Top pair production at lepton colliders

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Motivation

The top quark is the heaviest particle:

- Largest Yukawa coupling $\Rightarrow$ probes the Higgs sector

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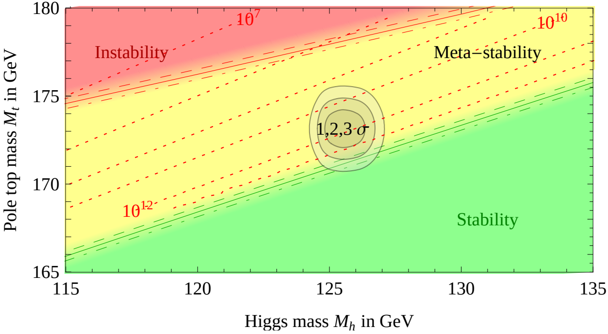
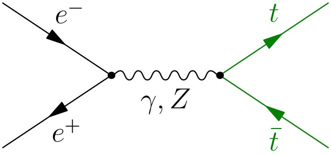


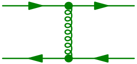
[Degrassi et al. 2012]

- "Decays before hadronisation"
- Not measured at lepton colliders yet

$e^+e^- \to t\bar{t}$ near threshold



- Kinematics: $v \ll 1, \quad E_{\text{kin}} \sim m_t v^2, \quad |\mathbf{p}| \sim m_t v$
- Dominant interaction:



$\Rightarrow$ Colour Coulomb potential $-\frac{C_F \alpha_s}{r}$

- $t\bar{t}$ "decays during bound state formation":

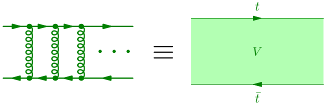
$$v \sim \alpha_s \Rightarrow E_{\text{kin}} \sim m_t \alpha_s^2 \sim -E_1$$
$$\alpha \sim \alpha_s^2 \Rightarrow \Gamma \sim m_t \alpha \sim -E_1$$

Potential non-relativistic QCD

[Pineda, Soto 97; Beneke, Signer, Smirnov 99; Brambilla et al. 99]

$$\mathcal{L}_{\text{PNRQCD}} = \psi^\dagger \left(i\partial_0 + \frac{\partial^2 + i m_t \Gamma_t}{2m_t} \right) \psi + \mathcal{L}_{\text{anti-quark}}$$
$$- \int d^3\mathbf{r} \, [\psi^\dagger \psi](x + \mathbf{r}) \, \frac{C_F \alpha_s}{r} \, [\chi^\dagger \chi](x)$$
$$+ \{\text{NLO}\}$$

- Propagator:

  Coulomb Green Function 

- Production: coupling of current $\mathbf{j} = \psi^\dagger \sigma \chi$ to external γ, Z

PNRQCD at higher orders

Scales: $m_t \gg m_t v \gg m_t v^2 \gg \Lambda_{\text{QCD}}$

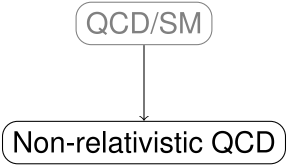
- hard modes: $k \sim m_t$
- soft modes: $k \sim m_t v$
- potential modes: $k_0 \sim m_t v^2, \vec{k} \sim m_t v$
- ultrasoft modes: $k \sim m_t v^2$

QCD/SM

PNRQCD at higher orders

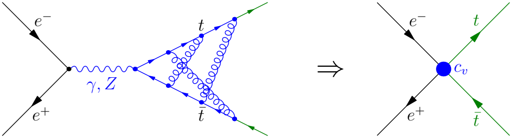
Scales: $m_t \gg m_t v \gg m_t v^2 \gg \Lambda_{\mathrm{QCD}}$

- hard modes: $k \sim m_t \rightarrow$ (local) effective vertices
- soft modes: $k \sim m_t v$
- potential modes: $k_0 \sim m_t v^2, \vec{k} \sim m_t v$
- ultrasoft modes: $k \sim m_t v^2$

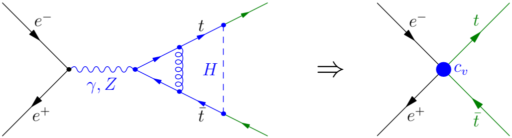


PNRQCD at higher orders

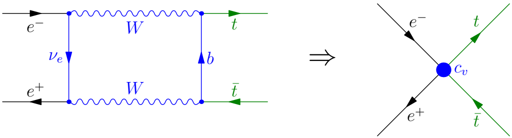
Hard matching



[Marquard, Piclum, Seidel, Steinhauer 2014]



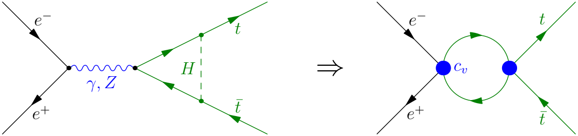
[Eiras, Steinhauer 2006]



[Grzadkowski, Kühn, Krawczyk, Stuart 1986]

[Guth, Kühn 1991]

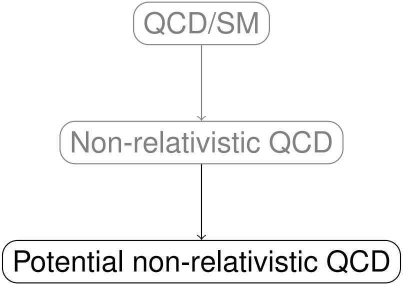
[Hoang, Reißer 2004 & 2006]



PNRQCD at higher orders

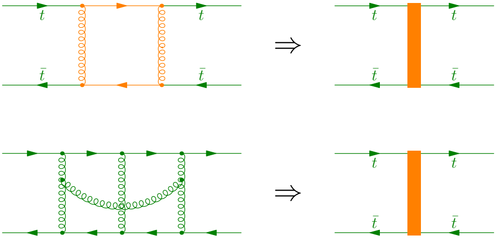
Scales: $m_t \gg m_t v \gg m_t v^2 \gg \Lambda_{\text{QCD}}$

- hard modes: $k \sim m_t \to$ (local) effective vertices
- soft modes: $k \sim m_t v \to$ (non-local) potentials
- potential light particle modes $\to$ (non-local) potentials
- potential top quark modes: $k_0 \sim m_t v^2, \vec{k} \sim m_t v$
- ultrasoft modes: $k \sim m_t v^2$



PNRQCD at higher orders

Soft matching



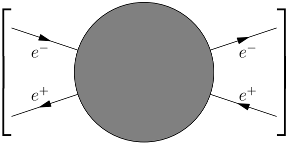
[Anzai, Kiyo, Sumino 2009]

[Smirnov, Smirnov, Steinhauser 2009]

[Lee, Smirnov, Smirnov, Steinhauser 2016]

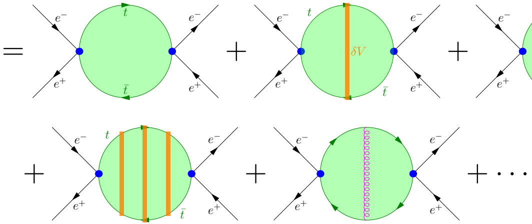
$e^+e^- \to t\bar{t}$ at NNNLO PNRQCD

$\sigma(e^+e^- \to t\bar{t}) \sim \text{Im}$



t or (W,b) cuts

$=$

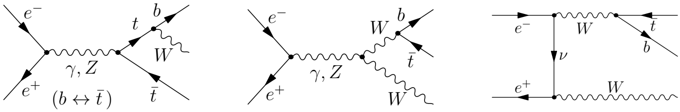


- $\sigma_{\text{LO}} \sim v \sum_k \left(\frac{\alpha_s}{v}\right)^k$
- N^{3}LO QCD + Higgs corrections
  $\sim \alpha_s^3 \sigma_{\text{LO}}, \alpha_s^2 v \sigma_{\text{LO}}, \alpha_s y_t^2 \sigma_{\text{LO}}, \ldots$
- N^{2}LO EW corrections $\sim \alpha \sigma_{\text{LO}}, \ldots$

Nonresonant production

Include top decay products (W^+b):

- Resonant production: $p_t^2 - m_t^2 \sim m_t\Gamma_t \hookrightarrow$ PNRQCD
- Nonresonant production: $p_t^2 - m_t^2 \sim m_t^2 \gg m_t\Gamma_t$
  or no top quark at all



Contributes at NLO $\sim \alpha/v \times \sigma_{\text{LO}}$

[Beneke, Jantzen, Ruiz-Femenía 2010]

QQbar_threshold

[Beneke, Kiyo, AM, Piclum 2016]

Public C++ library for $e^+e^- \to Q\bar{Q}$ near threshold:

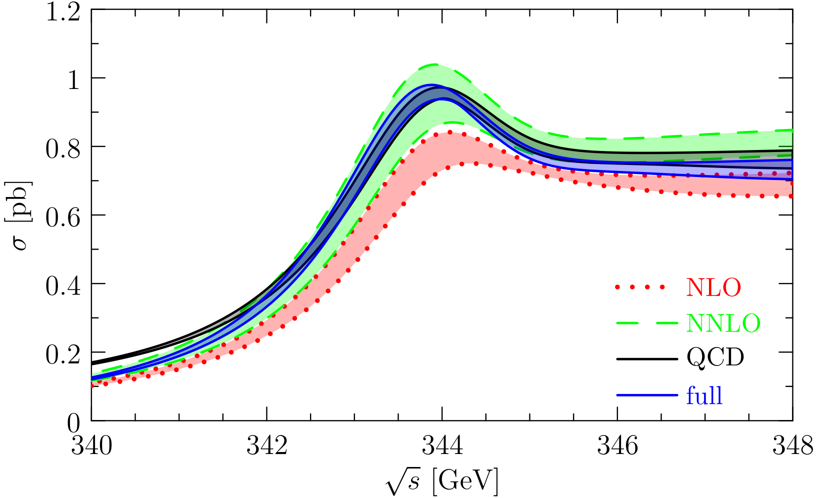
- N^{3}LO QCD + Higgs, N^{2}LO electroweak, NLO nonresonant production
- Top and bottom quarks
- Includes Mathematica package
- Extensive options:
  - Mass schemes: PS, 1S, $\overline{\text{MS}}$, pole
  - Loose invariant Wb mass cut: $(m_t - m_{Wb})^2 \gg \Gamma_t m_t$
  - Coarse and fine-grained control over contributions
  - …

```
Needs["QQbarThreshold‘"];
LoadGrid[GridDirectory <> "ttbar_grid.tsv"];
Plot[
   TTbarXSection[
      sqrts, {80., 350.}, {171.5, 1.33},
      "N3LO"
   ],
   {sqrts, 340, 348}
]
```

```
#include <iostream>
#include "QQbar_threshold/QQbar_threshold.hpp"
using namespace QQbar_threshold;
int main(){
  load_grid(grid_directory() + "ttbar_grid.tsv");
  std::cout << ttbar_xsection(
     344., {80., 350.}, {171.5, 1.33}, N3LO
  ) << '\n';
}
```

Results

Top-pair production cross section

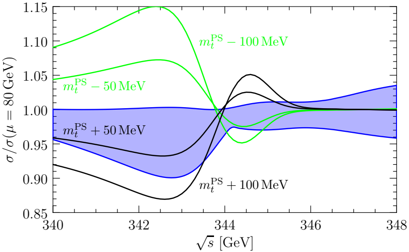


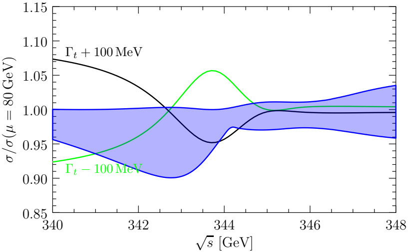
$m_t^{\mathsf{PS}}(20\,\mathsf{GeV}) = 171.5\,\mathsf{GeV}\,,\qquad \Gamma_t = 1.33\,\mathsf{GeV}\,,\qquad m_H = 125\,\mathsf{GeV}\,,$

$\alpha_s(m_Z) = 0.1177\,,\qquad \alpha(m_Z) = 1/128.944\,,\qquad m_W, m_Z$

Determination of top properties

Mass and width



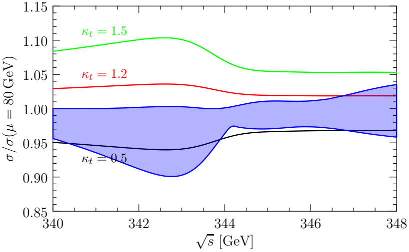


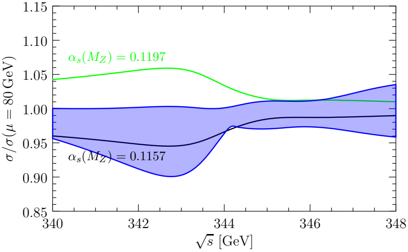
Preliminary experimental analysis [Simon 2016]:

- Symmetrised error $\Delta m_t = 45$ MeV
- Scale uncertainty dominant at 100 fb^{-1}

Determination of top properties

Yukawa coupling





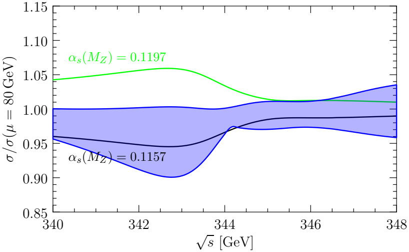
$$\kappa_t = y_t/y_t^{\mathsf{SM}}\,, \qquad y_t^{\mathsf{SM}} = \frac{\sqrt{2}m_t}{v}$$

from adding operator

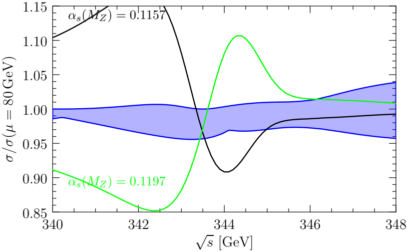
$$\Delta\mathcal{L} = -\frac{c_{\mathsf{NP}}}{\Lambda^2}(\phi^\dagger\phi)(\bar{Q}_3 i\sigma^2\phi^* t_R) + \text{h.c.}\,,$$

Determination of top properties

PS vs. $\overline{\text{MS}}$ scheme



$m_t^{\text{PS}}(20\,\text{GeV}) = 171.5\,\text{GeV}$

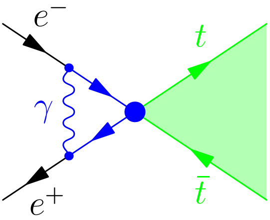


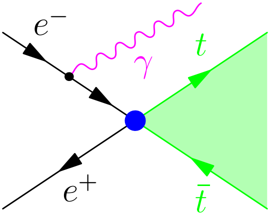
$\overline{m}_t(\overline{m}_t) = 163.3\,\text{GeV}$

Outlook

Initial state radiation

Photon corrections to initial state:





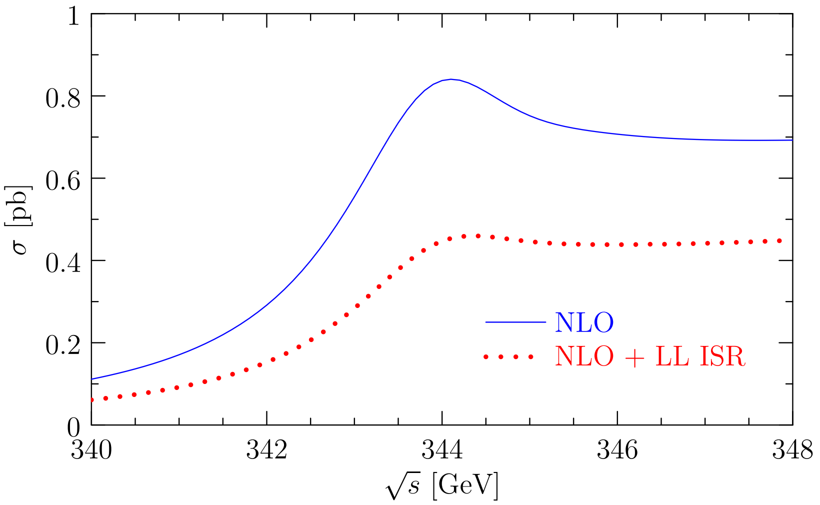
γ hard, hard-collinear γ ultrasoft, (ultra)soft-collinear

$\hookrightarrow$ large logarithms $\log^2 \frac{m_t}{m_e}$, resummed into structure functions

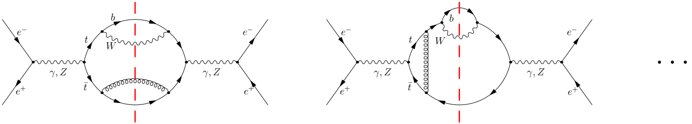
[Fadin, Kuraev 1985; Fadin, Khoze 1987]

$$\sigma(s) = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \, \Gamma_{ee}(x_1)\Gamma_{ee}(x_2)\hat{\sigma}(x_1 x_2 s) + \{\text{NNLO}\}$$

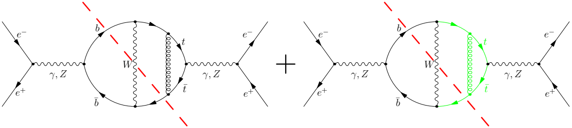
Initial state radiation



Nonresonant production at NNLO

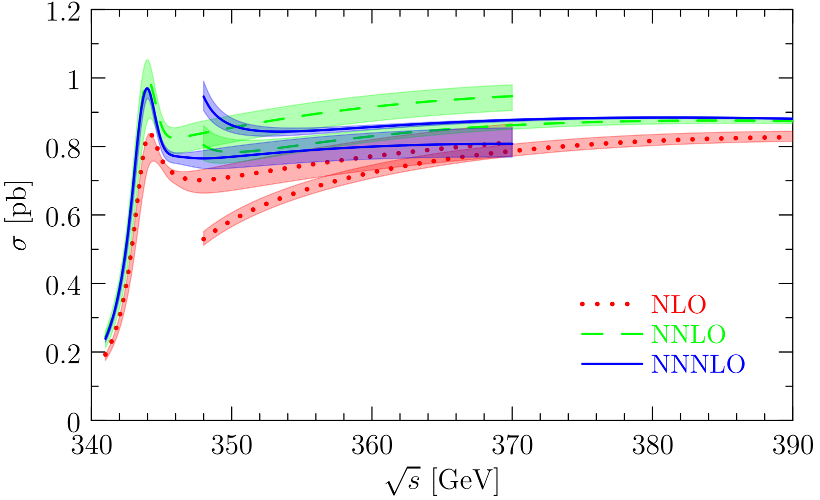


- Naïve expansion in $\Gamma_t \to 0$
  $\Rightarrow$ *endpoint divergences* as $p^2_{t,\bar{t}} \to m_t^2$
- Mostly automatised calculation (modified MadGraph 5)
  + manual calculation of endpoint divergent contributions in dimensional regularisation
- Divergences cancel against resonant part, [Jantzen, Ruiz-Femenía 2013]
  e.g. hard NNLO electroweak corrections:



$t\bar{t}$ production above threshold

Comparison to fixed-order perturbation theory (QCD only):



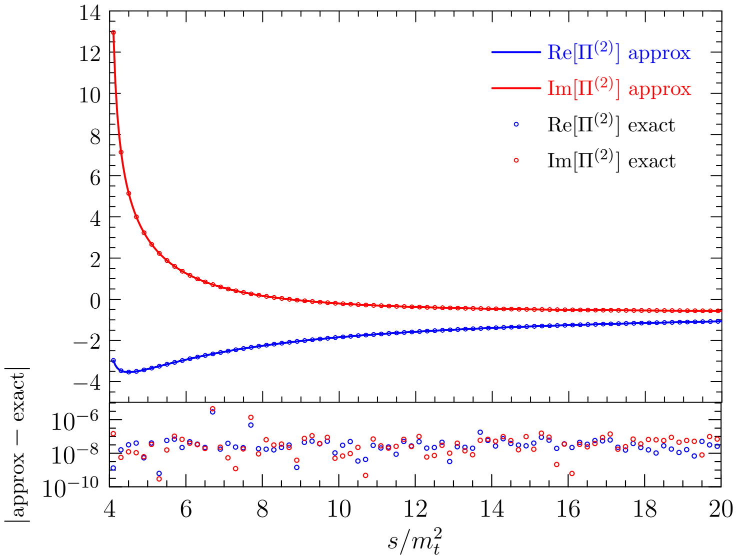
$t\bar{t}$ production above threshold

Fixed-order cross section

$$\sigma_{\mathsf{FO}}(s) \propto \mathrm{Im}\left[\sum\left(\frac{\alpha_s}{\pi}\right)^i \Pi^{(i)}(s+i\epsilon)\right]$$

- Analytic results for $\Pi^{(0)}(s), \Pi^{(1)}(s)$
- Approximate $\Pi^{(i)}(s), i > 1$ using known expansions
  [Hoang, Mateu, Zebarjad 2008; Kiyo, AM, Maierhöfer, Marquard 2009]
  - $s \to 0$: 30 terms for $\Pi^{(2)}(s)$, 3 terms for $\Pi^{(3)}(s)$
  - $s \to -\infty$: 30 terms for $\Pi^{(2)}(s)$, 2 (3) terms for $\Pi^{(3)}(s)$
  - $s \to 4m^2$: 3 terms for $\Pi^{(2,3)}(s)$ from NNLO PNRQCD
- Exact result for $\Pi^{(2)}(s)$:
  - Reduce to master integrals
  - Solve differential equations numerically

$t\bar{t}$ production above threshold



Conclusion

- $\sigma(e^+e^- \to t\bar{t})$ near threshold in PNRQCD with $\mathrm{N^3LO}$ QCD + $\mathrm{N^2LO}$ electroweak + NLO nonresonant corrections
- Allows precise measurement of width, mass with $\Delta m_t^{\mathrm{PS}} < 100\,\mathrm{MeV}$
- Precise and reliable value for α_s needed for extraction of Yukawa coupling and $\overline{\mathrm{MS}}$ mass
- Work in progress:
  - Initial state radiation
  - $\mathrm{N^2LO}$ nonresonant corrections
  - Matching to high-energy fixed-order prediction

Backup

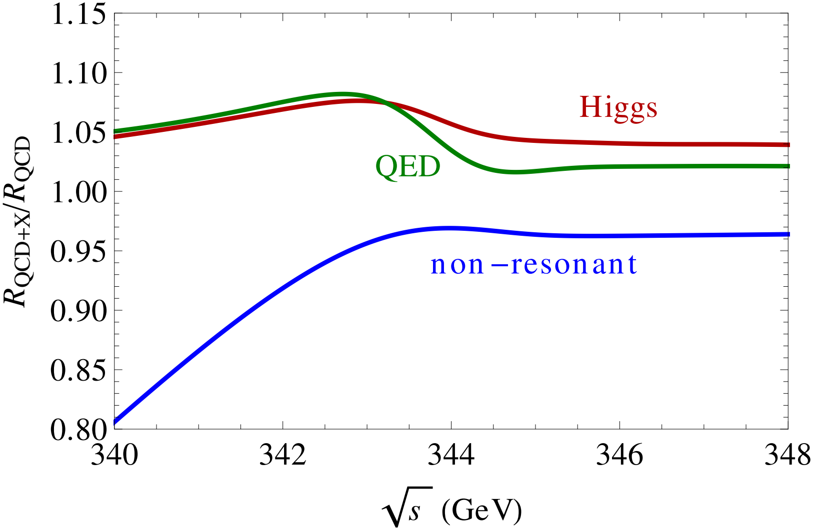
Width in nonrelativistic expansion

Position of complex pole: $p_{t,\bar{t}} = M_*^2 \equiv m_t^2 - i m_t \Gamma_t$

$\Rightarrow$ Kinetic terms in nonrelativistic theory:

$$\mathcal{L}_{\text{kin},\psi} = \psi^\dagger \left(i\partial^0 + \frac{\vec{\partial}^2 + i m_t \Gamma_t}{2m_t} + \frac{(\vec{\partial}^2 + i m_t \Gamma_t)^2}{8m_t^3} + \dots \right) \psi$$

Time dilatation: $\Gamma_{t\bar{t}} < 2\Gamma_t$


1.15
1.10
1.05
1.00
0.95
0.90
0.85
0.80
340
342
344
346
348
$R_{\text{QCD+X}}/R_{\text{QCD}}$
$\sqrt{s}$ (GeV)
Higgs
QED
non-resonant