Fragmentation functions, factorization, and non-perturbative effects

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- *Formal* proofs of factorization etc are by analysis of leading power behavior in every order of perturbation theory.
- How well do we know factorization from full QCD, beyond PT?
- I'll show a mechanism in hadronization that is not covered by factorization proofs.
- What are the implications?

(See JCC @ QCD Evolution 2016, arXiv:1610.09994)

Summary

- Factorization: Its significance
- Standard approach to proofs
- Beyond perturbation theory
- What are the problems? Why they matter?

Factorization and its uses

Basis:

- Factorization at high Q:

$$\sigma = \text{hard sc.} \otimes \text{pdfs and/or ffs} \ + \ \text{power-suppressed}$$

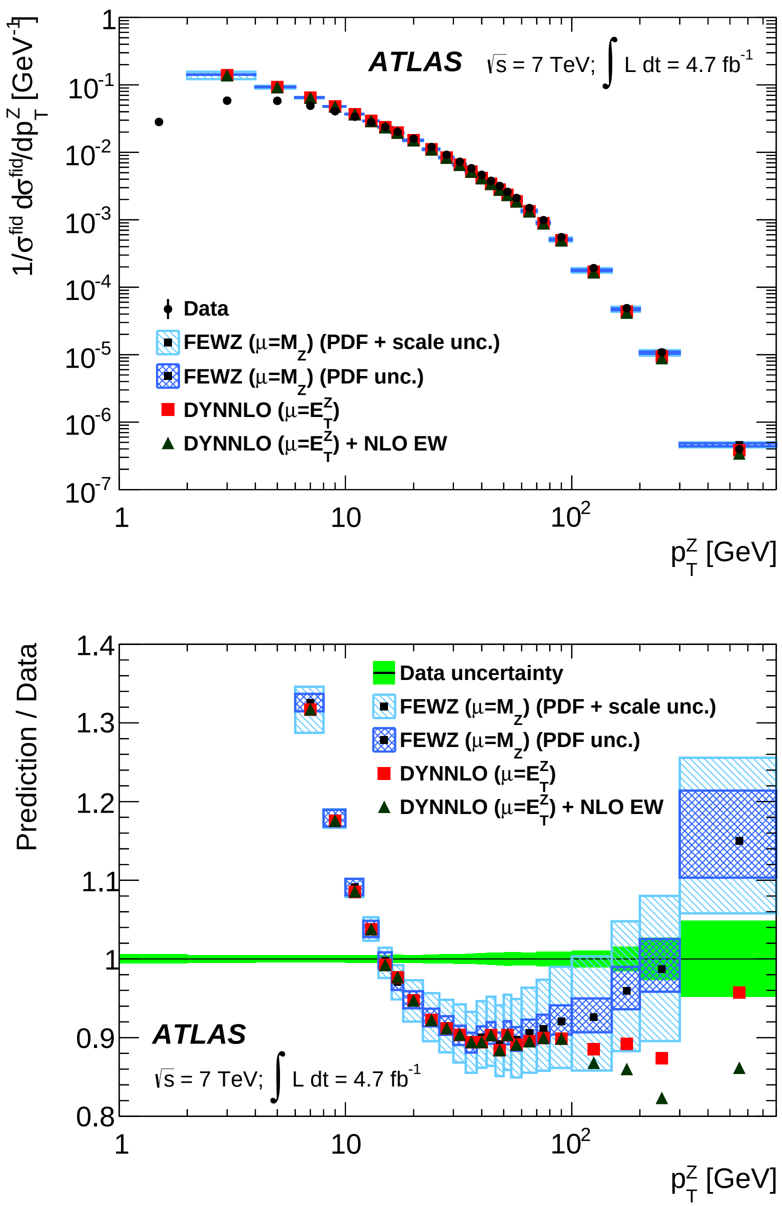
  and generalizations.

- Evolution equations for pdfs, ffs and α_s

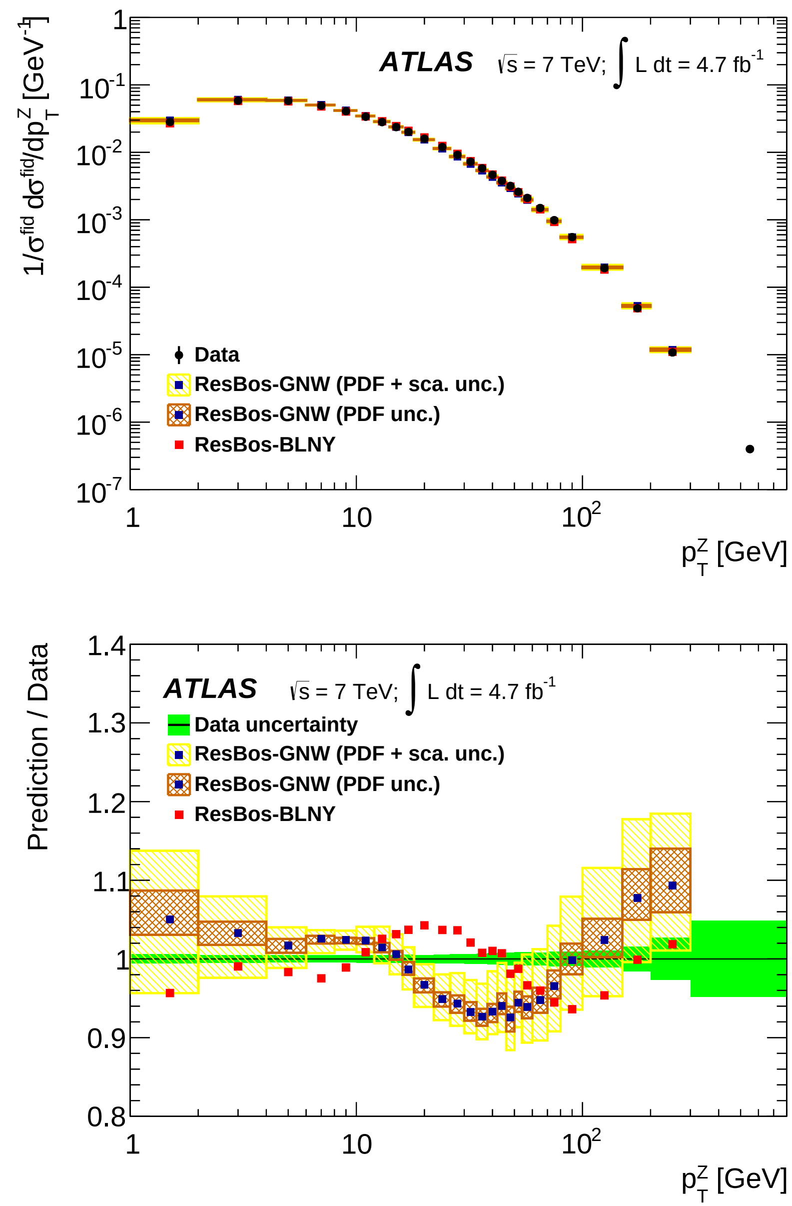
Predictive power (overcoming difficulty of calculations in low-scale regions):

- pQCD calculation of hard scattering, DGLAP kernels, etc
- Measurement of pdfs, ffs, Λ_{QCD} (etc) from a limited set of data.
- Universality of pdfs, ffs, etc gives predictions for many other processes at all (high enough) Q.
- Same issues with TMD pdfs and collinear pdfs, etc.

Successful predictions, e.g., of Z production at LHC



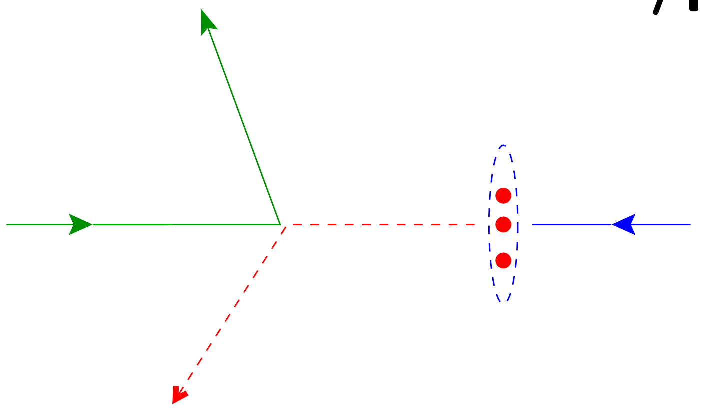
Collinear factorization



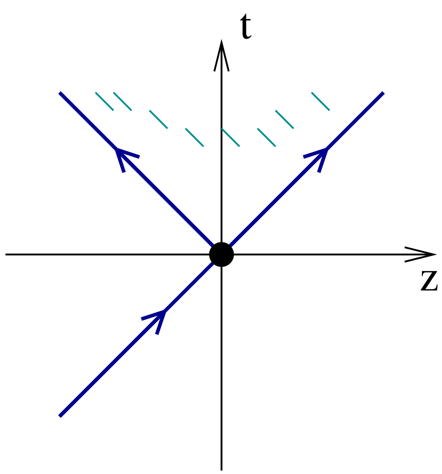
TMD factorization

From arXiv:1510.05427

Non-perturbative reasoning in coordinate-space motivates factorization/partons in DIS at high Q



Short distance collision of electron and constituent of fast moving hadron

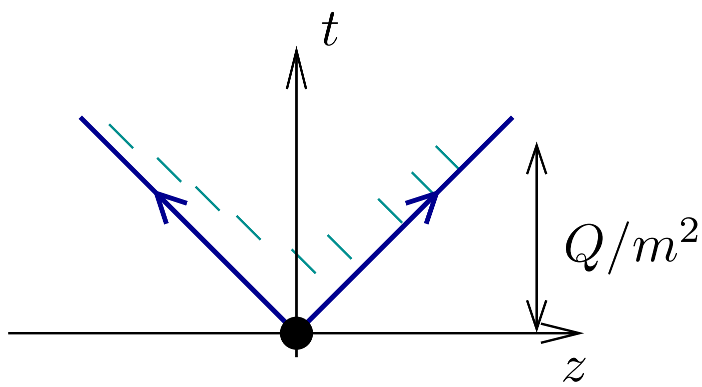


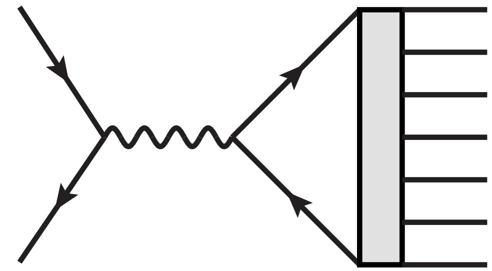
Use of

- Relativity
- Unitarity on final-state interactions

- $\mathrm{d}\sigma =$ Hard sc. $\otimes$ pdf
- (Extend to SIDIS with fragmentation)
- Coordinate space reasoning critical here. (N.B. Mismatch with mom. space work.)

Fragmentation $e^+ + e^- \to h_1 + h_2 + X$ at high Q



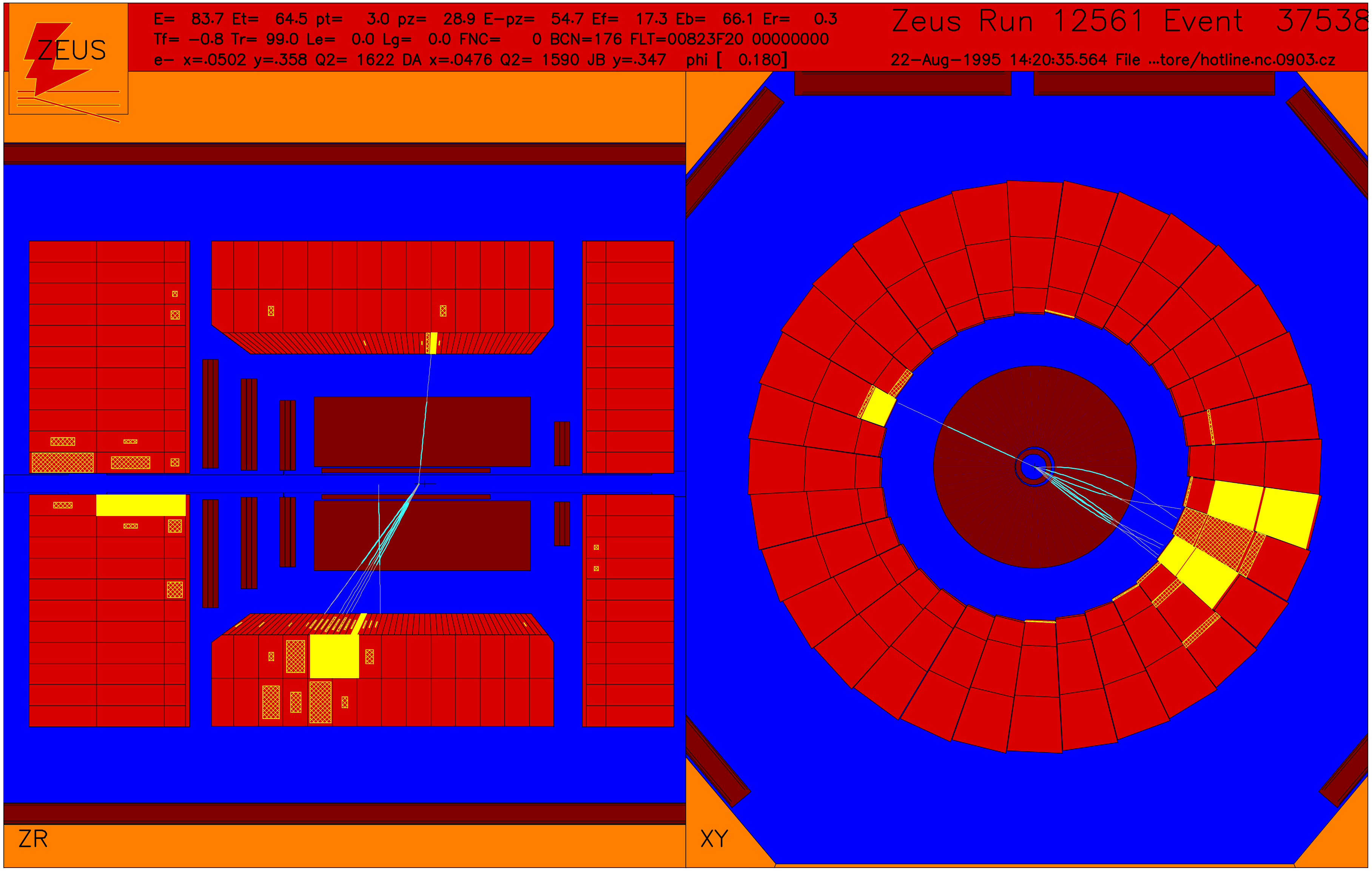


Space-like separation of “valence hadronization” *and* string-like fragmentation implies independent fragmentation

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z_1\,\mathrm{d}z_2} = \text{Hard sc.} \otimes \mathrm{ff}_1(z_1) \otimes \mathrm{ff}_2(z_2) \ + \ \text{power suppressed}$$

But rapidity gap filled in, so Feynman graphical structures couple the jets.

Jets show reality for jet hadronization of high energy parton



Standard factorization proofs

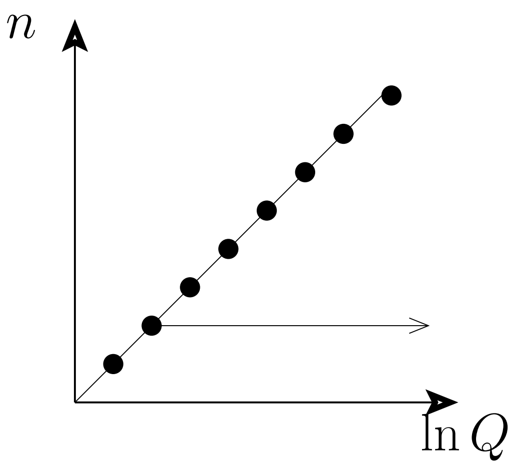
Motivation from trying to implement intuitive picture in QFT, and finding modifications.

Given some process with a large scale Q:

- Extract leading (usually) power large Q asymptote of each individual graph for process
- Deal with multiple regions per graph
- Organized in factorized form after sum over graphs

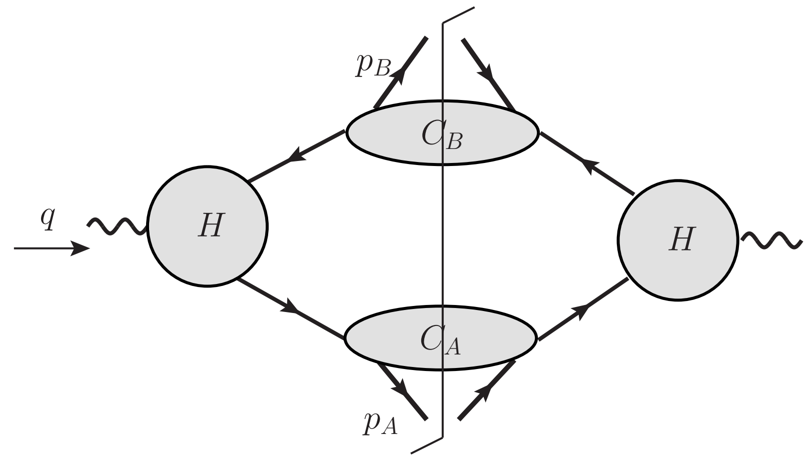
Issues in going beyond order-by-order perturbation theory:

- Perturbation theory is not literally convergent.
- Non-perturbative effects exist in QCD.
- Asymptote and infinite sum might not commute.

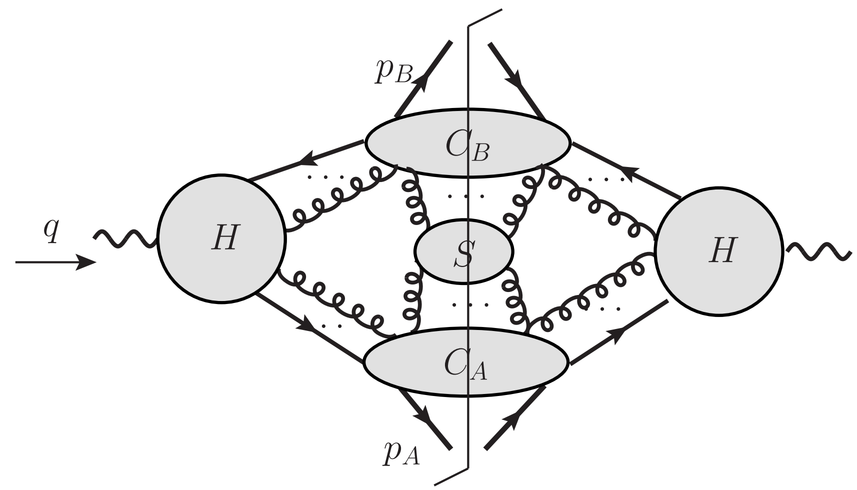


Feynman graphs, momentum regions

Basic "parton model"



But also have leading graphs



Canonical momentum regions in $(+,-,T)$ coordinates are, e.g.,

$$\text{Coll. A}: \left(Q, m^2/Q, m\right), \qquad \text{S}: (m,m,m) \text{ or } \left(m^2/Q, m^2/Q, m^2/Q\right)$$

All intermediate regions also contribute!

Steps in standard factorization proofs

1. Extract leading (usually) power large Q asymptote of each individual graph for process. (Libby-Sterman analysis)

2. Apply approximations. Aims:
   - Suitable for separation into factors
   - See item 4

3. Subtractions

4. *Ward identities. (Critical to disentangle gluons connecting subgraphs for different regions.)*

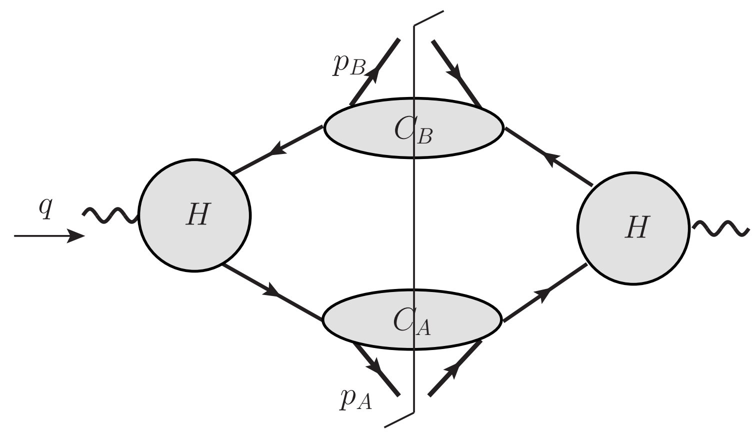
5. Final-state unitarity cancellations, etc.

6. Deduce factorized form after sum over graphs.

7. Similarly derive evolution equations (RG, DGLAP, CSS, etc).

Non-perturbative-compatible structures in factorization proof

- Overall leading-power analysis à la Libby-Sterman gives result that matches (extended) parton-model view, in coordinate space, etc.
- Approximations can be applied block-by-block rather than just individual-subgraph-by-individual-subgraph. E.g.:



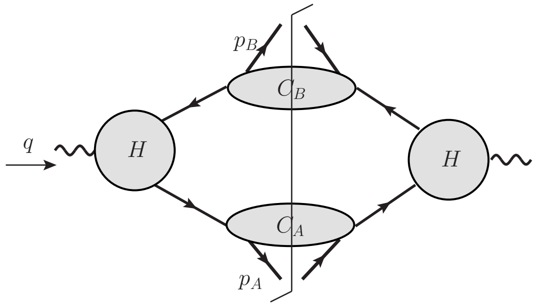
- Cancellation of spectator-spectator interactions in Drell-Yan was originally fully non-perturbative (De Tar, Ellis, Landshoff), with parton-model assumptions pre-QCD

  CSS updated it and included proper QCD interface. (. . .)

So the perturbative proofs cover *more* than just perturbation theory.

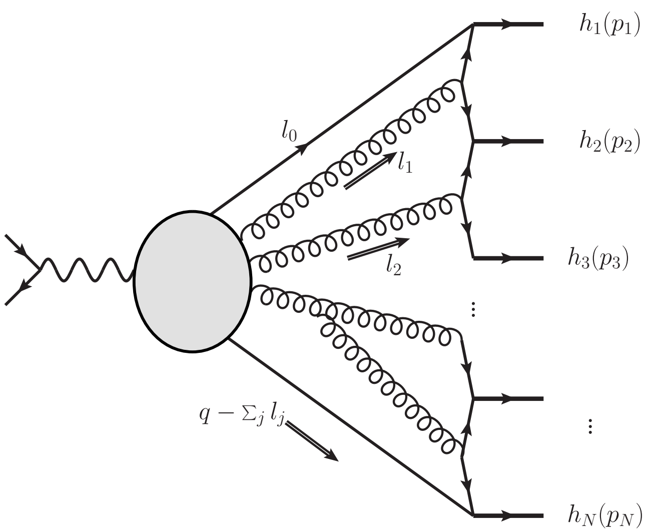
Filled-in rapidity gap

Parton-model-like graphs taken literally have large rapidity gap and fractionally charged particles:



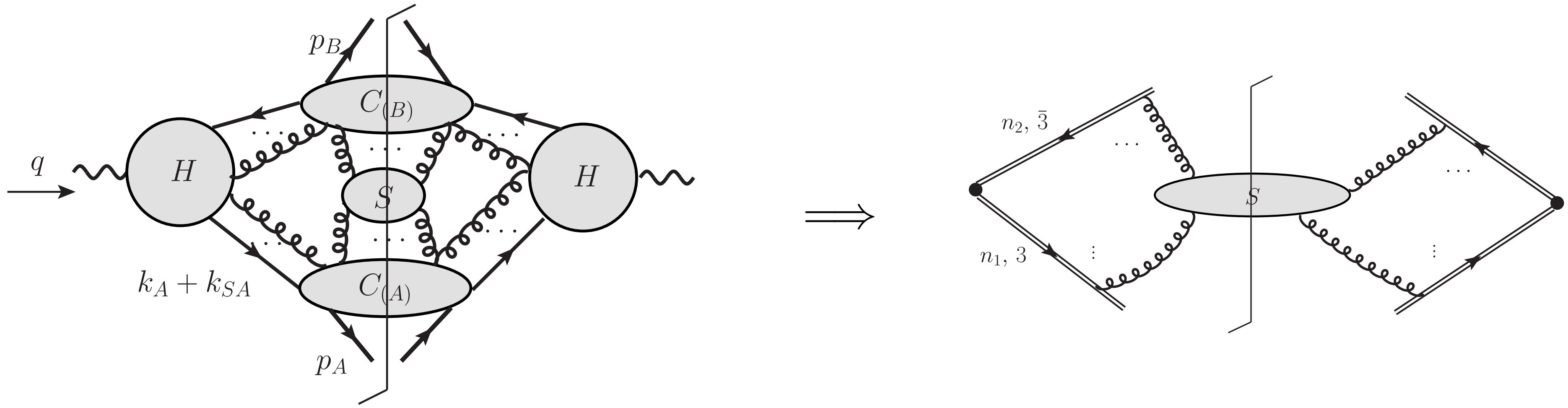
How does string-like (including cluster hadronization) match perturbative analysis?

Minimal Feynman graph model for string-like hadronization is cluster hadronization:



with order of graph $\propto \ln(Q^2/m^2)$

But usual approximations need large rapidity differences



E.g., for canonical soft gluon k attaching to collinear-A subgraph

$$k = (m, m, m) \text{ or } \left(m^2/Q, m^2/Q, m^2/Q\right) \qquad \text{Coll. A}: \left(Q, m^2/Q, m\right),$$

we use

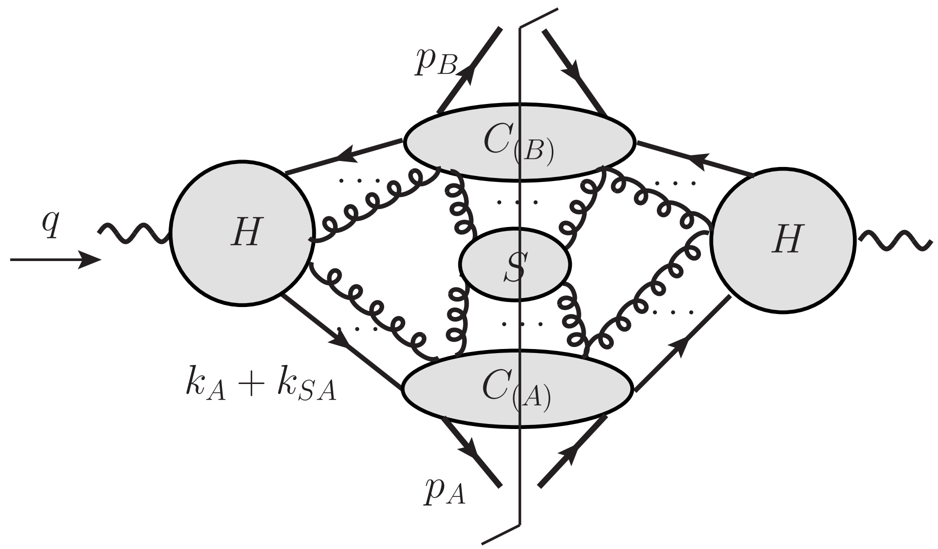
$$A(k,\dots)^\mu S(k,\dots)_\mu \simeq A(\hat{k},\dots)^+ S(k,\dots)^- = A(\hat{k},\dots)\cdot\hat{k}\frac{1}{k^-}S(k,\dots)^-$$

where $\hat{k} = (0, k^-, \mathbf{0}_\mathrm{T})$

Ward identities on $A(\hat{k},\dots)\cdot\hat{k}$ etc convert soft factor to Wilson line matrix element.

Etc (soft-to-B, collinear-to-H)

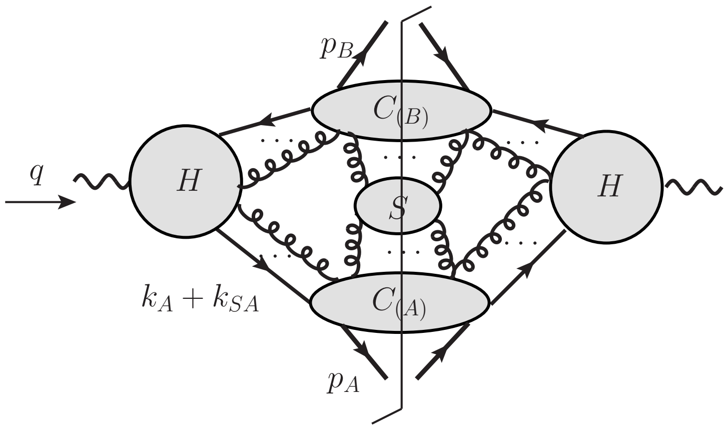
Why are Ward identities & associated approximations important?



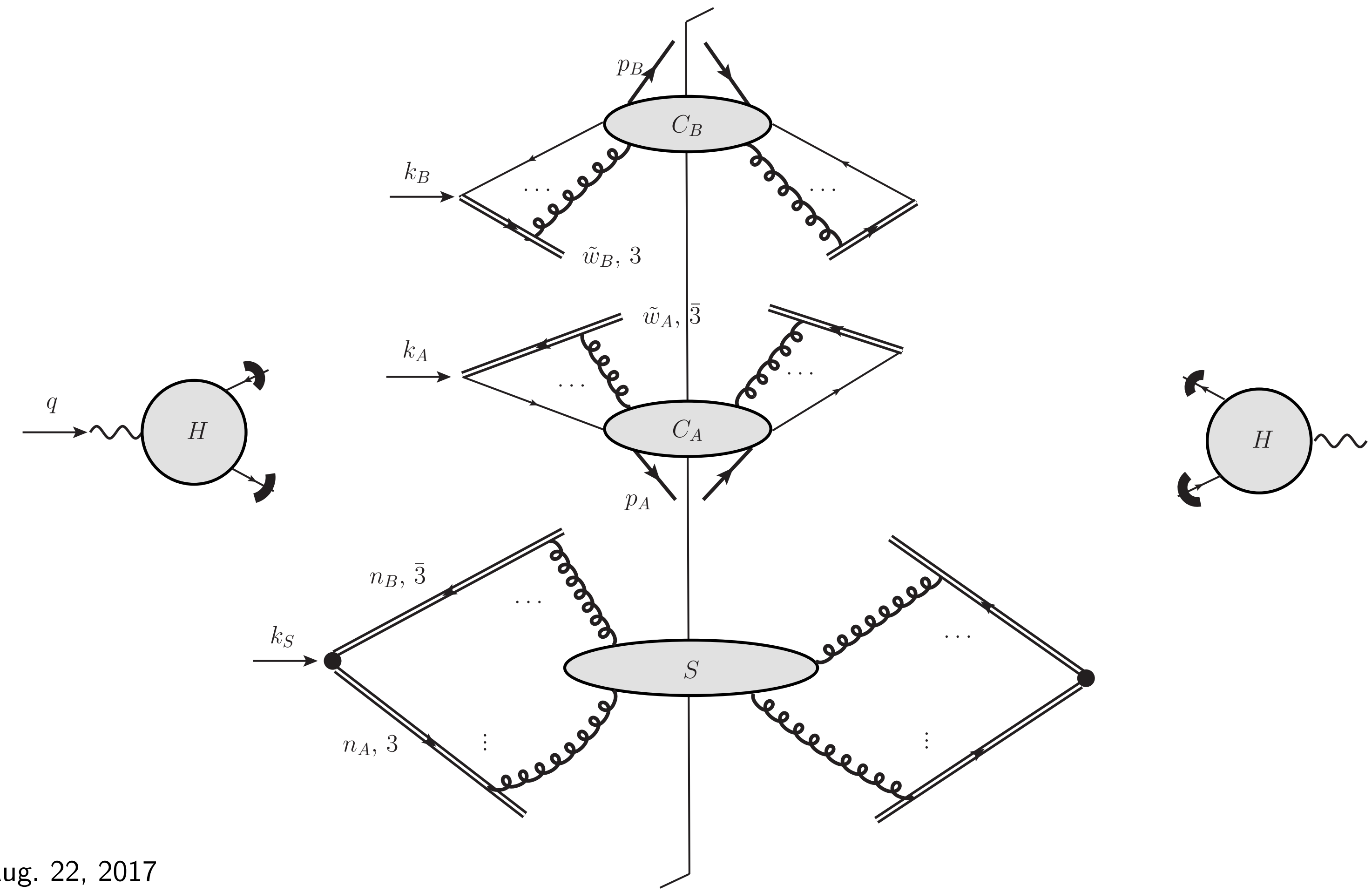
has indefinitely many variables joining factors.

But a useful factorization property has a fixed small number of integration variables joining factors (e.g., partonic ξ and/or partonic $\boldsymbol{k}_\mathrm{T}$, or $\boldsymbol{b}_\mathrm{T}$.

Why are Ward identities & associated approximations important?



gets replaced by (*subtracted* version of):

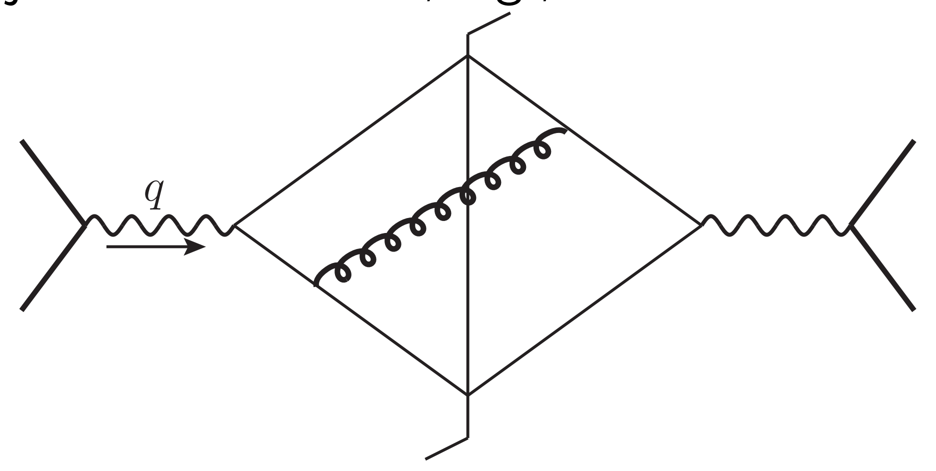


Subtractions etc

- Each graph has multiple leading regions.
- All intermediates between the canonical regions are equally important.
- When defining, e.g., a collinear factor (as in a pdf or frag. fn.), all internal loop momenta are integrated over.
- Therefore subtractions must be applied in each factor to ensure all relevant contributions are counted once, and the factors are individually finite.

Mismatch of calculated on-shell pQCD graphs and ideas of hadronization

Simple case: $e^+e^- \to$ jets. We calculate, e.g.,



with literally on-shell (and massless) final-state quarks and gluons (i.e., the quarks and gluons go to $t = +\infty$!).

We say that the partons turn into jets of hadrons (which happens at $t \propto Q/m^2$).

!!??!!

Correct physics is discussed, but the translation to Feynman-graphical notation is incomplete.

Summarize factorization and actual definition of TMD fragmentation function

(On board.)

N.B. Soft factor in numerator or denominator?

$$
\begin{aligned}
d\sigma &\propto f_{H_1/q} f_{H_2/\bar{q}} && \text{End result} \\
&\overset{?}{=} B_1^? B_2^? S && \text{Naive expectation} \\
&= \frac{f_1^{\text{unsubtr.}} f_2^{\text{unsubtr.}}}{S^{\text{unsubtr.}}} && \text{Actual}
\end{aligned}
$$

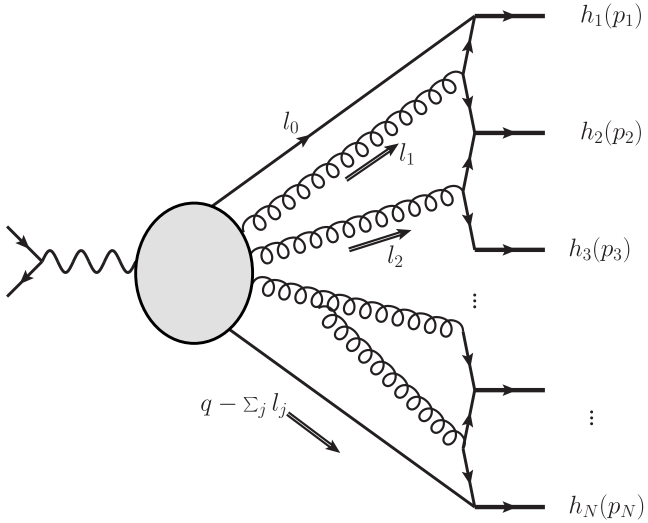
where "unsub." is "unsubtracted", and B_j are "beam/jet factors".

Final finite fragmentation functions are

$$
f = \frac{f^{\text{unsub.}}}{\sqrt{S^{\text{unsub.}}}} \times \text{U.V. ren.}
$$

in limit that regulators are removed. (Various related definitions are also available with same end product.)

Soft-to-collinear approximation fails in string-compatible graphs



Define

- Δy to be total available rapidity range ($\simeq \ln(Q^2/m^2)$)
- δy to be typical cluster separation, i.e., $\Delta y/\#$clusters
- Experimentally δy is typically small

Then

- Errors are now a power of $e^{-\delta y}$, not m/Q
- Order of relevant graphs increases with Q
- Quark, not gluon, exchange

Hence

- String-type hadronization doesn't match perturbative derivation of factorization, even in perturbative model.
- Problem applies everywhere with final-state detection.

But factorization is not (necessarily) falsified

- Opposite ends of a Lund string are at space-like separation, so independent hadronization (except for QM entanglement) is expected
- That already leads to some kind of fragmentation function, but without (yet) an explicit formal definition.
- The standard definition is almost certainly correct (IMO), but its justification needs to be improved.
- There may be modified physics relative to order-by-order perturbatively derived factorization.

Overall issues

- How well do we know that factorization (and generalizations) is valid?
- We have seen: String hadronization isn't allowed for in derivation.
- What other similar issues are there?
- What new phenomena could gaps in derivations uncover?

Future

- Sort out whether these issues actually do impact factorization formulae with fragmentation functions. (Collinear and TMD)
- Whether or not they do, find an improved formulation, perhaps involving a proper systematic interface between standard pQCD constructs and string-like constructs.
- Find a better way of analyzing graphs and amplitudes in coordinate space.
- Some implications, aims, etc:
  - Better interface between perturbative analysis and non-perturbative physics, especially for MCEG.
  - Definitions of pdfs, ffs, that can be used literally in non-perturbative calculations.
  - What else, e.g., new results?