Facets of Chiral Perturbation Theory

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Goal

systematic and quantitative treatment
of the Standard Model at low energies ($E < 1$ GeV)

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- Lattice Field Theory

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systematic and quantitative treatment of the Standard Model at low energies ($E < 1$ GeV)

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main objectives

- understand physics of the SM at low energies
- look for evidence of new physics

$E < 1$ GeV:

- strong-coupling regime of QCD $\longrightarrow$
  not accessible in standard perturbation theory

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key concept for EFT: **approximate chiral symmetry of QCD**

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}{\rm tr}(G_{\mu\nu}G^{\mu\nu}) + \sum_{f=1}^{6} \overline{q}_f \left(i\gamma^\mu D_\mu - m_f \mathbb{1}_c\right) q_f$$

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for $m_f = 0$: chiral components can be rotated separately

$$q_{fL} = \frac{1}{2}(1-\gamma_5)q_f\,, \qquad q_{fR} = \frac{1}{2}(1+\gamma_5)q_f$$

$\longrightarrow$ chiral symmetry $SU(n_F)_L \times SU(n_F)_R \times U(1)_V$

$m_f = 0$:

- very good approximation for $n_F = 2 \quad (u, d)$
- reasonable approximation for $n_F = 3 \quad (u, d, s)$

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in contrast to isospin $SU(2)$ or flavour $SU(3)$:

no sign of chiral symmetry in hadron spectrum

many other arguments in favour of

spontaneous breaking of chiral symmetry

$$SU(n_F)_L \times SU(n_F)_R \times U(1)_V \quad \longrightarrow \quad SU(n_F)_V \times U(1)_V$$

Goldstone theorem:

$\exists\ n_F^2 - 1$ massless (for $m_f = 0$) Goldstone bosons

Goldstone fields parametrize $SU(n_F)_L \times SU(n_F)_R \;/\; SU(n_F)_V$

n_F	$n_F^2 - 1$	Goldstone bosons
2	3	π
3	8	π, K, η

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nonlinear realization of chiral symmetry $\longrightarrow$

effective Lagrangian necessarily nonpolynomial

consequence:

EFT nonrenormalizable QFT: Chiral Perturbation Theory (CHPT)

Weinberg, Gasser, Leutwyler,...

nevertheless: CHPT fully renormalized QFT
(to next-to-next-to-leading order)

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systematic approach for low-energy hadron physics

- most advanced in meson sector (up to 2 loops)
- also single-baryon sector and few-nucleon systems
- electroweak interactions can be included

Effective chiral Lagrangian (meson sector)

$\mathcal{L}_{\text{chiral order}}$ (# of LECs)	loop order
$\mathcal{L}_{p^2}(2) + \mathcal{L}^{\text{odd}}_{p^4}(0) + \mathcal{L}^{\Delta S=1}_{G_F p^2}(2) + \mathcal{L}^{\text{emweak}}_{G_8 e^2 p^0}(1)$ $+ \mathcal{L}^{\text{em}}_{e^2 p^0}(1) + \mathcal{L}^{\text{leptons}}_{\text{kin}}(0)$	$L = 0$
$+ \mathcal{L}_{p^4}(10) + \mathcal{L}^{\text{odd}}_{p^6}(23) + \mathcal{L}^{\Delta S=1}_{G_8 p^4}(22) + \mathcal{L}^{\Delta S=1}_{G_{27} p^4}(28)$ $+ \mathcal{L}^{\text{emweak}}_{G_8 e^2 p^2}(14) + \mathcal{L}^{\text{em}}_{e^2 p^2}(13) + \mathcal{L}^{\text{leptons}}_{e^2 p^2}(5)$	$L \leq 1$
$+ \mathcal{L}_{p^6}(90)$	$L \leq 2$

LECs: low-energy constants $\equiv$ coupling constants of CHPT

in red: Lagrangians relevant for nonleptonic K decays

Nonleptonic kaon decays

dominant decays: $K \to 2\pi, 3\pi$

LO — Cronin

NLO — Kambor, Missimer, Wyler

NLO + isospin violation + rad. corrs. — Cirigliano, E., Neufeld, Pich; Bijnens, Borg

$\longrightarrow$ LO couplings G_8, G_{27} well known

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in red: LO Lagrangian for nonleptonic K decays

in blue: NLO Lagrangian — " —

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problem: 22 (octet) + 28 (27-plet) LECs

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Theorists' favourite nonleptonic decays

$K_S \to \gamma\gamma$, $K_L \to \pi^0\gamma\gamma$ [, $K_S \to \pi^0\pi^0\gamma\gamma$]

no LECs at all at NLO !

Status at $O(G_F p^4)$

$K_S \to \gamma\gamma$ D'Ambrosio, Espriu; Goity

$K_L \to \pi^0\gamma\gamma$ E., Pich, de Rafael; Cappiello, D'Ambrosio

$K_S \to \pi^0\pi^0\gamma\gamma$ Funck, Kambor

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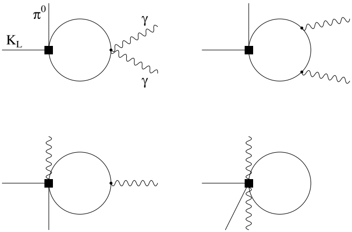

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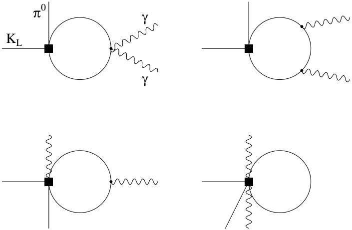
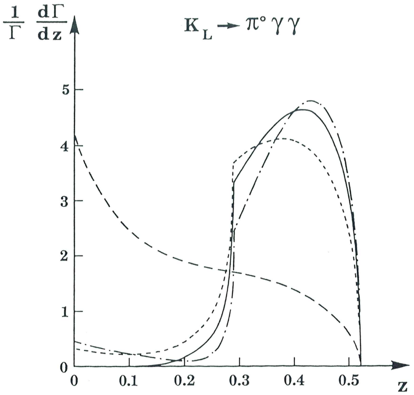


pre-CHPT: $K_L \to \pi^0\gamma\gamma$ vector-meson dominated

compare 2-photon spectra

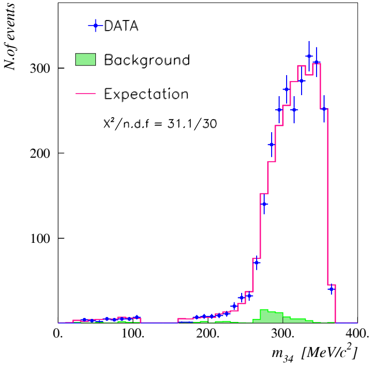


Normalized decay distribution in $z = M_{\gamma\gamma}^2/M_K^2$

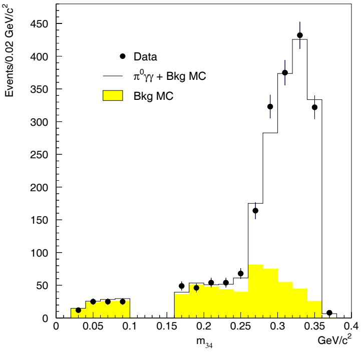
E., Pich, de Rafael

leading order [$O(p^4)$]	**full curve**
pure vector resonance exchange	**dashed curve**
[$a_V = -.32$	**dotted curve]**

$K_L \to \pi^0\gamma\gamma$ decay distribution in $m_{34} = M_{\gamma\gamma}$



NA48 (2002)



KTeV (2008)

higher-order corrections (starting at NNLO $= O(G_F p^6)$)

- rescattering (unitarity) corrections largely model independent

  Cappiello, D'Ambrosio, Miragliuolo; Cohen, E., Pich; Kambor, Holstein

$K_S \to \gamma\gamma$	"trivial" in terms of $K \to 2\pi$ rate
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- resonance contributions

  Cohen, E., Pich; D'Ambrosio, Portolés; Buchalla, D'Ambrosio, Isidori

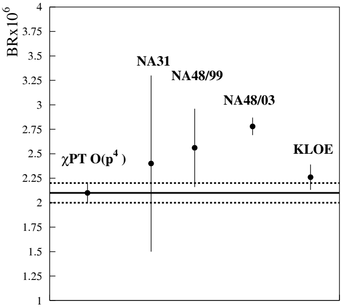
$K_S \to \gamma\gamma$	small (vector mesons cannot contribute)
$K_L \to \pi^0\gamma\gamma$	vector meson contribution model dependent
	good approximation: single parameter a_V

$K_S \to \gamma\gamma$

puzzling result of NA48 (2003):
rate substantially bigger than
$O(p^4)$ result

KLOE (2008):
$B(K_S \to \gamma\gamma) = 2.26(12)(06) \times 10^{-6}$

$\longrightarrow$ perfect agreement



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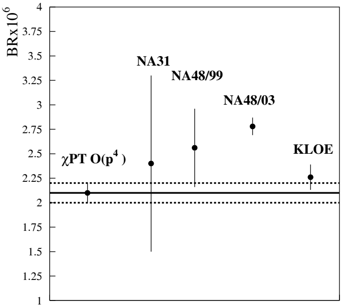

not a good idea:

PDG averages NA48/03 and KLOE $\longrightarrow$
$B(K_S \to \gamma\gamma) = 2.63(17) \times 10^{-6}$
??

$K_L \to \pi^0\gamma\gamma$

for reasonable values of a_V:

- pion loop dominates 2γ-spectrum
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$\Longrightarrow$ excellent agreement between theory and experiment

$$B(K_L \to \pi^0\gamma\gamma)\cdot 10^6 = \begin{cases} 1.27 \pm 0.04 \pm 0.01 & \text{NA48 (2002)} \\ 1.28 \pm 0.06 \pm 0.01 & \text{KTeV (2008)} \\ 1.273 \pm 0.033 & \text{PDG (2012)} \end{cases}$$

$$a_V = -0.43 \pm 0.06 \qquad \text{PDG (2012)}$$

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important consequence

CP-conserving contribution $K_L \to \pi^0\gamma^*\gamma^* \to \pi^0 e^+ e^-$ negligible in comparison with CP-violating amplitudes

Precision physics: $\Gamma(P \to e\nu_e)/\Gamma(P \to \mu\nu_\mu) \quad (P = \pi, K)$

$V - A$ structure of charged currents $\longrightarrow$

$R_{e/\mu}^{(P)} = \Gamma(P \to e\nu_e[\gamma])/\Gamma(P \to \mu\nu_\mu[\gamma])$ helicity suppressed

$\longrightarrow$ sensitive probe for new physics

(charged Higgs exchange, violation of lepton universality, . . .)

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	PDG 2012	Marciano, Sirlin 1993	Finkemeier 1996
$R^{(\pi)}_{e/\mu} \cdot 10^4$	1.230 ± 0.004	1.2352 ± 0.0005	1.2354 ± 0.0002
$R^{(K)}_{e/\mu} \cdot 10^5$	2.488 ± 0.012		2.472 ± 0.001

$P \to \ell \nu_\ell$ to $O(p^2)$

$$T_\ell^{p^2} = -2\, i\, G_F\, V_{ud}\, F\, m_\ell\, \bar{u}_L(p_\nu)\, v(p_\ell) \qquad (P = \pi)$$

$$O(p^{2n}), e = 0: \qquad F \to F_P^{(2n)} \quad \longrightarrow \quad R_{e/\mu}^P = \frac{m_e^2}{m_\mu^2} \left(\frac{M_P^2 - m_e^2}{M_P^2 - m_\mu^2} \right)^2$$

$\longrightarrow$ nontrivial corrections (hadronic structure) only for $e \neq 0$

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amplitudes of $O(e^2 p^2)$

one loop $\longrightarrow$ $T_\ell^{e^2 p^2}$ corresponds to point-like approximation

Kinoshita; Marciano, Sirlin
Knecht, Neufeld, Rupertsberger, Talavera

$O(e^2p^4)$ Cirigliano, Rosell

(up to) 2-loop diagrams $\longrightarrow$ involve meson structure

loop integration divergent $\longrightarrow$ $T_\ell^{e^2p^4}$ contains counterterm

Cirigliano, Rosell: counterterm fixed by

matching form factors with large-N_c QCD

- inclusion of real photon corrections $+$
  summation of leading logs $\alpha^n \log^n(m_\mu/m_e)$ (Marciano, Sirlin)

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[asymptotic behaviour of form factors incompatible with QCD]

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most recent experimental result for $R_{e/\mu}^{(K)}$

	NA62 (2013)	Cirigliano, Rosell
$R_{e/\mu}^{(K)} \cdot 10^5$	2.488 ± 0.010	2.477 ± 0.001

Low-energy constants and lattice QCD

Motto (Laurent Lellouch, John F. Kennedy)

Ask not what CHPT can do for the lattice,

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- CHPT $\longrightarrow$ lattice
  (chiral) extrapolation to physical quark (meson) masses
  still useful, but less needed than 5 years ago
- lattice $\longrightarrow$ CHPT
  determination of LECs (FLAG, ...)
  especially welcome for LECs multiplying quark mass terms
  advantage of lattice simulations compared to phenomenology:
  quark (and therefore meson) masses can be tuned

illustrative example:

chiral $SU(3)$ Lagrangian (strong interactions)

$$\mathcal{L}_{p^2}(2) = \frac{F_0^2}{4}\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U\rangle$$

$$\mathcal{L}_{p^4}(10) = \cdots + L_4\langle D_\mu U D^\mu U^\dagger\rangle\langle\chi U^\dagger + \chi^\dagger U\rangle + \ldots$$

$\langle\ldots\rangle$ flavour trace

$F_0 = \lim_{m_u,m_d,m_s\to 0} F_\pi$, $\chi = 2B_0\mathcal{M}_q$ ($B_0 \sim$ quark condensate)

$U = \mathbb{1} +$ meson fields

gauge-covariant derivative $D_\mu U$ (contains A_μ, $W_\mu^\pm$)

$$\mathcal{L}_{p^2}(2) + \mathcal{L}_{p^4}(10) = \frac{1}{4}\langle D_\mu U D^\mu U^\dagger \rangle \left[F_0^2 + 8 L_4 \left(2\mathring{M}_K^2 + \mathring{M}_\pi^2 \right) \right] + \ldots$$

$\mathring{M}_P$ lowest-order meson mass

$F_\pi^2/(16 M_K^2) = 2 \times 10^{-3} \quad \sim \quad$ typical size of NLO LEC

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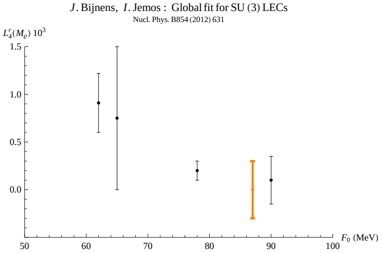
consequences

- strong anticorrelation between F_0 and L_4 in global fits

  Bijnens, Jemos



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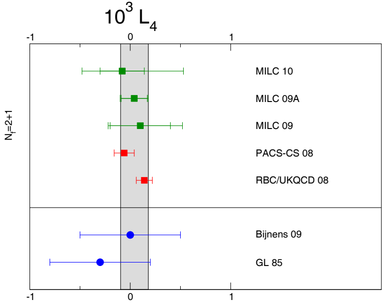
  FLAG (2011): published lattice determinations (for $L_4^r(M_\rho)$)



comparison between $SU(2)$ and $SU(3)$

$F = \lim_{m_u, m_d \to 0} F_\pi$

$$F_0 = F - F^{-1}\left\{\left(2M_K^2 - M_\pi^2\right)\left(4L_4^r(\mu) + \frac{1}{64\pi^2}\log\mu^2/M_K^2\right) + \frac{M_\pi^2}{64\pi^2}\right\} + O(p^6)$$

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"paramagnetic" inequality (Descotes-Genon, Girlanda, Stern)

$$F_0 < F \quad \longrightarrow \quad L_4^r(M_\rho) > -0.4 \times 10^{-3}$$

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FLAG (2011): $F = F_\pi/1.073(15)$

$\longrightarrow$ linear relation between F_0 and L_4

suggestion:

determine F_0, L_4 from $SU(3)$ lattice data for F_π

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essential: CHPT to NNLO $= O(p^6)$ (Amoros, Bijnens, Talavera)

drawback: only available in numerical form

suggestion:

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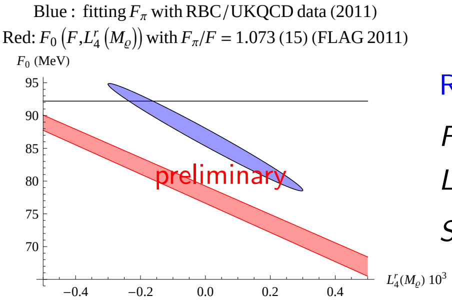

E., Masjuan, Neufeld

RBC/UKQCD data (2011)

$F_0 = (86.7 \pm 8.2)$ MeV

$L_4^r(M_\rho) = (0.0 \pm 0.3) \cdot 10^{-3}$

$SU(2)$ constraint ?

Carbogenesis: the Hoyle state

almost all carbon produced in stellar nucleosynthesis via

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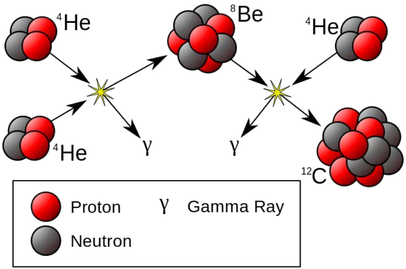

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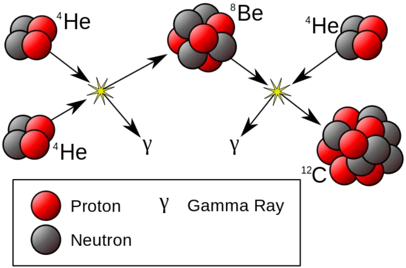

Hoyle (1954): to explain observed carbon abundance

$\longrightarrow$ $\exists$ excited 0^+ state of ^{12}C near ^{8}Be-α threshold

observed soon afterwards

properties of Hoyle state

$$\epsilon = 379.47(18)\ \text{keV} \qquad \text{(above } 3\alpha \text{ threshold)}$$

$$\Gamma_{\text{tot}} = 8.3(1.0)\ \text{eV}, \quad \Gamma_\gamma = 3.7(5)\ \text{meV}$$

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however:

more interesting issue:

dependence of ϵ on fundamental parameters of strong and electromagnetic interactions one-parameter (p) nuclear cluster model Oberhummer et al.

tolerances

$\Delta p/p \lesssim 0.5\%$ $\Delta F_{\mathrm{Coulomb}}/F_{\mathrm{Coulomb}} \lesssim 4\%$

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more recent development

nuclear lattice simulations (Muller, Lee, Borasoy, ...)

lattice dofs: nucleons (not quarks!) and pions

Monte-Carlo techniques

$\longrightarrow$ energies of low-lying states of ^{12}C (in MeV)

	0_1^+	$2_1^+(E^+)$	0_2^+
LO	$-96(2)$	$-94(2)$	$-89(2)$
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- we have gone some way in understanding the SM at low energies
- on the other hand: we have not found evidence for new physics
- but neither has the LHC !