Color-Kinematics Duality for QCD Amplitudes

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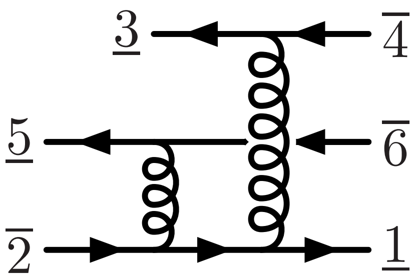
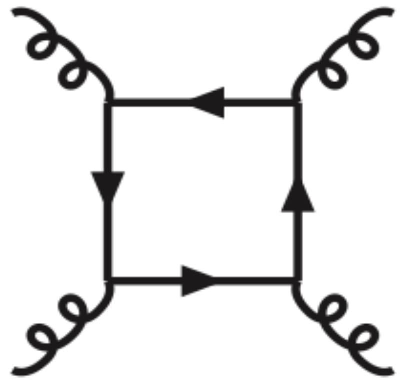


Nov. 26, 2015

DESY Zeuthen



work with Alexander Ochirov

arXiv: 1407.4772, 1507.00332

Outline

- Motivation & review: color-kinematics duality
  - Various gravity/gauge theories
- Generalization to QCD tree amplitudes
- New color decomposition
- Primitive amplitude relations for QCD
- Simple one loop application
  - One loop 4pt amplitude
- Conclusion

Color-kinematics duality

Color-kinematics duality for pure YM

YM theories are controlled by a hidden kinematic Lie algebra

- Amplitude expanded in terms of cubic graphs:

kinematic numerators

color factors

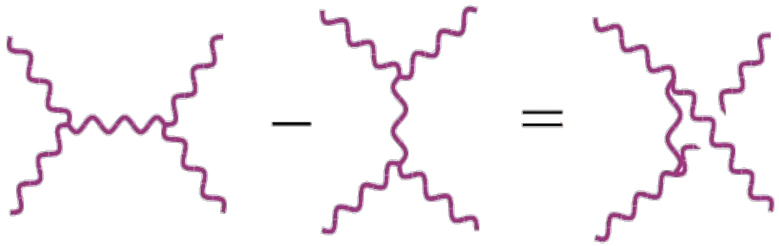
$$\mathcal{A}_n^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

propagators

Color & kinematic numerators satisfy same relations:

$$n_i - n_j = n_k \quad \Leftrightarrow \quad c_i - c_j = c_k$$

Bern, Carrasco, HJ

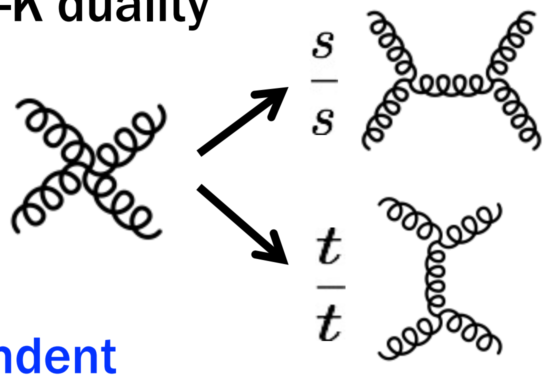


Jacobi identity

$$f^{dac} f^{cbe} - f^{dbc} f^{cae} = f^{abc} f^{dce} \quad \Rightarrow \quad c_i - c_j = c_k$$

Generalized gauge transformations

In general Feynman diagrams do not obey C-K duality



- Four-gluon vertex absorbed into cubic graphs → ambiguity

- Feynman diagrams are gauge-dependent
  → no reason to expect C-K duality to be present in all gauges

Amplitudes are invariant under "generalized gauge transformations"

$$n_i \to n_i + \Delta_i \qquad \text{such that} \qquad \sum_i \frac{c_i \Delta_i}{\prod_\alpha p_\alpha^2} = 0$$

but not duality: $\quad n_i - n_j \stackrel{?}{=} n_k \quad \Leftrightarrow \quad c_i - c_j = c_k$

Claim: starting from a general gauge there exists transformations Δ_i *that makes the numerators obey the duality !*

Bern, Carrasco, HJ ('08) ('10)

Gauge-invariant relations

$$A(1,2,\ldots,n-1,n) = A(n,1,2,\ldots,n-1)$$ Cyclicity → (n-1)! basis

$$\sum_{i=1}^{n-1} A(1,2,\ldots,i,n,i+1,\ldots,n-1) = 0$$ U(1) decoupling

$$A(1,\beta,2,\alpha) = (-1)^{|\beta|} \sum_{\sigma\in\alpha\sqcup\beta^T} A(1,2,\sigma)$$ Kleiss-Kuijf relations ('89)

$(n\text{-}2)!$ basis

$$\sum_{i=2}^{n-1}\Big(\sum_{j=2}^{i} s_{jn}\Big) A(1,2,\ldots i,n,i+1,\ldots,n-1) = 0$$

$$A(1,2,\alpha,3,\beta) = \sum_{\sigma\in S(\alpha)\sqcup\beta} A(1,2,3,\sigma)\prod_{i=1}^{|\alpha|}\frac{\mathcal{F}(3,\sigma,1|i)}{s_{2,\alpha_1,\ldots,\alpha_i}}$$

BCJ relations ('08)
$(n\text{-}3)!$ basis

BCJ rels. proven via string theory by Bjerrum-Bohr, Damgaard, Vanhove; Stieberger ('09)

and field theory proofs through BCFW: Feng, Huang, Jia; Chen, Du, Feng ('10 -'11)

Relations used in string calcs: Mafra, Stieberger, Schlotterer ('11 -'13)

Relations used by Cachazo, He, Yuan to motivate CHY and scattering eqns ('13)

Gravity is a double copy of YM

Gravity amplitudes obtained by replacing color with kinematics

$$\mathcal{A}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

$$\mathcal{M}_m^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i \tilde{n}_i}{p_{i_1}^2 p_{i_2}^2 p_{i_3}^2 \cdots p_{i_l}^2}$$

double copy

Bern, Carrasco, HJ

- The two numerators can differ by a generalized gauge transformation
  → only one copy needs to satisfy the kinematic algebra

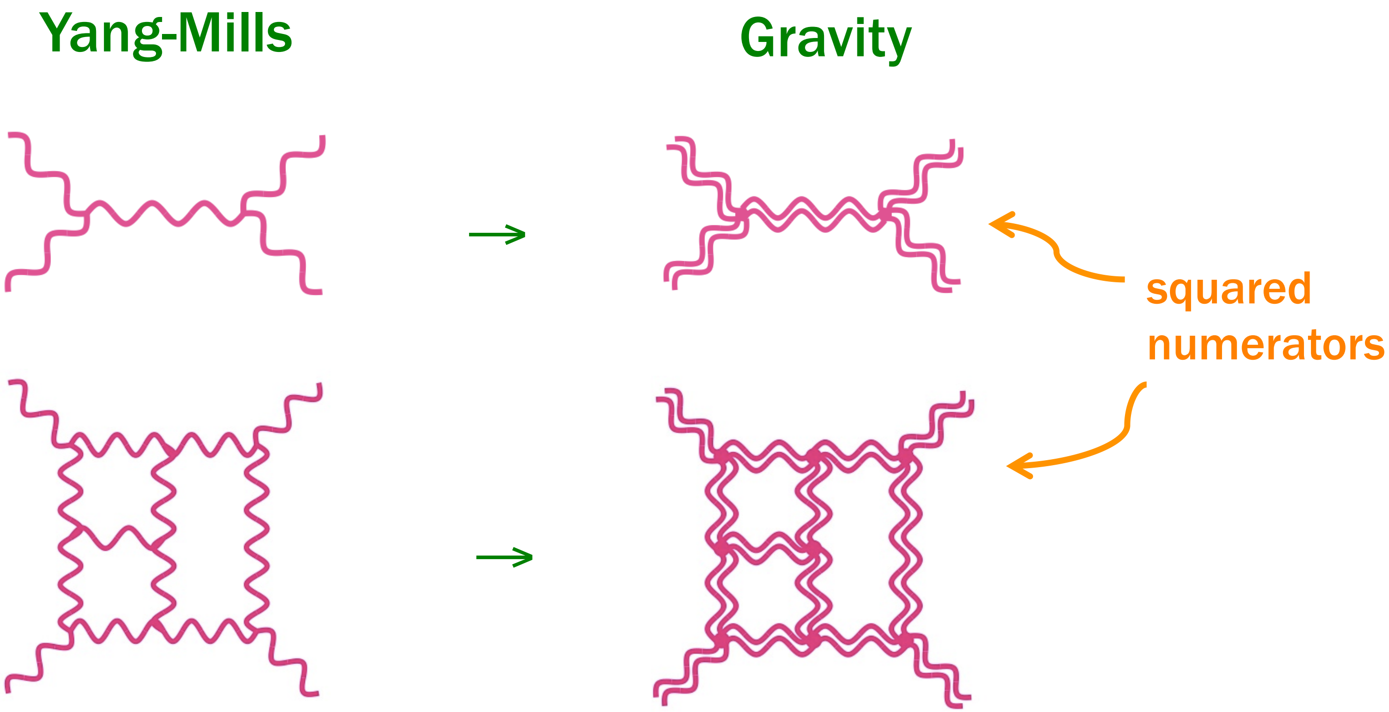
- The two numerators can differ by the external/internal states
  → graviton, dilaton, axion (*B*-tensor), matter amplitudes

- The two numerators can belong to different theories
  → give a host of different gravitational theories

Squaring of YM theory

Gravity processes = squares of gauge theory ones - entire S-matrix

Bern, Carrasco, HJ ('10)



E.g. pure Yang-Mills $\rightarrow$ Einstein gravity + dilaton + axion

$\mathcal{N}$=4 super-YM $\rightarrow$ $\mathcal{N}$=8 supergravity

Which "gauge" theories obey C-K duality

- Pure $\mathcal{N}$=0,1,2,4 super-Yang-Mills (any dimension) — Bern, Carrasco, HJ ('08); Bjerrum-Bohr, Damgaard, Vanhove; Stieberger; Feng et al. Mafra, Schlotterer, etc ('08-'11)
- Self-dual Yang-Mills theory O'Connell, Monteiro ('11)
- Heterotic string theory Stieberger, Taylor ('14)
- Yang-Mills + F^3 theory Broedel, Dixon ('12)
- QCD, super-QCD, higher-dim QCD HJ, Ochirov ('15)
- Generic matter coupled to $\mathcal{N}$= 0,1,2,4 super-Yang-Mills — Chiodaroli, Gunaydin, Roiban; HJ, Ochirov ('14)
- Spontaneously broken $\mathcal{N}$= 0,2,4 SYM Chiodaroli, Gunaydin, HJ, Roiban ('15)
- Yang-Mills + scalar φ^3 theory Chiodaroli, Gunaydin, HJ, Roiban ('14)
- Bi-adjoint scalar φ^3 theory — Bern, de Freitas, Wong ('99), Bern, Dennen, Huang; Du, Feng, Fu; Bjerrum-Bohr, Damgaard, Monteiro, O'Connell
- NLSM/Chiral Lagrangian Chen, Du ('13)
- *D*=3 Bagger-Lambert-Gustavsson theory (Chern-Simons-matter) Bargheer, He, McLoughlin; Huang, HJ, Lee ('12 -'13)

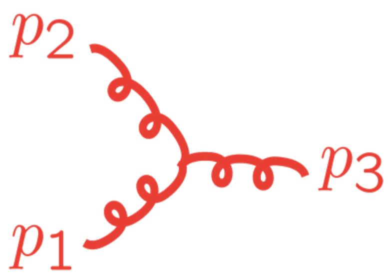
Which gravity theories are double copies

- Pure $\mathcal{N}$=4,5,6,8 supergravity (2 < D < 11) Bern, Carrasco, HJ ('08 -'10)
- Einstein gravity and pure $\mathcal{N}$=1,2,3 supergravity HJ, Ochirov ('14)
- Self-dual gravity O'Connell, Monteiro ('11)
- Closed string theories Mafra, Schlotterer, Stieberger ('11); Stieberger, Taylor ('14)
- Einstein + R^3 theory Broedel, Dixon ('12)
- Abelian matter coupled to supergravity HJ, Ochirov ('14 - '15)
- SYM coupled to supergravity Chiodaroli, Gunaydin, HJ, Roiban ('14)
- Spontaneously broken YM-Einstein gravity Chiodaroli, Gunaydin, HJ, Roiban ('15)
- *D*=3 supergravity (BLG Chern-Simons-matter theory)^2 $\left\{ \begin{array}{l} \text{Bargheer, He, McLoughlin;} \\ \text{Huang, HJ, Lee ('12 -'13)} \end{array} \right.$
- Born-Infeld, DBI, Galileon theories Cachazo, He, Yuan ('14)

Self-dual kinematic Lie algebra

Self dual YM in light-cone gauge:

Monteiro and O'Connell ('11)



Generators of diffeomorphism invariance:

$$L_k = e^{-ik\cdot x}(-k_w \partial_u + k_u \partial_w)$$

Lie Algebra:

YM vertex

$$[L_{p_1}, L_{p_2}] = iX(p_1, p_2) L_{p_1+p_2} = iF_{p_1 p_2}{}^k L_k$$

The $X(p_1, p_2)$ are YM vertices of type ++– helicity.

Diffeomorphism symmetry hidden in YM theory!

Self dual sector gives +++…+ amplitudes Boels, Isermann, Monteiro, O'Connell
(S-matrix is one-loop exact)

Color-Kinematics Duality for QCD

Defining QCD

'QCD' is taken to be the following theory:

$$\mathrm{SU}(N_c)\ \mathrm{YM}\ +\ N_f\ \text{massive quarks}$$

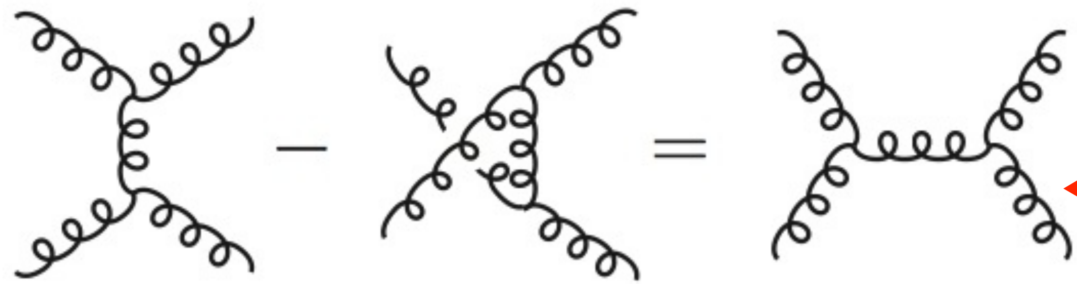
In fact, everything I say will also apply to:

$$G_c\ \mathrm{YM}\ +\ N_f\ \text{massive complex-rep. fermions} \\ /\text{scalars}$$

in D dimensions or SUSY extended SQCD

Only use two Lie-algebra properties

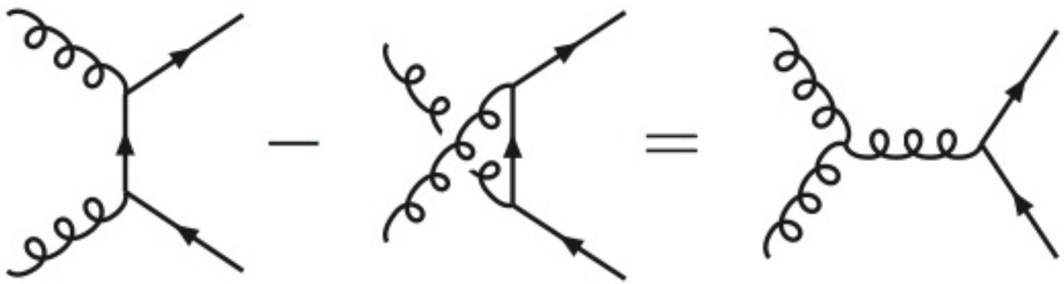
Jacobi Id.



adjoint repr.
or gluon, or
vector multipl.

$$\tilde{f}^{dac}\,\tilde{f}^{cbe} - \tilde{f}^{dbc}\,\tilde{f}^{cae} = \tilde{f}^{abc}\,\tilde{f}^{dce}$$

Commutation Id.



fund. repr.
or fermion, or
complex scalar,
or matter multipl.

$$T^{a}_{i\bar{k}}\,T^{b}_{k\bar{\jmath}} - T^{b}_{i\bar{k}}\,T^{a}_{k\bar{\jmath}} = \tilde{f}^{abc}\,T^{c}_{i\bar{\jmath}}\,.$$

Duality: $n_i - n_j = n_k \quad \Leftrightarrow \quad c_i - c_j = c_k$

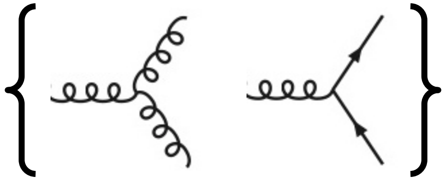
Amplitude presentation for QCD

QCD amplitude with *k* quark lines of distinct flavor:

HJ, Ochirov

$$\mathcal{A}^{(L)}_{n,k} = \sum_i \int \frac{d^{LD}\ell}{(2\pi)^{LD}} \frac{1}{S_i} \frac{n_i c_i}{D_i}$$

sum is over all cubic gluon–quark graphs with vertices

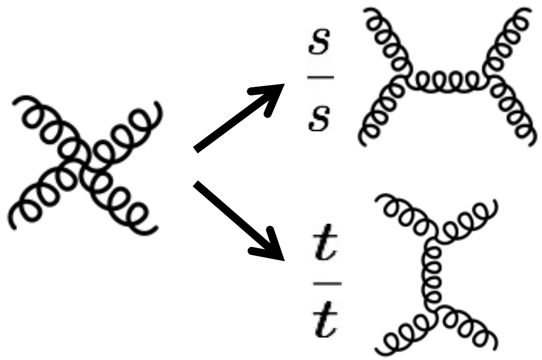


Color factors c_i are built out of $f^{abc}\,,\ T^a_{i\bar{\jmath}}$

Number of cubic tree-level graphs

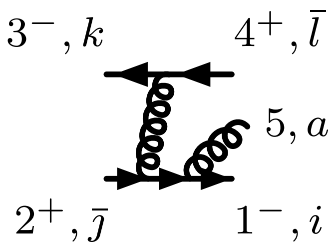
$k \setminus n$	3	4	5	6	7	8
0	1	3	15	105	945	10395
1	1	3	15	105	945	10395
2	-	1	5	35	315	3465
3	-	-	-	7	63	693
4	-	-	-	-	-	99

$$\nu(n,k) = \frac{(2n-5)!!}{(2k-1)!!} \text{ for } 2k \le n$$

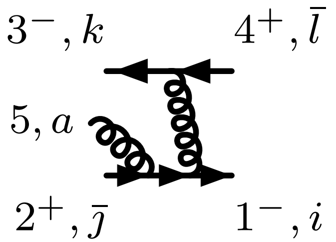


n=5 *k*=2 example

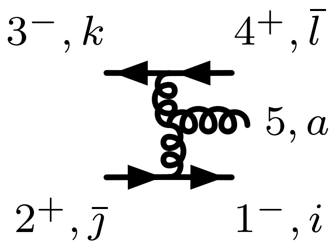
Look at 3 Feynman diagrams out of 5 in total:



$$= \frac{i}{\sqrt{2}} \frac{1}{s_{15} s_{34}} T^a_{i\bar{m}} T^b_{m\bar{\jmath}} T^b_{k\bar{l}} \, \langle 1|\varepsilon_5|1+5|3\rangle \, [24] = \frac{c_1 n_1}{D_1}$$



$$= -\frac{i}{\sqrt{2}} \frac{1}{s_{25} s_{34}} T^b_{i\bar{m}} T^a_{m\bar{\jmath}} T^b_{k\bar{l}} \, \langle 13\rangle \, [2|\varepsilon_5|2+5|4] = \frac{c_2 n_2}{D_2}$$



$$= \frac{i}{\sqrt{2}} \frac{1}{s_{12} s_{34}} \tilde{f}^{abc} T^b_{i\bar{\jmath}} T^c_{k\bar{l}} \Big(\langle 1| \varepsilon_5 |2] \, \langle 3| 5|4] - \langle 1| 5|2] \, \langle 3| \varepsilon_5 |4] - 2 \, \langle 13\rangle \, [24]((k_1+k_2)\cdot\varepsilon_5) \Big) = \frac{c_5 n_5}{D_5}$$

Not gauge invariant, but satisfy color-kinematics duality

$$c_1 - c_2 = -c_5 \qquad \Leftrightarrow \qquad n_1 - n_2 = -n_5$$

Color decomposition

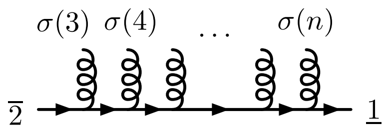
$\mathrm{SU}(N_c)$ **trace basis decomposition**

only gluons: $$\mathcal{A}^{\text{tree}}_{n,0} = \sum_{\sigma \in S_{n-1}(\{2,\ldots,n\})} \mathrm{Tr}\left(T^{a_1} T^{a_{\sigma(2)}} \ldots T^{a_{\sigma(n)}}\right) A(1, \sigma(2), \ldots, \sigma(n))$$

with quarks more complicated $$\sim \frac{1}{N_c^p} (T^{a_{2k+1}} \ldots T^{a_{l_1}})_{i_1 \bar{\alpha}_1} (T^{a_{l_1+1}} \ldots T^{a_{l_2}})_{i_2 \bar{\alpha}_2} \cdots (T^{a_{l_{k-1}+1}} \ldots T^{a_n})_{i_k \bar{\alpha}_k}$$

Mangano, ...

e.g. *k*=1 $$\mathcal{A}^{\text{tree}}_{n,1} = \sum_{\sigma \in S_{n-2}(\{3,\ldots,n\})} \left(T^{a_{\sigma(3)}} \ldots T^{a_{\sigma(n)}}\right)_{\bar{\jmath}_2 i_1} A(\underline{1}, \overline{2}, \sigma(3), \ldots, \sigma(n))$$



Del Duca, Dixon, Maltoni (DDM) basis

$$\mathcal{A}^{\text{tree}}_{n,0} = \sum_{\sigma \in S_{n-2}(\{3,\ldots,n\})} \tilde{f}^{\,a_2 a_{\sigma(3)} b_1} \tilde{f}^{\,b_1 a_{\sigma(4)} b_2} \ldots \tilde{f}^{\,b_{n-3} a_{\sigma(n)} a_1} A(1, 2, \sigma(3), \ldots, \sigma(n))$$

Properties: valid for any G, gives small *(n - 2)!* basis

Dyck words

Basis of planar (color-ordered) tree amplitudes:

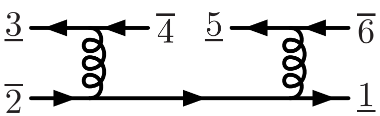
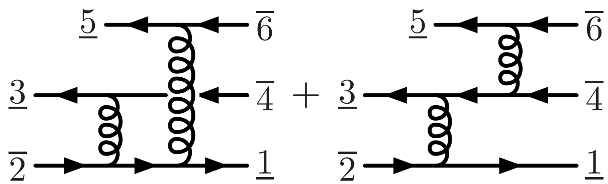
only quarks $\quad \left\{ A(\underline{1},\overline{2},\sigma) \;\middle|\; \sigma \in \mathrm{Dyck}_{k-1} \right\}$ **T. Melia**

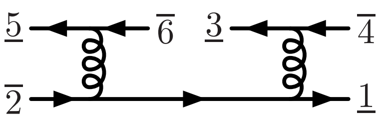
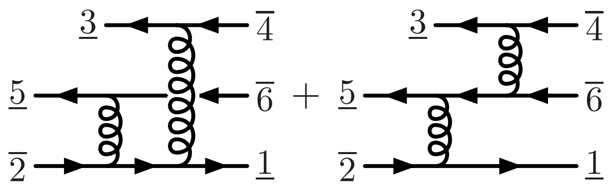
six-point example:

$$\begin{aligned}
\mathrm{XYXY} \quad &\Rightarrow \quad (\underline{3},\overline{4},\underline{5},\overline{6}),\ (\underline{5},\overline{6},\underline{3},\overline{4}) \quad \Leftrightarrow \quad \{3\,4\}\{5\,6\},\ \{5\,6\}\{3\,4\}\,, \\
\mathrm{XXYY} \quad &\Rightarrow \quad (\underline{3},\underline{5},\overline{6},\overline{4}),\ (\underline{5},\underline{3},\overline{4},\overline{6}) \quad \Leftrightarrow \quad \{3\{5\,6\}4\},\ \{5\{3\,4\}6\}\,.
\end{aligned}$$

basis: $A(\underline{1},\overline{2},\underline{3},\overline{4},\underline{5},\overline{6})\,,\ A(\underline{1},\overline{2},\underline{5},\overline{6},\underline{3},\overline{4})\,,\ A(\underline{1},\overline{2},\underline{3},\underline{5},\overline{6},\overline{4})$ and $A(\underline{1},\overline{2},\underline{5},\underline{3},\overline{4},\overline{6})$

Color coefficients:

$C_{\underline{1}\overline{2}\underline{3}\overline{4}\underline{5}\overline{6}} =$  ,
$C_{\underline{1}\overline{2}\underline{3}\underline{5}\overline{6}\overline{4}} =$ 

$C_{\underline{1}\overline{2}\underline{5}\overline{6}\underline{3}\overline{4}} =$  ,
$C_{\underline{1}\overline{2}\underline{5}\underline{3}\overline{4}\overline{6}} =$ 

HJ, Ochirov

Melia basis

Basis of planar (color-ordered) tree amplitudes:

gluons & quarks $\left\{ A(\underline{1},\overline{2},\sigma) \;\middle|\; \sigma \in \mathrm{Dyck}_{k-1} \times \{\text{gluon insertions}\}_{n-2k} \right\}$

T. Melia

size of basis:

$$\varkappa(n,k) = \underbrace{\overbrace{\frac{(2k-2)!}{k!(k-1)!}}^{\text{empty brackets}} \times (k-1)!}_{\text{dressed quark brackets}} \times \underbrace{(2k-1)(2k)\dots(n-2)}_{\text{insertions of } (n-2k) \text{ gluons}} = \frac{(n-2)!}{k!}$$

$k \setminus n$	3	4	5	6	7	8
0	1	2	6	24	120	720
1	1	2	6	24	120	720
2	-	1	3	12	60	360
3	-	-	-	4	20	120
4	-	-	-	-	-	30

Color decomposition, any $G_c, k,$ any rep.

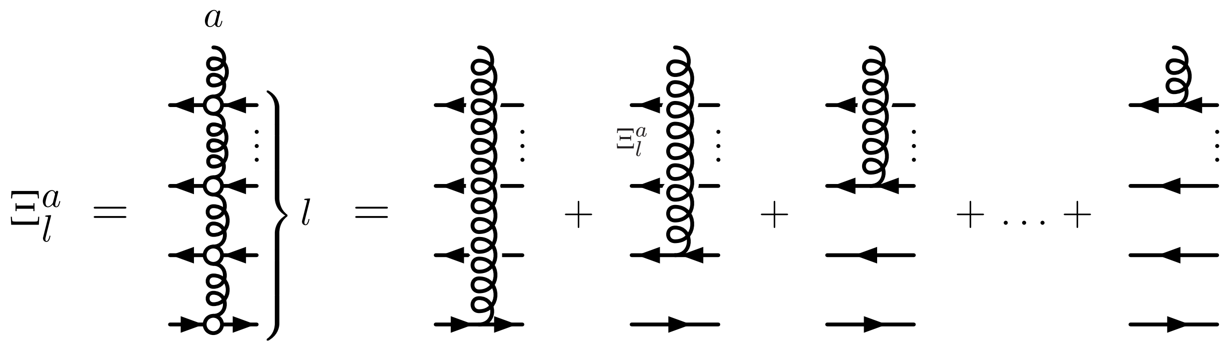
$$\mathcal{A}_{n,k}^{\text{tree}} = \sum_{\sigma \in \text{Melia basis}}^{\varkappa(n,k)} C(\underline{1},\overline{2},\sigma)\, A(\underline{1},\overline{2},\sigma)$$

HJ, Ochirov

Tensor representations

Tensor l copies of the gauge group Lie algebra:

$$\Xi_l^a = \sum_{s=1}^{l} \underbrace{1 \otimes \cdots \otimes 1 \otimes \overbrace{T^a \otimes 1 \otimes \cdots \otimes 1 \otimes \overline{1}}^{s}}_{l}$$



The Ξ_l^a are Lie algebra generators

$$\left[\Xi_l^a, \Xi_l^b\right] = \tilde{f}^{abc}\, \Xi_l^c$$

Color coefficients

Color coefficients are given by 'sandwich' formulas:

color wave function

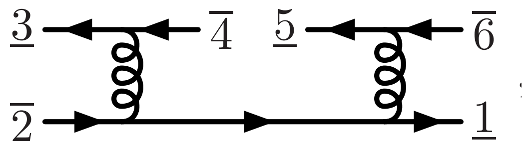
$$C(\underline{1},\overline{2},\sigma) = (-1)^{k-1}\,\{2|\sigma|1\}\Bigg|\begin{array}{lcl} \underline{q} & \to & \{q|\,T^b\otimes\Xi^b_{l-1} \\ \overline{q} & \to & |q\} \\ g & \to & \Xi^{a_g}_l \end{array}$$

HJ, Ochirov

(proof by Melia)

For example, consider:

$$C_{\underline{1}\overline{2}\underline{3}\overline{4}\underline{5}\overline{6}} =$$



$$C_{\underline{1}\overline{2}\underline{3}\overline{4}\underline{5}\overline{6}} = \{2|\{3|T^a\otimes\Xi^a_1|4\}\{5|T^b\otimes\Xi^b_1|6\}|1\} = \{2|\{3|T^a\otimes\overline{T}^a|4\}\{5|T^b\otimes\overline{T}^b|6\}|1\}$$

$$= \{2|\overline{T}^a\overline{T}^b|1\}\{3|T^a|4\}\{5|T^b|6\} = (T^bT^a)_{i_1\bar{\imath}_2}T^a_{i_3\bar{\imath}_4}T^b_{i_5\bar{\imath}_6}\,,$$

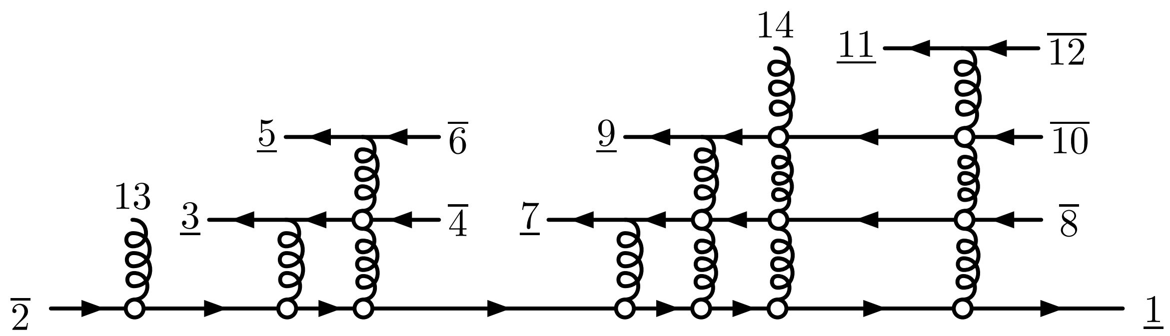
Color coefficient diagrams

Consider a high-multiplicity example:

$$A(\underline{1},\overline{2},13,\underline{3},\underline{5},\overline{6},\overline{4},\underline{7},\underline{9},14,\underline{11},\overline{12},\overline{10},\overline{8})$$

bra-(c)-ket structure

$$\{2\;13\{3\{5\;6\}4\}\{7\{9\;14\{11\;12\}10\}8\}1\}$$

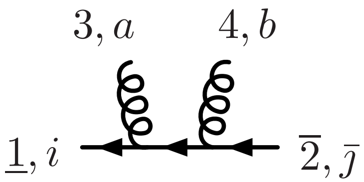


2 × 2×3 × 3 = 36

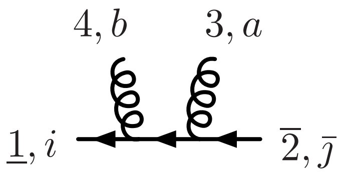
$$C_{\underline{1},\overline{2},13,\underline{3},\underline{5},\overline{6},\overline{4},\underline{7},\underline{9},14,\underline{11},\overline{12},\overline{10},\overline{8}} = -\{2|\Xi_1^{a_{13}}\{3|T^b\otimes\Xi_1^b\{5|T^c\otimes\Xi_2^c|6\}|4\}$$

$$\times\{7|T^d\otimes\Xi_1^d\{9|(T^e\otimes\Xi_2^e)\Xi_3^{a_{14}}\{11|T^f\otimes\Xi_3^f|12\}|10\}|8\}|1\}$$

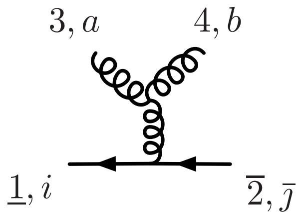
Amplitude relations: example



$$= -\frac{i}{2}\,\frac{T^a_{i\bar{k}}T^b_{k\bar{\jmath}}}{s_{13}-m^2}\,(\bar{u}_1 \not{\epsilon}_3(\not{k}_{1,3}+m)\not{\epsilon}_4 v_2) = \frac{c_1 n_1}{D_1}$$



$$= \frac{c_2 n_2}{D_2}$$



$$= \frac{c_3 n_3}{D_3}$$

commutation rel. holds: $c_1 - c_2 = c_3 \qquad n_1 - n_2 = n_3$

$$\mathcal{A}^{\text{tree}}_{4,1} = \sum_{i=1}^{3} \frac{c_i n_i}{D_i} = \left\{ c_1 \left(\frac{n_1}{D_1} + \frac{n_3}{D_3} \right) + c_2 \left(\frac{n_2}{D_2} - \frac{n_3}{D_3} \right) \right\} \equiv c_2 A_{\underline{1}\overline{2}34} + c_1 A_{\underline{1}\overline{2}43}$$

Gaussian elimination of n_1 $\quad A_{\underline{1}\overline{2}34} = \underbrace{\left(\frac{1}{D_2} + \frac{1}{D_3} - \frac{D_1}{(D_1+D_3)D_3} \right)}_{=0} n_2 - \frac{D_1}{D_1+D_3} A_{\underline{1}\overline{2}43}\,.$

→ BCJ amplitude rel. $\quad (s_{14} - m^2) A_{\underline{1}\overline{2}34} = (s_{13} - m^2) A_{\underline{1}\overline{2}43}$

Amplitude relations & basis

BCJ relations for pure-gluon amplitudes:

Bern, Carrasco, HJ

$$\sum_{i=2}^{n-1}\Big(\sum_{j=2}^{i} s_{jn}\Big) A(1,2,\ldots i,n,i+1,\ldots,n-1)=0$$

BCJ relations for quark-gluon QCD amplitudes:

HJ, Ochirov

$$\sum_{i=2}^{n-1}\Big(\sum_{j=2}^{i} s_{jn}-m_j^2\Big) A(1,2,\ldots i,n,i+1,\ldots,n-1)=0$$

gluon !

proof by: de la Cruz, Kniss, Weinzierl

Basis:

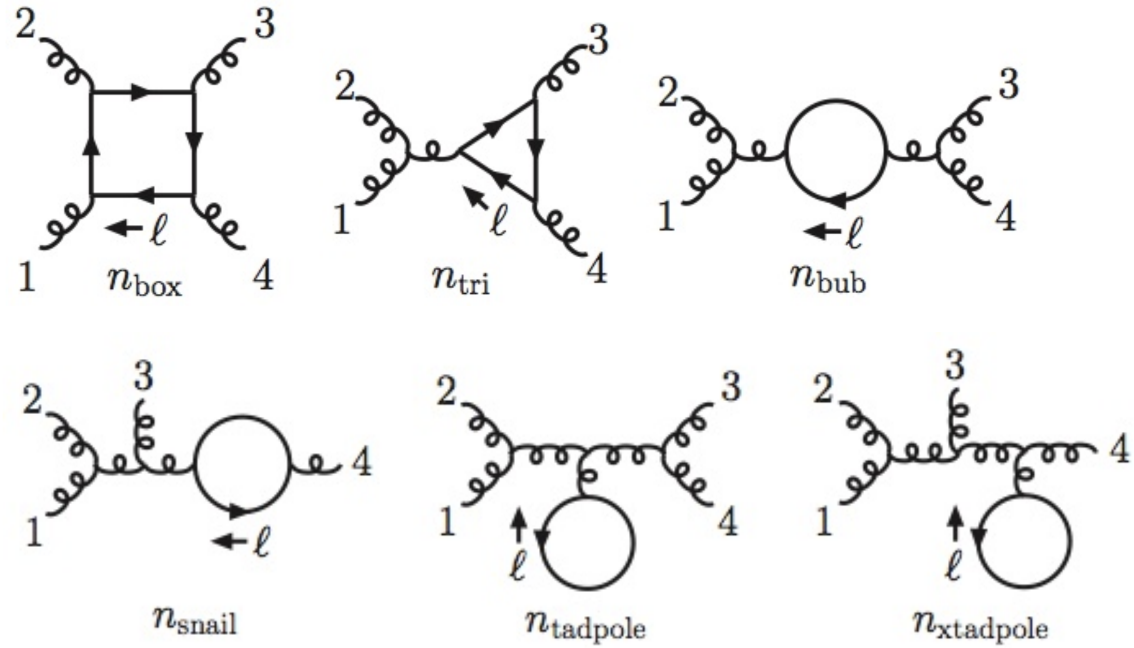
$k \setminus n$	3	4	5	6	7	8
0	1	1	2	6	24	120
1	1	1	2	6	24	120
2	-	1	2	6	24	120
3	-	-	-	4	16	80
4	-	-	-	-	-	30

$$\begin{array}{ll} (n-3)! & \text{for } k=0,1 \\ (n-3)!(2k-2)/k! & \text{for } 2<2k\le n \end{array}$$

Simple 1-loop examples

One-loop calculations

diagrams:



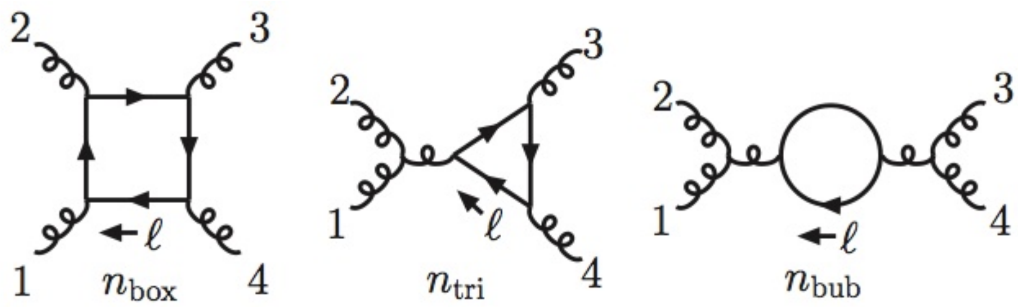
vanish
after
integration

kinematic algebra:

$$
\begin{aligned}
n_{\text{tri}}(1,2,3,4,\ell) &= n_{\text{box}}([1,2],3,4,\ell)\,, \\
n_{\text{bub}}(1,2,3,4,\ell) &= n_{\text{box}}([1,2],[3,4],\ell)\,, \\
n_{\text{snail}}(1,2,3,4,\ell) &= n_{\text{box}}([[1,2],3],4,\ell)\,, \\
n_{\text{tadpole}}(1,2,3,4,\ell) &= n_{\text{box}}([[1,2],[3,4]],\ell)\,, \\
n_{\text{xtadpole}}(1,2,3,4,\ell) &= n_{\text{box}}([[[1,2],3],4],\ell)\,.
\end{aligned}
$$

Ansatz for the box numerator: $\mathcal{N}$=0,1,2 SQCD

diagrams:



ansatz for 4pt MHV amplitude with internal matter, in any SYM theory: HJ, Ochirov

$$n_{\text{box}}(1,2,3,4,\ell) = \sum_{1\le i<j\le 4} \frac{\kappa_{ij}}{s_{ij}^N}\Big(\sum_k a_{ij;k} M_k^{(N)} + \epsilon(1,2,3,\ell)\sum_k \tilde{a}_{ij;k} M_k^{(N-2)}\Big)$$

power-counting factor: $N = 4 - \mathcal{N}$ ← SUSY

momentum monomials: $M^{(N)} = \Big\{ \prod_{i=1}^{N} m_i \;\Big|\; m_i \in \{s,\, t,\, \ell\cdot k_j,\, \ell^2,\, \mu^2\}\Big\}$

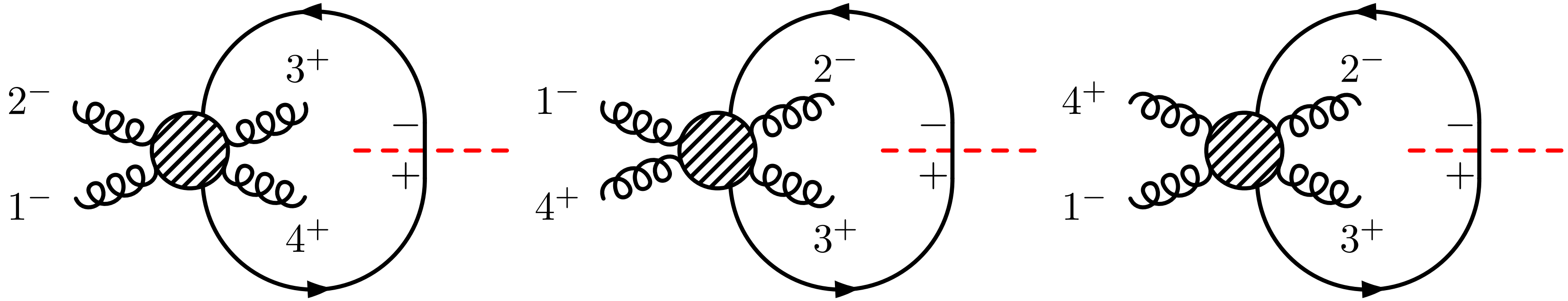
state dependence: $\kappa_{ij} = \frac{[1\,2]\,[3\,4]}{\langle 1\,2\rangle\,\langle 3\,4\rangle}\delta^{(2\mathcal{N})}(Q)\,\langle i\, j\rangle^{4-\mathcal{N}}\,\theta_i\theta_j$

(vector multiplet: $\mathcal{V}_{\mathcal{N}} = V_{\mathcal{N}} + \overline{V}_{\mathcal{N}}\,\theta$)

Unitarity cuts

Parameters in ansatz fixed by unitarity cuts (unitarity method)

Bern, Dixon, Dunbar, Kosower



***N*=2 SQCD:** Carrasco, Chiodaroli, Gunaydin, Roiban; Nohle; Ochirov, Tourkine, HJ, Ochirov

$$n_{\text{box}}^{\mathcal{N}=2,\text{fund}} = (\kappa_{12}+\kappa_{34})\frac{(s-\ell_s)^2}{2s^2} + (\kappa_{23}+\kappa_{14})\frac{\ell_t^2}{2t^2} + (\kappa_{13}+\kappa_{24})\frac{st+(s+\ell_u)^2}{2u^2}$$
$$- 2i\epsilon(1,2,3,\ell)\frac{\kappa_{13}-\kappa_{24}}{u^2} + \mu^2\left(\frac{\kappa_{12}+\kappa_{34}}{s} + \frac{\kappa_{23}+\kappa_{14}}{t} + \frac{\kappa_{13}+\kappa_{24}}{u}\right)$$

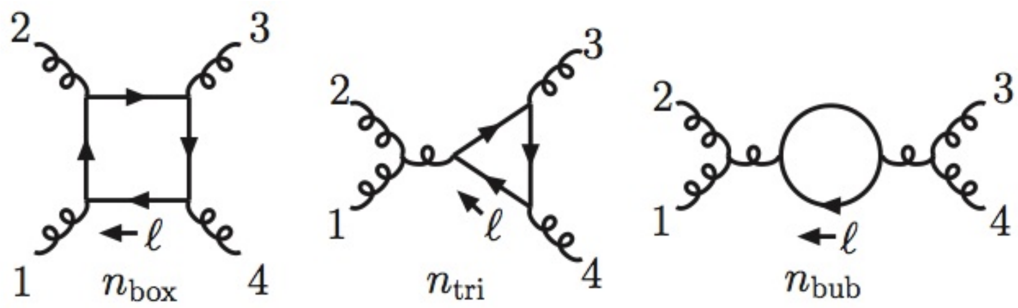
***N*=1 SQCD:** HJ, Ochirov

$$n_{\text{box}}^{\mathcal{N}=1,\text{odd}} = (\kappa_{12}-\kappa_{34})\frac{(\ell_s-s)^3}{2s^3} + (\kappa_{23}-\kappa_{14})\frac{\ell_t^3}{2t^3} + (\kappa_{13}-\kappa_{24})\frac{1}{2}\left(\frac{\ell_u^3}{u^3} + \frac{3s\ell_u^2}{u^3} - \frac{3s\ell_u}{u^2} + \frac{s}{u}\right)$$
$$- 2i\epsilon(1,2,3,\ell)(\kappa_{13}+\kappa_{24})\frac{2\ell_u-u}{u^3} - a\mu^2(\kappa_{13}-\kappa_{24})\frac{s-t}{u^2}\,,$$

YM + scalar: Nohle; HJ, Ochirov

QCD: HJ, Ochirov

$\mathcal{N}$=2 SQCD



$\mathcal{N}$=2 SQCD integrated amplitude:

Bern, Dixon, Dunbar, Kosower; Bern, Morgan

$$A_4^{\mathcal{N}=2,\mathrm{fund}}(1^-,2^-,3^+,4^+) = \frac{i\langle 12\rangle^2[34]^2}{(4\pi)^{D/2}}\left\{-\frac{1}{st}I_2(t)\right\},$$

$$A_4^{\mathcal{N}=2,\mathrm{fund}}(1^-,2^+,3^-,4^+) = \frac{i\langle 13\rangle^2[24]^2}{(4\pi)^{D/2}}\left\{-\frac{r_\Gamma}{2u^2}\left(\ln^2\left(\frac{-s}{-t}\right)+\pi^2\right)+\frac{1}{su}I_2(s)+\frac{1}{tu}I_2(t)\right\}$$

$$I_2(t) = \frac{r_\Gamma}{\epsilon(1-2\epsilon)}(-t)^{-\epsilon}$$

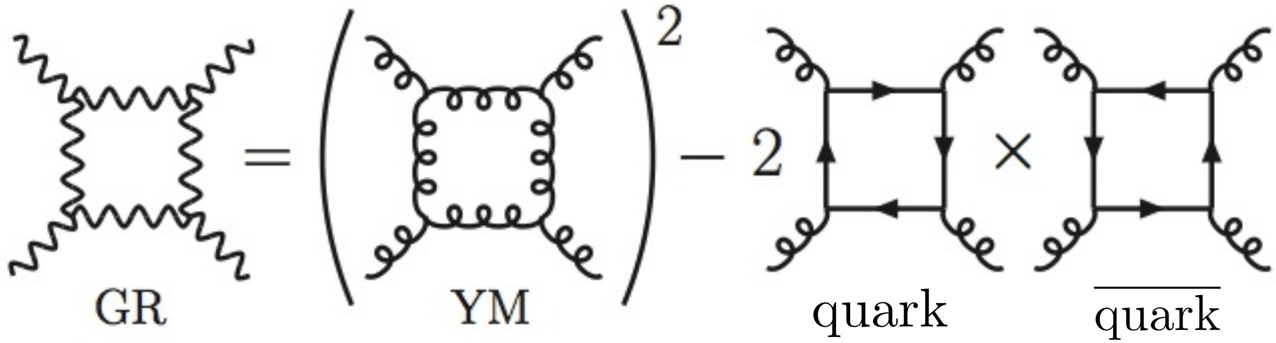
$$r_\Gamma = \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}$$

Using the QCD numerators to get GR

Pure Einstein gravity can be obtained from the QCD numerators:

$$\mathcal{M}_4^{(1)} = \sum_{\mathcal{S}_4} \sum_{i=\{B,t,b\}} \int \frac{d^D\ell}{(2\pi)^D} \frac{1}{S_i} \frac{n_i^V {n_i}^{V\prime} - \overline{n}_i^m {n_i}^{m\prime} - n_i^m {\overline{n}_i}^{m\prime}}{D_i}$$

The YM square contains dilaton & axion, which has to be subtracted out



HJ, Ochirov

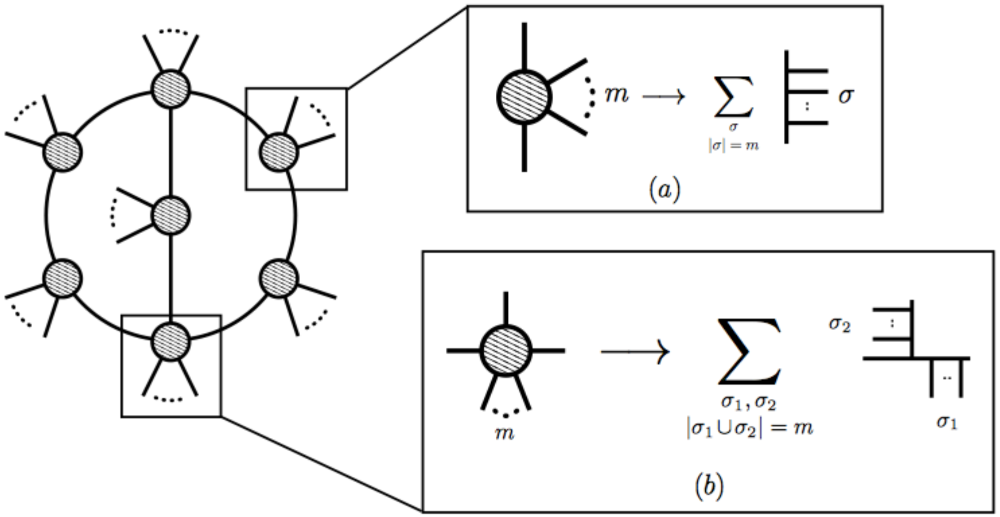
...and similarly for triangle and bubble

Gives correct pure GR amplitude (cf. Dunbar & Norridge)

Loop-level application QCD

Two-loop 5pt all-plus-helicity amplitude in pure YM computed to all orders in N_c using the DDM basis and BCJ relations:

Badger, Mogull, Ochirov, O'Connell (arXiv:1507.08797)



Summary

- Color-kinematics duality implies kinematic Lie algebra relations satisfied by the numerators of gauge theory amplitudes
- Generalized color-kinematics duality to QCD tree amplitudes
- New color decomposition of QCD tree amplitudes
- BCJ amplitude relations between primitives of QCD
- Checks: Explicitly up to 8pts tree level, proof color decompositon (Melia)
  proof BCJ relations (Weinzierl, et al.)
- Constructed one-loop 4pt amplitude in $\mathcal{N}$=1,2 SQCD and QCD such that the duality is manifest.
- Useful for construction of QCD loop amplitudes as well as pure Einstein gravity amplitudes