Non-local effects in exclusive $b \rightarrow s\ell\ell$ decays

Danny van Dyk
Technische Universität München
Theorieseminar
Humboldt-Universität zu Berlin
The $B$ Anomalies
in the Standard Model (SM), rare $b$ decays are suppressed

- CKM suppressed: $V_{tb}V_{ts}^* \sim \lambda^2$
  - ✓
- loop suppressed: $1/(16\pi^2)$
  - ✓
- (partially) GIM suppressed
  - ✓

even small New Physics (NP) contributions might yield significant effects amongst the suppressed SM “background”

- much theoretical and phenomenological interest in $b \to s\ell\ell$ decays
Motivation (experiment)

experimental measurements on $b \rightarrow s \ell \ell$

- LHCb measurements $B \rightarrow K\mu\mu$, $B \rightarrow K^*\mu\mu$, $B_s \rightarrow \phi\mu\mu$

- analogous measurements by Belle, ATLAS and CMS

- test of Lepton-Flavor Non-Universality ($\mu$ vs $e$)

raised a lot of interest, lot of work from theory + experiment

- gave rise to the so-called $b$ Anomalies

  ▶ $P'_5$: one coefficient in the angular distribution of $B \rightarrow K^*(\rightarrow K\pi)\ell^+\ell^-$ decays

  ▶ $R_{K^(*)}$: ratio of branching ratios $B \rightarrow K^{(*)}\mu^+\mu^- / B \rightarrow K^{(*)}e^+e^-$
Motivation (experiment)

intriguing "anomalies" in some observables


less significant yet intriguing deviations in branching ratios

[LHCb JHEP06(2014)133]

[LHCb JHEP09(2015)179]
significant SM pulls in global fits

[Descotes-Genon, Hofer, Matias, Virto 2015 + others]

decays

- $B \rightarrow K \ell^+ \ell^-$
- $B \rightarrow K^* \ell^+ \ell^-$
- $B_S \rightarrow \phi \ell^+ \ell^-$
- $B_S \rightarrow \ell^+ \ell^-$

observables

- 96 ($\ell = \mu$ only)
- 101 ($\ell = \mu, e$)

significance already at the level of $\sim 5\sigma$

caveat: Systematic uncertainty due to non-local contributions
State of the Art
for $\Lambda_{\text{EW}}, \Lambda_{\text{NP}} \gg M_B : b \to s$ FCNC mediated by $D = 6$ ops:

$$\mathcal{L}_W = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{QED}} + \frac{4G_F}{\sqrt{2}} \left[ \lambda_t \sum_i C_i \mathcal{O}_i + \lambda_c \sum_i C_i^c \mathcal{O}_i^c + \lambda_u \sum_i C_i^u \mathcal{O}_i^u \right]$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_R b) F_{\mu\nu}$$

$$\mathcal{O}_8 = \frac{g_s}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_R T^A b) G_{\mu\nu}^A$$

$$\mathcal{O}_9 \ell = \frac{\alpha}{4\pi} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \ell)$$

$$\mathcal{O}_{10} \ell = \frac{\alpha}{4\pi} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

$$\mathcal{O}_1^c = (\bar{c} \gamma_\mu P_L b) (\bar{s} \gamma^\mu P_L c)$$

$$\mathcal{O}_2^c = (\bar{c} \gamma_\mu P_L T^a b) (\bar{s} \gamma^\mu P_L T^a c)$$

$$\mathcal{O}_1^u = (\bar{u} \gamma_\mu P_L b) (\bar{s} \gamma^\mu P_L u)$$

$$\mathcal{O}_2^u = (\bar{u} \gamma_\mu P_L T^a b) (\bar{s} \gamma^\mu P_L T^a u)$$

$$\mathcal{O}_i = (\bar{s} \gamma_\mu P_X b) \sum_q (\bar{q} \gamma^\mu q)$$

SM contributions to $C_i(\mu_b)$ known to NNLL [Bobeth, Misiak, Urban '99; Misiak, Steinhauser '04, Gorbahn, Haisch '04; Gorbahn, Haisch, Misiak '05; Czakon, Haisch, Misiak '06]
Amplitudes in a Nutshell

\[ A_{\lambda}^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \pm C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2m_bM_B}{q^2} \left[ C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\} \]

- local form factors: \( \mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{\lambda}(k)|\bar{\lambda}^{(T)} b|\bar{B}(k+q)\rangle \)
- non-local: \( \mathcal{H}_\lambda(q^2) = i \mathcal{P}_\mu^\lambda \int d^4x \, e^{iq \cdot x} \langle \bar{M}_\lambda(k)|T\{\mathcal{J}_{em}(x), C_7\mathcal{O}_i(0)\}|\bar{B}(q+k)\rangle \)
- CKM structure: \( \mathcal{H}_\lambda = -\frac{\lambda_u}{\lambda_t} \mathcal{H}^{(u)}(q) - \frac{\lambda_c}{\lambda_t} \mathcal{H}^{(c)}(q) \quad \lambda_q = V_{qb}V_{qs}^* \)
Local Form Factors

- computable on the lattice
  - $B \to K \checkmark$
  - $B \to K^* \checkmark$
  - $\Lambda_b \to \Lambda \checkmark$
  - accessing small $q^2$ computationally expensive $\to$ extrapolate

- accessed through Light-Cone Sum Rules
  - $B \to K \checkmark$
  - $B \to K^* \checkmark$
  - $\Lambda_b \to \Lambda \checkmark$

- simultaneous fit to both theory inputs available

- will not further discuss local form factors in this talk
\( \mathcal{H}^\mu(q, k) \equiv i \int d^4x \ e^{iq \cdot x} \langle \overline{K}^*(k, \eta) | T\{ \mathcal{J}^\mu_{\text{em}}(x), C_i \mathcal{O}_i(0) \} | \overline{B}(k + q) \rangle 
\equiv M_B^2 \eta^*_\alpha \left[ S^{\alpha \mu}_\perp \mathcal{H}_\perp(q^2) - S^{\alpha \mu}_\parallel \mathcal{H}_\parallel(q^2) - S^{\alpha \mu}_0 \mathcal{H}_0(q^2) \right] 

- \( S^{\alpha \mu}_\lambda \) – basis of Lorentz structures (carefully chosen)
- \( \mathcal{H}_\lambda \) – Lorentz invariant correlation functions
- \( \lambda \) – polarization states (\( \perp, \parallel, 0 \))
Calculation: local OPE for $q^2$ below the $J/\psi$

- $q^2$: mass square of the lepton system
- $E_{K^*}$: energy of the $K^*$ in the $B$ rest frame
  - QCD Factorization shown for small $q^2$, large $E_{K^*} \sim m_b$

[sketch from Blake, Gershon, Hiller 2015]
Calculation: local OPE for $q^2$ below the $J/\psi$

- **QCD Factorization** (to NLO in $\alpha_s$)  
  
  $\mathcal{H}_\lambda = C_\lambda \mathcal{F}_\lambda + \sum \int \frac{d\omega}{\omega} \Phi^B_\pm(\omega) \int_0^1 du T^\pm_\lambda(u, \omega) \phi^\pm_M(u) + \mathcal{O}(\Lambda/m_B, \Lambda/E)$

\begin{align*}
\mathcal{H}_\lambda(q^2) &= \mathcal{H}_{\lambda;\text{fact,LO}}(q^2) + \mathcal{H}_{\lambda;\text{fact,NLO}}(q^2) + \mathcal{H}_{\lambda;\text{spect}}(q^2) + \mathcal{H}_{\lambda;\text{WA}}(q^2) + \cdots
\end{align*}
Calculation: Light-Cone OPE for \( q^2 \ll 4m_c^2 \)

- \( q^2 \): mass square of the lepton system
- Light-Cone OPE includes power corrections to QCDF for \( q^2 \ll 4m_c^2 \)

[sketch from Blake, Gershon, Hiller 2015]
Calculations: Light-Cone OPE for $q^2 \ll 4m_c^2$

- LCSRs with $B$-meson DAs

[Khodjamirian, Mannel, Pivovarov, Wang 2010]

LC exp. of charm prop.  

$\mathcal{H}_\lambda = (\text{matching coeff}) \times F^{LCSR}_\lambda$

$\frac{q^2}{4m_c^2} \rightarrow \left( \frac{C_1}{3} + C_2 \right) g(m_c^2, q^2) \left[ \Sigma \Gamma b \right] + \cdots$

[Balitsky, Braun 1989]
Calculation: Light-Cone OPE for $q^2 \ll 4m_c^2$

**LCSRs with $B$-meson DAs**

[Khodjamirian, Mannel, Pivovarov, Wang 2010]

3-particle correction to $\mathcal{F}_\lambda \longrightarrow$

LC exp. of charm prop. [Balitsky, Braun 1989]

\[
q^2 \ll 4m_c^2 \rightarrow \left( \frac{C_1}{3} + C_2 \right) g(m_c^2, q^2) \left[ \bar{s} \Gamma b \right] + \\
\text{matching coeff} \\
+ \text{(coeff)} \times \left[ \bar{s}_L \gamma^\alpha (i n_+ \cdot D)^n \bar{\sigma}_{\beta \gamma} b_L \right] + \cdots
\]
Calculation: Light-Cone OPE for $q^2 \ll 4m_c^2$

At the end of the day

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda;\text{fact,LO}(q^2) + \mathcal{H}_\lambda;\text{fact,NLO}(q^2) + \mathcal{H}_\lambda;\text{spect}(q^2) + \mathcal{H}_\lambda;\text{WA}(q^2) + \mathcal{H}_\lambda;\text{soft}(q^2) + \mathcal{H}_\lambda;\text{soft,0}_8(q^2) + \cdots$$

- $\mathcal{H}_\lambda;\text{soft}$ and $\mathcal{H}_\lambda;\text{fact,LO}$ cancel to large extent
  - reason: $\mathcal{H}_\lambda;\text{fact,LO}$ is color suppressed
- $\mathcal{H}_\lambda;\text{soft,0}_8$ contributions negligible
A different approach
Analytic structure

(a) 

(b) 

\[ B \rightarrow O_i \rightarrow K^* \]

\[ J/\psi, \psi(2S) \]

\[ J^\mu_{em} \]

\[ \bar{B} \rightarrow O_i \rightarrow K^* \]

\[ \bar{D} \]

\[ \bar{D}^* \]

\[ D^* \]

\[ \bar{B} \rightarrow O_i \rightarrow K^* \]

\[ J^\mu_{em} \]

\[ \pi \]

\[ J^\mu_{em} \]

\[ \bar{B} \rightarrow O_i \rightarrow K^* \]

Graph showing the analytic structure with two diagrams labeled (a) and (b). Diagrams illustrate the decay processes of \( B \) mesons into \( K^* \) mesons through intermediate states involving \( J/\psi, \psi(2S) \), \( \bar{D} \), and \( \bar{D}^* \) mesons. The graphs also include a real part \( Re q^2 \) and an imaginary part \( Im q^2 \) for the decay parameter.
 Strategy

- **calculate** non-local matrix elements at $q^2 < 0$
- **extrapolate** to $q^2 > 0$ via some type of analytic continuation
- **constrain** two narrow resonances at $q^2 > 0$ from data on $B \to \psi_n K^*$

[sketch from Blake, Gershon, Hiller 2015]
Accessing $q^2 > 0$: dispersion relations

Dispersion relation relating $H(q_0^2 < 0)$ to $H(q^2 > 0)$

[Khodjamirian, Mannel, Pivovarov, Wang 2010] [Hambrock, Khodjamirian, Rusov 2015]

\[
H^{(p)}(q^2) - H^{(p)}(q_0^2) = (q^2 - q_0^2) \left[ \sum_V \frac{f_V A^p(B \rightarrow VM)}{(m_V^2 - q_0^2)(m_V^2 - q^2 - im_V \Gamma_V^{tot})} + \int_{s_h}^{\infty} ds \frac{\rho_h^{(p)}(s)}{(s^2 - q_0^2)(s - q^2 - i\epsilon)} \right]
\]

- $V = \rho, \omega, \phi, J/\psi, \psi(2S)$
- for $b \rightarrow s$ ⇒ Neglect $\lambda_u$ and OZI suppressed contributions
  \[\Rightarrow A_c^c(B \rightarrow VM_s) \sim A(B \rightarrow \psi_n M_s)\] can be determined from data.
- for $b \rightarrow d$ both $A^u, c(B \rightarrow VM)$ important ⇒ Need extra theory input (QCDF)
- light-hadron spectral density ⇒ QH-Duality
- integral over Open-charm spectral density $\simeq a_p + b_p \frac{q^2}{4m_D^2}$ (expansion for $q^2 < m_{J/\psi}^2$)
Accessing $q^2 > 0$: $z$ expansion

Ansatz in $z$ valid below the $D\bar{D}$ threshold

Motivated by "$z$-parametrization" of form factors. [Boyd et al '94, Bourelly et al '08]

1. Extract the poles: $\mathcal{H}_\lambda(q^2) = (q^2 - M^2_{J/\psi})(q^2 - M^2_{\psi(2S)}) \mathcal{H}_\lambda(q^2)$

2. $\mathcal{H}_\lambda(q^2)$ is analytic except for $D\bar{D}$ cut.

3. perform conformal mapping $q^2 \mapsto z(q^2)$.

4. $\mathcal{H}_\lambda(z)$ analytic within unit circle.

5. Taylor expand $\mathcal{H}_\lambda(z)$ around $z = 0$.

6. reasonable convergence expected since $|z| < 0.52$ for $-7\text{GeV}^2 \leq q^2 \leq 14\text{GeV}^2$
Accessing $q^2 > 0$: $z$ expansion

Some details for actual parametrisation

- try to capture most features of the expansion (better convergence)
- parametrize the ratios $\mathcal{H}_\lambda(q^2)/\mathcal{F}_\lambda(q^2)$ instead
- the poles should not modify the asymptotic behaviour at $|q^2| \to \infty$

\[
\mathcal{H}_\lambda(z) = \frac{1 - ZZ_j^*}{Z - Z_j} \frac{1 - Z Z_{\psi}^*(2S)}{Z - Z_{\psi}(2S)} \mathcal{H}_\lambda(z)
\]

\[
\hat{\mathcal{H}}_\lambda(z) = \left[ \sum_{k=0}^{K} \alpha_k^{(\lambda)} z^k \right] \mathcal{F}_\lambda(z)
\]

where $\alpha_k^{(\lambda)}$ are complex coefficients, and the expansion is truncated after the term $z^K$ (we use $K = 2$, i.e. 16 real-valued parameters)

- the modified EOS source code is available upon request (public repo and web page should be updated soon!)
Experimental constraints on \( z \) parametrisation

The residues of the poles are given by \( B \to K^* \psi_n \):

\[
\mathcal{H}_\lambda(q^2 \to M_{\psi_n}^2) \sim \frac{M_{\psi_n} f_{\psi_n}^{*} A_{\lambda}^{\psi_n}}{M_B^2 (q^2 - M_{\psi_n}^2)} + \ldots
\]

Angular analyses determine

\[
|r_{\perp}^{\psi_n}|, \ |r_{\parallel}^{\psi_n}|, \ |r_{0}^{\psi_n}|, \ \text{arg}\{r_{\perp}^{\psi_n} r_{0}^{\psi_n}^{*}\}, \ \text{arg}\{r_{\parallel}^{\psi_n} r_{0}^{\psi_n}^{*}\},
\]

where

\[
r_{\lambda}^{\psi_n} \equiv \text{Res}_{q^2 \to M_{\psi_n}^2} \frac{\mathcal{H}_\lambda(q^2)}{\mathcal{F}_\lambda(q^2)} \sim \frac{M_{\psi_n} f_{\psi_n}^{*} A_{\lambda}^{\psi_n}}{M_B^2 \mathcal{F}_\lambda(M_{\psi_n}^2)}
\]

We produce correlated pseudo-observables from a fit (5+5).
Prior Fit to $z$ parametrisation

(Prior) Fit to Experimental and theoretical pseudo-observables

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

<table>
<thead>
<tr>
<th>$k$</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Re[$\alpha_k^{(\perp)}$]</td>
<td>$-0.06 \pm 0.21$</td>
<td>$-6.77 \pm 0.27$</td>
<td>$18.96 \pm 0.59$</td>
</tr>
<tr>
<td>Re[$\alpha_k^{(</td>
<td></td>
<td>)}$]</td>
<td>$-0.35 \pm 0.62$</td>
</tr>
<tr>
<td>Re[$\alpha_k^{(0)}$]</td>
<td>$0.05 \pm 1.52$</td>
<td>$17.26 \pm 1.64$</td>
<td></td>
</tr>
<tr>
<td>Im[$\alpha_k^{(\perp)}$]</td>
<td>$-0.21 \pm 2.25$</td>
<td>$1.17 \pm 3.58$</td>
<td>$-0.08 \pm 2.24$</td>
</tr>
<tr>
<td>Im[$\alpha_k^{(</td>
<td></td>
<td>)}$]</td>
<td>$-0.04 \pm 3.67$</td>
</tr>
<tr>
<td>Im[$\alpha_k^{(0)}$]</td>
<td>$-0.05 \pm 4.99$</td>
<td>$4.29 \pm 3.14$</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Mean values and standard deviations (in units of $10^{-4}$) of the prior PDF for the parameters $\alpha_k^{(\lambda)}$. 

[10x254]Re
[128x254]0:06
[140x254]0:21
[154x254]6:77
[157x254]0:27
[190x254]0:27
[195x254]18:96
[197x254]0:59
[63x152]Re
[74x152]∥
[82x152]0:35
[84x152]0:62
[88x152]3:13
[90x152]0:41
[93x152]12:20
[95x152]1:34
[63x134]Re
[74x134]0
[82x138]0:05
[84x139]1:52
[85x131]17:26
[87x132]1:64
[89x134]–
[63x96]Im
[74x96]∥
[82x100]0:21
[84x102]2:25
[88x103]1:17
[90x104]3:58
[93x105]–0:08
[95x106]2:24
[63x77]Im
[74x77]0
[82x81]0:05
[84x82]4:99
[85x83]0:05
[87x84]4:99
[89x85]4:29
[90x86]3:14
[93x87]–
Confronting LHCb Data
parametrisation does not provide enough freedom in the SM fit in order to deviate substantially from the prior.
SM predictions and Fit including $B \rightarrow K^* \mu^+ \mu^-$ data and $C_9^{\text{NP}}$

$$r_{A_1} \equiv \frac{A_1(q^2)}{V(q^2)} \times \text{kinematics}$$

- expected to be 1 in limit $m_b, E_{K^*} \rightarrow \infty$

- in fit to all $B \rightarrow K^* \ell^+ \mu^-$ data, fit prefers to change local form factor $V$ over non-local correlators

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]
New Physics Analysis

SM predictions and Fit including $B \to K^* \mu^+ \mu^-$ data and $C_9^{\text{NP}}$

[Bobeth, Chrzaszcz, van Dyk, Virto 2017]

the NP hypothesis with $C_9^{\text{NP}} \sim -1$ is strongly favoured by the fit

- pulls $> 3.4\sigma$ in 1D posterior of the parameter
- posterior odds (for some fits strongly) in favour of NP interpretation
Future experimental analyses

Sensitivity to New Physics in $C_9$ in $B \rightarrow K^* \mu^+ \mu^-$ from an unbinned fit

[Chrzaszcz, Mauri, Serra, Silva Coutinho, van Dyk w.i.p]

- close collaboration with LHCb members
  - preparation for unbinned analysis within LHCb
  - sensitivity study ongoing for LHCb and Belle II prospects
  - extracting $C_9$ in presence of $z^3$ will require simultaneous analysis of theory constraints + data
- first contacts by Belle II members
Future experimental analyses

Sensitivity to New Physics in $C_9$ in $B \to K^* \mu^+ \mu^-$ from an unbinned fit

Preliminary

[Chraszcz, Mauri, Serra, Silva Coutinho, van Dyk w.i.p]

$\sigma$ vs. $N_{\text{sig}}^{B^0 \to K^{0*} \mu^+ \mu^-} \times 10^3$ for $z^2$ fit only

- Use $C_9^{\text{NP}} = -1$ as benchmark point
- Use theory inputs exactly as in pheno analysis [Bobeth, Chraszcz, van Dyk, Virto 2017]
- Sensitivity to coefficients of $z^3$
- (Some) sensitivity to $z$ coefficients in absence of any theory priors!!
- Large increase in $C_9^{\text{NP}}$ uncertainty
non-local effects are (figuratively) half of the amplitude, must include them!

technically challenging, but good progress since 2001

theory calculations most reliable at spacelike $q^2$

- LCSRs [Khodjamirian, Mannel, Pivovarov, Wang 2010]

access timelike $q^2$ via $z$-parametrizations

- aiming for global analyses to exploit parametrical correlations
- experimental colleagues have begun work to incorporate $z$-parametrization in their analyses

did not discuss large $q^2 > M_{\psi(2S)}^2$!
Backup slides
Calculations at negative $q^2$

**LCSRs with $B$-meson DAs** [Khodjamirian, Mannel, Wang 2012]

Soft gluon correction to $O_8$ contribution

Simpler calculation than charm-loop

Numerically very small
Calculations at negative $q^2$

Results for $\mathcal{H}^{(c)}$

$B^- \to \pi^- \ell\ell$

$B^- \to K^- \ell\ell$

[Khodjamirian, Mannel, Wang 2012], [Hambrock, Khodjamirian, Rusov 2015]
Calculations at negative $q^2$

▶ Results for $\mathcal{H}^{(u)}$

$B^- \rightarrow \pi^- \ell \ell$

$B^0 \rightarrow \pi^0 \ell \ell$

[Hambrock, Khodjamirian, Rusov 2015]