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Non-local effects in exclusive $b \to s\ell\ell$ decays

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The *B* Anomalies

Motivation (theory)

- in the Standard Model (SM), rare b decays are suppressed
  - ▷ CKM suppressed: $V_{tb}V_{ts}^* \sim \lambda^2$ ✓
  - ▷ loop suppressed: $1/(16\pi^2)$ ✓
  - ▷ (partially) GIM suppressed ✓
- even small New Physics (NP) contributions might yield significant effects amongst the suppressed SM “background”
  - ▷ much theoretical and phenomenological interest in $b \to s\ell\ell$ decays

Motivation (experiment)

experimental measurements on $b \to s\ell\ell$

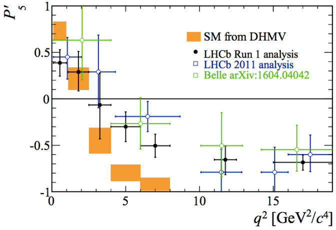
- LHCb measurements $B \to K\mu\mu$, $B \to K^*\mu\mu$, $B_s \to \phi\mu\mu$
- analogous measurements by Belle, ATLAS and CMS
- test of Lepton-Flavor Non-Universality (μ vs e)

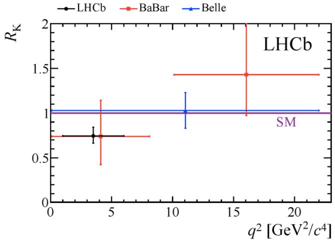
raised a lot of interest, lot of work from theory + experiment

- gave rise to the so-called b Anomalies
  - P_5': one coefficient in the angular distribution of $B \to K^*(\to K\pi)\ell^+\ell^-$ decays
  - $R_{K^{(*)}}$: ratio of branching ratios $B \to K^{(*)}\mu^+\mu^-$ / $B \to K^{(*)}e^+e^-$

Motivation (experiment)

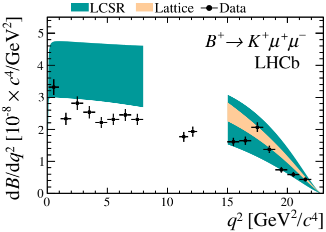
intriguing "anomalies" in some observables



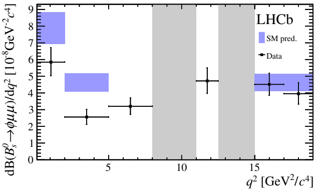


[Phys. Rev. Lett. 113, 151601 (2014)]

less significant yet intriguing deviations in branching ratios



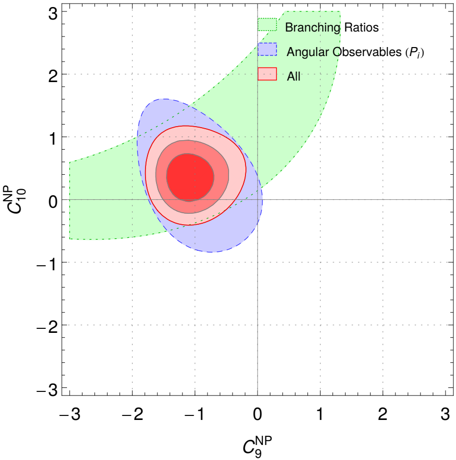
[LHCb JHEP06(2014)133]



[LHCb JHEP09(2015)179]

Motivation (metrology)

significant SM pulls in global fits [e.g. Descotes-Genon, Hofer, Matias, Virto 2015 + others]



decays

- $B \to K\ell^+\ell^-$
- $B \to K^*\ell^+\ell^-$
- $B_s \to \phi\ell^+\ell^-$
- $B_s \to \ell^+\ell^-$

observables

- 96 ($\ell = \mu$ only)
- 101 ($\ell = \mu, e$)

significance already at the level of $\sim 5\sigma$

caveat: Systematic uncertainty due to non-local contributions

State of the Art

Effective Theory

for $\Lambda_{\rm EW}, \Lambda_{\rm NP} \gg M_B$: $b \to s$ FCNC mediated by $D = 6$ ops :

$$\mathcal{L}_W = \mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm QED} + \frac{4G_F}{\sqrt{2}}\left[\lambda_t \sum_i \mathcal{C}_i\, \mathcal{O}_i + \lambda_c \sum_i \mathcal{C}_i^{\,c}\, \mathcal{O}_i^{\,c} + \lambda_u \sum_i \mathcal{C}_i^{\,u}\, \mathcal{O}_i^{\,u}\right]$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_R b) F_{\mu\nu} \qquad \mathcal{O}_8 = \frac{g_s}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_R T^A b) G^A_{\mu\nu}$$

$$\mathcal{O}_{9\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell) \qquad \mathcal{O}_{10\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

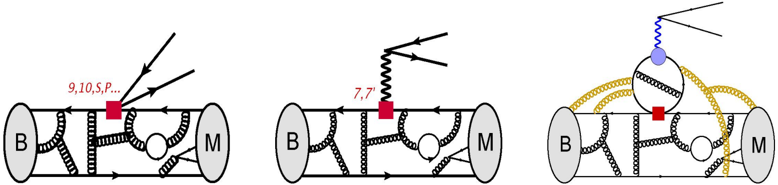
$$\mathcal{O}_1^{\,c} = (\bar{c}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L c) \qquad \mathcal{O}_2^{\,c} = (\bar{c}\gamma_\mu P_L T^a b)(\bar{s}\gamma^\mu P_L T^a c)$$

$$\mathcal{O}_1^{\,u} = (\bar{u}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L u) \qquad \mathcal{O}_2^{\,u} = (\bar{u}\gamma_\mu P_L T^a b)(\bar{s}\gamma^\mu P_L T^a u)$$

$$\mathcal{O}_i = (\bar{s}\gamma_\mu P_X b) \sum_q (\bar{q}\gamma^\mu q)$$

SM contributions to $\mathcal{C}_i(\mu_b)$ known to NNLL [Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorbahn, Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06]

$B \to M\ell\ell$ Amplitudes in a Nutshell



$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

- local form factors : $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) |\, \bar{s}\, \Gamma_\lambda^{(T)}\, b \,| \bar{B}(k+q) \rangle$
- non-local : $\mathcal{H}_\lambda(q^2) = i\, \mathcal{P}_\mu^\lambda \int d^4x\, e^{iq\cdot x} \langle \bar{M}_\lambda(k) | T\{ \mathcal{J}^\mu_{\mathrm{em}}(x), \mathcal{C}_i \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$
- CKM structure : $\mathcal{H}_\lambda = -\frac{\lambda_u}{\lambda_t} \mathcal{H}_\lambda^{(u)} - \frac{\lambda_c}{\lambda_t} \mathcal{H}_\lambda^{(c)}$ $\qquad \lambda_q = V_{qb} V_{qs}^*$

Local Form Factors

- computable on the lattice
  - $B \to K$ ✓ [Bailey *et al.* 2015 (FERMILAB/MILC)]
  - $B \to K^*$ ✓ [Horgan, Liu, Meinel, Wingate 2013]
  - $\Lambda_b \to \Lambda$ ✓ [Detmold, Meinel 2016]
  - accessing small q^2 computationally expensive $\to$ extrapolate
- accessed through Light-Cone Sum Rules
  - $B \to K$ ✓ [Khodjamirian, Mannel, Offen 2006]
  - $B \to K^*$ ✓ [Khodjamirian, Mannel, Offen 2006][Bharuch,Straub,Zwicky 2015]
  - $\Lambda_b \to \Lambda$ ✓ [Mannel, Wang 2011]
- simultaneous fit to both theory inputs available [Bharucha, Straub, Zwicky 2015]
- will not further discuss local form factors in this talk

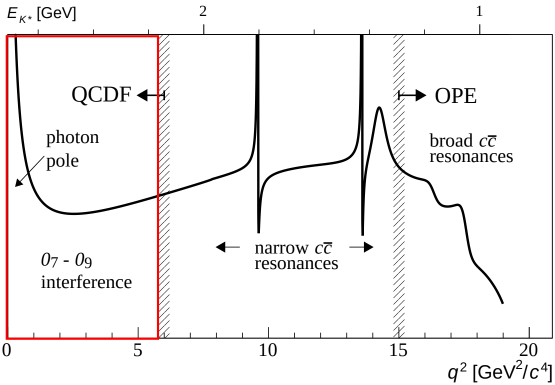
Lorentz Decomposition

$$\mathcal{H}^\mu(q,k) \equiv i \int \mathrm{d}^4x \, e^{iq\cdot x} \, \langle \bar{K}^*(k,\eta)|T\{\mathcal{J}^\mu_{\mathrm{em}}(x), \mathcal{C}_i\mathcal{O}_i(0)\}|\bar{B}(k+q)\rangle$$

$$\equiv M_B^2 \, \eta^*_\alpha \, \left[S_\perp^{\alpha\mu}\,\mathcal{H}_\perp(q^2) - S_\parallel^{\alpha\mu}\,\mathcal{H}_\parallel(q^2) - S_0^{\alpha\mu}\,\mathcal{H}_0(q^2)\right]$$

- $S_\lambda^{\alpha\mu}$ – basis of Lorentz structures (carefully chosen)
- $\mathcal{H}_\lambda$ – Lorentz invariant correlation functions
- λ – polarization states ($\perp$, $\parallel$, 0)

Calculation: local OPE for q^2 below the J/ψ



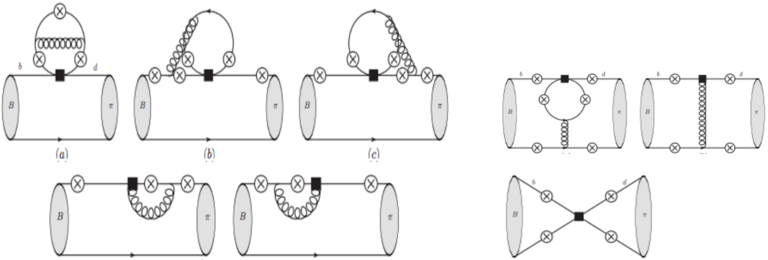
[sketch from Blake, Gershon, Hiller 2015]

- q^2: mass square of the lepton system
- $E_{K^{(*)}}$: energy of the $K^{(*)}$ in the B rest frame
  - QCD Factorization shown for small q^2, large $E_{K^{(*)}} \sim m_b$

Calculation: local OPE for q^2 below the J/ψ

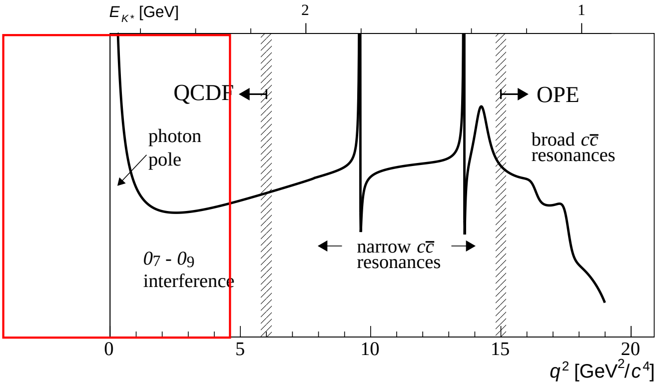
- **QCD Factorization** (to NLO in α_s) [Beneke, Feldmann, Seidel 2001 & 2004]

$$\mathcal{H}_\lambda = C_\lambda \, \mathcal{F}_\lambda + \sum_{\pm} \int \frac{d\omega}{\omega} \Phi^B_\pm(\omega) \int_0^1 du \, T^\pm_\lambda(u,\omega) \, \phi^\pm_M(u) + \mathcal{O}(\Lambda/m_B, \Lambda/E)$$



$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_{\lambda;\mathrm{fact,LO}}(q^2) + \mathcal{H}_{\lambda;\mathrm{fact,NLO}}(q^2) + \mathcal{H}_{\lambda;\mathrm{spect}}(q^2) + \mathcal{H}_{\lambda;\mathrm{WA}}(q^2) + \cdots$$

Calculation: Light-Cone OPE for $q^2 \ll 4m_c^2$

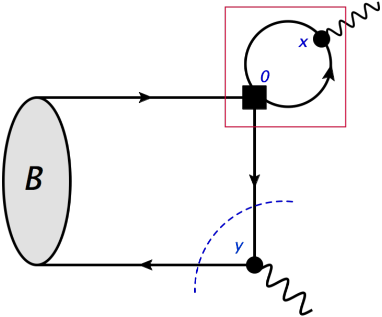


[sketch from Blake, Gershon, Hiller 2015]

- q^2: mass square of the lepton system
  - Light-Cone OPE includes power corrections to QCDF for $q^2 \ll 4m_c^2$

Calculations: Light-Cone OPE for $q^2 \ll 4m_c^2$

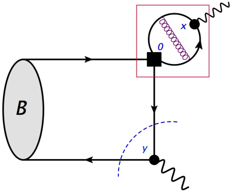
- **LCSRs with B-meson DAs** [Khodjamirian, Mannel, Pivovarov, Wang 2010]

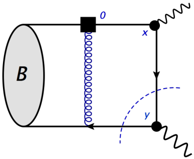


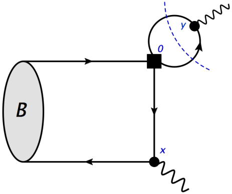
LC exp. of charm prop. [Balitsky, Braun 1989]

$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{matching coeff}} [\bar{s}\,\Gamma\, b] + \cdots$$

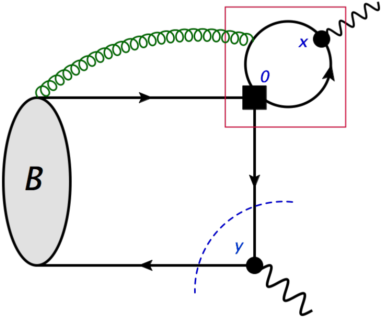
$$\Rightarrow \mathcal{H}_\lambda = (\text{matching coeff}) \times \mathcal{F}_\lambda^{LCSR}$$







- LCSRs with B-meson DAs [Khodjamirian, Mannel, Pivovarov, Wang 2010]

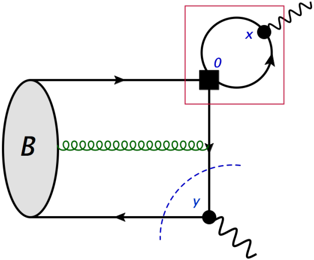


LC exp. of charm prop. [Balitsky, Braun 1989]

$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{matching coeff}} [\bar{s}\,\Gamma\, b] +$$

$$+ \,(\text{coeff}) \times [\bar{s}_L \gamma^\alpha (in_+ \cdot \mathcal{D})^n \tilde{G}_{\beta\gamma} b_L] \; + \cdots$$

3-particle correction to $\mathcal{F}_\lambda \longrightarrow$



Calculation: Light-Cone OPE for $q^2 \ll 4m_c^2$

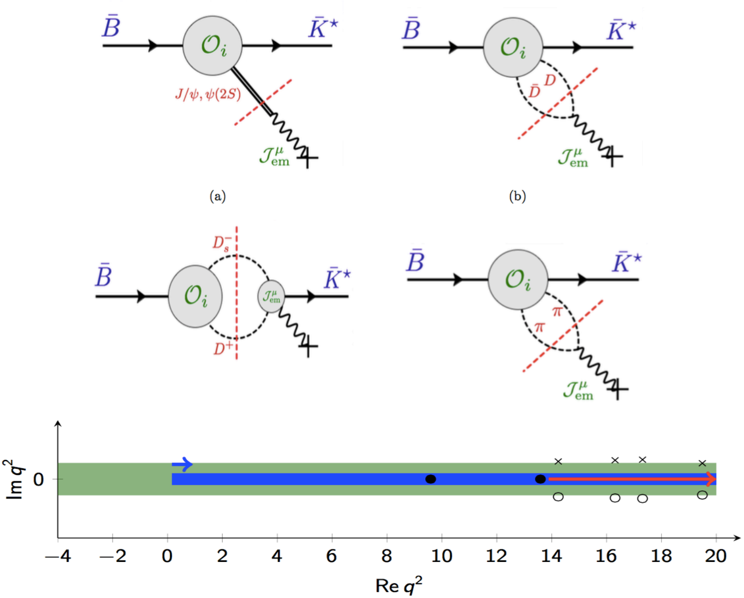
- At the end of the day

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_{\lambda;\mathrm{fact,LO}}(q^2) + \mathcal{H}_{\lambda;\mathrm{fact,NLO}}(q^2) + \mathcal{H}_{\lambda;\mathrm{spect}}(q^2) + \mathcal{H}_{\lambda;\mathrm{WA}}(q^2) + \\ + \mathcal{H}_{\lambda;\mathrm{soft}}(q^2) + \mathcal{H}_{\lambda;\mathrm{soft},O_8}(q^2) + \cdots$$

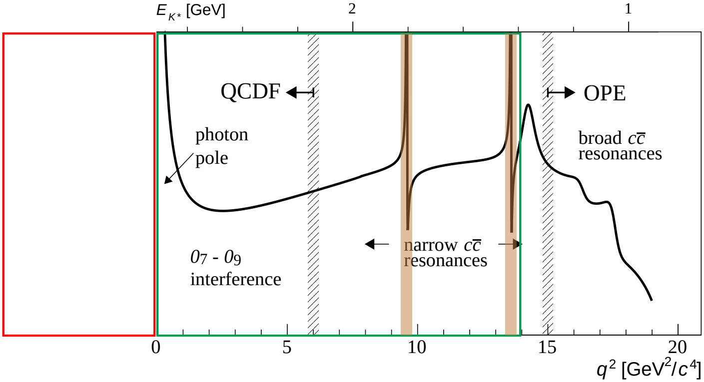
  - $\mathcal{H}_{\lambda;\mathrm{soft}}$ and $\mathcal{H}_{\lambda;\mathrm{fact,LO}}$ cancel to large extent
    - reason: $\mathcal{H}_{\lambda;\mathrm{fact,LO}}$ is color suppressed
  - $\mathcal{H}_{\lambda;\mathrm{soft},,O_8}$ contributions negligible

A different approach

Analytic structure



Strategy



[sketch from Blake, Gershon, Hiller 2015]

- calculate non-local matrix elements at $q^2 < 0$
- extrapolate to $q^2 > 0$ via some type of analytic continuation
- constrain two narrow resonances at $q^2 > 0$ from data on $B \to \psi_n K^*$

Accessing $q^2 > 0$: dispersion relations

Dispersion relation relating $\mathcal{H}(q_0^2 < 0)$ to $\mathcal{H}(q^2 > 0)$

[Khodjamirian, Mannel, Pivovarov, Wang 2010] [Hambrock, Khodjamirian, Rusov 2015]

$$\mathcal{H}^{(p)}(q^2) - \mathcal{H}^{(p)}(q_0^2) = (q^2 - q_0^2)\Bigg[\sum_V \frac{f_V\, \mathcal{A}^p(B \to VM)}{(m_V^2 - q_0^2)(m_V^2 - q^2 - i m_V \Gamma_V^{\rm tot})} + \int_{s_h}^{\infty} ds \frac{\rho_h^{(p)}(s)}{(s^2 - q_0^2)(s - q^2 - i\epsilon)}\Bigg]$$

$$(p = u, c)$$

- $V = \rho, \omega, \phi, J/\psi, \psi(2S)$
- for $b \to s \Rightarrow$ Neglect λ_u and OZI suppressed contributions

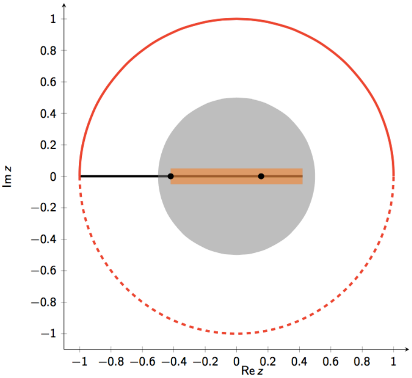
  $\Rightarrow \mathcal{A}^c(B \to VM_s) \sim \mathcal{A}(B \to \psi_n M_s)$ can be determined from data.
- for $b \to d$ both $\mathcal{A}^{u,c}(B \to VM)$ important $\Rightarrow$ Need extra theory input (QCDF)
- light-hadron spectral density $\Rightarrow$ QH-Duality
- integral over Open-charm spectral density $\simeq a_p + b_p \dfrac{q^2}{4m_D^2}$ (expansion for $q^2 < m_{J/\psi}^2$)

Ansatz in z valid below the $D\bar{D}$ threshold

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

Motivated by "z-parametrization" of form factors. [Boyd et al '94, Bourelly et al '08]

1. Extract the poles : $\hat{\mathcal{H}}_\lambda(q^2) = (q^2 - M^2_{J/\psi})(q^2 - M^2_{\psi(2S)})\,\mathcal{H}_\lambda(q^2)$



2. $\hat{\mathcal{H}}_\lambda(q^2)$ is analytic except for $D\bar{D}$ cut.
3. perform conformal mapping $q^2 \mapsto z(q^2)$.
4. $\hat{\mathcal{H}}_\lambda(z)$ analytic within unit circle.
5. Taylor expand $\hat{\mathcal{H}}_\lambda(z)$ around $z = 0$.
6. reasonable convergence expected since $|z| < 0.52$ for $-7\,\text{GeV}^2 \leq q^2 \leq 14\text{GeV}^2$

Some details for actual parametrisation

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

- try to capture most features of the expansion (better convergence)
- parametrize the ratios $\mathcal{H}_\lambda(q^2)/\mathcal{F}_\lambda(q^2)$ instead
- the poles should not modify the asymptotic behaviour at $|q^2| \to \infty$

$$\mathcal{H}_\lambda(z) = \frac{1 - z z^*_{J/\psi}}{z - z_{J/\psi}} \frac{1 - z z^*_{\psi(2S)}}{z - z_{\psi(2S)}} \hat{\mathcal{H}}_\lambda(z)$$

$$\hat{\mathcal{H}}_\lambda(z) = \left[\sum_{k=0}^{K} \alpha_k^{(\lambda)} z^k\right] \mathcal{F}_\lambda(z)$$

where $\alpha_k^{(\lambda)}$ are complex coefficients, and the expansion is truncated after the term z^K (we use $K = 2$, i.e. 16 real-valued parameters)

- the modified EOS source code is available upon request (public repo and web page should be updated soon!)

Experimental constraints on z parametrisation $[B \to K^* \ell\ell]$

Experimental constraints

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

- the residues of the poles are given by $B \to K^* \psi_n$:

$$\mathcal{H}_\lambda(q^2 \to M_{\psi_n}^2) \sim \frac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2 (q^2 - M_{\psi_n}^2)} + \cdots$$

- angular analyses determine

[Belle, Babar, LHCb]

$$|r_\perp^{\psi_n}|,\ |r_\parallel^{\psi_n}|,\ |r_0^{\psi_n}|,\ \arg\{r_\perp^{\psi_n} r_0^{\psi_n *}\},\ \arg\{r_\parallel^{\psi_n} r_0^{\psi_n *}\},$$

where $$r_\lambda^{\psi_n} \equiv \operatorname*{Res}_{q^2 \to M_{\psi_n}^2} \frac{\mathcal{H}_\lambda(q^2)}{\mathcal{F}_\lambda(q^2)} \sim \frac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2\, \mathcal{F}_\lambda(M_{\psi_n}^2)}$$

- we produce correlated pseudo-observables from a fit (5+5).

(Prior) Fit to Experimental and theoretical pseudo-observables

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

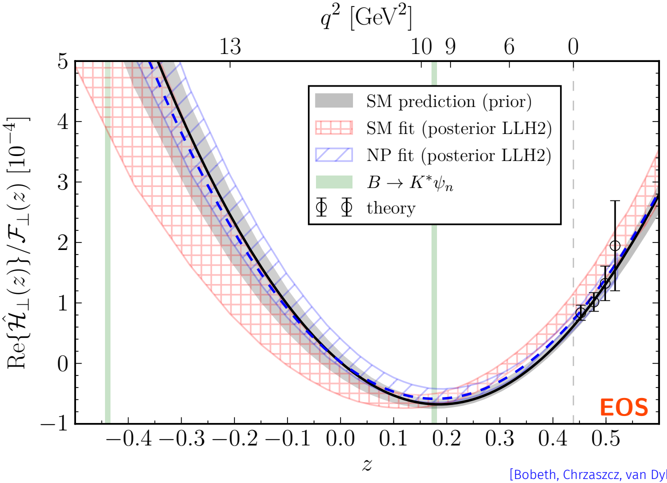
k	0	1	2
$\mathrm{Re}[\alpha_k^{(\perp)}]$	-0.06 ± 0.21	-6.77 ± 0.27	18.96 ± 0.59
$\mathrm{Re}[\alpha_k^{(\parallel)}]$	-0.35 ± 0.62	-3.13 ± 0.41	12.20 ± 1.34
$\mathrm{Re}[\alpha_k^{(0)}]$	0.05 ± 1.52	17.26 ± 1.64	–
$\mathrm{Im}[\alpha_k^{(\perp)}]$	-0.21 ± 2.25	1.17 ± 3.58	-0.08 ± 2.24
$\mathrm{Im}[\alpha_k^{(\parallel)}]$	-0.04 ± 3.67	-2.14 ± 2.46	6.03 ± 2.50
$\mathrm{Im}[\alpha_k^{(0)}]$	-0.05 ± 4.99	4.29 ± 3.14	–

Table 1: Mean values and standard deviations (in units of 10^{-4}) of the prior PDF for the parameters $\alpha_k^{(\lambda)}$.

Confronting LHCb Data

Fit to z parametrisation

SM predictions and Fit including $B \to K^* \mu^+ \mu^-$ data and $\mathcal{C}_9^{\text{NP}}$



[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

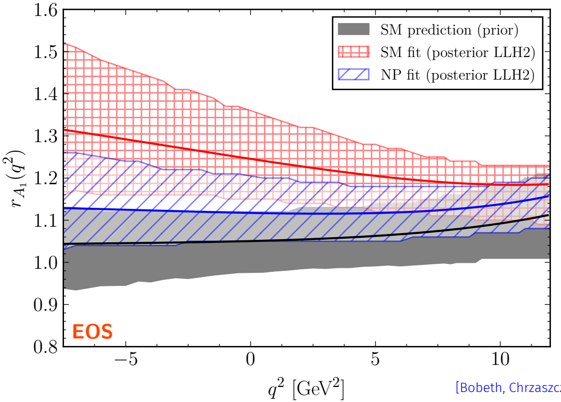
▶ parametrisation does not provide enough freedom in the SM fit in order to deviate substantially from the prior

Fit to z parametrisation

SM predictions and Fit including $B \to K^*\mu^+\mu^-$ data and $\mathcal{C}_9^{\text{NP}}$

$$r_{A_1} \equiv \frac{A_1(q^2)}{V(q^2)} \times \text{kinematics}$$

- expected to be 1 in limit $m_b, E_{K^*} \to \infty$

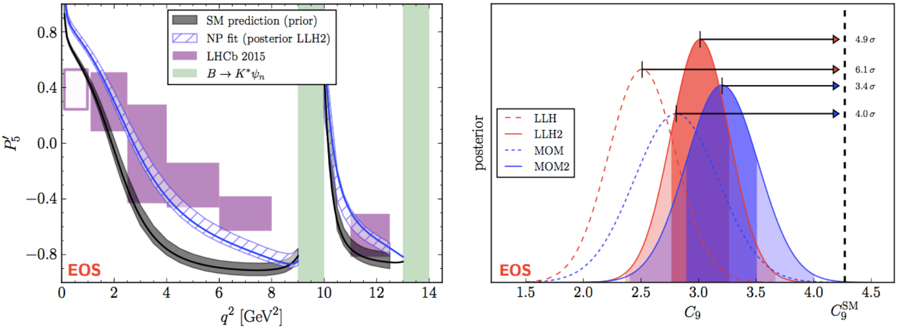


[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

- in fit to all $B \to K^*\ell^+\mu^-$ data, fit prefers to change local form factor V over non-local correlators

SM predictions and Fit including $B \to K^* \mu^+ \mu^-$ data and $\mathcal{C}_9^{\mathrm{NP}}$

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]



the NP hypothesis with $\mathcal{C}_9^{\mathrm{NP}} \sim -1$ is strongly favoured by the fit

- pulls $> 3.4\sigma$ in 1D posterior of the parameter
- posterior odds (for some fits strongly) in favour of NP interpretation

Prospects for the LHCb Upgrade and Belle II

Future experimental analyses $[B \to K^*\ell\ell]$

Sensitivity to New Physics in $\mathcal{C}_9$ in $B \to K^*\mu^+\mu^-$ from an unbinned fit

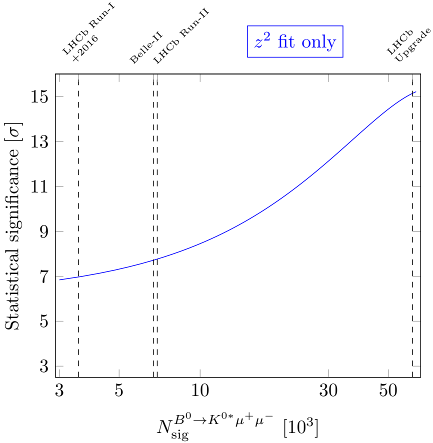
[Chrzaszcz, Mauri, Serra, Silva Coutinho, van Dyk w.i.p]

- close collaboration with LHCb members
  - preparation for unbinned analysis within LHCb
  - sensitivity study ongoing for LHCb and Belle II prospects
  - extracting C_9 in presence of z^3 will require simultaneous analysis of theory constraints + data
- first contacts by Belle II members

Sensitivity to New Physics in $\mathcal{C}_9$ in $B \to K^* \mu^+ \mu^-$ from an unbinned fit

[Chrzaszcz, Mauri, Serra, Silva Coutinho, van Dyk w.i.p]

Preliminary



- use $\mathcal{C}_9^{\rm NP} = -1$ as benchmark point
- use theory inputs exactly as in pheno analysis [Bobeth, Chrzaszcz, van Dyk, Virto 2017]
- sensitivity to coefficients of z^3
- (some) sensitivity to z coefficients in absence of any theory priors!!
  - large increase in $C_9^{\rm NP}$ uncertainty

Summary

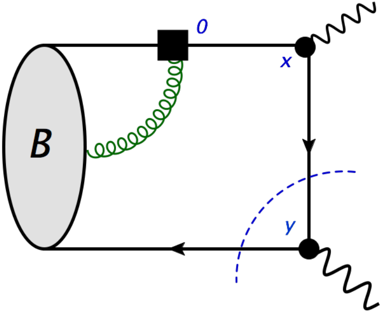
- non-local effects are (figuratively) half of the amplitude, must include them!
- technically challenging, but good progress since 2001
- theory calculations most reliable at spacelike q^2
  - QCD Factorization [Beneke, Feldmann, Seidel 2001 & 2004]
  - LCSRs [Khodjamirian, Mannel, Pivovarov, Wang 2010]
- access timelike q^2 via z-parametrizations
  - aiming for global analyses to exploit parametrical correlations
  - experimental colleagues have begun work to incorporate z-parametrization in their analyses
- did not discuss large $q^2 > M^2_{\psi(2S)}$!

Backup slides

Calculations at negative q^2

▶ LCSRs with B-meson DAs [Khodjamirian, Mannel, Wang 2012]

Soft gluon correction to O_8 contribution



Simpler calculation than charm-loop

Numerically very small

Calculations at negative q^2

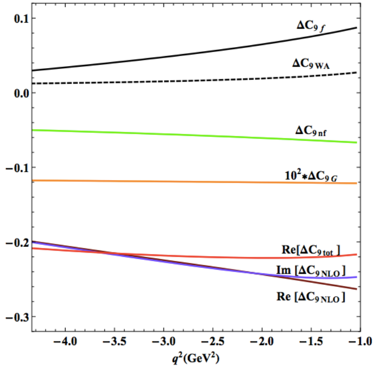
- **Results for $\mathcal{H}^{(c)}$**

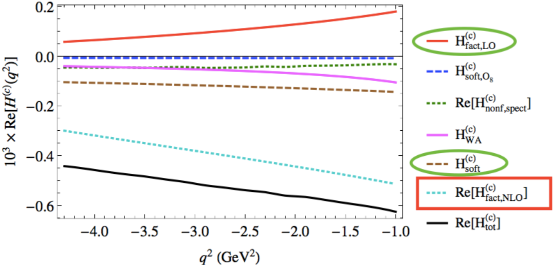
[Khodjamirian, Mannel, Wang 2012], [Hambrock, Khodjamirian, Rusov 2015]

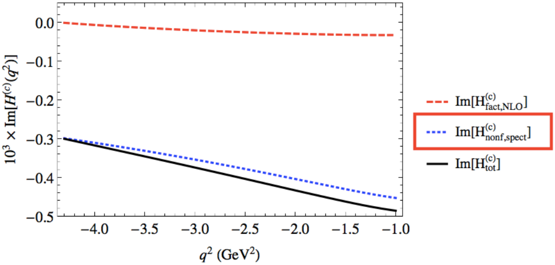
$B^- \to \pi^- \ell\ell \longrightarrow$

$B^- \to K^- \ell\ell$

$\downarrow$





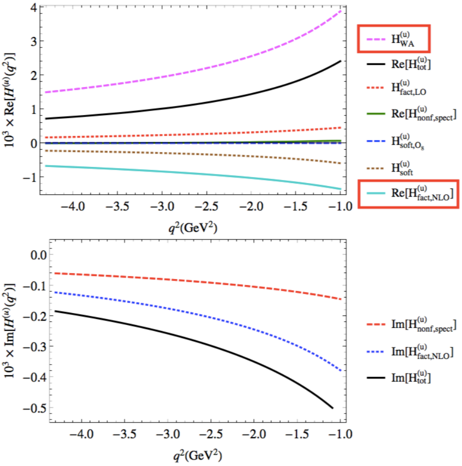


Calculations at negative q^2

- Results for $\mathcal{H}^{(u)}$

[Hambrock, Khodjamirian, Rusov 2015]

$B^- \to \pi^- \ell\ell$



$B^0 \to \pi^0 \ell\ell$