Towards four-loop renormalization of Standard Model and related theories

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Overview

Two sides of renormalization group

1. Renormalization group used to derive evolution equation
   - Need initial values for evolution
   - Small parameters are coupling constants

2. Renormalization group used to predict critical exponents near critical points
   - Fixed point determined from RG functions
   - Small parameter is ϵ, deviation from original space time dimension
   - Results need to be continued to $\epsilon = 1$

Such different problems, but can be attacked with the same technique

Motivation: Towards extremely high energies in the SM

- The SM being renormalizable can, in principle, be used to make predictions at scales $Q^2 \gg M_Z$, not accessible to colliders
- At such scales it is convenient to use running couplings $a(Q)$, which are obtained from a set of measured quantities $\{O\}$ by means of

1. Matching

- Relate running couplings $a(\mu \simeq M_Z)$ and $\{O\}$
- Initial values for evolution

2. Renormgroup

- Evolution of all SM couplings
- In $\overline{\text{MS}}$ scheme, β-functions are simple and polynomial in $a(\mu)$

PDG 20XX		Fixed μ_0		
$\{O\} = M_b, M_W, M_Z, M_H, M_t, G_F$	$\longrightarrow$	$g_i(\mu_0), y_i(\mu_0), \lambda(\mu_0)$ in $\overline{MS}$ scheme	$\longrightarrow$	Evolve from μ_0 to scale μ

Motivation: SM vacuum stability analysis at NNLO

- Three loop beta-functions for gauge Yukawa and self-coupling

[Mihaila,Salomon,Steinhauser'12;Bednyakov,AFP,Velizhanin'12,13;Chetyrkin,Zoller'13]

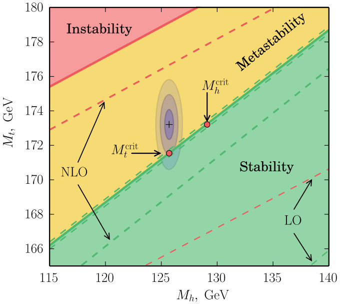
- Two loop full $\mathcal{O}(\alpha^2)$ threshold corrections

[Buttazzo et al.'13;Kniehl,AFP,Veretin'15]

$$V_{\text{eff}}(\phi \gg v) \simeq \frac{\lambda(\mu = \phi)}{4}\phi^4$$

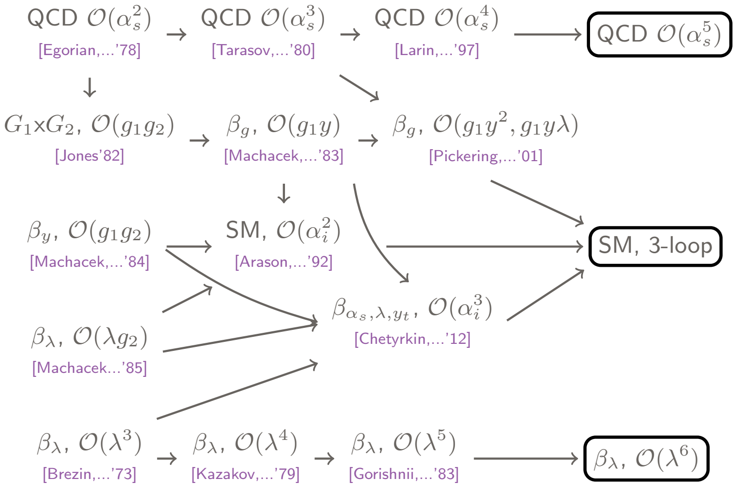
$$(4\pi)^2 \frac{d\lambda}{d\ln\mu^2} = 12\lambda + 6y_t^2\lambda - 3y_t^4 + \ldots$$

$$(4\pi)^2 \frac{dy_t}{d\ln\mu^2} = \frac{9}{4}y_t^3 - 4{g_s}^2 y_t + + \ldots$$



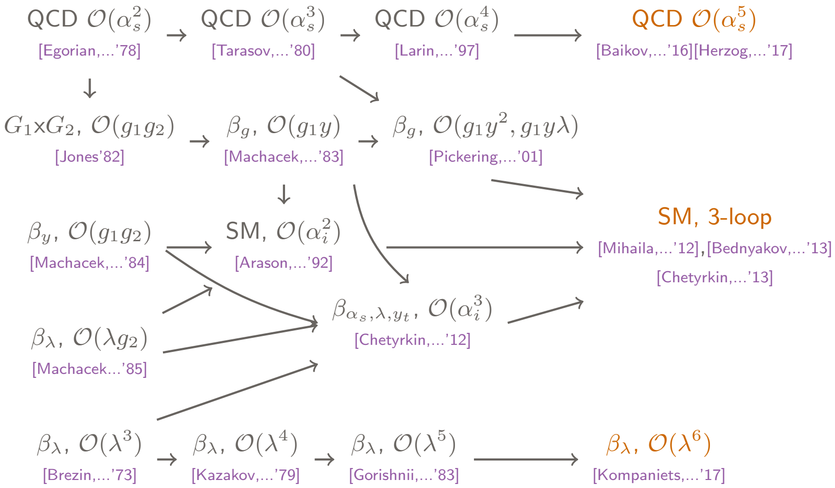
Main steps of beta-functions evaluation

Few years ago:



Main steps of beta-functions evaluation

Today:



Next step: calculation of four-loop SM beta-functions

What is known at four-loops?

- Strong coupling beta-function in the SM gaugeless limit — This talk

  [Bednyakov,AFP'15][Zoller'15]
- Leading QCD corrections to β_λ

  [Martin'15][Chetyrkin,Zoller'16]
- Generalization of QCD results on reducible fermion representaions

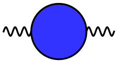
  [Chetyrkin,Zoller'17]
- Pure QCD corrections to β_y and Higgs field anomalous dimension

  [Larin,. . . '97][Chetyrkin'97][Chetyrkin'96]

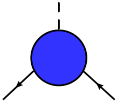
Are these corrections dominating?

Standard model: from Green functions to anomalous dimensions

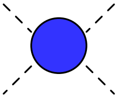
- Gauge couplings g_1, g_2, g_s [Mihaila,Salomon,Steinhauser'12] [Bednyakov,AP,Velizhanin'12]



- Yukawa couplings y_t, y_b, y_τ [Bednyakov,AP,Velizhanin'12]

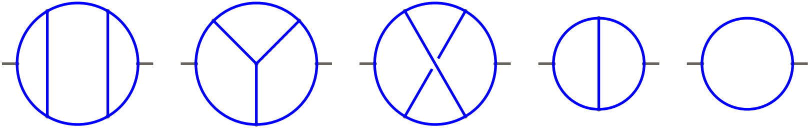


- Scalar potential parameters λ, m^2 [Chetyrkin,Zoller'13] [Bednyakov,AP,Velizhanin'13]



- Renormalization of all external legs

MINCER and efficient evaluation of two-point functions



1. Only two nontrivial master integrals:

$$\mathsf{NO}(1,1,1,1,1,1,1,1), \quad \mathsf{T1}(1,1,1,1,1+\epsilon)$$

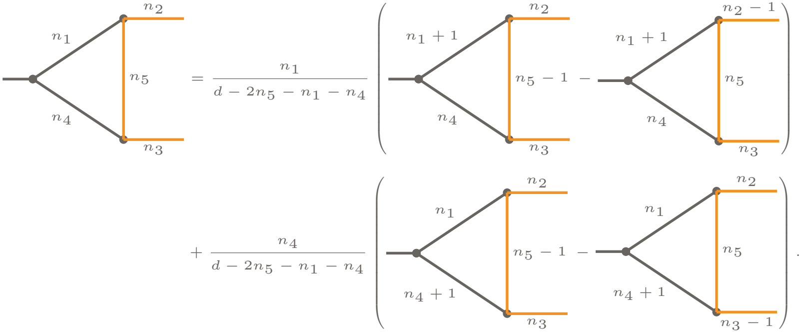
2. All other expressible through gamma functions:

$$G(1,1), \quad G(1,1+\epsilon), \quad G(1+\epsilon,1+\epsilon), \quad G(1,1+2\epsilon)$$

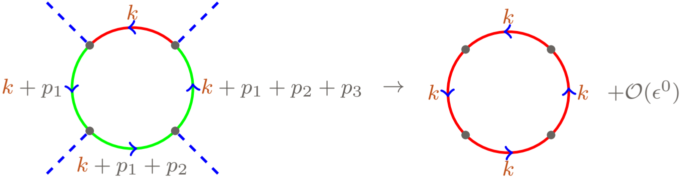
3. Possible to calculate higher moments of structure functions
4. Allow to keep full dependence on gauge fixing parameter ξ in gauge boson propagators
5. High speed due to local reduction rules

Triangle rule

$$T_1(n_1,\ldots,n_5) = \frac{\left[n_1(\mathbf{5}^- - \mathbf{2}^-)\mathbf{1}^+ + n_4(\mathbf{5}^- - \mathbf{3}^-)\mathbf{4}^+\right]}{d - 2n_5 - n_1 - n_4} T_1(n_1,\ldots,n_5)$$



Four-loop RGE from fully massive tadpoles



$$\underbrace{\frac{1}{(k+p)^2-M^2}}_{\omega=-2} = \underbrace{\frac{1}{k^2-m_A^2}}_{\omega=-2} + \underbrace{\frac{M^2-p^2-2kp-m_A^2}{k^2-m_A^2}\frac{1}{(k+p)^2-M^2}}_{\omega=-3}$$

- Four-loop QCD beta-function [Ritbergen,Vermaseren,Larin'97][Czakon'04]
- Anom. dim. of twist-2 opertors in QCD and N=4 SYM [Velizhanin'14]
- Renormalization of QCD with extended fermion sector [Chetyrkin,Zoller'17]

Gluon field renormalization constant Z_3

Arise non transverse part Π_M allowing to extract δZ_m

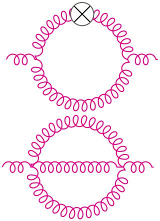
$$\Pi_{\mu\nu}(q^2) = q^2\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\Pi_T(q^2) + g_{\mu\nu} m_A^2 \Pi_M(q^2)$$

In one-loop:

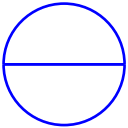
$$\Pi_M^{(1)}(q^2) = \frac{q^\mu q^\nu}{q^2 m_A^2}\Pi_{\mu\nu}^{(1)} = a_s\left(4T_F n_f + 4C_A - \frac{9}{4}\xi C_A\right) = \delta Z_m^{(1)}$$

$$Z_3 = 1 - \mathcal{K}R'\Pi_T(q^2) = Z_3 + \frac{a_s}{\epsilon}\delta Z_m^{(1)} C_A\left(\frac{11}{6} + \frac{1}{2}\xi\right)$$

$$+ \frac{a_s^2}{\epsilon}\left(-\frac{22}{3}T_F n_f C_A - 2T_F n_f C_A \xi - \frac{22}{3}C_A^2 + \frac{17}{8}C_A^2 \xi\right)$$



What structures in anomalous dimensions and tadpole integrals?

 $= \cdots + \text{D6} + \mathcal{O}(\epsilon)$,  $= \cdots + \frac{27}{2}\text{S2} + \epsilon\text{T1ep} + \mathcal{O}(\epsilon^2)$

$$\text{S2} = \frac{4}{9\sqrt{3}}\text{Cl}_2\left(\frac{\pi}{3}\right)$$

$$\text{T1ep} = -\frac{45}{2} - \frac{\pi\sqrt{3}\log^2 3}{8} - \frac{35\pi^3\sqrt{3}}{216} - \frac{9}{2}\zeta_2 + \zeta_3 + 6\sqrt{3}\text{Cl}_2\left(\frac{\pi}{3}\right) - 6\sqrt{3}\text{Im}\left(\text{Li}_3\left(\frac{e^{-i\frac{\pi}{6}}}{\sqrt{3}}\right)\right)$$

$$\text{D6} = 6\zeta_3 - 17\zeta_4 - 4\zeta_2\log^2 2 + \frac{2}{3}\log^4 2 + 16\text{Li}_4\left(\frac{1}{2}\right) - 4\left(\text{Cl}_2\left(\frac{\pi}{3}\right)\right)^2$$

- QCD, 3-loop: $\gamma_1, \gamma_2, \gamma_3, \gamma_3^c \sim \zeta_3 \quad \beta_g \sim \mathbb{Q}$ [Tarasov,Vladimirov,Zharkov'1980]
- QCD, 4-loop: $\gamma_1, \gamma_2, \gamma_3, \gamma_3^c \sim \zeta_3, \zeta_4, \zeta_5 \quad \beta_g \sim \zeta_3$ [Czakon'2004]

All three-loop tadpoles up to weight six expressed in terms of HPL with single argument $e^{\frac{i\pi}{3}}$ [Kniehl,AFP,Veretin'17]

Strong coupling — matching and running: known results

- Running in $\overline{\mathrm{MS}}$
  - 1-loop [Gross,Wilczek'73, Politzer'73]
  - 2-loop [Jones'74,Egorian,...'78] (QCD), [Machachek,Vaughn'83] (SM)
  - 3-loop [Tarasov,...'80] (QCD), [Mihaila,...'12,Bednyakov,...'12] (SM)
  - 4-loop [van Ritbergen,Vermaseren,Larin'97,Czakon'04] (QCD)
  - 5-loop [Baikov,...'16,Luthe,...'16,Herzog,...'17] (QCD)
- Matching in $\overline{\mathrm{MS}}$: we "match" effective five-flavor ($n_f = 5$) QCD and a more fundamental theory (usually, QCD with top quark $n_f = 6$)

$$\alpha_s^{(5)}(\mu) = \alpha_s(\mu)\xi_{\alpha_s}(\mu, M)$$

"integrate out" heavy fields with mass M

  - 1/2-loop [Bernreuther,Wetzel'81-83], [Bednyakov'15] (2-loop SM)
  - 3-loop [Chetyrkin,Kniehl,Steinhauser'97-98]
  - 4-loop [Schroder,...'05,Chetyrkin,...'05,Kniehl,...'06]

NB: L-loop running + (L-1)-loop matching are self-consistent

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NB: L-loop running + (L-1)-loop matching are self-consistent

- **Starting point:** Limit of vanishing $SU(2) \times U(1)$ gauge couplings
  Only the following SM parameters are considered

$$a_i = \left(\frac{g_s^2}{16\pi^2}, \frac{y_t^2}{16\pi^2}, \frac{\lambda}{16\pi^2}, \xi_G \right)$$

  Significantly reduce number of diagrams!
  NB: we keep track of gauge-parameter dependence!

- Easier to track γ_5 in dimensional regularization (see below):
  - no γ_5 in gauge vertices!
  - γ_5 appears only in (pseudo/charged) scalar couplings.

- In what follows (h counts powers of a_i)

$$\frac{d\ a_s}{d\ \log \mu^2} = \beta_{a_s} = - \sum_{i=0}^{3} \beta_i h^{i+2}$$

Further simplifications: background field gauge

- Split gauge fields $V = \tilde{V} + \hat{V}$ in
  - quantum $\tilde{V} = (\tilde{G}, \tilde{W}, \tilde{B}, \dots)$ and
  - background $\hat{V} = (\hat{G}, \hat{W}, \hat{B}, \dots)$
- Background fields do not propagate
- Modified Feynman rules [Abbot'80]
- QED-like connection between renormalization constants

$$Z_{a_s} = 1/Z_{\hat{G}}, \qquad Z_{\xi_G} = Z_{\tilde{G}}$$

- Only two-point functions are required (given all 3-loop Z-factors)
- Multiplicative renormalization by $a_{\rm bare} = Z_a a_{\rm ren}$

$$\Gamma^{(l)}_{\rm ren} = Z^{(l)}_{\Gamma} \left[1 + \Gamma^{(1)}_{\rm bare}(a_{\rm bare}) + \Gamma^{(2)}_{\rm bare}(a_{\rm bare}) + \dots + \Gamma^{(l)}_{\rm bare}(a_{\rm bare})\right]$$

- No need to apply IRR trick, have access to finite parts

RGE from propagator-type integrals

- Three-loop experience (different approaches to IRR [Vladimirov'78])
  - Massless propagators: gauge couplings, field renormalization constants
    using FORM based MINCER package
    NB: finite parts are available (in massless/unbroken theory)
  - Three-loop massive bubbles: Yukawa and Higgs self-coupling
    using FORM based MATAD package
    NB: aux mass renormalization
- Four-loop experience
  - QCD beta function
    massive vacuum integrals available, possible to calculate all types of renormalization constants.
  - Massless propagators. Difficult to prepare reduction. Easy to formulate the problem
    Independent tool for two-point Green functions renormalization constants calculation

Setup v.1

- Model file of the SM in BFG tested at lower loop calculations
- `DIANA/QGRAF` [Nogueira'93;Fleischer,Tentyukov'99]
  Diagram generation
- Prepared set of mappings to 3 auxiliary topos
  each topo with 11 denominators and 3 irreducible numerators
- Reduction
  - `LiteRed`
    IBP rules preparation [Lee'12]
  - `FIRE5, C++` version
    Integral reduction [Smirnov'14]
  - Master integrals
    4-loop propagators [Baikov,Chetyrkin'10;Lee,Smirnovs'11]

Setup v.2: improved reduction

We have four independent ways for reduction:

1. Reduction of massless propagators
   - `LiteRed`
     IBP rules preparation [Lee'12]
   - `FIRE5`, C++ version
     Integral reduction [Smirnov'14]
   - Master integrals
     4-loop propagators [Baikov,Chetyrkin'10;Lee,Smirnovs'11]
2. Reduction of massles propagators with `FORM` package `FORCER`
   Parametric integral reduction [Ruijl,Ueda,Vermaseren'17]
3. Reduction of fully massive tadpoles
   - `LiteRed`
     IBP rules preparation [Lee'12]
   - `FIRE5`, C++ version
     Integral reduction [Smirnov'14]
   - Master integrals
     4-loop fully massive tadpoles [Czakon'04]
4. Reduction of four-loop tadpoles with new `FORM` code `FMFT`
   details below [AFP'17]

4 loop QCD β-function and renormalization constants

- IRR with auxiliary mass
  fully massive four-loop tadpoles, possible to calculate all renormalization constants
  - From Z_g, Z_c, Z_{ccg} [Larin,Vermaseren,Ritbergen'97]

$$Z_{a_g} = \frac{Z_{ccg}^2}{Z_c^2 Z_g}$$

  - From Z_g, Z_q, Z_{qqg} [Czakon'04]

$$Z_{a_g} = \frac{Z_{qqg}^2}{Z_q^2 Z_g}$$

- Using 3-loop massless integrals [Chetyrkin'04]
  - From Z_c, Z_{ccg} and already known β_{a_g}
    Impossible to calculate Z_g, but independent calculation of other RCs

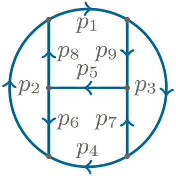
$$Z_g = \frac{Z_{ccg}^2}{Z_c^2 Z_{a_g}}$$

- Using 4-loop massless propagator type integrals and BFG

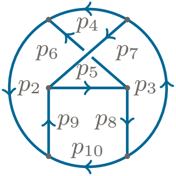
$$Z_{a_s} = 1/Z_{\hat{G}}, \qquad Z_{\xi_G} = Z_{\tilde{G}}$$

FMFT: four-loop tadpoles reduction

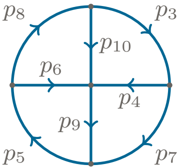
- Topolgies **H**,**X**,**BMW** need manual reduction rules



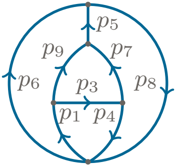
H



X



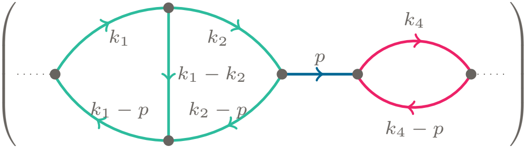
BMW



FG

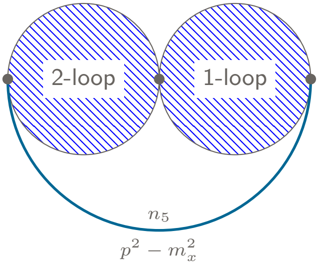
- Topolgy **FG** and all its subtopologies can be expressed as convolution

$$J_{\text{FG}} = \int d[p]$$



- One-loop and two-loop propagator type integrals with massive lines can be reduced separately

After reduction of all scalar one and two-loop subintegrals we can express all resulting integrals in form:



Where all 1-loop and 2-loop integrals are from the following set:

$$\begin{bmatrix} \texttt{F[11111]}, \texttt{V[1111]}, \texttt{J[211]}, \texttt{J[111]}, \texttt{T2[111]}, \\ \texttt{G[11]G[11]}, \texttt{G[11]T1[1]}, \texttt{T1[1]T1[1]} \end{bmatrix} \otimes \begin{bmatrix} \texttt{G[11]} \\ \texttt{T1[1]} \end{bmatrix}$$

All indices are fixed all masses are equal to m^2 and only missing part is reduction of line with index n_5 and arbitrary mass m_x

Difference equations from differential equations

We start from system of differential equations:

$$\frac{\partial J_i}{\partial m_x^2} = A_{ik} J_k, \quad \text{solution:} \quad J_i = \sum_{n=0}^{\infty} c_{i,n} z^n$$

To reduce diagrams with denominator $z = \frac{m_x^2 - m_i^2}{m^2}$:

$$\frac{1}{p^2 - m_x^2} = \frac{1}{p^2 - m_i^2} + \frac{1}{\left(p^2 - m_i^2\right)^2} \left(m_x^2 - m_i^2\right) + \frac{1}{\left(p^2 - m_i^2\right)^3} \left(m_x^2 - m_i^2\right)^2 + \ldots$$

To reduce diagrams with numerator $z = \frac{m^2}{m_x^2}$:

$$\frac{1}{p^2 - m_x^2} = -\left(\frac{1}{m_x^2}\right) - p^2 \left(\frac{1}{m_x^2}\right)^2 - p^4 \left(\frac{1}{m_x^2}\right)^3 + \ldots$$

Comparing terms with equal powers of z we can relate integrals with n-th power of denominator with n-th term of expansion

FMFT: topology FG and performance

- Using dimensional shifts we can reduce numerators of both one- and two-loop subintegrals [Tarasov,97]
- Remaining integrals with arbitrary power of denominator with momenta p reduced using one dimensional recurence relations

Available for download:

`http://git.io/fmft`

Nonplanar integral $X(-n,1,1,1,1,1,1,1,1,1)$ with numerator

$n =$	3	4	5	6	7	8
FMFT	0:00:11	0:00:27	0:01:55	0:07:35	0:25:31	01:30:31
FIRE	0:01:58	0:09:10	0:28:17	2:16:42	9:19:57	46:42:29

Time format **hh:mm:ss**, FIRE used with LiteRed rules

Renormalization constants in gaugeless limit of SM

- γ_{H} - Higgs field anomalous dimension from Z_{H}
- β_λ - Higgs self-coupling beta-function from Z_λ

$$Z_\lambda = \frac{Z_{\mathrm{HHHH}}}{Z_{\mathrm{H}}^2}$$

- β_{y_t} - top Yukawa coupling beta-function from Z_{y_t}

$$Z_{y_t} = \frac{Z_{\mathrm{uuH}}}{\sqrt{Z_{\mathrm{u,L}} Z_{\mathrm{u,R}} Z_{\mathrm{H}}}}$$

- β_{m^2} - Higgs mass parameter beta-function from Z_{m^2}

$$Z_{m^2} = \frac{Z_{\mathrm{HH[HH]}}}{Z_{\mathrm{H}}}$$

- Green functions calculated using massless propagators:

$$\Gamma_{\mathrm{HH}}, \Gamma_{\mathrm{uu}}, \Gamma_{\mathrm{uuH}}$$

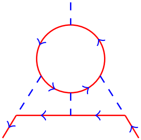
- Green functions calculated using massive tadpoles:

$$\Gamma_{\mathrm{HH}}, \Gamma_{\mathrm{uu}}, \Gamma_{\mathrm{uuH}}, \Gamma_{\mathrm{HHHH}}, \Gamma_{\mathrm{HH[HH]}}$$

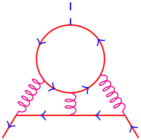
Traces at three-loop level

For γ_5 in dimensional regularization we use naive treatment, except diagrams with two fermionic traces with four contracted indices

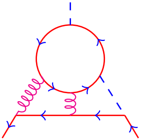
$$\mathrm{tr}[\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5]\,\mathrm{tr}[\gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta\gamma_5]$$



$\sim \mathrm{tr}[\gamma^\alpha\gamma^\beta\gamma^5]$



$2n+1$
γ matrices



$\sim \epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\delta}$

- Substitution is correct for $D = 4$

$$\mathrm{tr}[\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5] = -4i\epsilon_{\mu\nu\rho\sigma}$$
$$\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\delta} = -\mathcal{T}^{[\mu\nu\rho\sigma]}_{[\alpha\beta\gamma\delta]}, \qquad \mathcal{T}^{\mu\nu\rho\sigma}_{\alpha\beta\gamma\delta} = \delta^\mu_\alpha\delta^\nu_\beta\delta^\rho_\gamma\delta^\sigma_\delta\,,$$

- Apply in $D = 4 - 2\epsilon$, but for diagrams w/o subdivergencies

Number of diagrams up to four loops

- Total number of diagrams and additional diagrams with $\varepsilon \otimes \varepsilon$ contraction

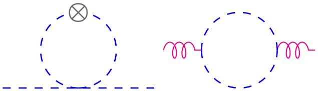
	1	2	$3/\gamma_5$	$4/\gamma_5$
$\Gamma_{\hat{g}\hat{g}}$	4	43	867	25374/72
Γ_{uu}	4	39	920	30035/94
Γ_{HH}	1	14	276	8822/18
Γ_{uuH}	3	102	4030/18	185981/2048
Γ_{HHHH}	5	47	1307	46536/74
$\Gamma_{\mathrm{HH[HH]}}$	3	27	616	23044/18

- Number of diagrams for Γ_{HHHH} can be reduced due to graph symmetries using `GraphState` package. Original numbers:

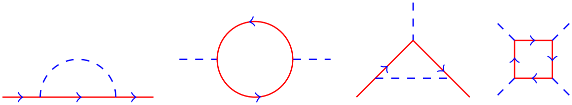
	1	2	3	4
Γ_{HHHH}	15	327	13212	685599

Uncertainty in contributions from lower loops

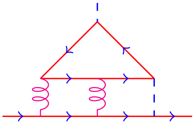
- No contribution from one-loop diagrams



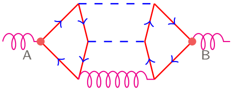
- Determinent contribution from one-loop renormalization (Z_{y_t} insertion)



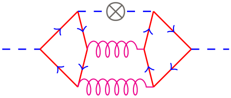
- Undetermined contribution from three-loop renormalization (Z_g, Z_{y_t} insertion)



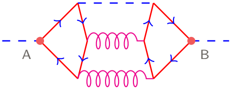
Four-loop diagrams and reading points



(**A** and **B**) or (**A** and not **B**) or (not **A** and not **B**)



fixed



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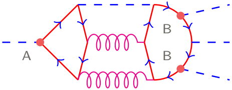


(**A** and **B**) or (**A** and not **B**) or
(not **A** and **B**) or (not **A** and not **B**)

For quark propagator and `uuH` vertex situation is more complicated

Keeping trace of uncertainties

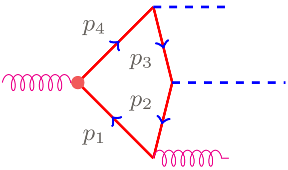
Two generalizations of trace taking procedures:

1. Use tensor reduction and move trace outside the integral

$$\int d[p_{1\dots n}]\,\mathrm{tr}[\not{p}_1 \dots \not{p}_k] \to \mathrm{tr}[\gamma_\alpha \dots \gamma_\beta] \int d[p_{1\dots n}](p_i \cdot p_j)$$

- Safe to apply $d = 4$ rules if integral have no higher poles
- Different reading points corresponds to different integrals combinations

2. Cut fermion line in all possible ways, but take trace under integral sign



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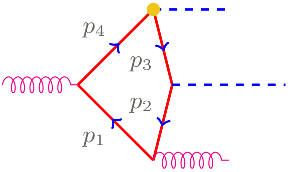

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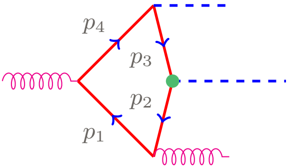

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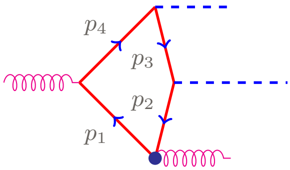
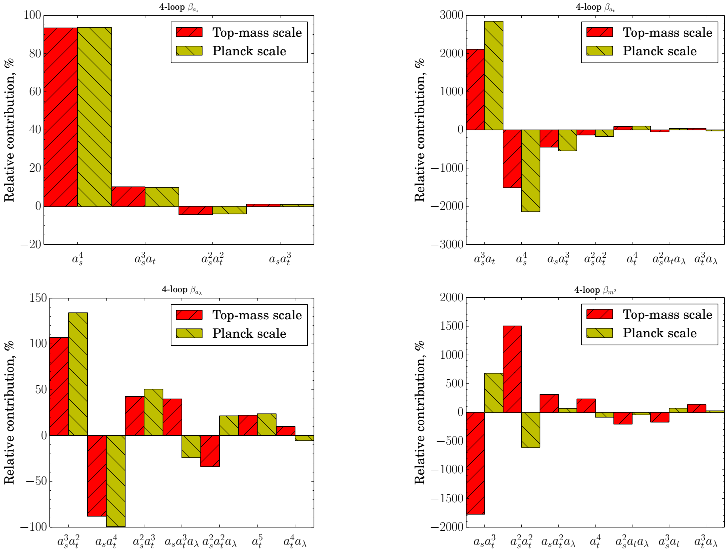


Relative contributions



Relative loop contributions

Substituting running couplings at scale $\mu = M_t$:

- Contribution from part with uncertainty due to γ_5 in β_{a_s} negligible
- Large cancellations, no uncertainty

$$\frac{\beta_{m^2}}{\beta_0} = \quad 1.h \quad + \quad 0.0264h^2 \quad + \quad 0.00266h^3 \quad - \quad [0.000027 - 0.000066]h^4$$

- Small cancellation and small contribution from piece with uncertainty

$$\frac{\beta_{a_\lambda}}{\beta_0} = \quad 1.h \quad + \quad 0.0089h^2 \quad - \quad 0.00025h^3 \quad - \quad [0.001323 \pm 0.000048]h^4$$

- Large cancelations and large contribution from piece with uncertainty

$$\frac{\beta_{a_t}}{\beta_0} = \quad 1.h \quad + \quad 0.1529h^2 \quad + \quad 0.00743h^3 \quad + \quad [0.000019 \pm 0.000612]h^4$$

Critical exponents and universality classes

We are interested in scaling laws near critical point:

- Fermion field

$$\langle \psi(x)\psi(0)\rangle \sim \frac{A\not{x}}{(x^2)^{\alpha}} \sim x^{-1-2\alpha} = \frac{1}{x^{2\Delta_\psi}}$$

- Scalar field

$$\langle \phi(x)\phi(0)\rangle \sim \frac{B}{(x^2)^{\beta}} \sim x^{-2\beta} = \frac{1}{x^{2\Delta_\phi}}$$

$$\Delta_\psi = d_\psi + \frac{\gamma_\psi^*}{2} = d_\psi + \frac{\eta_\psi}{2}, \quad \Delta_\sigma = d_\sigma + \frac{\gamma_\sigma^*}{2} = d_\sigma + \frac{\eta_\sigma}{2}, \quad \Delta_\epsilon = d - \frac{1}{\nu}$$

Where d_ψ,d_σ canonical dimensions and Δ_ψ,Δ_σ critical dimensions

Critical dimensions are equal for different systems but from one universality class

We introduce universal counter for number of fermion components

$$N = \mathrm{tr}[1] \cdot N_f$$

- Four-fermion interaction, renormalizable in $d = 2 + \epsilon$
- Generalization of Gross-Neveu model to the case of N_f Dirac fermions

$$\mathcal{L}_{\mathrm{GN}} = \bar{\psi}_j \, \partial\!\!\!/ \psi^j + \frac{g}{2} (\bar{\psi}_j \psi^j)^2$$

- Interction of fermion field with scalar and scalar field self-interaction, renormalizable in $d = 4 - \epsilon$
- UV completion for Gross-Neveu model

$$\mathcal{L}_{\mathrm{GNY}} = \frac{1}{2} (\partial_\mu \sigma)^2 + \bar{\psi}_j \, \partial\!\!\!/ \psi^j + y \phi \bar{\psi}_j \psi^j + \frac{1}{24} \lambda \phi^4$$

- In $d > 2$ renormalizable in $1/N$ expansion. Results of $1/N$ expansion for full sclaing dimensions coincide with both expansions near $d = 2$ and $d = 4$

Nambu-Jona-Lasinio and Nambu-Jona-Lasinio-Yukawa models

- Generalization to the case of N_f fermions chiral four-fermion interaction, renormalizable in $d = 2 + \epsilon$

$$\mathcal{L}_{\mathrm{NJL}} = \bar{\psi}_j \, \partial\!\!\!/ \psi^j + \frac{g}{2}\left((\bar{\psi}_j \psi^j)^2 - (\bar{\psi}_j \gamma_5 \psi^j)^2\right)$$

- Yukawa model with pseudo-scalar fields, renormalizable in $d = 4 - \epsilon$

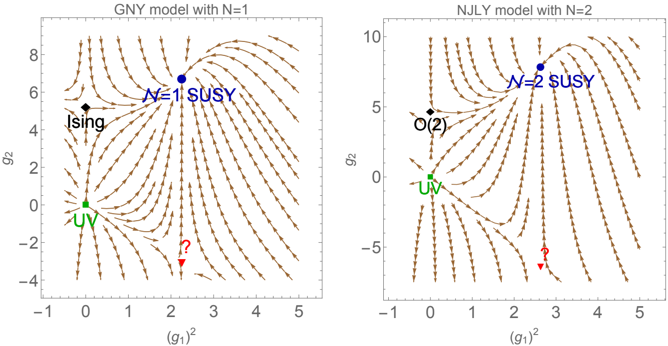
$$\mathcal{L}_{\mathrm{NJLY}} = \frac{1}{2}(\partial_\mu \phi_1)^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 + \bar{\psi}_j \partial\!\!\!/ \psi^j + y \bar{\psi}_j(\phi_1 + i\gamma_5 \phi_2)\psi^j + \frac{\lambda}{24}(\phi\bar{\phi})^2$$

- For the case $N = 1$ equal to Wess-Zumino SUSY model
- SUSY relations for anomalous dimensions and beta functions

RG flow and fixed points

One-loop RG flow with $N = 1$ for GNY model and $N = 2$ for NJLY model:

[Fei,Giombi,Klebanov,Tarnopolsky'16]



$$g_1^2 = y^2, g_2 = \lambda$$

- Gaussian fixed poin $(y^*, \lambda^*) = (0, 0)$
- Willson-Fisher fixed point with $y^* = 0$
- One stable with $y^* > 0, \lambda^* > 0$ and one unstable with $\lambda^* < 0$

Anomalous dimensions and critical exponents

- GN universality class
  - GN $2+\epsilon$ expansion four-loop results [Gracey et al.'16]

$$\eta_\psi = \gamma_\psi(g_c), \quad \eta_\phi = d + 2\gamma_m(g_c), \quad 1/\nu = -\beta'(g_c)$$

  - GNY four-loop results:this work, three-loop results from [Mihaila et al.17']

$$\eta_\psi = \gamma_\psi(y^*, \lambda^*), \quad \eta_\phi = \gamma_\phi(y^*, \lambda^*), \quad 1/\nu = 2 - \eta_\phi + \eta_{\phi^2}$$

  - $1/N$ expansion results: $\mathcal{O}\left(\frac{1}{N^3}\right)$ expansion for η_ψ and $\mathcal{O}\left(\frac{1}{N^2}\right)$ expansion for η_ϕ and $1/\nu$ [Gracey'91,92,94;Vasiliev'93]

1cm

- NJL universality class
  - NJLY four-loop:this work, three-loop results for $N = 2$ case [Zerf'16]
  - $1/N$ expansion results: $\mathcal{O}\left(\frac{1}{N^2}\right)$ expansion for η_ψ, η_ϕ and $1/\nu$ [Gracey'93,94]

SUSY relation between anomalous dimensions and critical exponents

- NJLY model with $N = 2$ equal to SUSY Wess-Zumino model:
  - Exact relation between anomalous dimensions and beta-functions, coincide with four-loop WZ beta-function [Avdeev,Gorishnii,Kamenshchik,Larin'82]

$$\beta_{\mathrm{WZ}} = 2\beta_\lambda = 2\beta_y = \frac{3}{2}\gamma_\psi = \frac{3}{2}\gamma_\phi$$

  - Fixed point is in $(y^*)^2 = \lambda^*$, and equal anomalous dimensions lead to:

$$\eta_\psi = \eta_\phi = \frac{\epsilon}{3}$$

  - Two posible ways to calculate $1/\nu$, one wo massive tadpoles

$$\omega = \frac{\partial \beta}{\partial g}, \quad \omega = \gamma_\phi^* - \gamma_{m^2}^*, \quad 1/\nu = 2 - \gamma_\phi^* + \gamma_{m^2}^* = 2 - \omega$$

- GNY model with $N = 1$ relations up to four-loop:
  - SUSY like relation for critical exponents: 4-loop valid

$$\eta_\psi = \eta_\phi$$

  - Other relations *looks like to be violated* at four-loop level

$$y^* \neq \lambda^*, \quad 1/\nu \neq 2 - \omega$$

Results for $d = 3$

- Four-loop results for Gross-Neveu model in $2 + \epsilon$ dimensions [Gracey et al.'16]
- Pade approximant do not have poles in interval $[2, 4]$ for two-side Pade, and in interval $[2, 3]$ for $2 + \epsilon$ expansion and $[3, 4]$ for $4 - \epsilon$ expansion

$N = 8$	η_ψ	η_ϕ	$1/\nu$	$N = 4$	η_ψ	η_ϕ	$1/\nu$
$2\text{Pade}_{[6,3]}$	0.047	0.735	0.98	$2\text{Pade}_{[7,2]}$	0.114	0.571	1.058
$(4-\epsilon)_{[3,1]}$	0.051	0.691	0.95	$(4-\epsilon)_{[4,0]}$	0.127	0.638	1.009
$(2+\epsilon)_{[2,2]}$	0.082	0.753	0.96	$(2+\epsilon)_{[2,2]}$	0.628	0.299	0.939
FRG	0.028	0.776	0.99	FRG	0.065	0.551	1.075
MC	0.38	0.62	1.20	MC	-	0.45	1.30

Two side Pade

$$G_{[n,m]}(d) = \frac{p_0 + dp_1 + \cdots + d^n p_n}{1 + dq_1 + \cdots + d^m q_m}$$

$$G_{[n,m]}(2+\epsilon) - F(2+\epsilon) = \mathcal{O}(\epsilon^{k_1})$$

$$G_{[n,m]}(4-\epsilon) - F(4-\epsilon) = \mathcal{O}(\epsilon^{k_2})$$

Where expansions for F known up to orders k_1, k_2: $k_1 + k_2 = n + m + 1$

GNY at smaller values of N

- $N = 1$ emergent SUSY

$N=1$	η_ψ	η_ϕ	$1/\nu$
$(4-\epsilon)_{[3,1]}$	0.170	0.170	1.415
FRG	0.180	0.180	1.408
conformal bootstrap	0.164	0.164	-

- Continuation to $N = 0$

$N=0$	η_ψ	η_ϕ	$1/\nu$
$(4-\epsilon)_{[3,1]}$	0.20	0.032	1.59

- Series are not convergent in both limits for $N = 0$

	$2+\epsilon$	$4-\epsilon$
η_ψ	$-0.125\epsilon^2 + 0.187\epsilon^3 - 0.195\epsilon^4$	$0.17\epsilon + 0.048\epsilon^2 - 0.047\epsilon^3 + 0.109\epsilon^4$
η_ϕ	$2 + 0.25\epsilon^2 + 1.5\epsilon^4$	$0.0185\epsilon^2 + 0.0187\epsilon^3 - 0.0083\epsilon^4$
$1/\nu$	$\epsilon + 0.5\epsilon^2 + 0.375\epsilon^3 + 8.038\epsilon^4$	$2 - 0.33\epsilon - 0.117\epsilon^2 + 0.124\epsilon^3 - 0.31\epsilon^4$

Conclusions - I

- We calculated universal contribution not affected by γ_5 definition in dimensional regularization to four-loop SM beta-functions in gaugeless limit

- Part dependent on γ_5 definition paramtrized for different reading points and need further analysis

- Four-loop package `FMFT` for fully massive tadpoles reduction created and successfully applied for calcualtion of all needed Green functions

Conclusions - II

- Calculated four-loop anomalous dimensions for Gross-Neveu-Yukawa and Nambu-Jona-Lasinio-Yukawa models
- For NJLY correctness of result checked with SUSY limit, which is equal to Wess-Zumino model
- For GNY model possible violation of emergent SUSY behaviour observed

Thank you for attention!