Factorization with Massive Quarks for Transverse Momentum Spectra at the LHC

Piotr Pietrulewicz

based on work

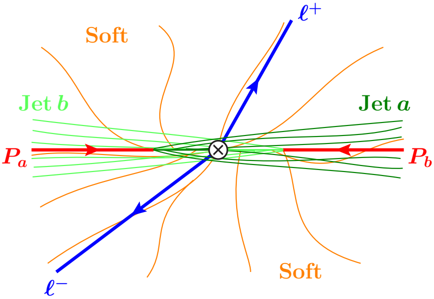
with Daniel Samitz, Anne Spiering, and Frank Tackmann
(arXiv: 1703.09702)

with Maximilian Stahlhofen (in progress)

Theory Seminar
Zeuthen, 20.04.2017

Drell-Yan at small q_T

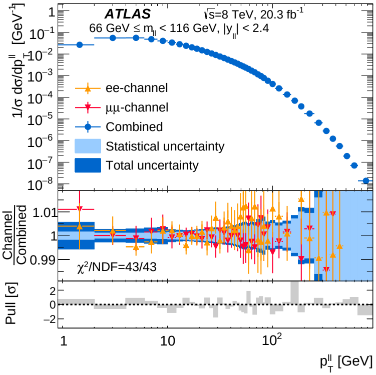
- m_b-effects frequently relevant for precision predictions at the LHC
- systematic treatment for many inclusive processes (4 vs. 5 flavor, VFNS)
- aim: systematic approach for exclusive processes (with an effective jet veto)
- here: focus on q_T-distribution for Drell-Yan ($pp \to Z/\gamma^* \to \ell\bar{\ell}$)
  effective veto on additional jets: $q_T^2 \equiv (\vec{p}_T^{\,\ell} + \vec{p}_T^{\,\bar{\ell}})^2 \ll Q^2 = (p_\ell + p_{\bar{\ell}})^2$



taken from: [Stewart, Tackmann, Waalewijn (2010)]

Drell-Yan at small q_T

- spectrum measured with high precision up to low q_T
- analytic high precision calculations, up to NNLL′+NNLO (soon N^{3}LL)
- no systematic description of b-mass effects at low q_T
  $\to$ e.g. important for m_W measurement
- in MC's: initial state splitting into massive quark pair not well understood



[ATLAS Collaboration (2015)]

$\Rightarrow$ Goal: Factorization framework for massive quark effects (using EFTs), explicit results at NNLL′ for Z/γ^* (NNLL resummation with FO ingredients at $\mathcal{O}(\alpha_s^2)$)

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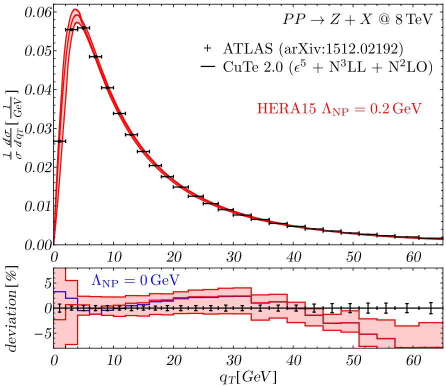

[Becher, Lübbert, Neubert, Wilhelm (2016)]

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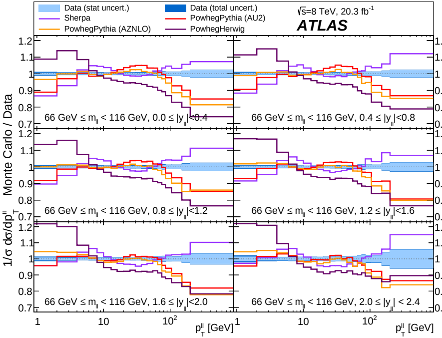

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explicit results at NNLL$'$ for Z/γ^* (NNLL resummation with FO ingredients at $\mathcal{O}(\alpha_s^2)$)

Outline

1. Massive quarks in inclusive DIS
2. Factorization with massless quarks for exclusive Drell-Yan
3. Factorization with massive quarks for exclusive Drell-Yan
4. Factorization with massive quarks for exclusive gluon fusion
5. Resummation with massive quarks
6. Conclusions & Outlook

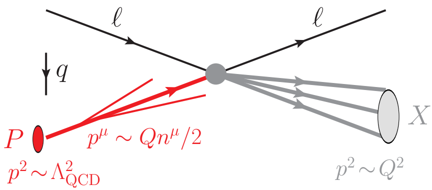
Outline

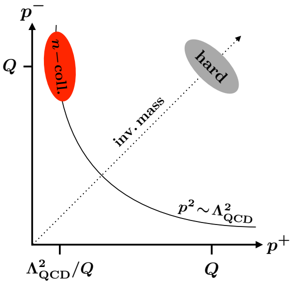
Inclusive DIS

- consider $\ell P \to \ell X$ (X inclusive)
- two relevant scales: $q^2 = -Q^2$, $\Lambda^2_{\rm QCD}$
- aim: factorization = separation of quantum fluctuations at different energy scales
  $\to$ conveniently achieved with EFTs (here: SCET)
- d.o.f. in the Breit frame ($q_0 = 0$): initial state n-collinear modes with $p^2 \sim \Lambda^2_{\rm QCD}$

$$p^\mu \sim (n \cdot p, \bar{n} \cdot p, p_\perp) \equiv (p^+, p^-, p_\perp) \sim (\Lambda^2_{\rm QCD}/Q, Q, \Lambda_{\rm QCD})$$

$$\text{with } n^\mu = (1, \vec{n}), \quad \bar{n}^\mu = (1, -\vec{n})$$



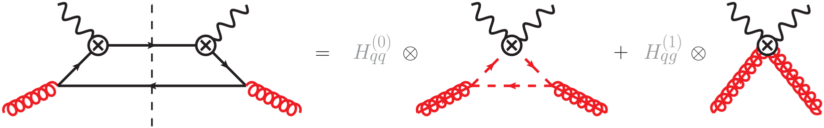


Factorization for massless quarks in inclusive DIS

Factorization ($\mu_H \equiv \mu_F = \mu_R$): [Collins, Soper, Sterman (1988); Bauer et al. (2002)]

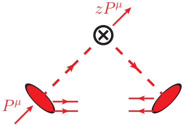
$$F_{1,2} \sim \sum_{i=q,\bar{q}} \sum_{j=q,\bar{q},g} H_{ij}(Q,\mu_H) \otimes U^f_{jk}(\mu_H,\mu_f) \otimes f_{k/P}(\mu_f) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{Q^2}\right)$$

- hard function $H_{ij}(\mu_H \sim Q)$: matching between full QCD and EFT, e.g.



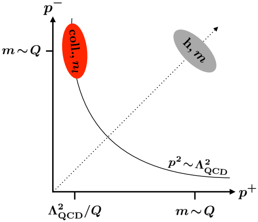
- parton distribution function (PDF) $f_{j/P}(\mu_f \sim \Lambda_{\rm QCD})$
  = coll. matrix element

$$f_{k/P}(x,\mu_f) = \langle P|\mathcal{O}_k(x,\mu_f)|P\rangle$$



- resum logs $\ln(\frac{\mu_H}{\mu_f}) \sim \ln(\frac{Q}{\Lambda_{\rm QCD}})$ via RG factor $U^f_{jk}(\mu_H,\mu_f)$ (implicit in the following)
- next: n_l light flavors + 1 heavy quark ($m \gg \Lambda_{\rm QCD}$)
  possible hierarchies: (i) $m \sim Q$, (ii) $m \ll Q \longrightarrow$ 2 EFTs (extreme hierarchies)

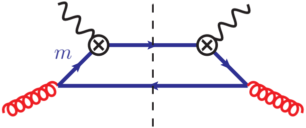
EFT description for $m \sim Q$ (“n_l-flavor scheme”)



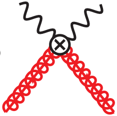
Factorization for $m \sim Q$:

$$F_{1,2} \sim \sum_{i=q,Q} \sum_{j=q,g} H^{(n_l)}_{ij}(Q,m,\mu_H) \otimes f^{(n_l)}_{j/P}(\mu_f)$$

- “integrate out" hard and massive quark fluctuations simultaneously at $\mu_H \sim Q$



$= H^{(1)}_{Qg}(m) \otimes$

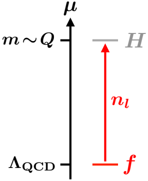


$\rightarrow$ no massive quark contributions in EFT

$\rightarrow$ automatic decoupling for $m \gg Q$: $H^{(n_l)}_{ij} \rightarrow 0$

$\rightarrow$ for $m \ll Q$: $H^{(n_l)}_{ij}(Q,m,\mu_H) \not\rightarrow H_{ij}(Q,\mu_H)$

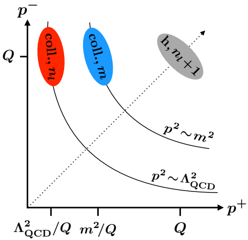
- PDF evolution always with n_l flavors



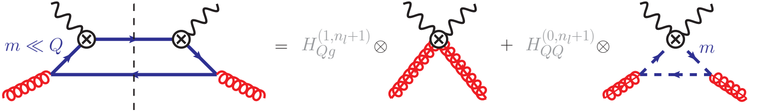
EFT description for $m \ll Q$ ("$n_l + 1$-flavor scheme")

Factorization for $m \ll Q$:

$$F_{1,2} \sim \sum_{i=q,Q} \sum_{j=q,Q,g} \sum_{k=q,g} H_{ij}^{(n_l+1)}(\mu_H) \otimes \mathcal{M}_{jk}(\mu_m) \otimes f_{k/P}^{(n_l)}(\mu_f) + \mathcal{O}(m^2/Q^2)$$



- "integrate out" hard modes at $\mu_H \sim Q$
  $\rightarrow H_{ij}$ with $n_l + 1$ (massless) flavors (IR mass dependence cancels in matching)



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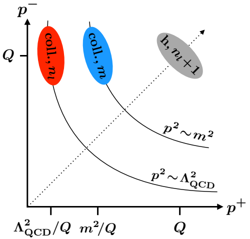
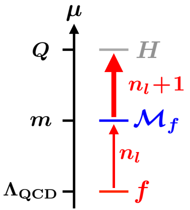
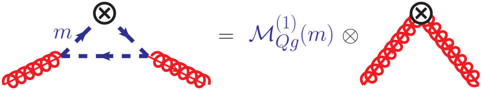


Factorization for $m \ll Q$:

$$F_{1,2} \sim \sum_{i=q,Q}\sum_{j=q,Q,g}\sum_{k=q,g} H_{ij}^{(n_l+1)}(\mu_H)\otimes\mathcal{M}_{jk}(\mu_m)\otimes f_{k/P}^{(n_l)}(\mu_f) + \mathcal{O}(m^2/Q^2)$$

- "integrate out" hard modes at $\mu_H \sim Q$
  $\rightarrow H_{ij}$ with n_l+1 (massless) flavors
- between μ_H and μ_m: evolution with n_l+1 flavors
- at $\mu_m \sim m$: "integrate out" mass modes
  $\rightarrow$ matching between PDFs (= "massive OME's")





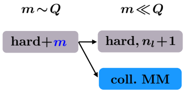
- below μ_m: PDF evolution with n_l flavors

Merging the two regimes

Comparison between regimes:

	$m \sim Q$	$m \ll Q$
FO content	full m/Q dependence	expansion in m^2/Q^2
resummation of $\ln(m^2/Q^2)$	–	✓
scale variations	μ_H	μ_H & μ_m

$$H_{ik}^{(n_l)}(Q,m,\mu) = \sum_{j=q,Q,g} H_{ij}^{(n_l+1)}(Q,\mu) \otimes \mathcal{M}_{jk}(m,\mu) + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$



$\Rightarrow$ combine both regimes to obtain full fixed-order content and resummation for $m \ll Q$

- "Variable-flavor number schemes" (e.g. ACOT [Aivazis, Collins, Olness, Tung (1994)])
- in SCET: merging of resummed predictions with fixed-order results is common

$$\mathrm{d}\sigma = \underbrace{\mathrm{d}\sigma_{m\ll Q}}_{\text{resummed, "singular"}} + \underbrace{(\mathrm{d}\sigma_{m\sim Q} - \mathrm{d}\sigma_{m\ll Q})}_{\mathcal{O}(m^2/Q^2),\text{ "nonsingular"}}\Big|_{\mu_m=\mu_H}$$

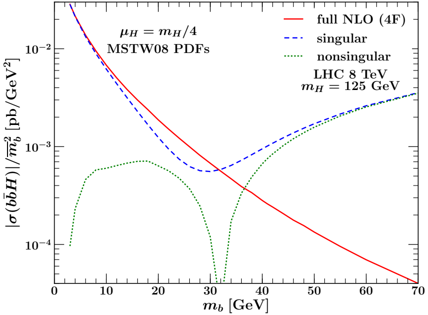
$\Rightarrow$ for $m \ll Q$: $\mathrm{d}\sigma \to \mathrm{d}\sigma_{m\ll Q}$

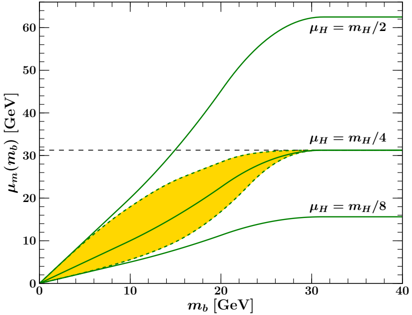
$\Rightarrow$ for $m \sim Q$: $\mathrm{d}\sigma \to \mathrm{d}\sigma_{m\sim Q}$ $\quad\to$ profiling of $\mu_m(m,Q)$

Example: inclusive $b\bar{b}H$-production

Inclusive $b\bar{b}H$-production at the LHC

- singular vs. nonsingular contributions as function of m_b
- profile scale μ_m as function of m_b

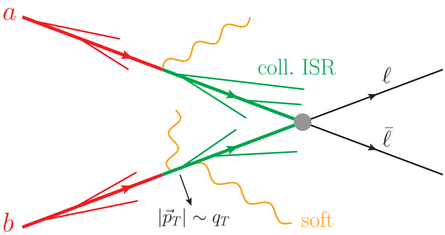


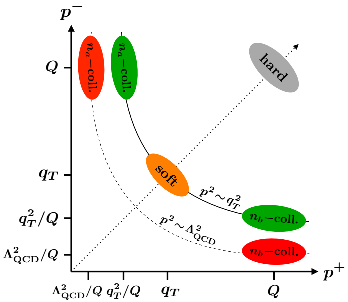


[Bonvini, Papanastasiou, Tackmann (2015)]

Outline

EFT modes for small q_T





DY with $q_T \ll Q = \sqrt{p_Z^2}$ is a multiscale process:

- hard process: $q\bar{q} \to Z/\gamma^* \to \ell\bar{\ell}$ at scale $\mu \sim Q$
- n_a-/n_b-collinear ISR at scale $\mu \sim q_T$
- wide-angle soft ISR at scale $\mu \sim q_T$
- nonperturbative collinear proton at scale $\mu \sim \Lambda_{\mathrm{QCD}}$

large scale hierarchies ⇒ large Sudakov logs $\alpha_s \ln^2(q_T/Q)$ in perturbation theory

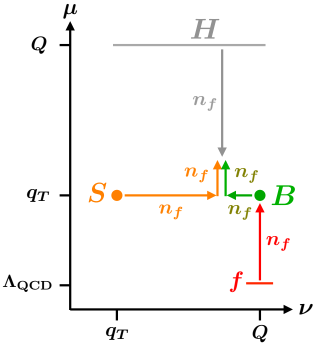
Factorization for small q_T

[Collins, Soper, Sterman (1985); Catani, de Florian, Grazzini (2001), Becher, Neubert (2011); Echevarria, Idilbi, Scimemi (2011); Chiu, Jain, Neill, Rothstein (2012); . . .]

Factorization theorem for n_f massless quarks: $(i, j, k \in \{q, \bar{q}, g\})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_f)} \times \Big[\sum_k \mathcal{I}_{ik}^{(n_f)} \otimes f_k^{(n_f)}\Big]^2 \otimes S^{(n_f)} + \mathcal{O}\left(\frac{q_T}{Q}, \frac{\Lambda_{\rm QCD}^2}{q_T^2}\right)$$

- hard function $H_{ij}(Q)$: process dependence
- beam function $B_i = \sum_k \mathcal{I}_{ik} \otimes_x f_k$
  → collinear ISR matching $\mathcal{I}_{ik}(\vec{q}_T, z)$
  → nonperturbative PDF $f_k(\Lambda_{\rm QCD}, x)$
- soft function $S(\vec{q}_T)$: wide-angle soft radiation
- resummation via evolution factors (implicit)
- rapidity divergences in S and B_i
  → associated rapidity logarithms
  → resummed via rapidity RGE
  [Chiu, Jain, Neill, Rothstein (2012)]



Rapidity divergences and logarithms

- unregulated divergences in soft and coll. matrix elements in DimReg

$$\text{in soft function}: \int_0^\infty \frac{\mathrm{d}k^-}{k^-}\,, \qquad \text{in beam function}: \int_0^1 \frac{\mathrm{d}k^-}{k^-}$$

- rapidity regularization, here: symmetric Wilson line regulator
  $\rightarrow$ additional factor at one loop: [Chiu, Jain, Neill, Rothstein (2012)]

$$\text{in soft function}: \left(\frac{\nu}{|k^+ - k^-|}\right)^\eta, \qquad \text{in beam function}: \left(\frac{\nu}{k^-}\right)^\eta$$

- soft and collinear contribution yield together rapidity log $\ln(q_T/Q)$
- for resummation: absorb $1/\eta$-divergences into counterterms
  $\rightarrow$ use rapidity RG evolution in ν

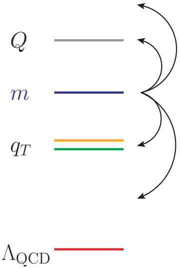
$$\nu \frac{\mathrm{d}}{\mathrm{d}\nu} S(\vec{q}_T, \mu, \nu) = \int \mathrm{d}^2 p_T\, \gamma_\nu(\vec{q}_T - \vec{p}_T, \mu)\, S(\vec{p}_T, \mu, \nu)$$

$$\nu \frac{\mathrm{d}}{\mathrm{d}\nu} B(Q, \vec{q}_T, \mu, \nu) = -\frac{1}{2} \int \mathrm{d}^2 p_T\, \gamma_\nu(\vec{q}_T - \vec{p}_T, \mu)\, B(Q, \vec{p}_T, \mu, \nu)$$

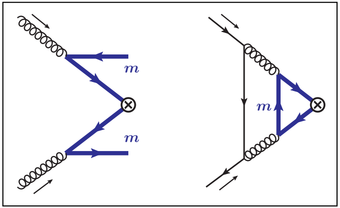
Outline

Massive quark effects

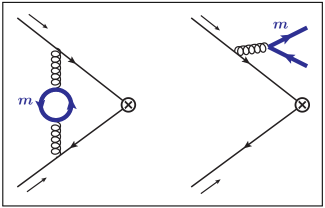
- additional scale m: different hierarchies possible
- required: systematic multi-scale factorization setup
  for e^+e^- event shapes, DIS at threshold:
  [Gritschacher, Hoang, Jemos, Mateu, P.P. (2014); Hoang, P.P., Samitz (2015)]
- for Z/γ^*: $\mathcal{O}(\alpha_s^2)$ corrections
  $\rightarrow$ relevant at NNLL$'$ (evolution affected already at LL)



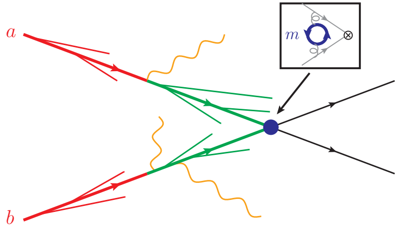
primary massive quark corrections:

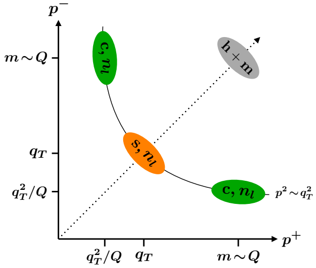


secondary massive quark corrections:



$\Lambda_{\rm QCD} \ll q_T \ll m \sim Q$



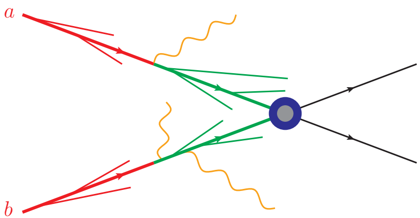


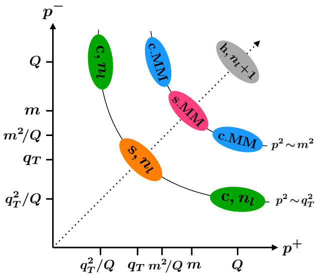
Factorization theorem: $(i,j,k \in \{q,\bar{q},g\})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}(m) \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{q_T^2}{m^2}\right)$$

- virtual massive quark corrections to hard function
  $\rightarrow$ decouple for $m \gg Q$ (for conserved vector current)
- beam and soft function with n_l massless flavors

$\Lambda_{\mathrm{QCD}} \ll q_T \ll m \ll Q$





Factorization theorem: $(i,j,k \in \{q,\bar{q},g\})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times |H_c(m)|^2 \times H_s(m) \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{q_T^2}{m^2}\right)$$

- two step matching: (1) QCD $\to$ SCET$^{(n_l+1)}$, (2) SCET$^{(n_l+1)}$ $\to$ SCET$^{(n_l)}$
- hard function with n_l+1 massless quarks
- beam and soft function with n_l massless quarks

$\Lambda_{\rm QCD} \ll q_T \ll m \ll Q$

Factorization theorem: $(i, j, k \in \{q, \bar{q}, g\})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times |H_c(m)|^2 \times H_s(m) \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{q_T^2}{m^2}\right)$$

- collinear & soft mass mode matching factors $H_c = |C_c|^2$, $H_s = |C_s|^2$ (at $\mu_m \sim m$)

$\rightarrow$ only secondary virtual corrections at $\mathcal{O}(\alpha_s^2)$

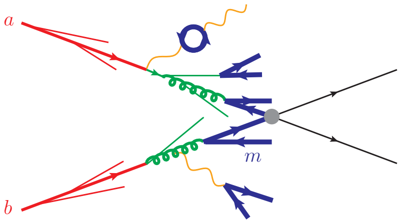
$\rightarrow$ independent of hard process (for 0 jets) and measurement

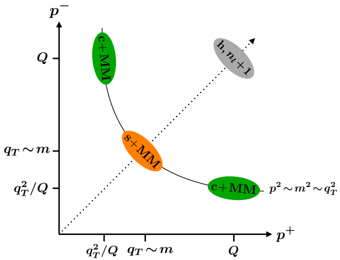
see also [Hoang, P.P., Samitz (2015); Hoang, Pathak, P.P, Stewart (2015)]

$\rightarrow$ yield together rapidity logarithm $\ln(Q/m)$

remark: same coefficient as for $[\frac{1}{1-z}]_+$-term in PDF matching [Hoang, P.P., Samitz (2015)]

$\Lambda_{\mathrm{QCD}} \ll m \sim q_T \ll Q$





Factorization theorem: $(i, j \in \{q, \bar{q}, Q, \bar{Q}, g\}, k \in \{q, \bar{q}, g\})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S(m) + \mathcal{O}\left(\frac{m}{Q}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

- primary and secondary massive quark corrections to beam functions/TMDs:

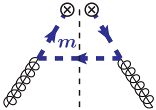
$$B_i^{(n_l+1)}(\vec{q}_T, x, m) = \sum_{k\in\{q,\bar{q},g\}} \mathcal{I}_{ik}(\vec{q}_T, x, m) \otimes f_k^{(n_l)}(x) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{m^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{q_T^2}\right)$$

- secondary massive quark corrections to soft function: $S^{(2)}(\vec{q}_T, m)$
- mass dependent rapidity divergences (and logs)

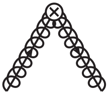
Massive quark corrections to beam and soft functions

primary and secondary massive quark corrections to beam functions/TMDs:

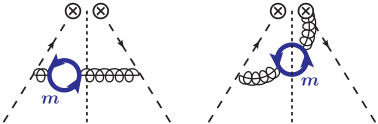
$\rightarrow \mathcal{O}(\alpha_s)$ primary massive: $\mathcal{I}^{(1)}_{Qg}(\vec{q}_T, x, m)$ ✓ [→ massive quark TMD]



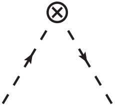
$\longrightarrow \mathcal{I}^{(1)}_{Qg}(m) \otimes$



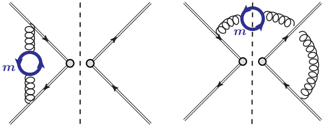
$\rightarrow \mathcal{O}(\alpha_s^2)$ secondary massive: $\mathcal{I}^{(2)}_{qq}(\vec{q}_T, x, m)$ ✓



$\longrightarrow \mathcal{I}^{(2)}_{qq}(m) \otimes$

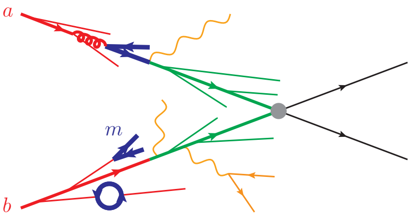


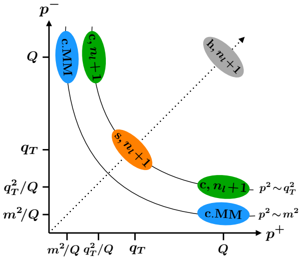
secondary massive quark corrections to soft function at $\mathcal{O}(\alpha_s^2)$: $S^{(2)}(\vec{q}_T, m)$ ✓



$\longrightarrow S^{(2)}(m)$

$\Lambda_{\text{QCD}} \ll m \ll q_T \ll Q$





Factorization theorem: $(i, j, m \in \{q, \bar{q}, Q, \bar{Q}, g\}, k \in \{q, \bar{q}, g\})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_k^{(n_l)} \Big]^2 \otimes S^{(n_l+1)} + \mathcal{O}\left(\frac{m^2}{q_T^2}, \frac{\Lambda_{\text{QCD}}^2}{m^2} \right)$$

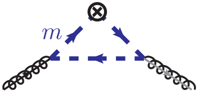
- hard fct, coll. ISR matching and soft fct with $n_l + 1$ massless flavors
- primary and secondary massive quark corrections in PDF matching

$$f_i^{(n_l+1)}(x, m) = \sum_{k \in \{q,\bar{q},g\}} \mathcal{M}_{ik}(x, m) \otimes f_k^{(n_l)}(x) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{m^2} \right)$$

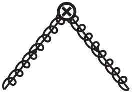
PDF matching for massive quarks (revisited)

primary and secondary massive quark corrections in PDF matching:

$\to \mathcal{O}(\alpha_s)$ primary massive: $\mathcal{M}_{Qg}^{(1)}(x,m)$ [$\to$ massive quark PDF]

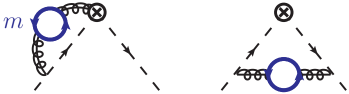


$= \mathcal{M}_{Qg}^{(1)}(m) \otimes$

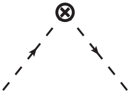


$\to \mathcal{O}(\alpha_s^2)$ secondary massive e.g. $\mathcal{M}_{qq}^{(2)}(x,m)$

[Buza, Matiounine, Smith, van Neerven (1998); Bierenbaum, Blümlein, Klein (2009)]

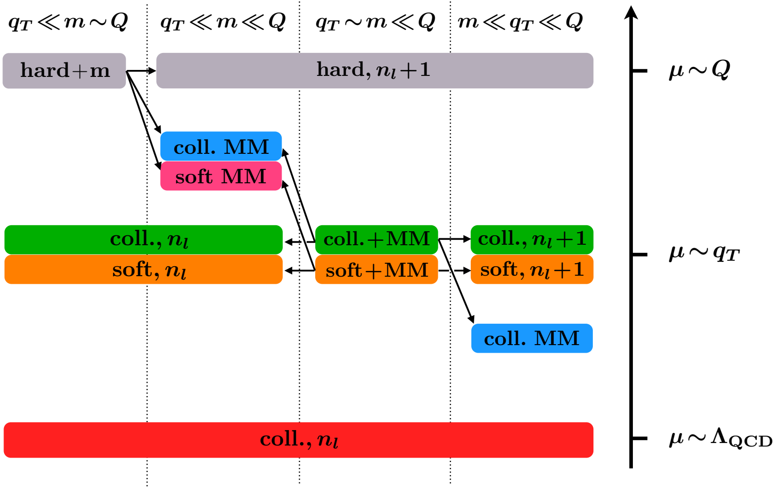


$= \mathcal{M}_{qq}^{(2)}(m) \otimes$



$\to$ corrections even known at $\mathcal{O}(\alpha_s^3)$ [Ablinger, Blümlein, ...]

Relations between hierarchies



Relations between hierarchies ($\longleftrightarrow$ inclusive cross section)

Factorization theorem for $m \sim q_T$:

$$\frac{d\sigma}{dq_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)} \right]^2 \otimes S(m)$$

Factorization theorem for $m \ll q_T$:

$$\frac{d\sigma}{dq_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \left[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_k^{(n_l)} \right]^2 \otimes S^{(n_l+1)} + \mathcal{O}\left(\frac{m^2}{q_T^2}\right)$$

$\Rightarrow$ Relations between ingredients:

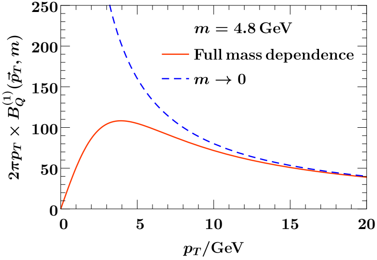
$$\mathcal{I}_{ik}(m) = \sum_j \mathcal{I}_{ij}^{(n_l+1)} \otimes \mathcal{M}_{jk}(m) \left[1 + \mathcal{O}\left(\frac{m^2}{q_T^2}\right)\right], \quad S(m) = S^{(n_l+1)} \left[1 + \mathcal{O}\left(\frac{m^2}{q_T^2}\right)\right]$$

- checked explicitly at $\mathcal{O}(\alpha_s^2)$ using known massless results for $\mathcal{I}_{ij}^{(n_f)}$ and $S^{(n_f)}$ ✓
  [Gehrmann, Luebbert, Yang (2012); Luebbert, Oredsson, Stahlhofen (2016)]
- resummation of $\ln(m^2/q_T^2)$ can be combined with power corrections of $\mathcal{O}(m^2/q_T^2)$
- similar relations between other hierarchies
  $\Rightarrow$ continuous description over complete spectrum possible

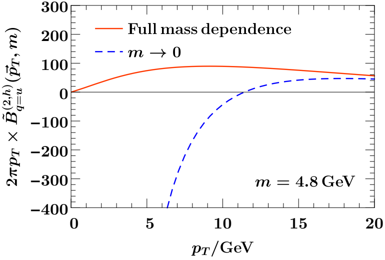
“Nonsingular” bottom mass corrections

- most relevant hierarchies for m_b effects: $m_b \ll q_T$, $m_b \sim q_T$
- here: full m_b-dependent TMD beam functions vs. “mass-singular” limit at FO
- note: secondary corrections are rather $\mathcal{O}\left(\frac{(2m_b)^2}{q_T^2}\right)$

spectrum ($\mu = m_b$):



$\mathcal{O}(\alpha_s T_F)$ primary



$\mathcal{O}(\alpha_s^2 C_F T_F)$ secondary

$[\tilde{B} = B \times \sqrt{S}]$

“Nonsingular” bottom mass corrections

- most relevant hierarchies for m_b effects: $m_b \ll q_T$, $m_b \sim q_T$
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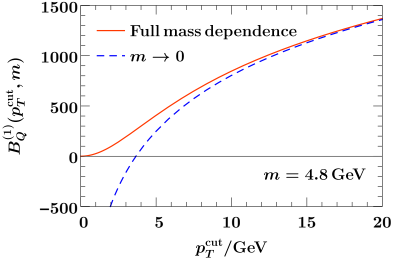
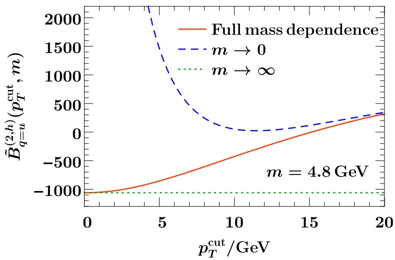


$\mathcal{O}(\alpha_s T_F)$ primary



$\mathcal{O}(\alpha_s^2 C_F T_F)$ secondary

$[\tilde{B} = B \times \sqrt{S}]$

Outline

Factorization for gluon fusion

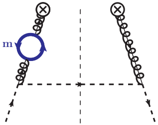
[Collins, Soper, Sterman (1985); Catani, de Florian, Grazzini (2001), Becher, Neubert (2011); Echevarria, Idilbi, Scimemi (2011); Chiu, Jain, Neill, Rothstein (2012); . . .]

Factorization theorem for n_f massless quarks: $(i, j, k \in \{q, \bar{q}, g\})$

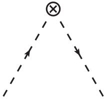
$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim H_{gg}^{(n_f)} \times \Big[\sum_k \mathcal{I}_{gk}^{\mu\nu(n_f)} \otimes f_k^{(n_f)}\Big]^2 \otimes S^{(n_f)} + \mathcal{O}\left(\frac{q_T}{Q}, \frac{\Lambda_{\rm QCD}^2}{q_T^2}\right)$$

- factorization with massive quarks → in analogy to DY
- ingredients for $m \gg q_T$ and $m \ll q_T$ already known at $\mathcal{O}(\alpha_s^2)$
- consider $m \sim q_T$:
  → soft function S satisfies Casimir scaling ✓
  → remaining ingredient for NNLL′: beam function matching $\mathcal{I}_{gk}^{\mu\nu}(m)$ at $\mathcal{O}(\alpha_s^2)$

$\mathcal{I}_{gq}^{(2)}(\vec{q}_T, m)$ ✓



$\longrightarrow \quad \mathcal{I}_{gq}^{(2)}(m) \otimes$



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- factorization with massive quarks $\rightarrow$ in analogy to DY
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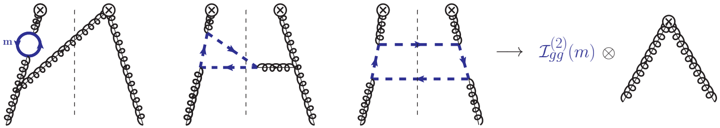
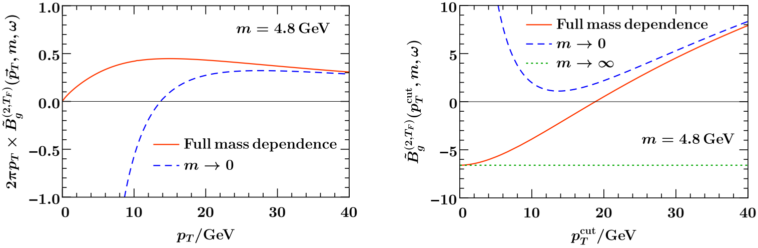
  $\rightarrow$ soft function S satisfies Casimir scaling ✓
  $\rightarrow$ remaining ingredient for NNLL′: beam function matching $\mathcal{I}_{gk}^{\mu\nu}(m)$ at $\mathcal{O}(\alpha_s^2)$

$\mathcal{I}_{gg}^{(2)}(\vec{q}_T, m)$ ✓



Massive quark corrections to gluon beam function

results for mass dependent beam function at $\mathcal{O}(\alpha_s^2 T_F)$: $\tilde{B} = B \times \sqrt{S}$

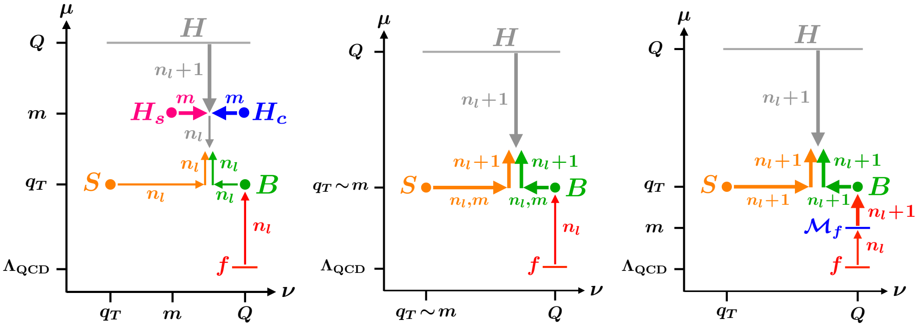


spectrum cumulant

nonsingular corrections sizable even for $q_T > 2m_b$

Outline

Resummation of logs



$q_T \ll m \ll Q$ $m \sim q_T \ll Q$ $m \ll q_T \ll Q$

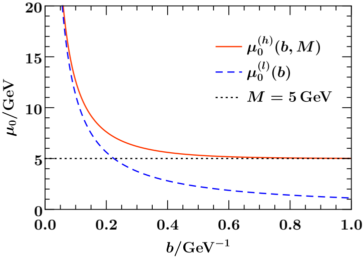
- μ-evolution with $n_l = 4$ quark flavors below the mass scale
- μ-evolution with $n_l + 1 = 5$ quark flavors above the mass scale
- for $q_T \ll m \ll Q$: additional ν-evolution at $\mu_m \sim m \rightarrow$ straightforward solution
- for $q_T \sim m$: ν-evolution modified by quark mass (due to secondary effects)

Resummation of rapidity logs for $q_T \sim m$

- convenient in impact parameter (= Fourier) space ($\vec{p}_T \leftrightarrow \vec{b}$)
- avoid (or otherwise resum) large logarithms in anomalous dimension $\gamma_\nu(b,\mu)$
- illustration at one-loop for $\gamma_\nu \equiv \gamma_{S,\nu}$ with massless/massive gluon:

$$M=0\text{: } \gamma_\nu(b,\mu) = -\frac{2\alpha_s(\mu)C_F}{\pi}\ln\left(\frac{b^2\mu^2 e^{2\gamma_E}}{4}\right) \quad \Rightarrow \mu \sim \mu_0(b) \equiv \frac{2e^{-\gamma_E}}{b}$$

$$M \neq 0\text{: } \gamma_\nu(b,M,\mu) = \frac{2\alpha_s(\mu)C_F}{\pi}\left(\ln\frac{M^2}{\mu^2} + 2K_0(b\,M)\right) \Rightarrow \mu \sim \mu_0(b,M) \equiv M e^{K_0(b\,M)}$$



$$\mu_0(b,M) \xrightarrow{b\to 0} \frac{2e^{-\gamma_E}}{b}$$

$$\mu_0(b,M) \xrightarrow{b\to\infty} M$$

$\Rightarrow$ mass introduces IR cutoff

$\Rightarrow$ no Landau pole for $b \to \infty$

- similar for massive quarks at $\mathcal{O}(\alpha_s^2)$ (with correct choices for α_s-scheme)

Outline

Summary

Conclusions:

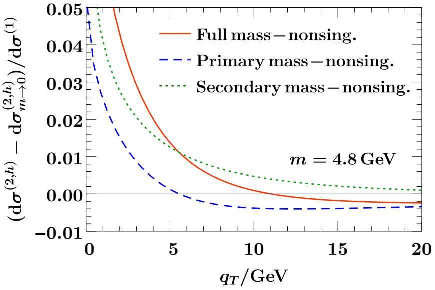
- factorization with massive quarks for Drell-Yan + 0 jets at low q_T ✓
- required ingredients for resummation of m_b-logs at NNLL$'$ accuracy ✓
- setup & ingredients also for beam thrust ✓
- application to Higgs q_T-spectrum

Outlook:

- phenomenological analysis of m_b effects for q_T-spectrum

quark mass corrections at $\mathcal{O}(\alpha_s^2)$ to Z-spectrum at FO

$\Rightarrow$ percent level effect for $q_T \sim m_b$



Summary

Conclusions:

- factorization with massive quarks for Drell-Yan + 0 jets at low q_T ✓
- required ingredients for resummation of m_b-logs at NNLL$'$ accuracy ✓
- setup & ingredients also for beam thrust ✓
- application to Higgs q_T-spectrum

Outlook:

- phenomenological analysis of m_b effects for q_T-spectrum
- charm quark effects for W-production at NNLL$'$ (need $\mathcal{O}(\alpha_s^2)$ primary corrections)
- application to other processes, e.g. $b\bar{b}H$-production

Outline

"Nonsingular" massive quark corrections for p_T-spectrum in DY

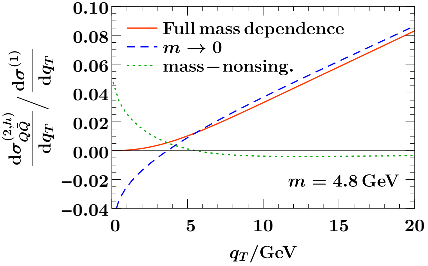
Factorization theorem for $m \sim q_T$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)} \Big]^2 \otimes S(m)$$

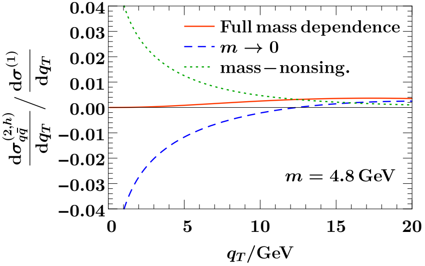
Factorization theorem for $m \ll q_T$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_{m,k} \mathcal{I}_{im}^{(n_l+1)} \otimes \mathcal{M}_{mk}(m) \otimes f_k^{(n_l)} \Big]^2 \otimes S^{(n_l+1)} + \mathcal{O}\left(\frac{m^2}{q_T^2}\right)$$

Primary corrections:



Secondary corrections:



Massless factorization theorem (explicit)

Factorization for $q_T \ll Q$: (quark initiated channels)

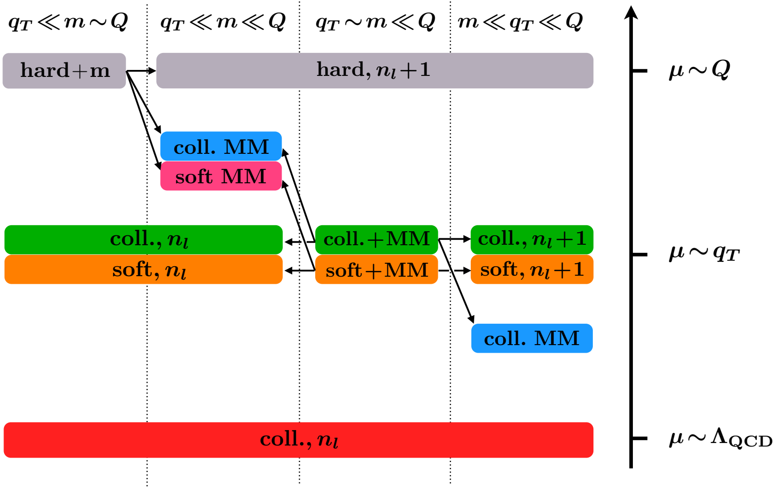
$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2\,\mathrm{d}Q^2\,\mathrm{d}Y} = \sum_{i,j\in\{q,\bar{q}\}} H_{ij}^{(n_f)}(Q,\mu) \int \mathrm{d}^2p_{T,a}\,\mathrm{d}^2p_{T,b}\,\mathrm{d}^2p_{T,s}\,\delta(q_T^2 - |\vec{p}_{T,a}+\vec{p}_{T,b}+\vec{p}_{T,s}|^2)$$
$$\times B_i^{(n_f)}\Big(\vec{p}_{T,a}, x_a, \mu, \frac{\nu}{\omega_a}\Big)\, B_j^{(n_f)}\Big(\vec{p}_{T,b}, x_b, \mu, \frac{\nu}{\omega_b}\Big)\, S^{(n_f)}(\vec{p}_{T,s}, \mu, \nu)$$
$$\times \Big[1 + \mathcal{O}\Big(\frac{q_T}{Q}, \frac{\Lambda_{\mathrm{QCD}}^2}{q_T^2}\Big)\Big]$$

with

$$\omega_a = Qe^{Y}\,, \quad \omega_b = Qe^{-Y}\,, \quad x_{a,b} = \frac{\omega_{a,b}}{E_{\mathrm{cm}}}\,.$$

$$B_i^{(n_f)}\Big(\vec{p}_T, x, \mu, \frac{\nu}{\omega}\Big) = \sum_k \underbrace{\int_x^1 \frac{\mathrm{d}z}{z}\, \mathcal{I}_{ik}^{(n_f)}\Big(\vec{p}_T, \frac{x}{z}, \mu, \frac{\nu}{\omega}\Big)\, f_k^{(n_f)}(z,\mu)}_{\equiv \mathcal{I}_{ik}^{(n_f)}(\vec{p}_T, x, \mu, \frac{\nu}{\omega}) \otimes f_k^{(n_f)}(x,\mu)} \Big[1 + \mathcal{O}\Big(\frac{\Lambda_{\mathrm{QCD}}^2}{p_T^2}\Big)\Big],$$

Relations between hierarchies



Relations between matrix elements:

Factorization theorem for $q_T \ll m \sim Q$: $(i,j,k \in \{q,\bar{q},g\})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}(m) \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{q_T^2}{m^2}\right)$$

Factorization theorem for $q_T \ll m \ll Q$: $(i,j,k \in \{q,\bar{q},g\})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times |H_c(m)|^2 \times H_s(m) \times \Big[\sum_k \mathcal{I}_{ik}^{(n_l)} \otimes f_k^{(n_l)}\Big]^2 \otimes S^{(n_l)} + \mathcal{O}\left(\frac{m^2}{Q^2}, \frac{q_T^2}{m^2}\right)$$

$\Rightarrow$ relation between hard functions:

$$H_{ij}(m) = H_{ij}^{(n_l+1)} \, |H_c(m)|^2 \, H_s(m) + \mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

Relations between matrix elements:

Factorization theorem for $q_T \ll m \ll Q$: $(i,j,k \in \{q,\bar{q},g\})$

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Factorization theorem: $(i,j \in \{q,\bar{q},Q,\bar{Q},g\}, k \in \{q,\bar{q},g\})$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T^2} \sim \sum_{i,j} H_{ij}^{(n_l+1)} \times \Big[\sum_k \mathcal{I}_{ik}(m) \otimes f_k^{(n_l)}\Big]^2 \otimes S(m) + \mathcal{O}\left(\frac{m}{Q}, \frac{\Lambda_{\mathrm{QCD}}^2}{m^2}\right)$$

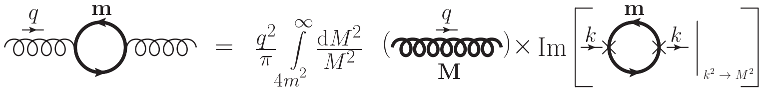
$\Rightarrow$ relations:

$$\mathcal{I}_{ik}(m) = H_c(m)\,\mathcal{I}_{ik}^{(n_l)} + \mathcal{O}\left(\frac{p_T^2}{m^2}\right), \quad S(m) = H_s(m)\,S^{(n_l)} + \mathcal{O}\left(\frac{p_T^2}{m^2}\right)$$

Dispersive technique for secondary corrections

dispersion relations for additive observables:

$$\frac{-i\,g^{\mu\rho}}{p^2+i\epsilon}\,\Pi_{\rho\sigma}(m^2,p^2)\,\frac{-i\,g^{\sigma\nu}}{p^2+i\epsilon} = \frac{1}{\pi}\int \frac{\mathrm{d}M^2}{M^2}\,\frac{-i\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)}{p^2-M^2+i\epsilon}\,\mathrm{Im}\big[\Pi(m^2,M^2)\big] - \frac{-i\left(g^{\mu\nu}-\frac{p^\mu p^\nu}{p^2}\right)}{p^2+i\epsilon}\,\Pi(m^2,0)\,.$$



- compute amplitudes for “massive gluon"
- convolve result with vacuum polarization (in d dimensions)
- applicable for both virtual and real bubble diagrams
- generic scaling $M \sim m$ is inherited, same field theoretic setup