An iterative Monte Carlo determination of spin-dependent PDFs and fragmentation functions

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Outline

- IMC methodology
  - Motivation
  - Toy Model
- Extraction of spin PDFs (arXiv:1601.07782)
  - Motivation
  - Target mass corrections/Higher Twist
  - Results
- Extraction of fragmentation functions
  - Background
  - IMC revised
  - Results

Background

- Collinear factorization → QCD observables split into hard scattering cross section and universal, non-perturbative functions
- Deep-inelastic scattering (DIS)

$$d\sigma = \sum_f \int d\xi f(\xi) d\hat{\sigma}$$

- Semi-inclusive DIS (SIDIS)

$$d\sigma = \sum_f \int d\xi d\zeta f(\xi) D(\zeta) d\hat{\sigma}$$

- Single-inclusive annihilation (SIA)

$$d\sigma = \sum_f \int d\zeta D(\zeta) d\hat{\sigma}$$

Background

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Parton distribution functions

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Fragmentation functions

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$$d\sigma = \sum_f \int d\xi f(\xi) d\hat{\sigma}$$

- Semi-inclusive DIS (SIDIS)

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PDFs and fragmentation functions must be determined through analyses of experimental data

- Single-inclusive annihilation (SIA)

$$d\sigma = \sum_f \int d\zeta D(\zeta) d\hat{\sigma}$$

Traditional Fitting Method

- Functional form for PDF and FF

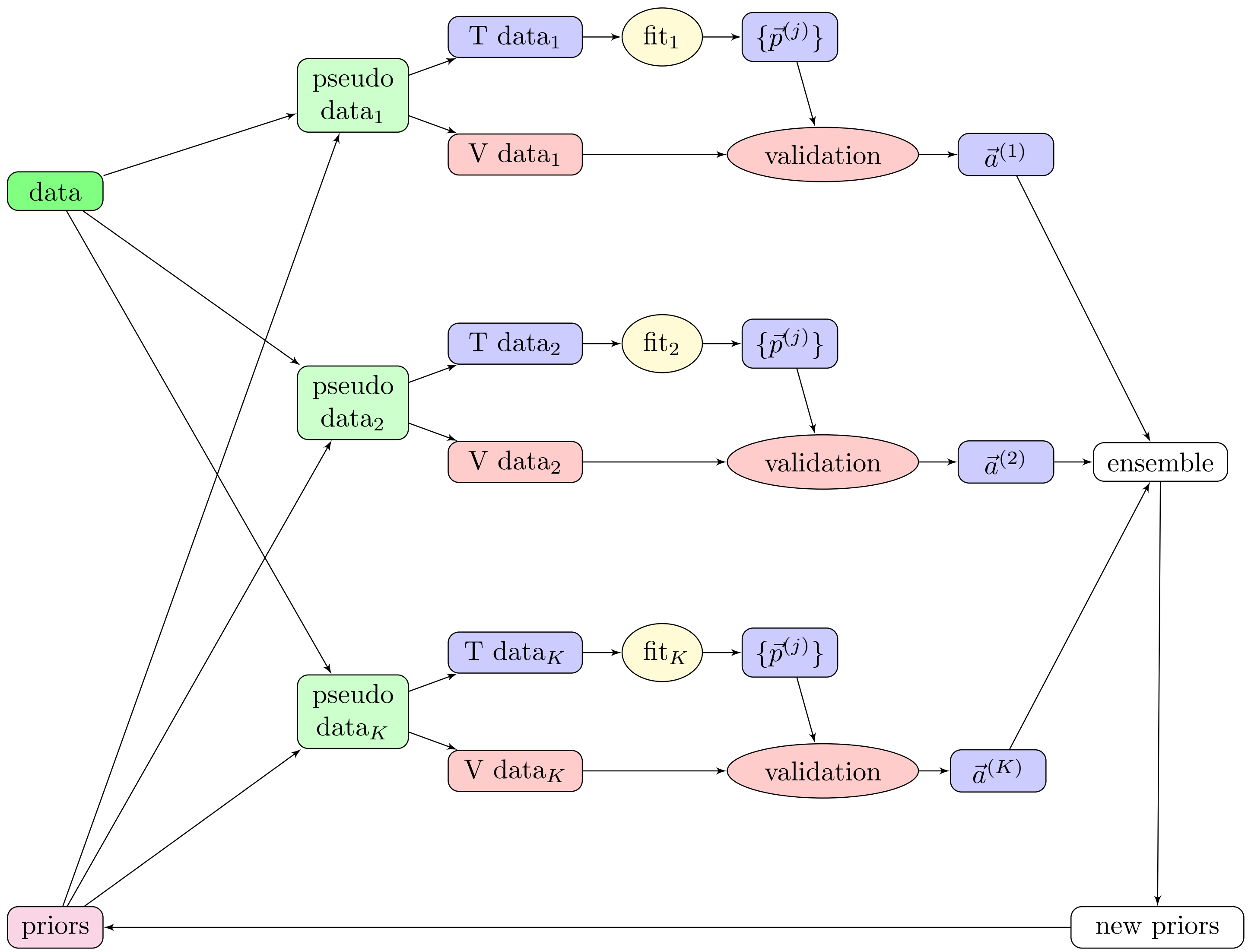
$$xf(x) = Nx^a(1-x)^b(1+c\sqrt{z}+dz)$$

- Single χ^2 fit of parameters
  - → Typically fix parameters that are difficult to constrain
- Uncertainties determined by Hessian or Lagrange multiplier methods
  - → Introduces tolerance criteria
- Since χ^2 is a highly non-linear function of the fit parameters, there can be various local minima
- How do we improve on this?

Iterative Monte Carlo (IMC) fitting!

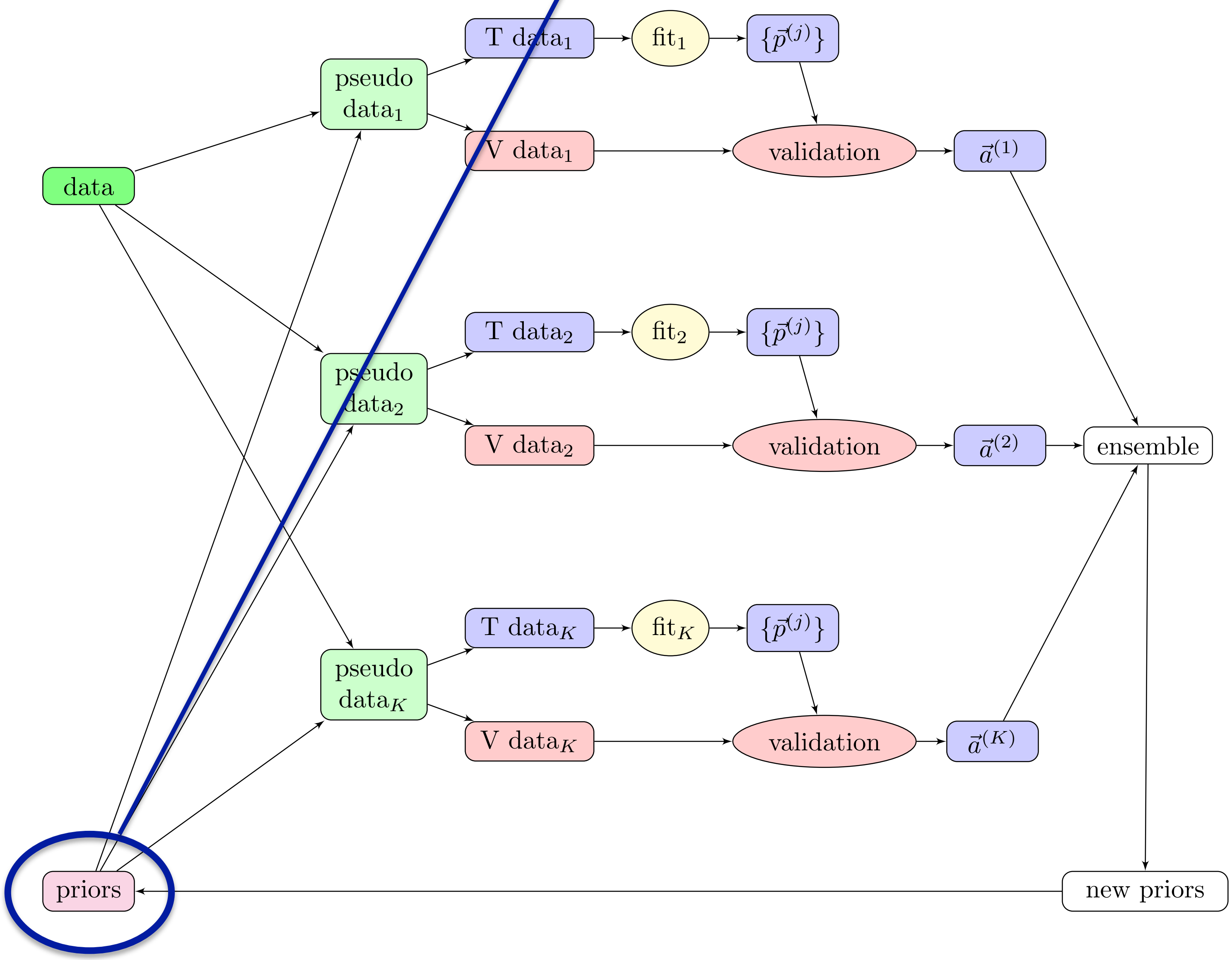
IMC Methodology

Same functional form for distributions



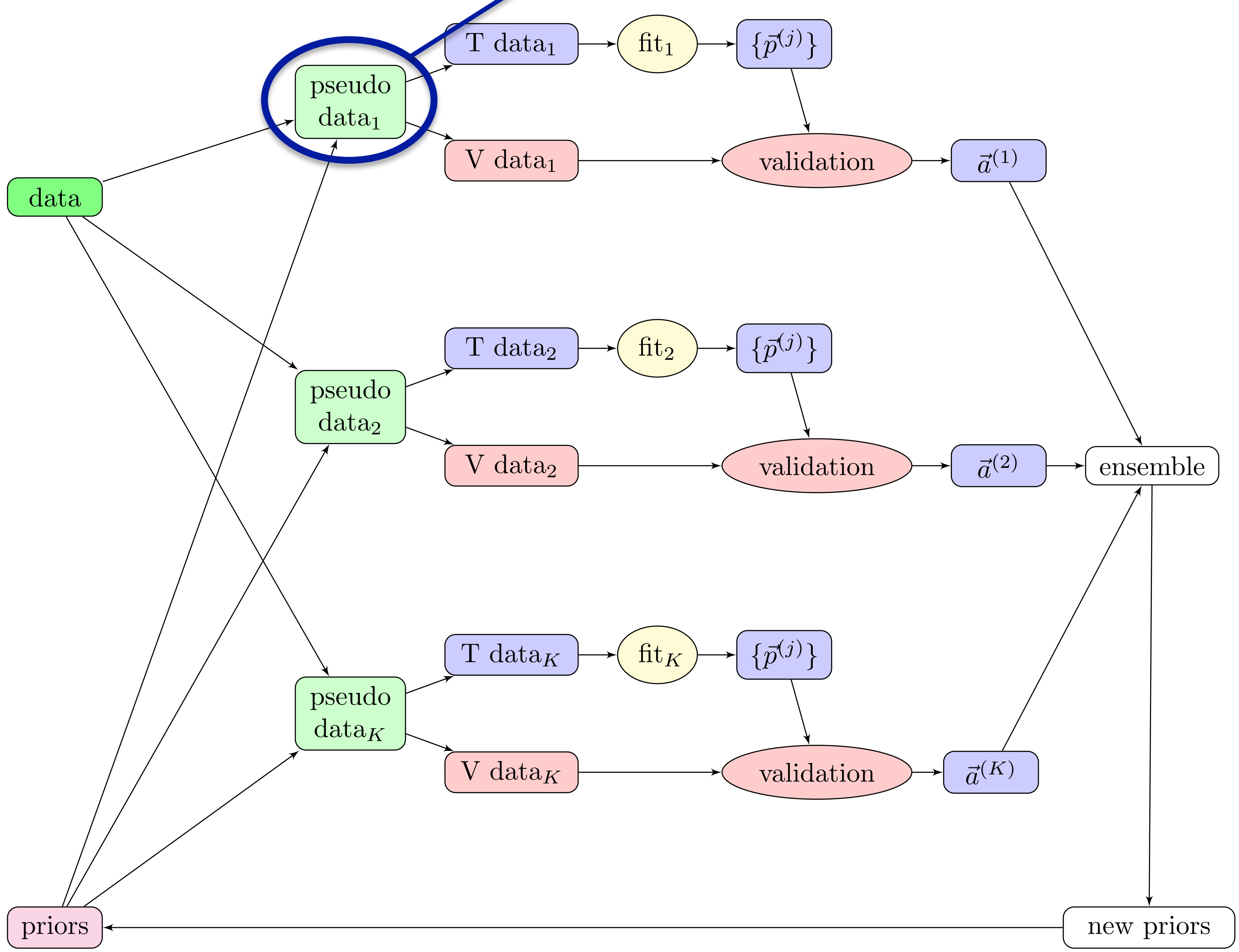
IMC Methodology

Flat sampling of priors → all parameters free



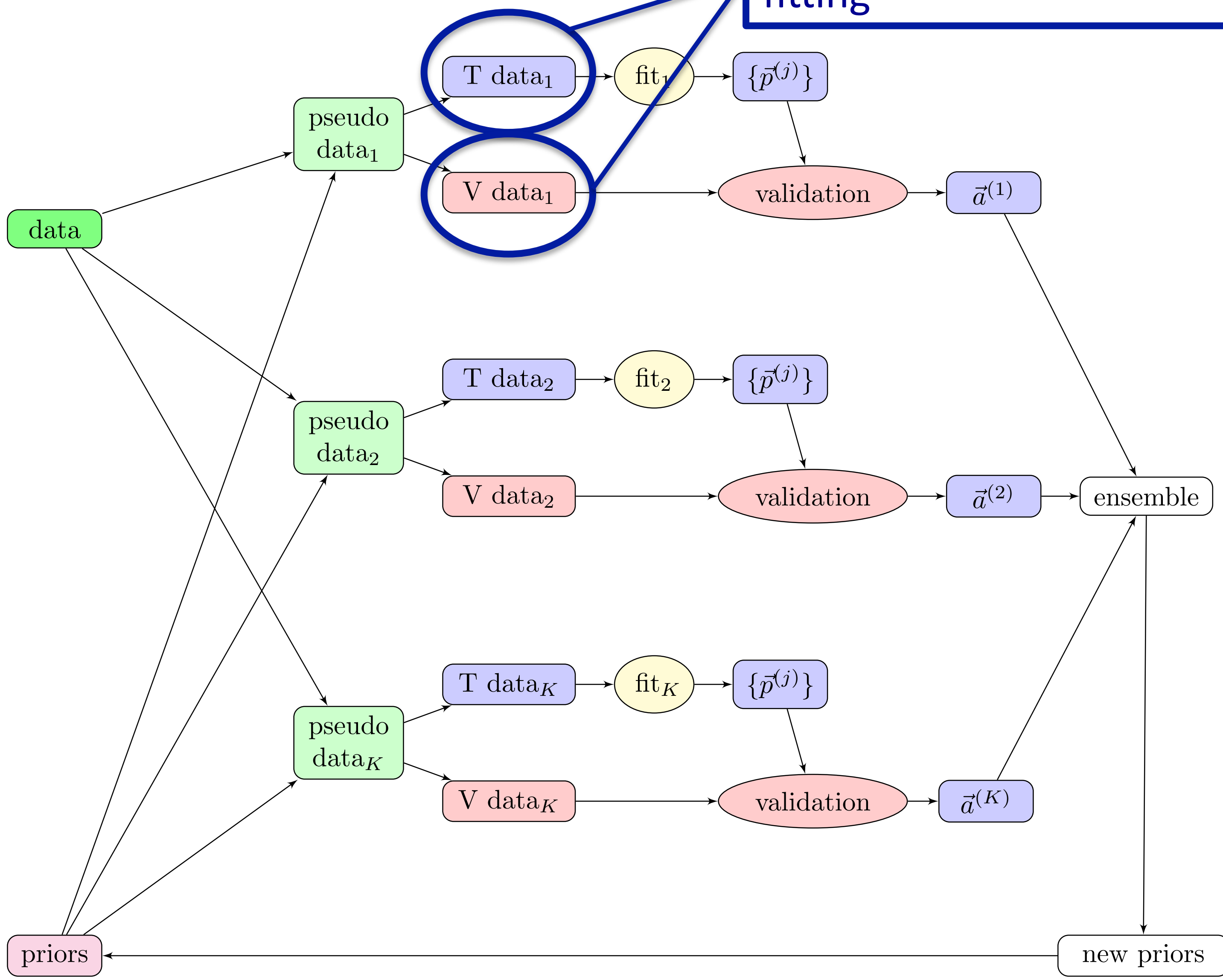
IMC Methodology

Bootstrap method of data sets

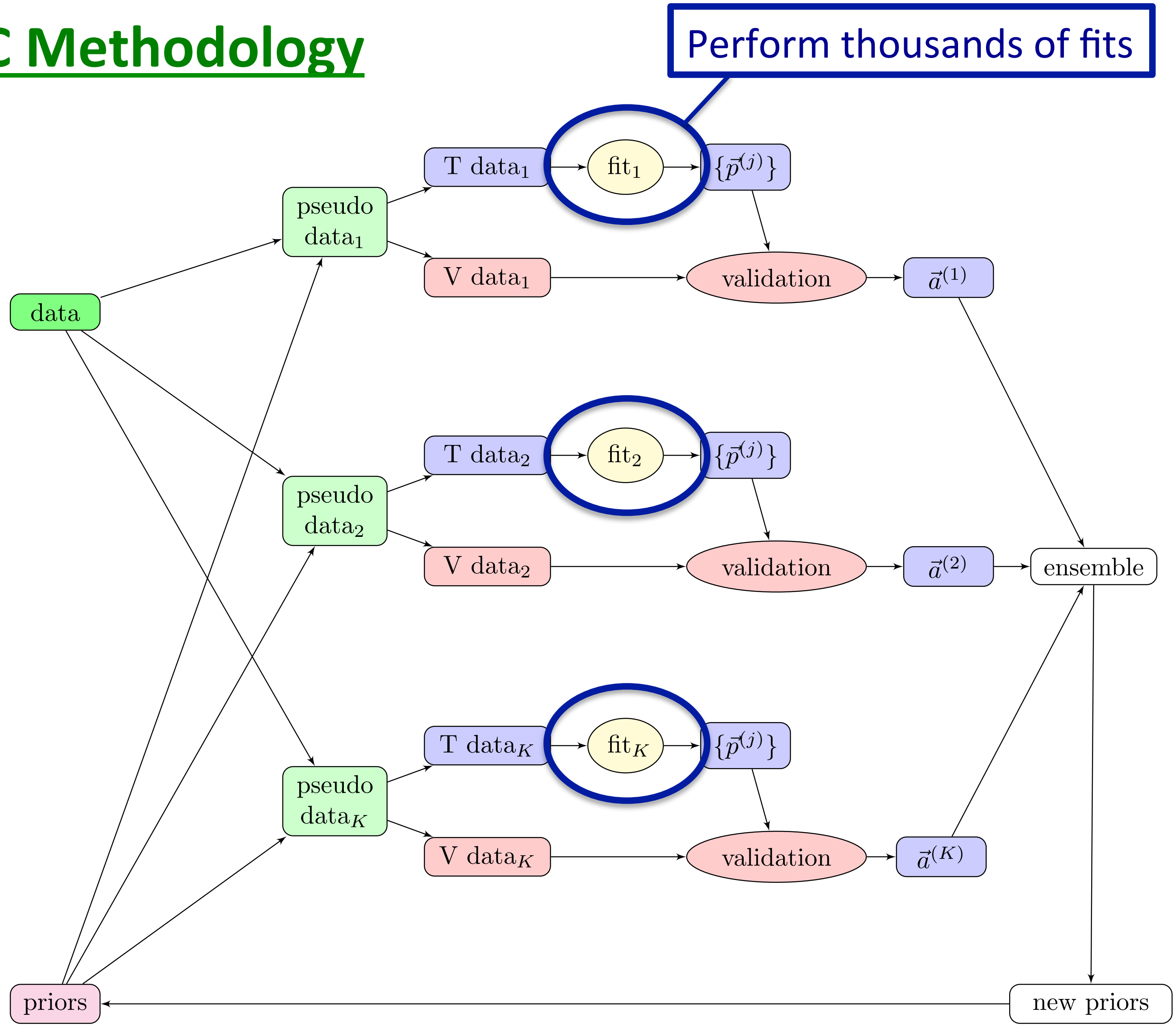


IMC Methodology

Cross-validation to prevent over-fitting

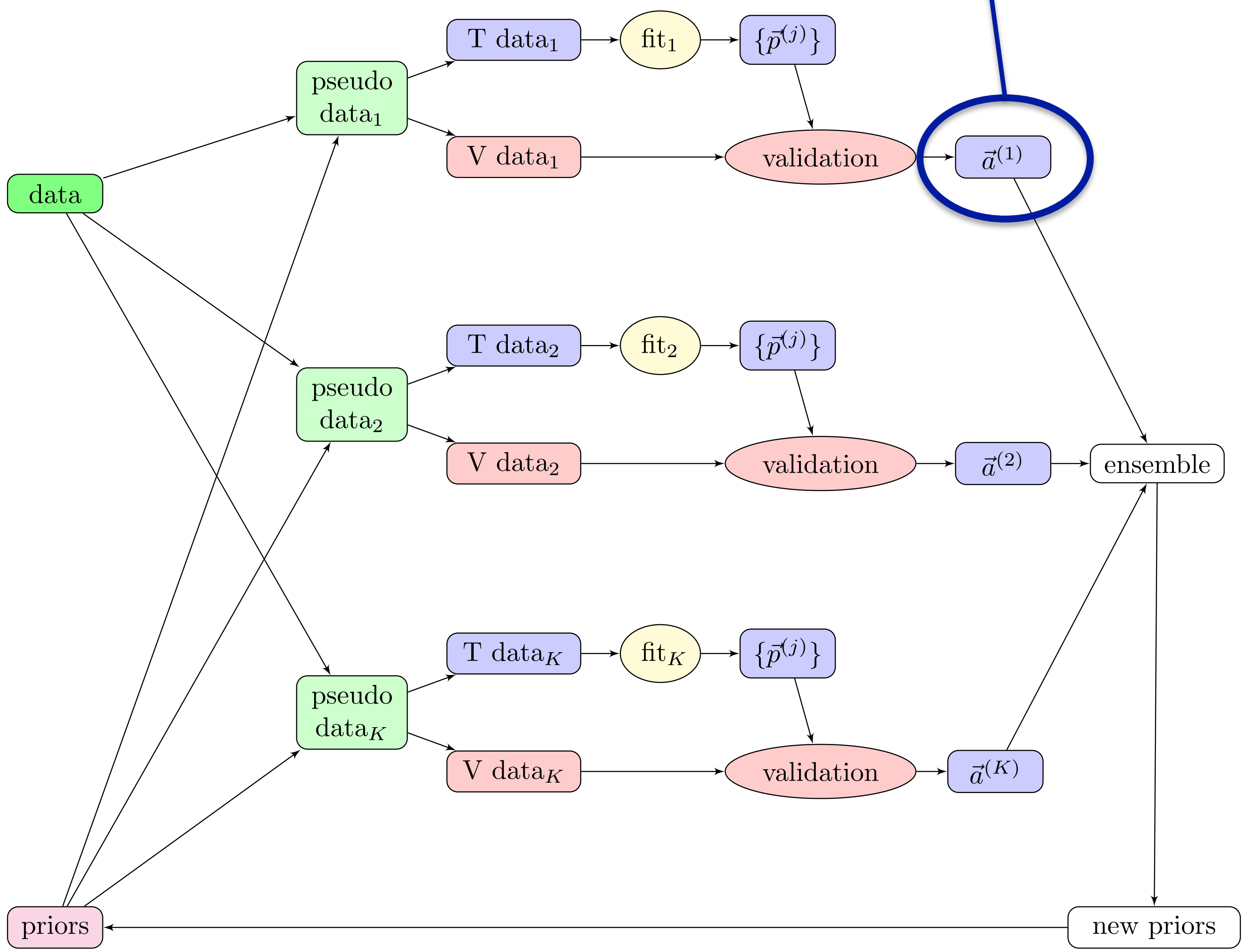


IMC Methodology



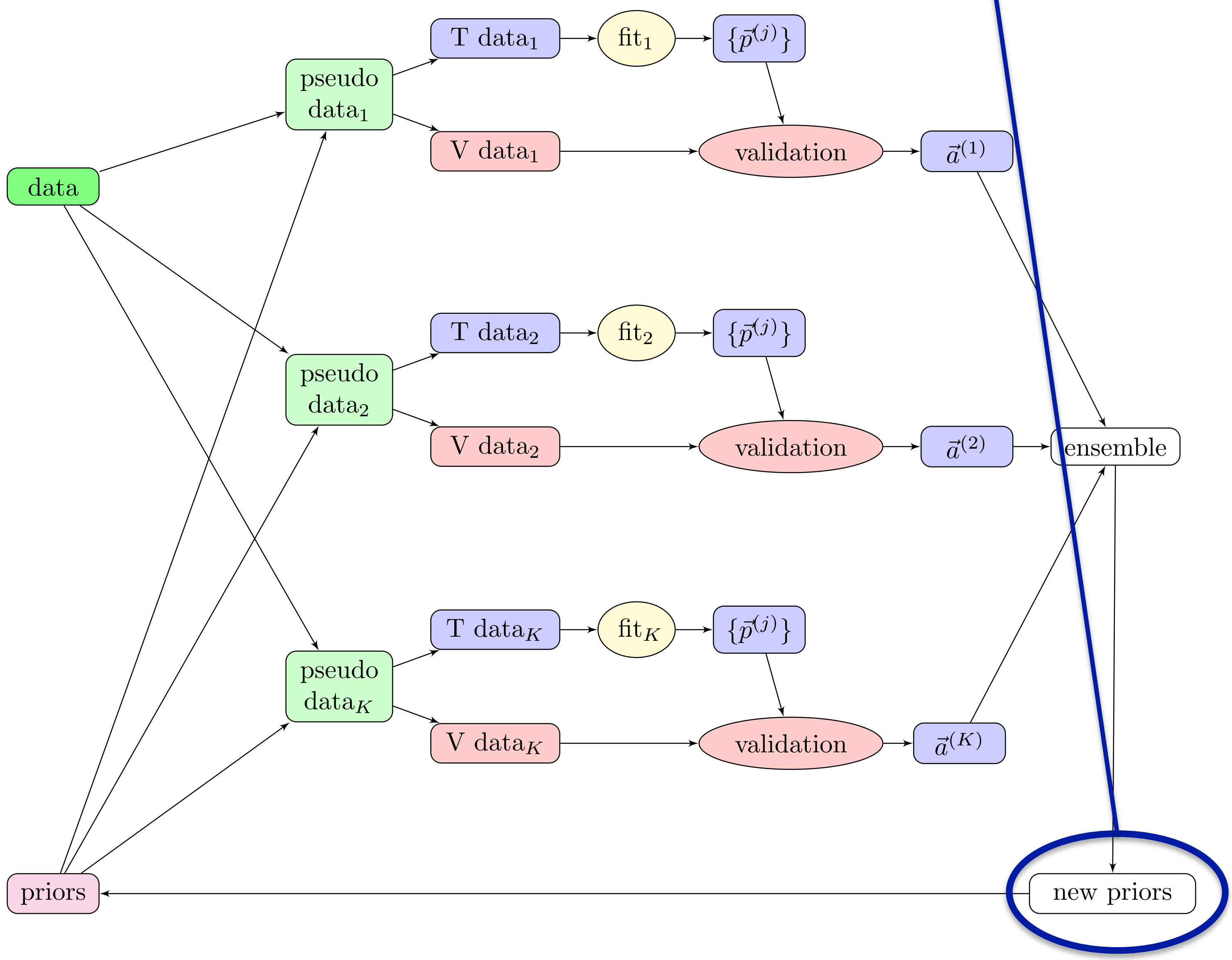
IMC Methodology

Choose final posteriors that minimize validation χ^2

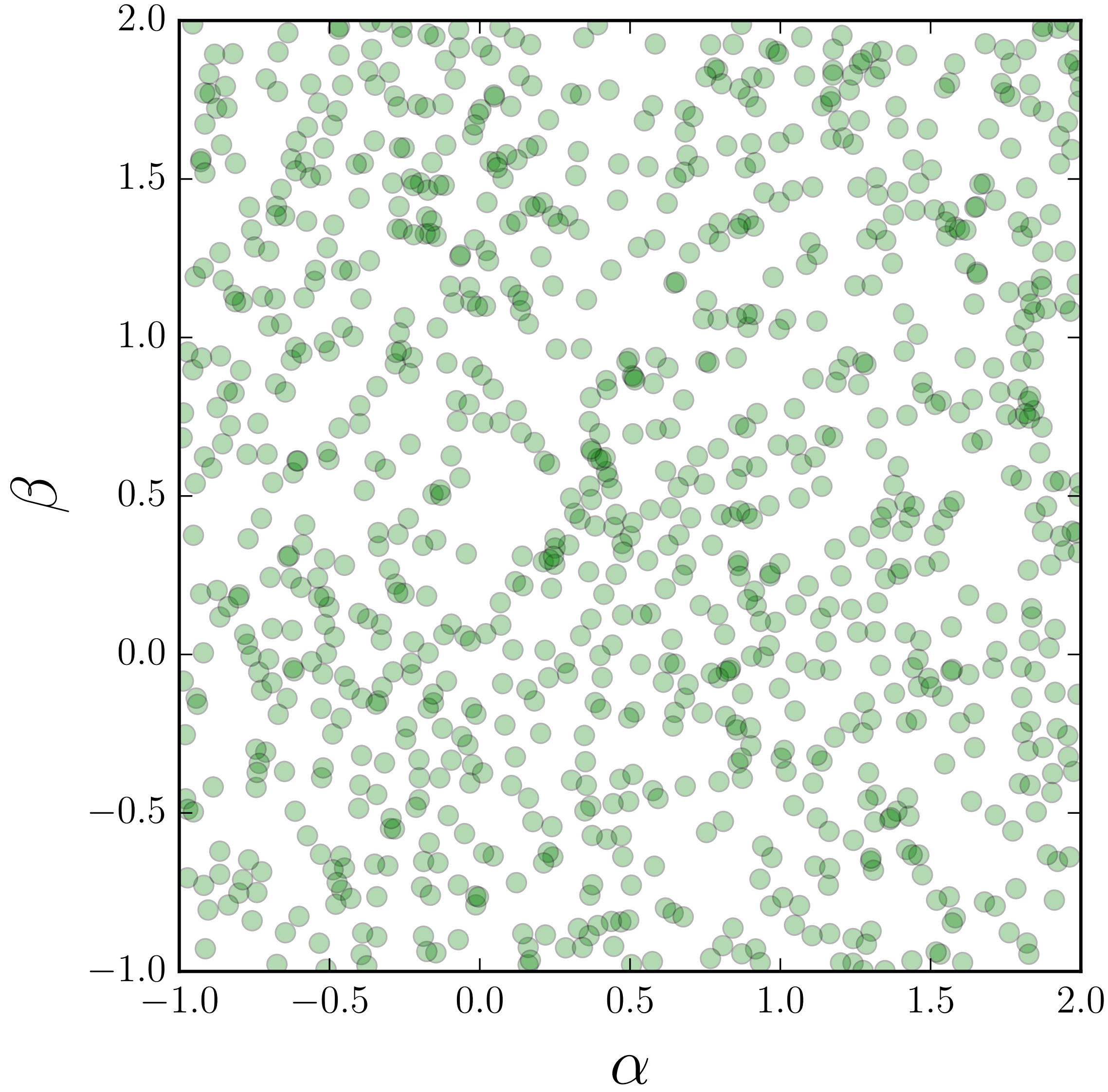


IMC Methodology

Iterate → use posteriors as priors for new set of fits



IMC Methodology

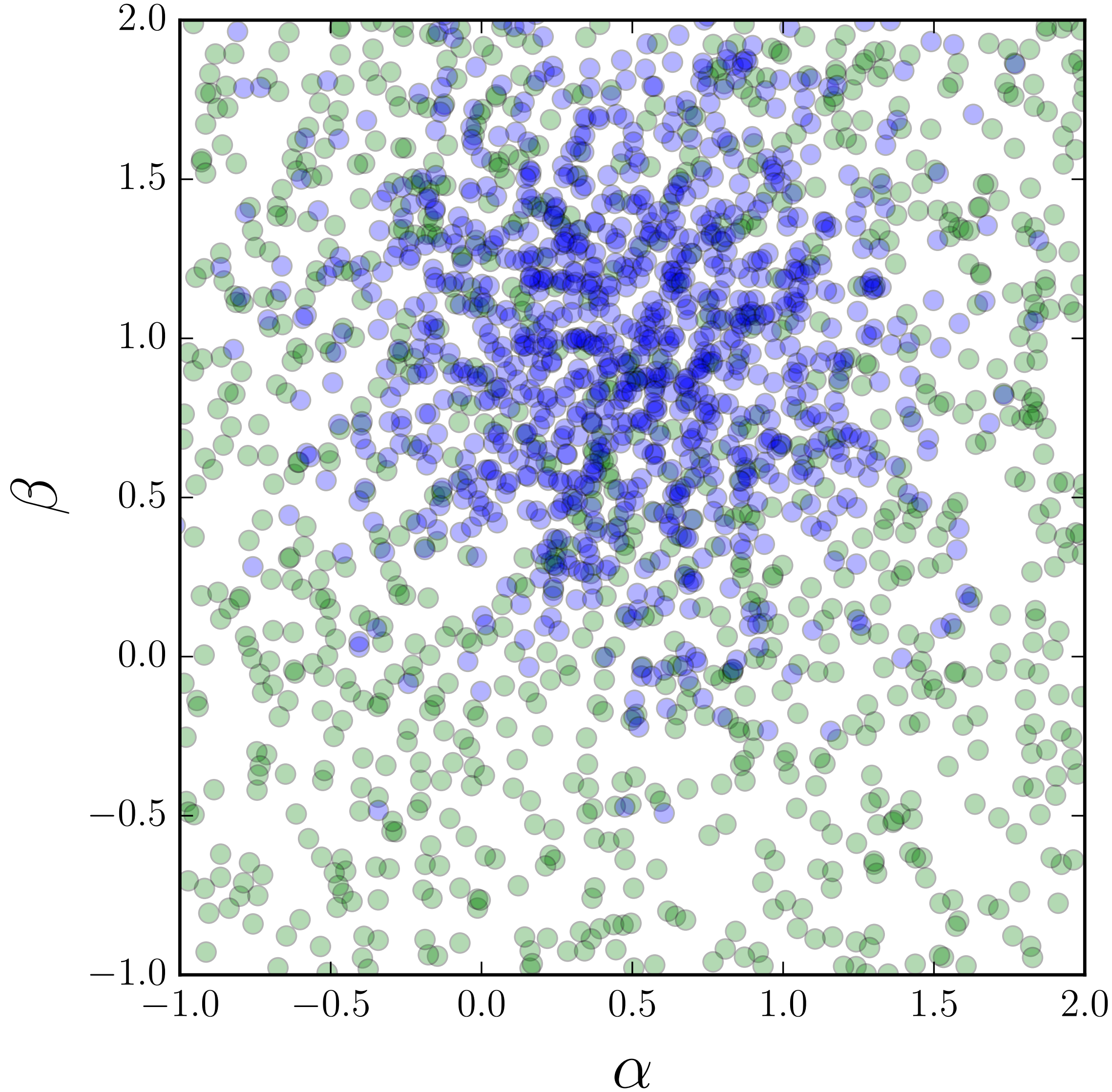


Toy Model

- Flat sampling of initial priors

$$\{\alpha, \beta\}$$

IMC Methodology



Toy Model

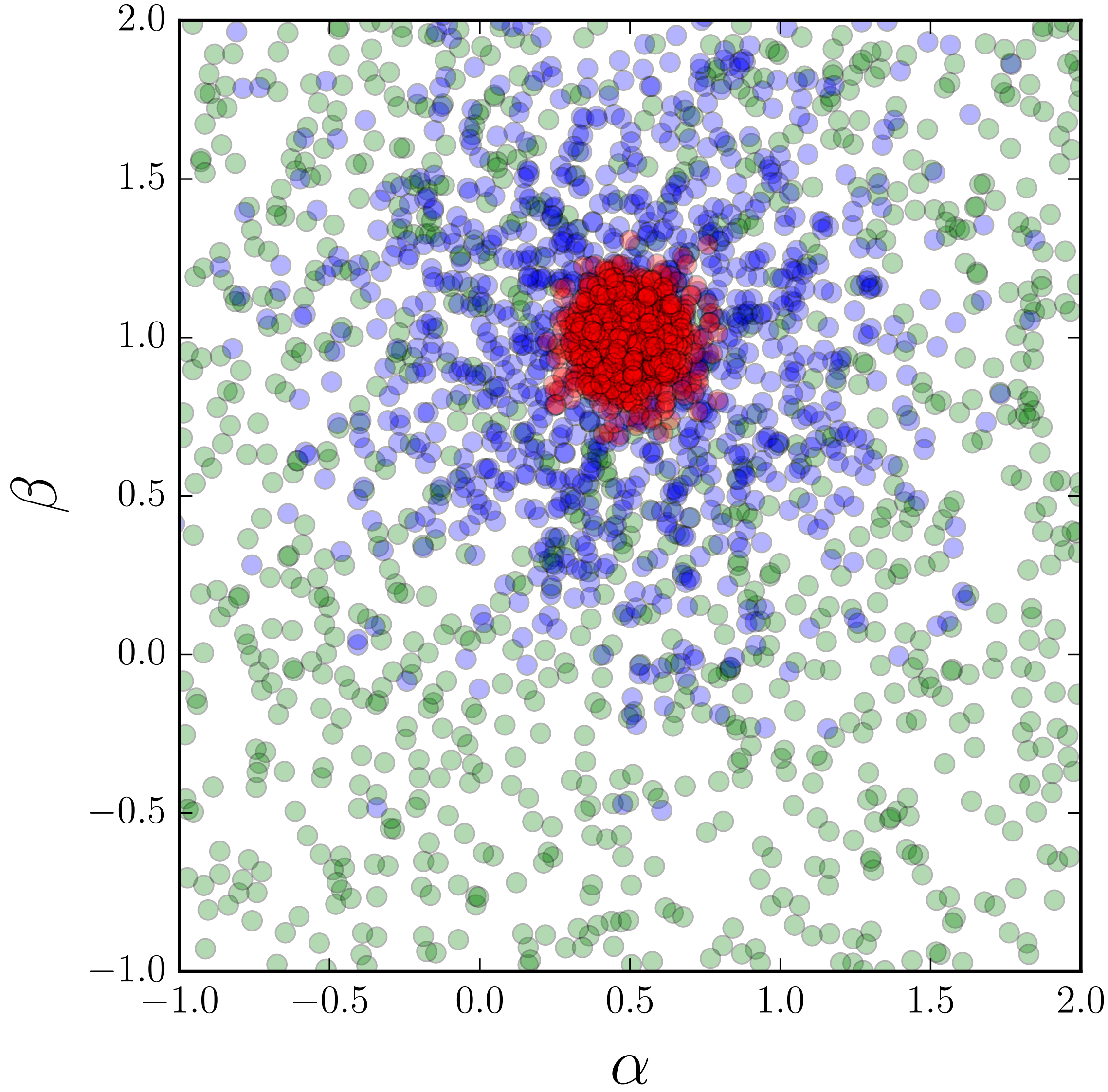
- Flat sampling of initial priors

$$\{\alpha, \beta\}$$

- Initial set of fits → posteriors

$$\{\alpha, \beta\} \rightarrow \{\alpha, \beta\}$$

IMC Methodology



Toy Model

- Flat sampling of initial priors

$$\{\alpha, \beta\}$$

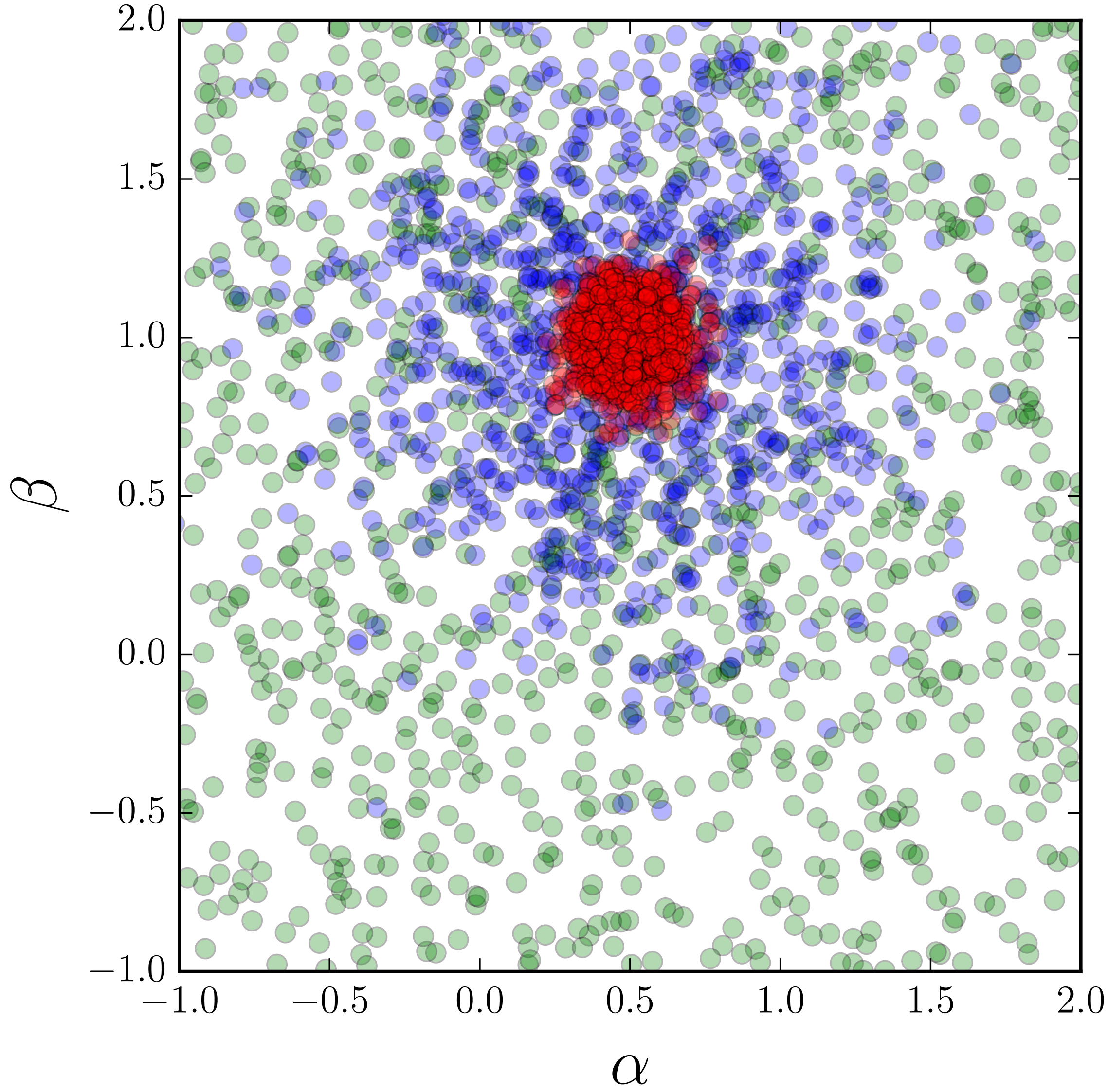
- Initial set of fits → posteriors

$$\{\alpha, \beta\} \to \{\alpha, \beta\}$$

- Posteriors → priors for first iteration → new posteriors

$$\{\alpha, \beta\} \to \{\alpha, \beta\}$$

IMC Methodology



Toy Model

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$$\{\alpha, \beta\} \rightarrow \{\alpha, \beta\}$$

- Posteriors → priors for first iteration → new posteriors

$$\{\alpha, \beta\} \rightarrow \{\alpha, \beta\}$$

- Repeat until convergence…

Spin PDFs

Proton spin

- Spin sum rule: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + \mathcal{L}$

$$\Delta q^+ = \Delta q + \Delta \bar{q}$$

→ Quark contribution: $\Delta\Sigma(Q^2) = \int_0^1 dx\, \left(\Delta u^+(x,Q^2) + \Delta d^+(x,Q^2) + \Delta s^+(x,Q^2)\right)$
$\approx 0.3_{[10^{-3},1]}$

→ Gluon contribution: $\Delta g(Q^2) = \int_0^1 dx \Delta g(x,Q^2) \approx 0.1_{[0.05,0.2]}$

→ Orbital angular momentum: determined from GPDs

- Higher Twist Information

→ d_2 matrix element: $d_2(Q^2) = \int_0^1 dx x^2 (2g_1^{\tau 3}(x,Q^2) + 3g_2^{\tau 3}(x,Q^2))$

→ Electric and magnetic color forces:

$$F_E = 2d_2 + f_2 \quad F_B = 4d_2 - f_2$$

Proton spin

- Spin sum rule: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + \mathcal{L}$ $\Delta q^+ = \Delta q + \Delta\bar{q}$

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From previous analyses

→ Orbital angular momentum: determined from GPDs

- Higher Twist Information

→ d_2 matrix element: $d_2(Q^2) = \int_0^1 dx x^2 (2g_1^{\tau 3}(x,Q^2) + 3g_2^{\tau 3}(x,Q^2))$

→ Electric and magnetic color forces:

$$F_E = 2d_2 + f_2 \quad F_B = 4d_2 - f_2$$

Polarized DIS

- We fit measurements of parallel and perpendicular spin asymmetries

$$A_{\parallel} = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} = D\,(A_1 + \eta A_2)$$

$$A_{\perp} = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\uparrow\Leftarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\uparrow\Leftarrow}} = d\,(A_2 + \zeta A_1)$$

$$A_1 = \frac{(g_1 - \gamma^2 g_2)}{F_1} \qquad A_2 = \gamma\frac{(g_1 + g_2)}{F_1} \qquad \gamma^2 = \frac{4M^2x^2}{Q^2}$$

- Define our polarized structure functions:

$$g_1(x,Q^2) = g_1^{\mathrm{LT+TMC}}(\Delta u^+, \Delta d^+, \Delta g, ...) + g_1^{\mathrm{T3+TMC}}(D_u, D_d) + g_1^{\mathrm{T4}}(H_{p,n})$$

$$g_2(x,Q^2) = g_2^{\mathrm{LT+TMC}}(\Delta u^+, \Delta d^+, \Delta g, ...) + g_2^{\mathrm{T3+TMC}}(D_u, D_d)$$

Leading Twist Structure Functions

- Leading twist defined as usual

$$g_1^{(\tau 2)}(x,Q^2) = \frac{1}{2}\sum_q e_q^2 \left[(\Delta C_q \otimes \Delta q^+)(x,Q^2) + \ (\Delta C_g \otimes \Delta g)(x,Q^2)\right]$$

- Leading twist + target mass corrections $\xi = \frac{2x}{1+\rho}, \qquad \rho^2 = 1+\gamma^2$

J. Blümlein and A. Tkabladze
Nucl. Phys. B553, 427 (1999)

$$g_2^{(\tau 2+\mathrm{TMC})}(x,Q^2) = -\frac{x}{\xi\rho^3}\, g_1^{(\tau 2)}(\xi,Q^2) + \frac{1}{\rho^4}\int_\xi^1 \frac{dz}{z}\left[\frac{x}{\xi} - (\rho^2-1) + \frac{3(\rho^2-1)}{2\rho}\ln\frac{z}{\xi}\right] g_1^{(\tau 2)}(z,Q^2)$$

$$g_1^{(\tau 2+\mathrm{TMC})}(x,Q^2) = \frac{x}{\xi\rho^3}\, g_1^{(\tau 2)}(\xi,Q^2) + \frac{(\rho^2-1)}{\rho^4}\int_\xi^1 \frac{dz}{z}\left[\frac{(x+\xi)}{\xi} - \frac{(3-\rho^2)}{2\rho}\ln\frac{z}{\xi}\right] g_1^{(\tau 2)}(z,Q^2)$$

- In Bjorken limit $(Q^2 \to \infty)$:

$$g_1^{(\tau 2+\mathrm{TMC})} = g_1^{(\tau 2)} \qquad g_2^{(\tau 2)}(x,Q^2) = -g_1^{(\tau 2)}(x,Q^2) + \int_x^1 \frac{dz}{z} g_1^{(\tau 2)}(z,Q^2)$$

Higher Twist Structure Functions

- Twist-3 polarized structure functions

$$g_1^{(\tau 3+\mathrm{TMC})}(x,Q^2)=\frac{(\rho^2-1)}{\rho^3}\,D(\xi,Q^2)-\frac{(\rho^2-1)}{\rho^4}\int_\xi^1\frac{dz}{z}\left[3-\frac{(3-\rho^2)}{\rho}\ln\frac{z}{\xi}\right]D(z,Q^2)$$

$$g_2^{(\tau 3+\mathrm{TMC})}(x,Q^2)=\frac{1}{\rho^3}\,D(\xi,Q^2)-\frac{1}{\rho^4}\int_\xi^1\frac{dz}{z}\left[3-2\rho^2+\frac{3(\rho^2-1)}{\rho}\ln\frac{z}{\xi}\right]D(z,Q^2)$$

- Twist-3 parton distributions:

$$D(x,Q^2)=\sum_q e_q^2 D_q(x,Q^2)$$

- In Bjorken limit $(Q^2\to\infty)$:

$$g_1^{(\tau 3)}=0 \qquad\qquad g_2^{(\tau 3)}(x,Q^2)=D(x,Q^2)-\int_x^1\frac{dz}{z}D(z,Q^2)$$

Twist-4 and Nuclear Structure Functions

- Twist-4 parameterized at the hadron level:

$$g_1^{\tau 4(p,n)}(x,Q^2) = \frac{H_{p,n}(x,Q^2)}{Q^2}$$

- Nuclear corrections:

$$g_i^A(x,Q^2) = \sum_{\tau=p,n} \int_x^A \frac{dz}{z} f_{ij}^{\tau/A}(z,\rho)\, g_j^\tau\left(\frac{x}{z},Q^2\right)$$

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Nuclear smearing function

Nucleon structure function

$$\rho = \sqrt{1+\gamma^2}$$

Parameterizations and Chi-square

- Parameterization: $xf(x) = Nx^a(1-x)^b(1+c\sqrt{z}+dz)$

  → Higher twist distributions parameterized the same ($D_{u,d}$; $H_{p,n}$)

  → Additional constraints from weak neutron and hyperon decay:

$$\int_0^1 dx\,\left(\Delta u^+ - \Delta d^+\right) = g_A = 1.269(3)$$

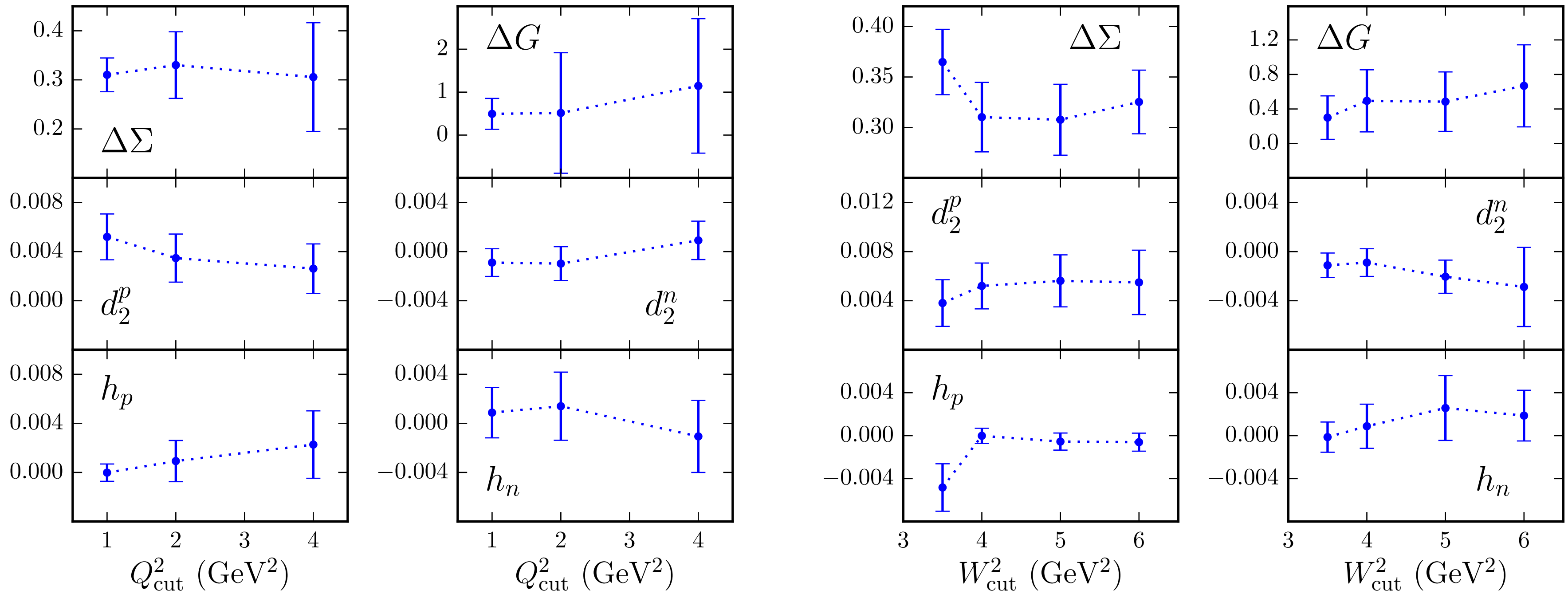
$$\int_0^1 dx\,\left(\Delta u^+ + \Delta d^+ - 2\Delta s^+\right) = a_8 = 0.586(31)$$

- Chi-squared definition:

$$\chi^2 = \sum_i \left(\frac{D_i - T_i(1-\sum_k r^k \beta_i^k / D_i)^{-1}}{\alpha_i}\right)^2 + \sum_k (r^k)^2$$

- Include correlated systematic uncertainties

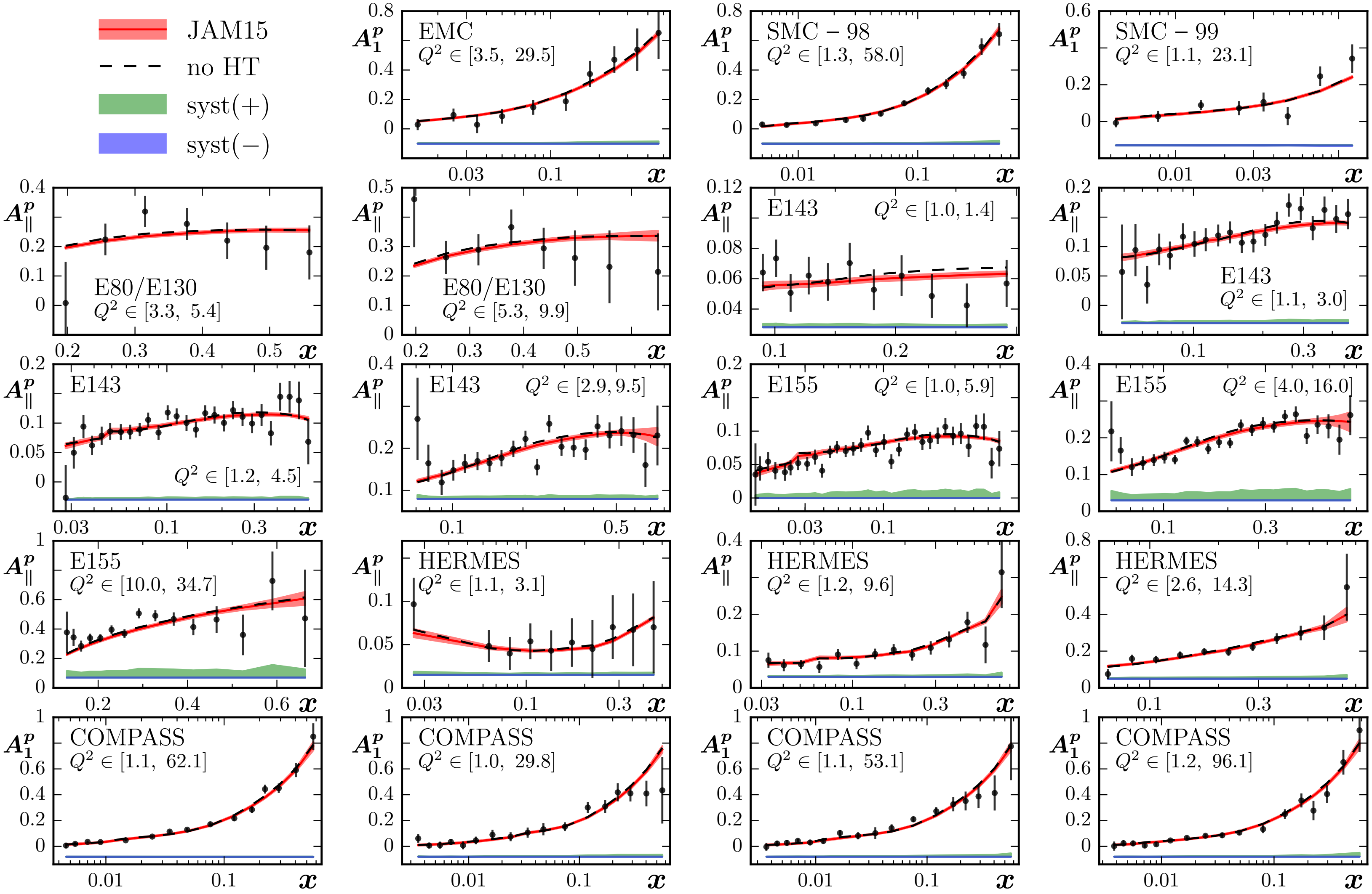
W2 and Q2 Cuts



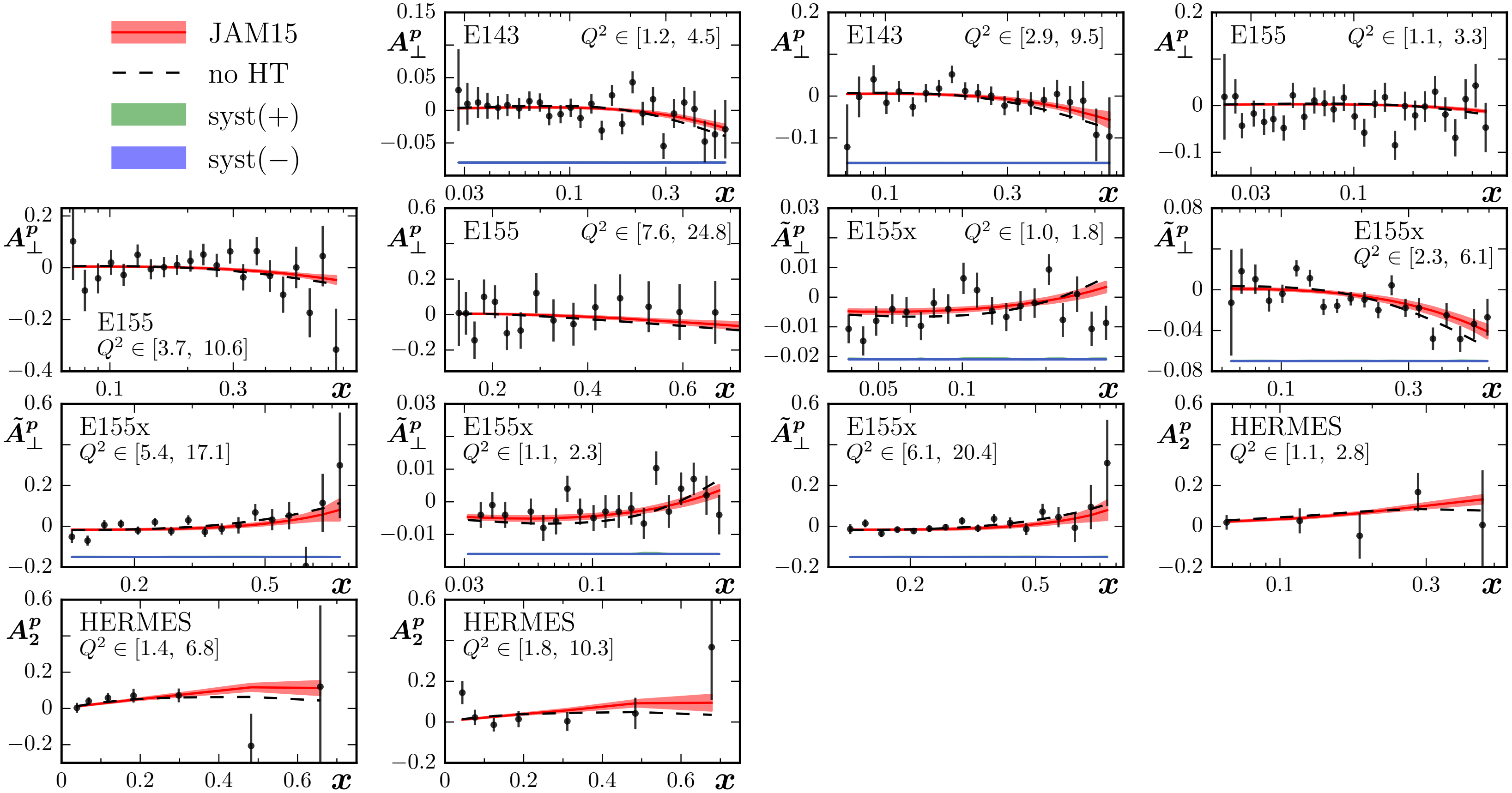
$Q^2_{\rm cut}$ (GeV^2)	**1.0**	2.0	4.0
# points	**2515**	1421	611
$\chi^2_{\rm dof}$	**1.07**	1.08	0.95

$W^2_{\rm cut}$ (GeV^2)	3.5	**4**	5	6	8	10
# points	2868	**2515**	1880	1427	943	854
$\chi^2_{\rm dof}$	1.20	**1.07**	1.03	1.02	0.99	0.97

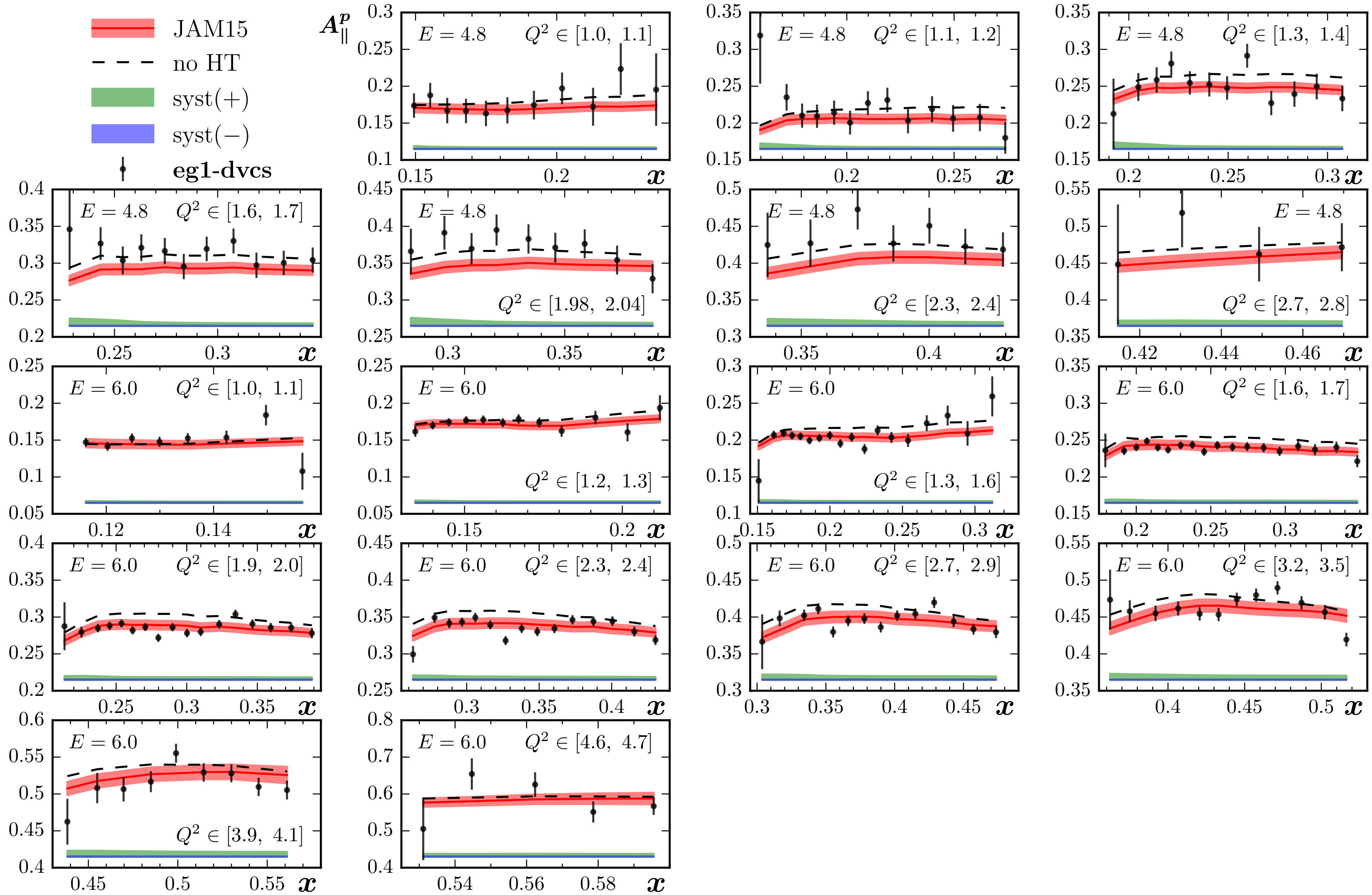
Data vs Theory: Proton parallel asymmetries



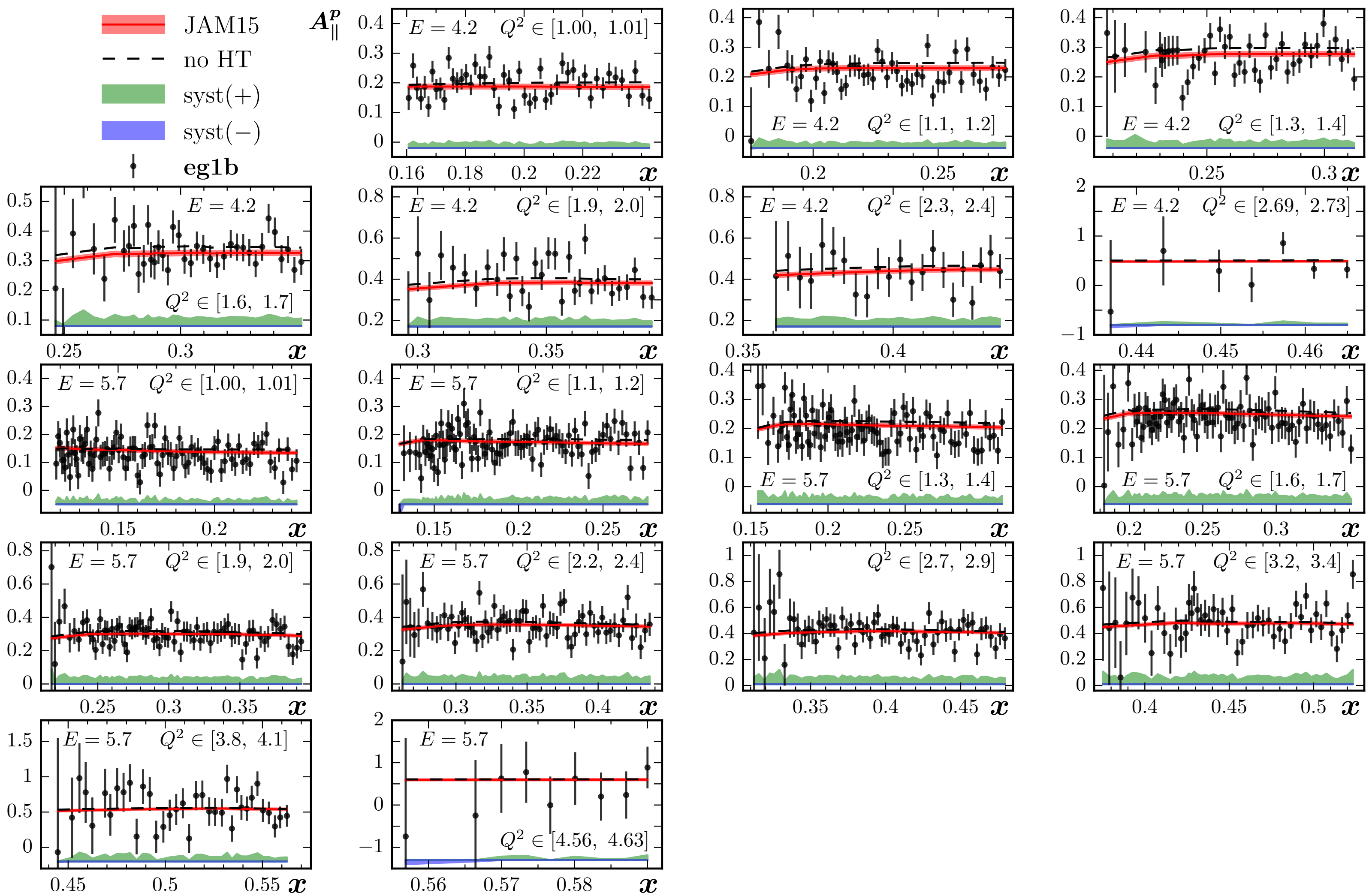
Data vs Theory: Proton perpendicular asymmetries



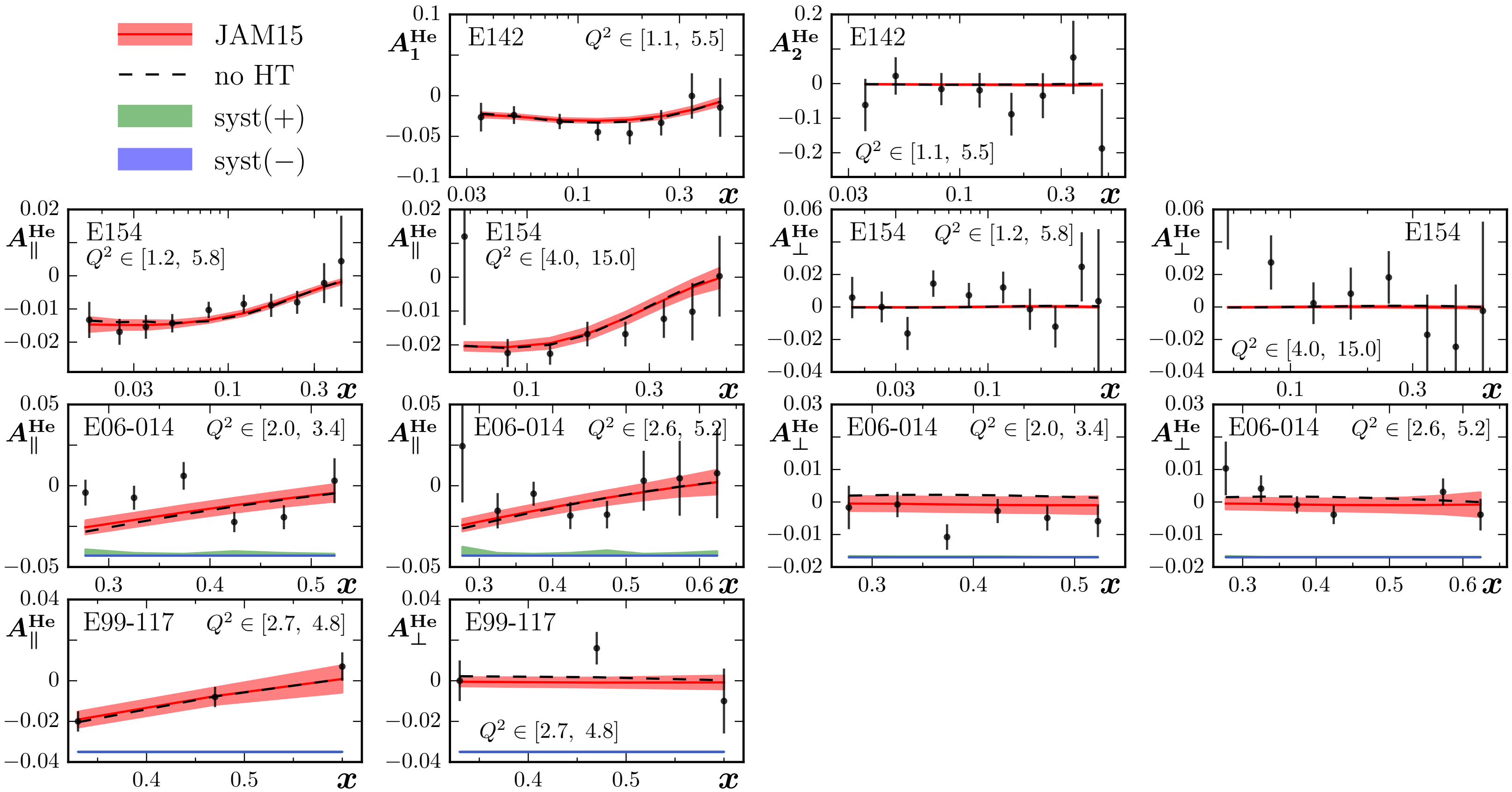
Data vs Theory: Proton eg1-dvcs data



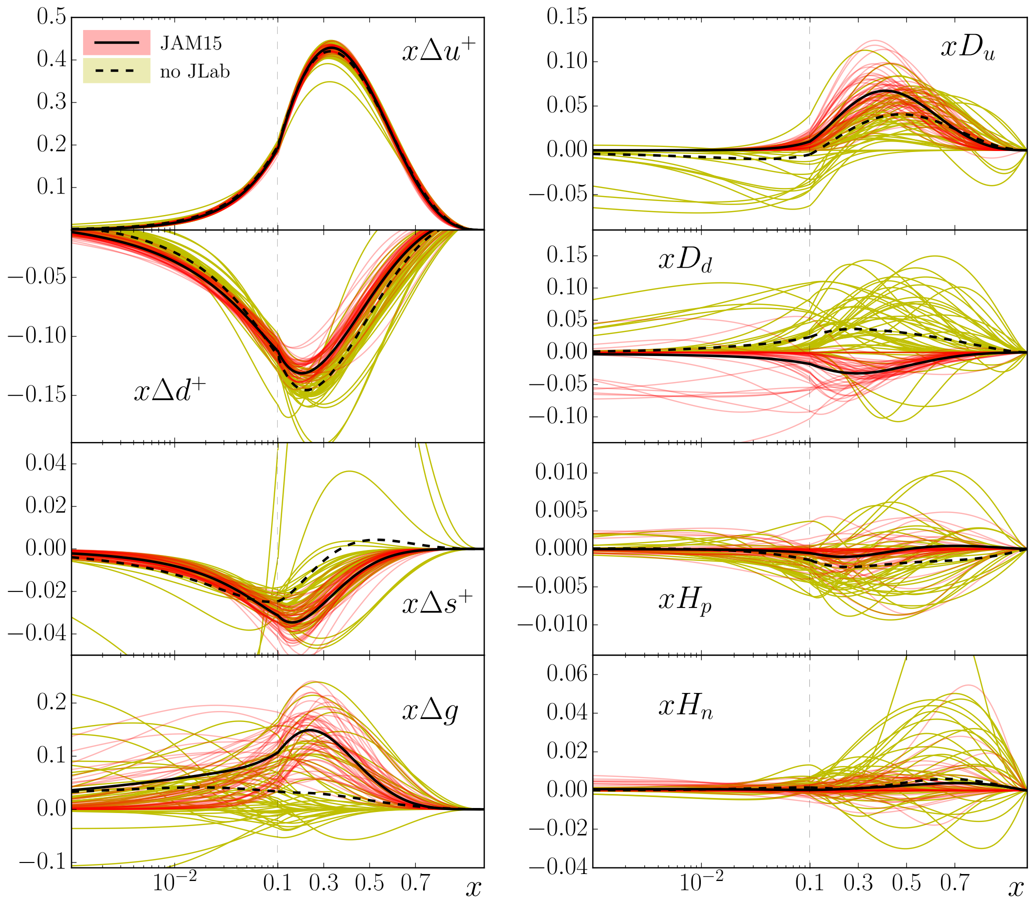
Data vs Theory: Proton EG1b data



Data vs Theory: ^{3}He Data

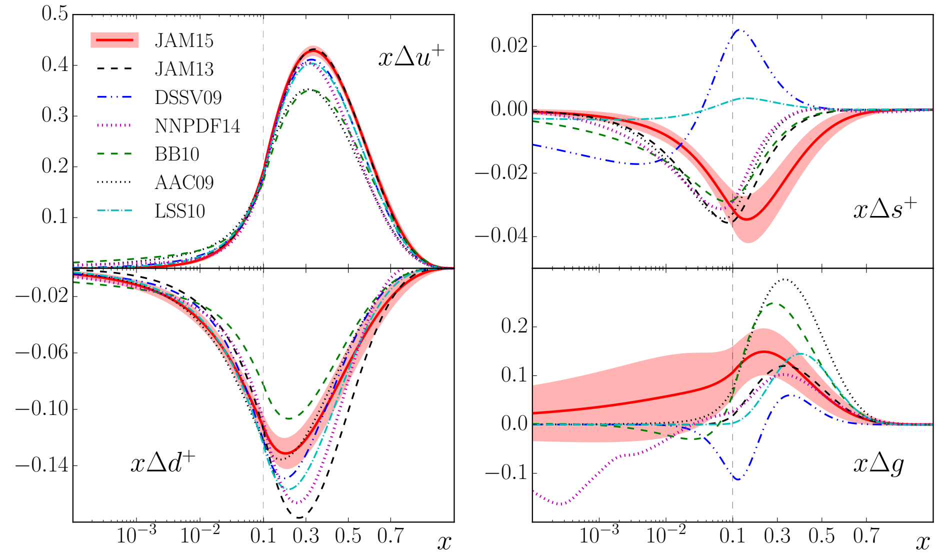


Impact of JLab Data

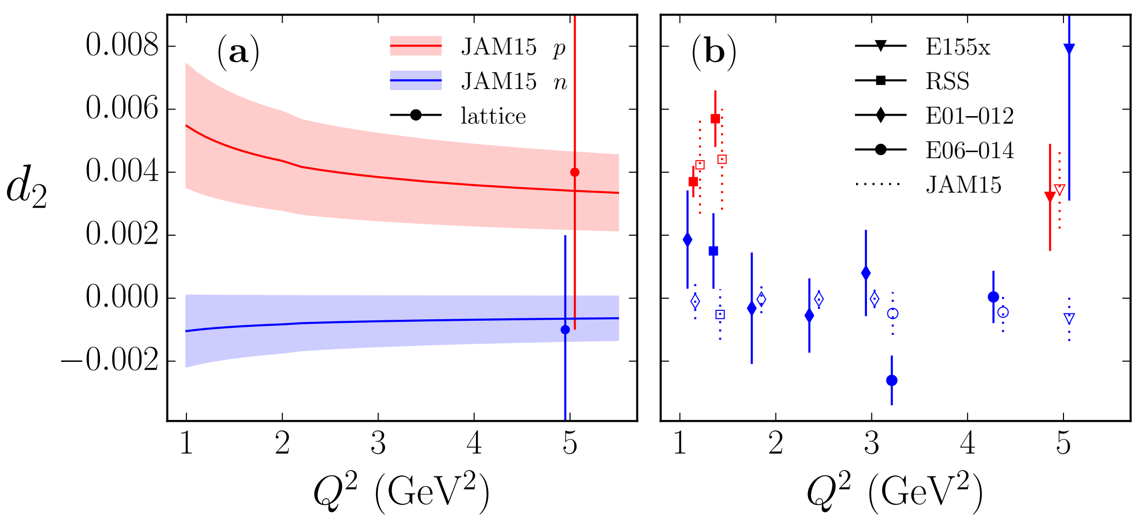


- Reduction of uncertainties in region 0.1 < x < 0.7
- Low-x constraints via weak baryon decay
- JLab data prefers positive glue for x > 0.1 → constraint through evolution
- Non-zero twist-3 quark distributions ; twist-4 consistent with zero

Final Results

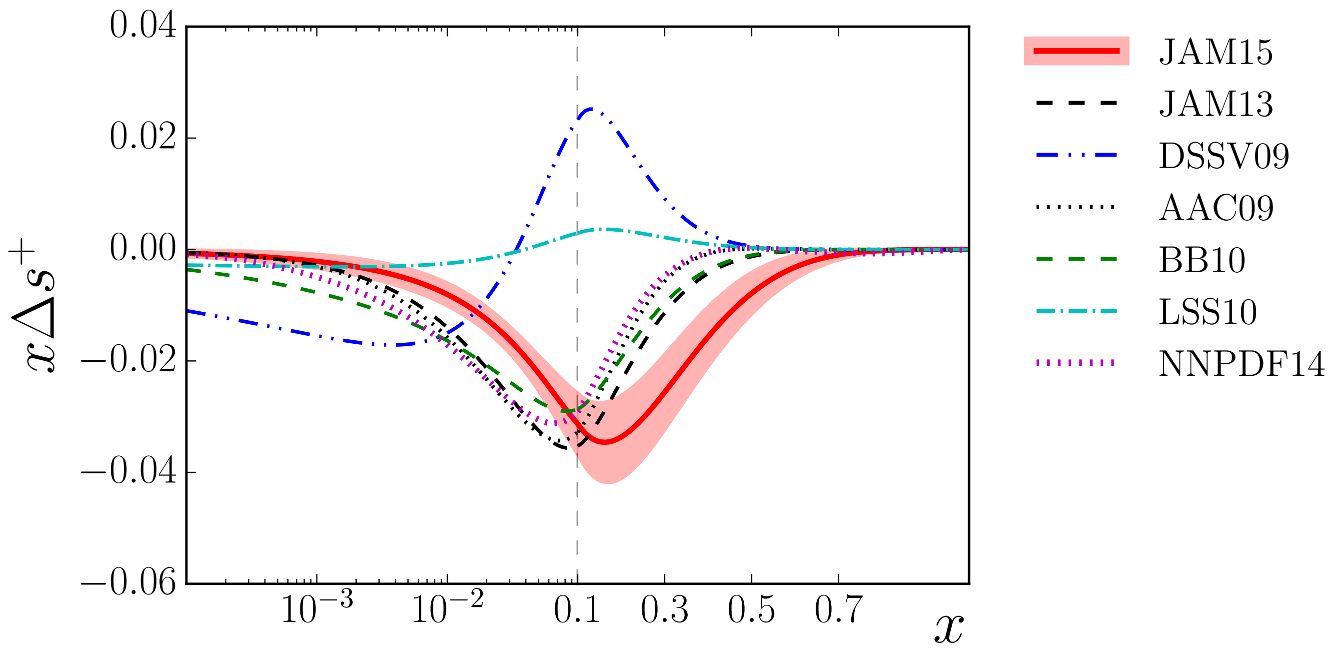


moment	truncated	full
Δu^+	0.82 ± 0.01	0.83 ± 0.01
Δd^+	-0.42 ± 0.01	-0.44 ± 0.01
Δs^+	-0.10 ± 0.01	-0.10 ± 0.01
$\Delta\Sigma$	0.31 ± 0.03	0.28 ± 0.04
ΔG	0.5 ± 0.4	1 ± 15
d_2^p	0.005 ± 0.002	0.005 ± 0.002
d_2^n	-0.001 ± 0.001	-0.001 ± 0.001
h_p	-0.000 ± 0.001	0.000 ± 0.001
h_n	0.001 ± 0.002	0.001 ± 0.003



- Up and down distributions consistent with previous analyses
- Strange peak shifted to larger-x
- d_2 moment "agrees" with experimental measurements

Strange polarization



$$\Delta \mathbf{s}^+(Q^2) = \int_0^1 dx \Delta s^+(x, Q^2)$$

JAM15: $\Delta s^+ = -0.10 \pm 0.01$ DSSV09: $\Delta s^+ = -0.11$ $Q^2 = 1\text{GeV}^2$

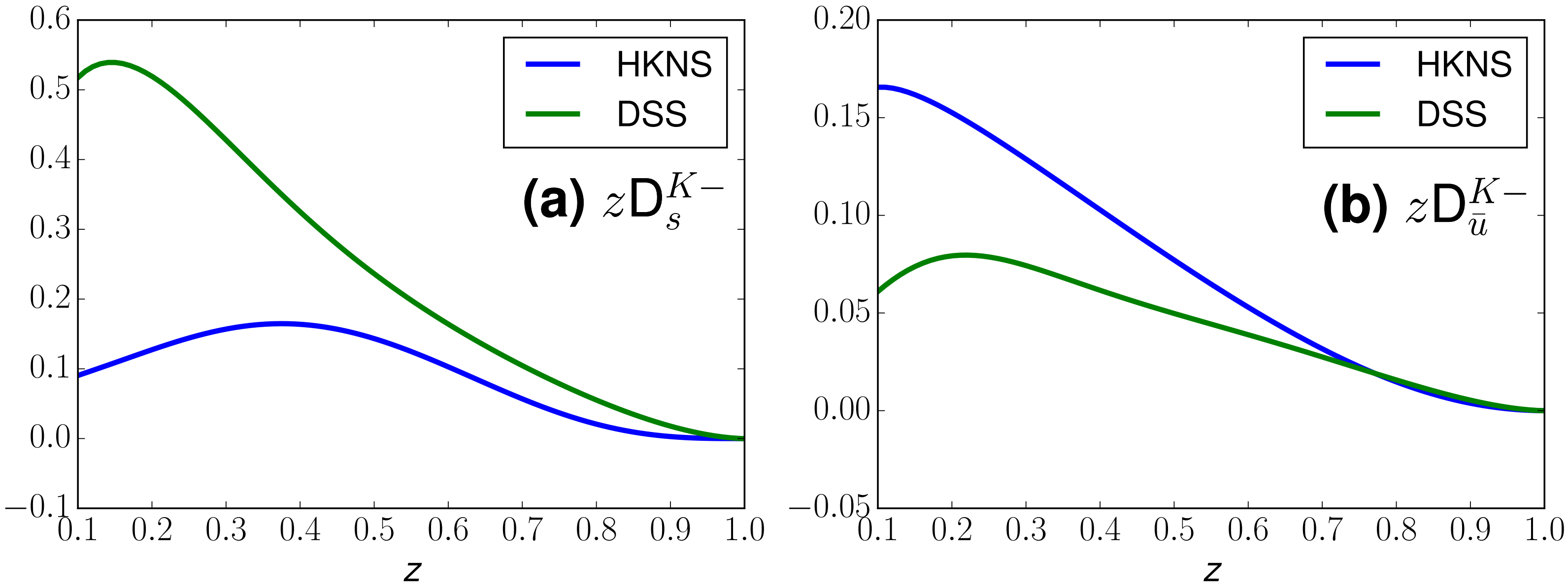
- Discrepancy in shape of strange polarization (DSSV, LSS)

  → Claim: analysis of SIDIS data changes strange polarization shape

Fragmentation Functions

- Large uncertainty between HKNS and DSS kaon fragmentation function parameterizations

→ Heavily influences shape of strange polarization density! (Leader, et al)



→ Constrained through global fits of single-inclusive annihilation (SIA), unpolarized/polarized SIDIS data, single inclusive pp collision data

Fragmentation Functions

Single-inclusive Annihilation (SIA)

- Cross sections for $e^+e^- \to hX$

$$\frac{d\sigma^h}{dz} = \frac{d\sigma_L^h}{dz} + \frac{d\sigma_T^h}{dz} \qquad z = \frac{2E_h}{\sqrt{s}}$$

- Transverse and longitudinal cross sections

$$\frac{d\sigma_T^h}{dz} = \sum_i \sigma_i \left[D_i(z,Q^2) + \frac{\alpha_s}{2\pi} \big(C_i^T \otimes D_i\big)(z,Q^2) \right] + \sigma_0 \, \frac{\alpha_s}{2\pi} \big(C_g^T \otimes D_g\big)(z,Q^2)$$

$$\frac{d\sigma_L^h}{dz} = \sum_i \sigma_i \, \frac{\alpha_s}{2\pi} \big(C_i^L \otimes D_i\big)(z,Q^2) + \sigma_0 \, \frac{\alpha_s}{2\pi} \big(C_g^L \otimes D_g\big)(z,Q^2)$$

Single-inclusive Annihilation (SIA)

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- Transverse and longitudinal cross sections

$$\frac{d\sigma^h_T}{dz} = \sum_i \sigma_i \left[D_i(z,Q^2) + \frac{\alpha_s}{2\pi}\big(C_i^T \otimes D_i\big)(z,Q^2)\right] + \sigma_0 \frac{\alpha_s}{2\pi}\big(C_g^T \otimes D_g\big)(z,Q^2)$$

$$\frac{d\sigma^h_L}{dz} = \sum_i \sigma_i \frac{\alpha_s}{2\pi}\big(C_i^L \otimes D_i\big)(z,Q^2) + \sigma_0 \frac{\alpha_s}{2\pi}\big(C_g^L \otimes D_g\big)(z,Q^2)$$

- Electroweak cross section $e^+e^- \to \gamma, Z \to q\bar{q}$:

$$\sigma_i(s) = \sigma_0 \Big[e_i^2 + 2e_i\, g_V^i g_V^e\, \rho_1(s) + \big(g_A^{e\,2} + g_V^{e\,2}\big)\,\big(g_A^{i\,2} + g_V^{i\,2}\big)\,\rho_2(s)\Big]$$

Single-inclusive Annihilation (SIA)

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$$\frac{d\sigma^h}{dz} = \frac{d\sigma_L^h}{dz} + \frac{d\sigma_T^h}{dz} \qquad\qquad z = \frac{2E_h}{\sqrt{s}}$$

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- Electroweak cross section $e^+e^- \to \gamma, Z \to q\bar{q}$:

$$\sigma_i(s) = \sigma_0\left[e_i^2 + 2e_i\, g_V^i g_V^e\, \rho_1(s) + \left(g_A^{e\,2} + g_V^{e\,2}\right)\left(g_A^{i\,2} + g_V^{i\,2}\right)\rho_2(s)\right]$$

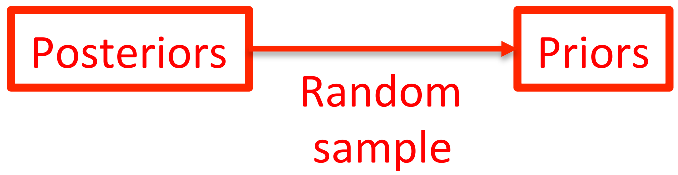
- Typically, observables are normalized by total hadronic cross section

$$\sigma_{\rm tot}(s) = \sum_i \sigma_i \left(1 + \frac{\alpha_s}{\pi}\right)$$

Refined Methodology

- Generation of priors after initial iteration:

Previous:



Current:

- Purpose of iteration:

Previous: convergence of chi-squared

Current: convergence of covariance matrix

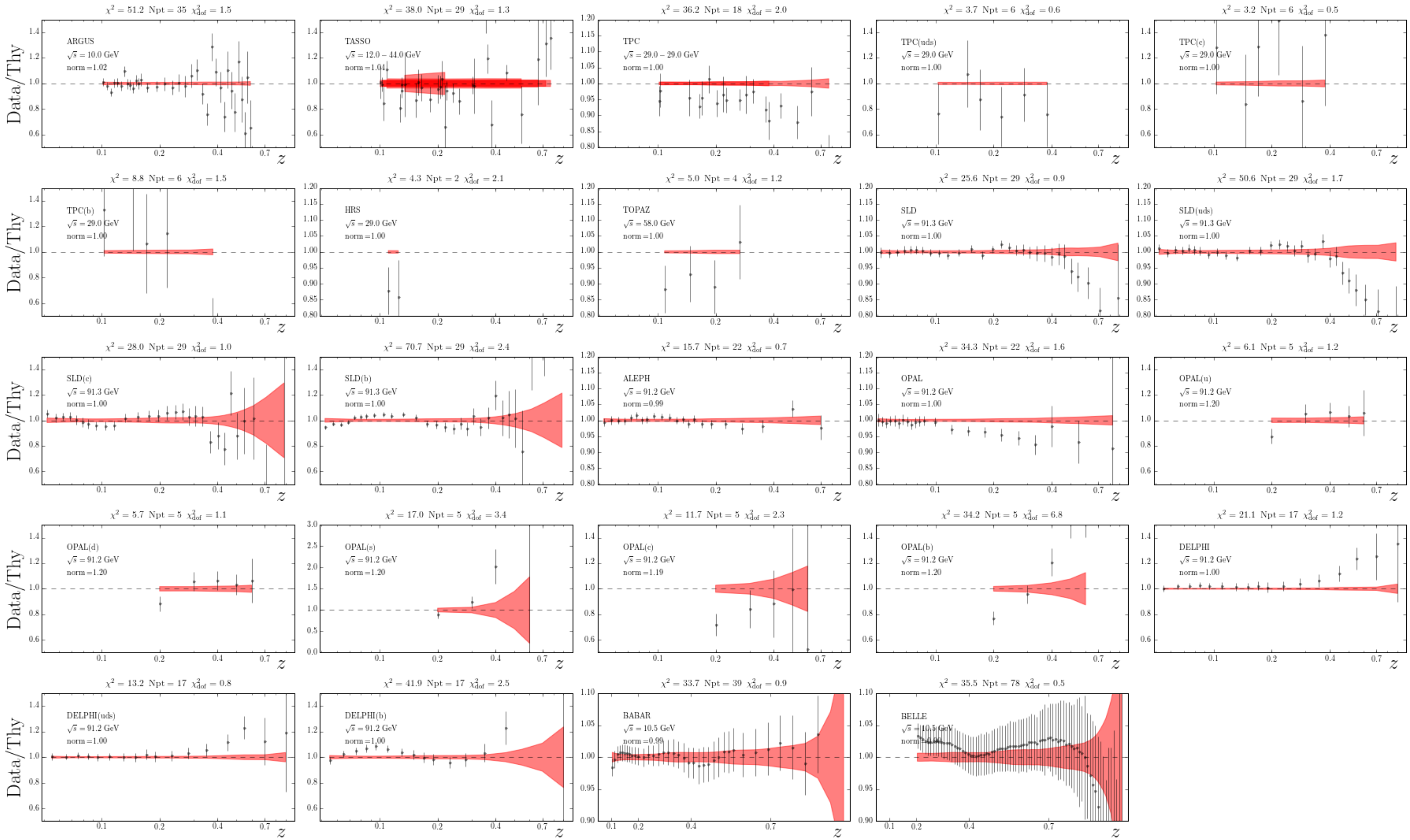
- Parameterization of FFs: $D_{q^+}(z, Q_0^2) = T(z, 1, \vec{p}) + T(z, 1, \vec{p'})$

$$\mathrm{T}(z; n, \vec{p}) = M \frac{z^{\alpha}(1-z)^{\beta}(1+\gamma\sqrt{z}+\delta z)}{\int_0^1 dz\, z^{n+\alpha}(1-z)^{\beta}(1+\gamma\sqrt{z}+\delta z)}$$

$$D_g(z, Q_0^2) = T(z, 1, \vec{p})$$

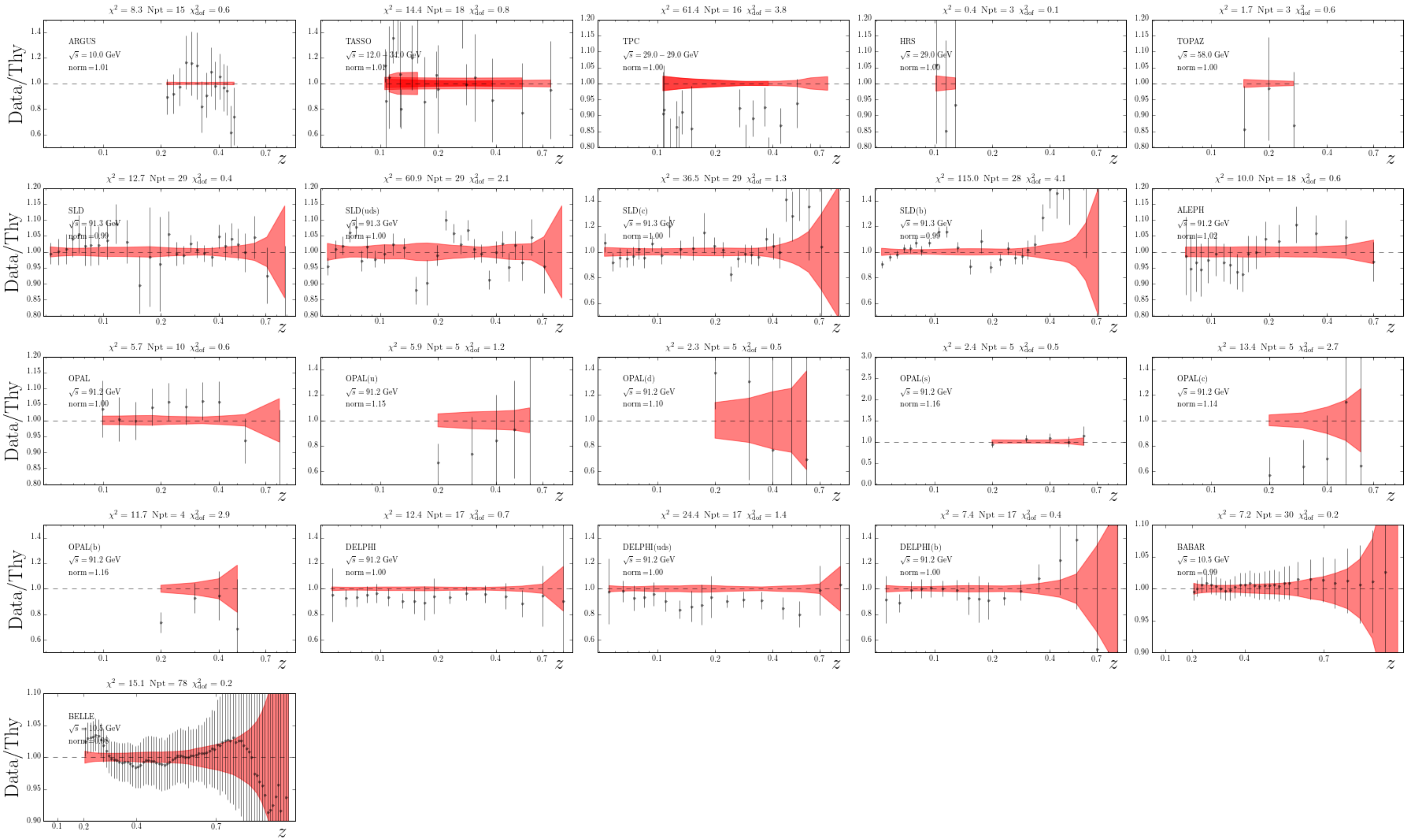
Data vs Theory: Pions

$N_{pts} = 459 \;\; \chi^2_{d.o.f.} = 1.30$

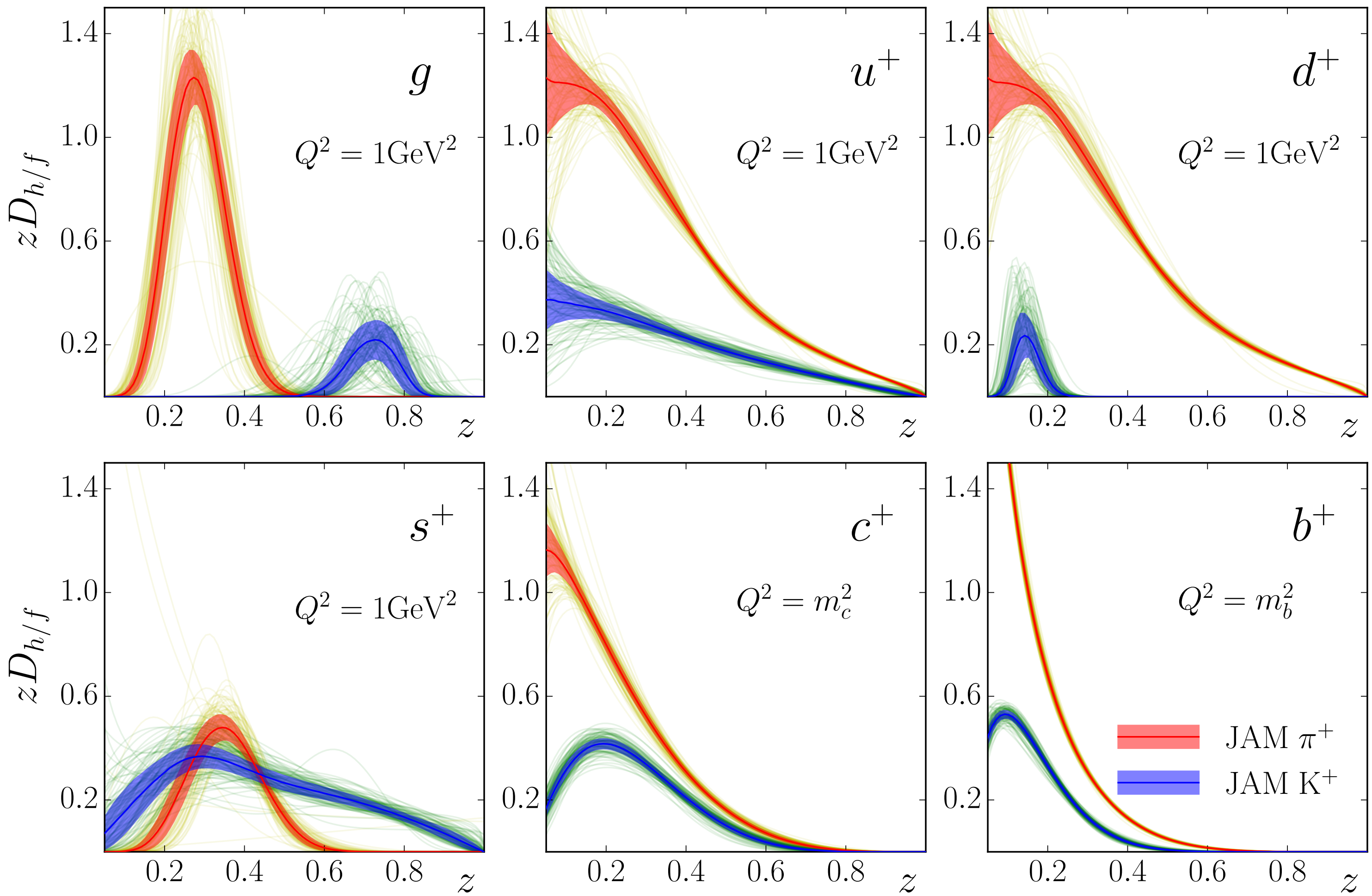


Data vs Theory: Kaons

$$N_{pts} = 391 \;\; \chi^2_{d.o.f.} = 1.11$$

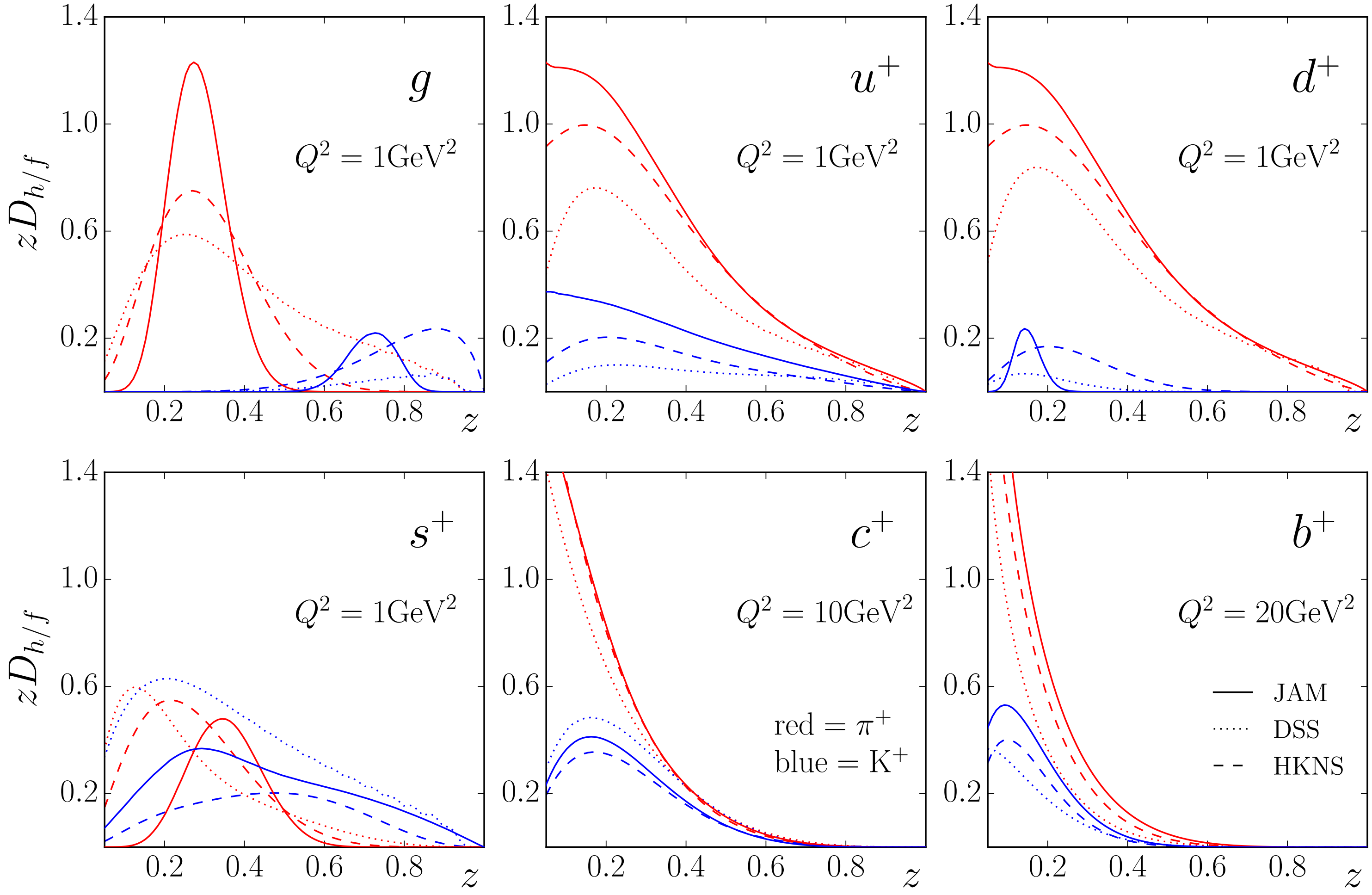


FF Results



g
$Q^2 = 1\mathrm{GeV}^2$
u^+
$Q^2 = 1\mathrm{GeV}^2$
d^+
$Q^2 = 1\mathrm{GeV}^2$
s^+
$Q^2 = 1\mathrm{GeV}^2$
c^+
$Q^2 = m_c^2$
b^+
$Q^2 = m_b^2$
JAM π^+
JAM K$^+$
$zD_{h/f}$
z
1.4
1.0
0.6
0.2
0.2
0.4
0.6
0.8

FF Results



1.4
1.0
0.6
0.2
0.2
0.4
0.6
0.8
z
$zD_{h/f}$
g
$Q^2 = 1\text{GeV}^2$
u^+
$Q^2 = 1\text{GeV}^2$
d^+
$Q^2 = 1\text{GeV}^2$
s^+
$Q^2 = 1\text{GeV}^2$
c^+
$Q^2 = 10\text{GeV}^2$
red = π^+
blue = K^+
b^+
$Q^2 = 20\text{GeV}^2$
JAM
DSS
HKNS

Future Work

- Flavor separation: combined DIS, SIA, and SIDIS analysis to begin soon!
  - Use posteriors from DIS and SIA as priors for combined analysis
  - Will also include polarized *pp* collisions and jets for gluon distribution
- Theoretical improvements needed:
  - Hadron mass corrections in SIDIS
  - Threshold resummation for PDFs and FFs (SCET vs Traditional)
- QCD observables yet to be implemented:
  - *W* asymmetries for spin PDFs (up and down sea constraints)
  - Unpolarized SIDIS and single-inclusive *pp* collision for FFs
- Working towards a universal fit of quark helicity distributions
  - Global analyses of unpolarized and polarized data together

Future Work

already underway!

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    - Hadron mass corrections in SIDIS
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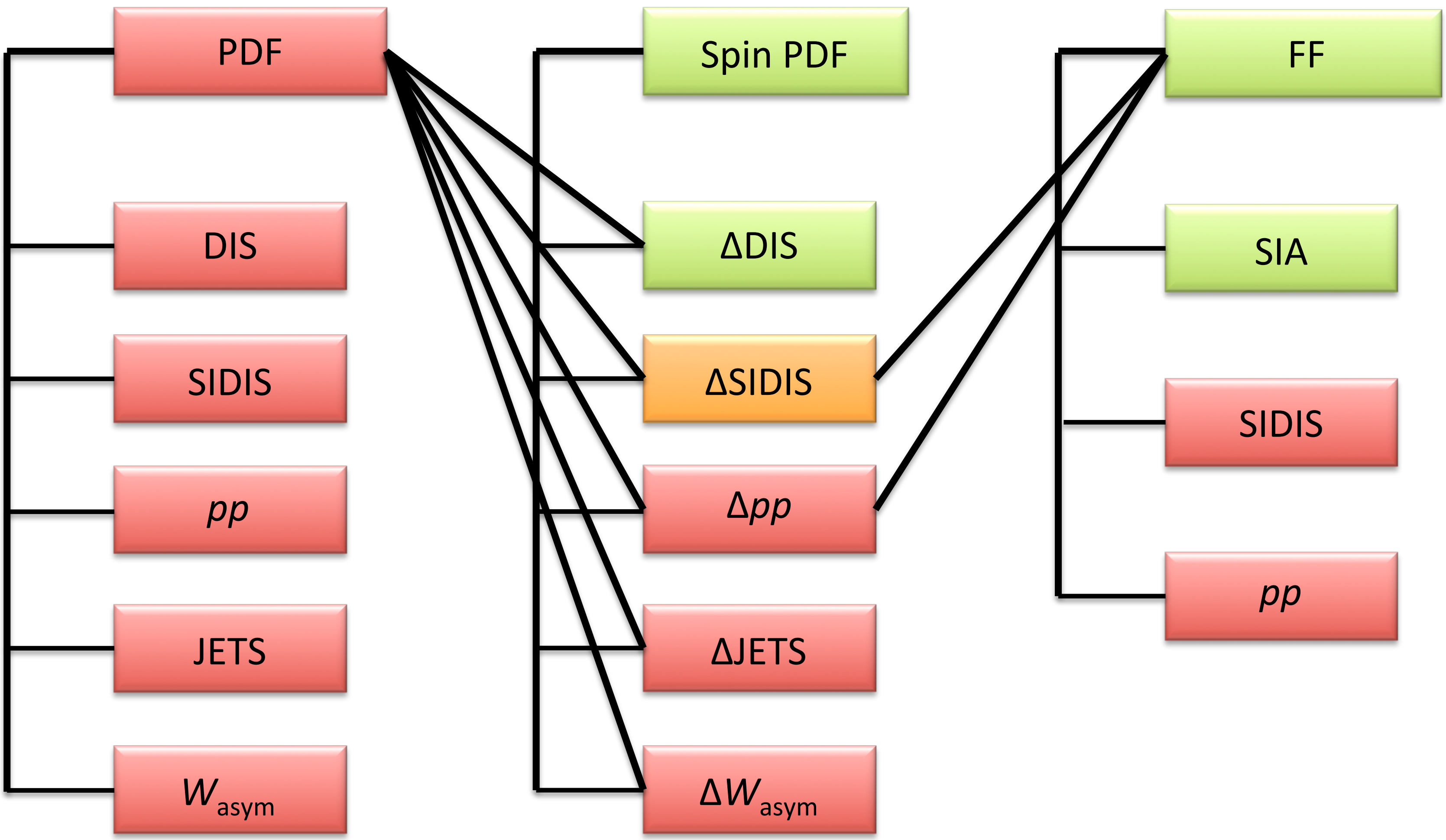
    - Global analyses of unpolarized and polarized data together

Future of JAM



Proof of Concept – First Ever Simultaneous Fit

- Procedure:

  → Single chi-squared fit to DIS data (with $W^2 > 10$ GeV^2) w/ r-values

  → Single chi-squared fit to SIA data (only M, a, b parameters) w/ r-values

  → Final parameters of separate fits used as starting point in single chi-squared fit of combined DIS, SIA, and SIDIS data

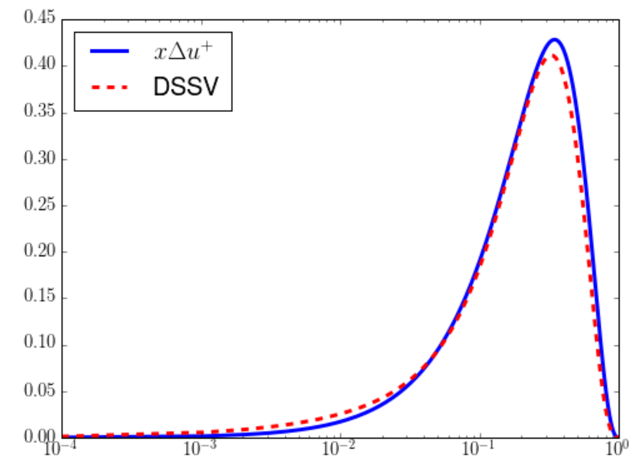
  → Generate priors for PDF and FF q- parameterization

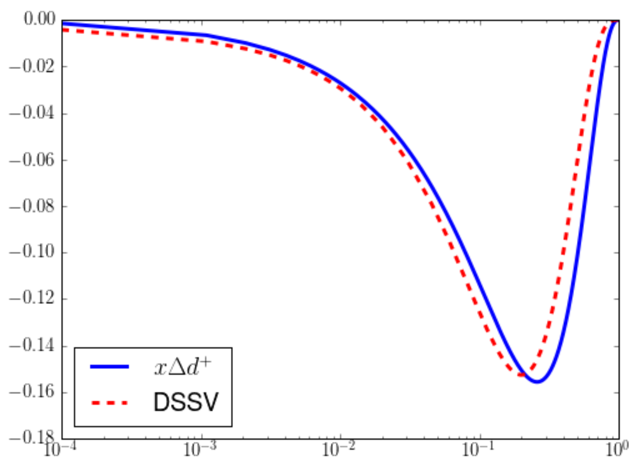
  → Fit only pion and kaon SIDIS data

- Total number of data points: 1833
- Number of parameters: 75 (r-values fixed to their separated fit values)
- Focus is on constraining shape of the strange polarization!

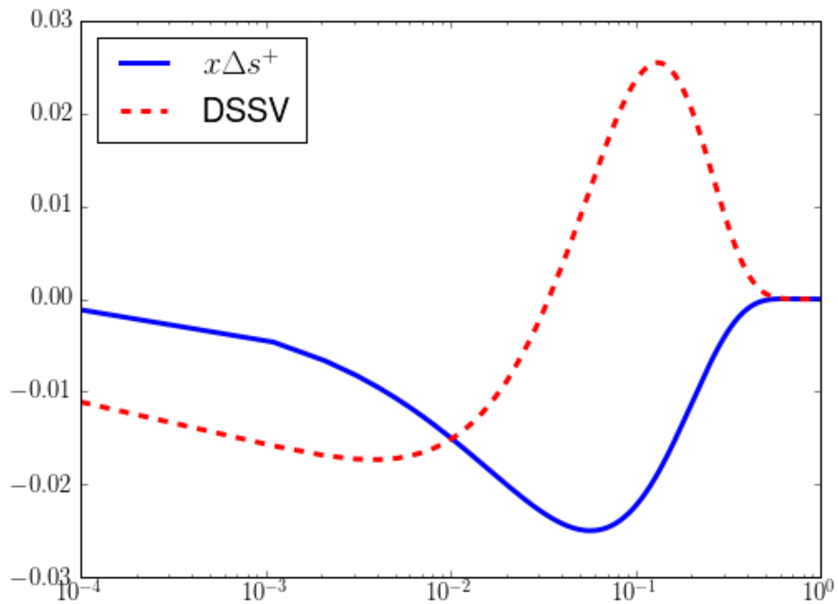
Proof of Concept – First Ever Simultaneous Fit

- Note: The following results are very preliminary!





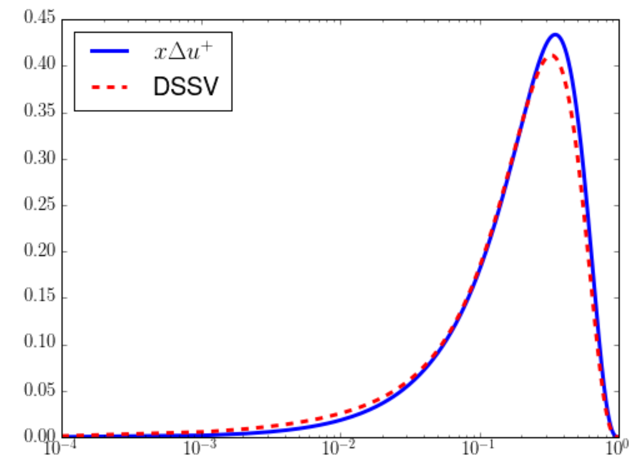
Strange is negative for all *x* as before

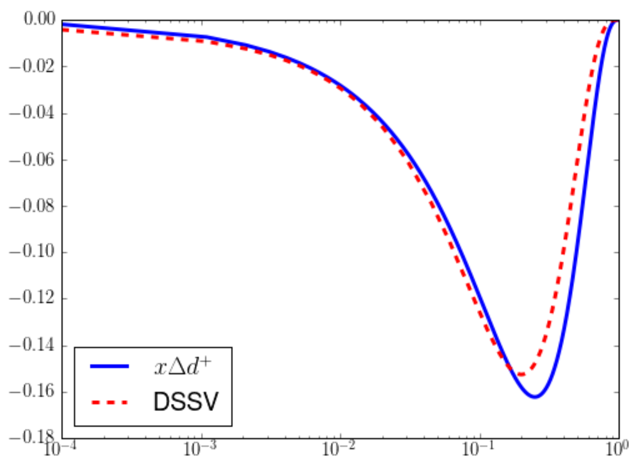


$$\int dx \Delta s^+ = -0.094$$

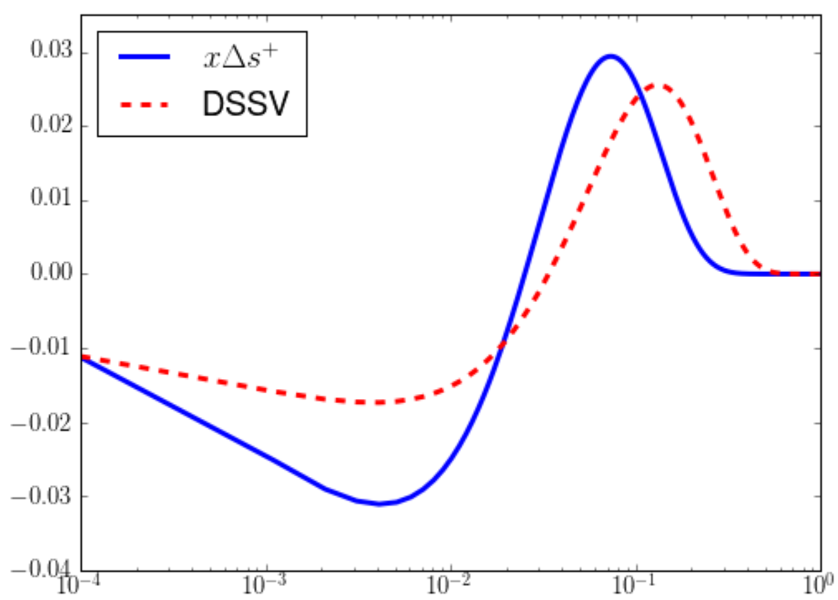
Proof of Concept – First Ever Simultaneous Fit

- Note: The following results are very preliminary!





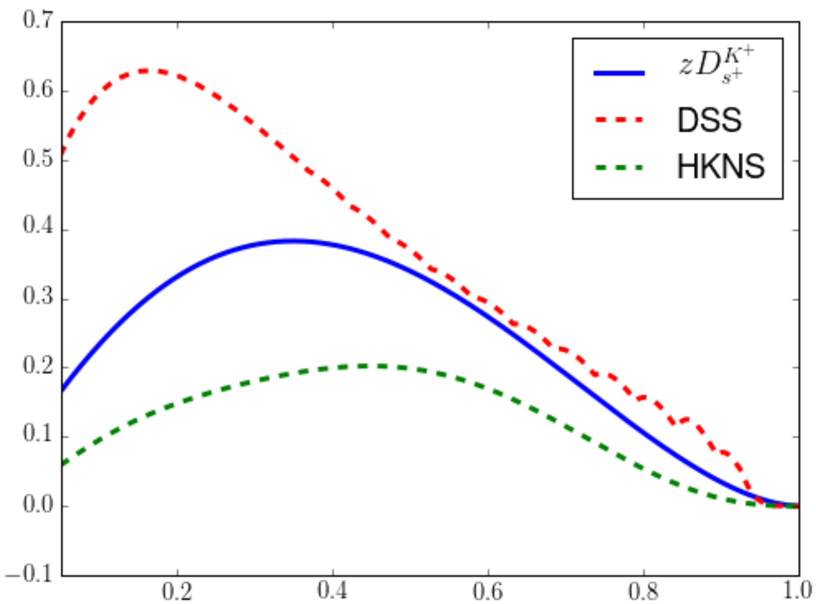
Strange now becomes positive!

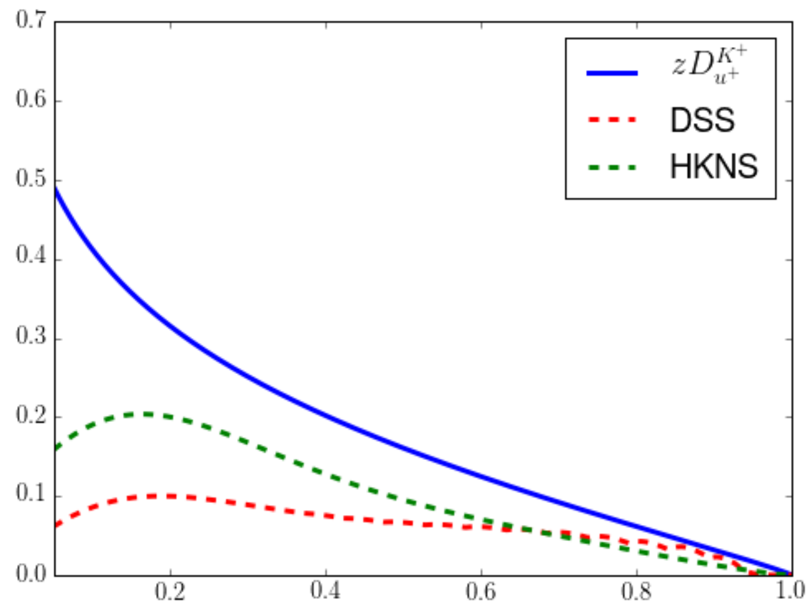


$$\int dx \Delta s^{+} = -0.108$$

Proof of Concept – First Ever Simultaneous Fit

- Note: The following results are very preliminary!
- SIDIS Pion Fit: $N_{pts} = 80$, $\chi^2_{d.o.f.} = 0.82$
- SIDIS Kaon Fit: $N_{pts} = 71$, $\chi^2_{d.o.f.} = 0.65$
- Fragmentation functions:



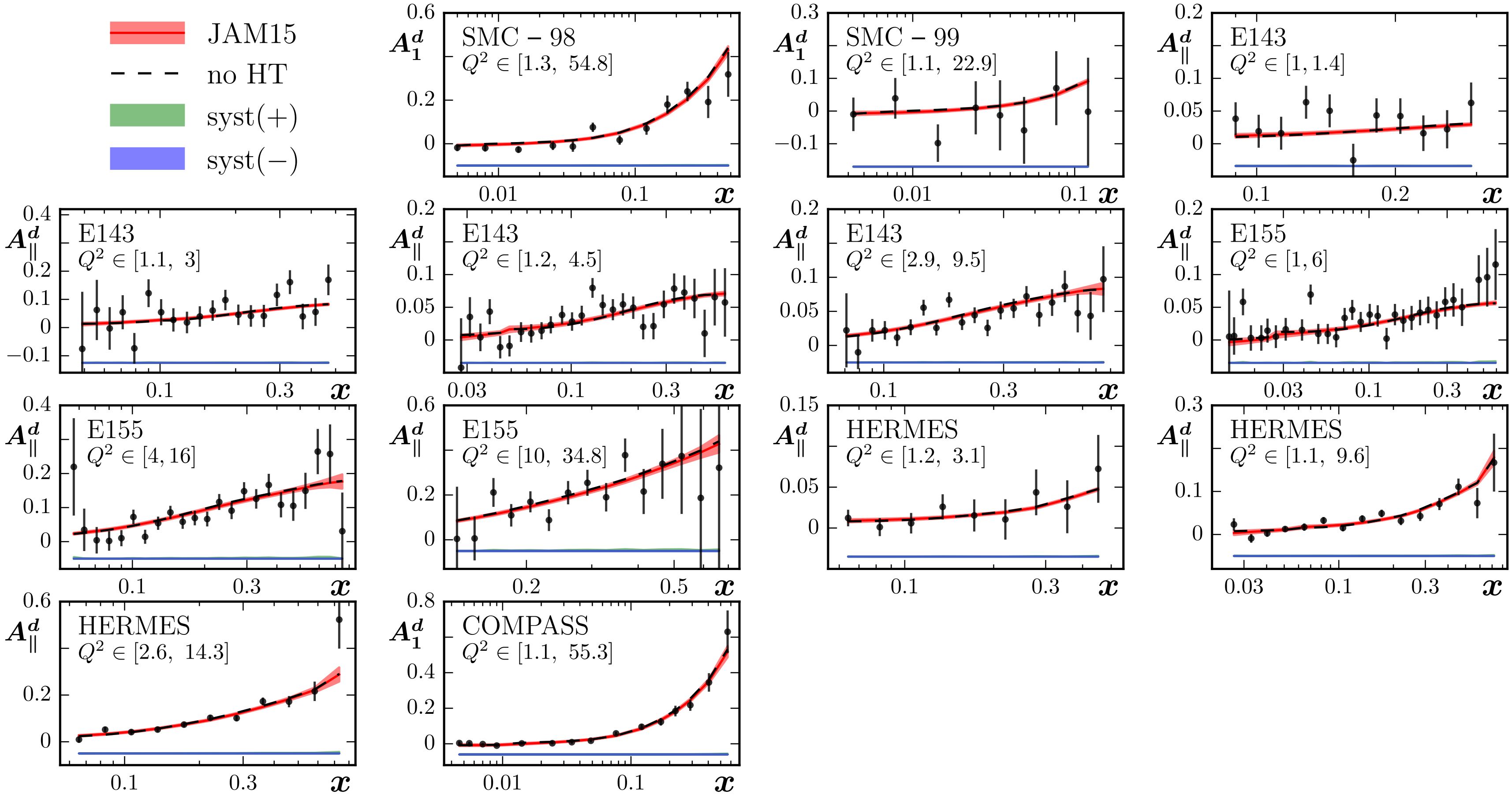


- Next: simultaneous fit using IMC technique, including higher twists

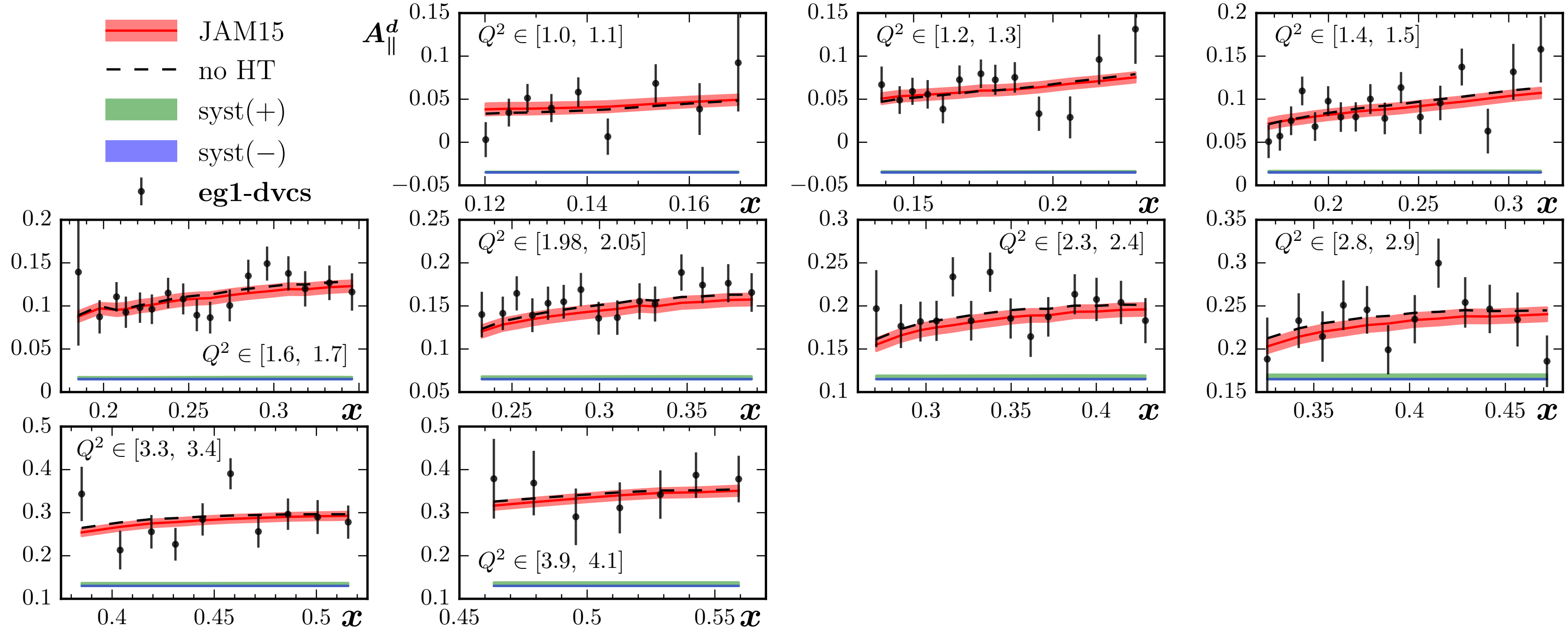
Thank You

Backup Slides

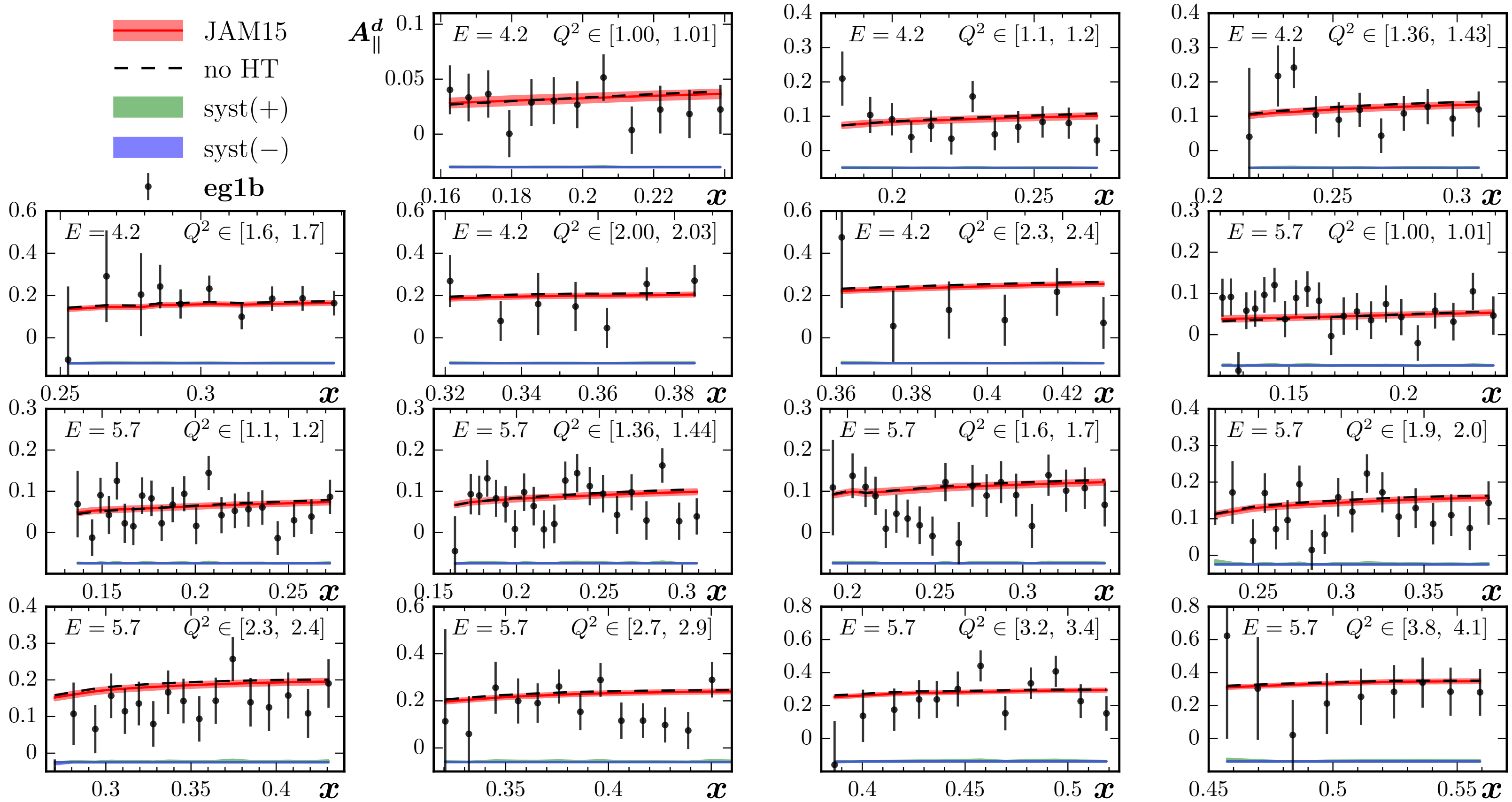
Data vs Theory: Deuteron parallel asymmetries



Data vs Theory: Deuteron eg1-dvcs data



Data vs Theory: Deuteron eg1b data



PDIS chi-squared table

experiment	reference	observable	target	# points	$\chi^2_{\rm dof}$
EMC	[?]	A_1	p	10	0.40
SMC	[?]	A_1	p	12	0.47
SMC	[?]	A_1	d	12	1.62
SMC	[?]	A_1	p	8	1.26
SMC	[?]	A_1	d	8	0.57
COMPASS	[?]	A_1	p	15	0.92
COMPASS	[?]	A_1	d	15	0.67
COMPASS	[?]	A_1	p	51	0.76
SLAC E80/E130	[?]	$A_\parallel$	p	22	0.59
SLAC E142	[?]	A_1	^{3}He	8	0.49
SLAC E142	[?]	A_2	^{3}He	8	0.60
SLAC E143	[?]	$A_\parallel$	p	81	0.80
SLAC E143	[?]	$A_\parallel$	d	81	1.12
SLAC E143	[?]	$A_\perp$	p	48	0.89
SLAC E143	[?]	$A_\perp$	d	48	0.91
SLAC E154	[?]	$A_\parallel$	^{3}He	18	0.51
SLAC E154	[?]	$A_\perp$	^{3}He	18	0.97
SLAC E155	[?]	$A_\parallel$	p	71	1.20
SLAC E155	[?]	$A_\parallel$	d	71	1.05
SLAC E155	[?]	$A_\perp$	p	65	0.99
SLAC E155	[?]	$A_\perp$	d	65	1.52
SLAC E155x	[?]	$\widetilde{A}_\perp$	p	116	1.27
SLAC E155x	[?]	$\widetilde{A}_\perp$	d	115	0.83
HERMES	[?]	A_1	"n"	9	0.25
HERMES	[?]	$A_\parallel$	p	35	0.47
HERMES	[?]	$A_\parallel$	d	35	0.94
HERMES	[?]	A_2	p	19	0.93
JLab E99-117	[?]	$A_\parallel$	^{3}He	3	0.27
JLab E99-117	[?]	$A_\perp$	^{3}He	3	1.58
JLab E06-014	[?]	$A_\parallel$	^{3}He	14	2.12
JLab E06-014	[?]	$A_\perp$	^{3}He	14	1.06
JLab eg1-dvcs	[?]	$A_\parallel$	p	195	1.52
JLab eg1-dvcs	[?]	$A_\parallel$	d	114	0.94
JLab eg1b	[?]	$A_\parallel$	p	890	1.11
JLab eg1b	[?]	$A_\parallel$	d	218	1.02
total				2515	1.07

Pion FF chi-squared table

	N	RS	chi2	exp	idx	npts
0	1.201535	91.20	34.17	OPAL(b)	1024	5
1	1.000000	91.20	21.07	DELPHI	1025	17
2	1.000000	91.20	13.24	DELPHI(uds)	1026	17
3	1.000000	91.20	41.94	DELPHI(b)	1027	17
4	0.993114	10.54	33.70	BABAR	1028	39
5	0.901166	10.52	35.48	BELLE	1029	78
6	1.023502	9.98	51.23	ARGUS	1030	35
7	1.147299	12.00	1.08	TASSO	1001	2
8	0.977585	14.00	15.04	TASSO	1002	7
9	1.046170	22.00	6.41	TASSO	1003	7
10	1.058221	34.00	8.07	TASSO	1005	8
11	1.038259	44.00	7.36	TASSO	1006	5
12	1.000000	29.00	6.21	TPC	1007	6
13	1.000000	29.00	29.98	TPC	1008	12
14	1.000000	29.00	3.70	TPC(uds)	1009	6
15	1.000000	29.00	3.22	TPC(c)	1010	6
16	1.000000	29.00	8.75	TPC(b)	1011	6
17	1.000000	29.00	4.28	HRS	1012	2
18	1.000000	58.00	4.95	TOPAZ	1013	4
19	1.002344	91.28	25.62	SLD	1014	29
20	1.002893	91.28	50.61	SLD(uds)	1015	29
21	0.997385	91.28	27.97	SLD(c)	1016	29
22	1.004594	91.28	70.68	SLD(b)	1017	29
23	0.987170	91.20	15.72	ALEPH	1018	22
24	1.000000	91.20	34.31	OPAL	1019	22
25	1.201011	91.20	6.12	OPAL(u)	1020	5
26	1.201646	91.20	5.67	OPAL(d)	1021	5
27	1.195654	91.20	16.95	OPAL(s)	1022	5
28	1.189695	91.20	11.74	OPAL(c)	1023	5

Kaon FF chi-squared table

	N	RS	chi2	exp	idx	npts
0	0.875937	12.00	3.28	TASSO	2001	3
1	0.964711	14.00	10.45	TASSO	2002	7
2	1.019881	22.00	0.38	TASSO	2003	4
3	1.009973	34.00	0.28	TASSO	2005	4
4	1.000000	29.00	20.18	TPC	2007	5
5	1.000000	29.00	41.23	TPC	2008	11
6	1.000000	29.00	0.35	HRS	2012	3
7	1.000000	58.00	1.74	TOPAZ	2013	3
8	0.994244	91.28	12.73	SLD	2014	29
9	0.995024	91.28	60.91	SLD(uds)	2015	29
10	1.000430	91.28	36.47	SLD(c)	2016	29
11	0.991026	91.28	114.97	SLD(b)	2017	28
12	1.015168	91.20	9.98	ALEPH	2018	18
13	1.000000	91.20	5.71	OPAL	2019	10
14	1.147011	91.20	5.87	OPAL(u)	2020	5
15	1.095078	91.20	2.26	OPAL(d)	2021	5
16	1.160967	91.20	2.38	OPAL(s)	2022	5
17	1.144994	91.20	13.36	OPAL(c)	2023	5
18	1.160419	91.20	11.67	OPAL(b)	2024	4
19	1.000000	91.20	12.38	DELPHI	2025	17
20	1.000000	91.20	24.45	DELPHI(uds)	2026	17
21	1.000000	91.20	7.44	DELPHI(b)	2027	17
22	0.988794	10.54	7.22	BABAR	2028	30
23	0.983820	10.52	15.12	BELLE	2029	78
24	1.005550	9.98	8.27	ARGUS	2030	15
25	1.000000	91.20	4.11	DELPHI	2031	10

Mellin Moment Calculation

- MC technique requires many fits → require optimized calculation speed
- Mellin moment technique:

$$I(x) = \int_x^1 \frac{dy}{y} f(y) \int_y^1 \frac{dz}{z} g(\frac{x}{yz})$$

$$= \frac{1}{2\pi i} \int_{\mathcal{C}} dN g_N \underbrace{\left[\int_x^1 \frac{dy}{y} f(y) \int_y^1 \frac{dz}{z} \left(\frac{x}{yz}\right)^{-N}\right]}_{=\mathcal{M}_{\mathcal{N}} \rightarrow \text{precomputed only once}}$$

$$= \sum_{i,k} w_i^k j^k Im \left(e^{i\phi} g_{N_j^k} \mathcal{M}_{N_j^k}\right)$$

Leading and Higher Twist Final Distributions