Flavour Symmetries and Neutrino Oscillations

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Plan

1. Summary of data on neutrino oscillations

2. How to extend the SM to incorporate neutrino masses?

3. Why neutrino masses are so small?
-- Purely Dirac neutrino masses
-- Neutrino masses from D=5 operator
-- The see-saw mechanism
-- Tests of D=5 operator

4. Why lepton mixing angles are different
from those of the quark sector?
-- Flavour symmetries

Some conventions

$$-\frac{g}{\sqrt{2}} W_\mu^- \bar{l}_L \gamma^\mu U_{PMNS} \nu_L$$

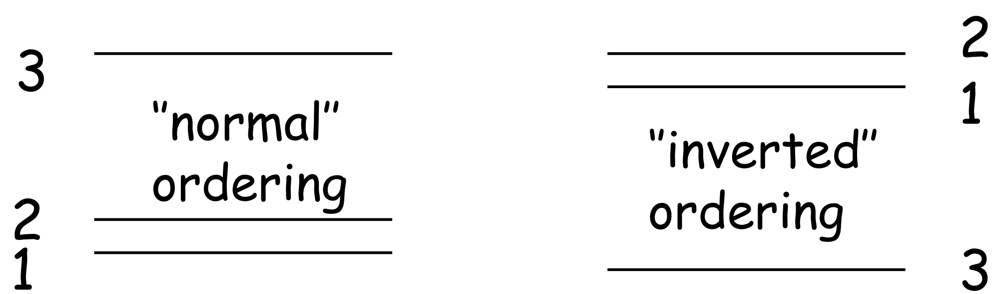
neutrino interaction eigenstates $\nu_f = \sum_{i=1}^{3} U_{fi} \nu_i \quad (f = e, \mu, \tau)$ neutrino mass eigenstates

$m_1 < m_2$

$\Delta m_{21}^2 < |\Delta m_{32}^2|, |\Delta m_{31}^2|$ $\quad [\Delta m_{ij}^2 \equiv m_i^2 - m_j^2]$

i.e. 1 and 2 are, by definition, the closest levels

two possibilities:



U_{PMNS} is a 3 x 3 unitary matrix

three mixing angles $\vartheta_{12}, \quad \vartheta_{13}, \quad \vartheta_{23}$

three phases (in the most general case) δ $\quad \underbrace{\alpha, \quad \beta}_{\text{do not enter}}$ $P_{ff'} = P(\nu_f \to \nu_{f'})$

oscillations can only test 6 combinations

$\Delta m_{21}^2, \Delta m_{32}^2, \quad \vartheta_{12}, \quad \vartheta_{13}, \quad \vartheta_{23} \quad \delta$

Summary of data

Summary of unkowns

$m_\nu < 2.2\, eV \quad (95\%\ CL)$ (lab)

$\sum_i m_i < 0.2 \div 1 \quad eV$ (cosmo)

absolute neutrino mass scale is unknown

$$\Delta m^2_{atm} \equiv \begin{cases} \Delta m^2_{31} = \left(2.47^{+0.069}_{-0.067}\right) \times 10^{-3}\ eV^2\ [\mathrm{NO}] \\ \Delta m^2_{32} = -\left(2.43^{+0.042}_{-0.065}\right) \times 10^{-3}\ eV^2\ [\mathrm{IO}] \end{cases}$$

[ordering
(either normal or inverted)
not known]

$$\Delta m^2_{sol} \equiv \Delta m^2_{21} = (7.50 \pm 0.185) \times 10^{-5} \quad eV^2$$

$$\sin^2 \vartheta_{13} = 0.023 \pm 0.0023$$

10σ away from 0

δ, α, β unknown

$$\sin^2 \vartheta_{23} = 0.41^{+0.037}_{-0.025} \oplus 0.59^{+0.021}_{-0.022}$$

hint for non maximal ϑ_{23} ?

[CP violation in lepton sector not yet established]

$$\sin^2 \vartheta_{12} = 0.30 \pm 0.013$$

[Gonzalez-Garcia, Maltoni, Salvado, Schwetz 1209.3023]

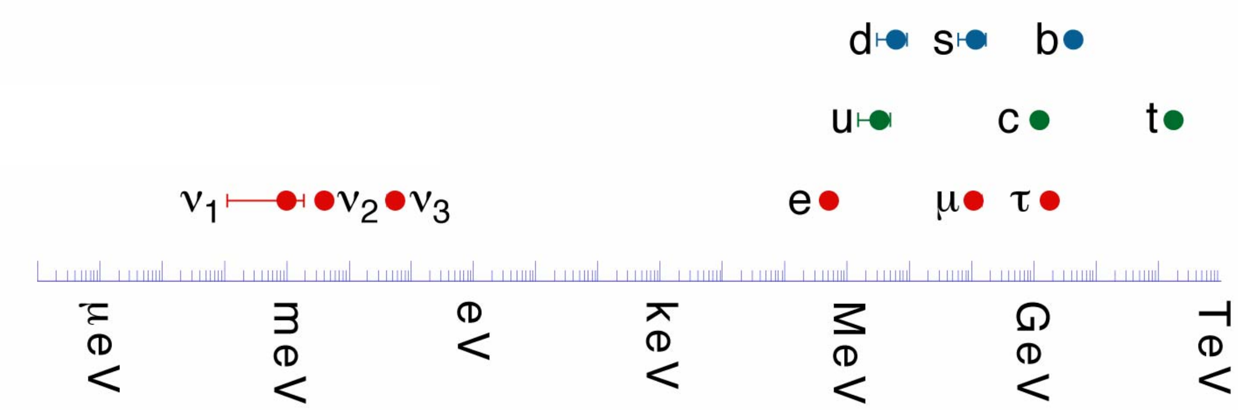
violation of individual lepton number implied by neutrino oscillations

violation of total lepton number not yet established a non-vanishing neutrino mass is evidence of the incompleteness of the SM

Questions

how to extend the SM in order to accommodate neutrino masses?

why neutrino masses are so small, compared with the charged fermion masses?



why lepton mixing angles are so different from those of the quark sector?

$$|U_{PMNS}| \approx \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.6 \\ 0.4 & 0.6 & 0.8 \end{pmatrix}$$

$$V_{CKM} \approx \begin{pmatrix} 1 & O(\lambda) & O(\lambda^4 \div \lambda^3) \\ O(\lambda) & 1 & O(\lambda^2) \\ O(\lambda^4 \div \lambda^3) & O(\lambda^2) & 1 \end{pmatrix}$$

$\lambda \approx 0.22$

How to modify the SM?

the SM, as a consistent RQFT, is completely specified by

0. invariance under local transformations of the gauge group G=SU(3)xSU(2)xU(1) [plus Lorentz invariance]

1. particle content

   three copies of (q, u^c, d^c, l, e^c)

   one Higgs doublet Φ

2. renormalizability (i.e. the requirement that all coupling constants g_i have non-negative dimensions in units of mass: $d(g_i) \geq 0$. This allows to eliminate all the divergencies occurring in the computation of physical quantities, by redefining a finite set of parameters.)

(0.+1.+2.) leads to the SM Lagrangian, L_{SM}, possessing an additional, accidental, global symmetry: (B-L)

0. We cannot give up gauge invariance! It is mandatory for the consistency of the theory. Without gauge invariance we cannot even define the Hilbert space of the theory [remember: we need gauge invariance to eliminate the photon extra degrees of freedom required by Lorentz invariance]!
   We could extend G, but, to allow for neutrino masses, we need to modify 1. (and/or 2.) anyway…

First possibility: modify (1), the particle content

there are several possibilities
one of the simplest one is to mimic the charged fermion sector

Example 1

- add (three copies of) right-handed neutrinos $\nu^c \equiv (1,1,0)$ full singlet under G=SU(3)xSU(2)xU(1)
- ask for (global) invariance under B-L (no more automatically conserved as in the SM)

the neutrino has now four helicities, as the other charged fermions, and we can build gauge invariant Yukawa interactions giving rise, after electroweak symmetry breaking, to neutrino masses

$$L_Y = d^c y_d (\Phi^+ q) + u^c y_u (\tilde{\Phi}^+ q) + e^c y_e (\Phi^+ l) + \nu^c y_\nu (\tilde{\Phi}^+ l) + h.c.$$

$$m_f = \frac{y_f}{\sqrt{2}} v \qquad f = u,d,e,\nu$$

with three generations there is an exact replica of the quark sector and, after diagonalization of the charged lepton and neutrino mass matrices, a mixing matrix U appears in the charged current interactions

$$-\frac{g}{\sqrt{2}} W_\mu^- \bar{e} \sigma^\mu U_{PMNS} \nu + h.c.$$

U_{PMNS} has three mixing angles and one phase, like V_{CKM}

a generic problem of this approach

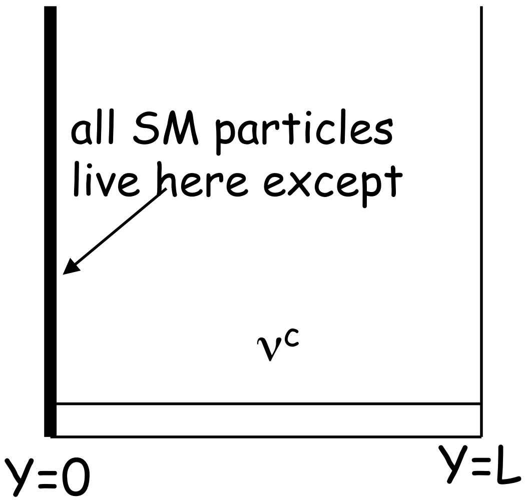
the particle content can be modified in several different ways
in order to account for non-vanishing neutrino masses
(additional right-handed neutrinos, new SU(2) fermion triplets, additional SU(2) scalar triplet(s), SUSY particles,…). Which is the correct one?

a problem of the above example

if neutrinos are so similar to the other fermions, why are so light?

$$\frac{y_\nu}{y_{top}} \le 10^{-12}$$

Quite a speculative answer:
neutrinos are so light, because the right-handed neutrinos have access to an extra (fifth) spatial dimension



neutrino Yukawa coupling

$$\nu^c(y=0)(\tilde{\Phi}^+ l) = \text{Fourier expansion}$$

$$= \frac{1}{\sqrt{L}}\nu_0^c(\tilde{\Phi}^+ l) + \ldots \quad \text{[higher modes]}$$

if L>>1 (in units of the fundamental scale)
then neutrino Yukawa coupling is suppressed

- additional KK states behave like sterile neutrinos
- at present no compelling evidence for sterile neutrinos

  hints [2σ level]

  - reactor anomaly: reevaluation of reactor antineutrino fluxes lead to indications of electron antineutrino disappearance in short BL experiments: $\Delta m^2 \approx eV^2$
  - LSND/MiniBoone: indication of electron (anti)neutrino appearance $\Delta m^2 \approx eV^2$

  disfavored by global fits

- eV sterile neutrino disfavored by energy loss of SN 1987A
- 1 extra neutrino preferred by CMB and LSS but its mass should be below 1 eV

Second possibility: abandon (2) renormalizability

Worth to explore. The dominant operators (suppressed by a single power of $1/\Lambda$) beyond L_{SM} are those of dimension 5. Here is a list of all d=5 gauge invariant operators

$$\frac{L_5}{\Lambda} = \frac{\left(\tilde{\Phi}^+ l\right)\left(\tilde{\Phi}^+ l\right)}{\Lambda} =$$

$$= \frac{v}{2}\left(\frac{v}{\Lambda}\right)\nu\nu + \ldots$$

a unique operator!
[up to flavour combinations]
it violates (B-L) by two units

it is suppressed by a factor (v/Λ) with respect to the neutrino mass term of Example 1:

$$\nu^c(\tilde{\Phi}^+ l) = \frac{v}{\sqrt{2}}\nu^c\nu + \ldots$$

it provides an explanation for the smallness of m_ν:
the neutrino masses are small because the scale Λ, characterizing (B-L) violations, is very large. How large? Up to about 10^{15} GeV

from this point of view neutrinos offer a unique window on physics at very large scales, inaccessible in present (and probably future) man-made experiments.

since this is the dominant operator in the expansion of L in powers of $1/\Lambda$, we could have expected to find the first effect of physics beyond the SM in neutrinos … and indeed this was the case!

L_5 represents the effective, low-energy description of several extensions of the SM

Example 2: see-saw

add (three copies of) $\nu^c \equiv (1,1,0)$

full singlet under G=SU(3)xSU(2)xU(1)

this is like Example 1, but without enforcing (B-L) conservation

$$L(\nu^c, l) = \nu^c y_\nu (\tilde{\Phi}^+ l) + \frac{1}{2}\nu^c M \nu^c + h.c.$$

mass term for right-handed neutrinos: G invariant, violates (B-L) by two units.

the new mass parameter M is independent from the electroweak breaking scale v. If M>>v, we might be interested in an effective description valid for energies much smaller than M. This is obtained by "integrating out" the field ν^c

$$L_{eff}(l) = -\frac{1}{2}(\tilde{\Phi}^+ l)\left[y_\nu^T M^{-1} y_\nu\right](\tilde{\Phi}^+ l) + h.c. + \ldots$$

terms suppressed by more powers of M^{-1}

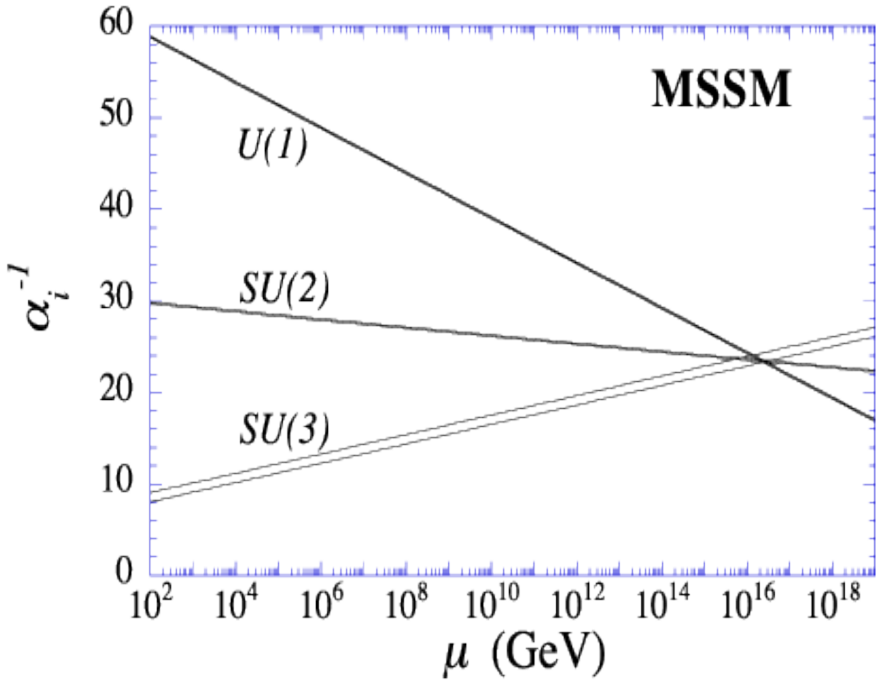
this reproduces L_5, with M playing the role of Λ. This particular mechanism is called (type I) see-saw.

Theoretical motivations for the see-saw

$\Lambda \approx 10^{15}$ GeV is very close to the so-called unification scale M_{GUT}.

an independent evidence for M_{GUT} comes from the unification of the gauge coupling constants in (SUSY extensions of) the SM.

such unification is a generic prediction of Grand Unified Theories (GUTs): the SM gauge group G is embedded into a simple group such as SU(5), SO(10),...



Particle classification: it is possible to unify all SM fermions (1 generation) into a single irreducible representation of the GUT gauge group. Simplest example: G_{GUT}=SO(10)

$$16 = (q, d^c, u^c, l, e^c, \nu^c)$$

a whole family plus a right-handed neutrino!

quite a fascinating possibility. Unfortunately, it still lacks experimental tests. In GUT new, very heavy, particles can convert quarks into leptons and the proton is no more a stable particle. Proton decay rates and decay channels are however model dependent. Experimentally we have only lower bounds on the proton lifetime.

2 additional virtues of the see-saw

The see-saw mechanism can enhance small mixing angles into large ones

$$m_\nu = -\left[y_\nu^T M^{-1} y_\nu\right] v^2$$

Example with 2 generations

$$y_\nu = \begin{pmatrix} \delta & \delta \\ 0 & 1 \end{pmatrix}$$

δ<<1
small mixing

$$M = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}$$

no mixing

$$y_\nu^T M^{-1} y_\nu = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{\delta^2}{M_1} + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \frac{1}{M_2}$$

$$\approx \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \frac{\delta^2}{M_1} \qquad \text{for} \quad \frac{M_1}{M_2} << \delta^2$$

The (out-of equilibrium, CP-violating) decay of heavy right-handed neutrinos in the early universe might generate a net asymmetry between leptons and anti-leptons. Subsequent SM interactions can partially convert it into the observed baryon asymmetry

$$\eta = \frac{(n_B - n_{\bar{B}})}{s} \approx 6 \times 10^{-10}$$

weak point of the see-saw

full high-energy theory is difficult to test

$$L(\nu^c, l) = \nu^c y_\nu (\tilde{\Phi}^+ l) + \frac{1}{2} \nu^c M \nu^c + h.c.$$

depends on many physical parameters:
3 (small) masses + 3 (large) masses
3 (L) mixing angles + 3 (R) mixing angles
6 physical phases = 18 parameters

the double of those describing (L_{SM})+L_5:
3 masses, 3 mixing angles and 3 phases

few observables to pin down the extra parameters: η,...
[additional possibilities exist under special conditions, e.g. Lepton Flavor Violation at observable rates]

easier to test the low-energy remnant L_5

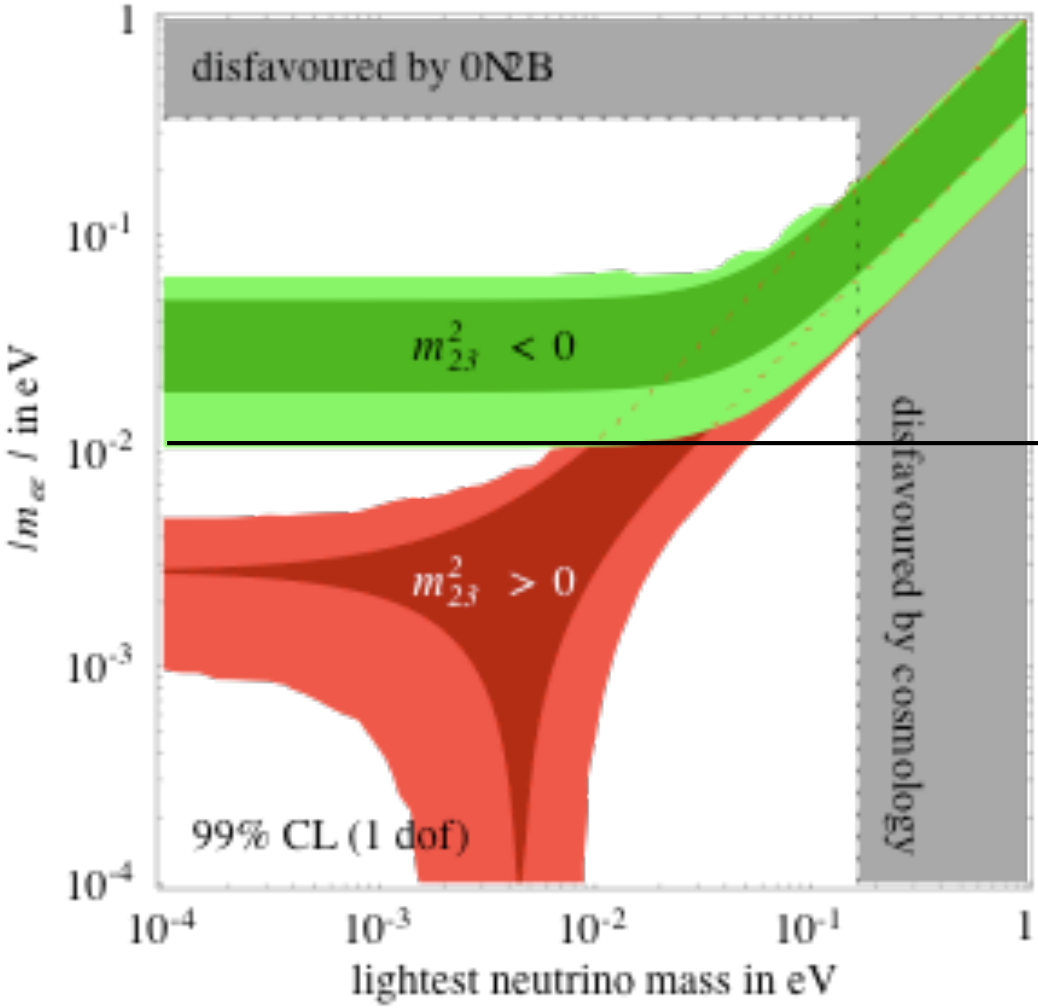
[which however is "universal" and does not implies the specific see-saw mechanism of Example 2]

look for a process where B-L is violated by 2 units. The best candidate is $0\nu\beta\beta$ decay: (A,Z)->(A,Z+2)+2e$^-$
this would discriminate L_5 from other possibilities, such as Example 1.

The decay in 0νββ rates depend on the combination $|m_{ee}| = \left|\sum_i U_{ei}^2 m_i\right|$

$$|m_{ee}| = \left|\cos^2\vartheta_{13}(\cos^2\vartheta_{12}\ m_1 + \sin^2\vartheta_{12}e^{2i\alpha}\ m_2) + \sin^2\vartheta_{13}e^{2i\beta}\ m_3\right|$$

[notice the two phases α and β, not entering neutrino oscillations]



from the current knowledge of $(\Delta m^2_{ij}, \vartheta_{ij})$ we can estimate the expected range of $|m_{ee}|$

future expected sensitivity on $|m_{ee}|$

10 meV

a positive signal would test both L_5 and the absolute mass spectrum at the same time!

Flavor symmetries

hierarchies in fermion spectrum

quarks

$$\frac{m_u}{m_t} << \frac{m_c}{m_t} << 1 \qquad \frac{m_d}{m_b} << \frac{m_s}{m_b} << 1 \qquad |V_{ub}| << |V_{cb}| << |V_{us}| \equiv \lambda < 1$$

leptons

$$\frac{m_e}{m_\tau} << \frac{m_\mu}{m_\tau} << 1$$

spontaneously broken $U(1)_{FN}$

[Froggatt,Nielsen 1979]

$$y_u = F_{U^c} Y_u F_Q$$

$$y_d = F_{D^c} Y_d F_Q$$

$$Y_{u,d} \approx O(1)$$

$$F_X = \begin{pmatrix} \lambda^{P(X_1)} & 0 & 0 \\ 0 & \lambda^{P(X_2)} & 0 \\ 0 & 0 & \lambda^{P(X_3)} \end{pmatrix} \qquad (X = Q, U^c, D^c)$$

$P(X_i)$ are $U(1)_{FN}$ charges [here $P(X_i) \geq 0$]

$$\lambda = \frac{\langle \vartheta \rangle}{\Lambda} \approx 0.2$$ [symmetry breaking parameter]

provides a qualitative picture of the existing hierarchies in the fermion spectrum compatible with SU(5) GUTs and realized in several different frameworks: FN, RS,....

Simple explanation of mixing angles

$$V_{CKM} \approx \begin{pmatrix} 1 & O(\lambda) & O(\lambda^3) \\ O(\lambda) & 1 & O(\lambda^2) \\ O(\lambda^3) & O(\lambda^2) & 1 \end{pmatrix} \Longleftrightarrow F_Q = \begin{pmatrix} \lambda^3 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$|U_{PMNS}| \approx \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.6 \\ 0.4 & 0.6 & 0.8 \end{pmatrix} \Longleftrightarrow F_L = \begin{pmatrix} O(1) & 0 & 0 \\ 0 & O(1) & 0 \\ 0 & 0 & O(1) \end{pmatrix}$$

for example:
$P(L_1)=P(L_2)=P(L_3)=0$
several variants are equally possible

mixing angles and mass ratios are O(1)
no special pattern beyond the data

Anarchy

large number of independent O(1) parameters
testable predictions beyond order-of-magnitude accuracy ?

orthogonal approach: discrete flavor symmetries

$$U_{PMNS} = U^0_{PMNS} + \text{corrections}$$

some simple pattern, exactly reproduced by a flavor symmetry

- well motivated before 2012

$$U^0_{PMNS} = U_{TB} \equiv \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Tribimaximal Mixing

discrete flavor symmetries showed very efficient to reproduce U^0_{PMNS}

- still justified today?

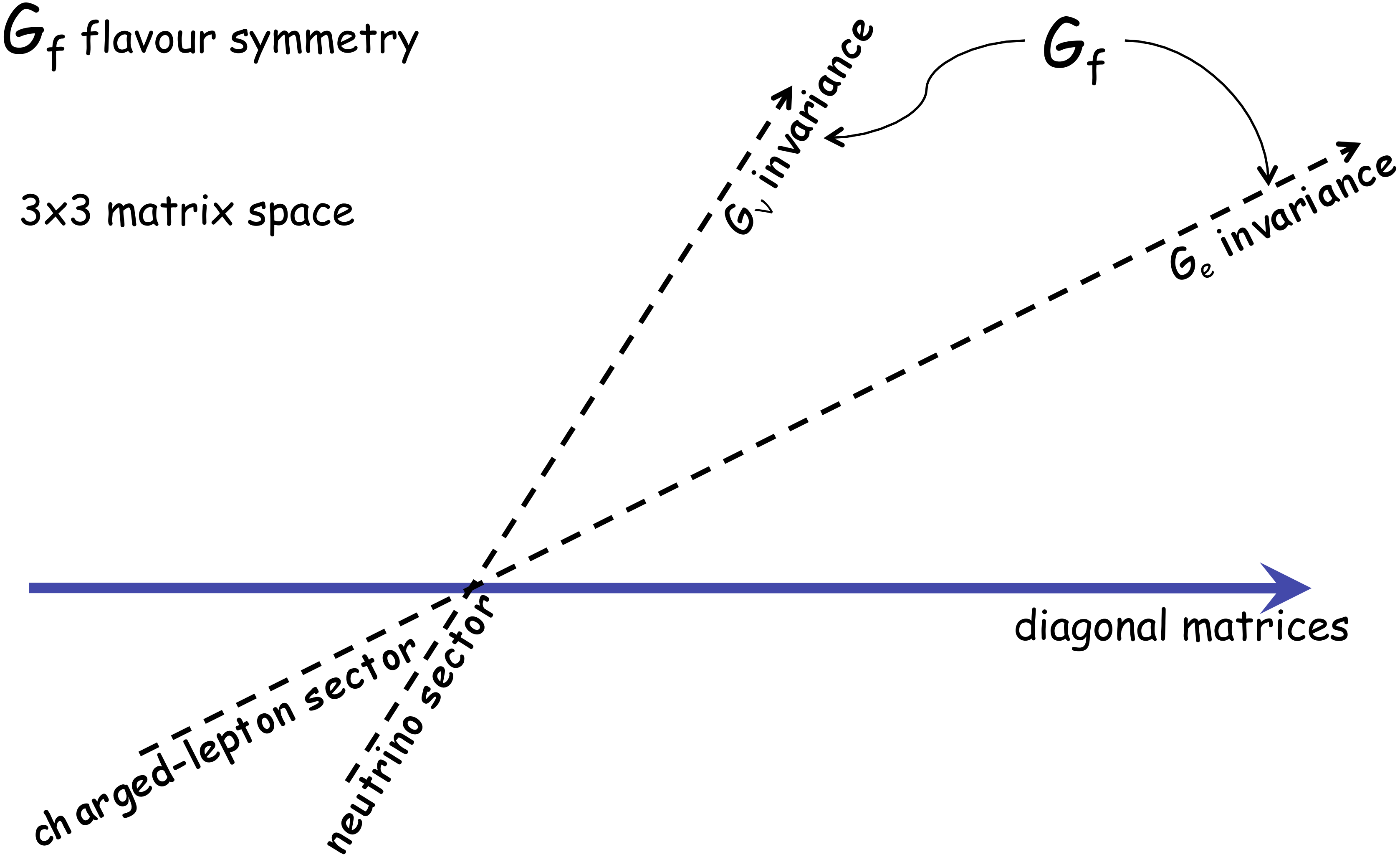
$$U_{TB} \approx \begin{pmatrix} 0.82 & 0.58 & 0 \\ -0.41 & 0.58 & -0.71 \\ -0.41 & 0.58 & 0.71 \end{pmatrix} \qquad |U_{PMNS}| = \begin{pmatrix} 0.80 \div 0.85 & 0.51 \div 0.59 & 0.13 \div 0.18 \\ 0.21 \div 0.54 & 0.42 \div 0.73 & 0.58 \div 0.81 \\ 0.22 \div 0.55 & 0.41 \div 0.73 & 0.57 \div 0.80 \end{pmatrix}$$

[3σ ranges from Gonzalez-Garcia, Maltoni, Salvado, Schwetz 1209.3023]

Mixing patterns U^0_{PMNS} from discrete symmetries

G_f flavour symmetry

3x3 matrix space



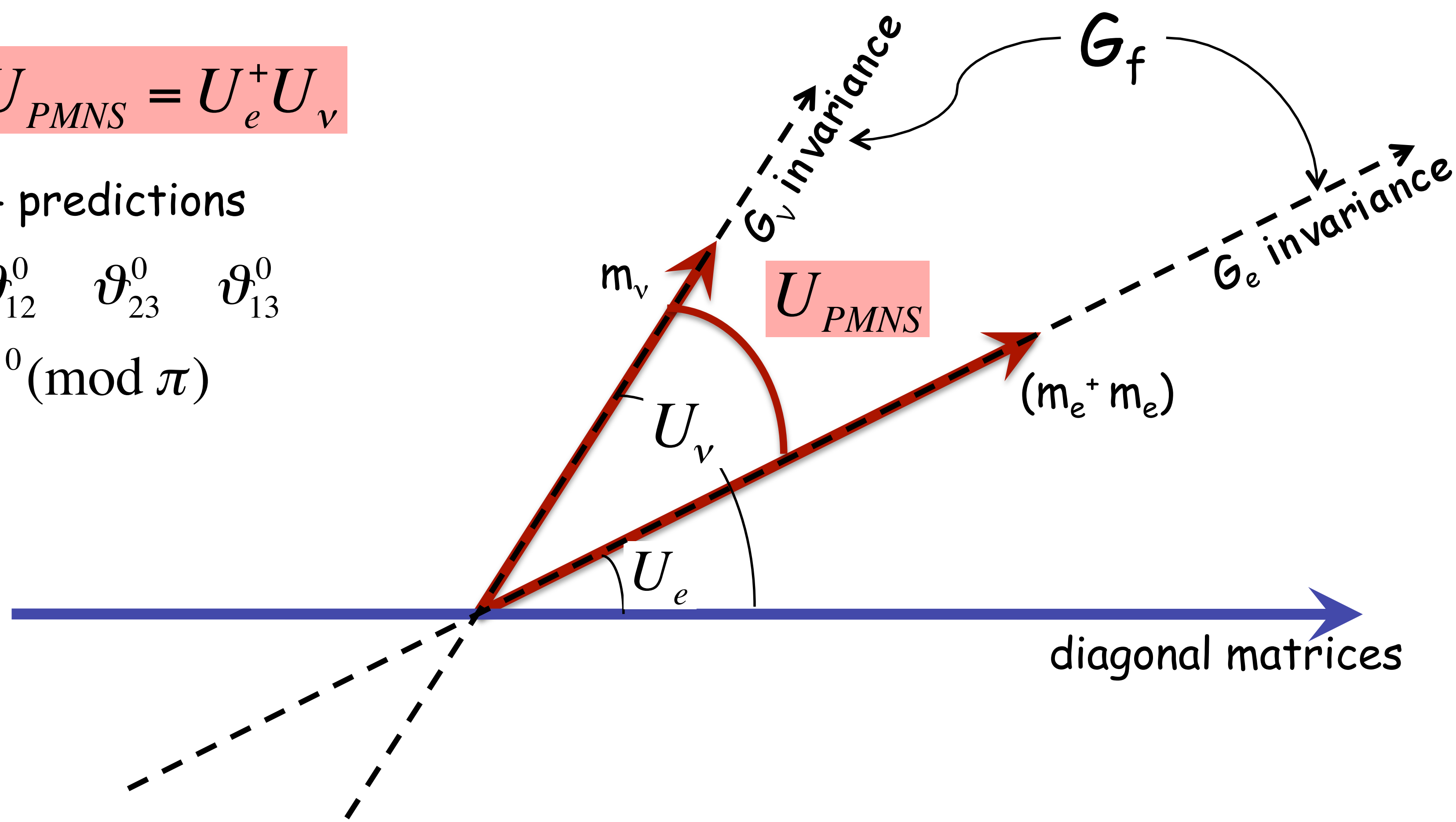
misalignment in flavour space from symmetry breaking

$$U_{PMNS} = U_e^+ U_\nu$$

4 predictions

$\vartheta_{12}^0 \quad \vartheta_{23}^0 \quad \vartheta_{13}^0$

$\delta^0 (\mathrm{mod}\ \pi)$



- the most general group leaving $\nu^T m_\nu \nu$ invariant, and m_i unconstrained

$$G_\nu = Z_2 \times Z_2$$

Majorana neutrinos imply G_ν discrete!

- G_e can be continuous but the simplest choice is G_e discrete

$$G_e = \begin{cases} Z_2 \times Z_2 \\ Z_n \quad n \geq 3 \end{cases}$$

Some mixing patterns

$$G_\nu = Z_2 \times Z_2$$

G_f	G_e	U_{PMNS}	$\sin^2\vartheta_{23}$	$\sin\vartheta_{13}$	$\sin^2\vartheta_{12}$	
A_4	Z_3	[M]	$1/2$	$1/\sqrt{3}$	$1/2$	
S_4	Z_3	[TB]	$1/2$	0	$1/3$	
	Z_4 $(Z_2 \times Z_2)'$	[BM]	$1/2$	0	$1/2$	
A_5	Z_3	[GR_1]	$1/2$	0	0.127	
	Z_5	[GR_2]	$1/2$	0	0.276	
	$(Z_2 \times Z_2)'$	[GR_3]	0.276	0.309	0.276	
		[Exp 3σ]	$0.34 \div 0.67$	$0.13 \div 0.17$	$0.27 \div 0.34$	

[Lam 1104.0055
F., Hagedorn, Toroop]

[TB <->Harrison,
Perkins and Scott]

[GR_2<-> Kajiyama,
Raidal, Strumia 2007]

-- a long way to promote a candidate pattern to a complete model

-- general feature $U_{PMNS} = U^0_{PMNS} + O(u)$ $\quad u \equiv \dfrac{\langle\varphi\rangle}{\Lambda} < 1$

-- neutrino masses fitted, not predicted.

expectation for $U^0_{PMNS}=U_{TB}$

$$\begin{cases}\vartheta^0_{13} = 0 \\ \vartheta^0_{23} = \dfrac{\pi}{4}\end{cases} \Rightarrow \begin{cases}\vartheta_{13} = \mathrm{O(few\ degrees)} \\ \vartheta_{23} = \text{close to } \dfrac{\pi}{4}\end{cases}$$

not to spoil the agreement with ϑ_{12}

wrong!

possibilities

1 add large corrections $O(\vartheta_{13})\approx 0.2$

- predictability is lost since in general correction terms are many
- new dangerous sources of FC/CPV if NP is at the TeV scale

2 change discrete group G_f

- solutions exist
  special forms of Trimaximal mixing

$$U^0 = U_{TB} \times \begin{pmatrix} \cos\alpha & 0 & e^{i\delta}\sin\alpha \\ 0 & 1 & 0 \\ -e^{-i\delta}\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

G_f	$\Delta(96)$	$\Delta(384)$	$\Delta(600)$
α	$\pm\pi/12$	$\pm\pi/24$	$\pm\pi/15$
$\sin^2\vartheta^0_{13}$	0.045	0.011	0.029

F.F., C. Hagedorn, R. de A.Toroop
hep-ph/1107.3486 and hep-ph/1112.1340
Lam 1208.5527 and 1301.1736
Holthausen1, Lim and Lindner 1212.2411

$\delta^0 = 0,\pi$ (no CP violation) and
α "quantized" by group theory

too big groups?

3 relax symmetry requirements [Hernandez,Smirnov 1204.0445]

G_e as before

$G_\nu = Z_2$

2 predictions:
2 combinations of ϑ_{12}^0 ϑ_{23}^0 ϑ_{13}^0

leads to testable sum rules

$$\sin^2\vartheta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}}\sin\vartheta_{13}\cos\delta_{CP} + O(\sin^2\vartheta_{13})$$

[He, Zee 2007 and 2011, Grimus, Lavoura 2008, Grimus, Lavoura, Singraber 2009, Albright, Rodejohann 2009, Antusch, King, Luhn, Spinrath 2011, King, Luhn 2011, G. Altarelli, F.F., L. Merlo and E. Stamou hep-ph/1205.4670]

4 include CP in the SB pattern

$$G_{CP} = G_f \rtimes CP$$

[F. F, C. Hagedorn and R. Ziegler 1211.5560]

$$G_e \qquad G_\nu = Z_2 \times CP$$

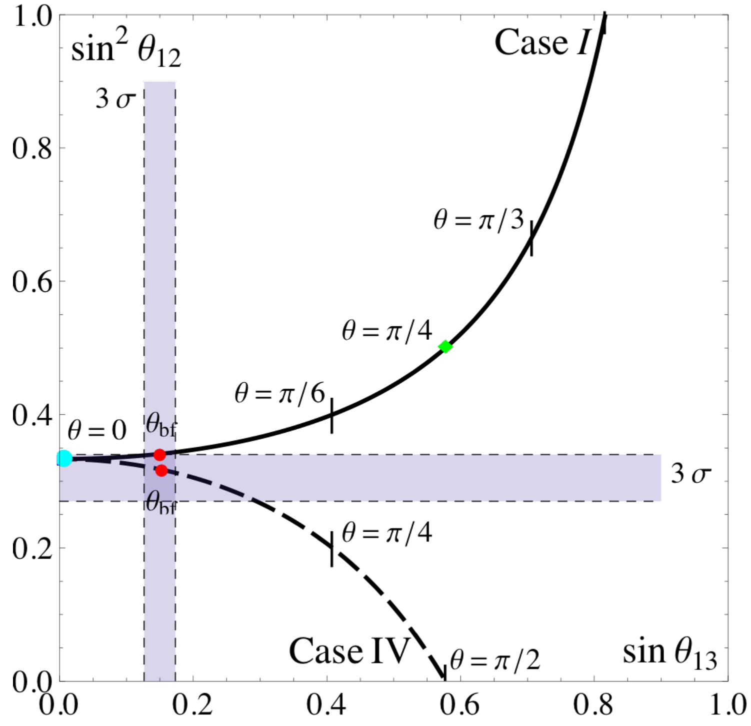
mixing angles and CP violating phases

$$(\vartheta_{12}^0, \vartheta_{23}^0, \vartheta_{13}^0, \delta^0, \alpha^0, \beta^0)$$

predicted in terms of a single real parameter $0 \leq \vartheta \leq 2\pi$

2 examples with $G_f = S_4$ $G_e = Z_3$

$$\sin^2\vartheta_{23}^0 = \frac{1}{2} \qquad |\sin\delta^0| = 1 \qquad \sin\alpha^0 = 0 \qquad \sin\beta^0 = 0$$



Conclusion

- big progress on the experimental side:
  -- precisely measured ϑ_{13}: many σ away from zero!
  -- potentially interesting implications on ϑ_{23}

- on the theory side:
  neutrino masses represent a unique window on high-energy physics (such as GUTs, B-L violation, leptogenesis,...) but the fundamental theory is hard to identify.

  flavour symmetries:
  no compelling and unique picture have emerged so far
  present data can be described within widely different frameworks

- models based on "anarchy" and/or its variants - $U(1)_{FN}$ models - in good shape:
  neutrino mass ratios and mixing angles just random O(1) quantities

- models based on discrete symmetries are less supported by data now and modifications of simplest realizations are required
  -- add large corrections $O(\vartheta_{13})\approx 0.2$
  -- move to large discrete symmetry groups G_f such as $\Delta(96)$ $\Delta(384)$ …
  -- relax symmetry requirements
  -- include CP in the SB pattern

Backup slides

Mixing matrix $U=U_{PMNS}$ (Pontecorvo,Maki,Nakagawa,Sakata)

neutrino interaction eigenstates

$$\nu_f = \sum_{i=1}^{3} U_{fi} \nu_i \qquad (f = e, \mu, \tau)$$

neutrino mass eigenstates

U is a 3 x 3 unitary matrix
standard parametrization

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{-i\delta} & c_{13}s_{23} \\ -c_{12}s_{13}c_{23}e^{-i\delta} + s_{12}s_{23} & -s_{12}s_{13}c_{23}e^{-i\delta} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

$$c_{12} \equiv \cos\vartheta_{12}, \ldots$$

three mixing angles $\vartheta_{12}, \quad \vartheta_{13}, \quad \vartheta_{23}$

three phases (in the most general case) δ $\qquad \underbrace{\alpha, \quad \beta}_{\text{do not enter}}$ $P_{ff'} = P(\nu_f \rightarrow \nu_{f'})$

oscillations can only test 6 combinations

$\Delta m^2_{21}, \Delta m^2_{32}, \quad \vartheta_{12}, \quad \vartheta_{13}, \quad \vartheta_{23} \quad \delta$

2011/2012 breakthrough

- from LBL experiments searching for $\nu_\mu \rightarrow \nu_e$ conversion

T2K: muon neutrino beam produced at JPARC [Tokai] E=0.6 GeV and sent to SK 295 Km apart [1106.2822]

MINOS: muon neutrino beam produced at Fermilab [E=3 GeV] sent to Soudan Lab 735 Km apart [1108.0015]

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2\vartheta_{23} \sin^2 2\vartheta_{13} \sin^2 \frac{\Delta m^2_{32} L}{4E} + \ldots$$

both experiments favor $\sin^2 \vartheta_{13} \sim$ few %

- from SBL reactor experiments searching for anti-ν_e disappearance

Double Chooz (far detector): $\sin^2 2\vartheta_{13} = 0.109 \pm 0.039$

Daya Bay (near + far detectors): $\sin^2 \vartheta_{13} = 0.089 \pm 0.011$

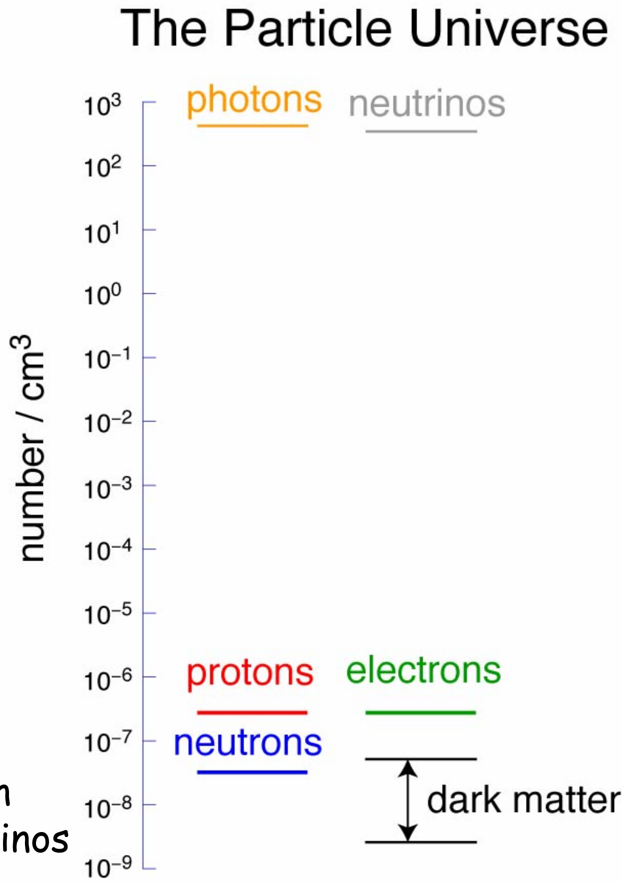
RENO (near + far detectors): $\sin^2 \vartheta_{13} = 0.113 \pm 0.023$

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\vartheta_{13} \sin^2 \frac{\Delta m^2_{32} L}{4E} + \ldots$$

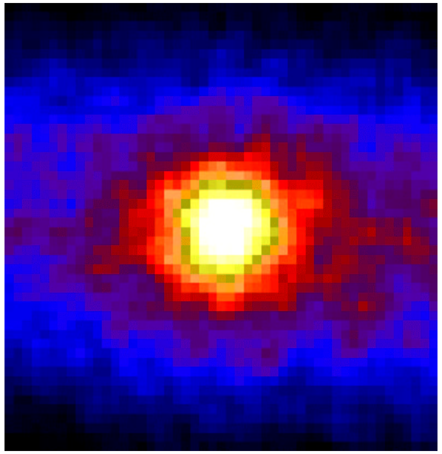
- SBL reactors are sensitive to ϑ_{13} only
  LBL experiments anti-correlate $\sin^2 2\vartheta_{13}$ and $\sin^2 \vartheta_{23}$
  also breaking the octant degeneracy $\vartheta_{23} \leftrightarrow (\pi-\vartheta_{23})$

General remarks on neutrinos

the more abundant particles in the universe after the photons: about 300 neutrinos per cm^3



produced by stars: about 3% of the sun energy emitted in neutrinos. As I speak more than 1 000 000 000 000 solar neutrinos go through your bodies each second.

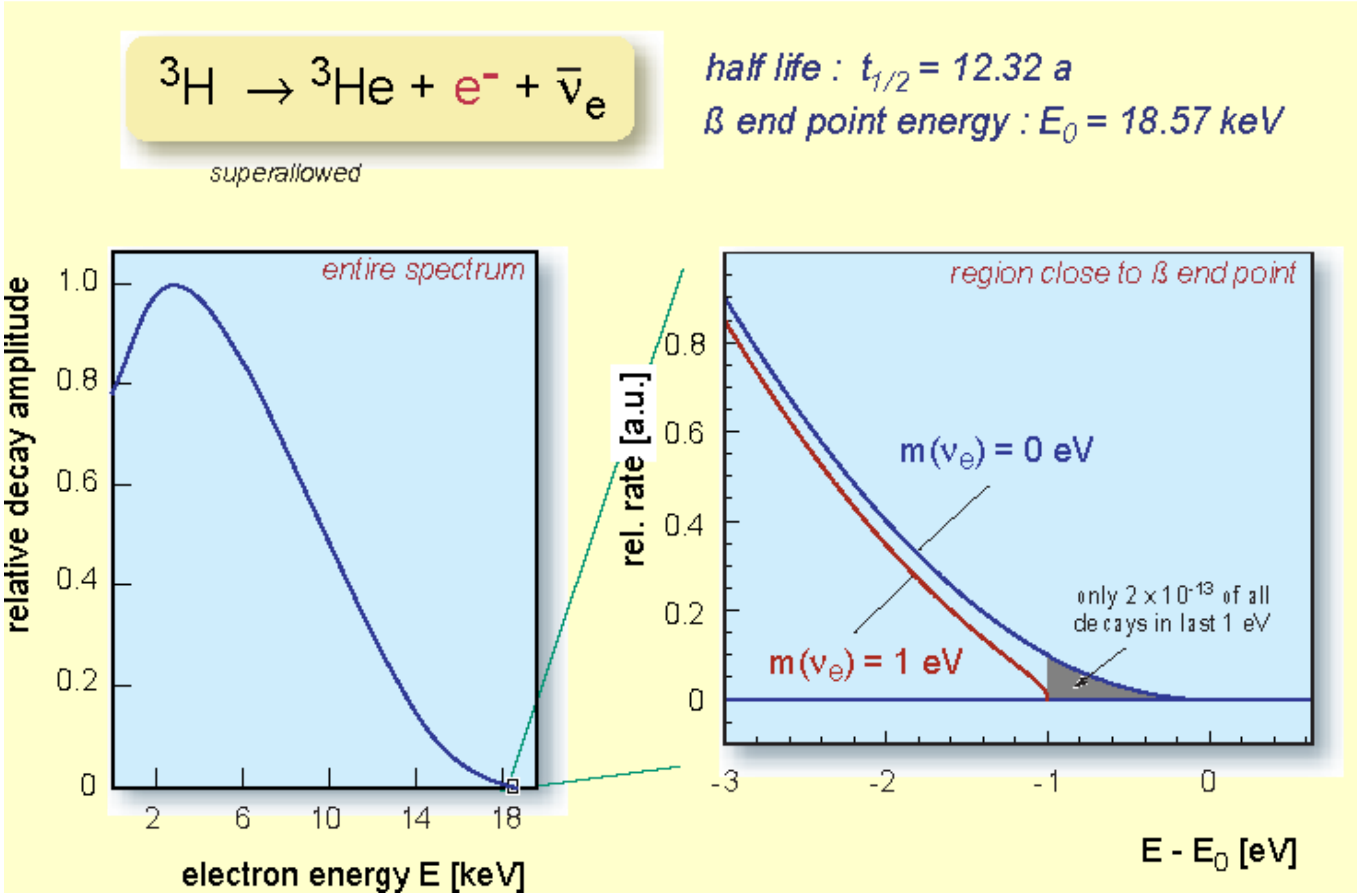


this is a picture of the sun reconstructed from neutrinos

electrically neutral and extremely light:
they can carry information about extremely large length scales
e.g. a probe of supernovae dynamics: neutrino events from a supernova explosion first observed 23 years ago

in particle physics:
they have a tiny mass (1 000 000 times smaller than the electron's mass)
the discovery that they are massive (twelve anniversary now!) allows us to explore, at least in principle, extremely high energy scales, otherwise inaccessible to present laboratory experiments (more on this later on...)

Upper limit on neutrino mass (laboratory)



$$m_\nu < 2.2\, eV \quad (95\%\ CL)$$

Upper limit on neutrino mass (cosmology)

massive ν suppress the formation of small scale structures

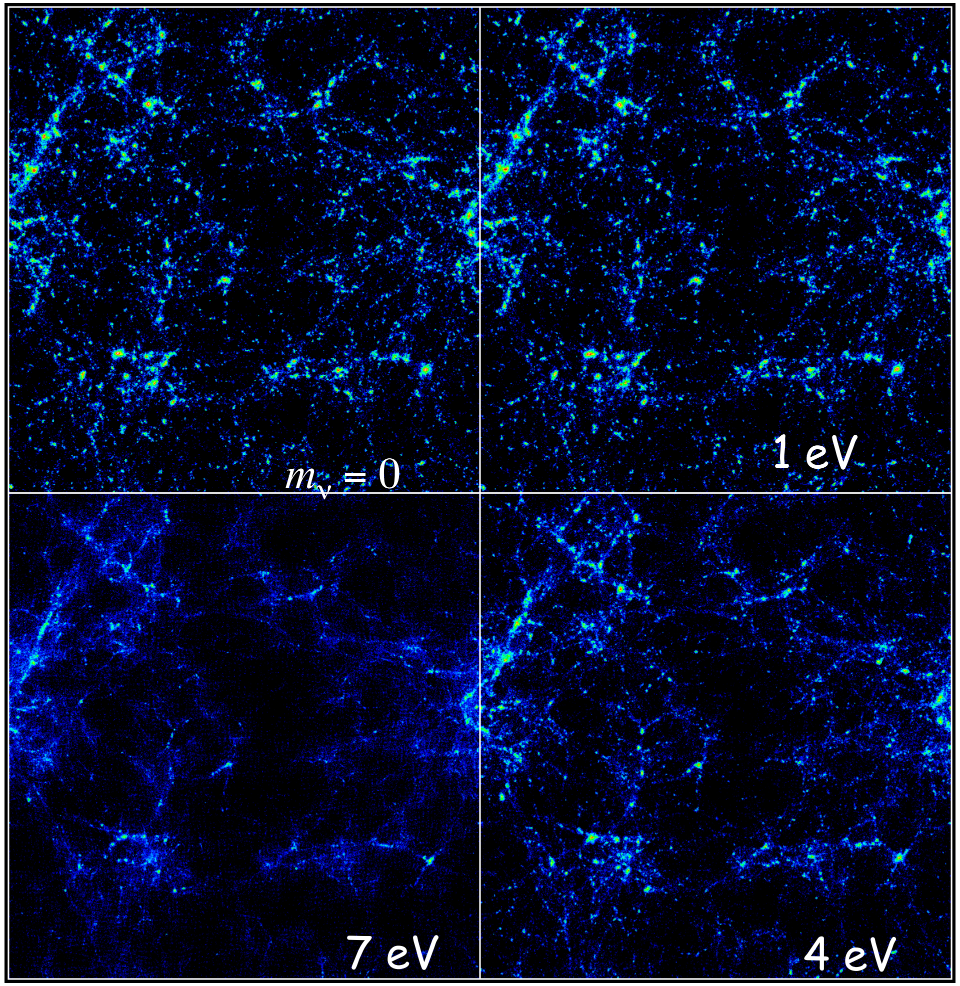
$$\sum_i m_i < 0.2 \div 1 \quad eV$$

depending on
- assumed cosmological model
- set of data included
- how data are analyzed

$$k_{\rm nr} \approx 0.026 \left(\frac{m_\nu}{1\,{\rm eV}}\right)^{1/2} \Omega_m^{1/2} h\,{\rm Mpc}^{-1}.$$

The small-scale suppression is given by

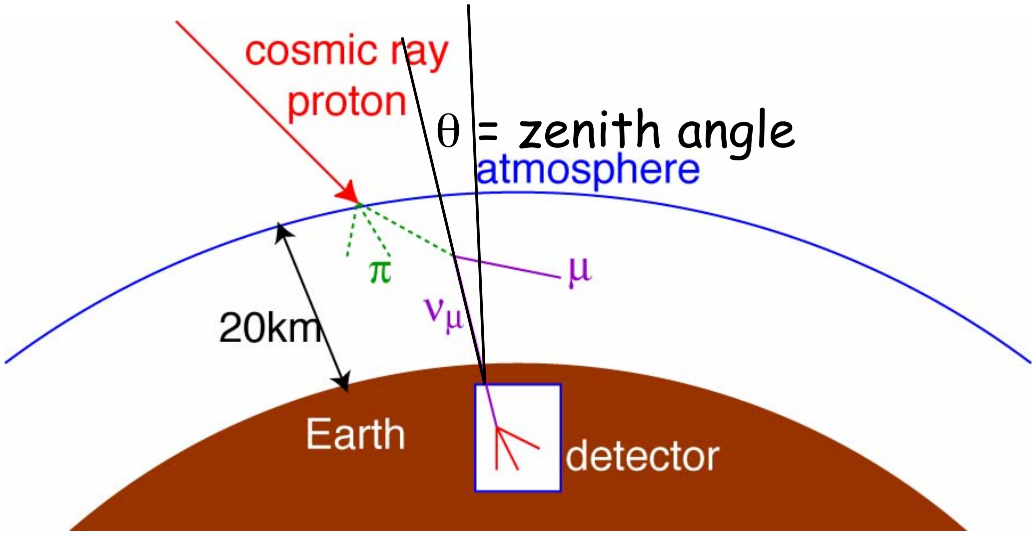
$$\left(\frac{\Delta P}{P}\right) \approx -8\frac{\Omega_\nu}{\Omega_m} \approx -0.8\left(\frac{m_\nu}{1\,{\rm eV}}\right)\left(\frac{0.1N}{\Omega_m h^2}\right)$$



$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2)\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)} P(\vec{k})$$

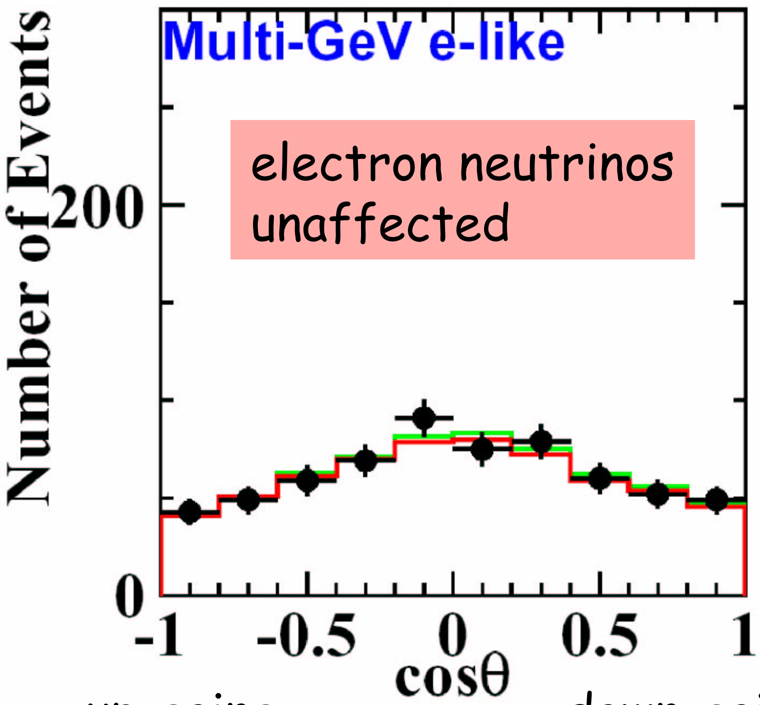
Atmospheric neutrino oscillations



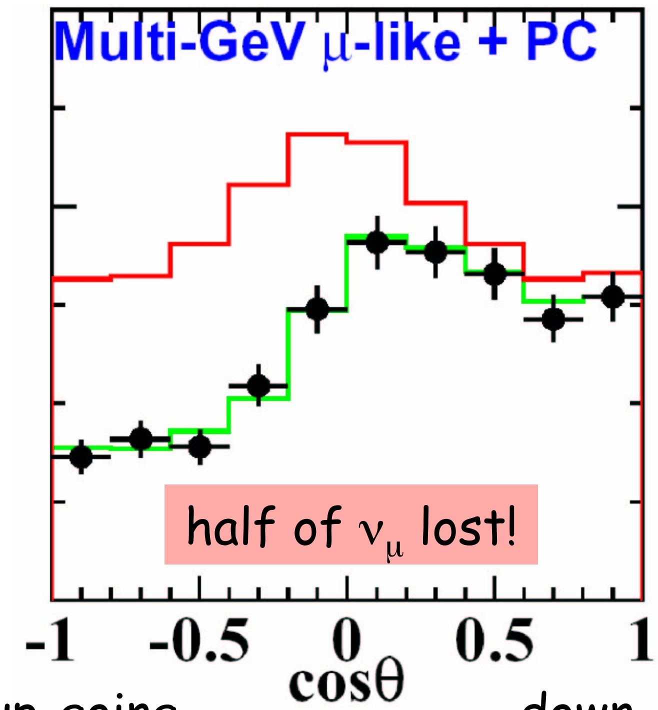
[this year: 10th anniversary]

Electron and muon neutrinos (and antineutrinos) produced by the collision of cosmic ray particles on the atmosphere

Experiment:
SuperKamiokande (Japan)



up-going down-going



up-going down-going

electron neutrinos do not oscillate

by working in the approximation $\Delta m^2_{21} = 0$

$$P_{ee} = 1 - \underbrace{4\left|U_{e3}\right|^2 (1 - \left|U_{e3}\right|^2)}_{\sin^2 2\vartheta_{13}} \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right) \approx 1$$

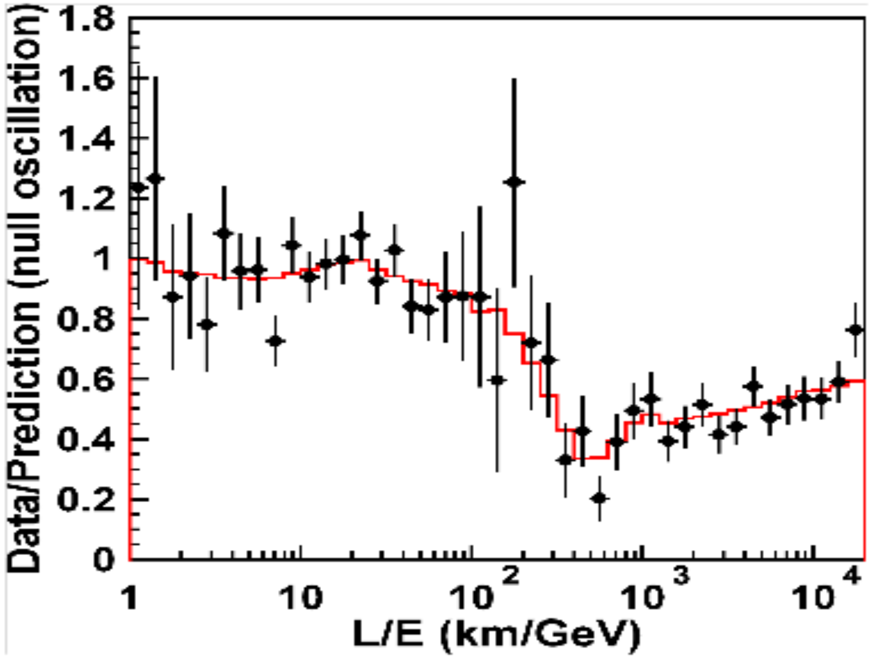
for $U_{e3} = \sin\vartheta_{13} \approx 0$

muon neutrinos oscillate

$$P_{\mu\mu} = 1 - \underbrace{4\left|U_{\mu 3}\right|^2 (1 - \left|U_{\mu 3}\right|^2)}_{\sin^2 2\vartheta_{23}} \sin^2\left(\frac{\Delta m^2_{32} L}{4E}\right)$$

$$\left|\Delta m^2_{32}\right| \approx 2 \cdot 10^{-3} \quad eV^2$$

$$\sin^2\vartheta_{23} \approx \frac{1}{2}$$



$$U_{PMNS} = \begin{pmatrix} \cdot & \cdot & \boxed{0} \\ \cdot & \cdot & -\frac{1}{\sqrt{2}} \\ \cdot & \cdot & \frac{1}{\sqrt{2}} \end{pmatrix} + \text{(small corrections)}$$

maximal mixing!
not a replica of the quark
mixing pattern

this picture is supported by other terrestrial esperiments such as
K2K (Japan, from KEK to Kamioka mine L ≈ 250 Km E ≈ 1 GeV)
and MINOS (USA, from Fermilab to Soudan mine L ≈ 735 Km E ≈ 5 GeV)
that are sensitive to Δm_{32}^2 close to 10^{-3} eV^2,

KamLAND

previous experiments were sensitive to Δm^2 close to 10^{-3} eV^2
to explore smaller Δm^2 we need larger L and/or smaller E

KamLAND experiment exploits the low-energy electron anti-neutrinos ($E \approx 3$ MeV) produced by Japanese and Korean reactors at an average distance of $L \approx 180$ Km from the detector and is potentially sensitive to Δm^2 down to 10^{-5} eV^2

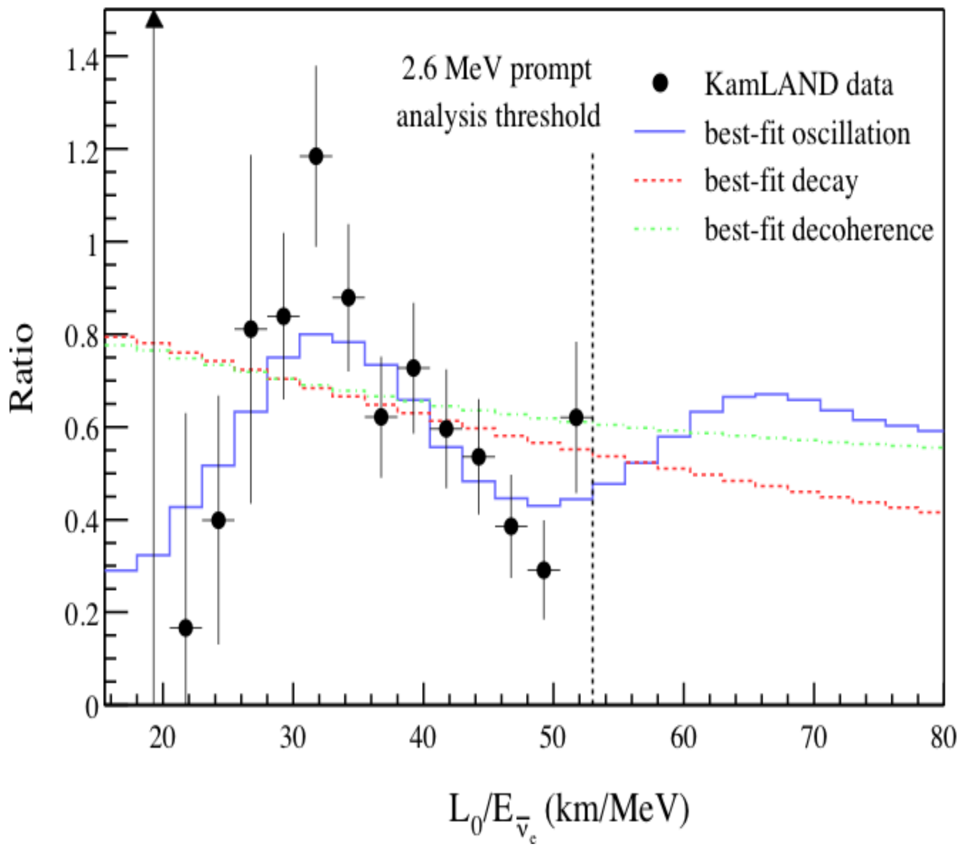
by working in the approximation

$U_{e3} = \sin\vartheta_{13} = 0$ we get

$$P_{ee} = 1 - \underbrace{4|U_{e1}|^2|U_{e2}|^2}_{\sin^2 2\vartheta_{12}} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

$$\Delta m_{21}^2 \approx 8 \cdot 10^{-5} \quad eV^2$$

$$\sin^2\vartheta_{12} \approx \frac{1}{3}$$



TB mixing from symmetry breaking

it is easy to find a symmetry that forces $(m_e^+ m_e)$ to be diagonal;
a "minimal" example (there are many other possibilities) is

$$G_T=\{1,T,T^2\}$$

$$T=\begin{pmatrix}1 & 0 & 0\\ 0 & \omega^2 & 0\\ 0 & 0 & \omega\end{pmatrix} \qquad \omega=e^{i\frac{2\pi}{3}}$$

[$T^3=1$ and mathematicians call a group with this property Z_3]

$$T^+(m_e^+ m_e)\,T=(m_e^+ m_e) \quad \Longrightarrow \quad \left(m_e^+ m_e\right)=\begin{pmatrix}m_e^2 & 0 & 0\\ 0 & m_\mu^2 & 0\\ 0 & 0 & m_\tau^2\end{pmatrix}$$

in such a framework TB mixing should arise entirely from m_ν

$$m_\nu(TB) \equiv \frac{m_3}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix} + \frac{m_2}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \frac{m_1}{6}\begin{pmatrix} 4 & -2 & -2 \\ -2 & 1 & 1 \\ -2 & 1 & 1 \end{pmatrix}$$

most general neutrino mass matrix giving rise to TB mixing

easy to construct from the eigenvectors:

$$m_3 \leftrightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \qquad m_2 \leftrightarrow \frac{1}{\sqrt{3}}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad m_1 \leftrightarrow \frac{1}{\sqrt{6}}\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$$

a "minimal" symmetry guaranteeing such a pattern [C.S. Lam 0804.2622]

$$G_S \times G_U \quad G_S=\{1,S\} \quad G_U=\{1,U\}$$

$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix} \qquad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

[this group corresponds to $Z_2 \times Z_2$ since $S^2=U^2=1$]

$$S^T m_\nu S = m_\nu \qquad U^T m_\nu U = m_\nu \quad \Rightarrow \quad m_\nu = m_\nu(TB)$$

Algorithm to generate TB mixing

- start from a flavour symmetry group G_f containing G_T, G_S, G_U
- arrange appropriate symmetry breaking

$$\begin{tikzcd} & & G_f \arrow[dl] \arrow[dr] & & \\ \text{charged lepton sector} & G_T & & G_S \times G_U & \text{neutrino sector} \end{tikzcd}$$

if the breaking is spontaneous, induced by $\langle \varphi_T \rangle$,$\langle \varphi_S \rangle$,… there is a vacuum alignment problem

$\sin^2\theta_{23}$

$\delta(\sin^2\theta_{23})$ reduced by future LBL experiments from $\nu_\mu \rightarrow \nu_\mu$ disappearance channel

$$\vartheta_{23} \approx \frac{\pi}{4}$$

$$P_{\mu\mu} \approx 1 - \sin^2 2\vartheta_{23} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

$$\delta\vartheta_{23} \approx \frac{\sqrt{\delta P_{\mu\mu}}}{2}$$

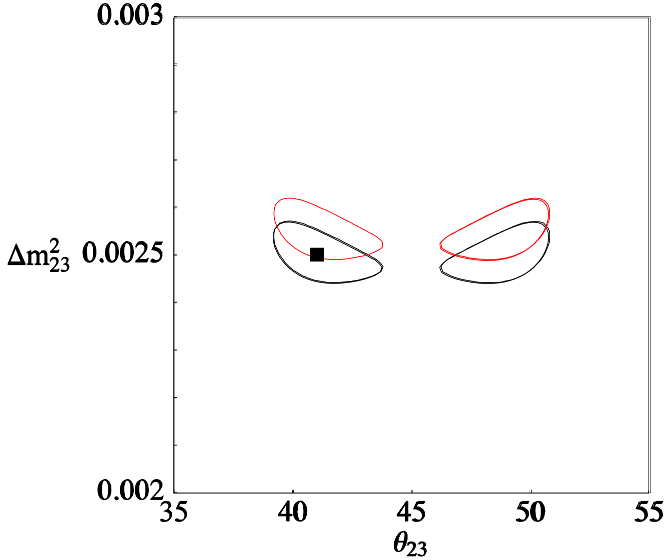
i.e. a small uncertainty on $P_{\mu\mu}$ leads to a large uncertainty on θ_{23}

- no substantial improvements from conventional beams
- superbeams (e.g. T2K in 5 yr of run)

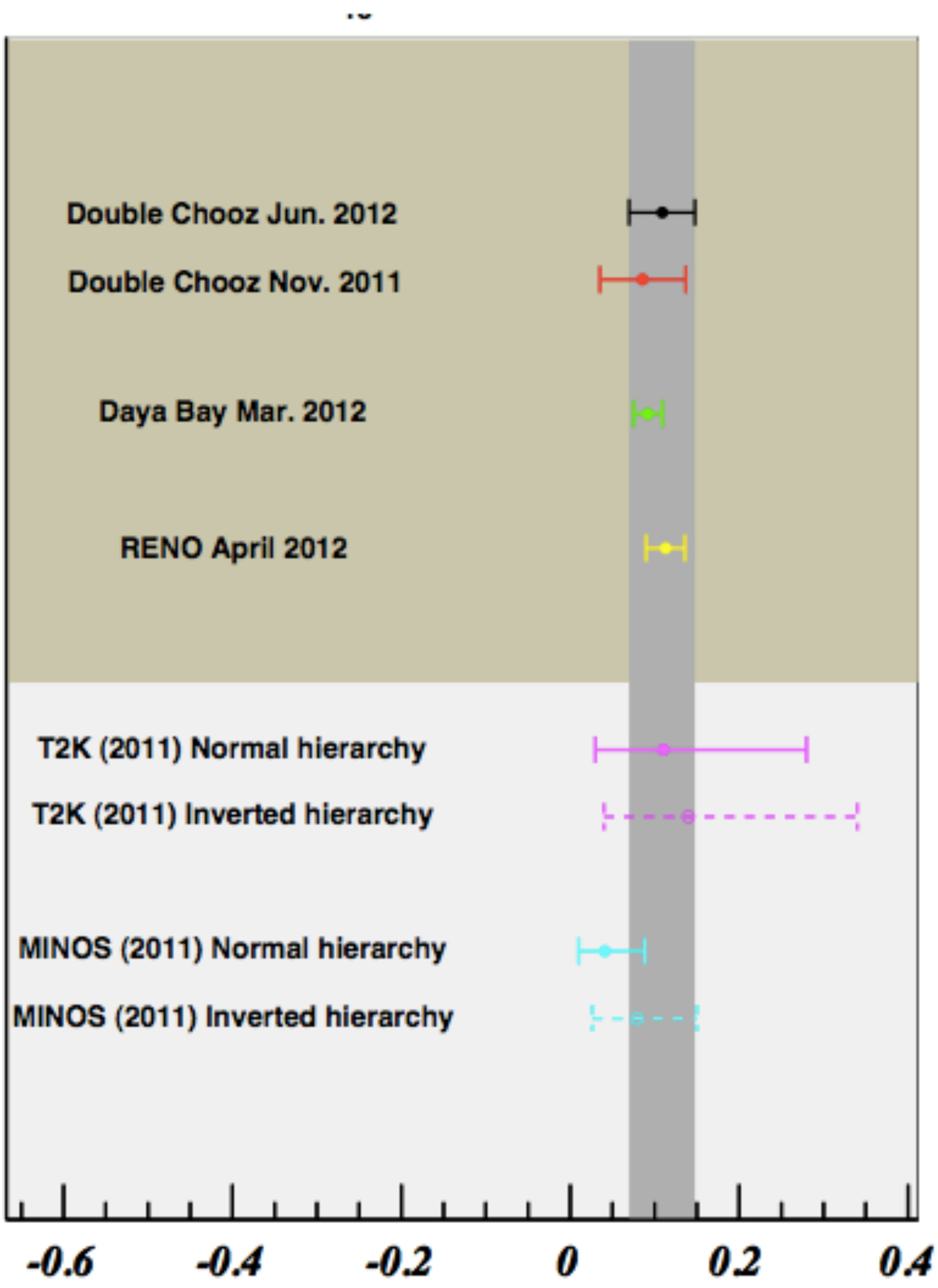
$$\delta P_{\mu\mu} \approx 0.01$$

$$\delta\vartheta_{23} \approx 0.05 \text{ rad} \leftrightarrow 2.9^0$$

improvement by about a factor 2



T2K-1
90% CL
black = normal hierarchy
red = inverted hierarchy
true value 41^0
[courtesy by Enrique Fernandez]


Double Chooz Jun. 2012
Double Chooz Nov. 2011
Daya Bay Mar. 2012
RENO April 2012
T2K (2011) Normal hierarchy
T2K (2011) Inverted hierarchy
MINOS (2011) Normal hierarchy
MINOS (2011) Inverted hierarchy
-0.6
-0.4
-0.2
0
0.2
0.4