummation Sterman ‘87; Catani, Trentadue ‘88; …
- Same for weak radiative corrections: positive corrections for large \( p_T \gg M_W \) Dittmaier, Huss, Speckner ‘12
Electroweak corrections at NLO

- Electroweak correction factors for inclusive jet cross-section as function of jet $p_T$ for all $|y|$ bins \cite{Dittmaier:2012}.  
  - negative corrections $\mathcal{O}(1 - 3\%)$ for $p_T$ in range $\mathcal{O}(\text{few 100})$ GeV  
  - sizeable (positive) effect $\mathcal{O}(10\%)$ at large $p_T \gg M_W$
Jet radius logarithms (I)

- Collinear singularity when the jet becomes very narrow
  - partons radiated outside of jet (not recombined with jet by chosen jet algorithm) become more and more collinear to emitter

- Example
  - loss of transverse momentum for leading jet
  - quasi-collinear branching of quark with transverse momentum $p_T$

\[
\delta p_T = (1 - z)p_T - p_T = -zp_T \quad \text{for} \quad (1 - z) > z
\]

\[
\delta p_T = (1 - z)p_T - p_T = -zp_T \quad \text{for} \quad z > (1 - z)
\]

- Perturbative radiation loss for average $\langle \delta p_T \rangle_q$

\[
\langle \delta p_T \rangle_q = p_T \frac{\alpha_s}{2\pi} \int \frac{d\theta^2}{\theta^2} \int z \left( \max[z, 1 - z] - 1 \right) P_{qq}(z)
\]
Jet radius logarithms (II)

- Leading order result for quark and gluon jets
  
  \[ \langle \delta p_T \rangle_q = C_F \frac{\alpha_s}{\pi} p_T \ln(R) \left( 2 \ln 2 - \frac{3}{8} \right) = 0.43 \alpha_s \ln(R) \]

  \[ \langle \delta p_T \rangle_q = \frac{\alpha_s}{\pi} p_T \ln(R) \left[ C_A \left( 2 \ln 2 - \frac{43}{96} \right) + T_f n_f \frac{7}{48} \right] = 1.02 \alpha_s \ln(R) \]

- Large single-logarithmic corrections \( \ln(1/R) \gg 1 \) for small \( R \)
  - **negative** corrections decrease partonic cross sections \( \hat{\sigma}_{i_1 i_2} \)
  - resummation …
Joint resummation

SCET factorization

- Factorization in small-$R$ and $z \to 0$ threshold limit
  - assume anti-$k_t$ jet algorithm, $z \sim R$, and small finite mass of jet

\[
\frac{d^2 \hat{\sigma}_{i_1 i_2}}{dv \, dz} = s \int ds_X \, ds_c \, ds_G \, \delta(zs - s_X - s_G - s_c) \\
\times \text{Tr} \left[ H_{i_1 i_2} (v, p_T, \mu_h, \mu) S_G (s_G, \mu_{sG}, \mu) \right] \, J_X (s_X, \mu_X, \mu) \\
\times \sum_m \text{Tr} \left[ J_m (p_T R, \mu_J, \mu) \otimes \Omega \right] S_{c,m} (s_c R, \mu_{sc}, \mu)
\]

- Specific functions for individual kinematic regions
  - hard functions for $2 \to 2$ scattering $H_{i_1 i_2}$ (known to 2-loops Broggio, Ferroglia, Pecjak, Zhang '14)
  - inclusive jet function $J_X (s_X)$ dependent on invariant mass $s_X$ of the recoiling collimated radiation (known to order $\alpha_s^2$ Becher, Neubert '06, Becher, Bell '10)
  - global soft function $S_G$ accounts for wide-angle soft radiation which cannot resolve the small radius $R$ (known to NLO Liu, S.M., Ringer '17)
Joint resummation

SCET factorization

- Factorization in small-$R$ and $z \to 0$ threshold limit
  - assume anti-$k_t$ jet algorithm, $z \sim R$, and small finite mass of jet

$$\frac{d^2 \hat{\sigma}_{i_1i_2}}{dv \, dz} = s \int ds_X \, ds_c \, ds_G \, \delta(zs - s_X - s_G - s_c)$$
  $$\times \text{Tr} \left[ H_{i_1i_2}(v, p_T, \mu_h, \mu) \, S_G(s_G, \mu_{sG}, \mu) \right] \, J_X(s_X, \mu_X, \mu)$$
  $$\times \sum_m \text{Tr} \left[ J_m(p_T R, \mu_J, \mu) \otimes_\Omega S_c m(s_c R, \mu_{sc}, \mu) \right]$$

- Specific functions for individual kinematic regions
  - signal-jet function $J(p_T R)$ accounts for energetic radiation inside jet
    Becher, Neubert, Rothen, Shao ‘15
  - soft-collinear (“coft”) function $S_c(s_c R)$ captures soft radiation near jet boundary
    Becher, Neubert, Rothen, Shao ‘15, Chien, Hornig, Lee ‘15

- Sum runs over all collinear splittings and traces taken in color space
- ‘$\otimes_\Omega$’ denotes associated angular integrals
  Becher, Neubert, Rothen, Shao ‘15
Phenomenology

- Evaluation of cross section in SCET (resummation) with renormalization group equations
  - evolution of all functions from their natural scales $\mu_i$ to common hard scale $\mu = p_T^{\text{max}}$
- Matching of NLL resummed results with full NLO calculation (need to avoid double counting)
  \[ d\sigma = d\sigma_{\text{NLL}} - d\sigma_{\text{NLO}_{\text{sin}}} + d\sigma_{\text{NLO}} \]
- resummed cross section $d\sigma_{\text{NLL}}$
- fixed order NLO result in singular limit $d\sigma_{\text{NLO}_{\text{sin}}}$
- complete fixed order NLO result $d\sigma_{\text{NLO}}$
Validation

- Comparison of fixed order NLO result in singular limit $d\sigma_{NLO_{\text{sin}}}$ with complete fixed order NLO result $d\sigma_{NLO}$

- Ratio $K_R = \frac{d\sigma_{NLO_{\text{sin}}}}{d\sigma_{NLO}}$ as function of jet $p_T$ for $1.5 < |\eta| < 2$ at $\sqrt{S} = 13$ TeV (error bars for numerical uncertainty)
Jet radius dependence (I)

Resummation for various $R$

- Ratio $K_R$ of NLO + NLL and NLO cross sections for different jet radii
  
  \[ K_R = \frac{\sigma_{\text{NLL+NLO}}(R)}{\sigma_{\text{NLO}}(R)} \]

- LHC at $\sqrt{s} = 7$ TeV (left) and 13 TeV (right) with NLO PDF set of MMHT

\[ \begin{array}{ll}
0.0 \leq |y| < 0.5 @ 7\text{TeV} & \quad 0.0 \leq |y| < 0.5 @ 13\text{TeV}
\end{array} \]

- Significant effect for small jet radii; reduction of $\mathcal{O}(20\%)$ for $R = 0.2$ in entire range of $p_T$
Jet radius dependence (II)

Impact of joint resummation

- Cross section ratio $D_R = \frac{\sigma(R)}{\sigma(R_{\text{fixed}})}$

- $D_R$ for NLO + NLL (left) and NLO (right) cross sections for $R_{\text{fixed}} = 0.5$ at $\sqrt{s} = 7$ TeV with NLO PDF set of MMHT14

- Single-inclusive jet data from collected at $\sqrt{s} = 7$ TeV with $R = 0.7$ by CMS [arXiv:1406.0324] (with NP correction factors)

- NLO result overshoots; joint resummation agrees with data for $D_{R=0.7}$
Data vs. theory (I)

LHC data at $\sqrt{S} = 7$ TeV

- Ratio $\frac{\sigma_{\text{Data}}}{\sigma_{\text{Theory}}}$ with $R = 0.5$ (left) and $R = 0.7$ (right) to theoretical results at NLO (black dots) and at NLO + NLL (red dots) accuracy
- NLO PDF set of MMHT14
- data at $\sqrt{S} = 7$ TeV by CMS [arXiv:1406.0324]

- Joint resummation agrees well with data; ratio with NLO predictions undershoots
Data vs. theory (II)

LHC data at $\sqrt{S} = 13$ TeV

- Ratio $\frac{\sigma_{\text{Data}}}{\sigma_{\text{Theory}}}$ with $R = 0.4$ (left) and $R = 0.7$ (right) to theoretical results at NLO (black dots) and at NLO + NLL (red dots) accuracy
  - NLO PDF set of MMHT14
  - data at $\sqrt{S} = 13$ TeV by CMS arXiv:1605.04436

0.0 $\leq |y| < 0.5$, $R = 0.4$ @ 13TeV

$0.0 \leq |y| < 0.5$, $R = 0.7$ @ 13TeV

- Same trend as for data at $\sqrt{S} = 7$ TeV, but still large experimental uncertainties
**Data vs. theory (III)**

**Experimental and theoretical uncertainties**

- Ratio $\sigma_{\text{Data}}/\sigma_{\text{Theory}}$ with $R = 0.5$ to theoretical results at NLO + NLL (left) and at NLO (right) accuracy at $\sqrt{s} = 7$ TeV using the NLO PDF set by CT10 for data collected by CMS [arXiv:1406.0324](http://arxiv.org/abs/1406.0324) (with NP correction factors)

  - error bars represent experimental statistical errors
  - solid (brown) lines represent experimental systematic ones
  - band (yellow) indicates theoretical scale uncertainties

![Graphical representation](attachment://graph.png)
PDF dependence

- Cross sections $\sigma_{NLO+NLL}$ at NLL + NLO accuracy with $R = 0.5$ at $\sqrt{s} = 7 \text{ TeV}$ normalized to the one with the NLO PDF set by CT10.
  - central NLO PDF sets HERAPDF2.0, MMHT14 and NNPDF3.1 (left)
  - central NLO PDF sets ABMP16, CT14 and NNPDF3.1 (right)
  - data collected by CMS[arXiv:1406.0324] (with NP correction factors) is superimposed.

NLO+NLL, $0.0 \leq |y| < 0.5 @ 7\text{ TeV}$
Summary

Theory framework

• Joint resummation of threshold logarithms $\alpha_s^n (\ln^{2n-1}(z)/z)_+$ and small jet-radius logarithms $\alpha_s^n \ln^n(R)$ to all orders at NLL accuracy
• Resummation at NLO + NLL accuracy in good agreement with CMS data
• Residual theoretical uncertainties from scale variation still of similar size as experimental systematics

Uses of jet measurements

• Determinations of strong coupling constant $\alpha_s(M_Z)$ and gluon PDF at medium to large-$x$ based on results fixed order perturbation theory likely to incur bias
• Precision of $\lesssim 1\%$ makes resummed theoretical predictions at NNLO + NNLL accuracy in QCD mandatory