The Higgs boson as Inflaton

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Outline of Talk:

- Introduction
- The Standard Model up to the Planck scale
- Low Energy Effective SM (LEESM)
- Cosmology and Inflation
- The Role of Quadratic Divergences in the SM
- The Cosmological Constant in the SM
- The Higgs Boson as Inflaton!
- Reheating and Baryogenesis
- Conclusion
Introduction

LHC ATLAS&CMS Higgs discovered ⇒ the SM completion

Higgs mass found by ATLAS and CMS agrees perfectly with the indirect bounds

LEP 2005 +++ LHC 2012

Englert&Higgs Nobel Prize 2013

Higgs mass found in very special mass range $125.9 \pm 0.4$ GeV
Common Folklore: hierarchy problem requires supersymmetry (SUSY) extension of the SM (no quadratic/quartic divergences) \( \text{SUSY} = \text{infinity killer!} \)

Do we need new physics? Stability bound of Higgs potential in SM:

\[
V = \frac{m^2}{2} H^2 + \frac{\lambda}{24} H^4
\]

Riesselmann, Hambye 1996

\[ M_H < 180 \text{ GeV} \]

– first 2-loop analysis, knowing \( M_t \) –

SM Higgs remains perturbative up to scale \( \Lambda \) if it is light enough (upper bound=avoiding Landau pole) and Higgs potential remains stable \( (\lambda > 0) \) if Higgs mass is not too light [parameters used: \( m_t = 175[150 - 200] \text{ GeV} ; \ \alpha_s = 0.118 \) ]
The Standard Model completed

SM – Fermions: 28 per family ⇒ 3x28=84 ; Gauge-Bosons: 1+3+8=12 ; Scalars: 1 Higgs

Before Higgs mechanism [symmetric phase]: $W^\pm$, $Z$ and all fermions massless
Higgs “ghosts” $\phi^\pm$, $\phi^0$ physical, heavy degenerate with the Higgs!

At “low” energy [likely up to $10^{16}$ GeV]:

$$V = \frac{m^2}{2} H^2 + \frac{\lambda}{24} H^4 ; \quad m^2 = -\mu^2 < 0$$

SM in broken phase: $H$, $W^\pm$, $Z$ and all fermions massive [each mass requires separate new interaction via the Higgs: 2+12+1 decay channels]; 3 Higgs “ghosts” $\phi^\pm$, $\phi^0$ disappear and transmute into longitudinal DOFs of $W^\pm$, $Z$

Basic parameters: gauge couplings $g' = g_1$, $g = g_2$, $g_3$, top quark Yukawa coupling $y_t$, Higgs self-coupling $\lambda$ and Higgs VEV $v$, besides smaller Yukawas.

Note: $1/(\sqrt{2}v^2) = G_F$ is the Fermi constant! [$v = (\sqrt{2}G_F)^{-1/2}$]
SSB $\Rightarrow$ mass $\propto$ interaction strength $\times$ Higgs VEV $v$

$$
\begin{align*}
M_W^2 &= \frac{1}{4} g^2 v^2 ; \\
m_f^2 &= \frac{1}{2} y_f^2 v^2 ; \\
M_Z^2 &= \frac{1}{4} (g^2 + g'^2) v^2 ; \\
M_H^2 &= \frac{1}{3} \lambda v^2
\end{align*}
$$

Effective parameters depend on renormalization scale $\mu$ [normalization reference energy!], scale at which ultraviolet (UV) singularities are subtracted

- couplings change substantially with energy and hence as a function of time during evolution of the universe!
- high energy behavior governed by $\overline{\text{MS}}$ Renormalization Group (RG) [$E \gg M_i$]
- key input matching conditions between $\overline{\text{MS}}$ and physical parameters!

energy scale $\leftrightarrow$ center of mass energy of a physical process

e.g. at Large Electron Positron Collider [LEP] (pre LHC $e^+e^-$ storage ring)
The Cosmic Bridge

**$e^+e^-$ Annihilation at LEP**

45.5 GeV 45.5 GeV

Electron (Matter) Positron (Antimatter)

$$E = 2m_{\text{eff}}c^2$$

Mini Big Bang!

$$e^+e^- \leftrightarrow \gamma^* \leftrightarrow XX \text{ new forms of matter}$$

Energy versus temperature correspondence:

$$T = \frac{E}{k_B} \degree K$$

$$1 \degree K \equiv 8.6 \times 10^{-5} \text{ eV}$$

(Boltzmann constant $k_B$)

↓

temperature of an event

$$T \sim 1.0 \times 10^{15} \degree K$$

In nature such temperatures only existed in the very early universe:

$$t = \frac{2.4}{\sqrt{g^*(T)}} \left(\frac{1 \text{MeV}}{k_B T}\right)^2 \text{ sec.}$$

↓

$$t \sim 0.3 \times 10^{-10} \text{ sec. after B.B.}$$

early universe

$g^*(T)$ number of relativistic degrees of freedom at given $T$
Universe is expanding, began with the Big Bang! began very hot and dense Early cosmology is Particle Physics!

At Start a Light-Flash: Big-Bang (fireball)
Light quanta very energetic, all matter totally ionized, all nuclei disintegrated. Elementary particles only!: $\gamma, e^+, e^-, p, \bar{p}, \cdots$

Processes:
\[
2\gamma \leftrightarrow e^+ + e^- \\
2\gamma \leftrightarrow \bar{p} + p \\
\vdots
\]
Collider energy temperature time after B.B.

<table>
<thead>
<tr>
<th>Collider</th>
<th>$E_{cm}$</th>
<th>$T$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEP I:</td>
<td>$E_{cm} \sim 100$ GeV</td>
<td>$1.16 \times 10^{15}$ °K</td>
<td>$t_{LEPI} \sim 2.58 \times 10^{-11}$ seconds</td>
</tr>
<tr>
<td>LEP II:</td>
<td>$E_{cm} \sim 200$ GeV</td>
<td>$2.33 \times 10^{15}$ °K</td>
<td>$t_{LEPII} \sim 6.46 \times 10^{-12}$ seconds</td>
</tr>
<tr>
<td>LHC :</td>
<td>$E_{cm} \sim 14$ TeV</td>
<td>$1.63 \times 10^{17}$ °K</td>
<td>$t_{LHC} \sim 1.185 \times 10^{-15}$ seconds</td>
</tr>
</tbody>
</table>

The key question: does SM physics describe physics up to the Planck scale?

or do we need new physics beyond the SM to understand the early universe?
or does the SM collapse if there is no new physics?

“collapse”: Higgs potential gets unstable below the Planck scale; actually several groups claim to have proven vacuum stability break down!

Shaposhnikov et al, Degrassi et al, Maina, Hamada et al, ...

Shaposhnikov et al. arXiv:1412.3811 say about Vacuum Stability

Although the present experimental data are perfectly consistent with the absolute stability of Standard Model within the experimental and theoretical uncertainties, one should not exclude the possibility that other experiments will be able to establish the metastability of the electroweak vacuum in the future.
Although other evaluations of the matching conditions seem to favor the metastability of the electroweak vacuum within the experimental and theoretical uncertainties, one should not exclude the possibility that other experiments and improved matching conditions will be able to establish the absolute stability of Standard Model in the future.

Scenarios this talk: Higgs vacuum remains stable up and beyond the Planck scale ⇒ seem to say we do not need new physics affecting the evolution of SM couplings to investigate properties of the early universe. In the focus:

- does Higgs self-coupling stay positive $\lambda > 0$ up to $\Lambda_{\text{Pl}}$?
- the key question/problem concerns the size of the top Yukawa coupling $y_t$ decides about stability of our world! — $[\lambda = 0$ would be essential singularity!]

Will be decided by:
- more precise input parameters
- better established EW matching conditions
The SM running parameters

The SM dimensionless couplings in the $\overline{\text{MS}}$ scheme as a function of the renormalization scale for $M_H = 124 - 127$ GeV.

- perturbation expansion works up to the Planck scale!
- no Landau pole or other singularities $\Rightarrow$ Higgs potential remains Stable!
- $U(1)_Y$ screening (IR free), $SU(2)_L$, $SU(3)_c$ antiscreening (UV free) [asymptotic freedom (AF)] – $g_1, g_2, g_3$

- Right – as expected

- Top Yukawa $y_t$ and Higgs $\lambda$: screening (IR free, like QED)

- Wrong!!! – transmutation from IR free to AF

- running top Yukawa – QCD takes over: IR free $\Rightarrow$ UV free

- running Higgs self-coupling – top Yukawa takes over: IR free $\Rightarrow$ UV free

- Higgs coupling decreases up to the zero of $\beta_\lambda$ at $\mu_\lambda \sim 3.5 \times 10^{17}$ GeV, where it is small but still positive and then increases up to $\mu = \Lambda_{\text{Pl}}$

The Higgs is special: before the symmetry is broken: all particles massless protected by gauge or chiral symmetry except the Higgses. Two quantities affected: Higgs mass and Higgs vacuum energy
Comparison of $\overline{\text{MS}}$ parameters at various scales: Running couplings for $M_H = 126$ GeV and $\mu_0 \approx 1.4 \times 10^{16}$ GeV.

<table>
<thead>
<tr>
<th>coupling \ scale</th>
<th>$M_Z$</th>
<th>$M_t$</th>
<th>$\mu_0$</th>
<th>$M_{\text{Pl}}$</th>
<th>my findings</th>
<th>Degrassi et al. 2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_3$</td>
<td>1.2200</td>
<td>1.1644</td>
<td>0.5271</td>
<td>0.4886</td>
<td>1.1644</td>
<td>0.4873</td>
</tr>
<tr>
<td>$g_2$</td>
<td>0.6530</td>
<td>0.6496</td>
<td>0.5249</td>
<td>0.5068</td>
<td>0.6483</td>
<td>0.5057</td>
</tr>
<tr>
<td>$g_1$</td>
<td>0.3497</td>
<td>0.3509</td>
<td>0.4333</td>
<td>0.4589</td>
<td>0.3587</td>
<td>0.4777</td>
</tr>
<tr>
<td>$y_t$</td>
<td>0.9347</td>
<td>0.9002</td>
<td>0.3872</td>
<td>0.3510</td>
<td>0.9399</td>
<td>0.3823</td>
</tr>
<tr>
<td>$\sqrt{\lambda}$</td>
<td>0.8983</td>
<td>0.8586</td>
<td>0.3732</td>
<td>0.3749</td>
<td>0.8733</td>
<td>i 0.1131</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.8070</td>
<td>0.7373</td>
<td>0.1393</td>
<td>0.1405</td>
<td>0.7626</td>
<td>- 0.0128</td>
</tr>
</tbody>
</table>

Most groups find just unstable vacuum at about $\mu \sim 10^9$ GeV! [not independent, same $\overline{\text{MS}}$ input]

Note: $\lambda = 0$ is an essential singularity and the theory cannot be extended beyond a possible zero of $\lambda$: remind $v = \sqrt{6m^2/\lambda}$ !!! i.e. $v(\lambda) \to \infty$ as $\lambda \to 0$

besides the Higgs mass $m_H = \sqrt{2} m$ all masses $m_i \propto g_i v \to \infty$ different cosmology
The SM’s naturalness problems and fine-tuning problems

Thematized by ’t Hooft 1979 as a relationship between macroscopic phenomena which follow from microscopic physics (condensed matter inspired), immediately the “hierarchy problem” has been dogmatized as a kind of fundamental principle.

- the Higgs mass: [note bare parameters parametrize the true Lagrangian]

\[ m_{\text{Higgs, bare}}^2 = m_{\text{Higgs, ren}}^2 + \delta m^2 ; \quad \delta m^2 = \frac{\Lambda_{\text{Pl}}^2}{(16\pi^2)} C(\mu) \]

coefficient typically \( C = O(1) \). To keep the renormalized mass at the observed small value \( m_{\text{ren}} = O(100 \text{ GeV}) \), \( m_{\text{bare}}^2 \) has to be tuned to compensate the huge term \( \delta m^2 \): about 35 digits must be adjusted in order to get the observed value.

Hierarchy Problem!
the vacuum energy density:

\[ \rho_{\text{vac, bare}} = \rho_{\text{vac, ren}} + \delta \rho; \quad \delta \rho = \frac{\Lambda_{\text{Pl}}^4}{(16\pi^2)^2} X(\mu) \]

SM predicts huge CC at \( \Lambda_{\text{Pl}} \):

\[ \rho_{\text{vac, bare}} \approx V(0) + \Delta V(\phi) \sim 2.77 \Lambda_{\text{Pl}}^4 \sim 6.13 \times 10^{76} \text{ GeV}^4 \quad \text{vs.} \quad \rho_{\text{vac}} = (0.002 \text{ eV})^4 \text{ today} \]

Cosmological Constant Problem!

Note: the only trouble maker is the Higgs!

Also note: naive arguments do not take into account that quantities compared refer to very different scales! \( m_{\text{Higgs, bare}}^2 \) short distance, \( m_{\text{Higgs, ren}}^2 \) long distance observables. Need UV-completion of SM: prototype lattice SM as true(r) system
Emergence Paradigm and UV completion

The SM is a low energy effective theory of a unknown Planck medium [the “ether”], which exhibits the Planck energy as a physical cutoff: i.e. the SM emerges from a system shaped by gravitation

\[ \Lambda_{Pl} = (G_N)^{-1/2} \approx 1.22 \times 10^{19} \text{ GeV} \]

\[ G_N \text{ Newton’s gravitational constant} \]

- SM works up to Planck scale, this mean that in makes sense to consider the SM as the Planck medium as seen from far away i.e. the SM is emergent at low energies. Expand in \( E/\Lambda_{Pl} \Rightarrow \) see renormalizable tail only.

- looking at shorter and shorter distances (higher energies) we can see the bare Planck system as it was evolving after the Big Bang!

- the tool for accessing early cosmology is the RG solution of SM parameters: we can calculate the bare parameters from the renormalized ones determined at low (accelerator) energies.
In the symmetric phase at very high energy we see the bare system:

the Higgs field is a collective field exhibiting an effective mass generated by radiative effects

\[ m^2_{\text{bare}} \approx \delta m^2 \text{ at } M_{\text{Pl}} \]

eliminates fine-tuning problem at all scales!

Many example in condensed matter systems.

“free lunch” in Low Energy Effective SM (LEESM) scenario:

- renormalizability of long range tail automatic!
- so are all consequences of renormalizability
- non-Abelian gauge symmetries, anomaly cancellation, fermion families etc
- last but not least the existence of the Higgs boson!
### The low energy expansion at a glance

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Operator</th>
<th>Scaling Behavior</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d=6$</td>
<td>$(\Box \phi)^2, (\bar{\psi}\psi)^2, \cdots$</td>
<td>$(E/\Lambda_{Pl})^2$</td>
</tr>
<tr>
<td>$d=5$</td>
<td>$\bar{\psi}\sigma^{\mu\nu} F_{\mu\nu} \psi, \cdots$</td>
<td>$(E/\Lambda_{Pl})$</td>
</tr>
<tr>
<td>$d=4$</td>
<td>$(\partial \phi)^2, \phi^4, (F_{\mu\nu})^2, \cdots$</td>
<td>$\ln(E/\Lambda_{Pl})$</td>
</tr>
<tr>
<td>$d=3$</td>
<td>$\phi^3, \bar{\psi}\psi$</td>
<td>$(\Lambda_{Pl}/E)$</td>
</tr>
<tr>
<td>$d=2$</td>
<td>$\phi^2, (A_{\mu})^2$</td>
<td>$(\Lambda_{Pl}/E)^2$</td>
</tr>
<tr>
<td>$d=1$</td>
<td>$\phi$</td>
<td>$(\Lambda_{Pl}/E)^3$</td>
</tr>
</tbody>
</table>

Note: $d=6$ operators at LHC suppressed by $(E_{LHC}/\Lambda_{Pl})^2 \approx 10^{-30}$

⇒ require **chiral symmetry, gauge symmetry, supersymmetry??**
Gravitation and Cosmological Models

Gravitation \iff\ all masses and even massless particle attract each other

Einstein’s General Relativity Theory (GRT): masses (energy density) determine the geometry of space-time (Riemannian Geometry)

Mass tells space how to curve – curved space tells bodies how to move

\Rightarrow\text{Einstein’s equation!}
Cosmology

- Weyl’s postulate (ideal fluid)
- Cosmological principle (isotropy implying homogeneity)

fix the form of the metric and of the energy-momentum tensor:

1. The metric (spaces of constant curvature \( k = \pm 1, 0 \))

\[
ds^2 = (cdt)^2 - a^2(t) \left( \frac{dr^2}{1-kr^2} + r^2 \ d\Omega^2 \right)
\]

2. The energy-momentum tensor

\[
T^\mu_\nu = (\rho + p)(t) \ u^\mu u^\nu - p(t) \ g^\mu_\nu ; \ u^\mu \equiv \frac{dx^\mu}{ds}
\]

where in the comoving frame \( ds = c \ dt \).

Need \( \rho(t) \) and \( p(t) \) to get \( a(t) \) radius of the universe
\[ G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} \]

Einstein Tensor \iff geometry of space-time
Gravitational interaction strength \[ \kappa = \frac{8\pi G_N}{3c^2} \]
Energy-Momentum Tensor \iff deriving from the Lagrangian of the SM

Cosmological solution: universe as a fluid of galaxies \implies Friedmann-Equations:

\[ 3 \frac{\dot{a}^2 + kc^2}{c^2 a^2} - \Lambda = \kappa \rho \]
\[ -2 \frac{\ddot{a} + \dot{a}^2 + kc^2}{c^2 a^2} + \Lambda = \kappa p \]

\[ a(t) \text{ Robertson-Walker radius of the universe} \]
\[ \Lambda \text{ Cosmological Constant} \]

- universe must be expanding, **Big Bang**
- Hubble’s law [galaxies: velocity\_recession = H \text{ Distance }], \( H \) Hubble constant
- temperature, energy density, pressure huge at begin, decreasing with time
\[ G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa \quad T_{\mu\nu}^{\text{tot}} ; \quad T_{\mu\nu}^{\text{tot}} = T_{\mu\nu} + \rho \Lambda g_{\mu\nu} ; \quad \rho \Lambda = \kappa \Lambda \]

Einstein Tensor \iff geometry of space-time

Gravitational interaction strength \[ \kappa = \frac{8\pi G_N}{3c^2} \]

Energy-Momentum Tensor \iff deriving from the Lagrangian of the SM

Cosmological solution: universe as a fluid of galaxies \Rightarrow Friedmann-Equations:

\[ \frac{3}{c^2 a^2} \frac{\ddot{a}^2 + kc^2}{c^2 a^2} = \kappa (\rho + \rho \Lambda) \]
\[ \frac{2}{c^2 a^2} \frac{\ddot{a}a + \dot{a}^2 + kc^2}{c^2 a^2} = \kappa (p + p \Lambda) \]

\[ a(t) \quad \text{Robertson-Walker radius of the universe} \]
\[ p \Lambda = -\rho \Lambda \quad \text{Dark Energy} \]

\[ \square \text{universe must be expanding, Big Bang,} \]
\[ \square \text{Hubble’s law [galaxies: } \text{velocity}_{\text{recession}} = H \text{ Distance } \text{], } H \text{ Hubble constant} \]
\[ \square \text{temperature, energy density, pressure huge at begin, decreasing with time} \]
Curvature: closed $k = 1$ [Ω₀ > 1], flat $k = 0$ [Ω₀ = 1] and open $k = -1$ [Ω₀ < 1]

Interesting fact: flat space geometry $\iff$ specific critical density, “very unstable”

$$\rho_{0,\text{crit}} = \rho_{\text{EdS}} = \frac{3H_0^2}{8\pi G_N} = 1.878 \times 10^{-29} h^2 \text{ gr/cm}^3,$$

where $H_0$ is the present Hubble constant, and $h$ its value in units of 100 km s$^{-1}$ Mpc$^{-1}$. Ω expresses the energy density in units of $\rho_{0,\text{crit}}$. Thus the present density $\rho_0$ is represented by

$$\Omega_0 = \frac{\rho_0}{\rho_{0,\text{crit}}}$$

Forms of energy:

- radiation: photons, highly relativistic particles $p_{\text{rad}} = \rho_{\text{rad}}/3$
normal and dark matter (non-relativistic, dilute) $p_{\text{matter}} \approx 0, \rho_{\text{matter}} > 0$

dark energy (cosmological constant) $p_{\text{vac}} = -\rho_{\text{vac}} < 0$

findings from Cosmic Microwave Background (COBE, WMAP, PLANCK)

- the universe is flat! $\Omega_0 \approx 1$. How to get this for any $k = \pm 1, 0$? $\Rightarrow$ inflation
The Cosmic Microwave Background

Cosmic black-body radiation of 3 °K  Penzias, Wilson NP 1978

The CMB fluctuation pattern: imprinted on the sky when the universe was just 380 000 years (after B.B.) old. Photons red-shifted by the expansion until the cannot ionize atoms (Hydrogen) any longer (snapshot of surface of last scattering). Smoot, Mather, NP 2006
Inflation

Need inflation! universe must blow up exponentially for a very short period, such that we see it to be flat! [switch on antigravity for very short period of time]
Solves:

- **Flatness problem** i.e. why $\Omega \approx 1$ (although unstable)?
- **Horizon problem** i.e. what does it mean **homogeneous** or **isotropic** for causally disconnected parts of the universe? Initial value problem required initial data on space-like plane. Data on space-like plane are causally uncorrelated! Finite age $t$ of universe, finite speed of light $c$: $D_{\text{Hor}} = c t$ what we can see at most?

CMB sky much larger than causally connected patch, but no shadow seen

- **Problem of fluctuations** magnitude, various components (dark matter, baryons, photons, neutrinos) related: same fractional perturbations $\Rightarrow$ quantum fluctuations at Planck time?

As we will see: - $\Omega = 1$ unstable only if not sufficient dark energy!
  - dark energy is provided by SM Higgs via $\kappa T_{\mu\nu}$
  - no extra cosmological constant $+\Lambda g_{\mu\nu}$ supplementing $G_{\mu\nu}$
  - i.e. all is standard GRT + SM (with minimal UV completion)
Flatness problem: observed today: (COBE, WMAP, PLANCK) \( \Omega_{\text{tot}} = 1.02 \pm 0.02 \)

Flat space unstable against perturbations: shown here initial data agreeing to 24 digits! CMB data say we are living in flat space!

\[
\frac{|\Omega_{\text{tot}}(t) - 1|_{\text{Pl}}}{|\Omega_{\text{tot}}(t) - 1|_0} = \frac{a^2(t_{\text{Pl}})}{a_0^2} \sim \frac{T_0^2}{T_{\text{Pl}}^2} \sim O(10^{60})
\]
Inflation at Work

Flatness, Causality, primordial Fluctuations ⇒ Solution: Guth 1980

Inflate the universe

Add an “Inflation term” to the r.h.s of the Friedmann equation, which dominates the very early universe blowing it up such that it looks flat afterwards

Need scalar field \( \phi(x) \equiv \text{“inflaton”} \): ⇒ inflation term \( \frac{8\pi}{3 M_{Pl}^2} \left( V(\phi) + \frac{1}{2} \dot{\phi}^2 \right) \)

Means: switch on strong anti-gravitation for an instant [sounds crazy]

Inflation: \( a(t) \propto e^{Ht} ; \quad H = H(t) \) Hubble constant = escape velocity \( v/\text{distance } D \)

\( \Rightarrow \quad N = \ln \frac{a_{\text{end}}}{a_{\text{initial}}} = H (t_e - t_i) \quad \text{automatic iff} \quad V(\phi) \gg \dot{\phi}^2 \)

“flattenization” by inflation: curvature term \( k/a^2(t) \sim k \exp(-2Ht) \to 0 \) \( (k = 0, \pm 1 \) the normalized curvature)
Energy-momentum tensor of SM $T_{\mu\nu} \equiv \Theta_{\mu\nu} = V(\phi) g_{\mu\nu} + \text{derivative terms}$

\[
\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi) ; \quad p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi)
\]

Equation of state: $w = \frac{p}{\rho} = \frac{\frac{1}{2} \dot{\phi}^2 - V(\phi)}{\frac{1}{2} \dot{\phi}^2 + V(\phi)}$

- small kinetic energy $\Rightarrow w \rightarrow -1$ is dark energy $p_\phi = -\rho_\phi < 0$!

indeed Planck (2013) finds $w = -1.13^{+0.13}_{-0.10}$.

Friedmann equation: $H^2 = \frac{8\pi G_N}{3} \left[ V(\phi) + \frac{1}{2} \dot{\phi}^2 \right]$

Field equation: $\ddot{\phi} + 3H \dot{\phi} = -V'(\phi)$

- Substitute energy density and pressure into Friedmann and fluid equation

- Expansion when potential term dominates

\[
\ddot{a} > 0 \iff p < -\frac{\rho}{3} \iff \dot{\phi}^2 < V(\phi)
\]
\[
N \equiv \ln \frac{a(t_{\text{end}})}{a(t_{\text{initial}})} = \int_{t_i}^{t_e} H(t) dt \simeq -\frac{8\pi}{M_{\text{Pl}}^2} \int_{\phi_i}^{\phi_e} \frac{V}{V'} d\phi
\]

- Need \( N \gtrsim 60 \) so called \( e \)-folds (CMB causal cone)

Key object of our interest: the Higgs potential

\[
V = \frac{m^2}{2} H^2 + \frac{\lambda}{24} H^4
\]

- Higgs mechanism
- When \( m^2 \) changes sign and \( \lambda \) stays positive \( \Rightarrow \) first order phase transition
- Vacuum jumps from \( v = 0 \) to \( v \neq 0 \)
The issue of quadratic divergences in the SM

Veltman 1978 [NP 1999] modulo small lighter fermion contributions, one-loop coefficient function $C_1$ is given by

$$
\delta m_H^2 = \frac{\Lambda^2_{\text{Pl}}}{16\pi^2} C_1 ; \quad C_1 = \frac{6}{v^2}(M_H^2 + M_Z^2 + 2M_W^2 - 4M_t^2) = 2 \lambda + \frac{3}{2} g'^2 + \frac{3}{2} g^2 - 12 y_t^2
$$

Key points:

- $C_1$ is universal and depends on dimensionless gauge, Yukawa and Higgs self-coupling only, the RGs of which are unambiguous. At two loops $C_2 \approx C_1$ numerically [Hamada et al 2013] stable under RCs!

- Couplings are running! $C_i = C_i(\mu)$

- the SM for the given running parameters makes a prediction for the bare effective mass parameter in the Higgs potential:
The phase transition in the SM. Left: the zero in $C_1$ and $C_2$ for $M_H = 125.9 \pm 0.4$ GeV. Right: shown is $X = \text{sign}(m^2_{\text{bare}}) \times \log_{10}(|m^2_{\text{bare}}|)$, which represents $m^2_{\text{bare}} = \text{sign}(m^2_{\text{bare}}) \times 10^X$.

Jump in vacuum energy: wrong sign and 50 orders of magnitude off $\Lambda_{\text{CMB}}$ !!!

$$\Delta V(\phi_0) = -\frac{m^2_{\text{eff}}v^2}{8} = -\frac{\lambda v^4}{24} \sim -9.6 \times 10^8 \text{ GeV}^4$$

⇒ one version of CC problem
in the broken phase $m_{\text{bare}}^2 = \frac{1}{2} m_H^2$, which is calculable!

- the coefficient $C_n(\mu)$ exhibits a zero, for $M_H = 126$ GeV at about $\mu_0 \sim 1.4 \times 10^{16}$ GeV, not far below $\mu = M_{\text{Planck}}$ !!!

- at the zero of the coefficient function the counterterm $\delta m^2 = m_{\text{bare}}^2 - m^2 = 0$ ($m$ the \text{MS} mass) vanishes and the bare mass changes sign

- this represents a phase transition (PT), which triggers the Higgs mechanism as well as cosmic inflation as $V(\phi) \gg \dot{\phi}^2$

- at the transition point $\mu_0$ we have $v_{\text{bare}} = v(\mu_0^2); \ m_{H,\text{bare}} = m_H(\mu_0^2)$, where $v(\mu^2)$ is the \text{MS} renormalized VEV

In any case at the zero of the coefficient function there is a phase transition, which corresponds to a restoration of the symmetry in the early universe.
Hot universe ⇒ finite temperature effects:

- finite temperature effective potential $V(\phi, T)$:

\[ V(\phi, T) = \frac{1}{2} \left( g_T T^2 - \mu^2 \right) \phi^2 + \frac{\lambda}{24} \phi^4 + \cdots \]

Usual assumption: Higgs is in the broken phase $\mu^2 > 0$ and $\mu \sim v$ at EW scale

EW phase transition is taking place when the universe is cooling down below the critical temperature $T_c = \sqrt{\mu^2 / g_T}$.

My scenario: above PT at $\mu_0$ SM in symmetric phase $-\mu^2 \rightarrow m^2 = (m_H^2 + \delta m_H^2)/2$

\[ m^2 \sim \delta m^2 \simeq \frac{M_{Pl}^2}{32\pi^2} C(\mu = M_{Pl}) \simeq (0.0295 M_{Pl})^2 , \ or \ m^2(M_{Pl})/M_{Pl}^2 \approx 0.87 \times 10^{-3} . \]

In fact with our value of $\mu_0$ almost no change of phase transition point by FT effects (see Plot below)
The Cosmological Constant in the SM

- in symmetric phase $SU(2)$ is a symmetry: $\Phi \rightarrow -U(\omega)\Phi$ and $\Phi^+\Phi$ singlet;

$$\langle 0|\Phi^+\Phi|0 \rangle = \frac{1}{2}\langle 0|H^2|0 \rangle \equiv \frac{1}{2} \Xi ; \quad \Xi = \frac{\Lambda_{Pl}^2}{16\pi^2} .$$

just Higgs self-loops

$$\langle H^2 \rangle = : ; \quad \langle H^4 \rangle = 3 \langle \langle H^2 \rangle \rangle^2 = :$$

$\Rightarrow$ vacuum energy $V(0) = \langle V(\phi) \rangle = \frac{m^2}{2} \Xi + \frac{A}{8} \Xi^2$; mass shift $m'^2 = m^2 + \frac{A}{2} \Xi$

$\square$ for our values of the $\overline{\text{MS}}$ input parameters

$$\mu_0 \approx 1.4 \times 10^{16} \text{ GeV} \rightarrow \mu'_0 \approx 7.7 \times 10^{14} \text{ GeV},$$

- potential of the fluctuation field $\Delta V(\phi) .$

$\Rightarrow$ quasi-constant vacuum density $V(0)$ representing the cosmological constant
fluctuation field eq. \( 3H\dot{\phi} \approx -(m^2 + \frac{1}{6} \phi^2) \phi \), \( \phi \) decays exponentially, must have been very large in the early phase of inflation

we adopt \( \phi_0 \approx 4.51 M_{\text{Pl}} \), big enough to provide sufficient inflation

Note: The Hubble constant in our scenario, in the symmetric phase, during the radiation dominated era is given by (Stefan-Boltzmann law)

\[
H = \ell \sqrt{\rho} \approx 1.66 (k_B T)^2 \sqrt{102.75} M_{\text{Pl}}^{-1}
\]

such that at Planck time (SM predicted)

\[
H_i \approx 16.83 M_{\text{Pl}}.
\]

\( V(0) \) very weakly scale dependent (running couplings): how to get ride of?

Note total energy density as a function of time

\[
\rho(t) = \rho_{0,\text{crit}} \left\{ \Omega_\Lambda + \Omega_{0,k} (a_0/a(t))^2 + \Omega_{0,\text{mat}} (a_0/a(t))^3 + \Omega_{0,\text{rad}} (a_0/a(t))^4 \right\}
\]
reflects a present-day snapshot. Cosmological constant is constant! Not quite!
intriguing structure again: the effective CC counterterm has a zero, which again is a point where renormalized and bare quantities are in agreement:

$$\rho_{\Lambda \text{ bare}} = \rho_{\Lambda \text{ ren}} + \frac{M_{\text{Pl}}^4}{(16\pi^2)^2} X(\mu)$$

with $X(\mu) \approx 2C(\mu) + \lambda(\mu)$ which has a zero close to the zero of $C(\mu)$ when $2C(\mu) = -\lambda(\mu)$.

Again we find a matching point between low energy and high energy world:

$$\rho_{\Lambda \text{ bare}} = \rho_{\Lambda \text{ ren}}$$

where memory of quartic Planck scale enhancement gets lost!

**Cosmological constant problem goodbye!**

Crucial point $X = 2C + \lambda = 5\lambda + 3g'^2 + 9g^2 - 24y_t^2$ acquires positive bosonic contribution and negative fermionic ones, with different scale dependence. $X$ can change a lot (pass a zero), while individual couplings are weakly scale dependent $y_t(M_Z)/y_t(M_{\text{Pl}}) \sim 2.7$ biggest, $g_1(M_Z)/g_1(M_{\text{Pl}}) \sim 0.76$ smallest.
Effect of finite temperature on the phase transition: bare \([m^2, C_1]\) vs effective from vacuum rearrangement \([m^2, C_1' = C_1 + \lambda]\) in case \(\mu_0\) sufficiently below \(M_{\text{Pl}}\) finite temperature effects affect little position of PT; vacuum rearrangement is more efficient:

\[
\mu_0 \approx 1.4 \times 10^{16} \text{ GeV} \rightarrow \mu'_0 \approx 7.7 \times 10^{14} \text{ GeV},
\]
SM predicts huge CC at $M_{Pl}$: $\rho_\phi \approx V(\phi) \sim 2.77 \, M_{Pl}^4 \sim 6.13 \times 10^{76} \, \text{GeV}^4$

how to tame it?

At Higgs transition: $m^\prime_2(\mu < \mu_0') < 0$ vacuum rearrangement of Higgs potential

How can it be: $V(0) + \Delta V \sim (0.002 \, \text{eV})^4$ ??? $\Rightarrow$ the zero of $X(\mu)$ makes $
\rho_{\Lambda, \text{bare}} = \rho_{\Lambda, \text{ren}}$ to be identified with observed value! (like the Higgs boson mass another free SM parameter to be fixed by experiment?). Naturally small, since $\Lambda_{Pl}^4$ term nullified at matching point.
Note: in principle, like the Higgs mass in the LEESM, also $\rho_{\Lambda\text{ren}}$ is expected to be a free parameter to be fixed by experiment. However, there is a big difference: inflation forces

$$\Omega_{\text{tot}} = \Omega_\Lambda + \Omega_{\text{mat}} + \Omega_{\text{rad}} = 1$$

and since $1 > \Omega_{\text{mat}}, \Omega_{\text{rad}} > 0$ actually $\Omega_\Lambda$ is fixed once we know dark matter, baryonic matter and the radiation density:

$$\Omega_\Lambda = 1 - \Omega_{\text{mat}} - \Omega_{\text{rad}}$$

So, where is the miracle to have CC of the magnitude corresponding to the critical density of a flat universe? Also this then is a prediction of the LEESM!

**effective Higgs mass square [left] and effective dark energy density [right] as predicted by SM**
The Higgs is the inflaton!

- after electroweak PT, at the zeros of quadratic and quartic “divergences”, memory of cutoff lost: renormalized low energy parameters match bare parameters

- in symmetric phase (early universe) bare effective mass and vacuum energy dramatically enhanced by quadratic and quartic cutoff effects

- slow-roll inflation condition $\frac{1}{2} \dot{\phi}^2 \ll V(\phi)$ satisfied

- Higgs potential provides huge dark energy in early universe which triggers inflation

The SM predicts dark energy and inflation!!!

dark energy and inflation are unavoidable consequences of the SM Higgs
(provided new physics does not disturb it substantially)
The evolution of the universe before the EW phase transition:

Inflation Times: the mass-, interaction- and kinetic-term of the bare Lagrangian in units of $M_{Pl}^4$ as a function of time.
The evolution of the universe before the EW phase transition:

Evolution until symmetry breakdown and vanishing of the CC. After inflation the scene is characterized by a free damped harmonic oscillator behavior.
Expansion before the Higgs transition: the FRW radius and its derivatives for $k = 1$ as a function of time, all in units of the Planck mass, i.e. for $M_{\text{Pl}} = 1$. Here LEESM versus Artwork.

Crucial: minimal leading UV completion by quadratic and quartic cut-off effects
Reheating and Baryogenesis

- inflation: exponential growth = exponential cooling

- reheating: pair created heavy states $X, \bar{X}$ in originally hot radiation dominated universe decay into lighter matter states which reheat the universe

- baryogenesis: $X$ particles produce particles of different baryon-number $B$ and/or different lepton-number $L$
“Baryogenesis in the Annihilation Drama of Matter”

\[ X, \bar{X} \text{−Decay: } \Rightarrow \{ \begin{align*} q : \bar{q} & = 1,000 \, 000 \, 001:1 \\ e^- : e^+ & = 1,000 \, 000 \, 001:1 \end{align*} \]
Sacharow condition for baryogenesis:

- small $\mathcal{B}$ is natural in LEESM scenario due to the close-by dimension 6 operators (Weinberg 1979, Buchmüller, Wyler 1985, Grzadkowski et al. 2010)

- suppressed by $(E/\Lambda_{\text{Pl}})^2$ in the low energy expansion. At the scale of the EW phase transition the Planck suppression factor is $1.3 \times 10^{-6}$.

- six possible four-fermion operators all $B - L$ conserving!

  - CP, out of equilibrium

$X$ is the Higgs! – “unknown” $X$ particles now known very heavy Higgs in symmetric phase of SM: Primordial Planck medium Higgses

All relevant properties known: mass, width, branching fractions, CP violation properties!
Stages: $k_B T > m_X \Rightarrow$ thermal equilibrium $X$ production and $X$ decay in balance

- $H \approx \Gamma_X$ and $k_B T < m_X \Rightarrow X$-production suppressed, out of equilibrium

- $H \rightarrow t\bar{t}, b\bar{b}, \cdots$ predominantly (largest Yukawa couplings)

- CP violating decays: $H^+ \rightarrow t\bar{d}$ [rate $\propto y_t y_d V_{td}$] $H^- \rightarrow b\bar{u}$ [rate $\propto y_b y_u V_{ub}$] and after EW phase transition: $t \rightarrow d e^+ \nu$ and $b \rightarrow u e^- \nu_e$ etc.

- Note: before Higgs mechanism bosonic triple couplings like $HWW, HZZ$ are absent (induced by SSB).

- Preheating absent! Reheating via $\phi \rightarrow f f\bar{f}$ while all bosonic decays heavily suppressed (could obstruct reheating)!
Higgses decay into heavy quarks afterwards decaying into light ones

Note: large CP violation in $V_{td}$ and $V_{ub}$

Seems we are all descendants of four heavy Higgses via top-bottom stuff!

Baryogenesis most likely a “SM + dim 6 operators” effect!

Unlikely: $B + L$ violating instanton effects $\propto \exp \left[ -\frac{8\pi^2}{g^2(\mu)} + \cdots \right] \approx e^{-315.8}$ too small.

⇒ observed baryon asymmetry $\eta_B \sim 10^{-10}$ cannot be a SM prediction, requires unknown $B$ violating coupling. But order of magnitude should be explainable.
Conclusion

- The LHC made tremendous step forward in SM physics and cosmology: the discovery of the Higgs boson, which fills the vacuum of the universe first with dark energy and latter with the Higgs condensate, thereby giving mass to quarks leptons and the weak gauge bosons, but also drives inflation, reheating and all that.

- Higgs not just the Higgs: its mass $M_H = 125.9 \pm 0.4 \text{ GeV}$ has a very peculiar value!! tailored such that strange exotic phenomena like inflation and likely also the continued accelerated expansion of the universe are a direct consequence of LEESM physics.

- ATLAS and CMS results may “revolution” particle physics in an unexpected way, namely showing that the SM has higher self-consistency (conspiracy) than expected and previous arguments for the existence of new physics may turn out not to be compelling.
SM as a low energy effective theory of some cutoff system at $M_{Pl}$ consolidated; crucial point $M_{Pl} >>>>> ...$ from what we can see!

change in paradigm:

Natural scenario understands the SM as the “true world” seen from far away

Methodological approach known from investigating condensed matter systems. (QFT as long distance phenomenon, critical phenomena)
Wilson 1971, NP 1982

cut-offs in particle physics are important to understand early cosmology, i.e. inflation, reheating, baryogenesis and all that.

the LEESM scenario, for the given now known parameters, the SM predicts dark energy and inflation, i.e. they are unavoidable

also note that the LEESM scenario is stable under radiative corrections (Hamada et al.)
Paths to Physics at the Planck Scale

M–theory (Brain world) candidate TOE exhibits intrinsic cut-off

↓

STRINGS

↓

SUGRA

↓

SUSY–GUT

↓

SUSY

Energy scale

Planck scale

E–theory (Real world)

“chaotic” system with intrinsic cut–off

top-down approach

bottom-up approach

Planck scale

parallel

$10^{19}$ GeV

$10^{16}$ GeV

1 TeV

QFT

“??SM??”

SM

symmetry high → → → symmetry low

?? symmetry ≡ blindness for details ??

the closer you look the more you can see when approaching the cut-off scale

soft SB only

SB soft at low/hard at high energies
Concluding remarks

- Conspiracy between SM couplings the new challenge
- Very delicate on initial values as we run over 16 orders of magnitude from the EW 250 GeV scale up to the Planck scale!
- Running couplings likely have dramatic impact on cosmology! The existence of the world in question?
- LHC and ILC will dramatically improve on Higgs self-coupling $\lambda$ (Higgs factory) as well as on top Yukawa $y_t$ ($t\bar{t}$ factory)
- for running $\alpha_{\text{em}}$ and $\sin^2\Theta_{\text{eff}} \leftrightarrow g_1$ and $g_2$ need more information from low energy hadron production facilities, improving QCD predictions and EW radiative corrections! Lattice QCD will play key role for sure.
Last but not least: today’s dark energy = relict Higgs vacuum energy?

WHAT IS DARK ENERGY?
Well, the simple answer is that we don’t know.
It seems to contradict many of our understandings about the way the universe works.

... Something from Nothing?
It sounds rather strange that we have no firm idea about what makes up 74% of the universe.
Afterglow Light Pattern 380,000 yrs.

Dark Ages

Development of Galaxies, Planets, etc.

Inflation

Quantum Fluctuations

1st Stars about 400 million yrs.

Big Bang Expansion

13.7 billion years

the Higgs at work
Thanks for your attention!

References:

“The Standard model as a low-energy effective theory: what is triggering the Higgs mechanism?,”

“The hierarchy problem of the electroweak Standard Model revisited,”

“Higgs inflation and the cosmological constant,”

Krakow/Durham Lectures:
http://www-com.physik.hu-berlin.de/~fjeger/SMcosmology.html
My view is anti-infinity!

- Infinities in Physics are the result of idealizations and show up as singularities in formalisms or models.

- A closer look usually reveals infinities to parametrize our ignorance or mark the limitations of our understanding or knowledge.

- My talk is about taming the infinities we encounter in the theory of elementary particles i.e. quantum field theories.

- I discuss a scenario of the Standard Model (SM) of elementary particles in which ultraviolet singularities which plague the precise definition as well as concrete calculations in quantum field theories are associated with a physical cutoff, represented by the Planck length.

- Thus in my talk infinities are replaced by eventually very large but finite
numbers, and I will show that sometimes such huge effects are needed in describing reality. Our example is inflation of the early universe.

**Limiting scales from the basic fundamental constants: \( c, \hbar, G_N \)**

\[ \Rightarrow \text{Relativity and Quantum physics married with Gravity yield} \]

Planck length: \( \ell_{\text{Pl}} = \sqrt{\frac{\hbar G_N}{c^3}} = 1.616252(81) \times 10^{-33} \text{ cm} \)

Planck time: \( t_{\text{Pl}} = \ell_{\text{Pl}}/c = 5.4 \times 10^{-44} \text{ sec} \)

Planck (energy) scale: \( M_{\text{Pl}} = \sqrt{c \hbar G_N} = 1.22 \times 10^{19} \text{ GeV} \)

Planck temperature: \( \frac{M_{\text{Pl}} c^2}{k_B} = \sqrt{\frac{\hbar c^5}{G_N k_B^2}} = 1.416786(71) \times 10^{32} \text{ K} \)

- shortest distance \( \ell_{\text{Pl}} \) and beginning of time \( t_{\text{Pl}} \)
- highest energy \( E_{\text{Pl}} = \Lambda_{\text{Pl}} \equiv M_{\text{Pl}} \) and temperature \( T_{\text{Pl}} \)
What about the hierarchy problem?

- In the Higgs phase:

There is no hierarchy problem in the SM!

In the broken phase, characterized by the non-vanishing Higgs field vacuum expectation value (VEV) $v(\mu)$, all the masses are determined by the well known mass-coupling relations

$$m^2_W(\mu) = \frac{1}{4} g^2(\mu) v^2(\mu) ; \quad m^2_Z(\mu) = \frac{1}{4} (g^2(\mu) + g'^2(\mu)) v^2(\mu) ;$$
$$m^2_f(\mu) = \frac{1}{2} y_f^2(\mu) v^2(\mu) ; \quad m^2_H(\mu) = \frac{1}{3} \lambda(\mu) v^2(\mu) .$$

- Higgs mass cannot by much heavier than the other heavier particles!

- Extreme point of view: all particles have masses $O(M_{Pl})$ i.e. $v = O(M_{Pl})$. 

F. Jegerlehner – Seminar, DESY Zeuthen, Apr. 23, 2015
This would mean the symmetry is not recovered at the high scale, notion of SSB obsolete! Of course this makes no sense.

- Higgs VEV $v$ is an order parameter resulting form long range collective behavior, can be as small as we like.

Prototype: magnetization in a ferromagnetic spin system

\[ M = M(T) \] and actually $M(T) \equiv 0$ for $T > T_c$ furthermore $M(T) \to 0$ as $T \to T_c$

- $v/M_{Pl} \ll 1$ just means we are close to a 2\textsuperscript{nd} order phase transition point.
In the symmetric phase at very high energy we see the bare system:

the Higgs field is a collective field exhibiting an effective mass generated by radiative effects

$\begin{align*} m_{\text{bare}}^2 &\approx \delta m^2 \text{ at } M_{\text{Pl}} \\ \text{eliminates fine-tuning problem at all scales!} \end{align*}$

Many example in condensed matter systems.
What rules the $\beta$-functions:

Naively:

- $U(1)_Y$ screening (IR free), $SU(2)_L$, $SU(3)_C$ antiscreening (UV free) [asymptotic freedom (AF)]
  
  Right – as expected

- Yukawa and Higgs: screening (IR free, like QED)
  
  Wrong!!! – transmutation from IR free to AF

At the $Z$ boson mass scale: $g_1 \approx 0.350$, $g_2 \approx 0.653$, $g_3 \approx 1.220$, $y_t \approx 0.935$ and $\lambda \approx 0.796$

Leading (one-loop) $\beta$-functions at $\mu = M_Z$: $[c = \frac{1}{16\pi^2}]$

- gauge couplings:

  $$\beta_1 = \frac{41}{6} g_1^3 c \approx 0.00185 ; \quad \beta_2 = -\frac{19}{6} g_2^2 c \approx -0.00558 ; \quad \beta_3 = -7 g_3^3 c \approx -0.08045 ,$$
\* top Yukawa coupling:

\[
\beta_{y_t} = \left( \frac{9}{2} y_t^3 - \frac{17}{12} g_1^2 y_t - \frac{9}{4} g_2^2 y_t - 8 g_3^2 y_t \right) c \\
\approx 0.02328 - 0.00103 - 0.00568 - 0.07046 \\
\approx -0.05389
\]

not only depends on \(y_t\), but also on mixed terms with the gauge couplings \(g', g\) and \(g_3\) which have a negative sign.

In fact the QCD correction is the leading contribution and determines the behavior. Notice the critical balance between the dominant strong and the top Yukawa couplings: QCD dominance requires \(g_3 > \frac{3}{4} y_t\) in the gaugeless limit.

\* the Higgs self-coupling

\[
\beta_{\lambda} = \left( 4 \lambda^2 - 3 g_1^2 \lambda - 9 \lambda g_2^2 + 12 y_t^2 \lambda + \frac{9}{4} g_1^4 + \frac{9}{2} g_1^2 g_2^2 + \frac{27}{4} g_2^4 - 36 y_t^4 \right) c \\
\approx 0.01606 - 0.00185 - 0.01935 + 0.05287 + 0.00021 + 0.00149 + 0.00777 - 0.17407 \\
\approx -0.11687
\]
dominated by $y_t$ contribution and not by $\lambda$ coupling itself. At leading order it is not subject to QCD corrections. Here, the $y_t$ dominance condition reads $\lambda < \frac{3(\sqrt{5}-1)}{2} y_t^2$ in the gaugeless limit.

- running top Yukawa QCD takes over: IR free $\Rightarrow$ UV free
- running Higgs self-coupling top Yukawa takes over: IR free $\Rightarrow$ UV free
Including all known RG coefficients (EW up incl 3–loop, QCD up incl 4–loop)

- except from $\beta_\lambda$, which exhibits a zero at about $\mu_\lambda \sim 10^{17}$ GeV, all other $\beta$-functions do not exhibit a zero in the range from $\mu = M_Z$ to $\mu = M_{Pl}$.

- so apart form the $U(1)_Y$ coupling $g_1$, which increases only moderately, all other couplings decrease and perturbation theory is in good condition.

- at $\mu = M_{Pl}$ gauge couplings are all close to $g_i \sim 0.5$, while $y_t \sim 0.35$ and $\sqrt{\lambda} \sim 0.36$.

- effective masses moderately increase (largest for $m_Z$ by factor $2.8$): scale like $m(\kappa)/\kappa$ as $\kappa = \mu'/\mu \to \infty$, i.e. mass effect get irrelevant as expected at high energies.
Non-zero dimensional $\overline{\text{MS}}$ running parameters: $m, v = \sqrt{6/\lambda} m$ and $G_F = 1/(\sqrt{2} v^2)$. Error bands include SM parameter uncertainties and a Higgs mass range $125.5 \pm 1.5$ GeV which essentially determines the widths of the bands. Note that $v$ increases by a factor about 2.5 before it jumps to zero at the transition point.
Gaussianity of Inflation

- The PLANCK mission power spectrum:
Planck data are consistent with Gaussian primordial fluctuations. There is no evidence for primordial Non Gaussian (NG) fluctuations in shapes (local, equilateral and orthogonal).

Shape non-linearity parameters:

\[
\begin{align*}
    f_{NL}^{\text{loc}} &= 2.7 \pm 5.8, \\
    f_{NL}^{\text{eq}} &= -42 \pm 75, \\
    f_{NL}^{\text{orth}} &= -25 \pm 39
\end{align*}
\]

(68% CL statistical)

- The scenario suggested by the present analysis is a Gaussian potential with small anharmonic perturbations, since \( m_{\text{bare}}^2 \) is predicted to be large while \( \lambda_{\text{bare}} \) remains small. Also the bare kinetic term is logarithmically “unrenormalized” only.

- Numbers depend sensibly on what \( \lambda(M_H) \) and \( y_t(M_t) \) are (LHC & future ILC!)
The SM renormalization group equations

References:


4–loop QCD: Ritbergen, Vermaseren, Larin 1997, Czakon 2005


3–loop QCD OS vs $\overline{\text{MS}}$ mass: Chetyrkin, Steinhauser 2000, Melnikov, Ritbergen 2000

$\beta_{g}^{(3)}, \beta_{g}^{(3)}$: Mihaila, Salomon, Steinhauser 2012, Bednyakov, Pikeln, Velizhanin 2012

$\beta_{y_{t}}^{(3)}, \beta_{\lambda}^{(3)}$: Chetyrkin, Zoller 2012/2013, Bednyakov, Pikeln, Velizhanin 2012/2013
Matching conditions for $\overline{\text{MS}}$ parameters in terms of physical parameters

References:

a) Higgs boson mass vs Higgs self-coupling:
The one-loop corrections give the dominant contribution in the matching relations Fleischer, F.J. 1981, Sirlin, Zucchini 1986

Two-loop results are partially known F.J., Kalmykov, Veretin 2002/.../2004.

b) Top quark mass vs top Yukawa coupling:
The QCD corrections
in the gaugeless-limit Martin 2005

Comparison of SM coupling evolution

Renormalization of the SM gauge couplings $g_1 = \sqrt{5/3} g_Y$, $g_2$, $g_3$, of the top, bottom and $\tau$ couplings ($y_t$, $y_b$, $y_\tau$), of the Higgs quartic coupling $\lambda$ and of the Higgs mass parameter $m$. We include two-loop thresholds at the weak scale and three-loop RG equations. The thickness indicates the $\pm 1\sigma$ uncertainties.
Some additional comments

- main unsolved problem: **dark matter** definitely requires a SM extension, maybe Majorana neutrinos, axions, $SU(4)$ bound states (hyper mesons)?

- **baryon asymmetry**
  detailed analysis missing as yet, must be reanalyzed in the new scenario!

- does vacuum stability and the Higgs transition point persist when input parameters are updated and additional higher order effects are included as my analysis suggests or do we still need new physics to “stabilize” the picture?

  such scenario essentially rules out SUSY, GUTs and Strings altogether! Also a 4th fermion family is definitely excluded, even so this would not affect renormalizability, but it would spoil SM running couplings pattern.
new physic (beyond cold dark matter etc.) still is expected to exist; however, even if needed help to stabilize vacuum, it should not deteriorate the gross features of the SM including MFV scenario. Axions still can help to solve strong CP problem and provide dark matter.

Keep in mind: the Higgs mass miraculously turns out to have a value as it was expected form vacuum stability. It looks like a tricky conspiracy with other couplings to reach this “purpose”. If it misses to stabilize the vacuum, why is it so close to stability?

Why not simple although it may well be more complicated?
A lot yet to be understood!

At least we now know why the top has to be so heavy together with the Higgs so “light” given the gauge couplings