RG analysis at higher orders in perturbativ QFTs in CMP

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[arXiv:1605.09423] & [arXiv:1703.08801]

Overview

Motivation

- Obtain <u>reliable</u> & precise predictions
- In HEP the SM (QFT with $\mathcal{L}_{\mathrm{SM}}$) provides collider observables
- New physics $\mathcal{L}_{\text{“real”}} = \mathcal{L}_{\mathrm{SM}} + \mathcal{L}_X$?
- New physics includes realization of $SUSY$? (MSSM, nMSSM, ...)
- Origin of $\mathcal{L}_{\mathrm{SM}}$?
- Origin of QFTs?

How to proceed?

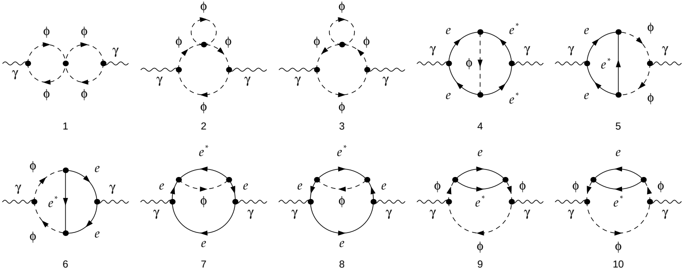
- Build and run LHC to find new physics
- Calculate observables at higher order in PT
- Investigate on various QFTs (may not have HEP relevancy)

Work in collaboration with...

- *University of Alberta (Edmonton)*
  Joseph Maciejko
  Chien-Hung Lin
- *University of Heidelberg*
  Michael Scherer ($\rightarrow$ Köln)
  Luminita Mihaila
  Bernhard Ihrig
- *DESY Zeuthen*
  Peter Marquard

Where to find QFTs?

On the blackboard at the office of my boss in Edmonton



A.P.: *"Some condensed matter physics guy asked me how to calculate these diagrams..."*

"Its only two loops. This should be easy for you. You can probably do it in a week or two."

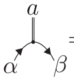
Where to find QFTs?

At the office of *Joseph Maciejko* in Edmonton

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi + |\partial_\mu\phi|^2 + m^2|\phi|^2 + \lambda^2|\phi|^4 + h(\phi^*\psi^T i\sigma_2\psi + h.c.)\,.$$

- Imaginary time Landau-Ginzburg Lagrangian with Lorenzian-symmetry.
- ψ: 2-component Dirac fermion
- ϕ: complex Cooper pair scalar-field
- $\partial\!\!\!/ = \gamma^\mu \cdot \partial_\mu$. +Clifford algebra: $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$.
  (Ex.Rep.: $\gamma^0 = \sigma_3, \gamma^1 = \sigma_1, \gamma^2 = \sigma_2$)
- "$i\sigma_2$" $\rightarrow \epsilon$! Invariant $SU(2)$ tensors: $\left(\overset{a}{T} = \frac{\sigma_a}{2}\right)$

$\alpha \rightarrow\!\blacktriangle\!\leftarrow \beta = \epsilon_{\alpha\beta}\,,\quad \alpha \leftarrow\!\blacktriangle\!\rightarrow \beta = \epsilon^{\alpha\beta}\,,$

$\alpha \longrightarrow \beta = \delta^\beta_\alpha\,,$  $= \overset{a}{T}{}^\beta_\alpha$.

LET for Superconductor

Landau-Ginzburg Lagrangian with Lorenzian-symmetry

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi + |\partial_\mu\phi|^2 + m^2|\phi|^2 + \lambda^2|\phi|^4 + h(\phi^*\psi^T i\sigma_2\psi + h.c.)\,.$$

- Semimetal phase Global $U(1)$ symmetry for $m^2 > 0$

$$\psi \to e^{i\theta}\psi\,, \qquad \phi \to e^{2i\theta}\phi\,, \qquad \langle 0|\phi|0\rangle = \phi_0 = 0\,.$$

- Phasetransition at Quantum Critical Point (QCP) for $m^2 = 0$
- Superconducting phase
  Spontaneously broken global $U(1)$ symmetry for $m^2 < 0$

$$\langle 0|\phi|0\rangle = \phi_0 \neq 0\,.$$

alias "non-vanishing order parameter ϕ_0"

LET for Superconductor

- Landau-Ginzburg Lagrangian with Lorenzian-symmetry

$$\mathcal{L} = i\bar{\psi}\partial\!\!\!/\psi + |\partial_\mu\phi|^2 + m^2|\phi|^2 + \lambda^2|\phi|^4 + h(\phi^*\psi^T i\sigma_2\psi + h.c.)\,.$$

- Behavior $\lambda(\mu), h(\mu)$ for small scale μ/low energy limit in $d = 2 + 1 = 3$ dimensions at the QCP?
- Results for $\overline{\mathrm{MS}}$ β-function at 1-loop PT in $d = 4 - \epsilon$ [Scott Thomas 05]: $\bar{h}^2 = h^2/(4\pi)^2, \bar{\lambda}^2 = \lambda^2/(4\pi)^2$

$$\beta_{h^2} = \frac{\mathrm{d}h^2}{\mathrm{d}\ln\mu} = -\epsilon\bar{h}^2 + 12\bar{h}^4,$$

$$\beta_{\lambda^2} = \frac{\mathrm{d}\lambda^2}{\mathrm{d}\ln\mu} = -\epsilon\bar{\lambda}^2 + 20\bar{\lambda}^4 + 8\bar{h}^2\bar{\lambda}^2 - 16\bar{h}^4,$$

$$\gamma_\psi = \frac{\mathrm{d}\ln Z_\psi}{\mathrm{d}\ln\mu} = 4\bar{h}^2\,,$$

$$\gamma_\phi = \frac{\mathrm{d}\ln Z_\phi}{\mathrm{d}\ln\mu} = 4\bar{h}^2\,.$$

Renormalization

- Do $\overline{MS}$ field and coupling redefinitions:

$$\phi \to \phi^0 = \sqrt{Z_\phi}\phi\,, \qquad \psi \to \psi^0 = \sqrt{Z_\psi}\psi\,,$$
$$\lambda \to \lambda^0 = \mu^{\epsilon/2} Z_\lambda \lambda\,, \qquad h \to h^0 = \mu^{\epsilon/2} Z_h h\,.$$

- Renormalized Landau-Ginzburg Lagrangian

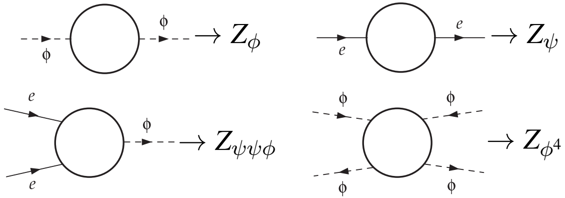
$$\begin{aligned}\mathcal{L} =& iZ_\psi \bar{\psi}\partial\!\!\!/\psi + Z_\phi |\partial_\mu \phi|^2 + Z_{\phi^4}\lambda^2 \mu^\epsilon |\phi|^4 \\ &+ Z_{\psi\psi\phi} h \mu^{\epsilon/2}(\phi^* \psi^T i\sigma_2 \psi + \text{h.c.})\,.\end{aligned}$$
$$Z_{\phi^4} = Z_\lambda^2 Z_\phi^2\,, \qquad Z_{\psi\psi\phi} = Z_h \sqrt{Z_\phi} Z_\psi\,.$$

- Bare parameters and fields do not depend on μ. $\to \beta, \gamma$.

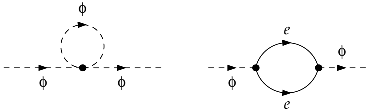
$$\frac{\mathrm{d}X^0}{\mathrm{d}\mu} = 0\,, \qquad X \in \{h^2, \lambda^2, \psi, \phi\}\,.$$

How to obtain Zs?

- Using DREG get poles in small $\epsilon = 4 - d$ of 1-PI n-point functions at $L = 1, 2, 3, ...$ loops for "any" kinematics



- Example Diagrams Z_ϕ $(L = 1)$



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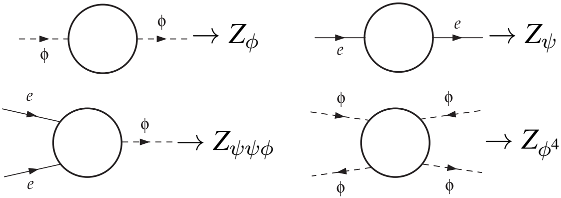

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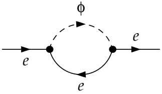

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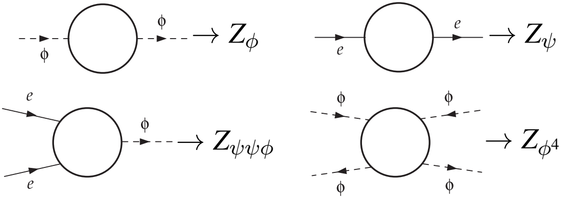

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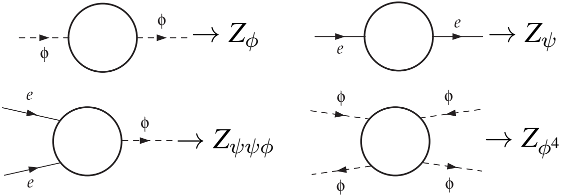

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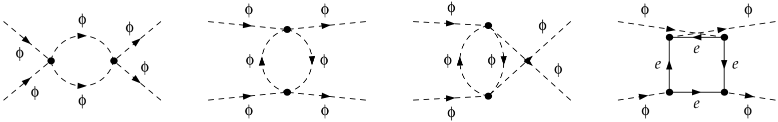

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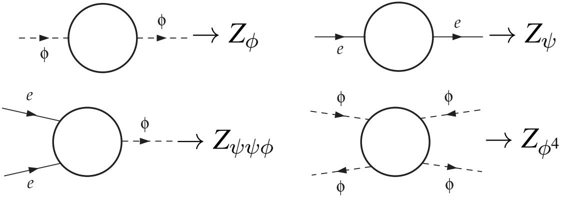

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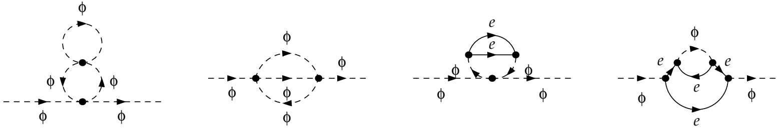

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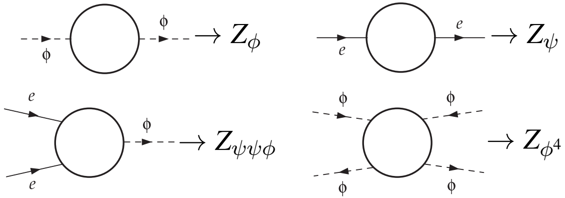

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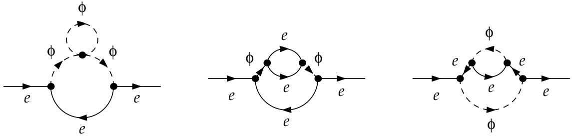

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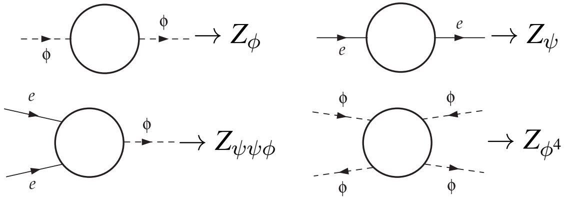

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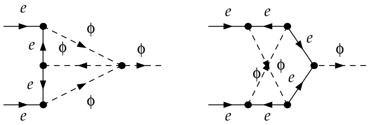

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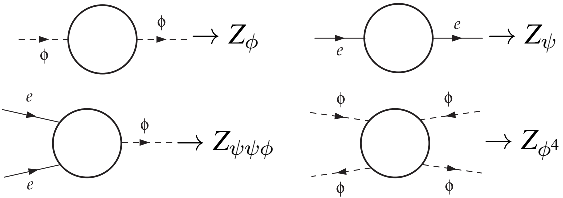

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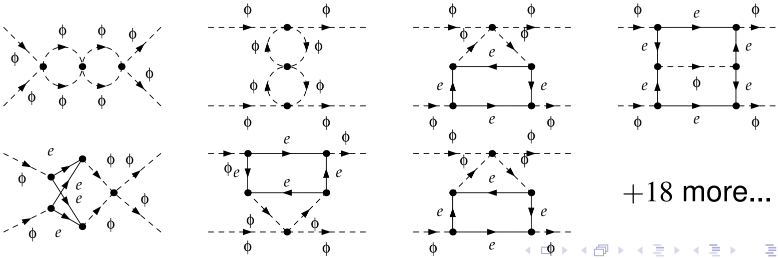

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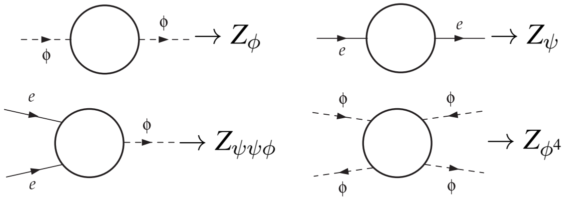

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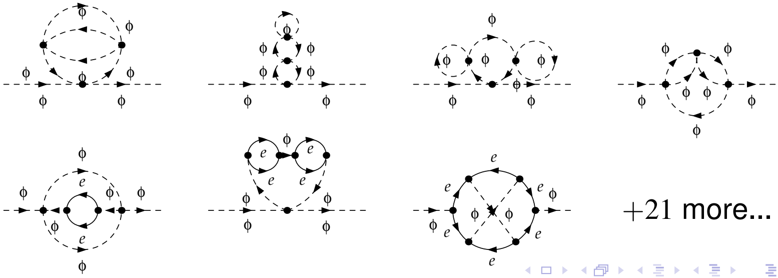

$+21$ more...

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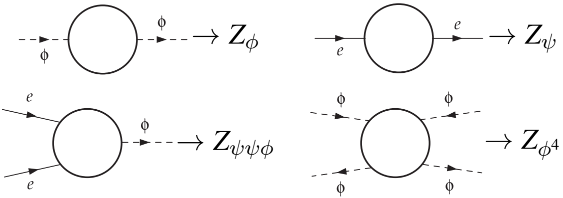

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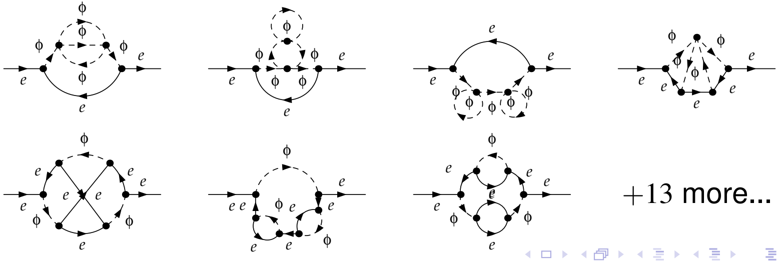

$+13$ more...

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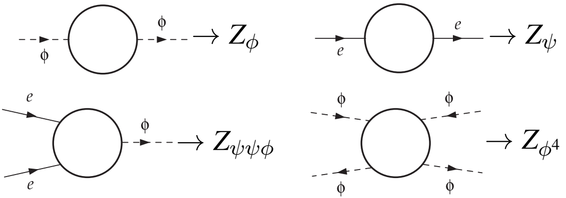

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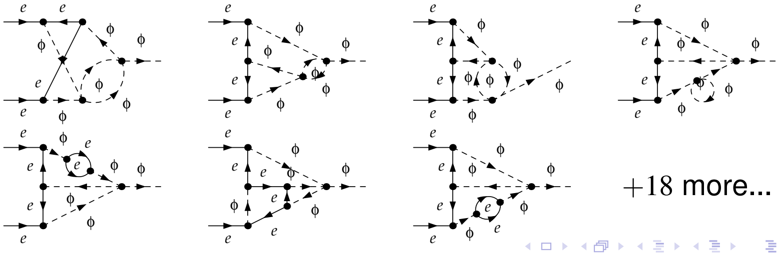

$+18$ more...

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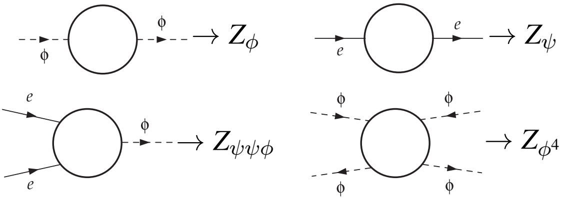

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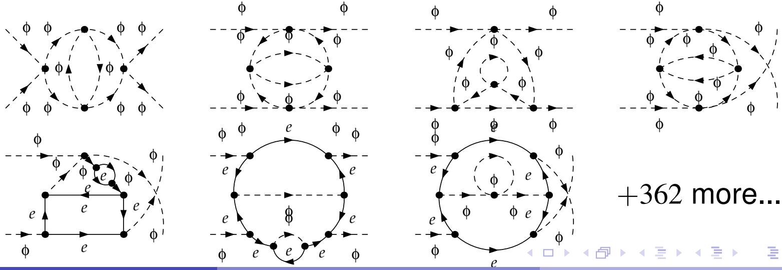

+362 more...

How to obtain Zs?

- Chosing “massiv tadpole kinematics”
  - Use single IR regulator mass M in every denominator
  - Expand in small external momenta $|p_i|/M \ll 1$
  - All integrals are 1-scale tadpole integrals
  - Use infrared rearrangement [Chetyrkin,Misiak,Munz] to consistently subtract artificial terms $\sim M^2$

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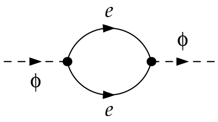
- Fully automatized setup
  - *QGRAF* [Nogueira]
  - *q2e* [Seidensticker]
  - *exp* [Seidensticker]
  - *FORM* [Vermaseren & Co]
  - *MATAD* [Steinhauser] OR *CRUSHER* [Marquard,Seidel]
  - modified *majoranas.pl* [Harlander]
  - *RecalcPrefac* [NZ]

Back in Wick's Garden...

Sign of Wick factor S is correlated with index order in $\epsilon_{\alpha\beta}$

$$\langle 0|\hat{a}_\phi \; (\tfrac{1}{2}\epsilon^{\alpha\beta}\phi^\star\psi_\alpha\psi_\beta)\; (\tfrac{1}{2}\epsilon_{\gamma\delta}\psi^{\dagger\,\gamma}\psi^{\dagger\,\delta}\phi)\; \hat{a}^\dagger_\phi|0\rangle$$

$$\to S = +\frac{1}{2}$$



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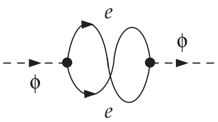
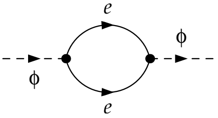


Back in Wick's Garden...

⚠ Sign of Wick factor S is correlated with index order in $\epsilon_{\alpha\beta}$

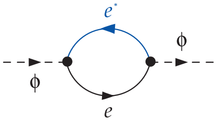
➡ Generate diagrams with equivalent fermions $\rightarrow |S|$



$\rightarrow |S| = \frac{1}{2}$

Back in Wick's Garden...

- Sign of Wick factor S is correlated with index order in $\epsilon_{\alpha\beta}$
- Generate diagrams with equivalent fermions $\to |S|$
- Chose a "Denner" current flow direction & recalculate sign of S



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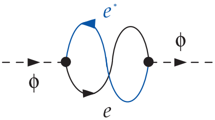

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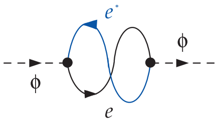

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+ using:

$$S_F = \psi\psi^\dagger \rightarrow (-1)\cdot S_F\,.$$

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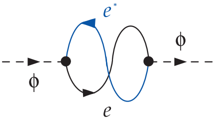

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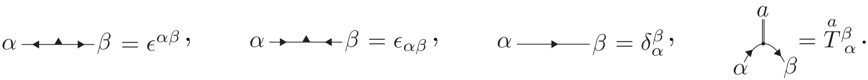
- ❓ How to treat $\epsilon^{\alpha\beta}$ in amplitudes with *FORM*?

How to $\epsilon^{\alpha\beta}$?

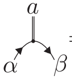
- **Shorthands**



- **Rider for sign controll** 

How to $\epsilon^{\alpha\beta}$?

- **Shorthands**

α  $\beta = \epsilon^{\alpha\beta}$, α  $\beta = \epsilon_{\alpha\beta}$, α  $\beta = \delta_\alpha^\beta$, 

$= \overset{a}{T}{}^\beta{}_\alpha$.

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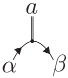
How to $\epsilon^{\alpha\beta}$?

Shorthands

$= \epsilon^{\alpha\beta}$,

$= \epsilon_{\alpha\beta}$,

$= \delta^{\beta}_{\alpha}$,



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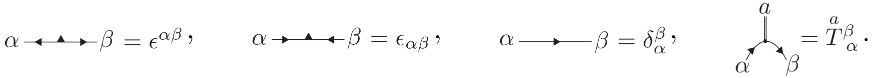

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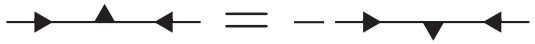

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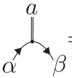
α  $\beta = \epsilon^{\alpha\beta}$, α  $\beta = \epsilon_{\alpha\beta}$, α  $\beta = \delta_\alpha^\beta$, 

$= \overset{a}{T}{}^\beta_{\ \alpha}$.

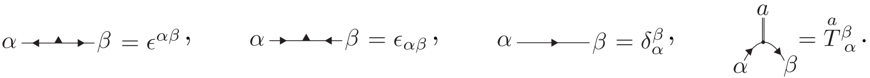
- **Rider for sign controll**  $= -$ 

- **Invariance conditions** $[G(\omega) = \exp(i\overset{a}{T}\omega^a)]$

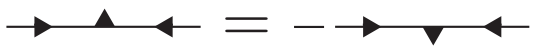
$$\epsilon_{\alpha_1\alpha_2} = G(\omega)_{\alpha_1}^{\gamma_1} G(\omega)_{\alpha_2}^{\gamma_2} \epsilon_{\gamma_1\gamma_2} \, , \qquad \epsilon^{\beta_1\beta_2} = \epsilon^{\delta_1\delta_2} G^{-1}(\omega)_{\delta_1}^{\beta_1} G^{-1}(\omega)_{\delta_2}^{\beta_2} \, .$$

How to $\epsilon^{\alpha\beta}$?

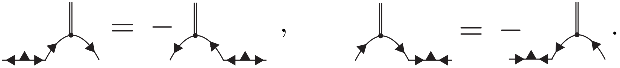
Shorthands



Rider for sign controll



Invariance condition $\sim \omega^1$



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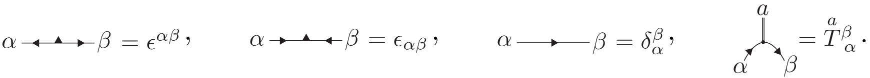
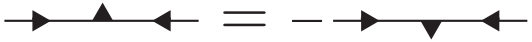
- Shorthands



- Rider for sign controll 

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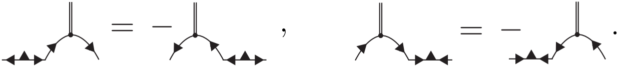
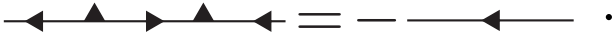


- Annihilation 

How to $\epsilon^{\alpha\beta}$?

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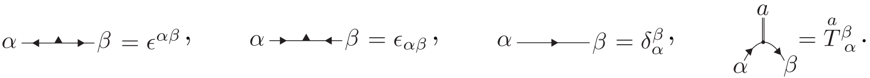

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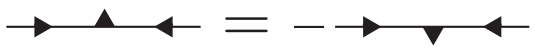

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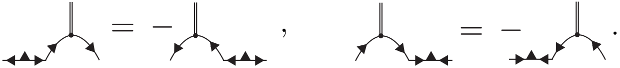

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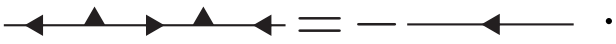

- ϵ can be emulated by naive anti-commuting γ_5:

$$\{\gamma^{\mu}, \gamma_5\} = 0\,.$$

β and γ @ 3 Loops

$$\bar{h}^2 = h^2/(4\pi)^2, \bar{\lambda}^2 = \lambda^2/(4\pi)^2$$

$$
\begin{aligned}
\beta_{h^2} &= -\epsilon\bar{h}^2 + 12\bar{h}^4 \\
&\quad +8\bar{h}^2\bar{\lambda}^4 - 64\bar{h}^4\bar{\lambda}^2 + 8\bar{h}^6 \\
&\quad -40\bar{h}^2\bar{\lambda}^6 + 300\bar{h}^4\bar{\lambda}^4 + 1632\bar{h}^6\bar{\lambda}^2 - [1652 - 576\zeta_3]\bar{h}^8, \\
\beta_{\lambda^2} &= -\epsilon\bar{\lambda}^2 + 20\bar{\lambda}^4 + 8\bar{h}^2\bar{\lambda}^2 - 16\bar{h}^4 \\
&\quad -80\bar{h}^2\bar{\lambda}^4 + 16\bar{h}^4\bar{\lambda}^2 + 256\bar{h}^6 - 240\bar{\lambda}^6 \\
&\quad +904\bar{h}^2\bar{\lambda}^6 + [3832 + 3264\zeta_3]\bar{h}^4\bar{\lambda}^4 - [8664 + 2688\zeta_3]\bar{h}^6\bar{\lambda}^2 \\
&\quad -[768 + 3072\zeta_3]\bar{h}^8 + [4936 + 3072\zeta_3]\bar{\lambda}^8, \\
\gamma_\phi &= 4\bar{h}^2 - 24\bar{h}^4 + 8\bar{\lambda}^4 \\
&\quad -40\bar{\lambda}^6 - 60\bar{h}^2\bar{\lambda}^4 + 160\bar{h}^4\bar{\lambda}^2 + [20 + 192\zeta_3]\bar{h}^6, \\
\gamma_\psi &= 4\bar{h}^2 - 16\bar{h}^4 \\
&\quad -44\bar{h}^2\bar{\lambda}^4 + 128\bar{h}^4\bar{\lambda}^2 + [192\zeta_3 - 4]\bar{h}^6.
\end{aligned}
$$

β and γ @ 3 Loops

when $\overline{\lambda} = \overline{h}$: $\overline{h}^2 = h^2/(4\pi)^2$

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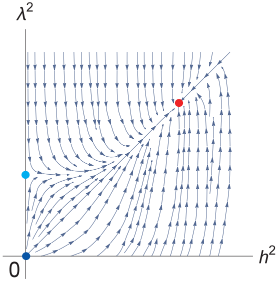
RG Fixed Points

- Searching for IR fixed points ($\mu \to 0$) at $d = 3$:

$$\left(\overline{h}_*^2, \overline{\lambda}_*^2\right) = (0,0)\,, \qquad \textit{(unstable)}$$

$$\left(\overline{h}_*^2, \overline{\lambda}_*^2\right) = \left(0, \frac{\epsilon}{20} + \frac{3\epsilon^2}{100} - \frac{384\zeta_3 - 103}{20000}\epsilon^3\right), \qquad \textit{(unstable)}$$

$$\overline{h}_*^2 = \overline{\lambda}_*^2 = \frac{\epsilon}{12} + \frac{\epsilon^2}{36} - \frac{4\zeta_3 - 1}{144}\epsilon^3. \qquad \textit{(stable)}$$



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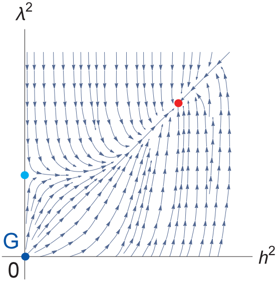

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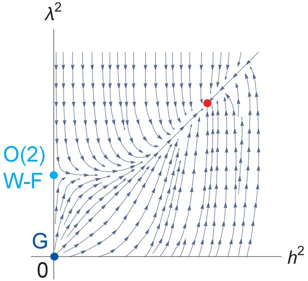

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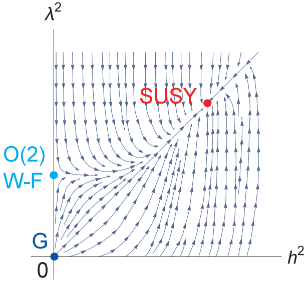

Emergent SUSY

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  supports dynamically emergent SUSY conjecture at $d = 3$

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- SUSY non-renormalization theorems
- At the SUSY fixed point we check: [Strassler'03]

$$\gamma^*_\phi = \gamma^*_\psi = \frac{\epsilon}{3} + \mathcal{O}(\epsilon^4).$$

Critical Exponents

- Stability exponent ($\omega \gtrless 0$ IR stable/unstable $*$)

$$\omega = \left.\frac{\mathrm{d}\beta_{h^2}(\lambda^2 = h^2)}{\mathrm{d}h^2}\right|_{h=h_*} = \quad \epsilon - \frac{1}{3}\epsilon^2 + \left(\frac{1}{18} + \frac{2}{3}\zeta_3\right)\epsilon^3 + \mathcal{O}(\epsilon^4)\,.$$

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$$\nu^{-1} = 2 + \gamma^*_{m^2}\,, \qquad \gamma_{m^2} = \frac{\mathrm{d}\ln Z_{m^2}}{\mathrm{d}\ln\mu}\,, \qquad Z_{m^2}m^2 = m_0^2\,.$$

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- $$\nu = \frac{1}{2} + \frac{1}{4}\epsilon + \frac{1}{24}\epsilon^2 + \left(\frac{1}{6}\zeta_3 - \frac{1}{144}\right)\epsilon^3 + \mathcal{O}(\epsilon^4)\,.$$

Critical Exponents

Naive extrapolation to $d = 2 + 1$ ($\epsilon = 1$):

$$\nu \overset{1-\mathrm{loop}}{=} 0.75\,, \qquad \nu \overset{2-\mathrm{loop}}{\approx} 0.792\,, \qquad \nu \overset{3-\mathrm{loop}}{\approx} 0.985\,.$$

Bootstrap result [Bobev,El-Showk,Mazac,Paulos‘15]:

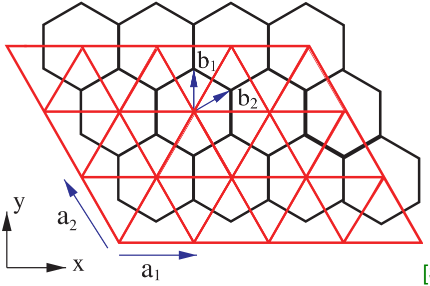
$$\nu \approx 0.917\,.$$

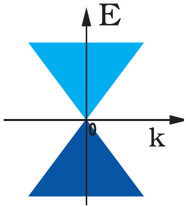
Where to find SUSY?

- @ LHC

Where to find SUSY?

- @ LHC
- As EFT on the surface of topological insulator @ the semimetal-superconductor QCP





[S.-S.Lee'07]

Gauging $U(1)$

What happens to SUSY when there is a dynamical photon?

➡ $\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$:

$$
\begin{aligned}
\mathcal{L} &= i\bar{\psi}\not{D}\psi + |D_\mu\phi|^2 + m^2|\phi|^2 + h(\phi^*\psi^T i\sigma_2\psi + \text{h.c.}) \\
&\quad + \lambda^2|\phi|^4 + \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2\xi}(\partial_\mu A_\mu)^2, \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu,
\end{aligned}
$$

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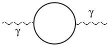
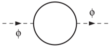
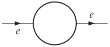
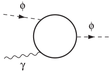
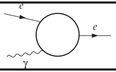
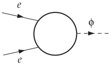
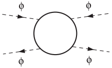
loops	1	2	3
	3	27	502
	4	27	455
	2	17	301
	5	107	3084
	3	69	1996
	3	64	1814
	20	683	26961

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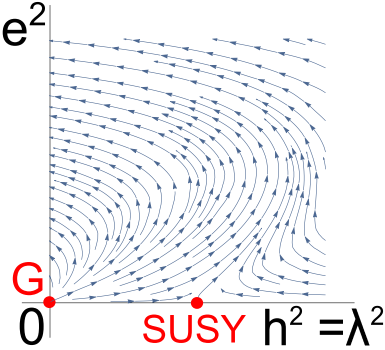


We can do fore...

NEW: 4-loop result for β and γ @ $e=0$ (here for $\lambda = h$):

$$\begin{aligned}\beta_{h^2} = \beta_{\lambda^2} &= -\epsilon\overline{h}^2 + 12\overline{h}^4 - 48\overline{h}^6 + 48\overline{h}^8(5+12\zeta_3) \\ &\quad -192\overline{h}^{10}(9+60\zeta_3 - 18\zeta_4 + 80\zeta_5)\,, \\ \gamma_\psi = \gamma_\phi &= 4\overline{h}^2 - 16\overline{h}^4 + 16\overline{h}^6(5+12\zeta_3) \\ &\quad -64\overline{h}^8(9+60\zeta_3 - 18\zeta_4 + 80\zeta_5)\,.\end{aligned}$$

- $h^2 = \lambda^2$ limit in agreement with Wess-Zumino model [Avdeev,Goroshni‘82]
- Stability?
- $\nu = 0.5 + 0.25\epsilon + 0.042\epsilon^2 + 0.193\epsilon^3 - 0.494\epsilon^4\,.$
- Asymptotic Series: More sophisticated extrapolations to $d=3$ required!
  Padé: $\nu_{[2,3]} \approx 0.918\,$....

Intermission

Upcoming next: Gross-Neveu-Yukawa Model

Where to find QFTs?

In Heidelberg, on the way to lunch @ Marstall with *Michael Scherer*
The Gross-Neveu-Yukawa Model:
"chiral Ising" Universality Class (UC)

$$\mathcal{L} = \bar{\psi}(\partial\!\!\!/ + g\phi)\psi + \frac{1}{2}\phi(m^2 - \partial_\mu^2)\phi + \lambda\phi^4 \,.$$

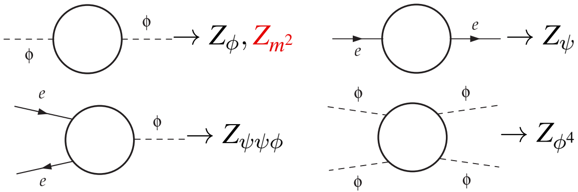
"chiral Heisenberg" UC ($T_F^a = \frac{1}{2}\sigma^a \in su(2)$)

$$\mathcal{L} = \bar{\psi}_\alpha(\partial\!\!\!/ + g[T_F^a]^\alpha_\beta\phi_a)\psi^\beta + \frac{1}{2}\phi_a(m^2 - \partial_\mu^2)\phi_a + \lambda(\phi_a\phi_a)^2 \,.$$

- Lorenzian-symmetry
- ψ: Dirac fermions. Result of lattice structure
- ϕ: real "spin-coupling" scalar-field
- $\partial\!\!\!/ = \gamma^\mu \cdot \partial_\mu$. +Clifford algebra: $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$.
- Chiral symmetry (massless fermions!)

Relevant Diagrams

- Using DREG get poles in small $\epsilon = 4 - d$ of 1-PI n-point functions at $L = 1, 2, 3, \ldots$ loops for “any” kinematics



- Example Diagrams Z_ϕ, Z_{m^2} $(L = 1)$

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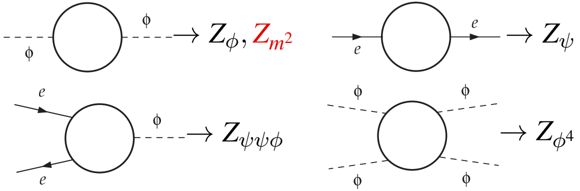

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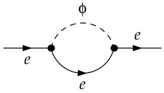

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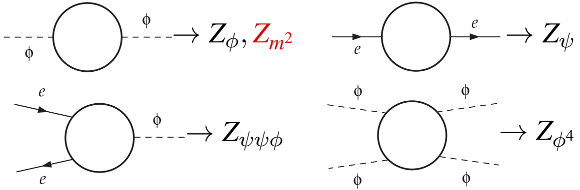
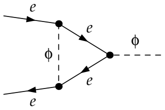
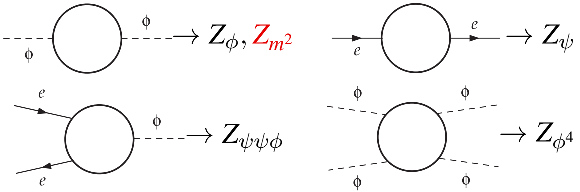


- Example Diagrams $Z_{\psi\psi\phi}$ $(L = 1)$

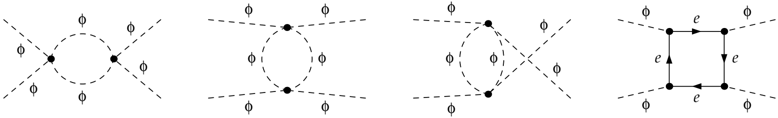


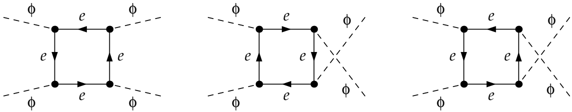
Relevant Diagrams

- Using DREG get poles in small $\epsilon = 4 - d$ of 1-PI n-point functions at $L = 1, 2, 3, \ldots$ loops for "any" kinematics



- Example Diagrams Z_{ϕ^4} $(L = 1)$





$+2$ more...

Relevant Diagrams

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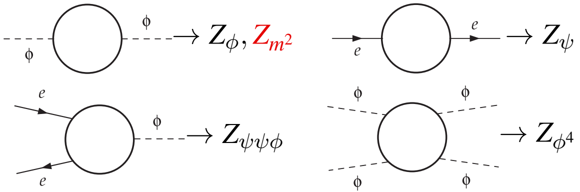
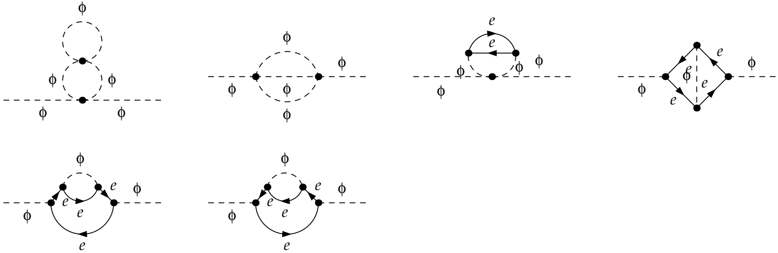


- Example Diagrams Z_ϕ, Z_{m^2} $(L = 2)$



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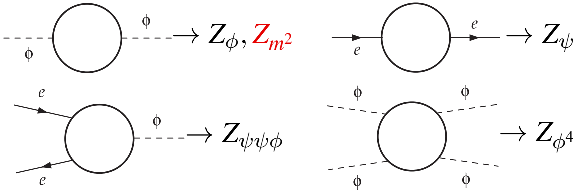
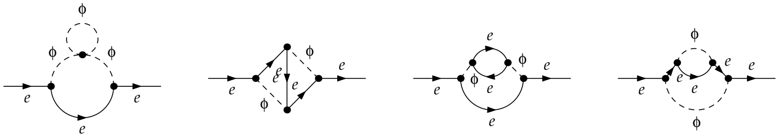


- Example Diagrams Z_ψ $(L = 2)$



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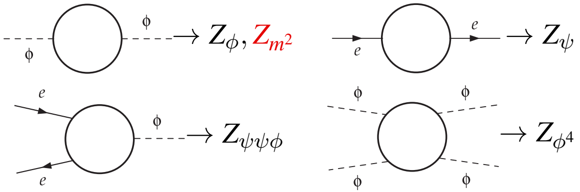

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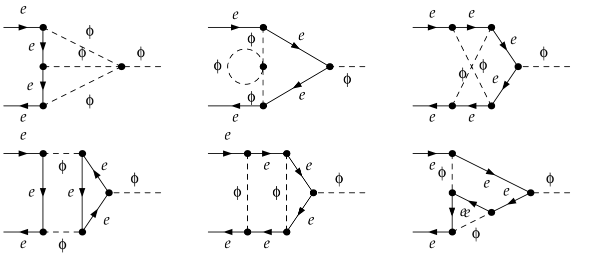

$+4$ more...

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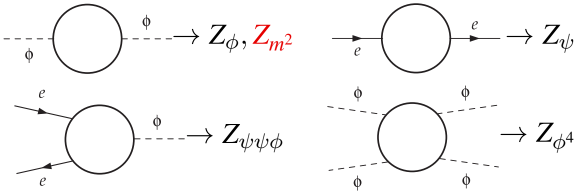

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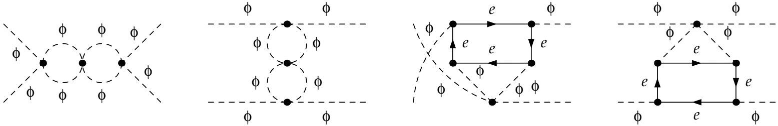

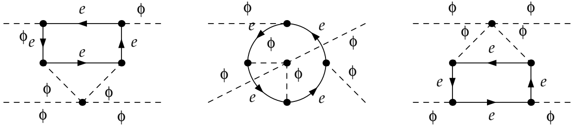

$+86$ more...

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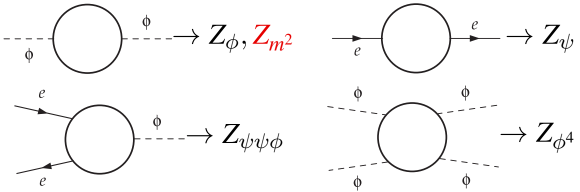

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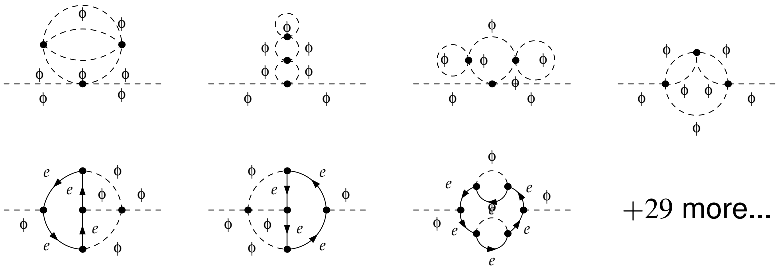

$+29$ more...

Relevant Diagrams

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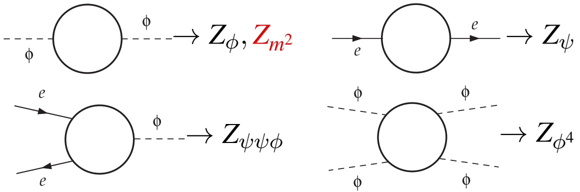
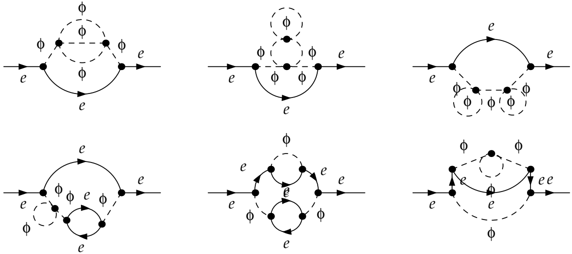


- Example Diagrams Z_ψ ($L = 3$)



$+24$ more...

Relevant Diagrams

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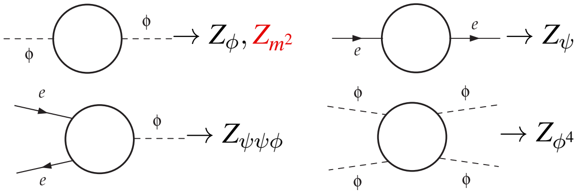

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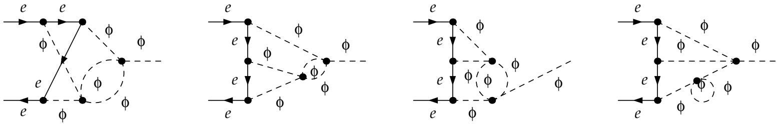

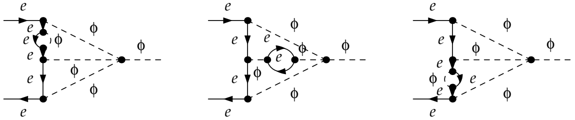

$+138$ more...

Relevant Diagrams

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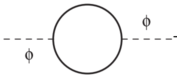
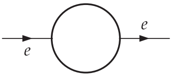
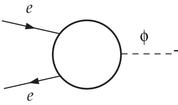
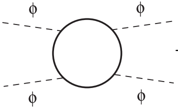

$\rightarrow Z_\phi, Z_{m^2}$


$\rightarrow Z_\psi$


$\rightarrow Z_{\psi\psi\phi}$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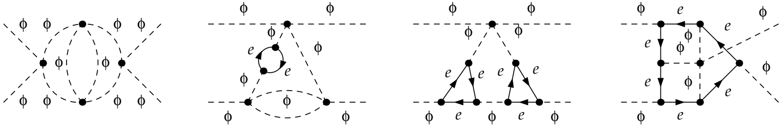
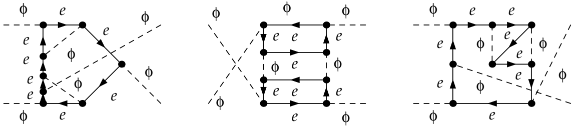
- Example Diagrams Z_{ϕ^4} $(L = 3)$

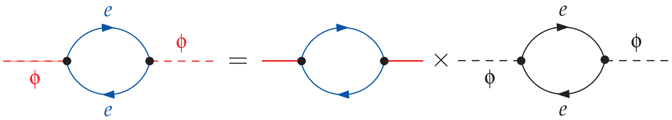




$+1469$ more...

Ising vs Heisenberg

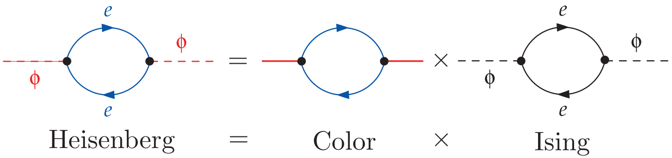
- Heisenberg diagrams factorize:



Heisenberg = Color × Ising

Ising vs Heisenberg

Heisenberg diagrams factorize:



Calculate color amplitudes for generic Lie algebra with *FORM* package *COLOR* [Ritbergen,Schellekens,Vermaseren‘98]

Ising vs Heisenberg

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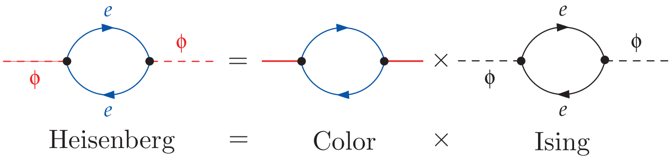
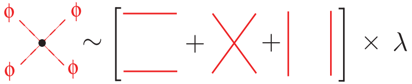


- Calculate color amplitudes for generic Lie algebra with *FORM* package *COLOR* [Ritbergen,Schellekens,Vermaseren'98]
- Color structure of ϕ^4-vertex:

$$\mathcal{L}_{\phi^4} = \lambda(\phi_a\phi_a)^2 = \frac{\lambda}{3}\phi_a\phi_b\phi_c\phi_d(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{cb}) \,.$$



What does *COLOR* do?

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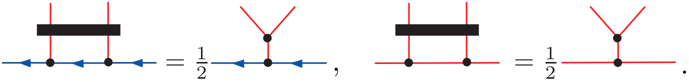
- Exploits the Lie algebra (adjoint rep.: Jacobi ID)



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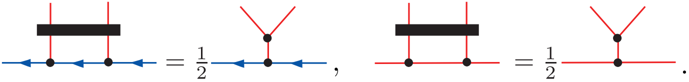
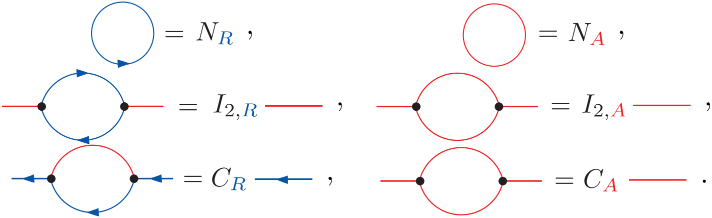


- to express scalar color amplitudes ($\rightarrow$ use projectors) via



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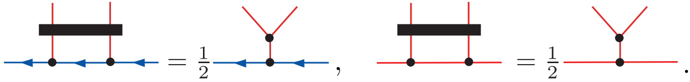

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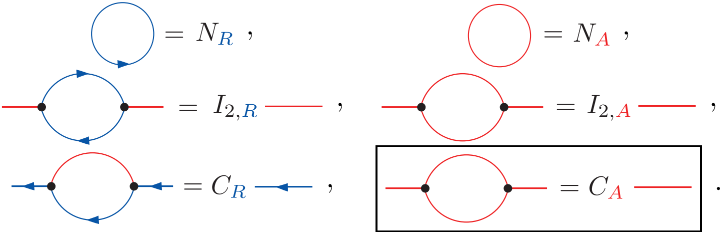

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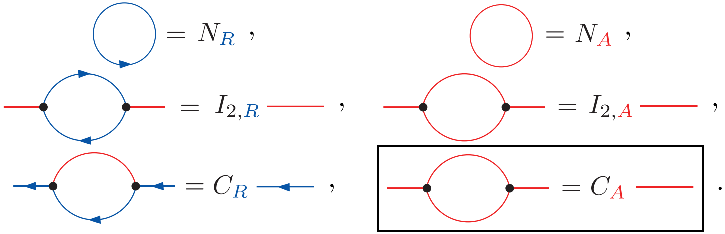
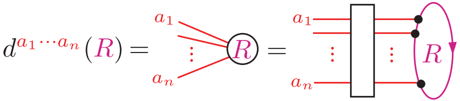
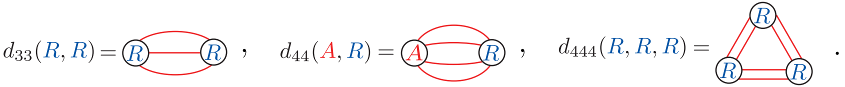


& stuff that is irreducible w.r.t to it

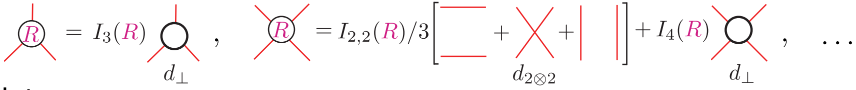


What does *COLOR* do?

- Explicit examples



- It does not use



Note:

$$d_\perp^{aab} = 0\,, \qquad d_\perp^{aabc} = 0\,, \qquad I_3(A) = 0\,.$$

Renormalizability

Is the QFT with

$$\mathcal{L} = \bar{\psi}_\alpha(\partial\!\!\!/ + g[T_R^a]^\alpha_\beta \phi_a)\psi^\beta - \frac{1}{2}\phi_a\partial_\mu^2\phi_a + \lambda(\phi_a\phi_a)^2\,,$$

a renormalizeable theory in $d = 4$ when we assume that T_R obey a finite dimensional simple Lie algebra?

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a renormalizeable theory in $d = 4$ when we assume that T_R obey a finite dimensional simple Lie algebra?

Lie algebra	Betti numbers
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G_2	$2, 6$
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Renormalizability

➡ Just calculate the Zs and see what happens... $Z_x = 1 + \sum_{n=1}^{\infty} \delta Z_x^{(n)}$

- ✓ 1-loop:

$$\delta Z_x^{(1)} = \sum_{i+j=1} \left(\frac{g^2}{(4\pi)^2} \right)^i \left(\frac{\lambda}{(4\pi)^2} \right)^j \frac{1}{\epsilon} N_{x,i,j}^{(1,1)} , \qquad N \in \mathbb{R} .$$

Renormalizability

- Just calculate the Zs and see what happens... $Z_x = 1 + \sum_{n=1}^{\infty} \delta Z_x^{(n)}$
  - 2-loop:

$$\delta Z_x^{(2)} = \sum_{i+j=2} \left(\frac{g^2}{(4\pi)^2} \right)^i \left(\frac{\lambda}{(4\pi)^2} \right)^j \left(\frac{1}{\epsilon} N_{x,i,j}^{(2,1)} + \frac{1}{\epsilon^2} N_{x,i,j}^{(2,2)} \right) , N \in \mathbb{R}$$

Renormalizability

Just calculate the Zs and see what happens... $Z_x = 1 + \sum_{n=1}^{\infty} \delta Z_x^{(n)}$

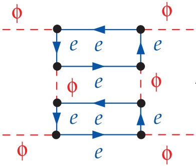
3-loop:

$$\delta Z_{\phi^4}^{(3)} = \frac{1}{\epsilon}\left(\frac{g^2}{(4\pi)^2}\right)^4 \left(\frac{\lambda}{(4\pi)^2}\right)^{-1} N_{\phi^4,4,-1}^{(3,1)} + \cdots ,$$
$$N_{\phi^4,4,-1}^{(3,1)} = N \cdot \log(\mu^2/M^2) + \ldots .$$

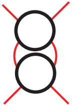
$\rightarrow$ Renormalization constant Z_{ϕ^4} is non-local
$\rightarrow$ Theory is not renormalizeable for a generic simple Lie algebra

Renormalizability

- Origin of the $N^{(3,1)}_{\phi^4,4,-2} + \sim d_{44}(R,R)\log(\mu^2/M^2)$ terms @ 3-loop:



$$+\cdots \sim$$



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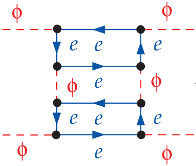
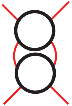
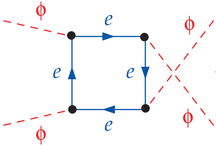


$+\cdots\sim$

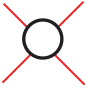


$+\ldots$

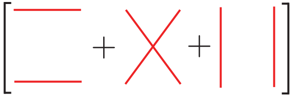
- Already at 1-loop color structure $\sim d_\perp^{abcd}$ divergent:



$+\cdots\sim x_1$



$+x_2$



Renormalizability

- For simple Lie algebras which can make up a fully symmetric $d_{\perp}^{abcd}$ the theory is not renormalizeable

Renormalizability

➡ For simple Lie algebras which can make up a fully symmetric $d_\perp^{abcd}$ the theory is not renormalizeable

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✓ For all exceptional algebras the theory is renormalizeable

Renormalizability

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- For all exceptional algebras the theory is renormalizeable
- For $SU(N), Sp(N), SO(N)$ with $N < 4, 4, 5$ the theory is renormalizeable. "The number of independent fully symmetric tensors is $\propto$ the rank of a simple Lie algebra"

Renormalizability

- Can we fix Renormalizability for $SU(4+), Sp(4+), SO(5+)$?

Renormalizability

- Can we fix Renormalizability for $SU(4+), Sp(4+), SO(5+)$?
- Yes, we can!

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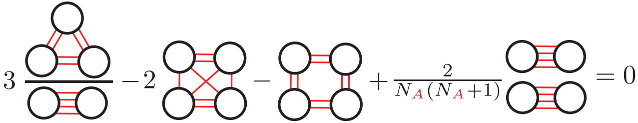
  - 2-loop: Z_{ϕ^4} Non-trivial $\log(\mu^2/M^2)$ cancellation



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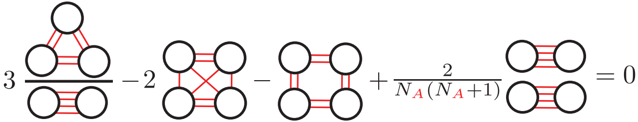

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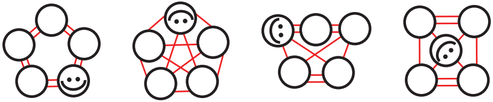

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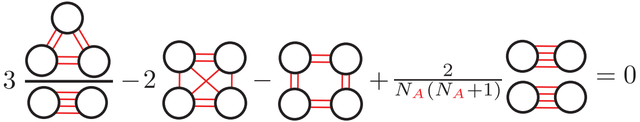

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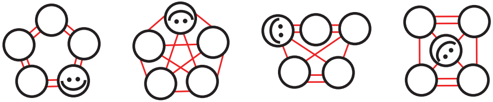


- Condensed matter physics: $T_R \in SU(2)$.

β for Ising

Number of Fermion flavours $= N$, $\frac{g^2}{8\pi^2} \to y$, $\frac{\lambda}{8\pi^2} \to \lambda$

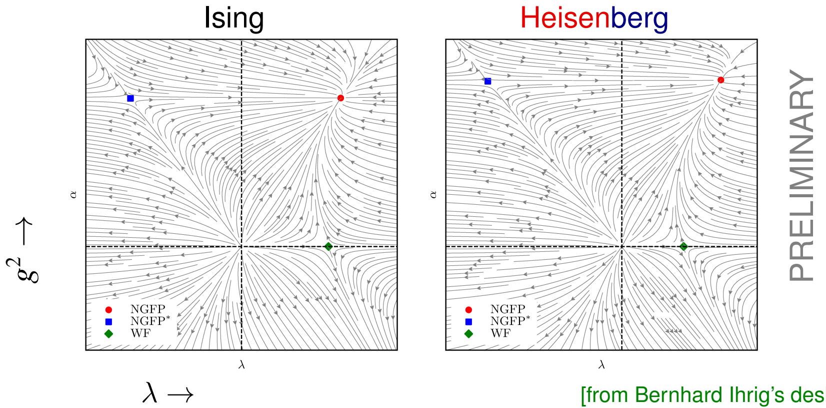
$$\begin{aligned}\beta_y = & -\epsilon y + (3+2N)y^2 + 24y\lambda(\lambda - y) - (\frac{9}{8} + 6N)y^3 \\ & + \frac{y}{64}\Big(1152(7+5N)y^2\lambda + 192(91-30N)y\lambda^2 \\ & + \big(912\zeta_3 - 697 + 2N(67 + 112N + 432\zeta_3)\big)y^3 \\ & - 13824\lambda^3\Big),\end{aligned}$$

$$\begin{aligned}\beta_\lambda = & -\epsilon\lambda + 36\lambda^2 + 4Ny\lambda - Ny^2 + 4Ny^3 + 7Ny^2\lambda \\ & -72Ny\lambda^2 - 816\lambda^3 + \frac{1}{32}\Big(6912(145+96\zeta_3)\lambda^4 \\ & + 49536Ny\lambda^3 - 48N(72N - 361 - 648\zeta_3)y^2\lambda^2 \\ & + 2N(1736N - 4395 - 1872\zeta_3)y^3\lambda \\ & + N(5 - 628N - 384\zeta_3)y^4\Big),\end{aligned}$$

✓ 2-loop results [Rosenstein,Kovner,Yu'93]

Fixed Point Analysis

- Number of fermion generations $N = 2$:



[from Bernhard Ihrig's desk]

Critical Exponents (Chiral Ising)

			$\eta_x = \gamma_x(y_*, \lambda_*)$
$N = 2:$	ν^{-1}	$\approx$	$2 - 0.952\epsilon + 0.00723\epsilon^2 - 0.0949\epsilon^3$
	η_ψ	$\approx$	$0.0714\epsilon - 0.00671\epsilon^2 - 0.0243\epsilon^3$
	η_ϕ	$\approx$	$0.571\epsilon + 0.124\epsilon^2 - 0.0278\epsilon^3$
$N = 1:$	ν^{-1}	$\approx$	$2 - 0.835\epsilon - 0.00571\epsilon^2 - 0.0603\epsilon^3$
	η_ψ	$\approx$	$0.1\epsilon + 0.0102\epsilon^2 - 0.033\epsilon^3$
	η_ϕ	$\approx$	$0.4\epsilon + 0.102\epsilon^2 - 0.0632\epsilon^3$
$N = 1/4:$	ν^{-1}	$\approx$	$2 - 0.571\epsilon - 0.0204\epsilon^2 + 0.024\epsilon^3$
	η_ψ	$\approx$	$0.143\epsilon + 0.0408\epsilon^2 - 0.048\epsilon^3$
	η_ϕ	$\approx$	$0.143\epsilon + 0.0408\epsilon^2 - 0.048\epsilon^3$
$N = 0:$	ν^{-1}	$\approx$	$2 - 0.333\epsilon - 0.117\epsilon^2 + 0.125\epsilon^3$
	η_ψ	$\approx$	$0.167\epsilon + 0.0478\epsilon^2 - 0.0469\epsilon^3$
	η_ϕ	$\approx$	$0.0185\epsilon^2 + 0.0187\epsilon^3$

Critical Exponents (Chiral Ising)

$N = 2$	$1/\nu$	η_ϕ	η_ψ
this work (Padé $[2/1]$)	1.048	0.672	0.0740
$(2+\epsilon)$, (ϵ^4, Padé)[Gracey,Luthe,Schroder'16]	0.931	0.745	0.082
functional RG[Knorr'16]	0.994(2)	0.7765	0.0276
Monte Carlo[Chandrasekharan,Li'13]	1.20(1)	0.62(1)	0.38(1)
$N = 1$	$1/\nu$	η_ϕ	η_ψ
this work (Padé $[2/1]$)	1.166	0.463	0.102
functional RG[Knorr'16]	1.075(4)	0.5506	0.0645
Monte Carlo[Li,Jiang, Yao'15]	1.30	0.45(3)	
$N = 1/4$	$1/\nu$	η_ϕ	η_ψ
this work (Padé $[2/1]$)	1.419	0.162	0.162
functional RG[Heilmann,ea'15]	1.408	0.180	0.180
conformal bootstrap[Iliesiu,ea'16]		0.164	0.164

Summary & Outlook

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- QFTs are ETs
- If SUSY is not found at LHC one may take a closer look at condensed matter system "on a desk"
- Kinematics $\leftrightarrow$ Symmetry
- Deep understanding of symmetries is required

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Outlook

- Heisenberg
- More sophisticated RG analysis (large order behavior?)
- Z_{m^2} for GNY at 4-loop
- XY-Model

To backup slides $\rightarrow$

Superspace

$$d^2\theta \equiv -\frac{1}{4} d\theta^\alpha d\theta^\beta \varepsilon_{\alpha\beta},$$
$$d^2\bar{\theta} \equiv -\frac{1}{4} d\bar{\theta}^\alpha d\bar{\theta}^\beta \varepsilon_{\alpha\beta},$$
$$\theta^2 \equiv \theta^\alpha \theta_\alpha = \theta^\alpha \varepsilon_{\alpha\beta} \theta^\beta,$$

$$\int d^2\theta d^2\bar{\theta}\, \Phi^\dagger \Phi = i\bar{\psi}\partial\!\!\!/ \psi + |\partial_\mu \phi|^2 + |F|^2. \qquad \textit{kinetic terms}$$

$$\int d^2\theta\, \frac{h}{3} \Phi^3 = h\phi^2 F + h\phi\psi^T i\sigma_2 \psi. \qquad \textit{super potential terms}$$

Auxiliary fields are eliminated using EQM: $F = -h\phi^{*2}$, $F^* = -h\phi^2$.