The Yang and Yin of Neutrinos

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Introduction

Neutrinos have a bright side (Yang) because they are essential in the nuclear processes which power the sun:

$$
\begin{aligned}
{}^1H + {}^1H &\to {}^2H + e^+ + \nu_e, \\
{}^1H + {}^2H &\to {}^3He + \gamma, \\
{}^3He + {}^3He &\to {}^4He + {}^1H + {}^1H.
\end{aligned}
$$

Do they have a dark side (Yin) as well?

The simplest idea is that neutrinos are Dirac fermions, with an interacting left-handed component ν_L and an inert right-handed component ν_R. This requires the imposition of global $U(1)$ lepton number.

Another idea is that $SU(2)_R$ exists as well as $SU(2)_L$, and ν_R interacts as a doublet under $SU(2)_R$, with ν_R joining ν_L to form a Dirac fermion.

In either case, neutrinos have both a dark side and a bright side, but they themselves have no connection to the observed dark matter of the Universe. [Recall neutrinos themselves could not be most of dark matter.]

The Mirror Manifestation

Postulate: Our world of fundamental particles has a mirror counterpart ($L \leftrightarrow R$). Foot,Lew,Volkas (1991): $SU(3)_C \times SU(2)_W \times U(1)_Y$ has a reflection $SU(3)'_C \times SU(2)'_W \times U(1)'_Y$ so that

$$(u,d)_L \sim (3,2,1/6), (1,1,0),$$
$$(u',d')_R \sim (1,1,0), (3,2,1/6),$$
$$(\nu,e)_L \sim (1,2,-1/2), (1,1,0),$$
$$(\nu',e')_R \sim (1,1,0), (1,2,-1/2), etc.$$

All primed fermions belong to the dark side, including ν'.

The communication between our world and the mirror world is possible through the kinetic mixing of the $U(1)_Y$ and $U(1)'_Y$ gauge factors [Holdom(1986)]. This idea of a dark photon and a possible dark neutrino has become popular in recent years, prompting many theoretical and some experimental studies.

In such a scenario, the neutrino is not special, because every standard-model particle has a dark companion.

The Left-Right Manifestation

Instead of $(u,d)_R$ and $(\nu,e)_R$ as $SU(2)_R$ doublets, it is possible to have $(u,h)_R$ and $(n,e)_R$, where h and n are distinct from d and ν. This is a natural decomposition of E_6 models, and known as the ALRM [Ma(1987)].

At the $SU(2)_L \times SU(2)_R$ level, an extra global $U(1)$ symmetry S is used to forbid the would-be allowed term $\bar{h}_L d_R$, etc. A linear combination of S and T_{3R} remains unbroken and serves as the dark symmetry. The new fermions n (which may be Majorana or Dirac) are the dark-matter companions of the observed neutrinos ν.

Khalil,Lee,Ma(2009): DLRM I: n is Majorana and has generalized $L = S - T_{3R} = 0$, with $R = (-1)^{(L+2j)}$ as dark parity [Ma(2015)].

Khalil,Lee,Ma(2010): DLRM II: n is Dirac with $L = S + T_{3R} = 2$.

In I(II), $W_R^\pm$ has $L = \mp 1(\pm 1)$. Neither mixes with $W_L^\pm$. In both, Z' gauge interactions are constrained by LHC data as well as $n\bar{n}$ annihilation. The former implies a much heavier Z' (up to 4 TeV) than is possible for the latter. Hence n itself is not likely to be dark matter. However, other particles in the dark sector could be.

Instead of $SU(5) \sim E_4$, or $SO(10) \sim E_5$, or E_6 unification, $[SU(N)]^k$ unification is an attractive alternative, with fermions transforming as

$$(N, N^*, 1, ...) + (1, N, N^*, ...) + ... + (N^*, 1, ..., N)$$

in a closed chain. Some interesting random facts:

- (1) $SU(3)_C \times SU(3)_L \times SU(3)_R$ is the maximum subgroup of E_6:
  $\underline{27} = (3, 3^*, 1) + (1, 3, 3^*) + (3^*, 1, 3)$

- (2) $[SU(3)]^4$ may contain leptonic color:

$$SU(3)_q \times SU(3)_L \times SU(3)_l \times SU(3)_R$$

- (3) $[SU(3)]^6$ may contain axial QCD and quark-lepton nonuniversality:
  $SU(3)_{CL} \times SU(3)_{qL} \times SU(3)_{lL} \times SU(3)_{lR} \times$
  $SU(3)_{qR} \times SU(3)_{CR}$

- (4) Supersymmetric $[SU(N)]^k$ is a finite field theory for 3 families independent of N and k:

$$b_i = -\frac{11}{3}N + \frac{2}{3}N + N_f\left(\frac{2}{3}+\frac{1}{3}\right)\frac{1}{2}(2N)$$

The ALRM is naturally embedded in the maximal subgroup $SU(3)_C \times SU(2)_L \times SU(3)_R$ of E_6.

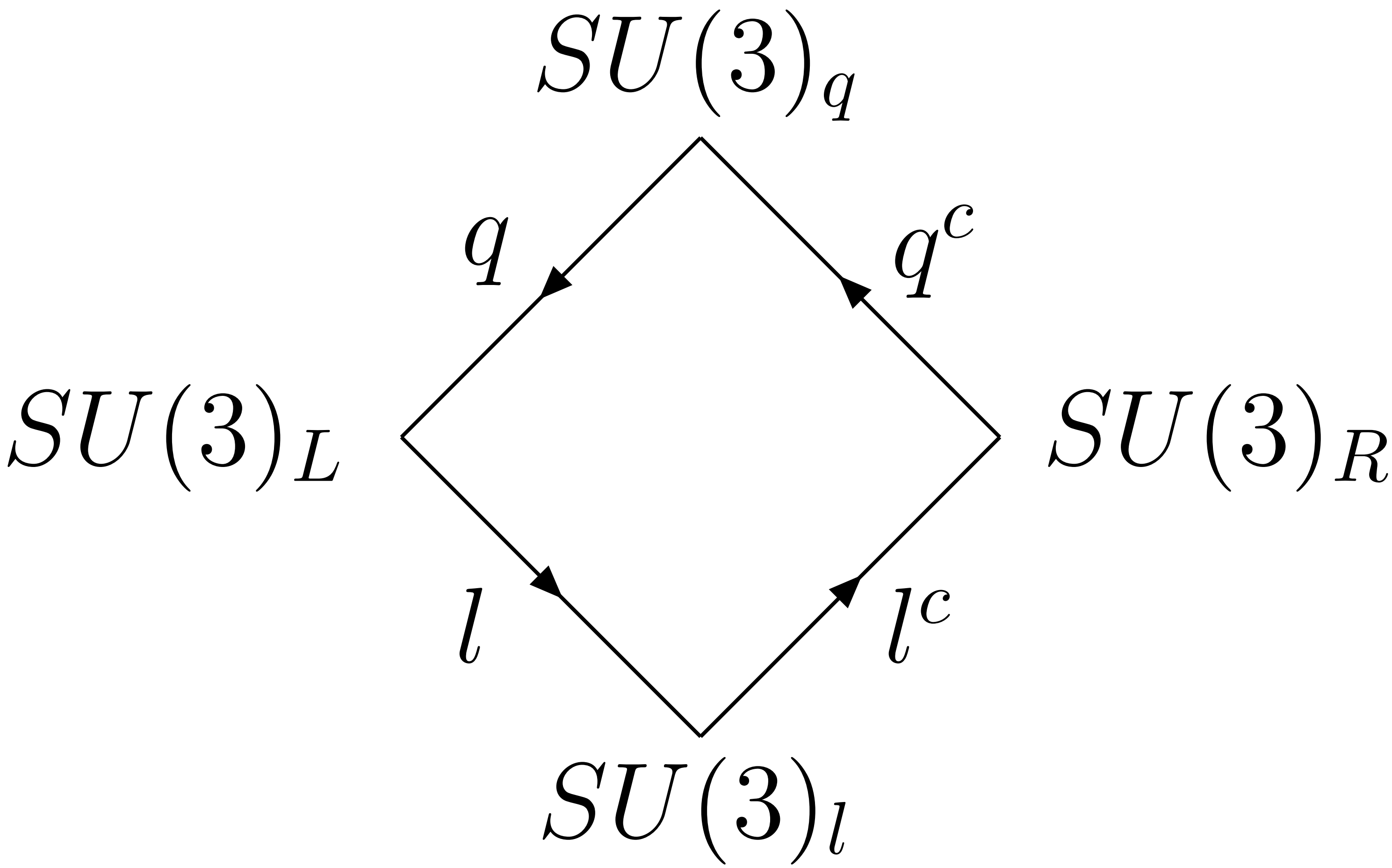
$$q \sim (3, 3^*, 1) \sim \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix},$$

$$\lambda \sim (1, 3, 3^*) \sim \begin{pmatrix} \nu & E^c & N \\ e & N^c & E \\ S & e^c & \nu^c \end{pmatrix}$$

$$q^c \sim (3^*, 1, 3) \sim \begin{pmatrix} h^c & h^c & h^c \\ u^c & u^c & u^c \\ d^c & d^c & d^c \end{pmatrix}.$$

As such, the Dirac partner of ν is ν^c through the vacuum expectation value of scalar N^c, but ν^c is a singlet under $SU(2)_R$. The $SU(2)_R$ doublet here is instead (e^c, S). Hence S is the right-handed interacting companion of ν_L. Note that $(\nu, e)_L$ is part of a bidoublet, so it has $SU(2)_R$ interactions as well in this scenario.

In the DLRM, $(\nu, e)_L$ is a pure $SU(2)_L$ doublet. It is thus naturally embedded in the $[SU(3)]^4$ extension of $[SU(3)]^3$, i.e. $SU(3)_q \times SU(3)_L \times SU(3)_l \times SU(3)_R$ [Babu,Ma,Willenbrock(2004)], with $SU(3)_l$ as the leptonic color counterpart of $SU(3)_q$ [Foot,Lew(1990)].


$SU(3)_q$
q
q^c
$SU(3)_L$
$SU(3)_R$
l
l^c
$SU(3)_l$

Kownacki,Ma,Pollard,Popov,Zakeri(2018):

$$q \sim (3,3^*,1,1) \sim \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix},$$

$$l \sim (1,3,3^*,1) \sim \begin{pmatrix} x & x & \nu \\ y & y & e \\ z & z & n \end{pmatrix},$$

$$l^c \sim (1,1,3,3^*) \sim \begin{pmatrix} z^c & y^c & x^c \\ z^c & y^c & x^c \\ n^c & e^c & \nu^c \end{pmatrix}$$

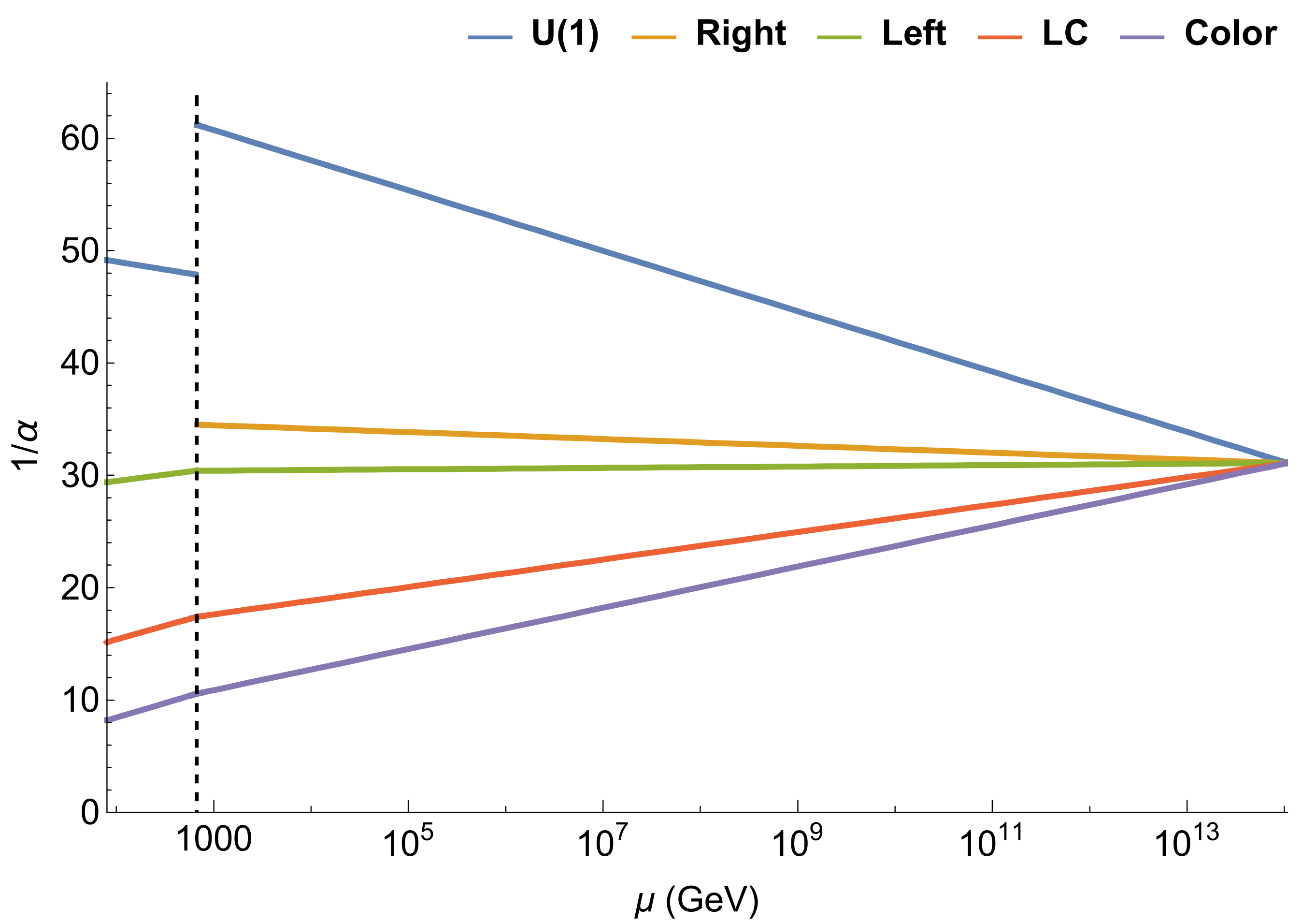
$$q \sim (3^*,1,1,3) \sim \begin{pmatrix} h^c & h^c & h^c \\ u^c & u^c & u^c \\ d^c & d^c & d^c \end{pmatrix}$$

Whereas $SU(3)_q$ is unbroken, $SU(3)_l$ breaks to unbroken $SU(2)_l$, which confines the exotic (x, y, z) hemions with charges $(1/2, -1/2, 1/2)$ and leaves the leptons free.
In general

$$Q = I_{3L} + I_{3R} - \frac{1}{\sqrt{3}}(Y_L + Y_R + Y_l).$$

At $M_U \sim 10^{14}$ GeV, $[SU(3)]^4$ breaks to $SU(3)_q \times SU(2)_l \times SU(2)_L \times SU(2)_R \times U(1)_X$.

At $M_R \sim 600$ GeV, $SU(2)_R \times U(1)_X$ breaks to $U(1)_Y$.


U(1)
Right
Left
LC
Color
60
50
40
30
20
10
0
1/α
1000
10^5
10^7
10^9
10^11
10^13
μ (GeV)

Consider the global symmetry

$$S = \frac{1}{\sqrt{3}}(Y_R - 2Y_L - 2Y_l).$$

It is unbroken (even though the gauge symmetry is broken) before $SU(2)_R$ breaking. It is broken after $SU(2)_R$ breaking, but the combination $I_{3R} + S$ remains unbroken.

This is in fact $B - L$ for the known quarks and leptons. Hence $I_{3R} + S$ may be defined to be generalized $B - L$ and serves as the dark symmetry of $[SU(3)]^4$.

Under $R = (-1)^{3B-3L+2j}$, the standard-model particles are even. The odd particles are the exotic h quark, the x, y hemions, the neutral n fermion, the $W_R^\pm$ gauge bosons, and the (η^0, η^-) and λ^0 scalars.

$$\phi \sim (1,3,1,3^*) \sim \begin{pmatrix} \eta^0 & \phi_2^+ & \phi_1^0 \\ \eta^- & \phi_2^0 & \phi_1^- \\ \chi^0 & \chi^+ & \lambda_0 \end{pmatrix},$$

$$\bar{\phi} \sim (1,3^*,1,3) \sim \begin{pmatrix} \bar{\eta}^0 & \eta^+ & \bar{\chi}^0 \\ \phi_2^- & \bar{\phi}_2^0 & \chi^- \\ \bar{\phi}_1^0 & \phi_1^+ & \bar{\lambda}^0 \end{pmatrix}.$$

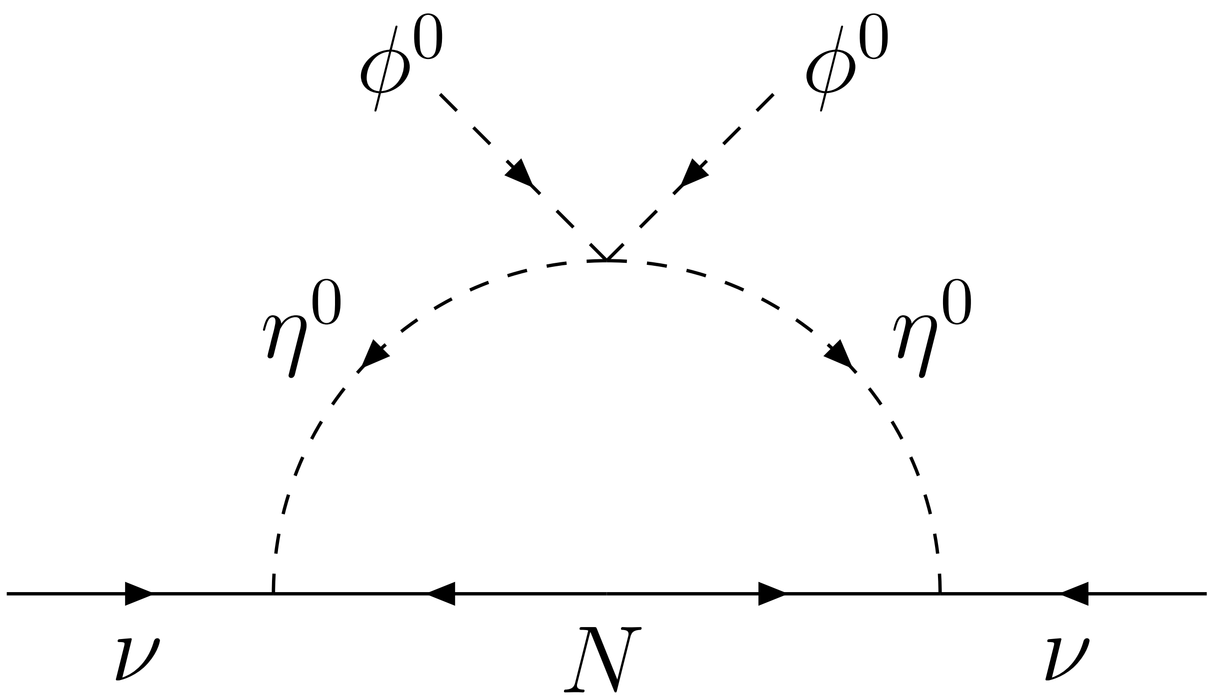
The neutrino ν_L has its dark companion n_R. They are not mass partners, but they do interact through the Yukawa term $\bar{\nu}_L n_R \eta^0$. Since n_R has Z' interactions, it is not likely to be dark matter. Instead, the singlet scalar λ^0 is a good choice. Its coupling to the standard-model Higgs has to be very small to be below direct-search limits. However, its annihilation to hemion pairs $x\bar{x}$ through z exchange should allow it to have the correct relic abundance.

The hemions form vector bound states (hemionia) in the same way that quarks form quarkonia, and could be observable at a future e^-e^+ collider, linear or circular.

The Scotogenic Manifestation

If a second scalar doublet (η^+, η^0) is added to the SM such that it is odd under a new exactly conserved Z_2 discrete symmetry [Deshpande/Ma(1978)], then η^0_R or η^0_I is absolutely stable. This simple idea for dark matter lay dormant for almost 30 years until it was used to generate neutrino mass [Ma(2006)] by adding three neutral singlet fermions N which are also odd under Z_2.

This Z_2 was recognized later [Ma(2015)] to be just $(-1)^{L+2j}$ with the assignment of $L = 0$ for N and $L = -1$ for η.


ϕ^0
ϕ^0
η^0
η^0
ν
N
ν

This is known as the scotogenic mechanism, from the Greek 'scotos' meaning darkness. Since N is odd, there is no $\nu N\phi^0$ coupling, but $h\nu N\eta^0$ as well as $(\lambda_5/2)(\Phi^\dagger\eta)^2 + H.c.$ are allowed, thus realizing a generic radiative mechanism already known [Ma(1998)].

$$(\mathcal{M}_\nu)_{ij} = \sum_k \frac{h_{ik}h_{jk}M_k}{16\pi^2}[f(M_k^2/m_R^2) - f(M_k^2/m_I^2)],$$

where $f(x) = \ln x/(x-1)$. Note that the λ_5 term splits the mass of $\eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$, so that $m_R^2 - m_I^2 = 2\lambda_5 v^2$, which makes the one-loop diagram finite. It also solves the problem of the direct detection of $\eta_{R,I}$ through Z exchange with nuclei. Since Z_μ couples to $\eta_R \partial^\mu \eta_I - \eta_I \partial^\mu \eta_R$, a mass gap of just a few hundred keV is enough to forbid its elastic scattering in underground dark-matter search experiments using nucleus recoil.

The dark companions of ν are N, both of which are Majorana. Whereas ν has $SU(2)_L$ gauge interactions, N has only Yukawa interactions $h \nu N \eta^0$. Their masses are linked in much the same way as in the Type I seesaw. There are numerous (more complicated) variations of this simple model: [Restrepo,Zapata,Yaguna(2013)].

Instead of very large M_N, there is another interesting limit, i.e. $M_N^2 << m_R^2, m_I^2$ [Ma(2012)]. In that case

$$(\mathcal{M}_\nu)_{ij} = \frac{\ln(m_R^2/m_I^2)}{16\pi^2} \sum_k h_{ik} h_{jk} M_k.$$

Now N could be warm dark matter at a few keV. This scenario restricts severely the neutrino mass spectrum and predicts it to be quasidegenerate at about 0.1 eV.

Here the breaking of lepton number L to $(-1)^L$ occurs with small M_N (much below the electron mass) instead of very large M_N as in the usual seesaw mechanism.

The Discrete Dirac Manifestation

Lepton number is usually thought of as being an integer L or a parity $(-1)^L$. In the latter case, neutrinos are Majorana, which is the default option. In the former case, they are Dirac, and in the persisting nonobservation of neutrinoless double beta decay, there is a theoretical resurgence of interest in them.

What is new in the last few years is the realization that lepton number may be based on Z_n. There are already explicit examples of Z_3 and Z_4 models.

Ma,Pollard,Srivastava,Zakeri(2015):

To gauge $U(1)_{B-L}$, consider the addition of the following singlet fermions:

$$\nu_R \sim 5, -4, -4, \quad N_{L,R} \sim -1, -1, -1.$$

Since $5 - 4 - 4 = -3$ and $(5)^3 - (4)^3 - (4)^3 = -3$, this model is anomaly-free. Let $U(1)_{B-L}$ be broken by two scalar singlets:

$$\chi_3 \sim 3, \quad \chi_6 \sim -6,$$

with the allowed $\chi_3^2 \chi_6$ term in the Higgs potential.

The allowed Yukawa terms are $\bar{\nu}_L N_R \bar{\phi}^0$, $\bar{N}_L \nu_{R1} \chi_6$, $\bar{N}_L \nu_{R2} \chi_3$, and $\bar{N}_L \nu_{R3} \chi_3$. As $\chi_{3,6}$ pick up nonzero vacuum expectation values, the 6×6 Dirac mass matrix for (ν, N) is of the form

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & \mathcal{M}_0 \\ \mathcal{M}_3 & \mathcal{M}_N \end{pmatrix}.$$

where $\mathcal{M}_N$ are the allowed invariant Dirac mass terms for N. The resulting theory preserves a global $U(1)_{B-L}$ symmetry, under which $\nu, N \sim -1$ and all the scalars are trivial.

Suppose an extra scalar $\chi_2 \sim 2$ is added, then the following new terms appear:

$$\chi_2 N_L N_L, \quad \chi_2 N_R N_R, \quad {\chi_2}^3 \chi_6.$$

As χ_6 acquire a nonzero vacuum expectation value, the residual symmetry of this model becomes Z_3, under which

$$\chi_2, N_{L,R}, \nu_{L,R} \sim \omega$$

where $\omega^3 = 1$, i.e. $\omega = -1/2 + i\sqrt{3}/2$.
This means that χ_2 has a discrete lepton number, so it

may decay to two antineutrinos, i.e.

$$\chi_2 \to \bar{\nu}_i \bar{\nu}_j$$

through the Dirac seesaw $\nu - N$ mixing. Note that it cannot decay to two positrons because of charge conservation. For $\mathcal{M}_N$ of order 10^{13} GeV, the decay lifetime of χ_2 is about the age of the Universe. This is acceptable because if the decay product consists of electrons or photons, then the lifetime has to be orders of magnitude greater because they would disturb the cosmic microwave background.

As for relic abundance, the χ_2 annihilation through the SM Higgs or the $B - L$ gauge boson is suppressed because of the constraint from direct-search experiments. However, χ_2 annihilation to the $L = 0$ scalars $\chi_{3,6}$ is available for its thermal freezeout. Since the latter could remain in thermal equilibrium with the SM Higgs, this offers a viable scenario for χ_2 to be dark matter.

In this discrete Dirac manifestation, the Dirac neutrino has a dark side because it is intimately related to dark matter as they share the same Z_3 discrete symmetry and one becomes the other's exclusive decay product.

Lineros,Ma(2018):

Under Z_3, let $(\nu, e)_L, e_R, \nu_R \sim \omega$. Add scalar $\zeta \sim \omega$ and Majorana fermion $\chi_R \sim 1$. Allowed Yukawa couplings are

$$f_1 \zeta \nu_R \nu_R, \quad f_2 \zeta^* \nu_R \chi_R.$$

Assume $f_{1,2}$ to be very small and $m_\zeta > m_\chi$, then ζ goes through thermal freezeout and converts its relic abundance to χ through the decay $\zeta \to \chi_R \nu_R$. As for χ_R, it decays through a virtual ζ to 3 neutrinos with a very long lifetime. Again its decay products would not disturb the CMB.

Concluding Remarks

Neutrinos are conjectured to be intimately connected to dark matter. There are many possible ways for this to happen.

- SM $+ (-1)^{L+2j}$ with $L = 0$ singlet neutral fermions and a leptonic scalar doublet, resulting in scotogenic Majorana neutrino masses. There are many possible variations of this simple idea.

- SM + gauge $B - L$ which results in Z_3 Dirac neutrinos. The dark matter candidate is a scalar with

Z_3 lepton number decaying to 2 antineutrinos, or a Majorana fermion decaying to 3 neutrinos.

- SM extended with gauge $SU(2)_R$ such that the neutral fermion partner to the electron in the $SU(2)_R$ doublet is not the Dirac mass partner of the left-handed neutrino, but a different particle which belongs to the dark sector.

- SM + its mirror image with kinetic mixing of $U(1)_Y$ and $U(1)'_Y$. This assigns a dark side to every known particle, including the neutrino.