NNLO cross sections for processes with jets

Nigel Glover

IPPP, Durham University

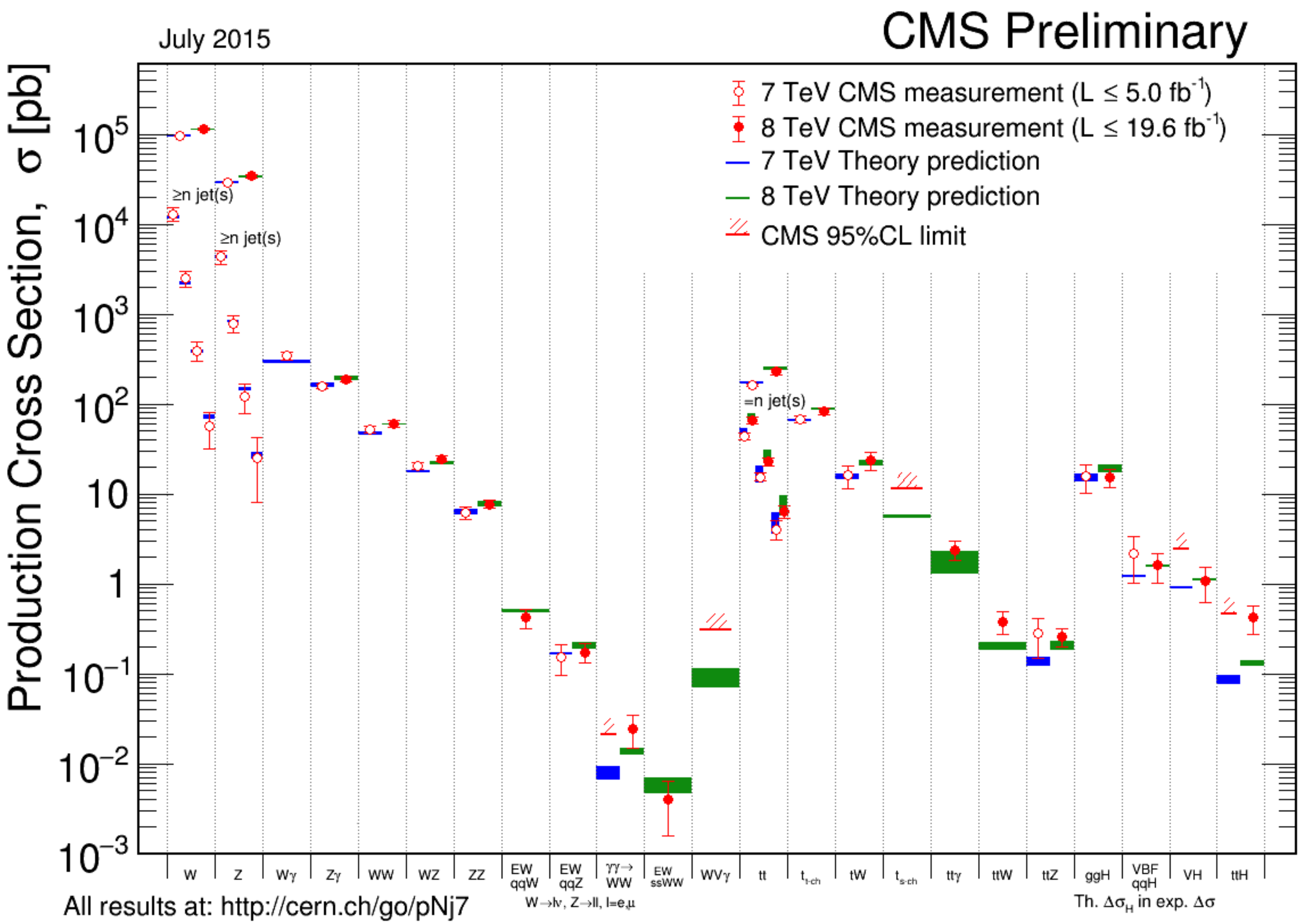
DESY, Zeuthen
26 May 2016

The challenge from the LHC

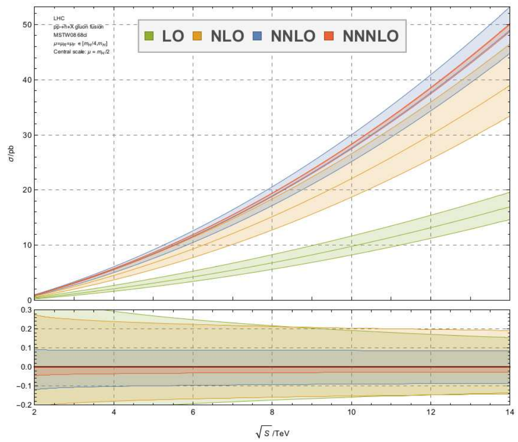
- ✓ Everything (signals, backgrounds, luminosity measurement) involves QCD
- ✓ Strong coupling is not small: $\alpha_s(M_Z) \sim 0.12$ and running is important
  - ⇒ events have high multiplicity of hard partons
  - ⇒ each hard parton fragments into a cluster of collimated particles jet
  - ⇒ higher order perturbative corrections can be large
  - ⇒ theoretical uncertainties can be large
- ✓ Processes can involve multiple energy scales: e.g. p_T^W and M_W
  - ⇒ may need resummation of large logarithms
- ✓ Parton/hadron transition introduces further issues, but for suitable (infrared safe) observables these effects can be minimised
  - ⇒ importance of infrared safe jet definition
  - ⇒ accurate modelling of underlying event, hadronisation, ...

✓✓ Nevertheless, excellent agreement between theory and experiment over a wide range of observables

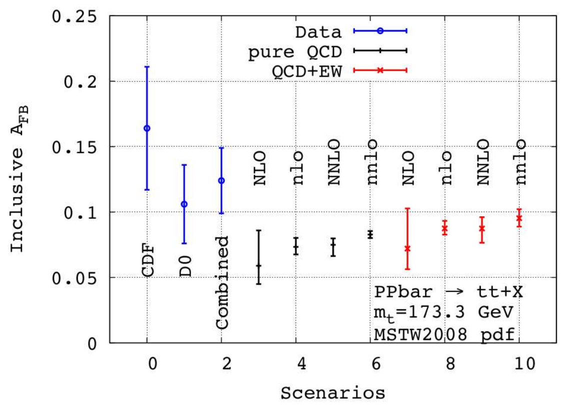
Cross Sections at the LHC



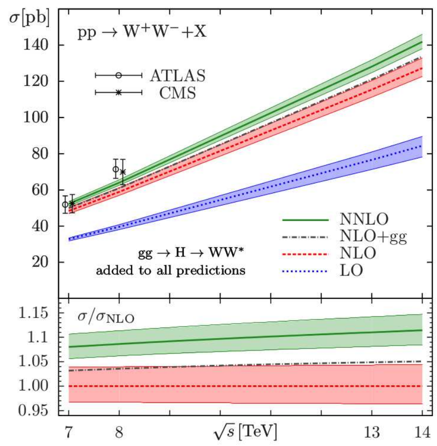
Discrepancies with data



Anastasiou, Duhr, Dulat, Herzog, Mistlberger



Czakon, Fiedler, Mitov

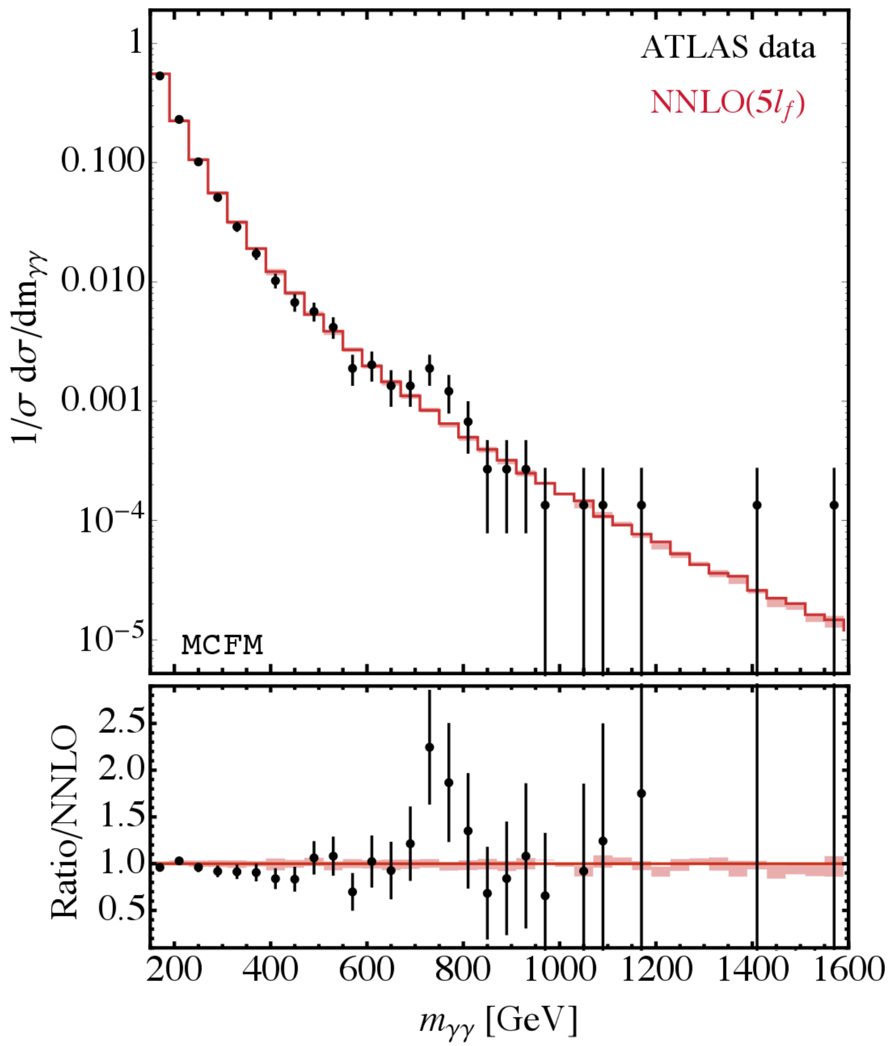


Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Ravlev, Tancredi

No BSM discovered yet. . . but plenty of BNLO

and a few interesting outliers

- ✓ bump hunting uses data driven methods
- ✓ extrapolation to region with little data
- ✓ can fit rate and compare to precise SM prediction
- ✓ more than 340 theory papers (since December 16)
- ✓ will survive . . . at least until ICHEP



Motivation for more precise theoretical calculations

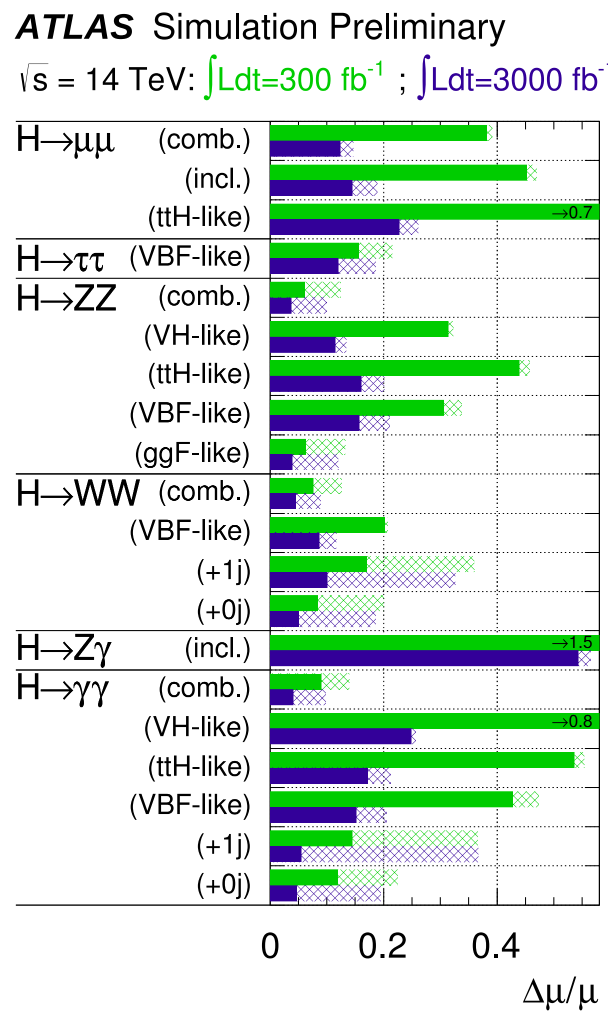
✓ Theory uncertainty has big impact on quality of measurement

✘ *NLO QCD is clearly insufficiently precise for SM, top (and even Higgs) measurements*, D. Froidevaux, HiggsTools School, 2015

⇒ Revised wishlist of theoretical predictions for

- Higgs processes
- Processes with vector bosons
- Processes with top or jets

Les Houches 2013, arXiv:1405.1067



Theoretical Uncertainties

- Missing Higher Order corrections (MHO)
  - truncation of the perturbative series
  - often estimated by scale variation - renormalisation/factorisation
  - ✓ systematically improvable by inclusion of higher orders
- Uncertainties in input parameters
  - parton distributions
  - masses, e.g., m_W, m_h, $[m_t]$
  - couplings, e.g., $\alpha_s(M_Z)$
  - ✓ systematically improvable by better description of benchmark processes
- Uncertainties in parton/hadron transition
  - fragmentation (parton shower)
  - ✓ systematically improvable by matching/merging with higher orders
  - hadronisation (model)
  - underlying event (tunes)

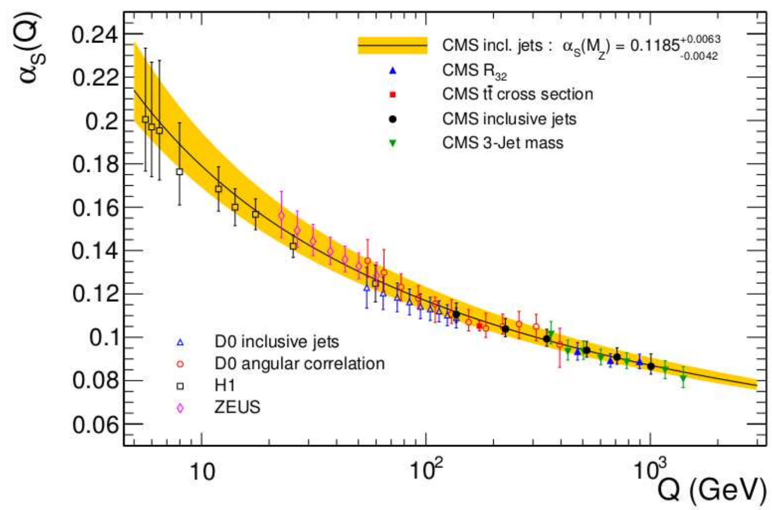
Goal: Reduce theory certainties by a factor of two compared to where we are now in next decade

The strong coupling

World Average

Year	$\alpha_s(M_Z)$
2008	0.1176 ± 0.0009
2012	0.1184 ± 0.0007
2014	0.1185 ± 0.0006

- ✓ Average of wide variety of measurements
  - ✓ τ-decays
  - ✓ e^+e^- annihilation
  - ✓ Z resonance fits
  - ✓ DIS
  - ✓ Lattice
- ✓ Generally stable to choice of measurements



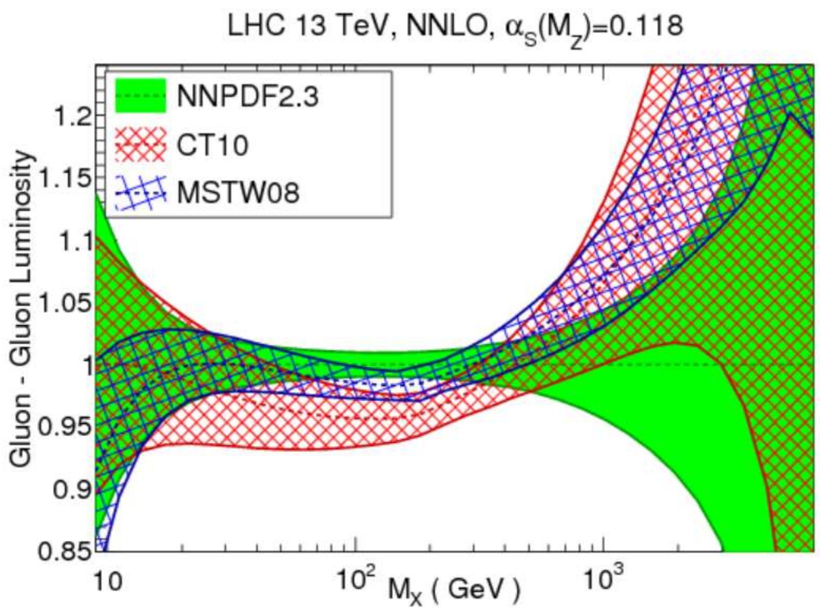
- ✓ Impressive demonstration of running of α_s to O(1 TeV)
- ✓ . . . but some outlier values from global PDF fits, e.g.,
  $\alpha_s(M_Z) \sim 0.1136 \pm 0.0004$ (G)JR
  $\alpha_s(M_Z) \sim 0.1132 \pm 0.0011$ ABM14
- ⇒ Still need to understand uncertainty and make more precise determination

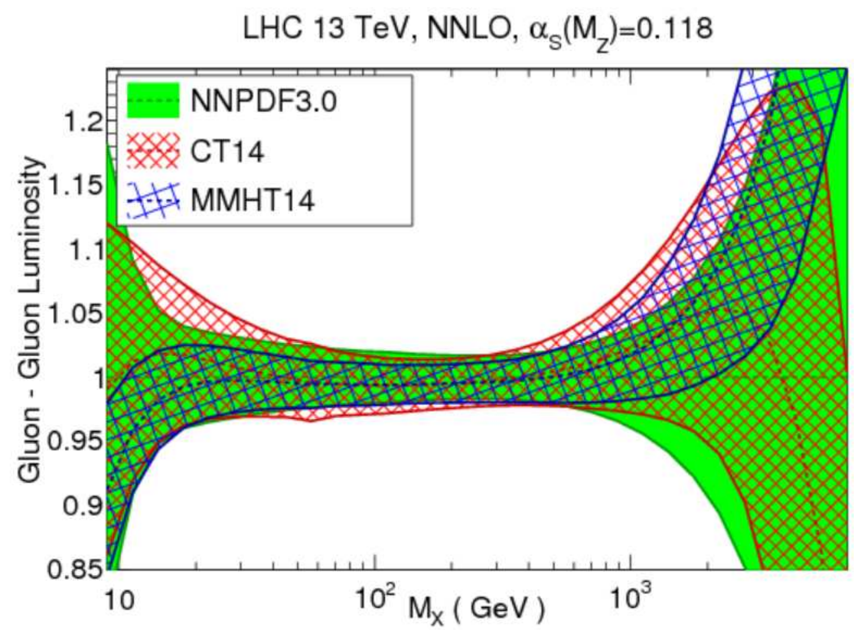
Parton Distribution Functions

All fits NNLO

Set	DIS	DY	jets	LHC	errors
MMHT14	✓	✓	✓	✓	hessian
CT14	✓	✓	✓	✓	hessian
NNPDF3.0	✓	✓	✓	✓	Monte Carlo
HeraPDF2.0	✓	✗	✗	✗	hessian
ABM14	✓	✓	✓	✗	hessian
G(JR)	✓	✓	✓	✗	hessian

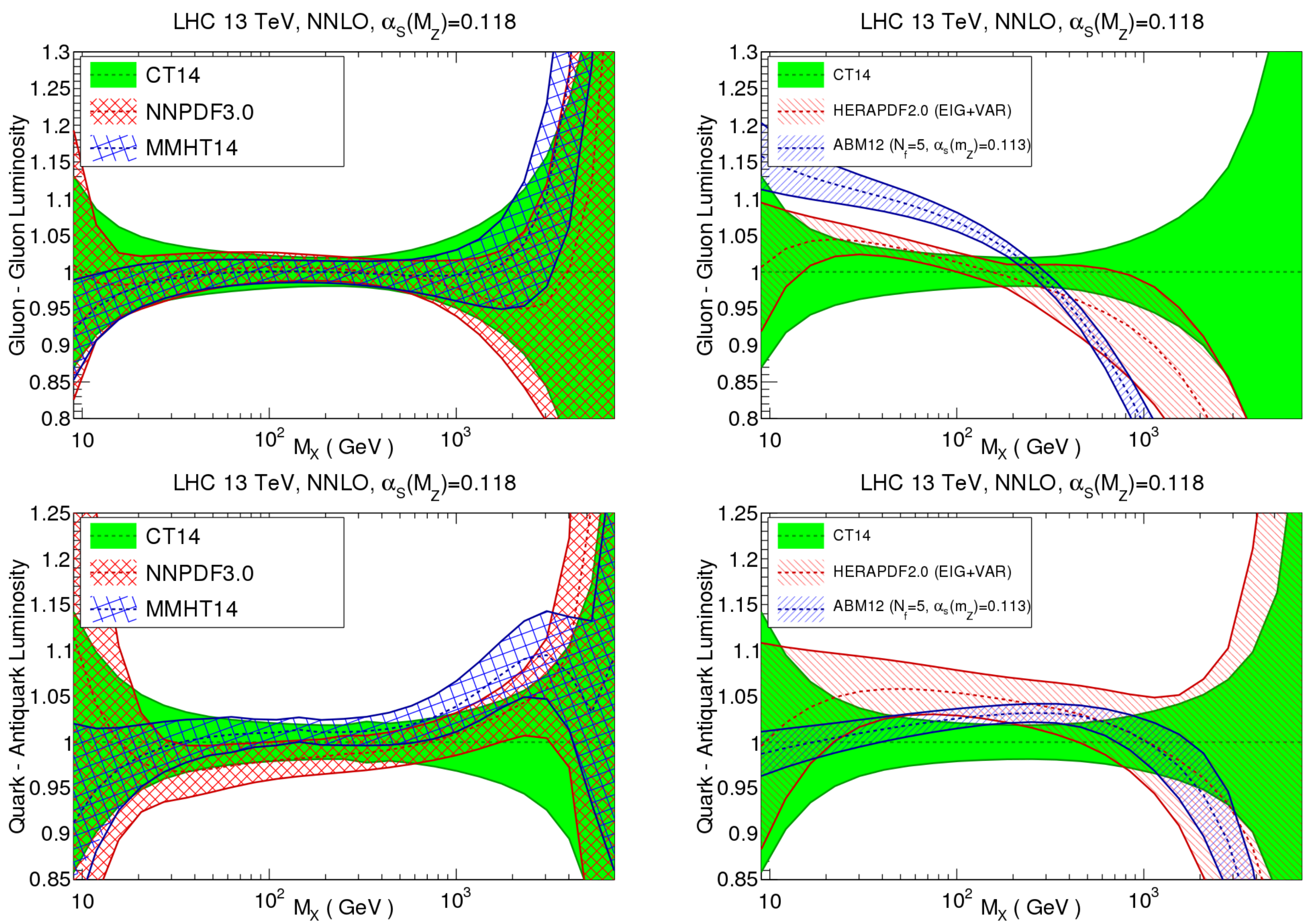
✓ Clear reduction in gluon-gluon luminosity for $M_X \sim 125$ GeV





✓ . . . with commensurate reduction in uncertainty on Higgs cross section

Parton Distribution Functions



but still differences of opinion

Partonic cross sections

$$\hat{\sigma} \sim \alpha_s^n \left(\hat{\sigma}^{LO} + \left(\frac{\alpha_s}{2\pi}\right) \hat{\sigma}^{NLO}_{QCD} \quad + \quad \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{NNLO}_{QCD} + \left(\frac{\alpha_s}{2\pi}\right)^3 \hat{\sigma}^{N3LO}_{QCD} + \ldots \right.$$

$$\left. \quad + \quad \left(\frac{\alpha_W}{2\pi}\right) \hat{\sigma}^{NLO}_{EW} + \ldots \right)$$

NLO QCD

✓ At least NLO is needed to obtain reliable predictions

NNLO QCD

✓ provides the first serious estimate of the theoretical uncertainty

NLO EW

✓ naively similar size to NNLO QCD

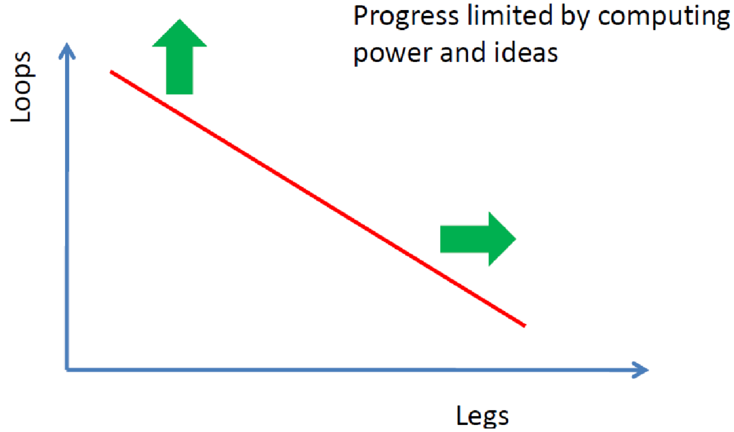
✓ particularly important at high energies/p_T and near resonances

N3LO QCD

✓ landmark result for Higgs production

What is the hold up?

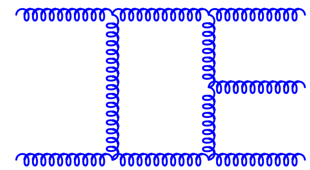
Rough idea of complexity of process $\sim$ #Loops + #Legs (+ #Scales)



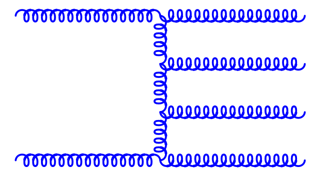
- loop integrals are ultraviolet/infrared divergent
- complicated by extra mass/energy scales
- loop integrals often unknown
  - ✓ completely solved at NLO
- real (tree) contributions are infrared divergent
- isolating divergences complicated
  - ✓ completely solved at NLO
- currently far from automation
  - ✓ mostly solved at NLO

Current standard: NLO

Anatomy of a NLO calculation



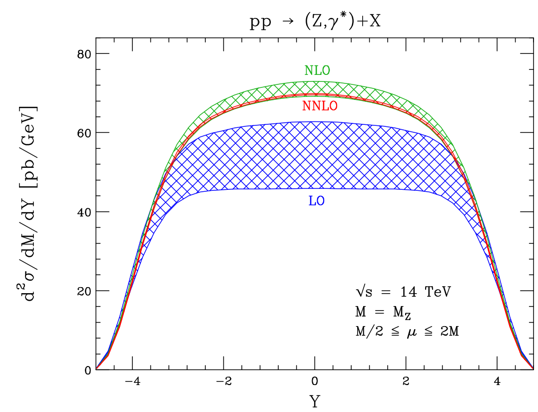
- ✓ one-loop $2 \to 3$ process
  - ✓ explicit infrared poles from loop integral
  - ✓ looks like 3 jets in final state



- ✓ tree-level $2 \to 4$ process
  - ✓ implicit poles from soft/collinear emission
  - ✓ looks like 3 or 4 jets in final state
- ✓ plus method for combining the infrared divergent parts
  - ✚ dipole subtraction — Catani, Seymour; Dittmaier, Trocsanyi, Weinzierl, Phaf
  - ✚ residue subtraction — Frixione, Kunszt, Signer
  - ✚ antenna subtraction — Kosower; Campbell, Cullen, NG; Daleo, Gehrmann, Maitre
  - ✚ phase space slicing — Giele, NG
  - ✚ sector decomposition — Hepp; Binoth, Heinrich
- ✓ NLO problem is solved in principle
- ✘ In practice, limitations in numerical accuracy for matrix elements and efficient phase space evaluation means that problems may occur with O(4-6) particles in final state

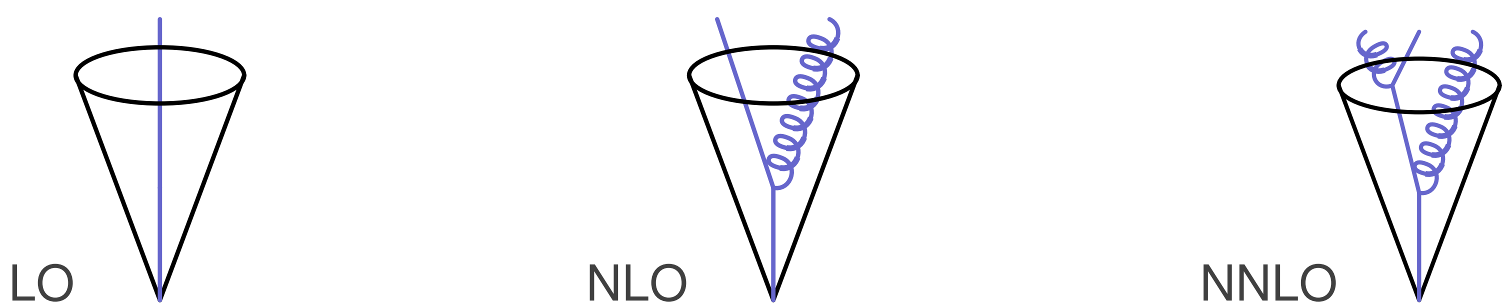
What NNLO might give you (1)

✓ Reduced renormalisation scale dependence



✓ Event has more partons in the final state so perturbation theory can start to reconstruct the shower

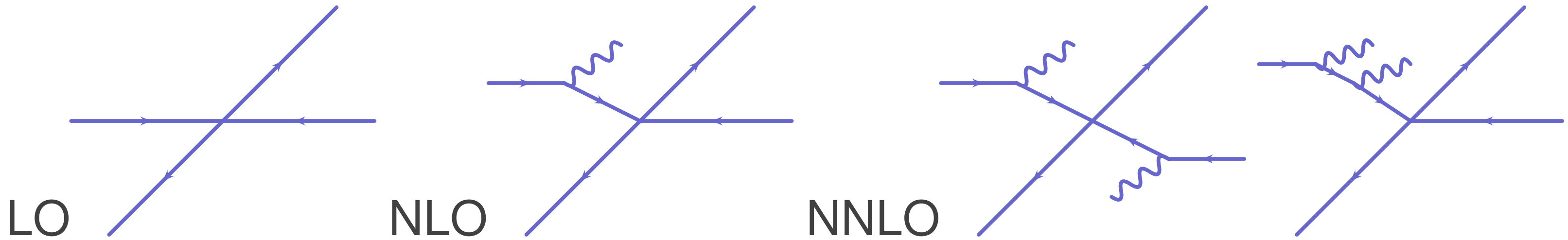
$\Rightarrow$ better matching of jet algorithm between theory and experiment



✓ Reduced power correction as higher perturbative powers of $1/\ln(Q/\Lambda)$ mimic genuine power corrections like $1/Q$

What NNLO might give you (2)

✓ Better description of transverse momentum of final state due to double radiation off initial state



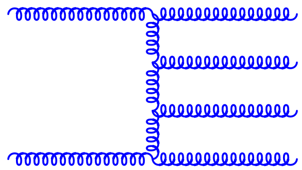
- ✓ At LO, final state has no transverse momentum
- ✓ Single hard radiation gives final state transverse momentum, even if no additional jet
- ✓ Double radiation on one side, or single radiation of each incoming particle gives more complicated transverse momentum to final state

✓ NNLO provides the first serious estimate of the theoretical uncertainty

✓✓✓ and most importantly, the volume and quality of the LHC data!!

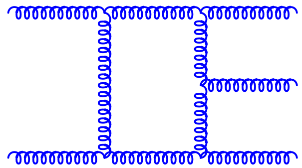
Anatomy of a NNLO calculation e.g. pp to JJ

✓ double real radiation matrix elements $\mathrm{d}\hat{\sigma}^{RR}_{NNLO}$



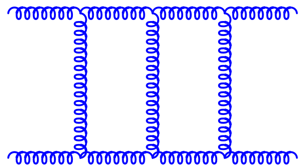
- ✚ implicit poles from double unresolved emission

✓ single radiation one-loop matrix elements $\mathrm{d}\hat{\sigma}^{RV}_{NNLO}$



- ✚ explicit infrared poles from loop integral
- ✚ implicit poles from soft/collinear emission

✓ two-loop matrix elements $\mathrm{d}\hat{\sigma}^{VV}_{NNLO}$



- ✚ explicit infrared poles from loop integral

$$\mathrm{d}\hat{\sigma}_{NNLO} \sim \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\hat{\sigma}^{RR}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\hat{\sigma}^{RV}_{NNLO} + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\hat{\sigma}^{VV}_{NNLO}$$

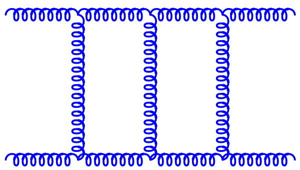
Anatomy of a NNLO calculation e.g. pp to JJ

✓ Double real and real-virtual contributions used in NLO calculation of X+1 jet

Can exploit NLO automation

...but needs to be evaluated in regions of phase space where extra jet is not resolved

✚ Two loop amplitudes - very limited set known



...currently far from automation

✚ Method for cancelling explicit and implicit IR poles - overlapping divergences

...currently not automated

Recap: IR subtraction at NLO

✓ To subtract the infrared singularities, we recast the NLO cross section in the form

$$\begin{aligned} \mathrm{d}\hat{\sigma}_{NLO} &= \int_{\mathrm{d}\Phi_{m+1}} \left[\mathrm{d}\hat{\sigma}^{R}_{NLO} - \mathrm{d}\hat{\sigma}^{S}_{NLO}\right] \\ &+ \int_{\mathrm{d}\Phi_{m}} \left[\mathrm{d}\hat{\sigma}^{V}_{NLO} - \mathrm{d}\hat{\sigma}^{T}_{NLO}\right] \end{aligned}$$

where the terms in each of the square brackets is finite, well behaved in the infrared singular regions and can be evaluated numerically.

$$\mathrm{d}\hat{\sigma}^{T}_{NLO} = -\int_{1} \mathrm{d}\hat{\sigma}^{S}_{NLO} + \mathrm{d}\hat{\sigma}^{MF}_{NLO}$$

✓ $\mathrm{d}\hat{\sigma}^{S}_{NLO}$

must cancel the implicit divergences in regions of phase space where $\mathrm{d}\hat{\sigma}^{R}_{NLO}$ is singular (subtraction)

or restrict the phase space to avoid these regions (slicing)

IR cancellation at NNLO

- ✓ The aim is to recast the NNLO cross section in the form

$$
\begin{aligned}
\mathrm{d}\hat{\sigma}_{NNLO} &= \int_{\mathrm{d}\Phi_{m+2}} \left[\mathrm{d}\hat{\sigma}^{RR}_{NNLO} - \mathrm{d}\hat{\sigma}^{S}_{NNLO}\right] \\
&+ \int_{\mathrm{d}\Phi_{m+1}} \left[\mathrm{d}\hat{\sigma}^{RV}_{NNLO} - \mathrm{d}\hat{\sigma}^{T}_{NNLO}\right] \\
&+ \int_{\mathrm{d}\Phi_{m}} \left[\mathrm{d}\hat{\sigma}^{VV}_{NNLO} - \mathrm{d}\hat{\sigma}^{U}_{NNLO}\right]
\end{aligned}
$$

  where the terms in each of the square brackets is finite, well behaved in the infrared singular regions and can be evaluated numerically.

- ✚ Much more complicated cancellations between the double-real, real-virtual and double virtual contributions
- ✚ intricate overlapping divergences

NNLO - IR cancellation schemes

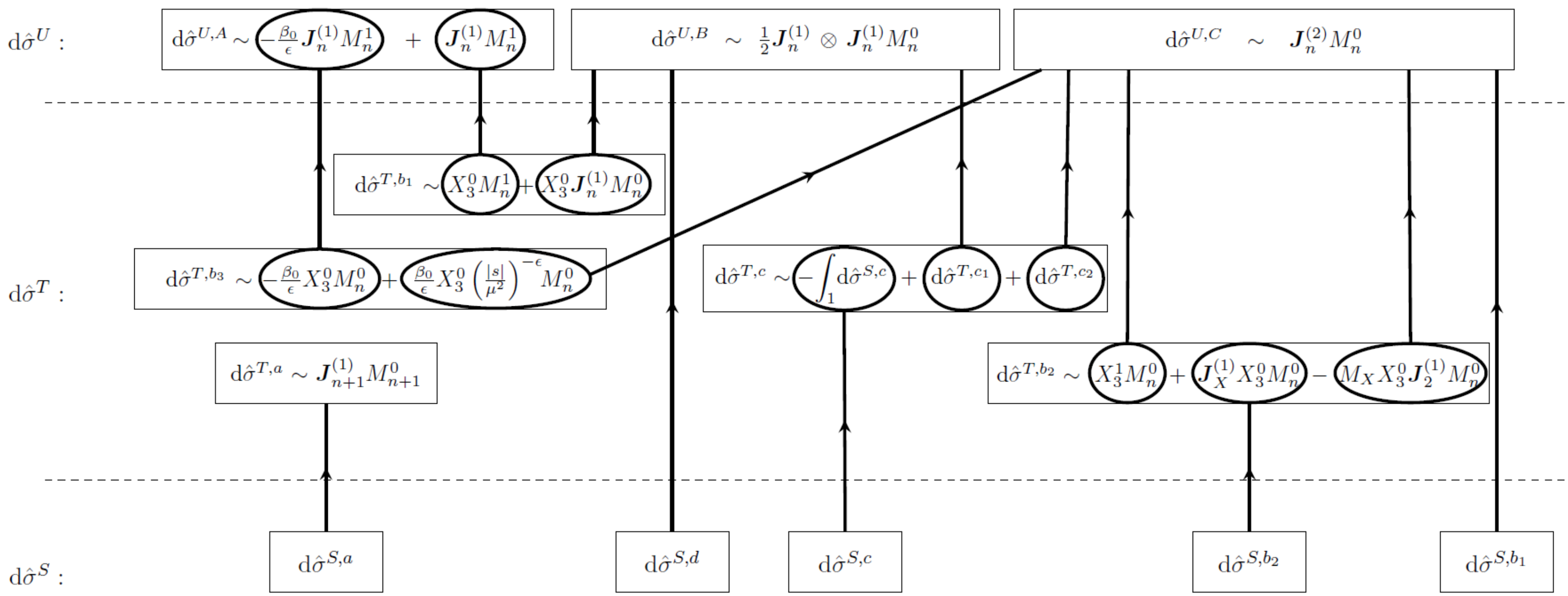
Unlike at NLO, we do not have a fully general NNLO IR cancellation scheme

- Antenna subtraction — Gehrmann, Gehrmann-De Ridder, NG (05)
- Colourful subtraction — Del Duca, Somogyi, Trocsanyi (05)
- q_T subtraction — Catani, Grazzini (07)
- STRIPPER (sector subtraction) — Czakon (10); Boughezal et al (11) Czakon, Heymes (14)
- N-jettiness subtraction — Boughezal, Focke, Liu, Petriello (15) Gaunt, Stahlhofen, Tackmann, Walsh (15)
- Projection to Born — Cacciari, Dreyer, Karlberg, Salam, Zanderighi (15)

Each method has its advantages and disadvantages

	Analytic	FS colour	IS colour	Azimuthal	Approach
Antenna	✓	✓	✓	✘	Subtraction
Colourful	✓	✓	✘	✓	Subtraction
q_T	✓	✘ (✓)	✓	–	Slicing
STRIPPER	✘	✓	✓	✓	Subtraction
N-jettiness	✓	✓	✓	–	Slicing
P2B	✓	✓	✓	–	Slicing

IR subtraction at NNLO



Currie, NG, Wells (13)

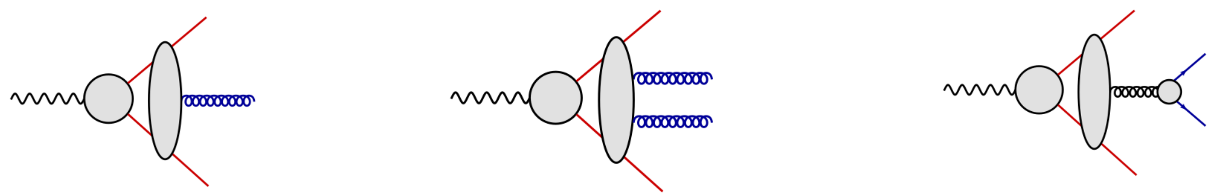
Antenna subtraction at NNLO

✓ Antenna subtraction exploits the fact that matrix elements already possess the intricate overlapping divergences

$$X_3^0(i,j,k) \sim \frac{|\mathcal{M}_3^0(i,j,k)|^2}{|\mathcal{M}_2^0(I,K)|^2}, \qquad X_4^0(i,j,k,l) \sim \frac{|\mathcal{M}_4^0(i,j,k,l)|^2}{|\mathcal{M}_2^0(I,L)|^2}$$

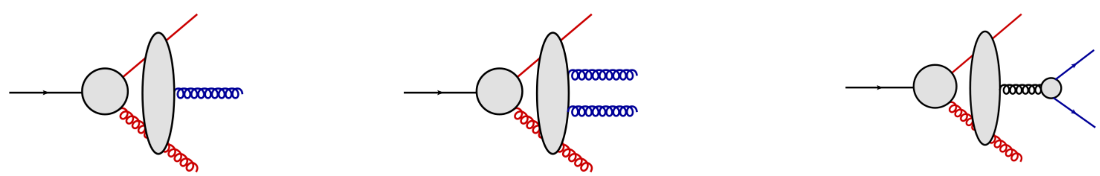
Quark-antiquark:

$\gamma^* \to q\bar{q} + \cdots$



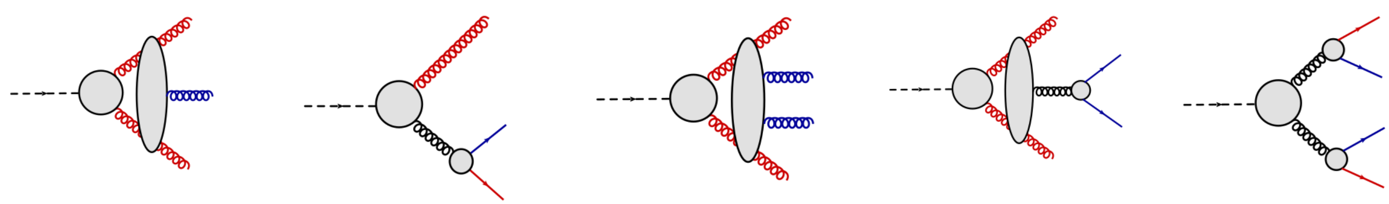
Quark-gluon:

$\tilde{\chi}^0 \to \tilde{g}g + \cdots$



Gluon-gluon:

$H \to gg + \cdots$



✓ plus mappings $i+j+k \to I+J$, $i+j+k+l \to I+L$

Antenna subtraction at NNLO

✓ Antenna mimics all singularities of QCD

$$A_4^0(1_q, 2_g, 3_g, 4_{\bar{q}})$$

1||2||3 $\rightarrow P^0_{qgg}$

2,3~0 $\rightarrow S^0_{1234}$

3||4 + 2~0 $\rightarrow S^0_{1;234}$

✓ Phase space map smoothly interpolates momenta for reduced matrix element between limits

$$\begin{aligned}
(\widetilde{123}) &= xp_1 + r_1 p_2 + r_2 p_3 + z p_4 \\
(\widetilde{234}) &= (1-x)p_1 + (1-r_1)p_2 + (1-r_2)p_3 + (1-z)p_4
\end{aligned}$$

Antenna subtraction at NNLO

- ✓ All unintegrated antennae available
  - ✓✓ Final-Final — Gehrmann-De Ridder, Gehrmann, NG, (05)
  - ✓✓ Initial-Final — Daleo, Gehrmann, Maitre, (07)
  - ✓✓ Initial-Initial — Daleo, Gehrmann, Maitre, (07); NG, Pires, (10)
- ✓ All antennae analytically integrated
  - ✓✓ Final-Final — Gehrmann-De Ridder, Gehrmann, NG, (05)
  - ✓✓ Initial-Final — Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, (10)
  - ✓✓ Initial-Initial — Gehrmann, Monni, (11); Boughezal, Gehrmann-De Ridder, Ritzmann, (11); Gehrmann, Ritzmann, (12)
- ✚ Laurent expansion in ϵ

NNLOJET

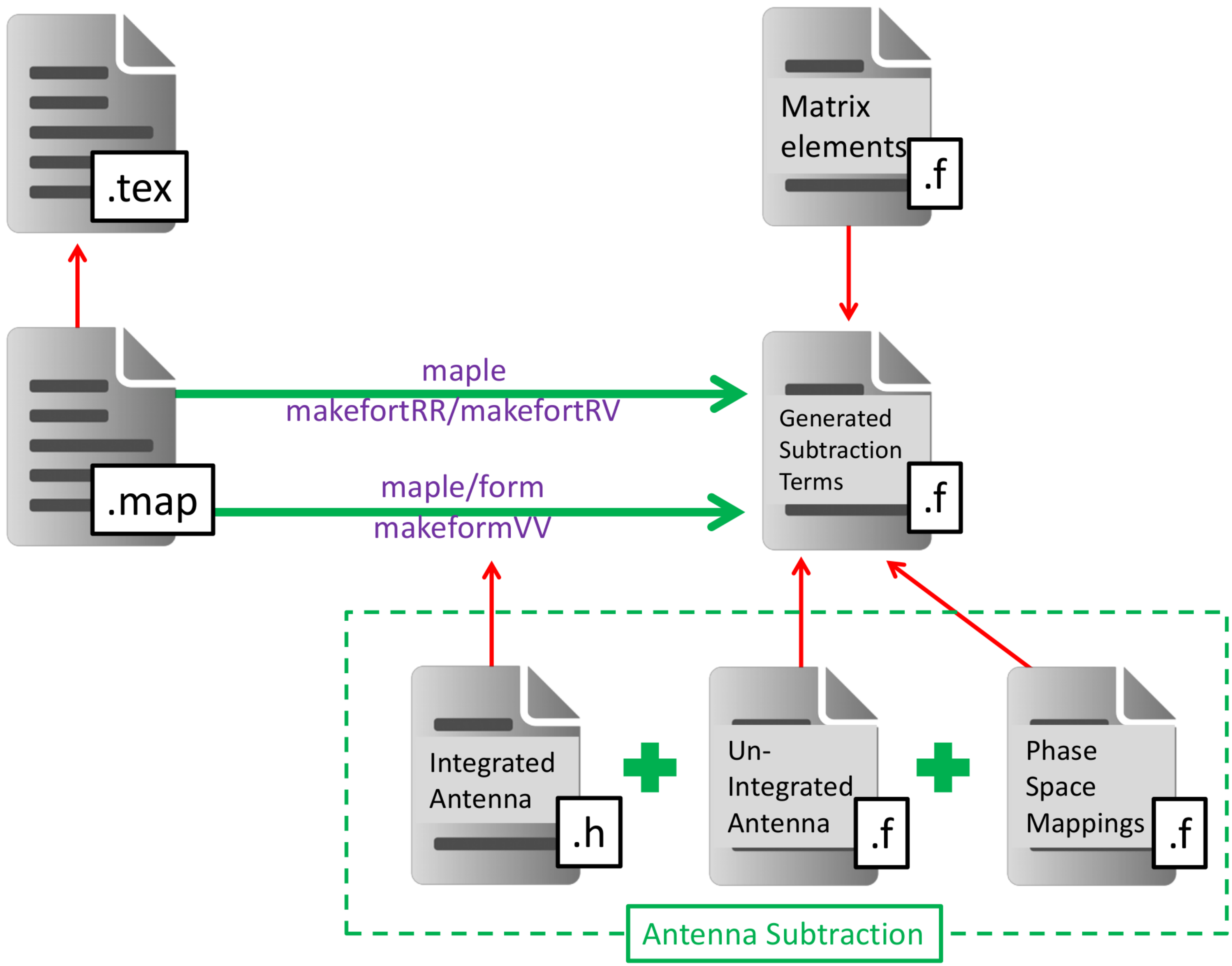
X. Chen, J. Cruz-Martinez, J. Currie, A. Gehrmann-De Ridder, T. Gehrmann, NG, A. Huss, M. Jaquier, T. Morgan, J. Niehues, J. Pires

UDUR, ETH, UZH, MPI, Peking University

Implementing NNLO corrections using Antenna subtraction for

- ✓ $pp \to H \to \gamma\gamma$ plus 0, 1, 2 jets
  1507.02850, 1601.04569, 1605.04295
- ✓ $pp \to e^+e^-$ plus 0, 1 jets
  1408.5325, 1604.04085
- ✓ $pp \to$ dijets
  1301.7310, 1310.3993
- ✓ $ep \to 2(+1)$ jets
  1605.XXX
- ✓ $\ldots$

Automatically generating the code (1)



Maple script: RR example


```
+F40a(i,j,k,l)*A4g0(1,2,[i,j,k],[j,k,l])
-f30FF(i,j,k)*f30FF([i,j],[j,k],l)
*A4g0(1,2,[[i,j],[j,k]],[[j,k],l])
...
```

$$+\,F_4^{0,a}(i,j,k,l)\,A_4^0(1,2,(\widetilde{ijk}),(\widetilde{jkl}))$$

$$-\,f_3^0(i,j,k)\,f_3^0((\widetilde{ij}),(\widetilde{jk}),l)\,A_4^0(1,2,[(\widetilde{ij}),(\widetilde{jk})],(\widetilde{(\widetilde{jk})l}))$$

...

- ✓ X_4^0, X_3^0 (and X_3^1 in RV) are unintegrated antennae
- ✓ `[i,j,k]` or $(\widetilde{ijk})$ are mapped momenta

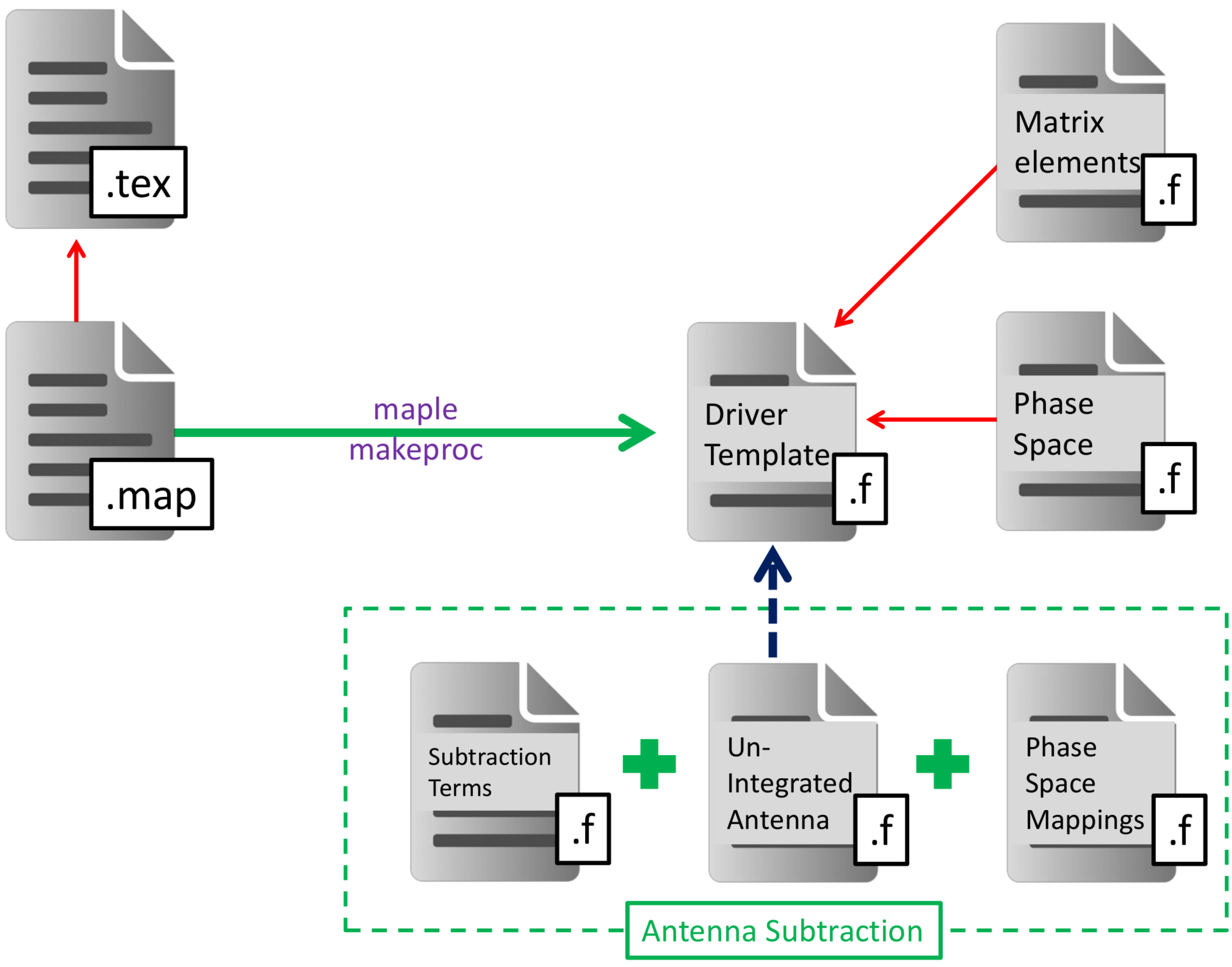
Maple script: VV example


```
-(+1/2*calgF40FI(2,3)
+1/2*calgF31FI(2,3)
+b0/e*1/2*QQ(s23)*calgF30FI(2,3)
-b0/e*1/2*calgF30FI(2,3)
-1/2*calgF30FI(2,3)*1/2*calgF30FI(2,3)
-1/2*gamma2gg(z2)
+b0/e*1/2*gamma1gg(z2)
)*A4g0(1,2,3,4)
...
```

$$
\begin{aligned}
+\Bigg[& -\frac{1}{2}\,\mathcal{F}^{0}_{4,g}(s_{23}) \\
& -\frac{1}{2}\,\mathcal{F}^{1}_{3,g}(s_{23}) \\
& -\frac{b_0}{2\epsilon}\left(\frac{s_{23}}{\mu_R^2}\right)^{-\epsilon}\mathcal{F}^{0}_{3,g}(s_{23}) \\
& +\frac{b_0}{2\epsilon}\,\mathcal{F}^{0}_{3,g}(s_{23}) \\
& +\frac{1}{4}\,\mathcal{F}^{0}_{3,g}(s_{23})\otimes\mathcal{F}^{0}_{3,g}(s_{23}) \\
& +\frac{1}{2}\,\Gamma^{(2)}_{gg}(z_2) \\
& -\frac{b_0}{2\epsilon}\,\Gamma^{(1)}_{gg}(z_2)\Bigg] A^{0}_{4}(1,2,3,4)
\end{aligned}
$$

✓ $\mathcal{X}_4^0$, $\mathcal{X}_3^0$ and $\mathcal{X}_3^1$ are integrated antennae

Automatically generating the code (2)



Maple script to produce driver template


```
R:=[
[A5g0,[g,g,g,g,g],1],
[B3g0,[qb,g,g,g,q],1/nc],
...
]:
```

$$
\begin{aligned}
d\sigma_{gg}^{R} = \mathcal{N}_{LO}\left(\frac{\alpha_s N}{2\pi}\right)\Bigg[\\
&+2\,\frac{1}{3!}\left(\sum_{12} \mathtt{A5g0}(1,2,3,4,5) - \mathtt{ggA5g0SNLO}(1,2,3,4,5)\right) \\
&+\frac{N_F}{N}\left(\sum_{6} \mathtt{B3g0}(3,1,2,4,5) - \mathtt{ggB3g0SNLO}(3,1,2,4,5)\right) \\
&\ldots\Bigg]
\end{aligned}
$$

✓ Have to link subtraction terms to automatically generated code (1)

Checks

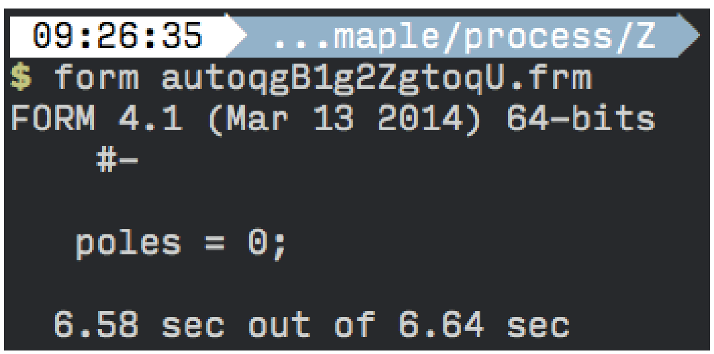
✓ Unresolved limits for RR, RV

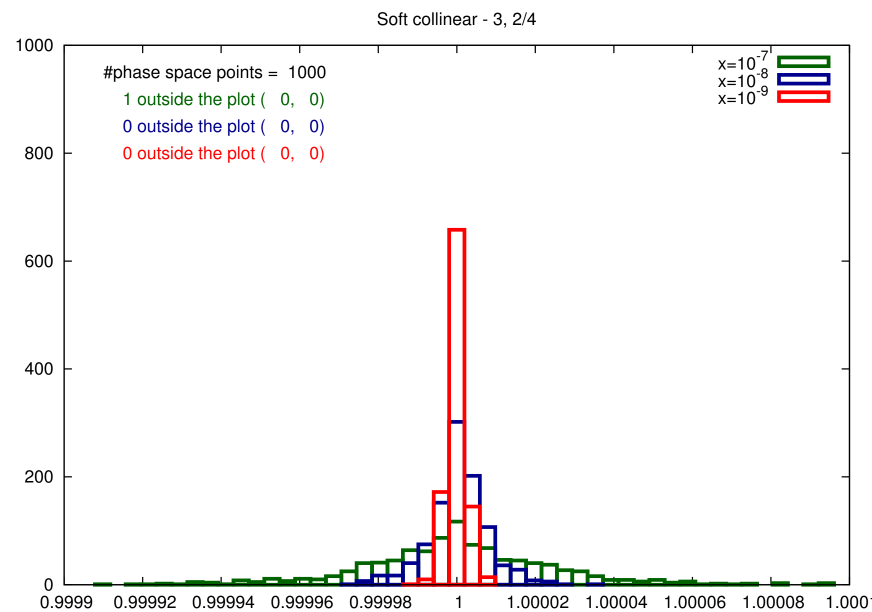
$$d\sigma^S \longrightarrow d\sigma^{RR}$$
$$d\sigma^T \longrightarrow d\sigma^{RV}$$

$q\bar{q} \to Z + g_3\ g_4\ g_5$ (g_3 soft & $g_4 \parallel \bar{q}$)

✓ Analytic pole cancellations for RV, VV

$$\text{Poles}\left(d\sigma^{RV} - d\sigma^{T}\right) = 0$$
$$\text{Poles}\left(d\sigma^{VV} - d\sigma^{U}\right) = 0$$





H + J production, large mass limit

Boughezal, Caola, Melnikov, Petriello, Schulze (13,15)
Chen, Gehrmann, NG, Jaquier (14,16)
Boughezal, Focke, Giele, Liu, Petriello (15)
Caola, Melnikov, Schulze (15)

- ✓ phenomenologically interesting
- ✓ large scale uncertainty
- ✓ large K-factor

$$\sigma_{NLO}/\sigma_{LO} \sim 1.6$$
$$\sigma_{NNLO}/\sigma_{NLO} \sim 1.3$$

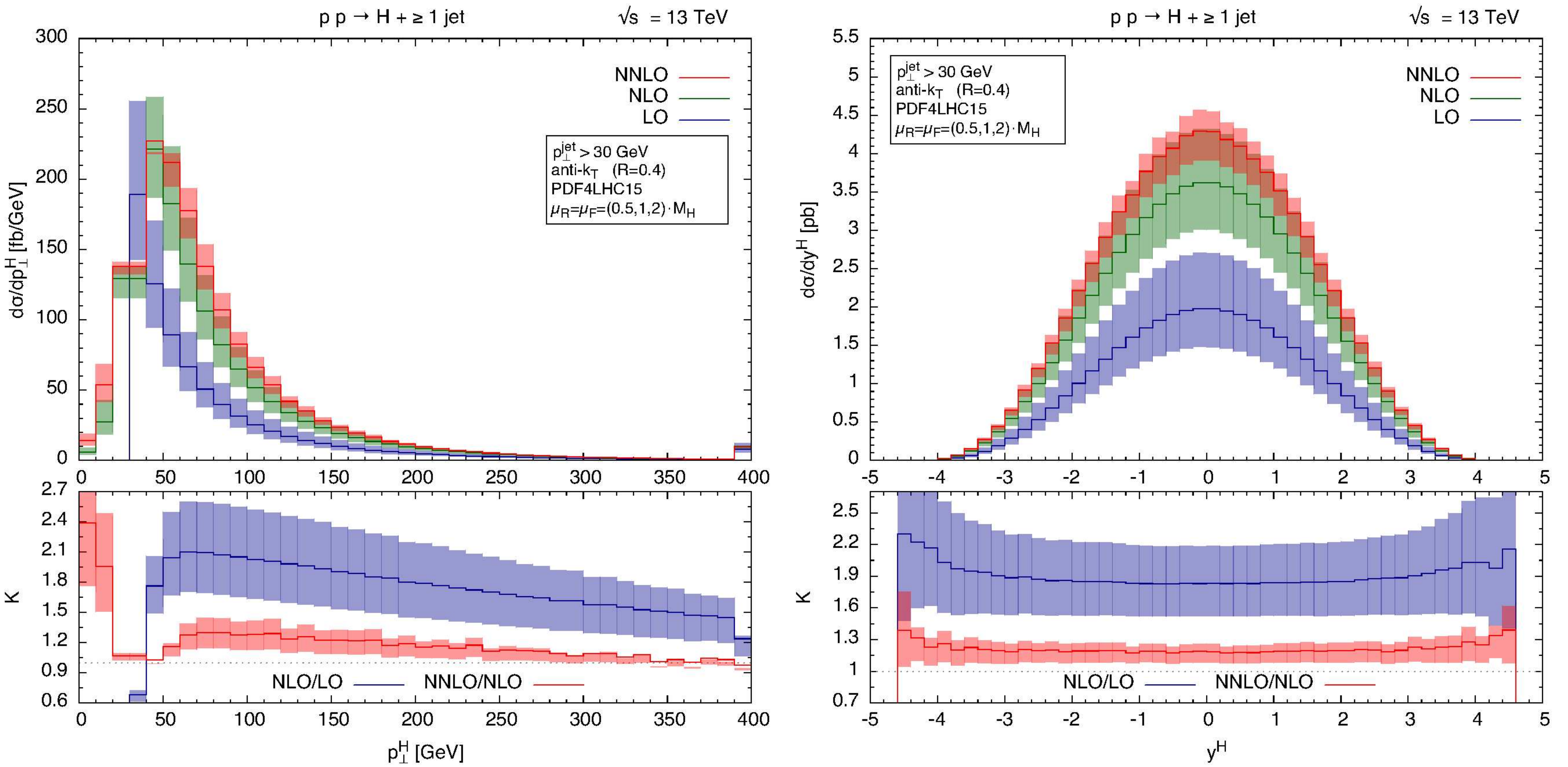
- ✓ significantly reduced scale dependence $\mathcal{O}(4\%)$

- ✓ Three independent computations:
  - ✚ STRIPPER
  - ✚ Antenna
  - ✚ N-jettiness
- ✓ allows for benchmarking of methods (for gg, qg and $\bar{q}g$ processes)
  - ✚ $\sigma^{NNLO} = 9.45^{+0.58}_{-0.82}$ fb

    Caola, Melnikov, Schulze (15)
  - ✚ $\sigma^{NNLO} = 9.44^{+0.59}_{-0.85}$ fb

    Chen, Gehrmann, NG, Jaquier (16)

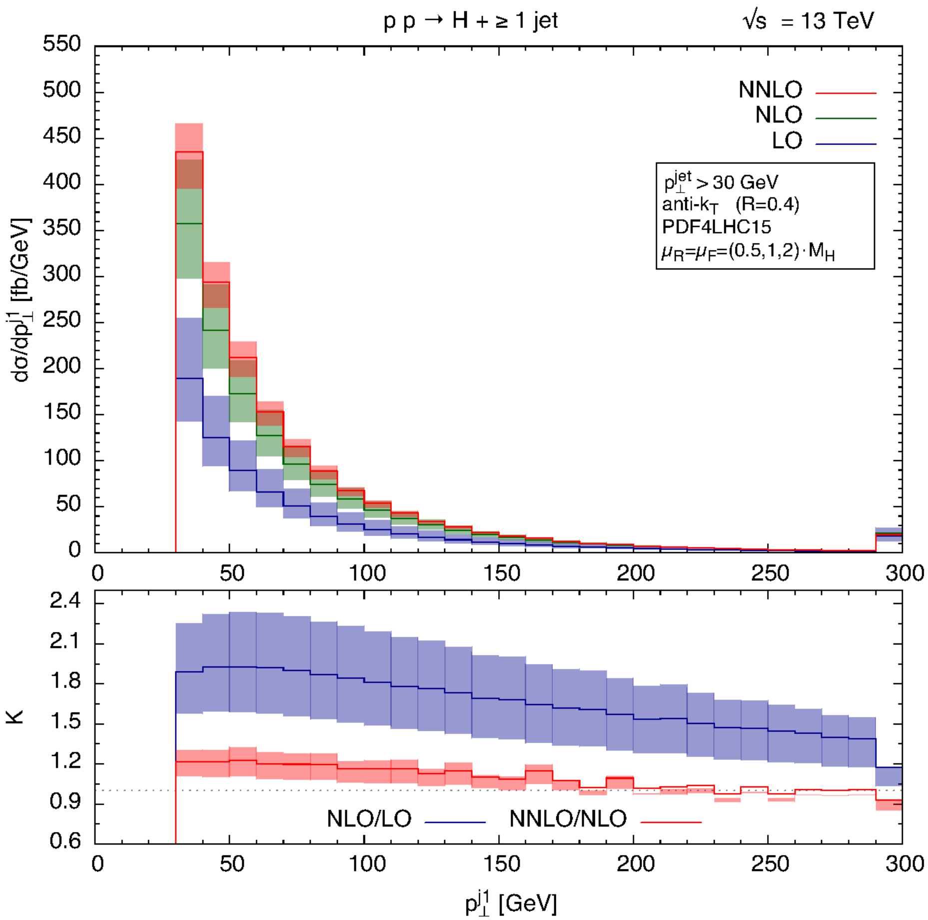
Higgs p_T and rapidity

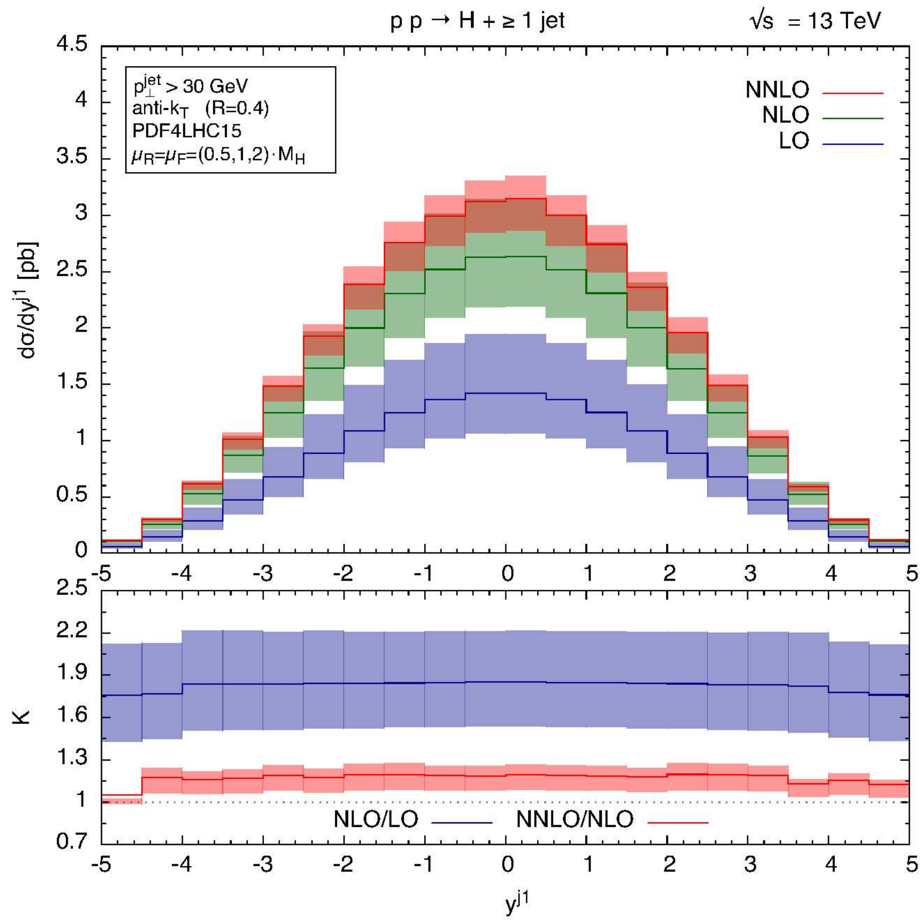


Higgs p_T and rapidity distributions

$\sqrt{s} = 13$ TeV, PDF4LHC15, $p_T^{jet} > 30$ GeV, anti-k_T, $R = 0.4$, $\mu_F = \mu_R = (0.5, 1, 2)m_H$

Jet p_T and rapidity

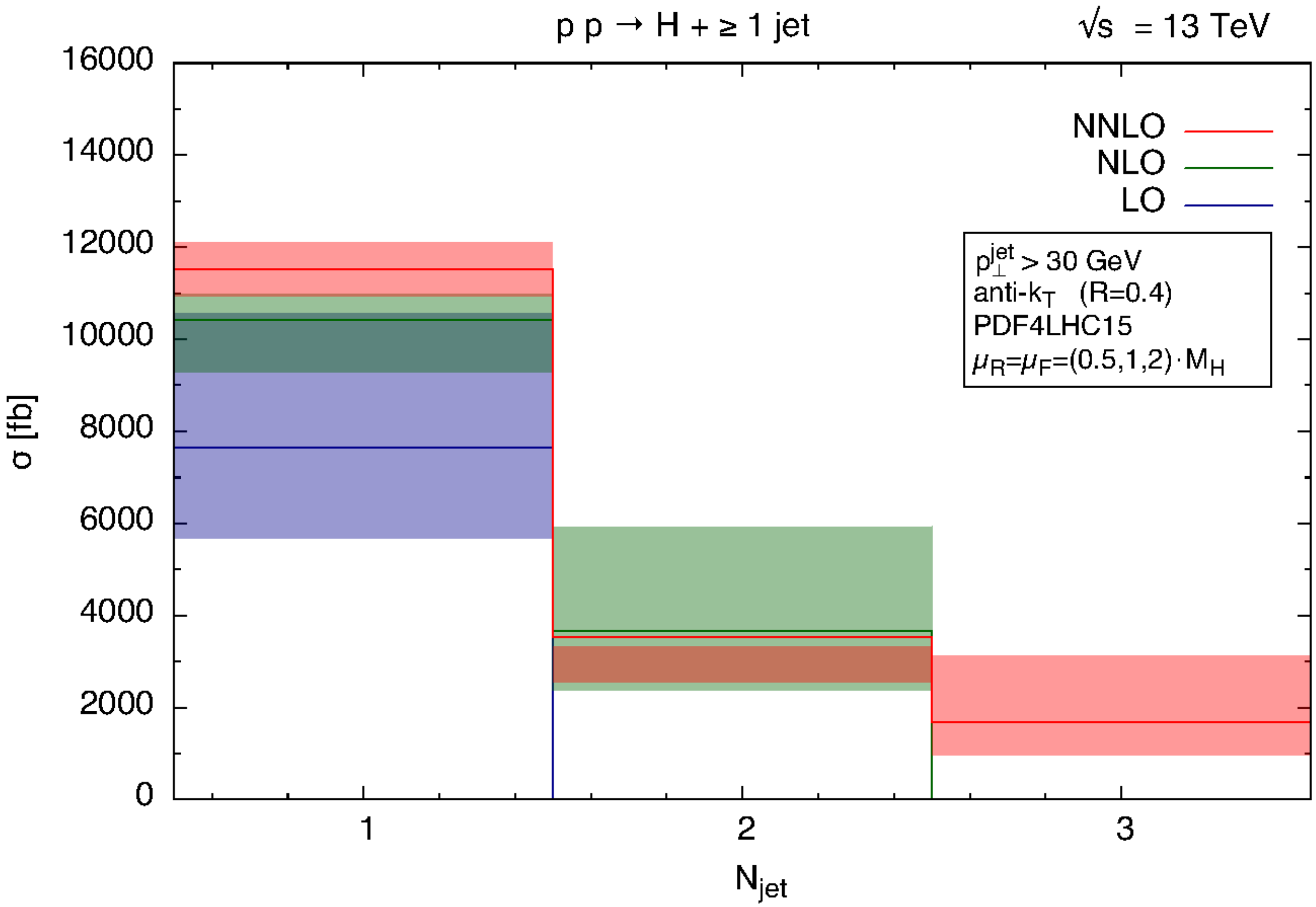




Leading jet p_T and rapidity distributions

$\sqrt{s} = 13$ TeV, PDF4LHC15, $p_T^{jet} > 30$ GeV, anti-k_T, $R = 0.4$, $\mu_F = \mu_R = (0.5, 1, 2)m_H$

Exclusive jet bins

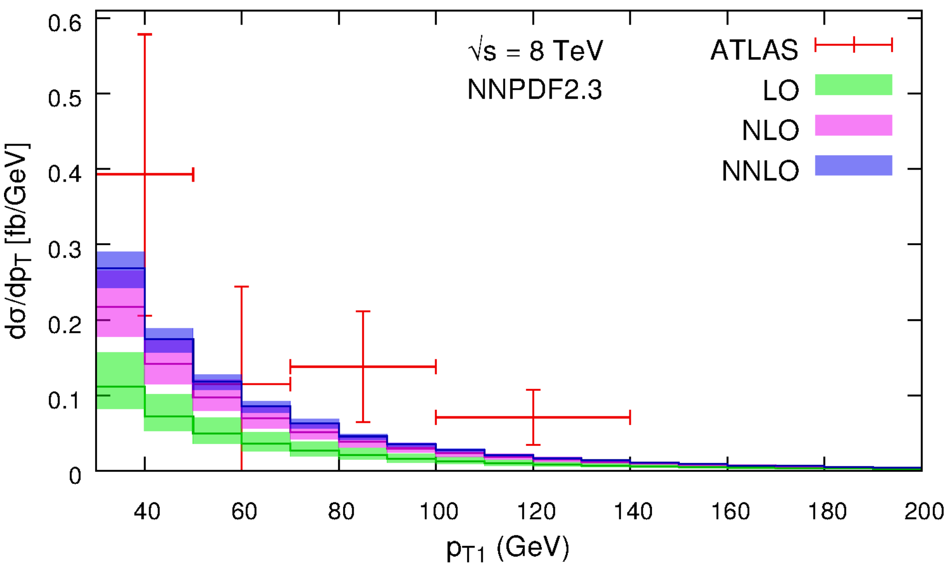


Comparison with Data

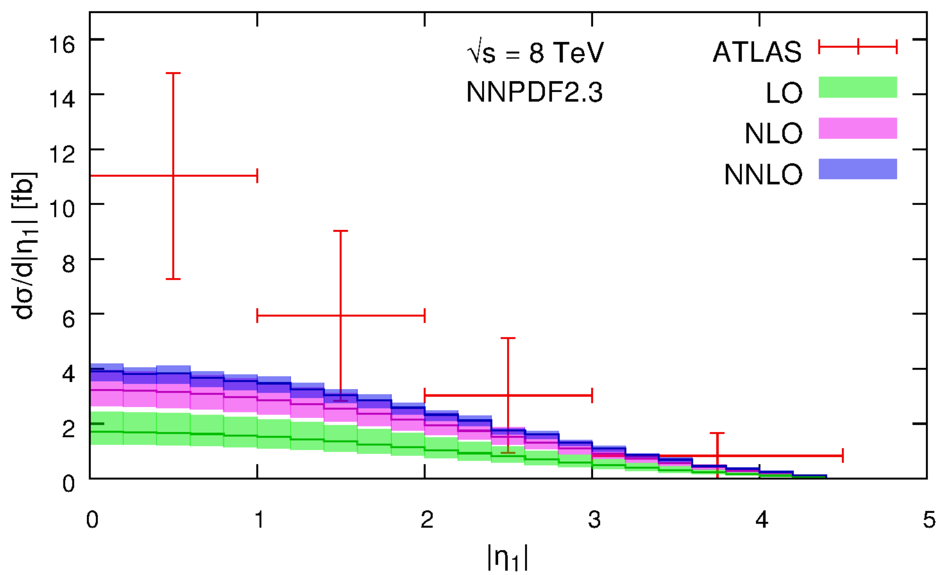
ATLAS setup arXiv:1407.4222

- ✓ H+J NNLO prediction undershoots ATLAS data
- ✓ statistical errors still quite large
- ✓ finite mass effects estimated to be 2-3% @NLO

Harlander, Neumann, Ozeren, Wiesemann (12)



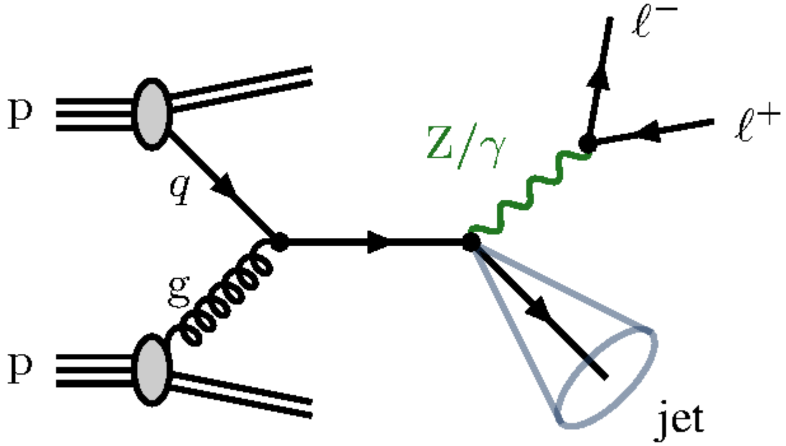
p_T of leading jet



Rapidity of leading jet

Z + J production

Gehrmann-De Ridder, Gehrmann, NG, Huss, Morgan (15)
Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello (15)
Boughezal, Liu, Petriello (16)



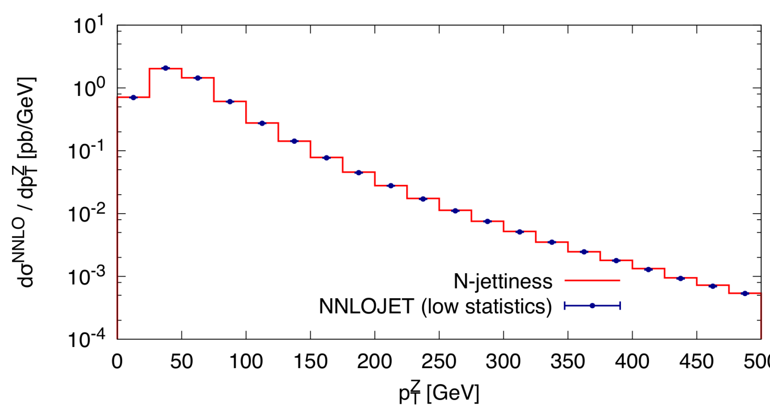
- ✓ clean leptonic signature
- ✓ good handle on jet energy scale
- ✓ significant NLO K-factor and scale uncertainty

$$\sigma_{NLO}/\sigma_{LO} \sim 1.4$$

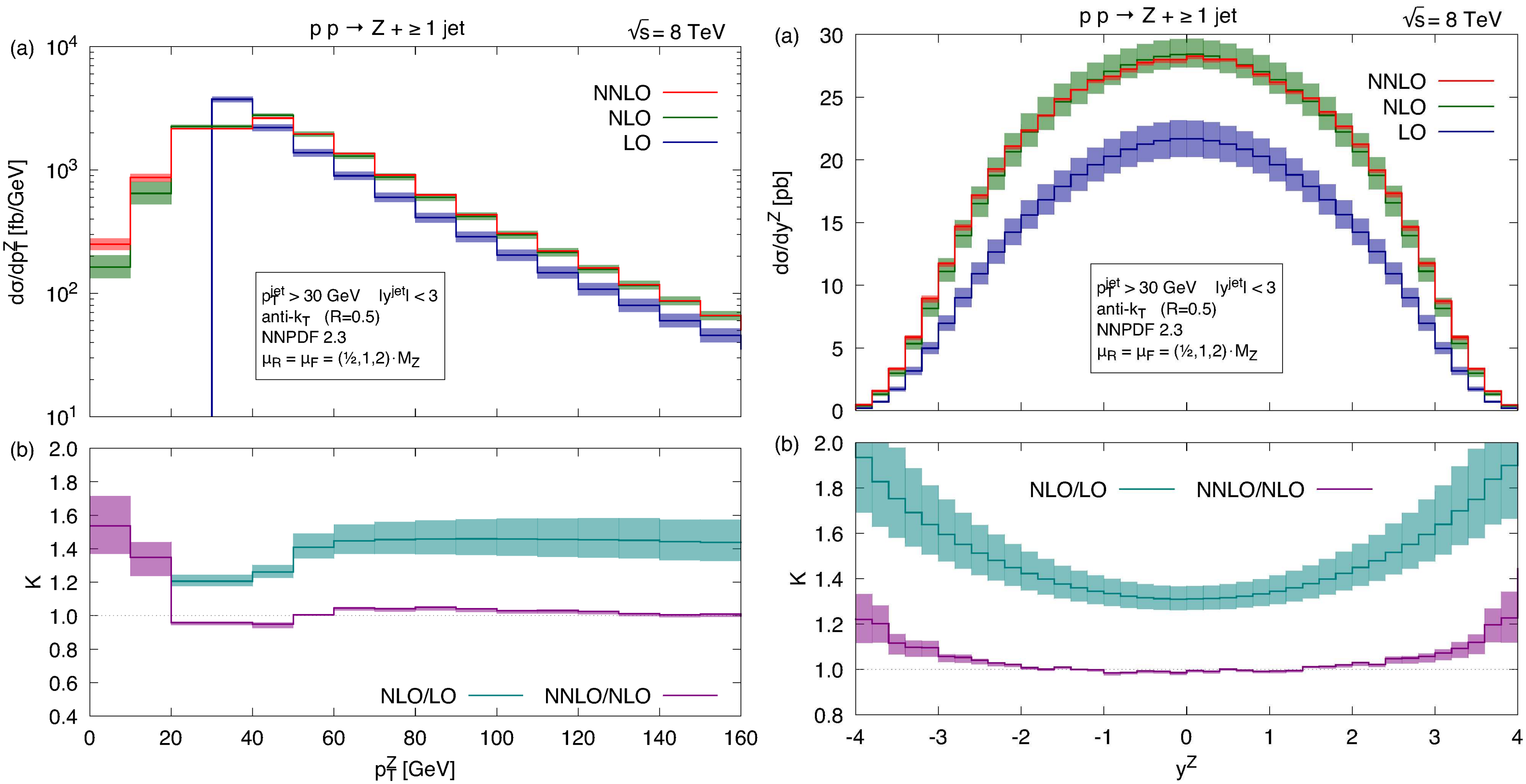
- ✓ Two independent computations:
- ✓ allows for benchmarking of methods
  - ✚ $\sigma^{NNLO} = 135.6^{+0.0}_{-0.4}$ fb

    Gehrmann-De Ridder, Gehrmann, NG, Huss, Morgan (15)
  - ✚ $\sigma^{NNLO} = 135.6^{+0.0}_{-0.4}$ fb

    Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello (15)



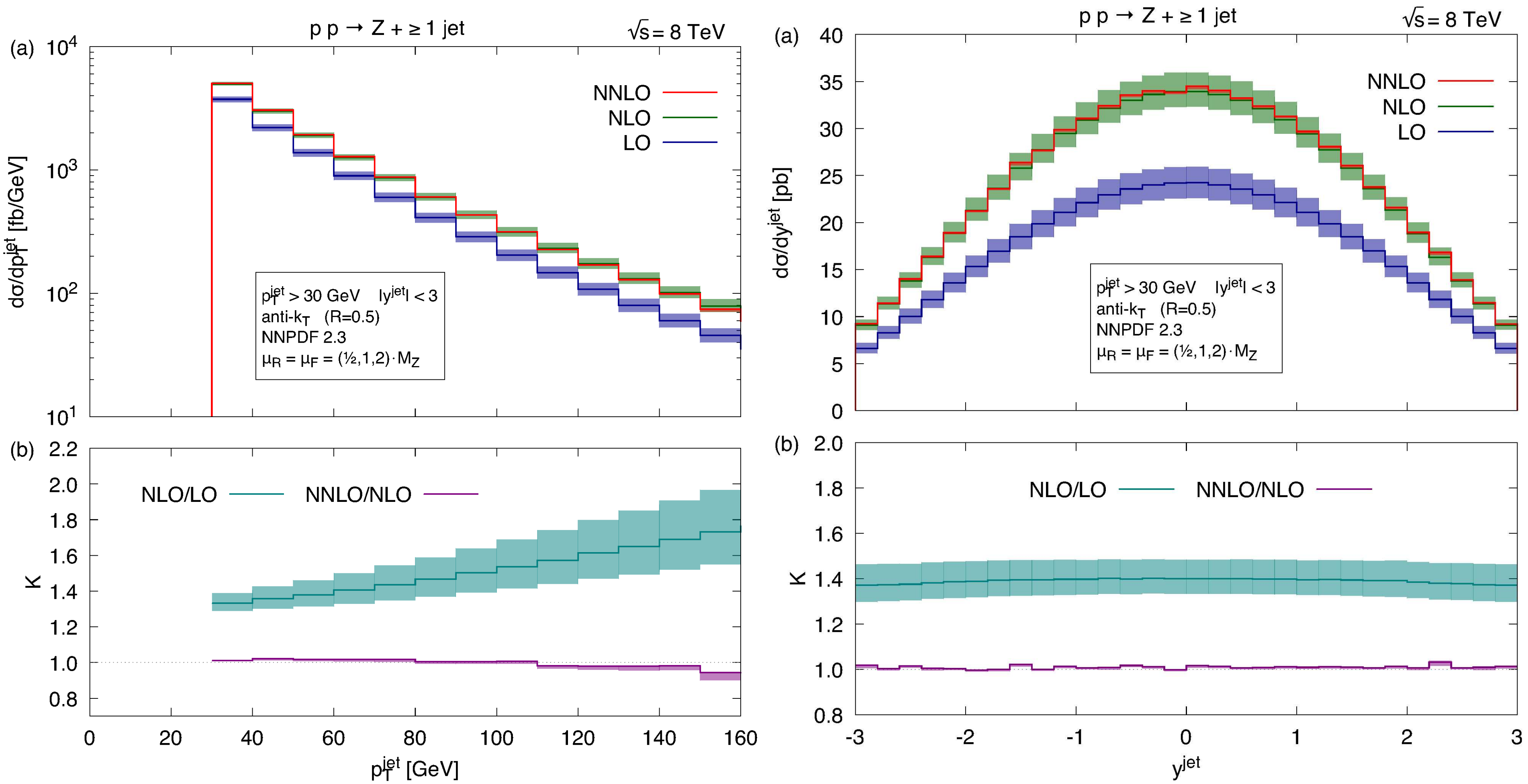
$Z\ p_T$ and rapidity



$Z\ p_T$ and rapidity distributions

$\sqrt{s} = 8$ TeV, NNPDF2.3, $p_T^{jet} > 30$ GeV, $|y^{jet}| < 3$, anti-k_T, $R = 0.5$, $80\ \mathrm{GeV} < m_{\ell\ell} < 100\ \mathrm{GeV}$, $\mu_F = \mu_R = (0.5, 1, 2)m_Z$

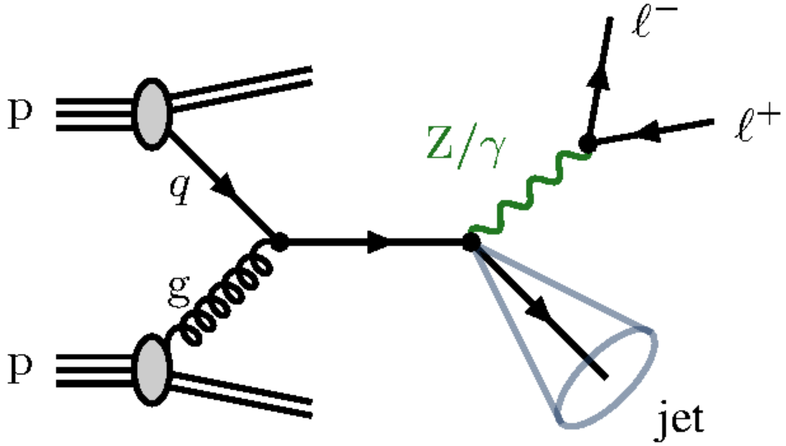
Jet p_T and rapidity



Leading jet p_T and rapidity distributions

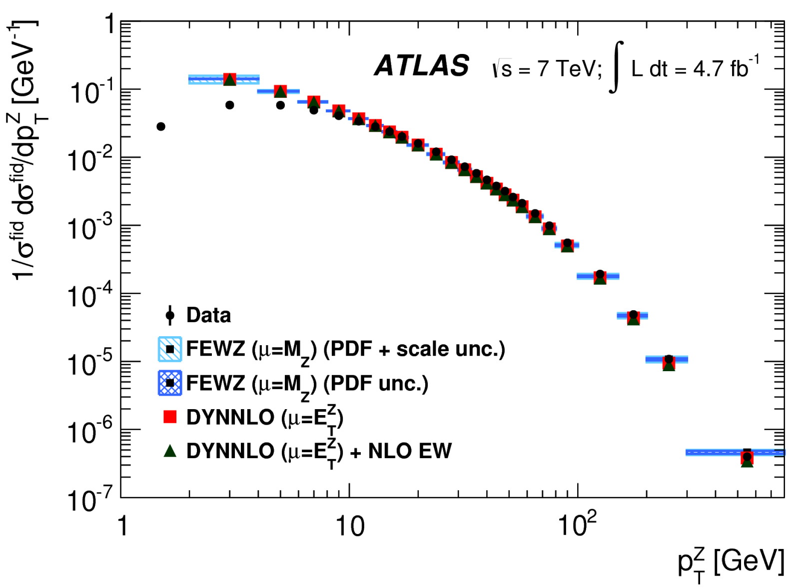
$\sqrt{s} = 8$ TeV, NNPDF2.3, $p_T^{jet} > 30$ GeV, $|y^{jet}| < 3$, anti-k_T, $R = 0.5$, $80\text{ GeV} < m_{\ell\ell} < 100\text{ GeV}$, $\mu_F = \mu_R = (0.5, 1, 2)m_Z$

Inclusive p_T spectrum of Z



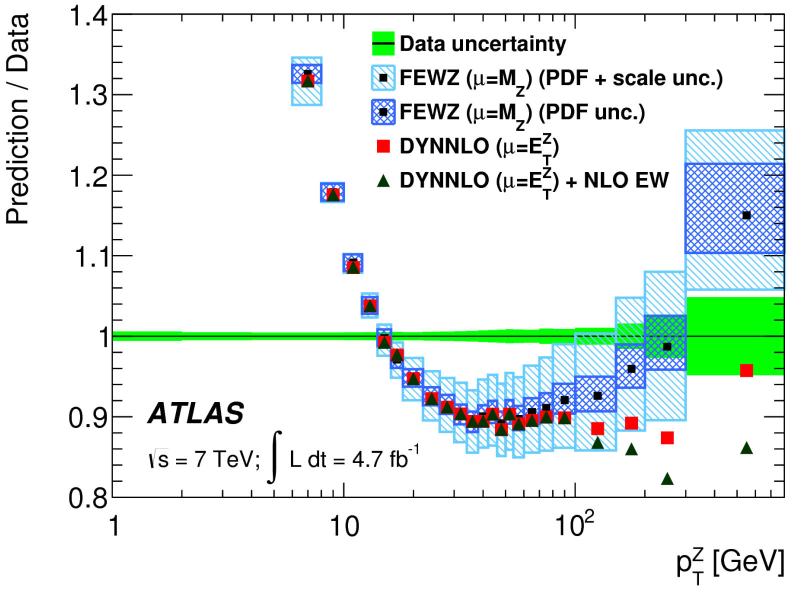
$$pp \to Z/\gamma^* \to \ell^+\ell^- + X$$

- large cross section
- clean leptonic signature



- fully inclusive wrt QCD radiation
- only reconstruct ℓ^+, ℓ^- so clean and precise measurement
- potential to constrain gluon PDFs

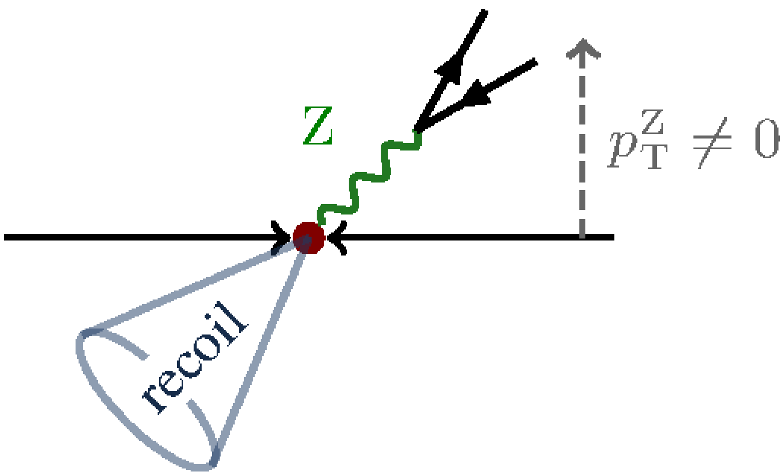
Inclusive p_T spectrum of Z



✚ low $p_T^Z \leq 10$ GeV, resummation required

✚ $p_T^Z \geq 20$ GeV, fixed order prediction about 10% below data

✘ *Very precise measurement of Z* p_T *poses problems to theory*, D. Froidevaux, HiggsTools School



FEWZ/DYNNLO are Z + 0 jet @ NNLO

✘ Only NLO accurate in this distribution

✓ Requiring recoil means Z + 1 jet @ NNLO required

Inclusive p_T^Z spectrum: Setup

Calculational setup

- ✓ LHC @ 8 TeV
- ✓ PDF: NNPDF2.3 $\alpha_s(M_Z)$ = 0.118
- ✓ fully inclusive wrt QCD radiation
- ✓ $p_T^Z > 20$ GeV
- ✓ $p_T^{\ell_1} > 20$ GeV, $p_T^{\ell_1} > 10$ GeV, $|y^{\ell^\pm}| < 2.4$, 12 GeV $< m_{\ell\ell} < 150$ GeV
- ✓ dynamical scale choice

$$\mu_R = \mu_F = \sqrt{m_{\ell\ell}^2 + p_{T,Z}^2} \times \left[\frac{1}{2}, 1, 2\right]$$

CMS setup arXiv:1504.03511

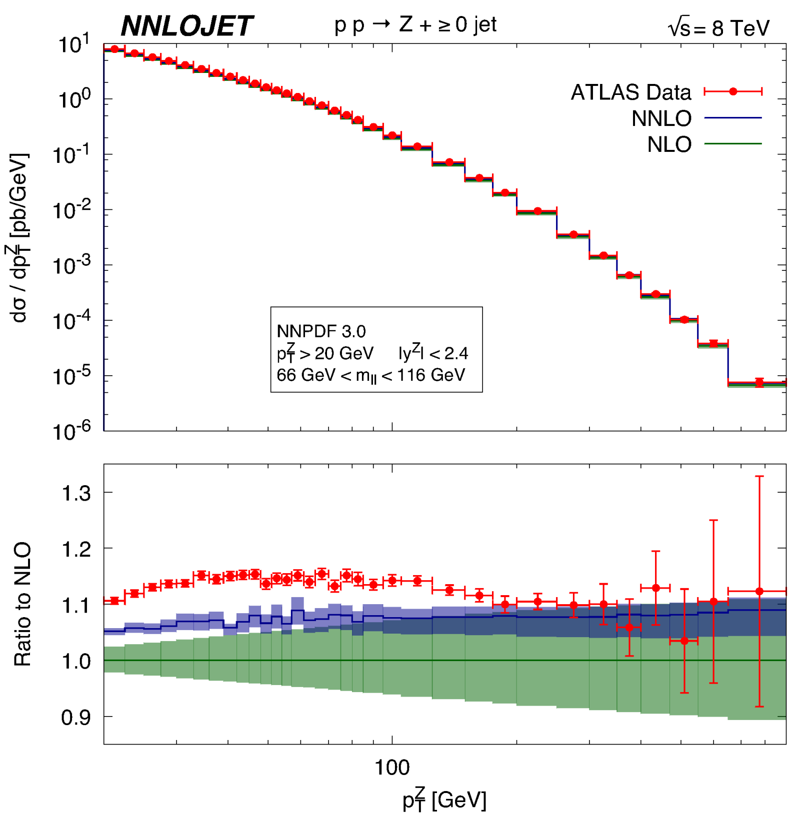
- $p_T^{\ell_1} > 25$ GeV, $|y^{\ell_1}| < 2.1$
- $p_T^{\ell_2} > 10$ GeV, $|y^{\ell_2}| < 2.4$
- 81 GeV $< m_{\ell\ell} < 101$ GeV+ binning in y^Z

ATLAS setup arXiv:1512.02192

- $p_T^{\ell^\pm} > 20$ GeV, $|y^{\ell^\pm}| < 2.4$
- 66 GeV $< m_{\ell\ell} < 116$ GeV + binning in y^Z
- $|y^Z| < 2.4$ + binning in $m_{\ell\ell}$

Inclusive p_T spectrum of Z

$$\left.\frac{\mathrm{d}\hat{\sigma}}{dp_T^Z}\right|_{p_T^Z>20\ \mathrm{GeV}} \equiv \frac{\mathrm{d}\hat{\sigma}_{LO}^{ZJ}}{dp_T^Z} + \frac{\mathrm{d}\hat{\sigma}_{NLO}^{ZJ}}{dp_T^Z} + \frac{\mathrm{d}\hat{\sigma}_{NNLO}^{ZJ}}{dp_T^Z} \tag{1}$$

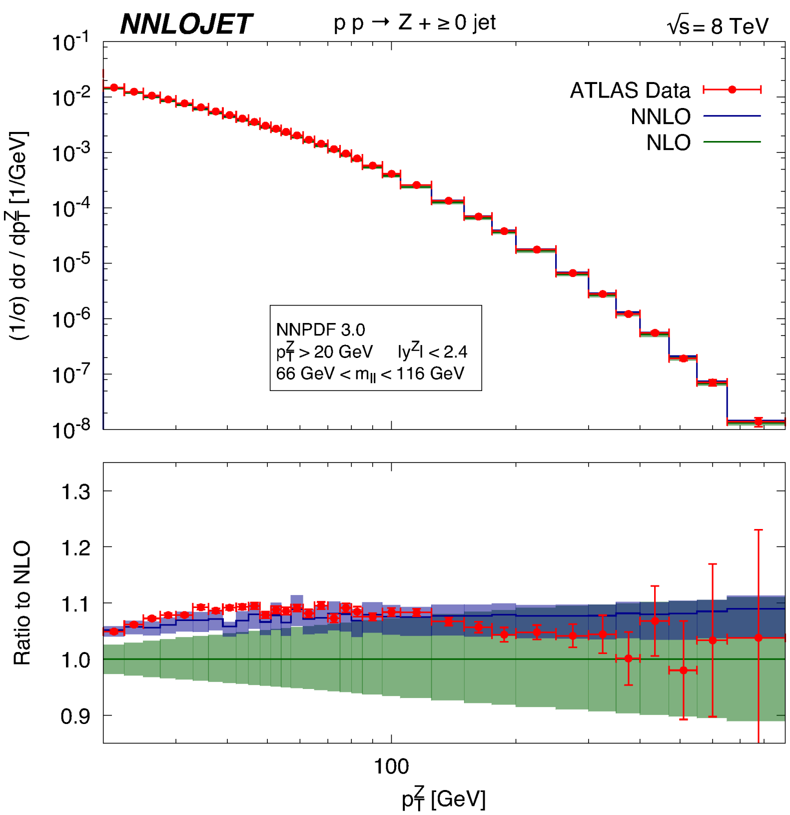


- ✓ NLO corrections $\sim 40 - 60\%$
- ✓ significant reduction of scale uncertainties NLO $\rightarrow$ NNLO
- ✓ NNLO corrections relatively flat $\sim 4 - 8\%$
- ✓ improved agreement, but not enough
- ✓ Note that for $66\ \mathrm{GeV} < m_{\ell\ell} < 116\ \mathrm{GeV}$

$$\sigma_{\mathrm{exp}} = 537.1 \pm 0.45\% \pm 2.8\%\ \mathrm{pb}$$

$$\sigma_{\mathrm{NNLO}} = 507.9^{+2.4}_{-0.7}\ \mathrm{pb}$$

Inclusive p_T spectrum of Z



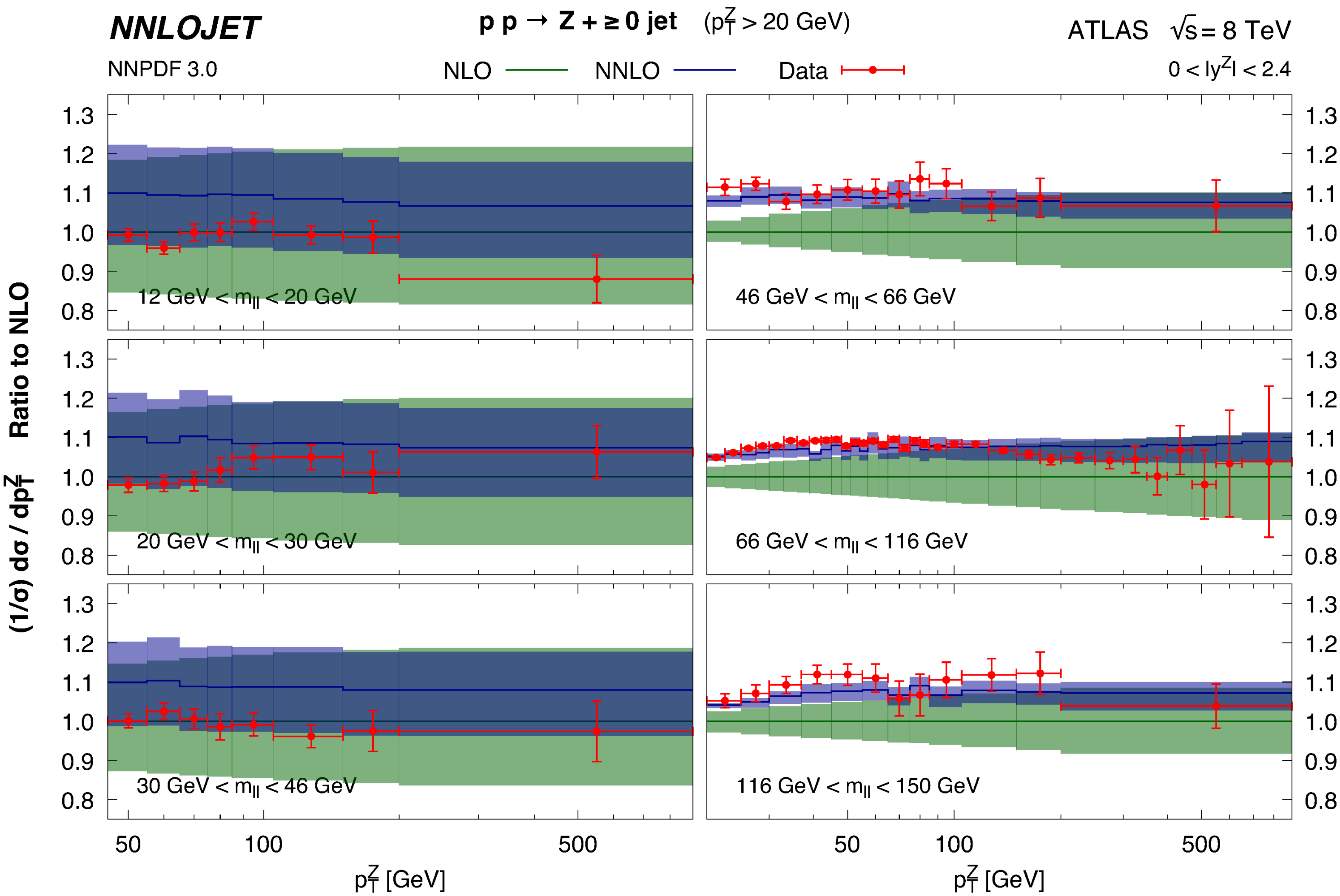
$$\frac{1}{\sigma} \cdot \frac{\mathrm{d}\hat{\sigma}}{dp_T^Z}\bigg|_{p_T^Z > 20\ \mathrm{GeV}}$$

with

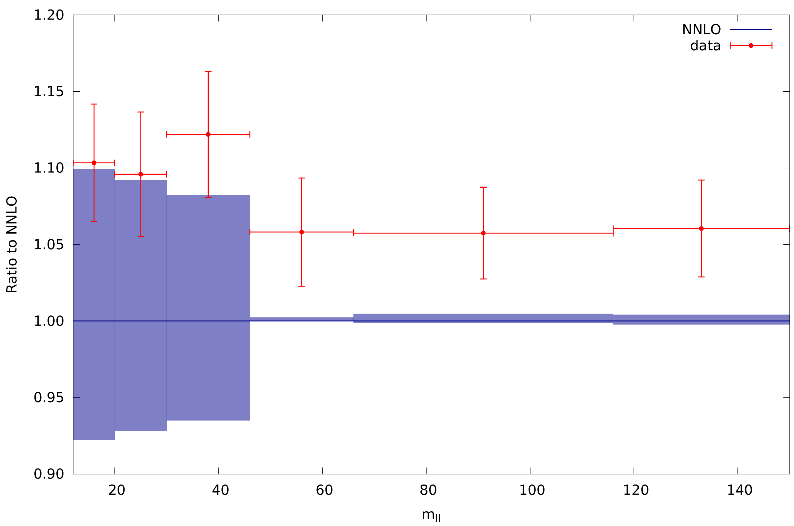
$$\sigma = \int_0^\infty \frac{\mathrm{d}\hat{\sigma}}{dp_T^Z} dp_T^Z \equiv \sigma_{LO}^Z + \sigma_{NLO}^Z + \sigma_{NNLO}^Z.$$

- ✓ Much improved agreement
- ✓ luminosity uncertainty reduced
- ✓ dependence on EW parameters reduced
- ✓ dependence on PDFs reduced $\Rightarrow$ study

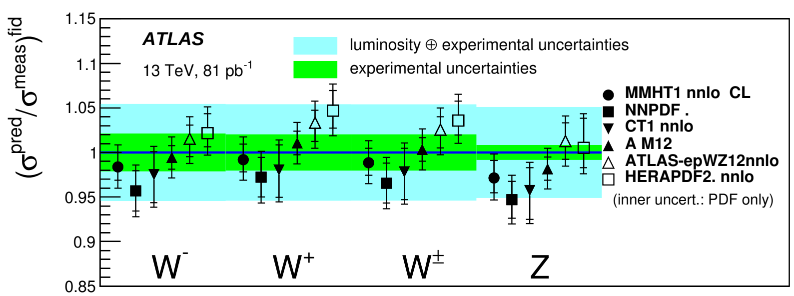
Inclusive p_T spectrum of Z



Inclusive p_T spectrum of Z

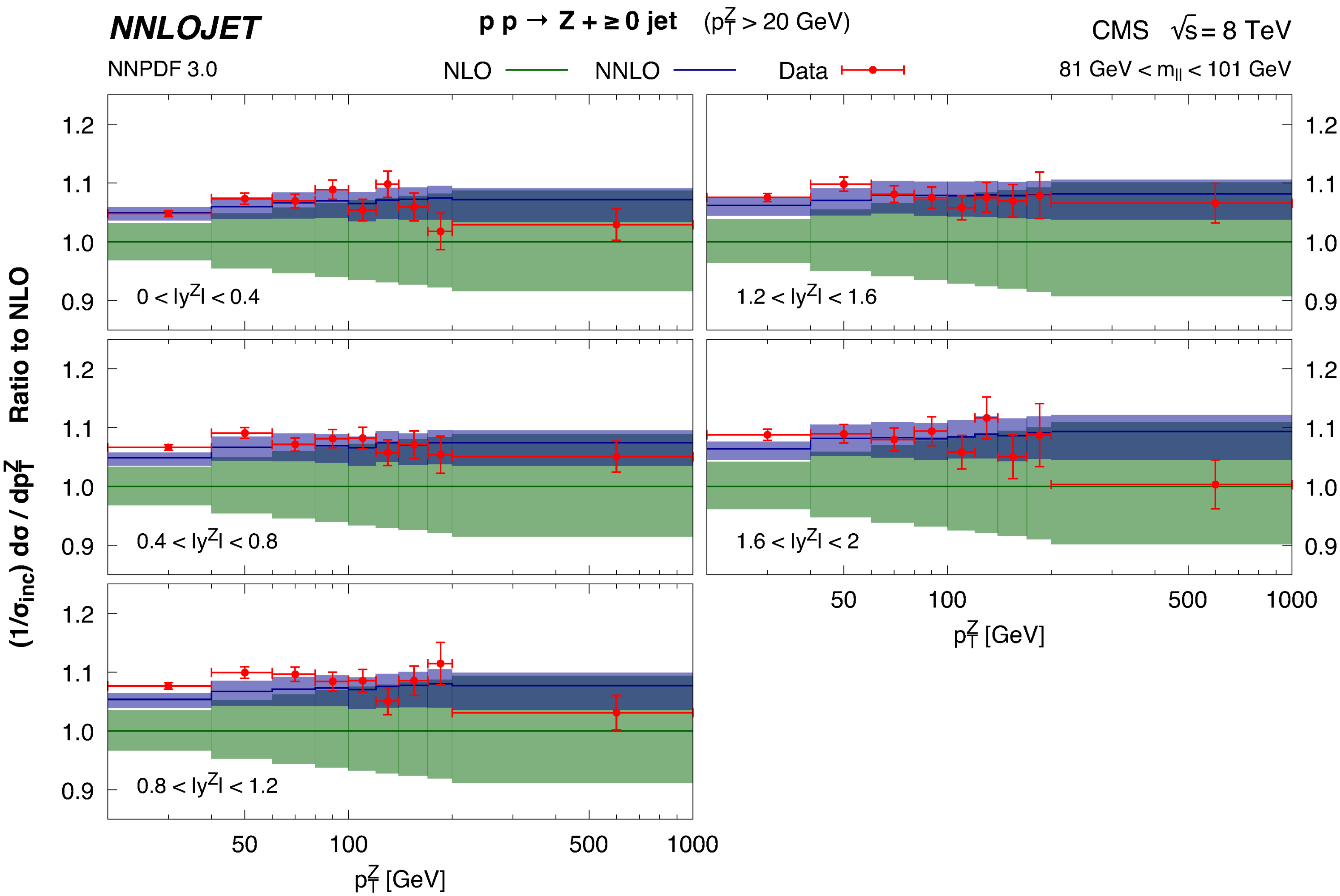


Significant difference between NNLO inclusive cross section and experimental data with NNPDF3.0 for different $m_{\ell\ell}$ bins



- ✓ Noted by ATLAS arXiv:1603.09222
- ✘ NNPDF3.0 doesnt fit the data very well
- ⇒ Sensitivity to PDFs

Inclusive p_T spectrum of Z



CMS Different rapidity slices

Summary

- ✓ NNLOJET is able to make fully differential NNLO predictions that can be compared with data
- ✓ H+jet
  - ✚ Validated against calculation using different IR subtraction
  - ✚ Large corrections, but still some tension with inclusive H+J data
- ✓ Z+jet
  - ✚ The inclusive p_T^Z spectrum is a powerful testing ground for QCD predictions, modelling of Z/W backgrounds, potential to constrain PDFs, ...
  - ✚ We have predicted this distribution to NNLO accuracy for $p_T^Z > p_{T,\mathrm{cut}}^Z$
  - ✚ We observe a reduction of the scale uncertainty and an improvement in the theory vs. data comparison
  - ✚ Normalised distributions show excellent agreement between data and NNLO

Work in progress:

- ✓ Including other processes, such as dijets, other Higgs decays, etc
- ✓ Studying potential of data to constrain PDF sets and interface to APPLgrid, fastNLO

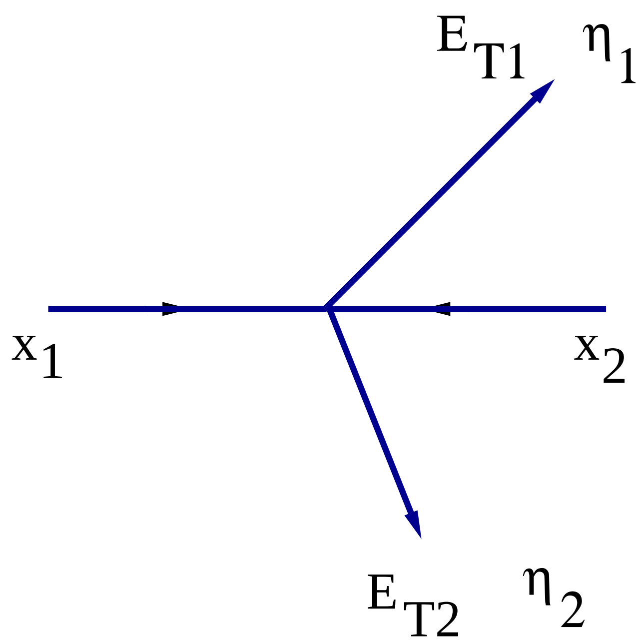
Maximising the impact of NNLO calculations

Triple differential form for a $2 \to 2$ cross section

$$\frac{d^3\sigma}{dE_T d\eta_1 d\eta_2} = \frac{1}{8\pi} \sum_{ij} x_1 f_i(x_1, \mu_F)\, x_2 f_j(x_2, \mu_F)\, \frac{\alpha_s^2(\mu_R)}{E_T^3} \frac{|\mathcal{M}_{ij}(\eta^*)|^2}{\cosh^4 \eta^*}$$

✓ Direct link between observables E_T, η_1, η_2 and momentum fractions/parton luminosities

$$x_1 = \frac{E_T}{\sqrt{s}} \left(\exp(\eta_1) + \exp(\eta_2)\right),$$
$$x_2 = \frac{E_T}{\sqrt{s}} \left(\exp(-\eta_1) + \exp(-\eta_2)\right)$$

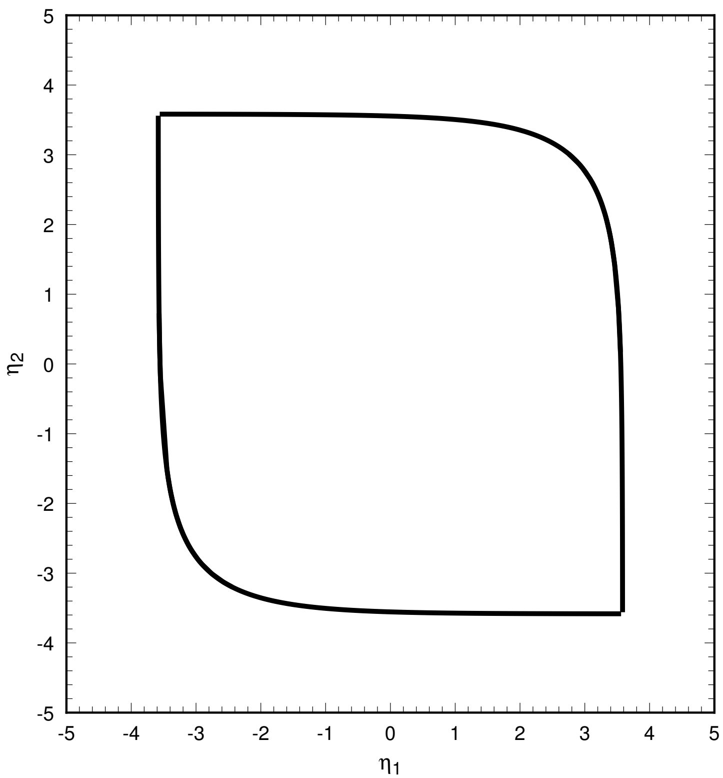


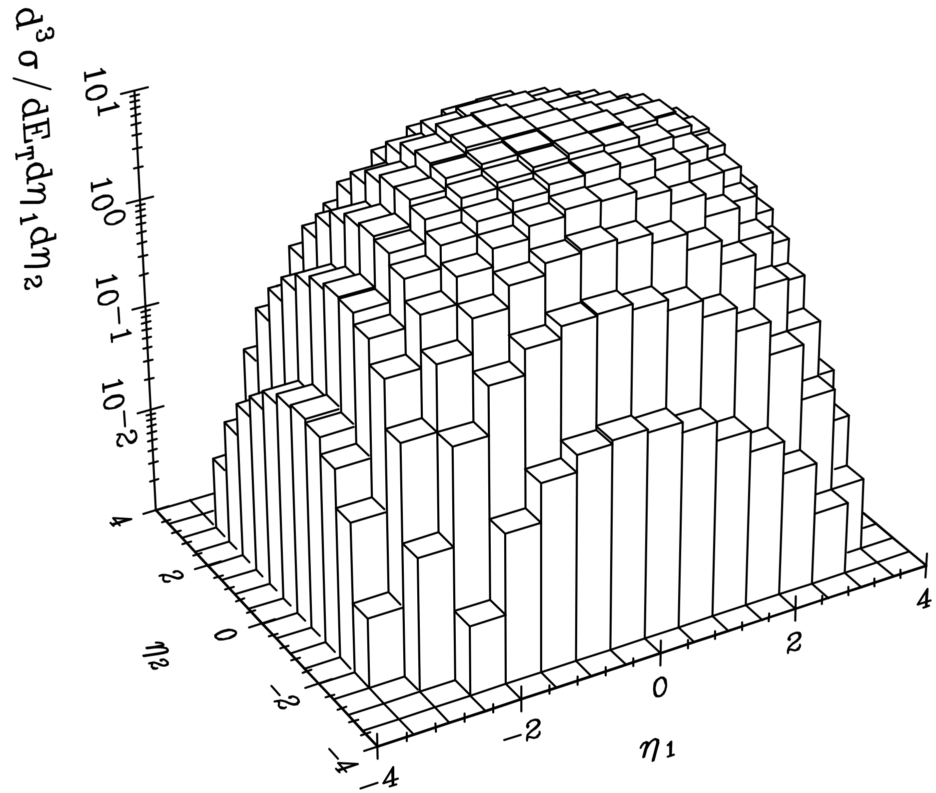
✓ and matrix elements that only depend on

$$\eta^* = \frac{1}{2}(\eta_1 - \eta_2)$$

Triple differential distribution

✓ Range of x_1 and x_2 fixed allowed LO phase space for jets $E_T \sim 200$ GeV at $\sqrt{s} = 7$ TeV





✓ Shape of distribution can be understood by looking at parton luminosities and matrix elements (in for example the single effective subprocess approximation)

Giele, NG, Kosower, hep-ph/9412338

Phase space considerations

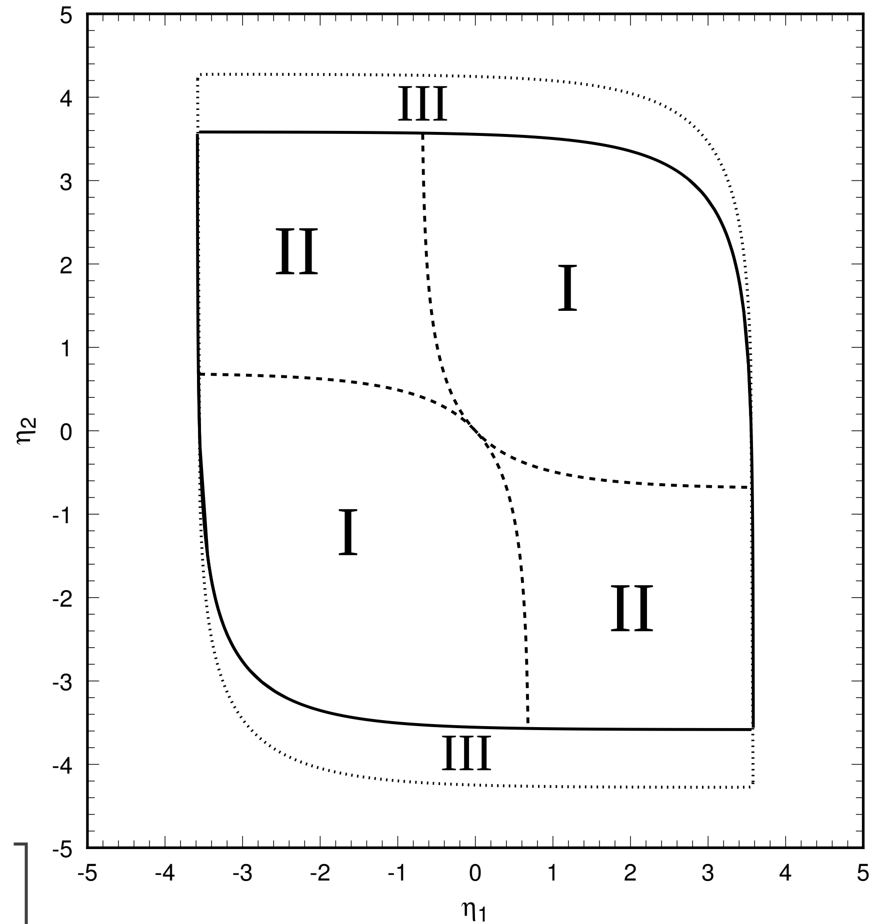
✓ Phase space boundary fixed when one or more parton fractions $\to 1$.

I $\eta_1 > 0$ and $\eta_2 > 0$ OR $\eta_1 < 0$ and $\eta_2 < 0$
$\Rightarrow$ one x_1 or x_2 is less than x_T
- small x

II $\eta_1 > 0$ and $\eta_2 < 0$ OR $\eta_1 < 0$ and $\eta_2 > 0$
$\Rightarrow$ both x_1 and x_2 are bigger than x_T
- large x

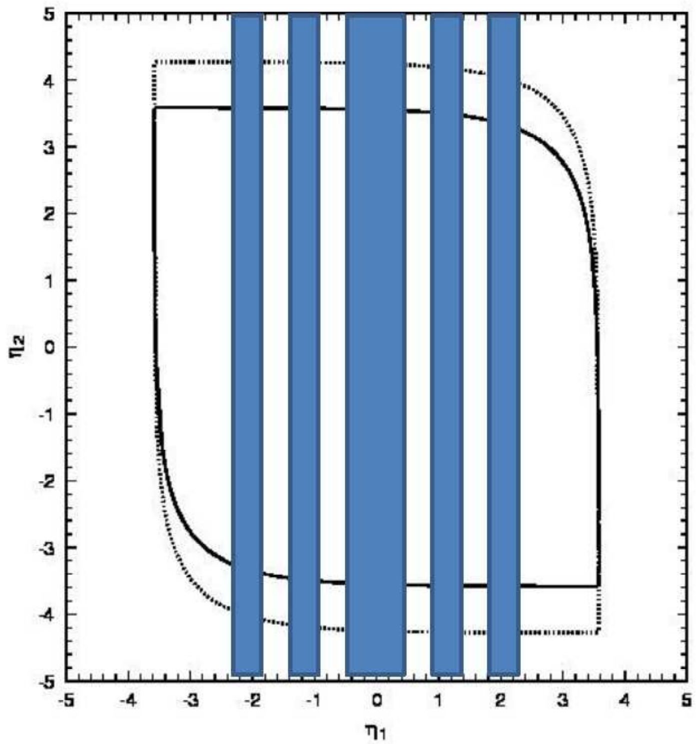
III growth of phase space at NLO (if $E_{T1} > E_{T2}$)

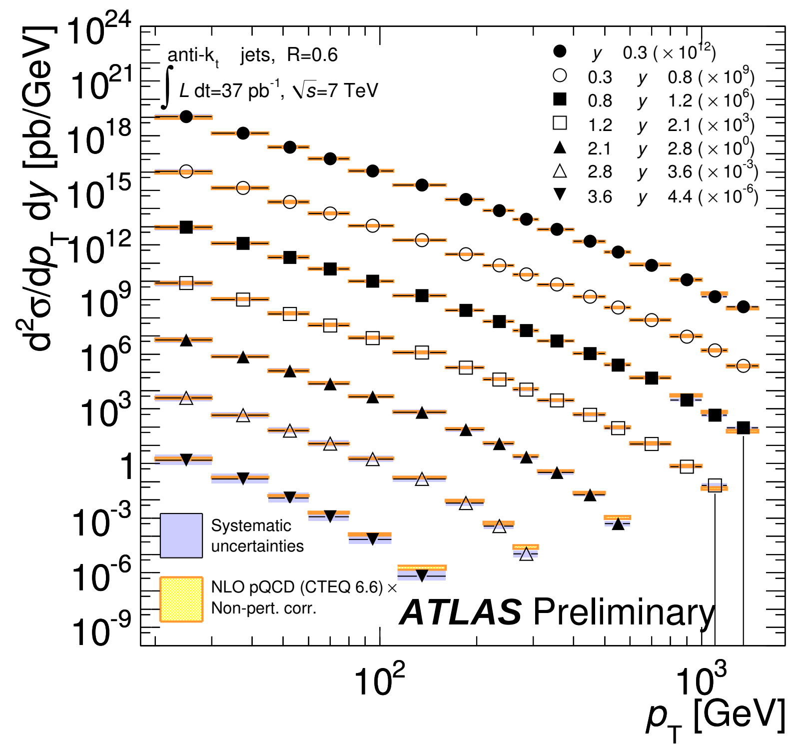
$$\left[x_T^2 < x_1 x_2 < 1 \qquad \text{and} \qquad x_T = 2E_T/\sqrt{s} \right]$$



Single Jet Inclusive Distribution

✓ Single Jet Inclusive Distribution is just a slice of the triple differential distribution, moving from $(x_1, x_2) = (1, x_T^2 \cosh^2(\eta^*))$ to $(x_T^2 \cosh^2(\eta^*), 1)$ where $\eta^* = \frac{1}{2}(\eta_1 - \eta_2)$





Measuring PDF's at the LHC?

Should be goal of LHC to be as self sufficient as possible!

Study triple differential distribution for as many $2 \to 2$ processes as possible!

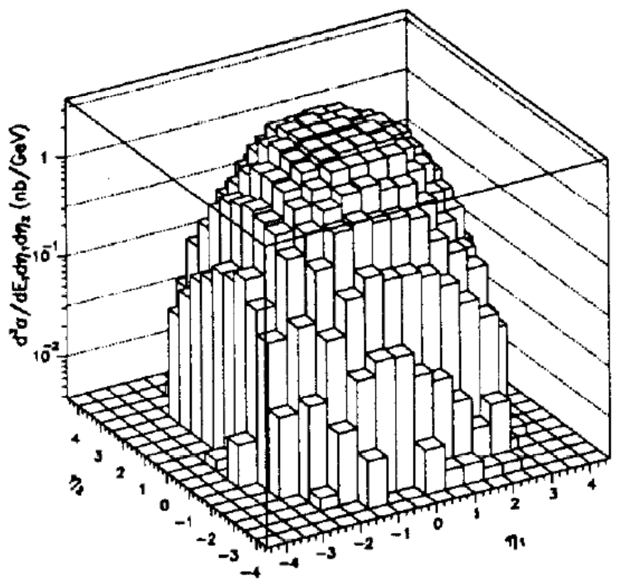
- ✓ Medium and large x gluon and quarks
  - ✓ $pp \to$ di-jets — dominated by gg scattering
  - ✓ $pp \to \gamma$ + jet — dominated by qg scattering
  - ✓ $pp \to \gamma\gamma$ — dominated by $q\bar{q}$ scattering
- ✓ Light flavours and flavour separation at medium and small x
  - ✓ Low mass Drell-Yan
  - ✓ W lepton asymmetry
  - ✓ $pp \to Z$+jet
- ✓ Strangeness and heavy flavours
  - ✓ $pp \to W^{\pm} + c$ — probes $s,\ \bar{s}$ distributions
  - ✓ $pp \to Z + c$ — probes c distribution
  - ✓ $pp \to Z + b$ — probes b distribution

Measurements of strong coupling

- ✓ With incredible jet energy resolution, the LHC can do better!!
- ✓ by simultaneously fitting the parton density functions and strong coupling
- ✓ If the systematic errors can be understood, the way to do this is via the triple differential cross section

Giele, NG, Yu, hep-ph/9506442

- ✓ and add NNLO $W^{\pm}$+jet, Z+jet, γ+jet calculations (with flavour tagging) as they become available



D0 preliminary, 1994