NNLO cross sections for processes with jets

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The challenge from the LHC

✓ Everything (signals, backgrounds, luminosity measurement) involves QCD

✓ Strong coupling is not small: $\alpha_s(M_Z) \sim 0.12$ and running is important
  ⇒ events have high multiplicity of hard partons
  ⇒ each hard parton fragments into a cluster of collimated particles jet
  ⇒ higher order perturbative corrections can be large
  ⇒ theoretical uncertainties can be large

✓ Processes can involve multiple energy scales: e.g. $p_T^W$ and $M_W$
  ⇒ may need resummation of large logarithms

✓ Parton/hadron transition introduces further issues, but for suitable (infrared safe) observables these effects can be minimised
  ⇒ importance of infrared safe jet definition
  ⇒ accurate modelling of underlying event, hadronisation, ...

✓✓ Nevertheless, excellent agreement between theory and experiment over a wide range of observables
Cross Sections at the LHC

CMS Preliminary

July 2015

Production Cross Section, $\sigma$ [pb]

7 TeV CMS measurement (L $\leq$ 5.0 fb$^{-1}$)
8 TeV CMS measurement (L $\leq$ 19.6 fb$^{-1}$)
7 TeV Theory prediction
8 TeV Theory prediction
CMS 95%CL limit

All results at: http://cern.ch/go/pNj7
Discrepancies with data

Anastasiou, Duhr, Dulat, Herzog, Mistlberger

Czakon, Fiedler, Mitov

Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Ravlev, Tancredi

No BSM discovered yet... but plenty of BNLO
and a few interesting outliers

✓ bump hunting uses data driven methods
✓ extrapolation to region with little data
✓ can fit rate and compare to precise SM prediction
✓ more than 340 theory papers (since December 16)
✓ will survive ... at least until ICHEP
Motivation for more precise theoretical calculations

✓ Theory uncertainty has big impact on quality of measurement

✗ NLO QCD is clearly insufficiently precise for SM, top (and even Higgs) measurements, D. Froidevaux, HiggsTools School, 2015

⇒ Revised wishlist of theoretical predictions for
+ Higgs processes
+ Processes with vector bosons
+ Processes with top or jets

Theoretical Uncertainties

- Missing Higher Order corrections (MHO)
  - truncation of the perturbative series
  - often estimated by scale variation - renormalisation/factorisation
  ✓ systematically improvable by inclusion of higher orders

- Uncertainties in input parameters
  - parton distributions
  - masses, e.g., \( m_W, m_h, [m_t] \)
  - couplings, e.g., \( \alpha_s(M_Z) \)
  ✓ systematically improvable by better description of benchmark processes

- Uncertainties in parton/hadron transition
  - fragmentation (parton shower)
  ✓ systematically improvable by matching/merging with higher orders
  - hadronisation (model)
  - underlying event (tunes)

Goal: Reduce theory certainties by a factor of two compared to where we are now in next decade
## The strong coupling

### World Average

<table>
<thead>
<tr>
<th>Year</th>
<th>$\alpha_s(M_Z)$</th>
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<tbody>
<tr>
<td>2008</td>
<td>0.1176 ± 0.0009</td>
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<tr>
<td>2012</td>
<td>0.1184 ± 0.0007</td>
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<tr>
<td>2014</td>
<td>0.1185 ± 0.0006</td>
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- Average of wide variety of measurements
- $\tau$-decays
- $e^+e^-$ annihilation
- $Z$ resonance fits
- DIS
- Lattice
- Generally stable to choice of measurements

- Impressive demonstration of running of $\alpha_s$ to $O(1$ TeV$)$
- ... but some outlier values from global PDF fits, e.g.,
  $\alpha_s(M_Z) \sim 0.1136 \pm 0.0004$ (G)JR
  $\alpha_s(M_Z) \sim 0.1132 \pm 0.0011$ ABM14

⇒ Still need to understand uncertainty and make more precise determination
All fits NNLO

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<tr>
<th>Set</th>
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<th>DY</th>
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✓ Clear reduction in gluon-gluon luminosity for $M_X \sim 125$ GeV

✓ … with commensurate reduction in uncertainty on Higgs cross section
Parton Distribution Functions

but still differences of opinion
Partonic cross sections

\[ \hat{\sigma} \sim \alpha_s^n \left( \hat{\sigma}^{LO} + \left( \frac{\alpha_s}{2\pi} \right)^2 \hat{\sigma}^{NLO}_{QCD} + \left( \frac{\alpha_s}{2\pi} \right)^3 \hat{\sigma}^{N3LO}_{QCD} + \ldots \right) + \left( \frac{\alpha_W}{2\pi} \right)^2 \hat{\sigma}^{NLO}_{EW} + \ldots \]

**NLO QCD**
- At least NLO is needed to obtain reliable predictions

**NNLO QCD**
- Provides the first serious estimate of the theoretical uncertainty

**NLO EW**
- Naively similar size to NNLO QCD
- Particularly important at high energies \( p_T \) and near resonances

**N3LO QCD**
- Landmark result for Higgs production
What is the hold up?

Rough idea of complexity of process $\sim \#\text{Loops} + \#\text{Legs} (+ \#\text{Scales})$

- Loop integrals are ultraviolet/infrared divergent
- Complicated by extra mass/energy scales
- Loop integrals often unknown
  ✓ completely solved at NLO
- Real (tree) contributions are infrared divergent
- Isolating divergences complicated
  ✓ completely solved at NLO
- Currently far from automation
  ✓ mostly solved at NLO

Current standard: NLO
Anatomy of a NLO calculation

✓ one-loop $2 \rightarrow 3$ process
✓ explicit infrared poles from loop integral
✓ looks like 3 jets in final state

✓ tree-level $2 \rightarrow 4$ process
✓ implicit poles from soft/collinear emission
✓ looks like 3 or 4 jets in final state

✓ plus method for combining the infrared divergent parts
  ✬ dipole subtraction
  ✬ residue subtraction
  ✬ antenna subtraction
  ✬ phase space slicing
  ✬ sector decomposition

✓ NLO problem is solved in principle

✘ In practice, limitations in numerical accuracy for matrix elements and efficient phase space evaluation means that problems may occur with $O(4-6)$ particles in final state
What NNLO might give you (1)

✓ Reduced renormalisation scale dependence

✓ Event has more partons in the final state so perturbation theory can start to reconstruct the shower
  ⇒ better matching of jet algorithm between theory and experiment

✓ Reduced power correction as higher perturbative powers of $1/\ln(Q/\Lambda)$ mimic genuine power corrections like $1/Q$
What NNLO might give you (2)

✓ Better description of transverse momentum of final state due to double radiation off initial state

✓ At LO, final state has no transverse momentum
✓ Single hard radiation gives final state transverse momentum, even if no additional jet
✓ Double radiation on one side, or single radiation of each incoming particle gives more complicated transverse momentum to final state
✓ NNLO provides the first serious estimate of the theoretical uncertainty

✓✓✓ and most importantly, the volume and quality of the LHC data!!
Anatomy of a NNLO calculation e.g. pp to JJ

- ✓ double real radiation matrix elements $d\hat{\sigma}_{NNLO}^{RR}$
- ✚ implicit poles from double unresolved emission
- ✓ single radiation one-loop matrix elements $d\hat{\sigma}_{NNLO}^{RV}$
- ✚ explicit infrared poles from loop integral
- ✚ implicit poles from soft/collinear emission
- ✓ two-loop matrix elements $d\hat{\sigma}_{NNLO}^{VV}$
- ✚ explicit infrared poles from loop integral

$$d\hat{\sigma}_{NNLO} \sim \int d\Phi_{m+2} d\hat{\sigma}_{NNLO}^{RR} + \int d\Phi_{m+1} d\hat{\sigma}_{NNLO}^{RV} + \int d\Phi_{m} d\hat{\sigma}_{NNLO}^{VV}$$
Anatomy of a NNLO calculation e.g. pp to JJ

✓ Double real and real-virtual contributions used in NLO calculation of X+1 jet

Can exploit NLO automation

…but needs to be evaluated in regions of phase space where extra jet is not resolved

✚ Two loop amplitudes - very limited set known

…the currently far from automation

✚ Method for cancelling explicit and implicit IR poles - overlapping divergences

…the currently not automated
Recap: IR subtraction at NLO

✓ To subtract the infrared singularities, we recast the NLO cross section in the form

\[
d\hat{\sigma}_{NLO} = \int d\Phi_{m+1} \left[ d\hat{\sigma}^R_{NLO} - d\hat{\sigma}^S_{NLO} \right] + \int d\Phi_m \left[ d\hat{\sigma}^V_{NLO} - d\hat{\sigma}^T_{NLO} \right]
\]

where the terms in each of the square brackets is finite, well behaved in the infrared singular regions and can be evaluated numerically.

\[
d\hat{\sigma}^T_{NLO} = -\int_1 d\hat{\sigma}^S_{NLO} + d\hat{\sigma}^{MF}_{NLO}
\]

✓ \(d\hat{\sigma}^S_{NLO}\) must cancel the implicit divergences in regions of phase space where \(d\hat{\sigma}^R_{NLO}\) is singular (subtraction)

or restrict the phase space to avoid these regions (slicing)
The aim is to recast the NNLO cross section in the form

\[
d\hat{\sigma}_{NNLO} = \int d\Phi_{m+2} \left[ d\hat{\sigma}_{\text{RR} \, NNLO} - d\hat{\sigma}_{\text{S} \, NNLO} \right] \\
+ \int d\Phi_{m+1} \left[ d\hat{\sigma}_{\text{RV} \, NNLO} - d\hat{\sigma}_{\text{T} \, NNLO} \right] \\
+ \int d\Phi_{m} \left[ d\hat{\sigma}_{\text{VV} \, NNLO} - d\hat{\sigma}_{\text{U} \, NNLO} \right]
\]

where the terms in each of the square brackets is finite, well behaved in the infrared singular regions and can be evaluated numerically.

Much more complicated cancellations between the double-real, real-virtual and double virtual contributions

intricate overlapping divergences
Unlike at NLO, we do not have a fully general NNLO IR cancellation scheme.

- **Antenna subtraction**
  - **Gehrmann, Gehrmann-De Ridder, NG (05)**

- **Colourful subtraction**
  - **Del Duca, Somogyi, Trocsanyi (05)**

- **$q_T$ subtraction**
  - **Catani, Grazzini (07)**

- **STRIPPER (sector subtraction)**
  - **Czakon (10); Boughezal et al (11)**
  - **Czakon, Heymes (14)**

- **N-jettiness subtraction**
  - **Boughezal, Focke, Liu, Petriello (15)**
  - **Gaunt, Stahlhofen, Tackmann, Walsh (15)**

- **Projection to Born**
  - **Cacciari, Dreyer, Karlberg, Salam, Zanderighi (15)**

Each method has its advantages and disadvantages.

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<thead>
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<th>IS colour</th>
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</table>
IR subtraction at NNLO

\[
d\sigma^U: \quad d\sigma^{U,A} \sim \frac{\alpha_s}{\epsilon} \frac{J^{(1)}_{n+1} M_{n+1}^0}{M_n^0} + J^{(1)}_{n} M_{n}^0
\]

\[
d\sigma^{U,B} \sim \frac{1}{2} J^{(1)}_{n} \otimes J^{(1)}_{n} M_{n}^0
\]

\[
d\sigma^{U,C} \sim J^{(2)}_{n} M_{n}^0
\]

\[
d\sigma^T: \quad d\sigma^{T,b_1} \sim \frac{\alpha_s}{\epsilon} X^0_{3} M_{n}^0 + \frac{\alpha_s}{\epsilon} X^0_{3} J^{(1)}_{n} M_{n}^0
\]

\[
d\sigma^{T,b_3} \sim \frac{\alpha_s}{\epsilon} X^0_{3} M_{n}^0 + \frac{\alpha_s}{\epsilon} X^0_{3} \left( \frac{L_{3}}{\mu^2} \right)^{-1} M_{n}^0
\]

\[
d\sigma^{T,c} \sim - \frac{1}{4} d\sigma^{S,c} + d\sigma^{T,c_1} + d\sigma^{T,c_2}
\]

\[
d\sigma^{T,b_2} \sim X^1_{3} M_{n}^0 + J^{(1)}_{X} X^0_{3} M_{n}^0 - J^{(1)}_{X} X^0_{3} J^{(1)}_{n} M_{n}^0
\]

\[
d\sigma^S: \quad d\sigma^{S,a}
\]

\[
d\sigma^{S,d}
\]

\[
d\sigma^{S,c}
\]

\[
d\sigma^{S,b_2}
\]

\[
d\sigma^{S,b_1}
\]

Currie, NG, Wells (13)
Antenna subtraction at NNLO

Antenna subtraction exploits the fact that matrix elements already possess the intricate overlapping divergences

\[ X_3^0(i, j, k) \sim \frac{|M_3^0(i, j, k)|^2}{|M_2^0(I, K)|^2}, \quad X_4^0(i, j, k, l) \sim \frac{|M_4^0(i, j, k, l)|^2}{|M_2^0(I, L)|^2} \]

Quark-antiquark:
\[ \gamma^* \to q\bar{q} + \cdots \]

Quark-gluon:
\[ \bar{\chi}^0 \to g\bar{g} + \cdots \]

Gluon-gluon:
\[ H \to gg + \cdots \]

\[ i + j + k \to I + J, \quad i + j + k + l \to I + L \]
Antenna subtraction at NNLO

✓ Antenna mimics all singularities of QCD

\[
A^0_4(1_q, 2_g, 3_g, 4_q) = \begin{cases} 
1||2||3 & 2,3\sim0 \\
S^0_{1234} & 3||4 + 2\sim0 \\
S^0_{1;234}
\end{cases}
\]

✓ Phase space map smoothly interpolates momenta for reduced matrix element between limits

\[
\begin{align*}
(123) &= xp_1 + r_1 p_2 + r_2 p_3 + z p_4 \\
(234) &= (1-x)p_1 + (1-r_1)p_2 + (1-r_2)p_3 + (1-z)p_4
\end{align*}
\]
Antenna subtraction at NNLO

✓ All unintegrated antennae available

✓✓ Final-Final  Gehrmann-De Ridder, Gehrmann, NG, (05)
✓✓ Initial-Final Daleo, Gehrmann, Maitre, (07)
✓✓ Initial-Initial Daleo, Gehrmann, Maitre, (07)
               NG, Pires, (10)

✓ All antennae analytically integrated

✓✓ Final-Final  Gehrmann-De Ridder, Gehrmann, NG, (05)
✓✓ Initial-Final Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni, (10)
✓✓ Initial-Initial Gehrmann, Monni, (11)
               Boughezal, Gehrmann-De Ridder, Ritzmann, (11)
               Gehrmann, Ritzmann, (12)

✚ Laurent expansion in $\epsilon$
Implementing NNLO corrections using Antenna subtraction for

- $pp \rightarrow H \rightarrow \gamma\gamma$ plus 0, 1, 2 jets
  - 1507.02850, 1601.04569, 1605.04295
- $pp \rightarrow e^+e^-$ plus 0, 1 jets
  - 1408.5325, 1604.04085
- $pp \rightarrow$ dijets
  - 1301.7310, 1310.3993
- $ep \rightarrow 2(+1)$ jets
  - 1605.XXX
- ...
Automatically generating the code (1)
Maple script: RR example

+ F40a(i, j, k, l) * A4g0(1, 2, [i, j, k], [j, k, l])
- f30FF(i, j, k) * f30FF([i, j], [j, k], l)
* A4g0(1, 2, [[i, j], [j, k]], [[j, k], l])

... + F40\alpha(i, j, k, l) A4^0(1, 2, (\tilde{ijk}), (\tilde{jk}l))
- f3^0(i, j, k) f3^0((\tilde{ij}), (\tilde{jk}), l) A4^0(1, 2, [(\tilde{ij}), (\tilde{jk})], (\tilde{jk}l))

✓ X^0_4, X^0_3 (and X^1_3 in RV) are unintegrated antennae
✓ [i, j, k] or (\tilde{ijk}) are mapped momenta
Maple script: VV example

\[ + \left[ - \frac{1}{2} \mathcal{F}_{4,g}^{0}(s_{23}) ight. \]
\[ - \frac{1}{2} \mathcal{F}_{3,g}^{1}(s_{23}) \]
\[ - \frac{b_{0}}{2\epsilon} \left( \frac{s_{23}}{\mu_{R}^{2}} \right)^{-\epsilon} \mathcal{F}_{3,g}^{0}(s_{23}) \]
\[ + \frac{b_{0}}{2\epsilon} \mathcal{F}_{3,g}^{0}(s_{23}) \]
\[ + \frac{1}{4} \mathcal{F}_{3,g}^{0}(s_{23}) \otimes \mathcal{F}_{3,g}^{0}(s_{23}) \]
\[ + \frac{1}{2} \Gamma_{gg}^{(2)}(z_{2}) \]
\[ - \frac{b_{0}}{2\epsilon} \Gamma_{gg}^{(1)}(z_{2}) \right] A_{4}^{0}(1, 2, 3, 4) \]

✓ $\mathcal{X}_{4}^{0}$, $\mathcal{X}_{3}^{0}$ and $\mathcal{X}_{3}^{1}$ are integrated antennae
Automatically generating the code (2)
Maple script to produce driver template

\[ R := [ \\
[A5g0, [g, g, g, g, g], 1], \\
[B3g0, [qb, g, g, g, q], 1/nc], \\
... \\
] : \\
\]

\[ d\sigma_{gg}^{R} = \mathcal{N}_{LO} \left( \frac{\alpha_s N}{2\pi} \right) \left[ \right. \\
+ 2 \frac{1}{3!} \left( \sum_{12} A5g0(1, 2, 3, 4, 5) - ggA5g0SNLO(1, 2, 3, 4, 5) \right) \\
+ \frac{N_F}{N} \left( \sum_{6} B3g0(3, 1, 2, 4, 5) - ggB3g0SNLO(3, 1, 2, 4, 5) \right) \\
\left. \right] \]

✓ Have to link subtraction terms to automatically generated code (1)
Checks

✓ Analytic pole cancellations for RV, VV

Poles \((d\sigma^{RV} - d\sigma^{T})\) = 0

Poles \((d\sigma^{VV} - d\sigma^{U})\) = 0

✓ Unresolved limits for RR, RV

\[ d\sigma^{S} \rightarrow d\sigma^{RR} \]
\[ d\sigma^{T} \rightarrow d\sigma^{RV} \]

\[ q\bar{q} \rightarrow Z + g_3 \ g_4 \ g_5 \ (g_3 \text{ soft} \ & \ g_4 \parallel \bar{q}) \]

09:26:35 ...maple/process/Z
$ form autoqgB1g2ZgtoqU.frm
FORM 4.1 (Mar 13 2014) 64-bits
#
   poles = 0;
6.58 sec out of 6.64 sec
H + J production, large mass limit

Boughezal, Caola, Melnikov, Petriello, Schulze (13,15)
Chen, Gehrmann, NG, Jaquier (14,16)
Boughezal, Focke, Giele, Liu, Petriello (15)
Caola, Melnikov, Schulze (15)

✓ phenomenologically interesting
✓ large scale uncertainty
✓ large $K$-factor

$$\frac{\sigma_{NLO}}{\sigma_{LO}} \sim 1.6$$
$$\frac{\sigma_{NNLO}}{\sigma_{NLO}} \sim 1.3$$
✓ significantly reduced scale dependence $\mathcal{O}(4\%)$

✓ Three independent computations:
  ✫ STRIPPER
  ✫ Antenna
  ✫ N-jettiness

✓ allows for benchmarking of methods (for $gg$, $qg$ and $\bar{q}g$ processes)
  ✫ $\sigma^{NNLO} = 9.45^{+0.58}_{-0.82}$ fb
    Caola, Melnikov, Schulze (15)
  ✫ $\sigma^{NNLO} = 9.44^{+0.59}_{-0.85}$ fb
    Chen, Gehrmann, NG, Jaquier (16)
Higgs $p_T$ and rapidity distributions

$\sqrt{s} = 13$ TeV, PDF4LHC15, $p_T^{jet} > 30$ GeV, anti-$k_T$, $R = 0.4$, $\mu_F = \mu_R = (0.5, 1, 2)m_H$
Jet $p_T$ and rapidity distributions

$\sqrt{s} = 13$ TeV, PDF4LHC15, $p_T^{jet} > 30$ GeV, anti-$k_T$, $R = 0.4$, $\mu_F = \mu_R = (0.5, 1, 2)m_H$
Exclusive jet bins

$p p \rightarrow H + \geq 1 \text{ jet}$

\(\sqrt{s} = 13 \text{ TeV}\)

\(p_T^{\text{jet}} > 30 \text{ GeV}\)

anti-\(k_T\) (\(R=0.4\))

PDF4LHC15

\(\mu_R = \mu_F = (0.5, 1, 2) \cdot M_H\)
Comparison with Data

ATLAS setup

✓ H+J NNLO prediction undershoots ATLAS data
✓ statistical errors still quite large
✓ finite mass effects estimated to be 2-3% @NLO

Harlander, Neumann, Ozeren, Wiesemann (12)
Z + J production

Gehrmann-De Ridder, Gehrmann, NG, Huss, Morgan (15)
Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello (15)
Boughezal, Liu, Petriello (16)

✓ clean leptonic signature
✓ good handle on jet energy scale
✓ significant NLO K-factor and scale uncertainty

\[ \frac{\sigma_{NLO}}{\sigma_{LO}} \sim 1.4 \]

✓ Two independent computations:
✓ allows for benchmarking of methods

+ \[ \sigma^{NNLO} = 135.6^{+0.0}_{-0.4} \text{ fb} \]
  Gehrmann-De Ridder, Gehrmann, NG, Huss, Morgan (15)

+ \[ \sigma^{NNLO} = 135.6^{+0.0}_{-0.4} \text{ fb} \]
  Boughezal, Campbell, Ellis, Focke, Giele, Liu, Petriello (15)
$Z$ $p_T$ and rapidity distributions

$\sqrt{s} = 8$ TeV, NNPDF2.3, $p_T^{jet} > 30$ GeV, $|y^{jet}| < 3$, anti-$k_T$, $R = 0.5$, 80 GeV $< m_{\ell\ell} < 100$ GeV, $\mu_F = \mu_R = (0.5, 1, 2) m_Z$
**Jet $p_T$ and rapidity**

**Leading jet $p_T$ and rapidity distributions**

\[ \sqrt{s} = 8 \text{ TeV}, \ \text{NNPDF2.3}, \ p_T^{\text{jet}} > 30 \text{ GeV}, \ |y^{\text{jet}}| < 3, \ \text{anti-}k_T, \ R = 0.5, \ 80 \text{ GeV} < m_{\ell\ell} < 100 \text{ GeV}, \ \mu_F = \mu_R = (0.5, 1, 2) m_Z \]
Inclusive $p_T$ spectrum of $Z$

$pp \rightarrow Z/\gamma^* \rightarrow \ell^+\ell^- + X$

- large cross section
- clean leptonic signature

- fully inclusive wrt QCD radiation
- only reconstruct $\ell^+, \ell^-$ so clean and precise measurement
- potential to constrain gluon PDFs
Inclusive $p_T$ spectrum of $Z$

- low $p_T^Z \leq 10$ GeV, resummation required
- $p_T^Z \geq 20$ GeV, fixed order prediction about 10% below data
- Very precise measurement of $Z$ $p_T$ poses problems to theory, D. Froidevaux, HiggsTools School

FEWZ/DYNNLO are $Z + 0$ jet @ NNLO
- Only NLO accurate in this distribution
- Requiring recoil means $Z + 1$ jet @ NNLO required

![Graph showing data and theoretical predictions for $Z$ boson transverse momentum spectrum.](image)
Inclusive $p_T^Z$ spectrum: Setup

Calculational setup
✓ LHC @ 8 TeV
✓ PDF: NNPDF2.3 $\alpha_s(M_Z) = 0.118$
✓ fully inclusive wrt QCD radiation
✓ $p_T^Z > 20$ GeV
✓ $p_T^{\ell_1} > 20$ GeV, $p_T^{\ell_2} > 10$ GeV, $|y^{\ell_\pm}| < 2.4$, $12$ GeV $< m_{\ell\ell} < 150$ GeV
✓ dynamical scale choice

$$\mu_R = \mu_F = \sqrt{m_{\ell\ell}^2 + p_{T,Z}^2} \times \left[ \frac{1}{2}, 1, 2 \right]$$

CMS setup arXiv:1504.03511
- $p_T^{\ell_1} > 25$ GeV, $|y^{\ell_1}| < 2.1$
- $p_T^{\ell_2} > 10$ GeV, $|y^{\ell_2}| < 2.4$
- $81$ GeV $< m_{\ell\ell} < 101$ GeV + binning in $y^Z$

ATLAS setup arXiv:1512.02192
- $p_T^{\ell\pm} > 20$ GeV, $|y^{\ell\pm}| < 2.4$
- $66$ GeV $< m_{\ell\ell} < 116$ GeV + binning in $y^Z$
- $|y^Z| < 2.4$ + binning in $m_{\ell\ell}$
Inclusive $p_T$ spectrum of $Z$

\[
\left. \frac{d\sigma}{dp_T^Z} \right|_{p_T^Z>20 \text{ GeV}} \equiv \frac{d\hat{\sigma}_{LO}^{Z}}{dp_T^Z} + \frac{d\hat{\sigma}_{NLO}^{Z}}{dp_T^Z} + \frac{d\hat{\sigma}_{NNLO}^{Z}}{dp_T^Z}
\]

✓ NLO corrections $\sim 40 - 60\%$

✓ significant reduction of scale uncertainties NLO $\rightarrow$ NNLO

✓ NNLO corrections relatively flat $\sim 4 - 8\%$

✓ improved agreement, but not enough

✓ Note that for $66 \text{ GeV} < m_{\ell\ell} < 116 \text{ GeV}$

\[
\sigma_{\text{exp}} = 537.1 \pm 0.45\% \pm 2.8\% \text{ pb}
\]

\[
\sigma_{\text{NNLO}} = 507.9^{+2.4}_{-0.7} \text{ pb}
\]
Inclusive $p_T$ spectrum of $Z$

\[ \frac{1}{\sigma} \cdot \frac{d\hat{\sigma}}{dp_T} \bigg|_{p_T^Z > 20 \text{ GeV}} \]

with

\[ \sigma = \int_0^\infty \frac{d\hat{\sigma}}{dp_T} dp_T \equiv \sigma_{LO}^Z + \sigma_{NLO}^Z + \sigma_{NNLO}^Z. \]

✓ Much improved agreement
✓ Luminosity uncertainty reduced
✓ Dependence on EW parameters reduced
✓ Dependence on PDFs reduced
⇒ Study
Inclusive $p_T$ spectrum of $Z$

**NNLOJET**

<table>
<thead>
<tr>
<th>ATLAS $\sqrt{s} = 8$ TeV</th>
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<tbody>
<tr>
<td>0 &lt;</td>
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**$p p \rightarrow Z + \geq 0 \text{ jet}$ ($p_T^Z > 20$ GeV)**

- NNPDF 3.0
- NLO
- NNLO
- Data

**Ratio to NLO**

- $12$ GeV < $m_{ll}$ < $20$ GeV
- $46$ GeV < $m_{ll}$ < $66$ GeV
- $20$ GeV < $m_{ll}$ < $30$ GeV
- $66$ GeV < $m_{ll}$ < $116$ GeV
- $30$ GeV < $m_{ll}$ < $46$ GeV
- $116$ GeV < $m_{ll}$ < $150$ GeV

**$(1/\alpha) \, d\sigma / dp_T^Z$**

- $50$ GeV < $p_T^Z$ < $500$ GeV
- $50$ GeV < $p_T^Z$ < $500$ GeV
Inclusive $p_T$ spectrum of $Z$

Significant difference between NNLO inclusive cross section and experimental data with NNPDF3.0 for different $m_{\ell\ell}$ bins

- Noted by ATLAS  
  arXiv:1603.09222

- NNPDF3.0 doesn't fit the data very well

⇒ Sensitivity to PDFs
Inclusive $p_T$ spectrum of $Z$

**NNLOJET**

$\text{NNPDF 3.0}$

$\text{p p \rightarrow Z + \geq 0 jet (p_T^Z > 20 \text{ GeV})}$

CMS $\sqrt{s} = 8 \text{ TeV}$

$81 \text{ GeV} < m_{ll} < 101 \text{ GeV}$

**CMS Different rapidity slices**
Summary

✓ **NNLOJET** is able to make fully differential NNLO predictions that can be compared with data

✓ **H+jet**
  + Validated against calculation using different IR subtraction
  + Large corrections, but still some tension with inclusive H+J data

✓ **Z+jet**
  + The inclusive $p_T^{Z}$ spectrum is a powerful testing ground for QCD predictions, modelling of $Z/W$ backgrounds, potential to constrain PDFs, …
  + We have predicted this distribution to NNLO accuracy for $p_T^{Z} > p_T^{Z,\text{cut}}$
  + We observe a reduction of the scale uncertainty and an improvement in the theory vs. data comparison
  + Normalised distributions show excellent agreement between data and NNLO

Work in progress:

✓ Including other processes, such as dijets, other Higgs decays, etc

✓ Studying potential of data to constrain PDF sets and interface to **APPLgrid**, **fastNLO**
Maximising the impact of NNLO calculations

Triple differential form for a $2 \rightarrow 2$ cross section

$$\frac{d^3 \sigma}{dE_T d\eta_1 d\eta_2} = \frac{1}{8\pi} \sum_{ij} x_1 f_i(x_1, \mu_F) x_2 f_j(x_2, \mu_F) \frac{\alpha_s^2(\mu_R)}{E_T^3} \frac{|M_{ij}(\eta^*)|^2}{\cosh^4 \eta^*}$$

✓ Direct link between observables $E_T$, $\eta_1$, $\eta_2$ and momentum fractions/parton luminosities

$$x_1 = \frac{E_T}{\sqrt{s}} \left( \exp(\eta_1) + \exp(\eta_2) \right),$$

$$x_2 = \frac{E_T}{\sqrt{s}} \left( \exp(-\eta_1) + \exp(-\eta_2) \right)$$

✓ and matrix elements that only depend on

$$\eta^* = \frac{1}{2} (\eta_1 - \eta_2)$$
Triple differential distribution

✓ Range of $x_1$ and $x_2$ fixed allowed LO phase space for jets
$E_T \sim 200$ GeV at $\sqrt{s} = 7$ TeV

Shape of distribution can be understood by looking at parton luminosities and matrix elements (in for example the single effective subprocess approximation)

Giele, NG, Kosower, hep-ph/9412338
Phase space considerations

✓ Phase space boundary fixed when one or more parton fractions $\to 1$.

I $\eta_1 > 0$ and $\eta_2 > 0$ OR $\eta_1 < 0$ and $\eta_2 < 0$
   $\Rightarrow$ one $x_1$ or $x_2$ is less than $x_T$
   - small $x$

II $\eta_1 > 0$ and $\eta_2 < 0$ OR $\eta_1 < 0$ and $\eta_2 > 0$
   $\Rightarrow$ both $x_1$ and $x_2$ are bigger than $x_T$
   - large $x$

III growth of phase space at NLO
   (if $E_{T1} > E_{T2}$)

$$x_T^2 < x_1 x_2 < 1 \quad \text{and} \quad x_T = 2E_T/\sqrt{s}$$
Single Jet Inclusive Distribution

Single Jet Inclusive Distribution is just a slice of the triple differential distribution, moving from \((x_1, x_2) = (1, x_T^2 \cosh^2(\eta^*))\) to \((x_T^2 \cosh^2(\eta^*), 1)\) where \(\eta^* = \frac{1}{2}(\eta_1 - \eta_2)\)
Measuring PDF’s at the LHC?

Should be goal of LHC to be as self sufficient as possible!

Study triple differential distribution for as many $2 \rightarrow 2$ processes as possible!

✓ Medium and large $x$ gluon and quarks
  ✓ $pp \rightarrow$ di-jets dominated by $gg$ scattering
  ✓ $pp \rightarrow \gamma + \text{jet}$ dominated by $qg$ scattering
  ✓ $pp \rightarrow \gamma\gamma$ dominated by $q\bar{q}$ scattering

✓ Light flavours and flavour separation at medium and small $x$
  ✓ Low mass Drell-Yan
  ✓ $W$ lepton asymmetry
  ✓ $pp \rightarrow Z+\text{jet}$

✓ Strangeness and heavy flavours
  ✓ $pp \rightarrow W^\pm + c$ probes $s$, $\bar{s}$ distributions
  ✓ $pp \rightarrow Z + c$ probes $c$ distribution
  ✓ $pp \rightarrow Z + b$ probes $b$ distribution
Measurements of strong coupling

✓ With incredible jet energy resolution, the LHC can do better!!
✓ by simultaneously fitting the parton density functions and strong coupling
✓ If the systematic errors can be understood, the way to do this is via the triple differential cross section

Giele, NG, Yu, hep-ph/9506442

✓ and add NNLO $W^{\pm} + \text{jet}$, $Z + \text{jet}$, $\gamma + \text{jet}$ calculations (with flavour tagging) as they become available

D0 preliminary, 1994