Threshold and jet radius joint resummation for single-inclusive jet production

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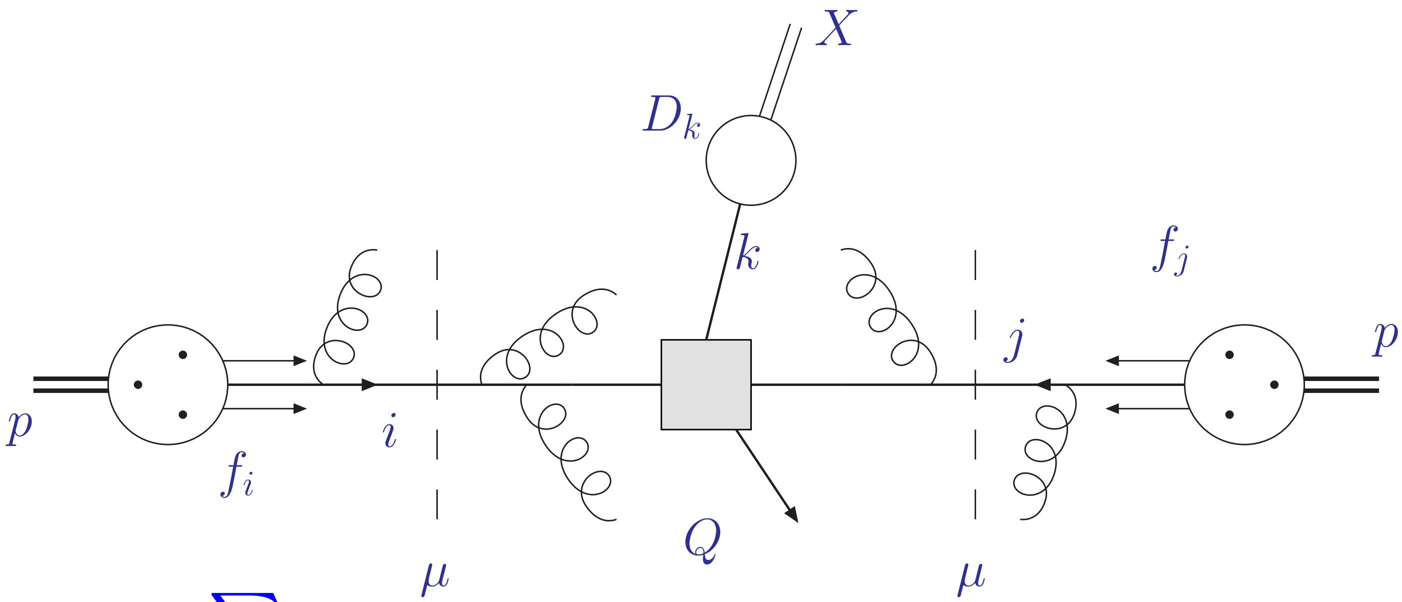
Based on work done in collaboration with:

- *Threshold and jet radius joint resummation for single-inclusive jet production*
  Xiaohui Liu, S. M. and Felix Ringer Phys.Rev.Lett. 119 (2017) no.21, 212001
  `arXiv:1708.04641`

- *Phenomenology of single-inclusive jet production with jet radius and threshold resummation*
  Xiaohui Liu, S. M. and Felix Ringer `arXiv:1801.07284`

QCD factorization

QCD factorization



$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu) \otimes f_j(\mu) \otimes \hat{\sigma}_{ij\to X}\left(\alpha_s(\mu), Q, \mu, m_X\right)$$

- Factorization at scale μ
  - separation of sensitivity to dynamics from long and short distances
- Hard parton cross section $\hat{\sigma}_{ij\to X}$ calculable in perturbation theory
  - cross section $\hat{\sigma}_{ij\to k}$ for parton types i, j and hadronic final state X
- Parton distribution functions f_i, strong coupling α_s, particle masses m_X
  - known from global fits to exp. data, lattice computations, . . .

Parton luminosity

- Long distance dynamics due to proton structure

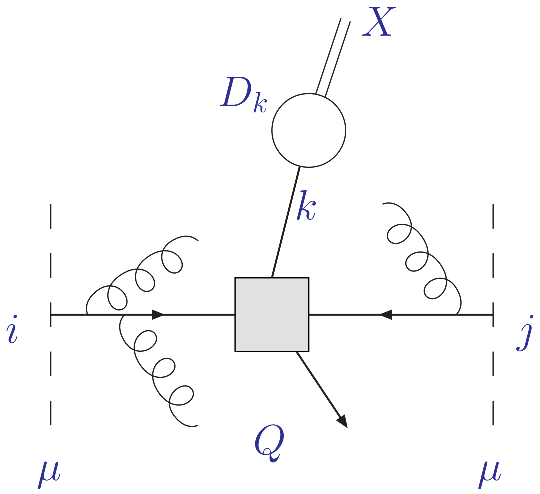
- Cross section depends on parton distributions f_i

$$\sigma_{pp\to X} = \sum_{ij} f_i(\mu) \otimes f_j(\mu) \otimes \Big[\dots \Big]$$

- Parton distributions known from global fits to exp. data
  - available fits accurate to NNLO
  - information on proton structure depends on kinematic coverage

Hard scattering cross section

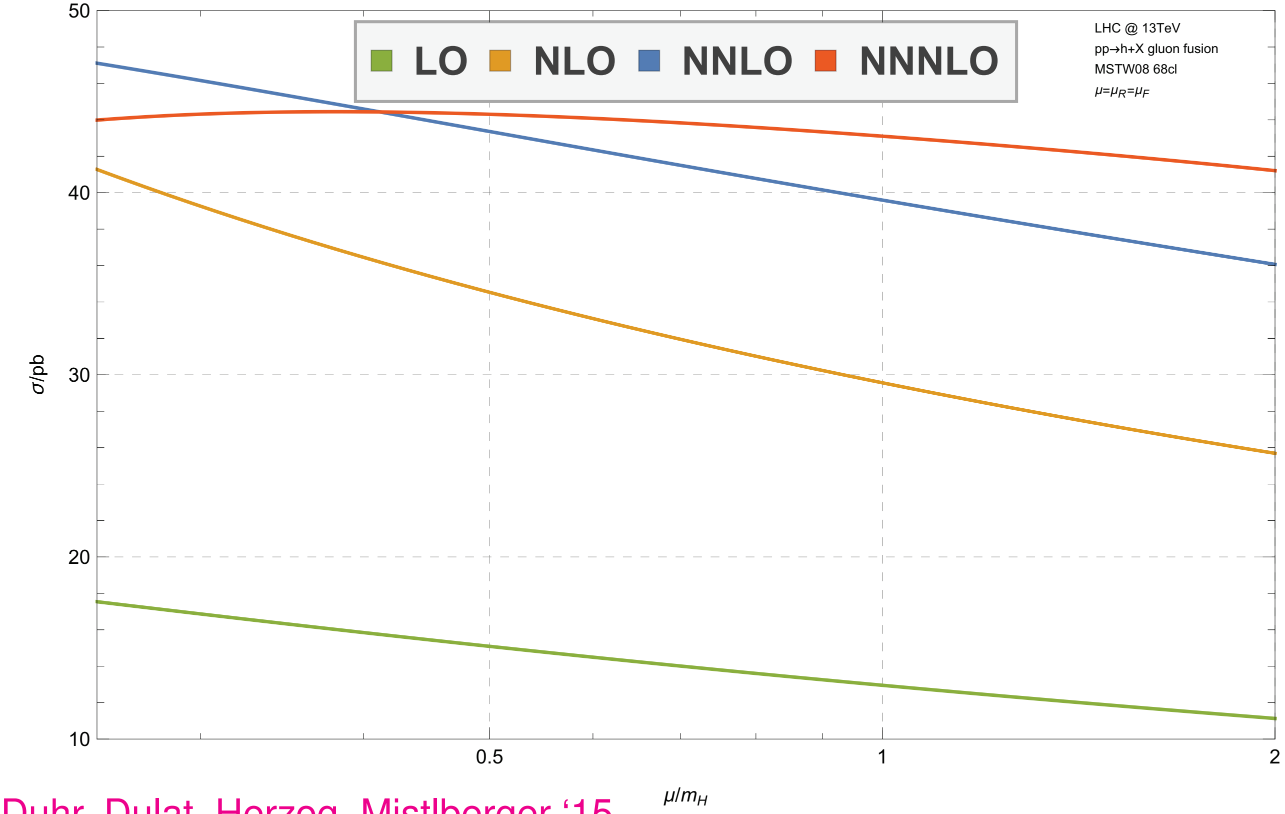
- Parton cross section $\hat{\sigma}_{ij \to k}$ calculable pertubatively in powers of α_s
  - known to NLO, NNLO, . . . ($\mathcal{O}(\mathrm{few}\%)$ theory uncertainty)



- Accuracy of perturbative predictions
  - LO (leading order) ($\mathcal{O}(50 - 100\%)$ unc.)
  - NLO (next-to-leading order) ($\mathcal{O}(10 - 30\%)$ unc.)
  - NNLO (next-to-next-to-leading order) ($\lesssim$ $\mathcal{O}(10\%)$ unc.)
  - N^{3}LO (next-to-next-to-next-to-leading order)
  - . . .

Perturbative QCD at higher orders (I)

Exact N^{3}LO Higgs cross section at LHC



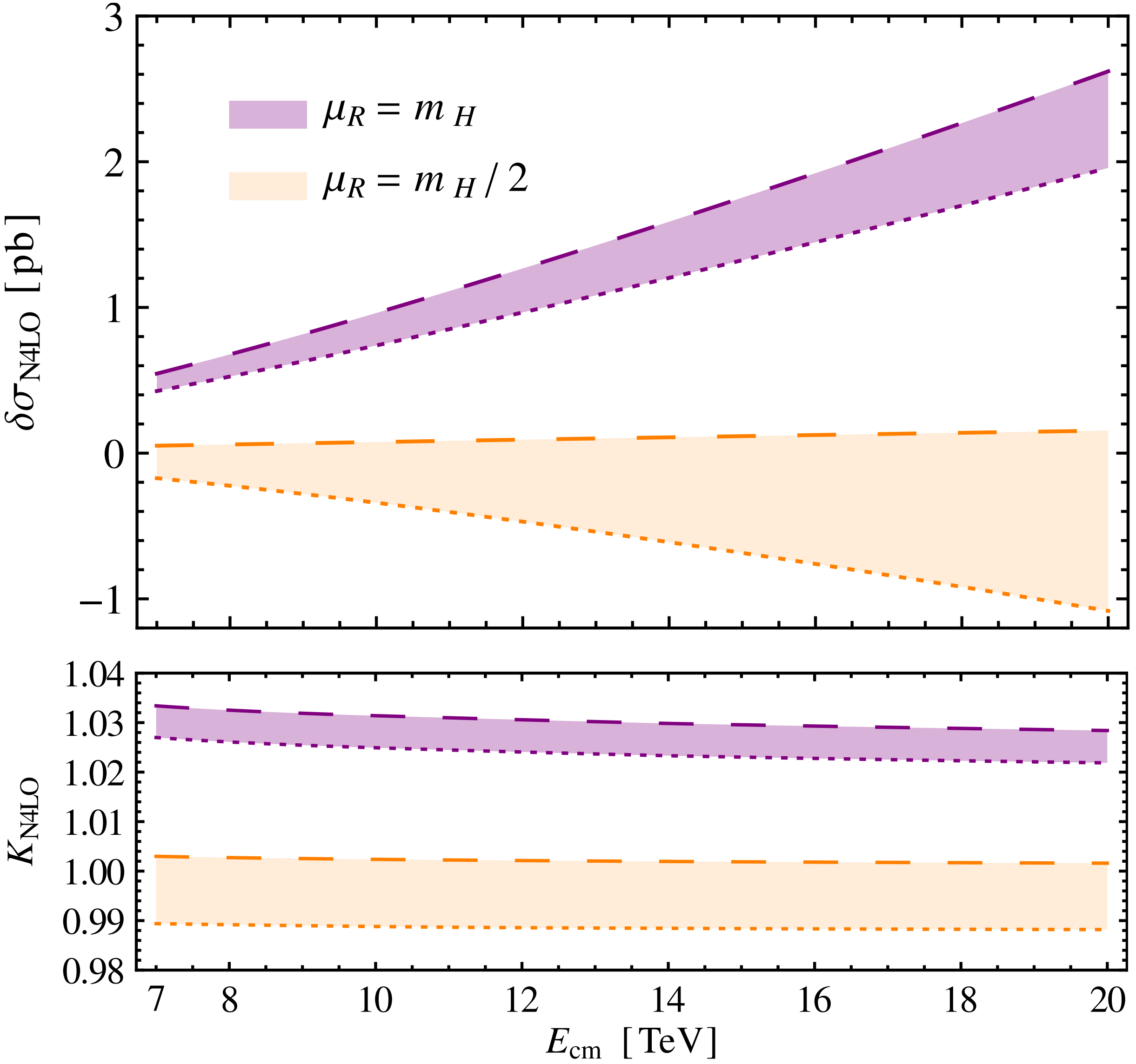
Anastasiou, Duhr, Dulat, Herzog, Mistlberger ‘15

- Apparent convergence of perturbative expansion
- Scale dependence of exact N^{3}LO prediction with residual uncertainty 3%
- Minimal sensitivity at scale $\mu = m_H/2$

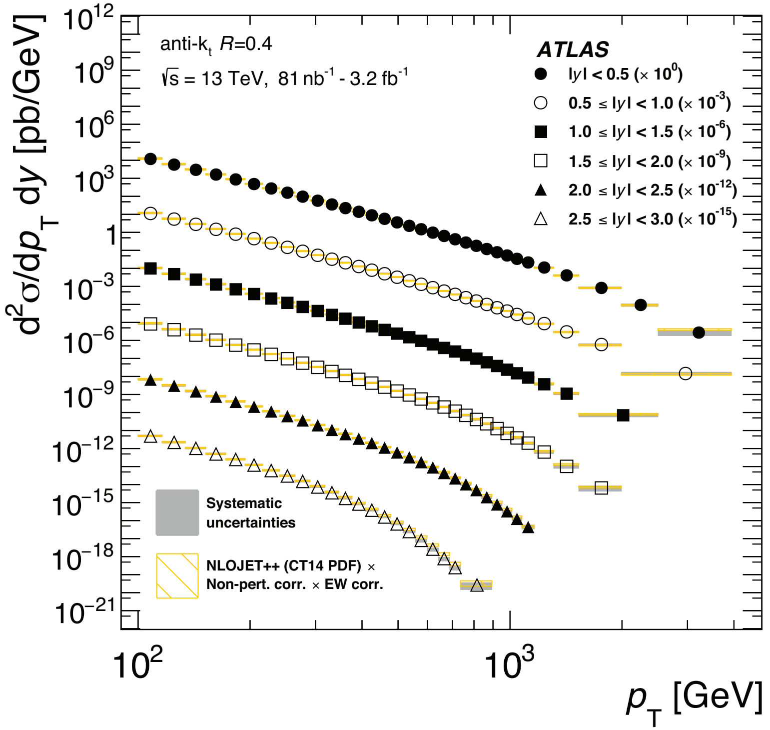
Perturbative QCD at higher orders (II)

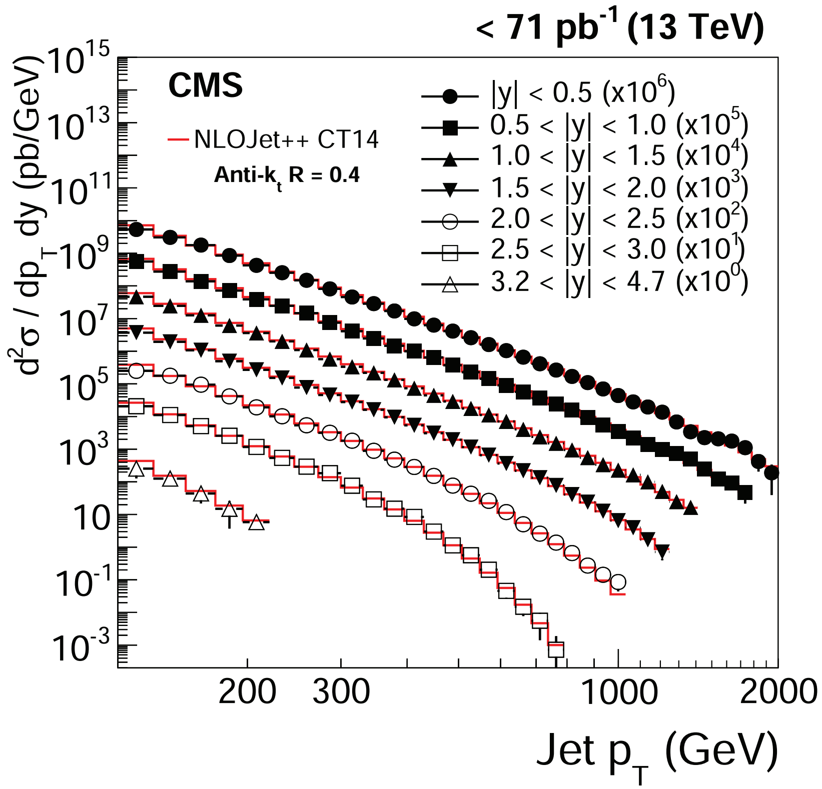
Approximate N^{4}LO Higgs cross section at LHC

- Threshold logarithms provide approximate N^{4}LO corrections at two scales $\mu = m_H$ and $\mu = m_H/2$
- K-factor $\simeq 1\%$ for $\mu = m_H/2$ with at $\sqrt{s} = 13$ TeV



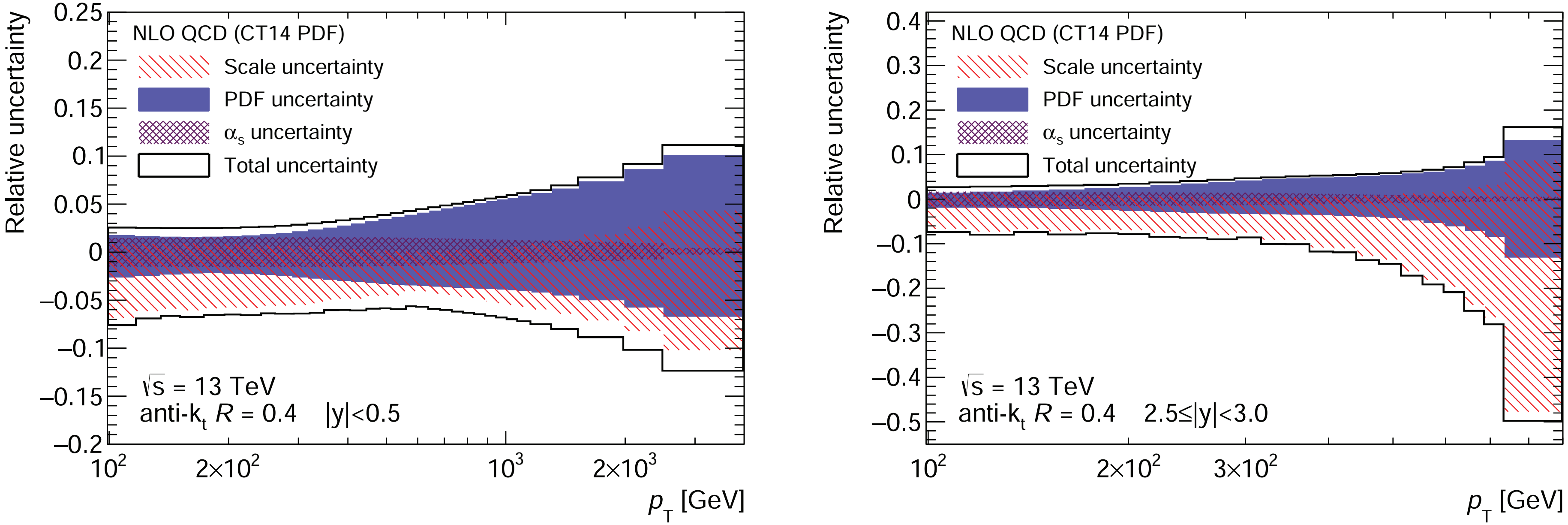
Single-inclusive jet production





- Double differential cross section for $pp \to$ jet $+ X$ at $\sqrt{s} = 13$ TeV
  - transverse momentum p_T and rapidity y of signal-jet
  - ATLAS arXiv:1711.02692 (left), CMS arXiv:1605.04436 (right)
- Comparison with NLO perturbative QCD predictions (NLOJET++ Nagy)
  - impressive agreement over several orders of magnitude

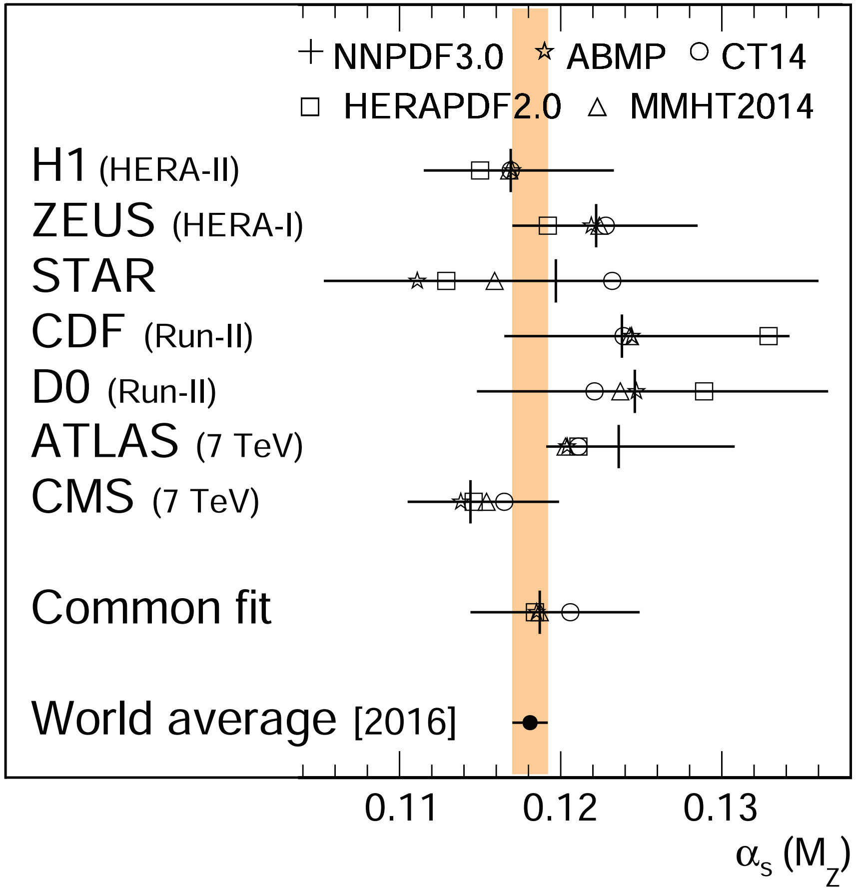
Uncertainties



- Relative QCD uncertainties in single-inclusive jet cross-sections at NLO
  - first and last $|y|$ bins for inclusive jet measurement
  - uncertainties due to renormalization and factorization scales, μ_R, μ_F, the strong coupling α_s with $\Delta\alpha_s = 0.0015$ and PDFs CT14
- Sizable uncertainties
  - $\mathcal{O}(10\%)$ for central rapidities $|y| < 0.5$
  - $\mathcal{O}(30 - 40\%)$ forward $2.5 \leq |y| < 3.0$ at large p_T

Uses of inclusive jet data

- Dermination of $\alpha_s(M_Z)$ and PDFs (gluon at medium to large x)
  - partonic cross sections $\hat{\sigma}_{ij\to\mathrm{jet}} \propto \alpha_s^2(\mu)$
  - jet cross section $d\sigma_{pp\to\mathrm{jet}} = \alpha_s^2(\mu) \sum_{ij} f_i(\mu) \otimes f_j(\mu) \otimes [\dots]$



- $\alpha_s(M_Z)$ at NLO in QCD from inclusive jet cross section data

  Britzger, Rabbertz, Savoiu, Sieber '17
- correlations between PDFs and $\alpha_s(M_Z)$ are important

Single-inclusive jet production

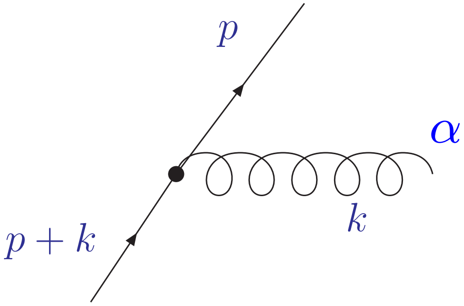
Theory status at NNLO

- Fully differential kinematics to match experimental cuts and fiducial cross section
- QCD corrections at NNLO

  Gehrmann-De Ridder, Gehrmann, Glover, Pires ‘13; Currie, Glover, Pires ‘16
  - all partonic channels; leading color contributions only

Soft and collinear singularities

- Soft/collinear regions of phase space
  - massless partons



$$\frac{1}{(p+k)^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_qE_g(1-\cos\theta_{qg})}$$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \longrightarrow \alpha_s \int dE_g\, d\theta_{qg}\, \frac{1}{2E_qE_g(1-\cos\theta_{qg})}$$

$$\longrightarrow \alpha_s \frac{1}{\epsilon^2} \times (\dots)$$ in dim. reg. $D = 4 - 2\epsilon$

- Regularization for real emissions required

Regularization

Subtraction

- removes all $\frac{1}{\epsilon^k}$ singularities analytically; $k \leq 4$ at NNLO
  - Antenna subtraction Gehrmann-De Ridder, Gehrmann, Glover '05
  - Colourful subtraction Del Duca, Somogyi, Trocsanyi '05
  - Sector subtraction
    Czakon '10; Boughezal, Melnikov, Petriello '11; Czakon, Heymes '14
  - Projection to Born Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15

Slicing

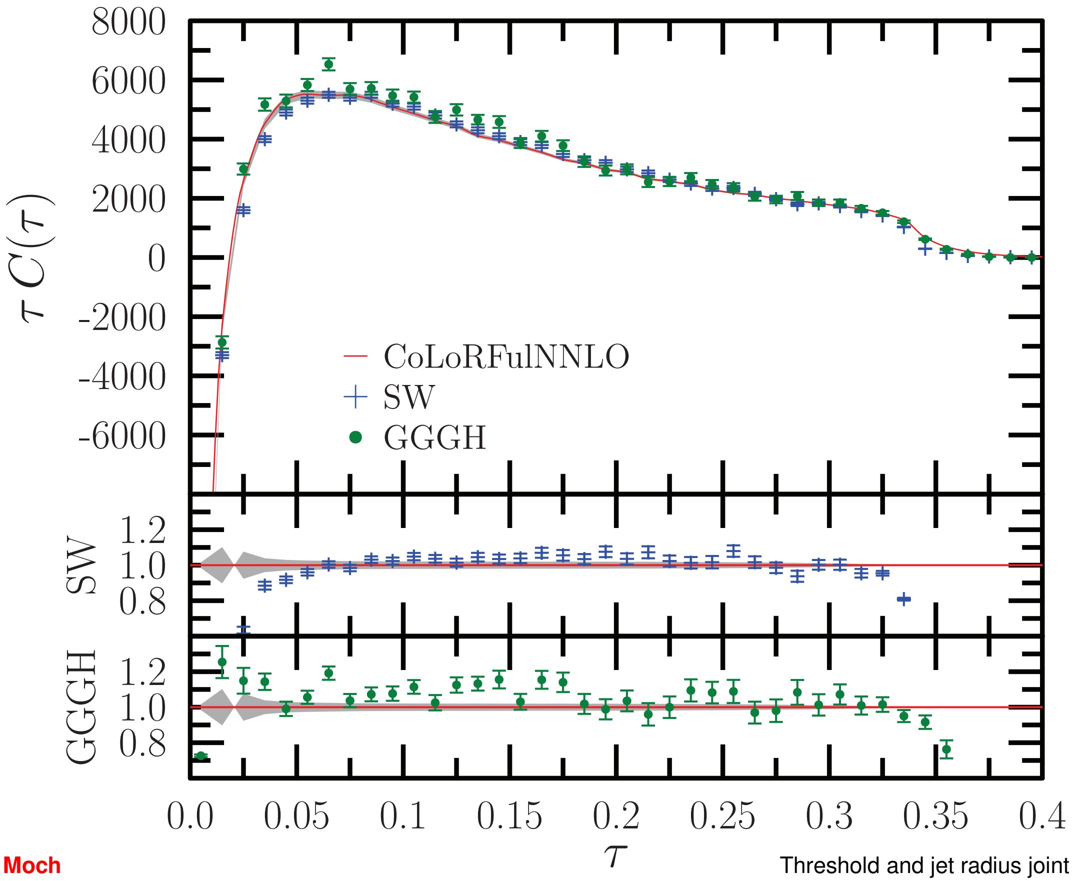
- Cuts imposed in phase space; singularities $\ln^k(\text{some cut})$ need to cancel numerically; $k \leq 4$ at NNLO
  - q_T subtraction Catani, Grazzini '07
  - N-jettiness subtraction
    Boughezal, Focke, Liu, Petriello '15; Gaunt, Stahlhofen, Tackmann, Walsh '15

Computing challenges at NNLO

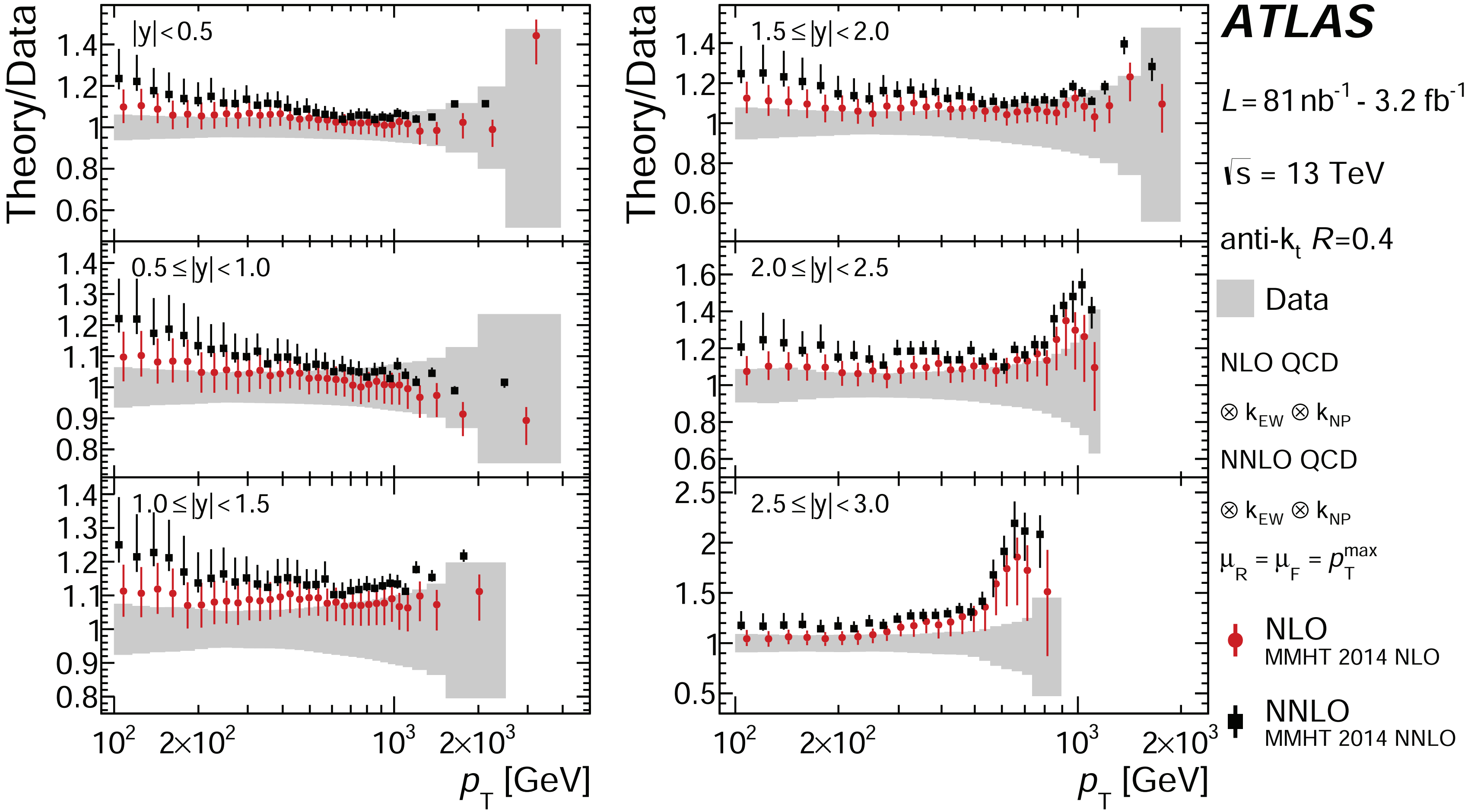
- NNLO differential computations require significant computational efforts
- Target for numerical accuracy is $\mathcal{O}(1 \text{ ppm})$

State-of-the-art (I): $e^+e^- \to 3$ jets

- Thrust $T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \right)$ for $e^+e^- \to 3$ jets at NNLO
- Comparison for NNLO coefficient of weighted $\tau = 1 - T$ distribution
  - colourful subtraction Del Duca, Duhr, Kardos, Somogyi, Trocsanyi '16
  - SW Weinzierl '09; GGGH Gehrmann-De Ridder, Gehrmann, Glover, Heinrich '07

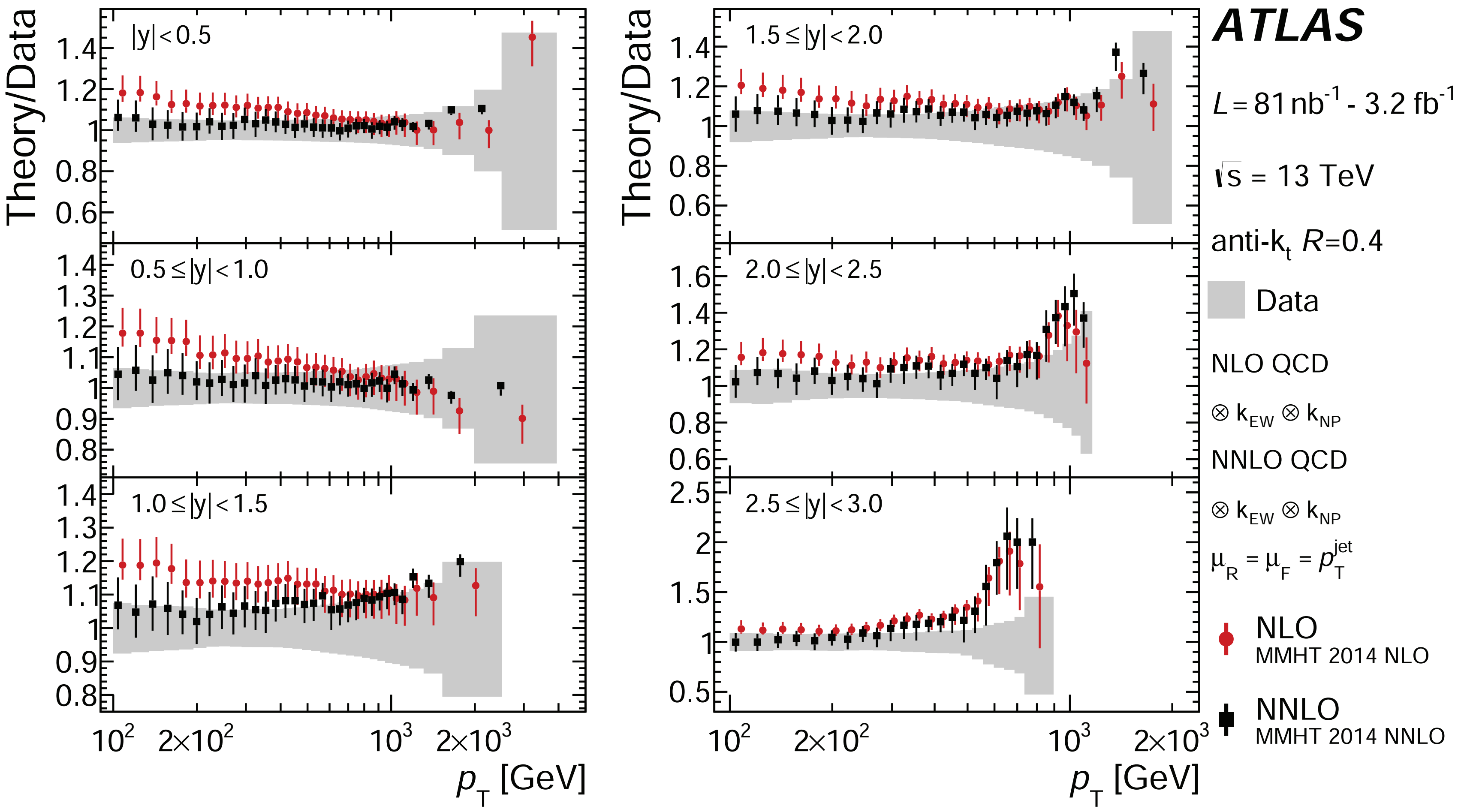


State-of-the-art (II): jet hadro-production



- Ratios of NLO and NNLO QCD predictions as function of jet p_T for anti-k_T jets with $R = 0.4$ to data at $\sqrt{s} = 13$ TeV ATLAS arXiv:1711.02692
- NNLO computation from standalone production run with $\sqrt{s}$, $\sqrt{R}$, PDFs fixed and three scale variations at CPU cost of $\mathcal{O}(350000)$ h
- Scale choice $\mu_R = \mu_F = p_{\mathrm{T}}^{\mathrm{max}}$ (natural hard scale)

State-of-the-art (III): jet hadro-production



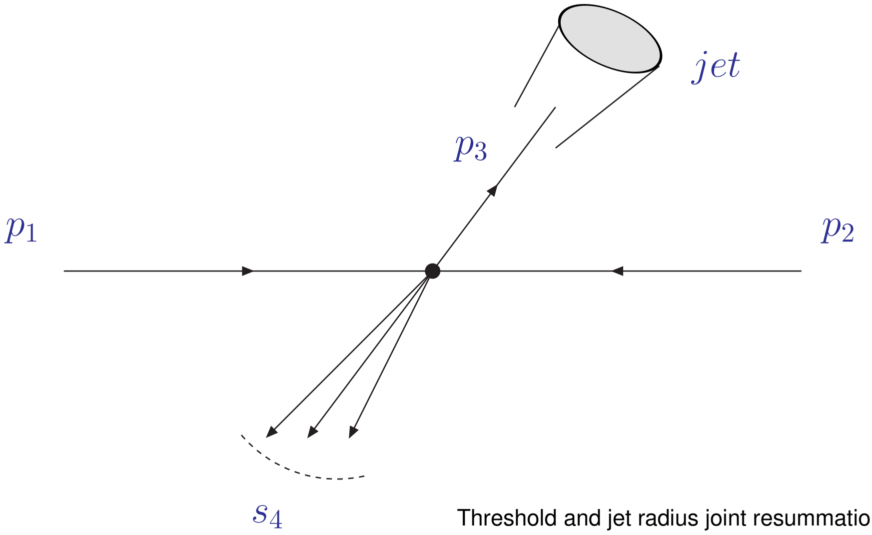
- Scale choice $\mu_R = \mu_F = p_{\mathrm{T}}^{\mathrm{jet}}$ with individual jet uses softer scale
  - generated events acquire different weight (softer jets with $\alpha_s(\mu_R)$)
- Ratios of theory to data display inverted hierarchy (NNLO corrections smaller than NLO)

QCD factorization

- Double differential cross section for $pp \to \text{jet} + X$
  - transverse momentum p_T and rapidity η of signal-jet

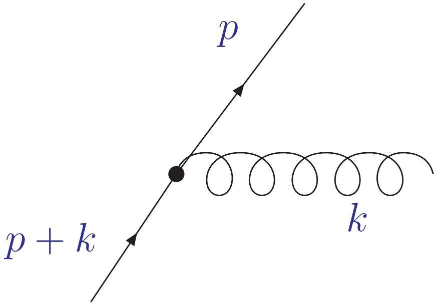
$$\frac{p_T^2 \mathrm{d}^2\sigma}{\mathrm{d}p_T^2 \mathrm{d}\eta} = \sum_{i_1 i_2} \int_0^{V(1-W)} \mathrm{d}z \int_{\frac{VW}{1-z}}^{1-\frac{1-V}{1-z}} \mathrm{d}v \, x_1^2 \, f_{i_1}(x_1) \, x_2^2 \, f_{i_2}(x_2) \, \frac{\mathrm{d}^2\hat{\sigma}_{i_1 i_2}}{\mathrm{d}v \, \mathrm{d}z}(v, z, p_T, R)$$

- PDFs f_i and variables $V = 1 - p_T e^{-\eta}/\sqrt{S}$, $VW = p_T e^{\eta}/\sqrt{S}$
- Partonic cross sections $\hat{\sigma}_{i_1 i_2}$ dependent on partonic kinematic variables on $s = x_1 x_2 S$, $v = u/(u+t)$ and partonic threshold $z = s_4/s \to 0$
- Mandelstam variables $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$ and $u = (p_2 - p_3)^2$ with kinematics constraint $s + t + u = s_4$



Threshold logarithms

- Soft and collinear regions of phase space
  - double logarithms from singular regions in Feynman diagrams
  - propagator vanishes for: $E_g = 0$, soft $\theta_{qg} = 0$ collinear



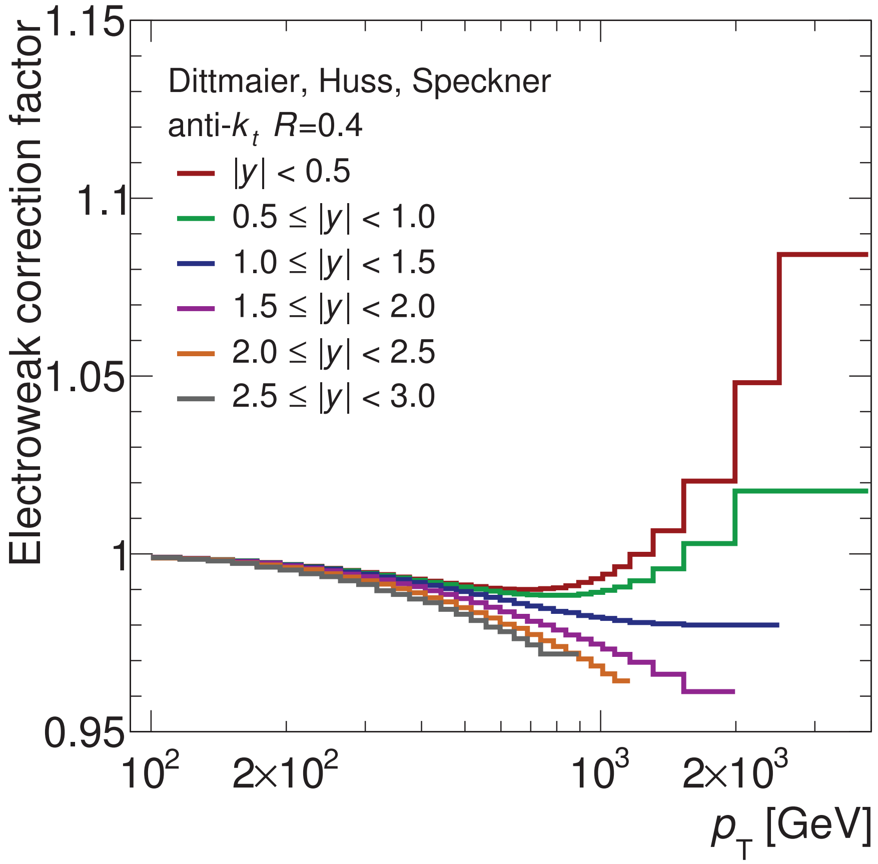
$$\frac{1}{(p+k)^2} = \frac{1}{2p\cdot k} = \frac{1}{2E_q E_g (1-\cos\theta_{qg})}$$

$$\alpha_s \int d^4k \frac{1}{(p+k)^2} \longrightarrow \alpha_s \int dE_g \, d\sin\theta_{qg} \frac{1}{2E_q E_g (1-\cos\theta_{qg})}$$

$$\longrightarrow \alpha_s \ln^2(\dots)$$

- Large double-logarithmic corrections $\ln(\dots) \gg 1$ near threshold
- Single-inclusive jet production with threshold logarithms $\alpha_s^n (\ln^{2n-1}(z)/z)_+$ for $z = s_4/s \to 0$
  - *positive* corrections enhance partonic cross sections $\hat{\sigma}_{i_1 i_2}$
  - long history of resummation Sterman '87; Catani, Trentadue '88; ...
- Same for weak radiative corrections: positive corrections for large $p_T \gg M_W$ Dittmaier, Huss, Speckner '12

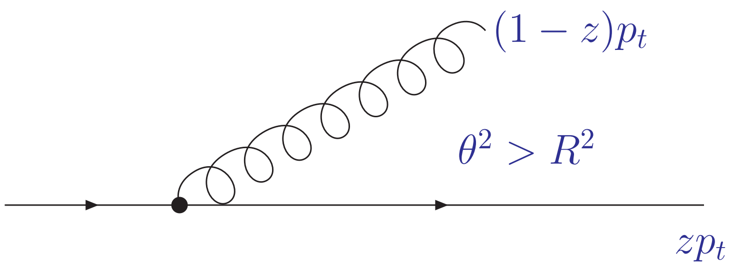
Electroweak corrections at NLO



- Electroweak correction factors for inclusive jet cross-section as function of jet p_T for all $|y|$ bins Dittmaier, Huss, Speckner '12
  - negative corrections $\mathcal{O}(1-3\%)$ for p_T in range $\mathcal{O}(\text{few } 100)$ GeV
  - sizeable (positive) effect $\mathcal{O}(10\%)$ at large $p_T \gg M_W$

Jet radius logarithms (I)

- Collinear singularity when the jet becomes very narrow
  - partons radiated outside of jet (not recombined with jet by chosen jet algorithm) become more and more collinear to emitter
- Example
  - loss of transverse momentum for leading jet
  - quasi-collinear branching of quark with transverse momentum p_T



$$\begin{aligned} \delta p_T &= (1-z)p_T - p_T &= -zp_T \text{ for } (1-z) > z \\ \delta p_T &= (1-z)p_T - p_T &= -zp_T \text{ for } z > (1-z) \end{aligned}$$

- Perturbative radiation loss for average $\langle \delta p_T \rangle_{\mathrm{q}}$

$$\langle \delta p_T \rangle_{\mathrm{q}} = p_T \frac{\alpha_s}{2\pi} \int_{R^2}^{1} \frac{d\theta^2}{\theta^2} \int dz \Big(\max[z, 1-z] - 1 \Big) P_{qq}(z)$$

Jet radius logarithms (II)

- Leading order result for quark and gluon jets Dasgupta, Magnea, Salam ‘07

$$\langle \delta p_T \rangle_{\mathrm{q}} = C_F \frac{\alpha_s}{\pi} p_T \ln(R) \left(2\ln 2 - \frac{3}{8} \right) = 0.43\, \alpha_s \ln(R)$$

$$\langle \delta p_T \rangle_{\mathrm{q}} = \frac{\alpha_s}{\pi} p_T \ln(R) \left[C_A \left(2\ln 2 - \frac{43}{96} \right) + T_f\, n_f \frac{7}{48} \right] = 1.02\, \alpha_s \ln(R)$$

- Large single-logarithmic corrections $\ln(1/R) \gg 1$ for small R
  - *negative* corrections decrease partonic cross sections $\hat{\sigma}_{i_1 i_2}$
  - resummation …

Joint resummation

SCET factorization

- Factorization in small-R and $z \to 0$ threshold limit
  - assume anti-k_t jet algorithm, $z \sim R$, and small finite mass of jet

$$
\begin{aligned}
\frac{\mathrm{d}^2 \hat{\sigma}_{i_1 i_2}}{\mathrm{d}v\,\mathrm{d}z} \quad = \quad & s \int \mathrm{d}s_X\,\mathrm{d}s_c \mathrm{d}s_G\,\delta(zs - s_X - s_G - s_c) \\
& \times \mathrm{Tr}\left[\mathbf{H}_{i_1 i_2}(v, p_T\,, \mu_h\,, \mu)\,\mathbf{S}_G(s_G\,, \mu_{sG}\,, \mu)\right]\;J_X(s_X\,, \mu_X\,, \mu) \\
& \times \sum_m \mathrm{Tr}\left[J_m(p_T R\,, \mu_J\,, \mu) \otimes_\Omega S_{c,m}(s_c R\,, \mu_{sc}\,, \mu)\right]
\end{aligned}
$$

- Specific functions for individual kinemtic regions
  - hard functions for $2 \to 2$ scattering $\mathbf{H}_{i_1 i_2}$ (known to 2-loops Broggio, Ferroglia, Pecjak, Zhang '14)
  - inclusive jet function $J_X(s_X)$ dependent on invariant mass s_X of the recoiling collimated radiation (known to order α_s^2 Becher, Neubert '06, Becher, Bell '10)
  - global soft function $\mathbf{S}_G$ accounts for wide-angle soft radiation which cannot resolve the small radius R (known to NLO Liu, S.M., Ringer '17)

Joint resummation

SCET factorization

- Factorization in small-R and $z \to 0$ threshold limit
  - assume anti-k_t jet algorithm, $z \sim R$, and small finite mass of jet

$$
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& \times \mathrm{Tr}\left[\mathbf{H}_{i_1 i_2}(v, p_T\,, \mu_h\,, \mu)\, \mathbf{S}_G(s_G\,, \mu_{sG}\,, \mu)\right]\; J_X(s_X\,, \mu_X\,, \mu) \\
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\end{aligned}
$$

- Specific functions for individual kinemtic regions
  - signal-jet function $J(p_T\, R)$ accounts for energetic radiation inside jet Becher, Neubert, Rothen, Shao ‘15
  - soft-collinear (“coft”) function $S_c(s_c R)$ captures soft radiation near jet boundary Becher, Neubert, Rothen, Shao ‘15, Chien, Hornig, Lee ‘15
- Sum runs over all collinear splittings and traces taken in color space
- ‘$\otimes_\Omega$’ denotes associated angular integrals Becher, Neubert, Rothen, Shao ‘15

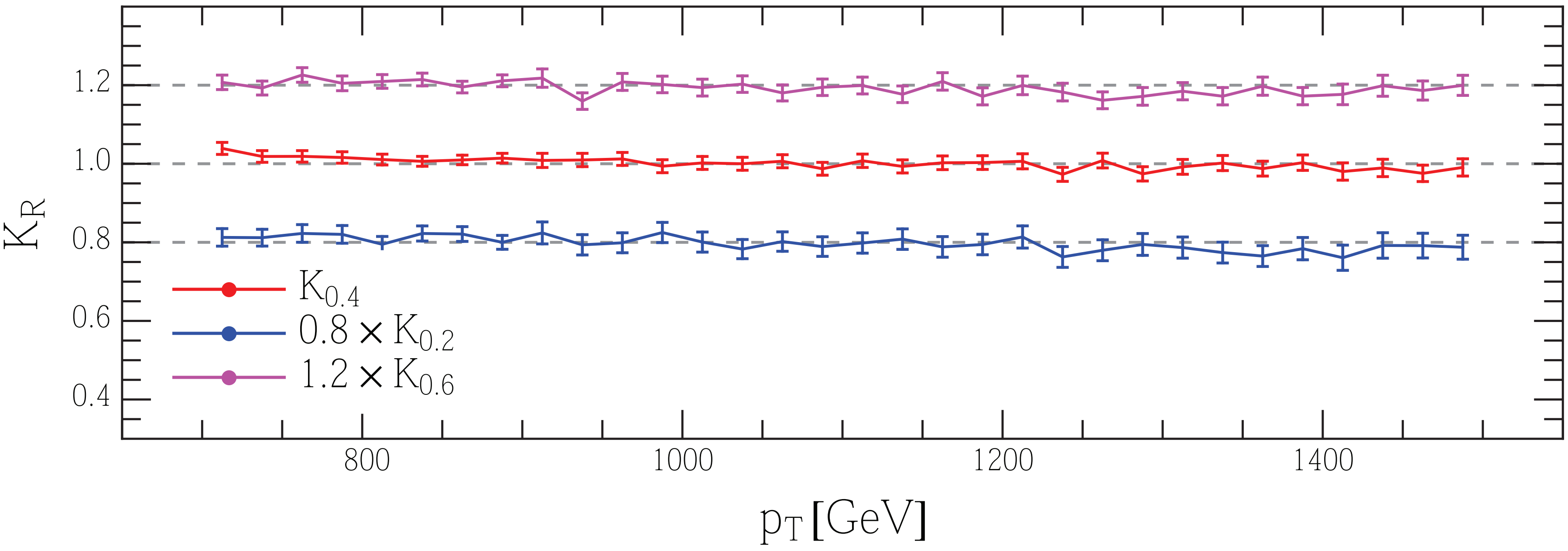
Phenomenology

- Evaluation of cross section in SCET (resummation) with renormalization group equations
  - evolution of all functions from their natural scales μ_i to common hard scale $\mu = p_T^{\max}$
- Matching of NLL resummed results with full NLO calculation (need to avoid double counting)

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{NLL}} - \mathrm{d}\sigma_{\mathrm{NLO}_{\mathrm{sin}}} + \mathrm{d}\sigma_{\mathrm{NLO}}$$

  - resummed cross section $\mathrm{d}\sigma_{\mathrm{NLL}}$
  - fixed order NLO result in singular limit $\mathrm{d}\sigma_{\mathrm{NLO}_{\mathrm{sin}}}$
  - complete fixed order NLO result $\mathrm{d}\sigma_{\mathrm{NLO}}$

Validation

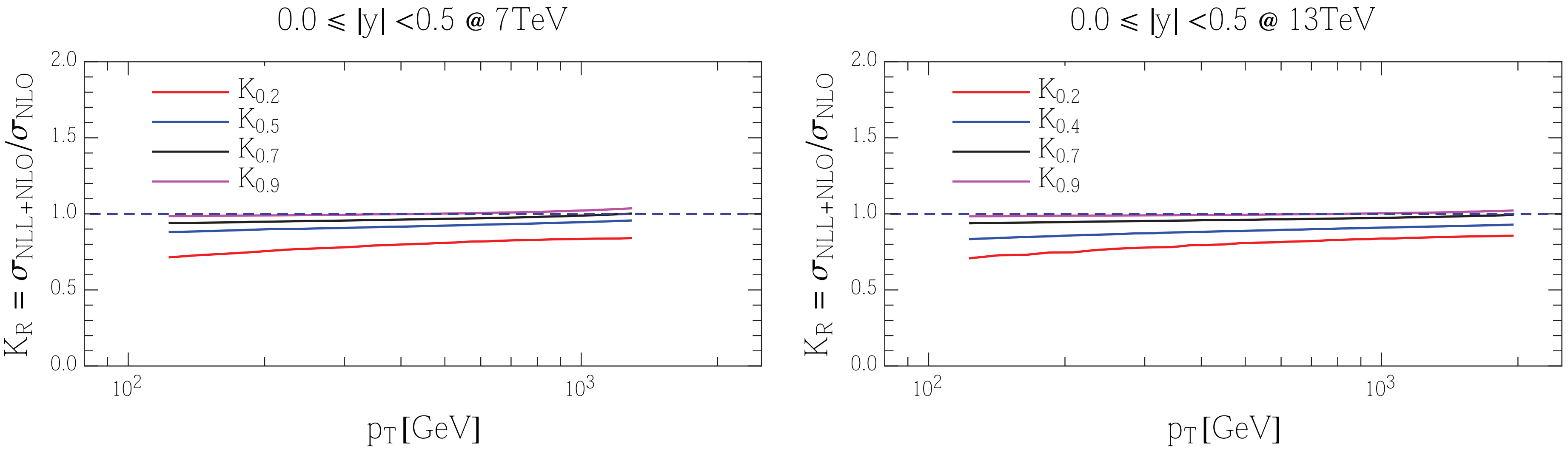


- Comparison of fixed order NLO result in singular limit $\mathrm{d}\sigma_{\mathrm{NLO}_{\mathrm{sin}}}$ with complete fixed order NLO result $\mathrm{d}\sigma_{\mathrm{NLO}}$
- Ratio $K_R = \dfrac{\mathrm{d}\sigma_{\mathrm{NLO}_{\mathrm{sin}}}}{\mathrm{d}\sigma_{\mathrm{NLO}}}$ as function of jet p_T for $1.5 < |\eta| < 2$ at $\sqrt{S} = 13$ TeV (error bars for numerical uncertainty)

Jet radius dependence (I)

Resummation for various R

- Ratio K_R of NLO + NLL and NLO cross sections for different jet radii
  $$K_R = \frac{\sigma_{\mathrm{NLL+NLO}}(R)}{\sigma_{\mathrm{NLO}}(R)}$$
- LHC at $\sqrt{S} = 7$ TeV (left) and 13 TeV (right) with NLO PDF set of MMHT

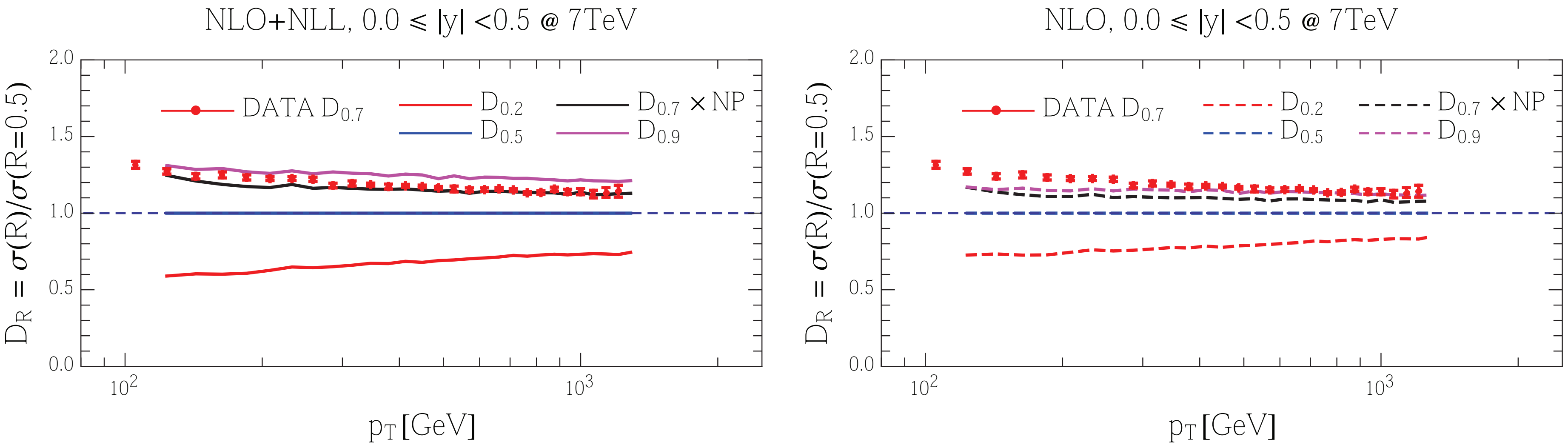


- Significant effect for small jet radii; reduction of $\mathcal{O}(20\%)$ for $R = 0.2$ in entire range of p_T

Jet radius dependence (II)

Impact of joint resummation

- Cross section ratio $D_R = \dfrac{\sigma(R)}{\sigma(R_{\text{fixed}})}$
- D_R for NLO + NLL (left) and NLO (right) cross sections for $R_{\text{fixed}} = 0.5$ at $\sqrt{S} = 7$ TeV with NLO PDF set of MMHT14
- Single-inclusive jet data from collected at $\sqrt{S} = 7$ TeV with $R = 0.7$ by CMS `arXiv:1406.0324` (with NP correction factors)

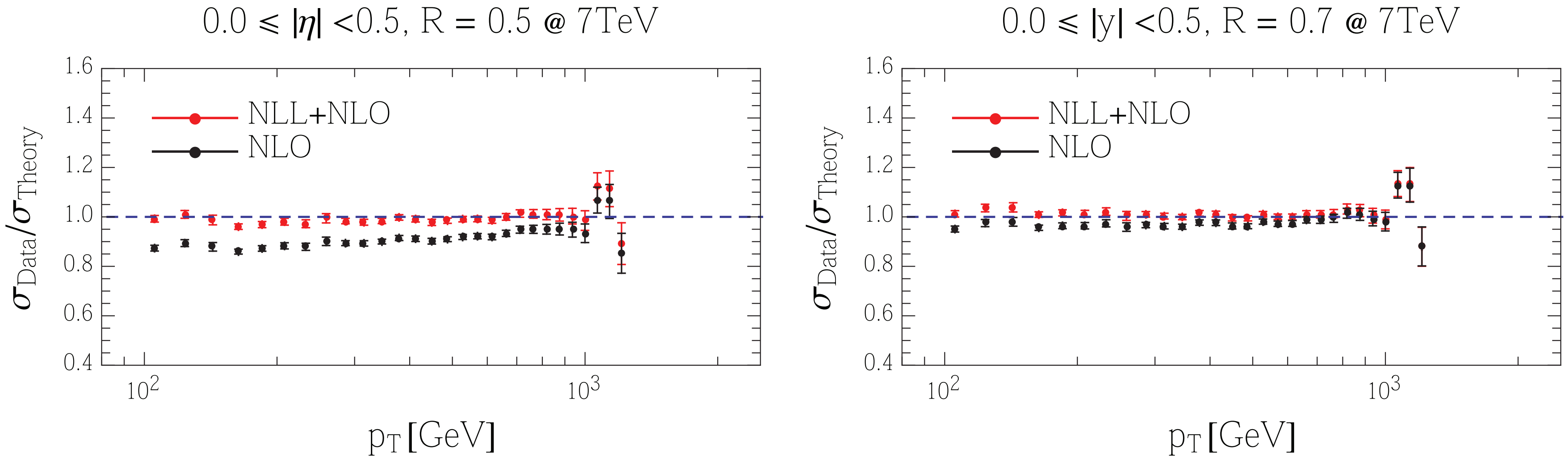


- NLO result overshoots; joint resummation agrees with data for $D_{R=0.7}$

Data vs. theory (I)

LHC data at $\sqrt{S} = 7$ TeV

- Ratio $\sigma_{\rm Data}/\sigma_{\rm Theory}$ with $R = 0.5$ (left) and $R = 0.7$ (right) to theoretical results at NLO (black dots) and at NLO $+$ NLL (red dots) accuracy
  - NLO PDF set of MMHT14
  - data at $\sqrt{S} = 7$ TeV by CMS `arXiv:1406.0324`

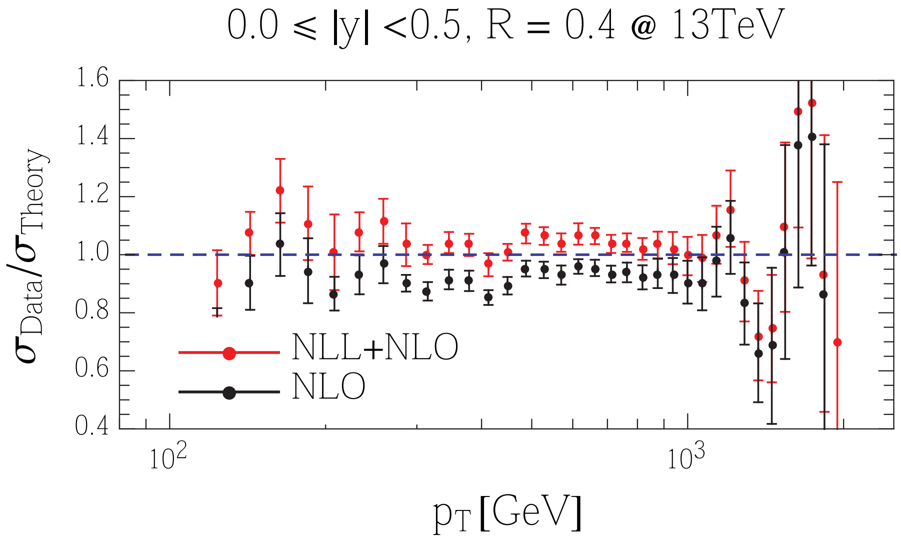


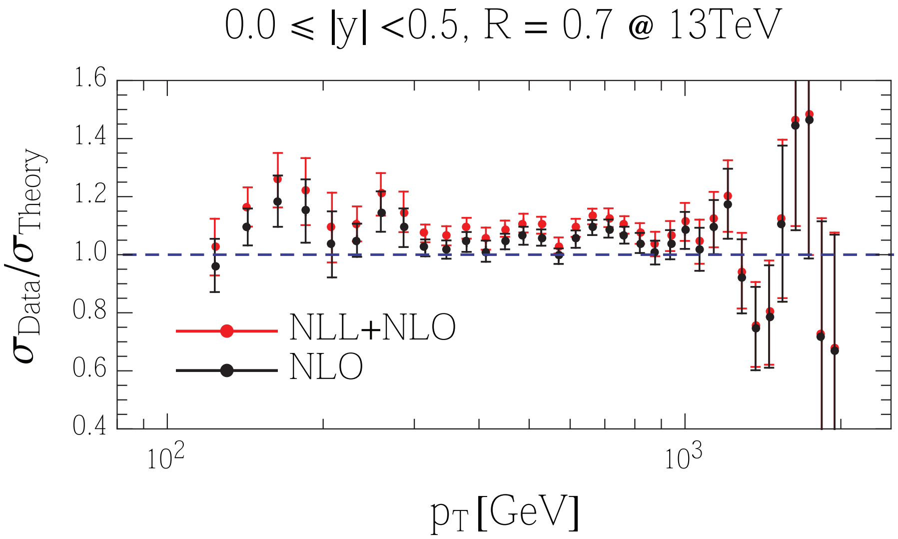
- Joint resummation agrees well with data; ratio with NLO predictions undershoots

Data vs. theory (II)

LHC data at $\sqrt{S} = 13$ TeV

- Ratio $\sigma_{\rm Data}/\sigma_{\rm Theory}$ with $R = 0.4$ (left) and $R = 0.7$ (right) to theoretical results at NLO (black dots) and at NLO $+$ NLL (red dots) accuracy
  - NLO PDF set of MMHT14
  - data at $\sqrt{S} = 13$ TeV by CMS `arXiv:1605.04436`



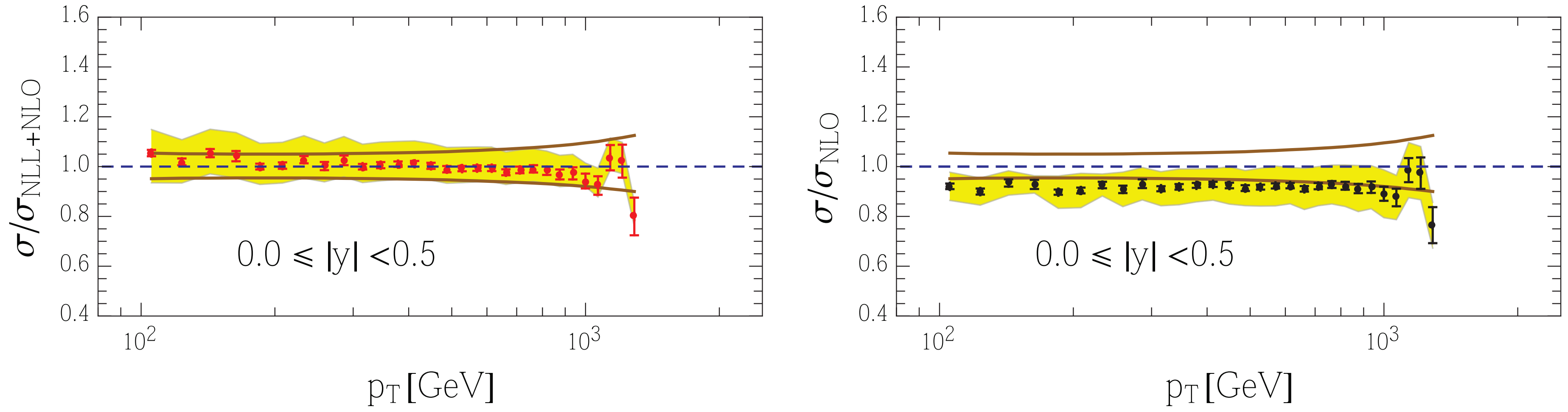


- Same trend as for data at $\sqrt{S} = 7$ TeV, but still large experimental uncertainties

Data vs. theory (III)

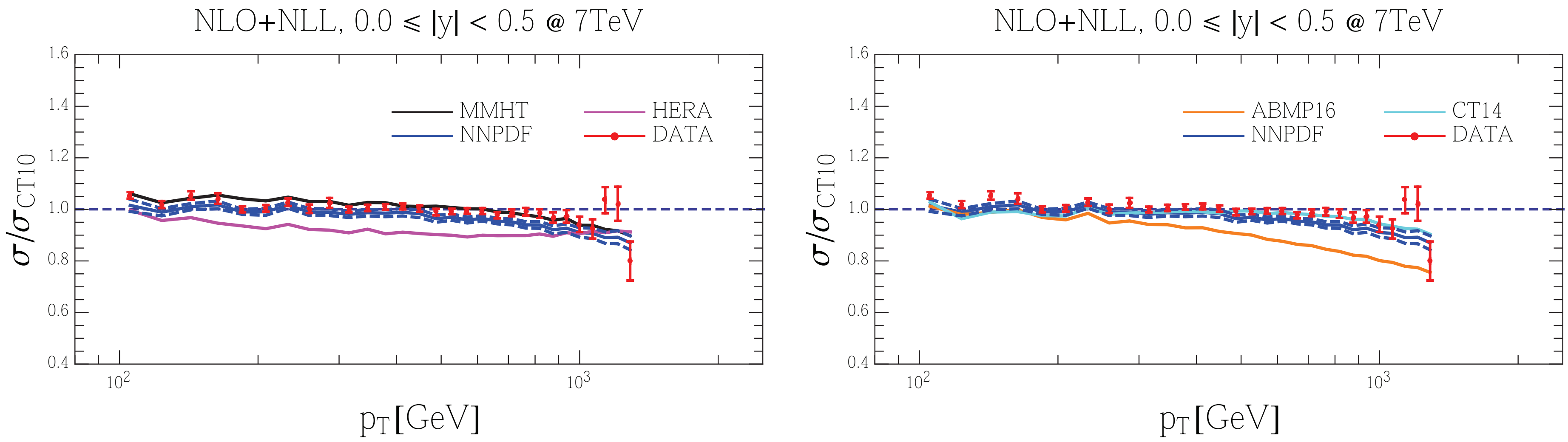
Experimental and theoretical uncertainties

- Ratio $\sigma_{\mathrm{Data}}/\sigma_{\mathrm{Theory}}$ with $R = 0.5$ to theoretical results at NLO $+$ NLL (left) and at NLO (right) accuracy at $\sqrt{S} = 7$ TeV using the NLO PDF set by CT10 for data collected by CMS `arXiv:1406.0324` (with NP correction factors)
  - error bars represent experimental statistical errors
  - solid (brown) lines represent experimental systematic ones
  - band (yellow) indicates theoretical scale uncertainties



PDF dependence

- Cross sections $\sigma_{\mathrm{NLO+NLL}}$ at NLL + NLO accuracy with $R = 0.5$ at $\sqrt{S} = 7$ TeV normalized to the one with the NLO PDF set by CT10
  - central NLO PDF sets HERAPDF2.0, MMHT14 and NNPDF3.1 (left)
  - central NLO PDF sets ABMP16, CT14 and NNPDF3.1 (right)
  - data collected by CMS `arXiv:1406.0324` (with NP correction factors) is superimposed



Summary

Theory framework

- Joint resummation of threshold logarithms $\alpha_s^n (\ln^{2n-1}(z)/z)_+$ and small jet-radius logarithms $\alpha_s^n \ln^n(R)$ to all orders at NLL accuracy
- Resummation at NLO $+$ NLL accuracy in good agreement with CMS data
- Residual theoretical uncertainties from scale variation still of similar size as experimental systematics

Uses of jet measurements

- Determinations of strong coupling constant $\alpha_s(M_Z)$ and gluon PDF at medium to large-x based on results fixed order perturbation theory likely to incur bias
- Precision of $\lesssim 1\%$ makes resummed theoretical predictions at NNLO $+$ NNLL accuracy in QCD mandatory