Determination of α_s from the QCD static energy

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Universität Bern
(work done with Alexei Bazavov, Nora Brambilla, Péter Petreczky, Joan Soto and Antonio Vairo)

arXiv:1205.6155 [hep-ph]

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Outline of the talk

- Introduction: the QCD static energy

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- Lattice

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- Perturbation theory

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- Conclusions

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Energy between a static quark and a static antiquark separated a distance r, *QCD static energy* $E_0(r)$. Basic object to understand the behavior of QCD

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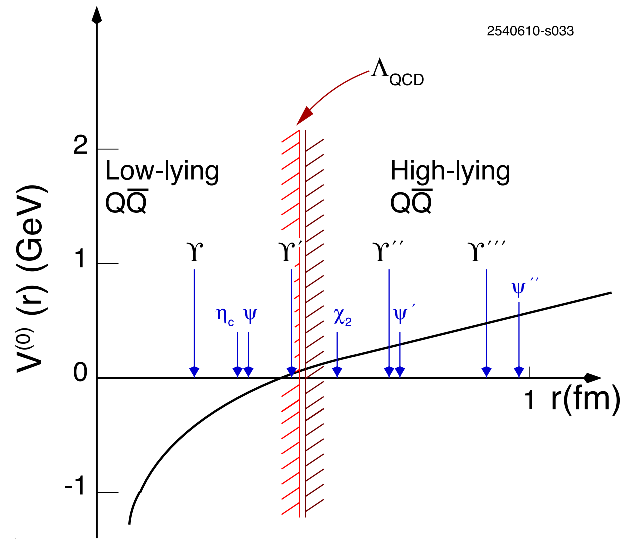

From N. Brambilla *et al.*, Eur. Phys. J. **C71** (2011) 1534

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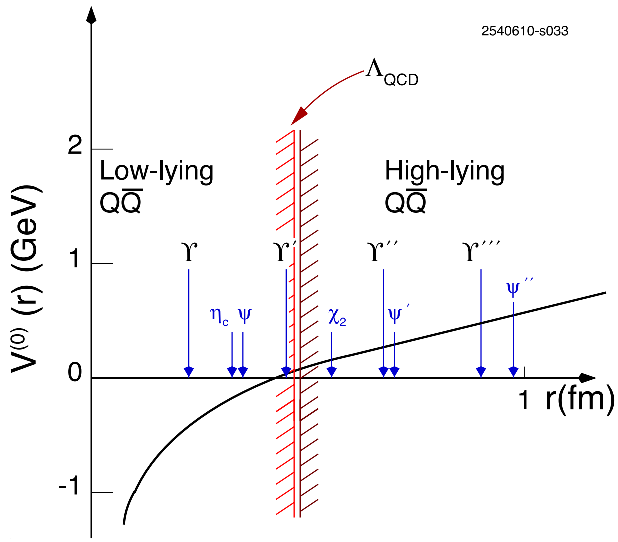

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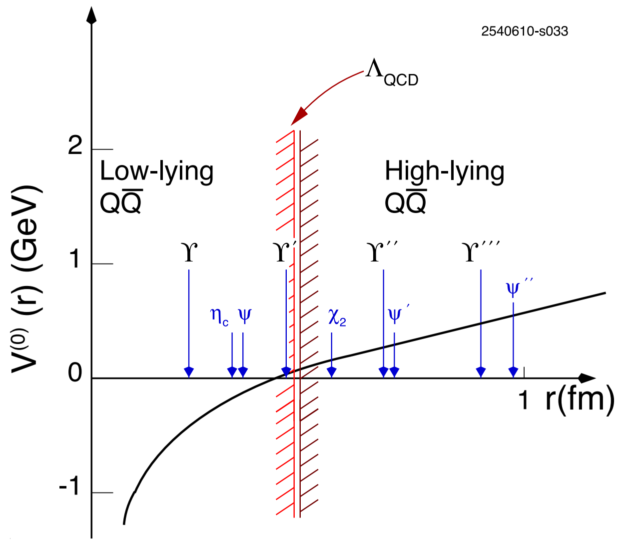

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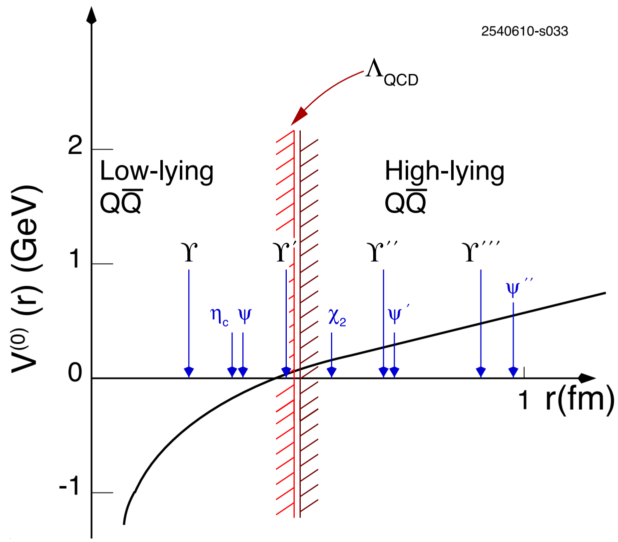

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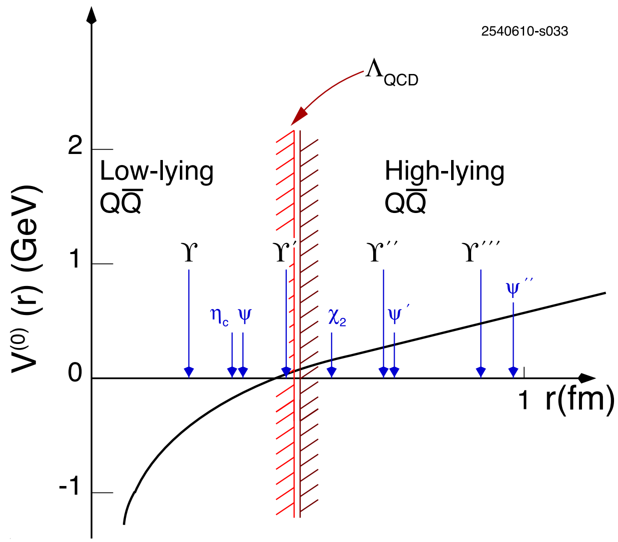

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$$\overbrace{\text{Short-distance part} \longleftrightarrow \text{Long-distance part}}^{\text{Lattice QCD}}$$

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Perturbation theory — Encodes confining dynamics

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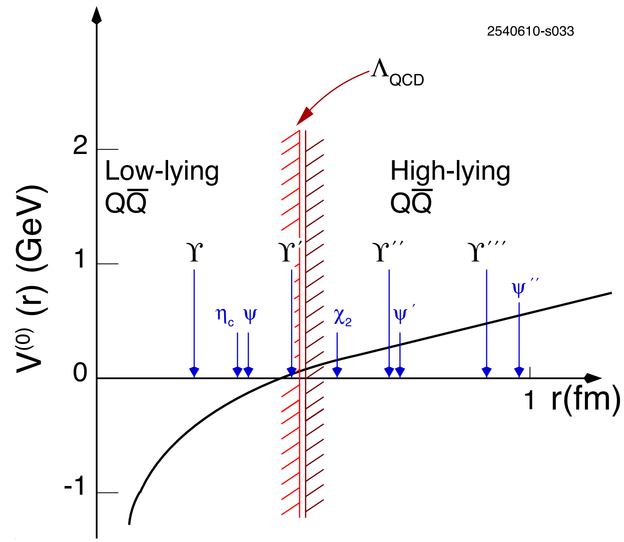

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↓ ↓

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Provides information on the region of validity of the perturbative weak-coupling approach

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Allows us to determine the strong coupling α_s

Lattice

$E_0(r)$ recently calculated on the lattice in 2+1 flavor QCD

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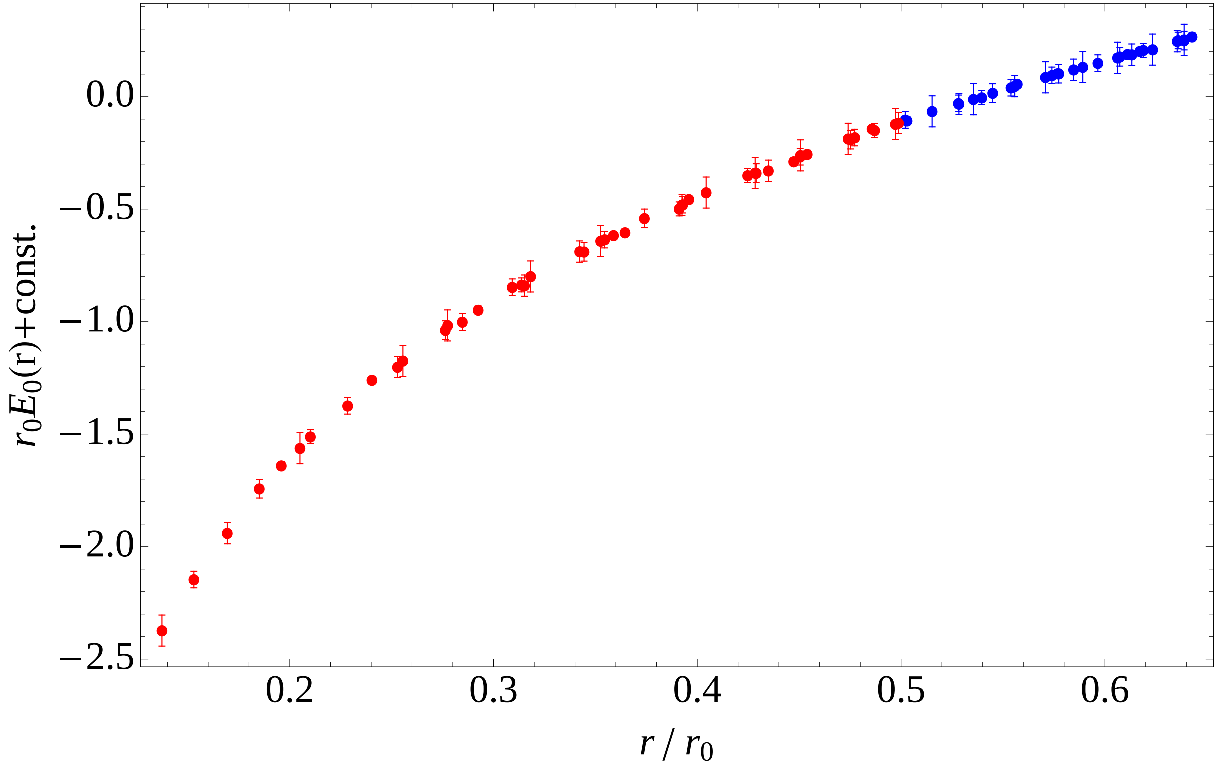

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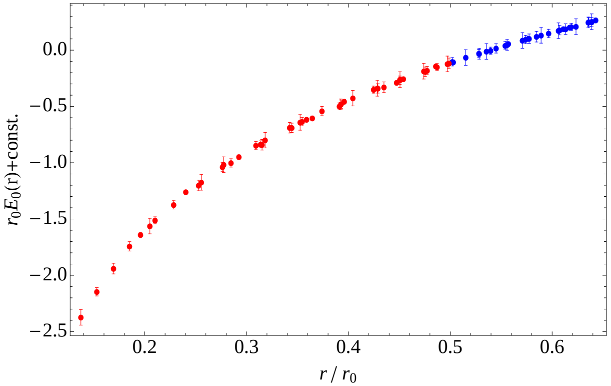

Combination of tree-level improved gauge action and HISQ action

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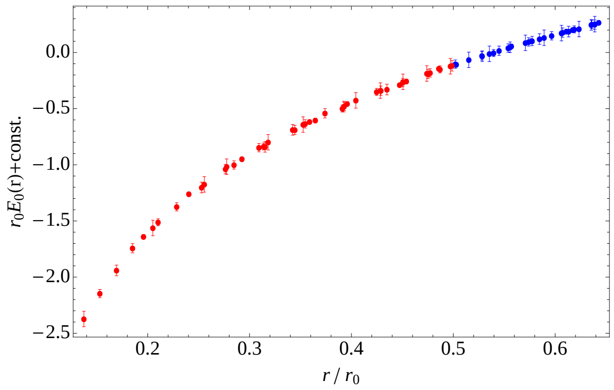

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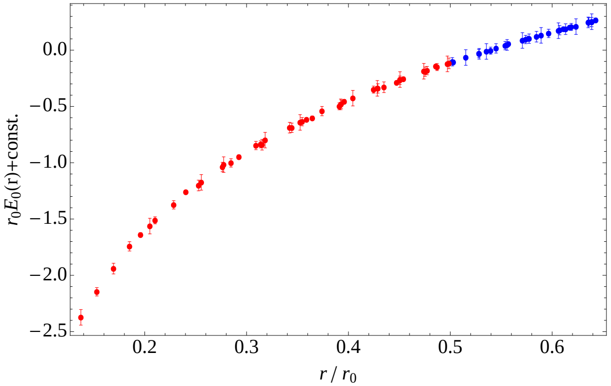

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Energy calculated in units of r_0 Sommer'93

$$r^2 \frac{dE_0(r)}{dr}|_{r=r_0} = 1.65$$

Calculation for a wide range of gauge couplings.
($\beta = 6.664, 6.740, 6.800, 6.880, 6.950, 7.030, 7.150, 7.280$; corresponds to lattice spacings $3.994/r_0 \leq a^{-1} \leq 6.991/r_0$)

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Corrected for lattice artifacts:

- Replace r by improved distance $r_I = (4\pi C_L(r))^{-1}$

  Necco Sommer'01

$$C_L(r) = \int \frac{d^3k}{(2\pi)^3} D_{00}(k_0 = 0, \vec{k}) e^{i\vec{k}\vec{r}}$$

(D_{00} is the tree-level gluon propagator)

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- Fit lattice data to the form $const - a/r + \sigma r + a'(1/r - 1/r_I)$ and subtract the last term from the lattice data

  Aubin *et al.*'04, Booth *et al.*'92

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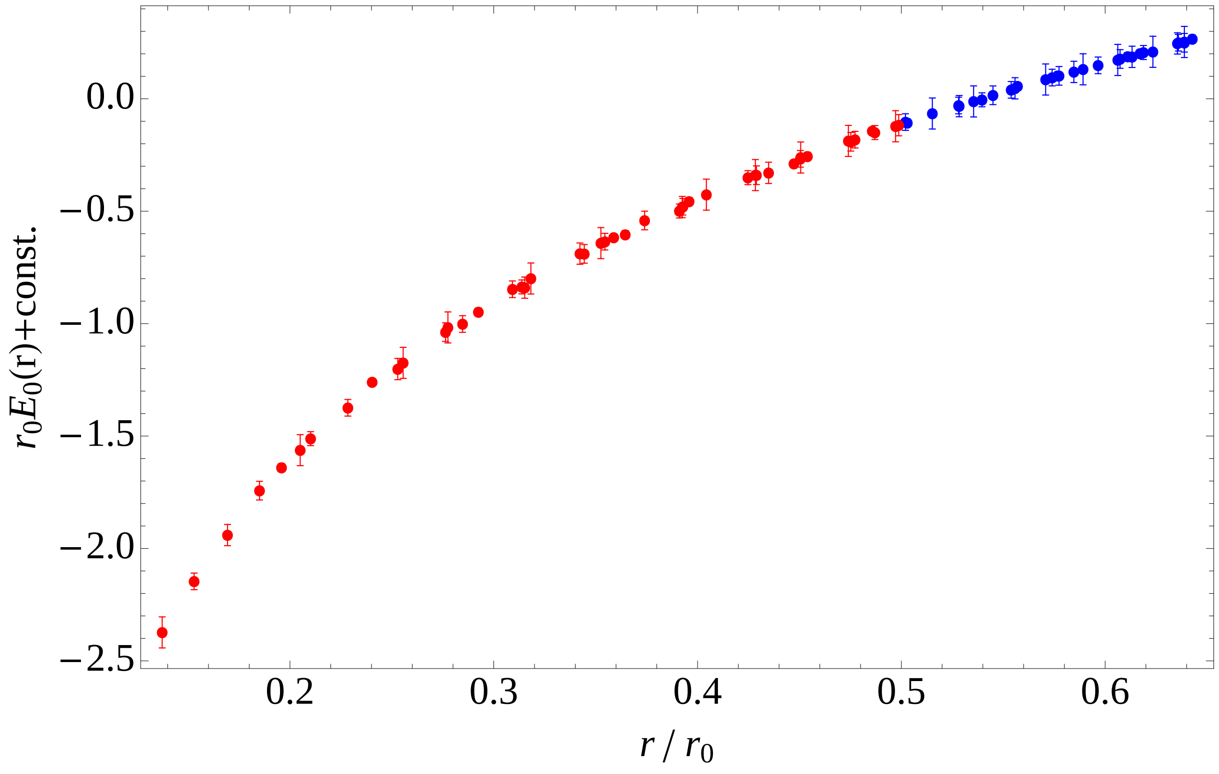
Both methods lead to the same results, within errors


0.0
−0.5
−1.0
−1.5
−2.0
−2.5
0.2
0.3
0.4
0.5
0.6
$r_0 E_0(r)$+const.
r / r_0

Perturbation theory

$$E_0(r) \sim -C_F \frac{\alpha_s(1/r)}{r}$$

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$$E_0(r) \sim -C_F \frac{\alpha_s(1/r)}{r} \left(1 + O(\alpha_s) + O(\alpha_s^2) + O(\alpha_s^3, \alpha_s^3 \ln \alpha_s) + \cdots\right)$$

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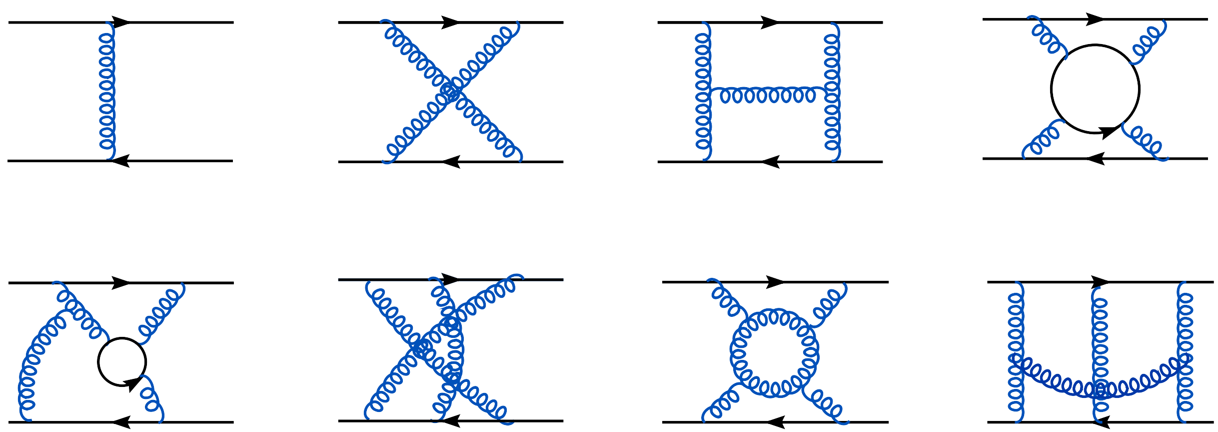

(Picture from A. V. Smirnov, V. A. Smirnov and M. Steinhauser, Phys.Rev.Lett. **104** (2010) 112002 [arXiv:0911.4742 [hep-ph]])

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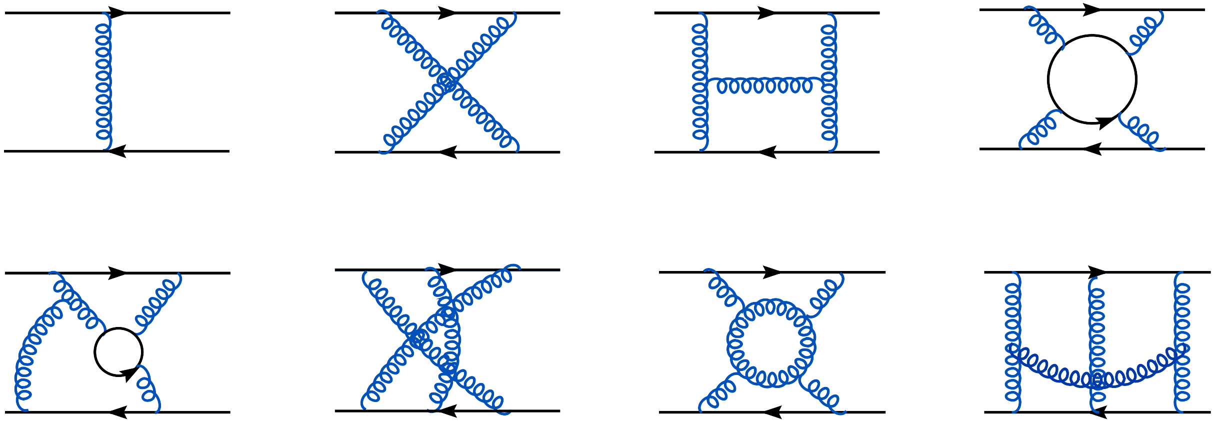

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Virtual emissions that change the color state of the pair (*Ultrasoft gluons*)

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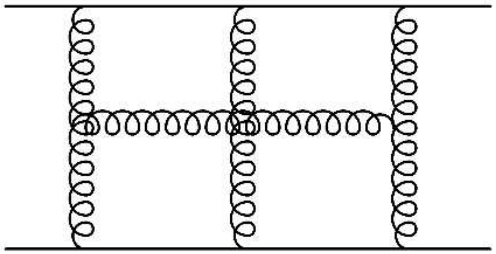
When calculated in perturbation theory infrared divergences are found, starting at three loops



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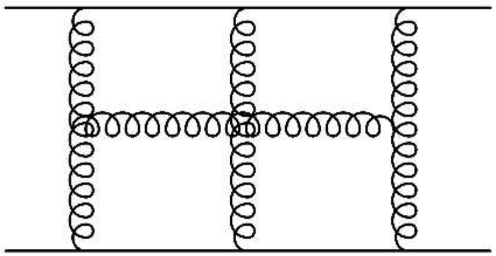
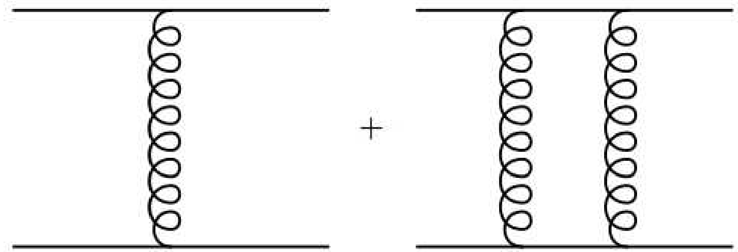
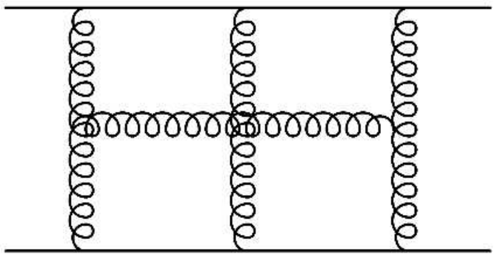
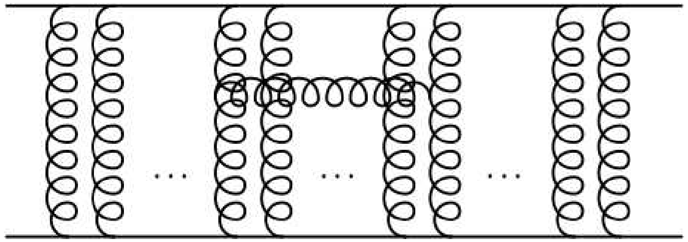


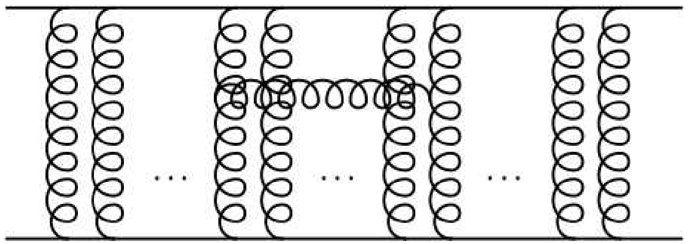
After selective resummation of certain type of diagrams (organize the expansion around the Coulombic state), logarithmic contributions are generated



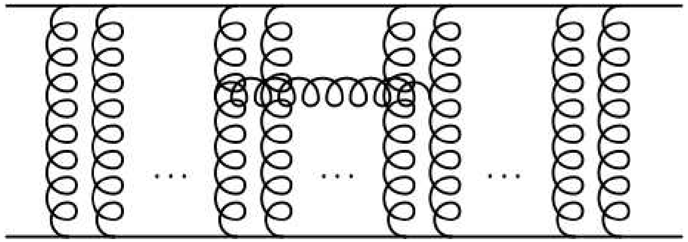
$$+\cdots \simeq \frac{1}{E-\left(\frac{p^2}{m}+V\right)}$$







Conveniently calculated in an effective theory framework. Disentangle *soft* effects at the scale $1/r$ from *ultrasoft* ones at the scale α_s/r



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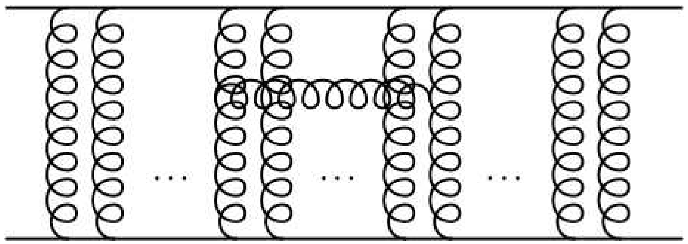
QCD



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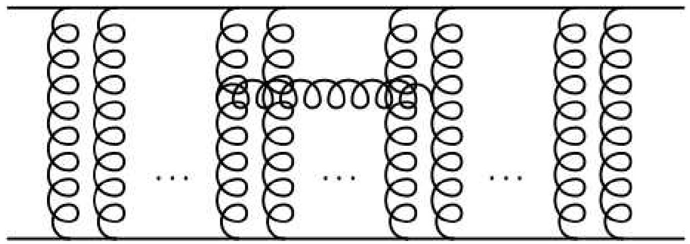
$$\mathrm{QCD} \longrightarrow \mathrm{NRQCD}$$



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$$\mathrm{QCD} \longrightarrow \mathrm{NRQCD} \xrightarrow{\frac{1}{r} \gg \frac{\alpha_s}{r} \gg \Lambda_{QCD}} \mathrm{pNRQCD}$$

potential Non-Relativistic QCD exploits the hierarchy of scales

Brambilla Pineda Soto Vairo'99

pNRQCD can be organized as a (multipole) expansion in r (and $1/m$)

$$\mathcal{L} = \int d^3\mathbf{r}\ \mathrm{Tr}\left\{ \mathrm{S}^\dagger\left[i\partial_0 - V_s(r;\mu)\right]\mathrm{S} + \mathrm{O}^\dagger\left[iD_0 - V_o(r;\mu)\right]\mathrm{O} \right\} +$$

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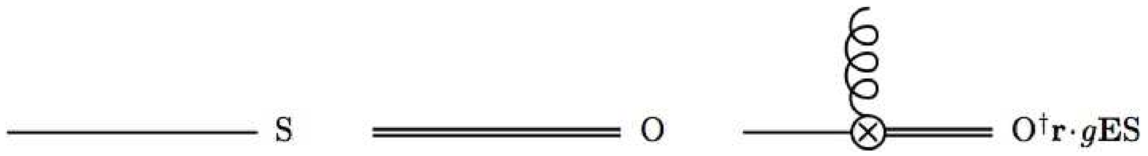
$$+\frac{V_B(r;\mu)}{2}\mathrm{Tr}\left\{\mathrm{O}^\dagger\mathbf{r}\cdot g\mathbf{E}\mathrm{O} + \mathrm{O}^\dagger\mathrm{O}\mathbf{r}\cdot g\mathbf{E}\right\} - \frac{1}{4}F^a_{\mu\nu}F^{\mu\nu\ a}$$



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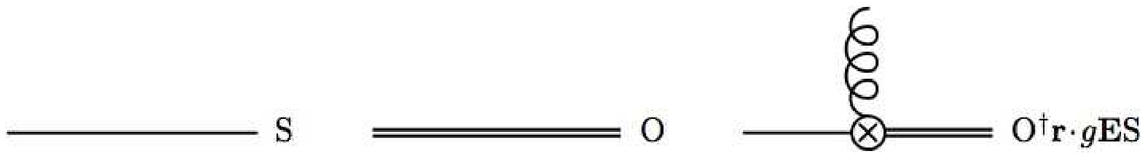

Potentials appear as Wilson coefficients in the EFT

We obtain the potential by matching NRQCD to pNRQCD. Schematically (at order r^2)

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$$E_0(r) = \lim_{T\to\infty} \frac{i}{T} \ln \left\langle P \, \exp\left\{ -ig \oint_{r\times T} dz^\mu A_\mu(z) \right\} \right\rangle = V_s(r;\mu) - i\frac{g^2}{N_c} V_A^2 \int_0^\infty dt\, e^{-it(V_o - V_s)} \langle \mathbf{r}\cdot\mathbf{E}\ \mathbf{r}\cdot\mathbf{E}\rangle(\mu)$$

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Expectation value of Wilson loop operator

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Expectation value of Wilson loop operator   Matching coefficient

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Expectation value of Wilson loop operator — Matching coefficient — Ultrasoft contribution (retardation effects)

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Expectation value of Wilson loop operator — Matching coefficient — Ultrasoft contribution (retardation effects)

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physical observable

IR divergent

UV divergent

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$$E_0(r) = \lim_{T\to\infty} \frac{i}{T} \ln \left\langle P \exp\left\{ -ig \oint_{r\times T} dz^\mu A_\mu(z) \right\} \right\rangle = V_s(r;\mu) - i\frac{g^2}{N_c} V_A^2 \int_0^\infty dt\, e^{-it(V_o - V_s)} \langle \mathbf{r}\cdot\mathbf{E}\ \mathbf{r}\cdot\mathbf{E}\rangle(\mu)$$

Expectation value of Wilson loop operator — Matching coefficient — Ultrasoft contribution (retardation effects)

Energy $\sim$ Potential $+$ Ultrasoft contribution

physical observable

IR divergent
UV divergent
} Require regularization. Scheme dependent

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Expectation value of Wilson loop operator — Matching coefficient — Ultrasoft contribution (retardation effects)

- The logarithmic contribution at three loops in the static energy can be deduced from the leading ultrasoft contribution Brambilla Pineda Soto Vairo'99, Kniehl Penin'99, the logarithmic terms at four loops from the sub-leading contribution Brambilla X.G.T. Soto Vairo'06 (using as ingredient NLO result for $\langle EE\rangle$ from Eidemüller Jamin'97)

$$
\begin{aligned}
V_s(r,\mu) = -\frac{C_F}{r}\alpha_s(1/r)\Bigg\{ & 1 + (a_1 + 2\gamma_E\beta_0)\frac{\alpha_s(1/r)}{4\pi} \\
& + \left[a_2 + \left(\frac{\pi^2}{3} + 4\gamma_E^2\right)\beta_0^2 + \gamma_E\left(4a_1\beta_0 + 2\beta_1\right)\right]\left(\frac{\alpha_s(1/r)}{4\pi}\right)^2 \\
& + \left[\frac{16\,\pi^2}{3}C_A^3\,\ln r\mu + \tilde{a}_3\right]\left(\frac{\alpha_s(1/r)}{4\pi}\right)^3 \\
& + \Bigg[a_4^{L2}\ln^2 r\mu + \left(a_4^L + \frac{16}{9}\pi^2\,C_A^3\beta_0(-5 + 6\ln 2)\right)\ln r\mu \\
& \qquad + \tilde{a}_4\Bigg]\left(\frac{\alpha_s(1/r)}{4\pi}\right)^4\Bigg\}
\end{aligned}
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Known **Not known**

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$$\begin{cases} \mu \frac{d}{d\mu} V_s = -\frac{2}{3} \frac{\alpha_s C_F}{\pi} \left(1 + 6 \frac{\alpha_s}{\pi} B\right) V_A^2 \left(V_o - V_s\right)^3 r^2 \\ \mu \frac{d}{d\mu} V_o = \frac{1}{N_c^2 - 1} \frac{2}{3} \frac{\alpha_s C_F}{\pi} \left(1 + 6 \frac{\alpha_s}{\pi} B'\right) V_A^2 \left(V_o - V_s\right)^3 r^2 \\ \mu \frac{d}{d\mu} \alpha_s = \alpha_s \beta(\alpha_s) \\ \mu \frac{d}{d\mu} V_A = 0 \\ \mu \frac{d}{d\mu} V_B = 0 \end{cases}$$

Pineda Soto'00, Brambilla X.G.T. Soto Vairo'09, Pineda Stahlhofen'11

pNRQCD can also be used to perform the resummation of the ultrasoft logarithms, via solving the renormalization group equations

$$V_s(\mu) = V_s(1/r) + 2\frac{N_c^2-1}{N_c^2}\left[(V_o - V_s)(1/r)\right]^3 r^2 \frac{\gamma_{os}^{(0)}}{\beta_0}\left\{\ln\frac{\alpha_s(\mu)}{\alpha_s(1/r)} + \left(-\frac{\beta_1}{4\beta_0} + \frac{\gamma_{os}^{(1)}}{\gamma_{os}^{(0)}}\right)\left[\frac{\alpha_s(\mu)}{\pi} - \frac{\alpha_s(1/r)}{\pi}\right]\right\}$$

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Pineda Soto '00 Brambilla X.G.T. Soto Vairo '09

The result for $E_0(r)$ is obtained by adding the potential and the ultrasoft contribution $\delta_{\rm US}$

$$\delta_{\rm US} = C_F \frac{C_A^3}{24}\frac{1}{r}\frac{\alpha_{\rm s}(\mu)}{\pi}\alpha_{\rm s}^3(1/r)\left(-2\log\frac{C_A\alpha_s(1/r)}{2r\mu} + \frac{5}{3} - 2\log 2\right) + \cdots$$

In summary, the static energy $E_0(r)$ is currently known at *3 loop +sub-leading ultrasoft log res.* (N^3LL) accuracy

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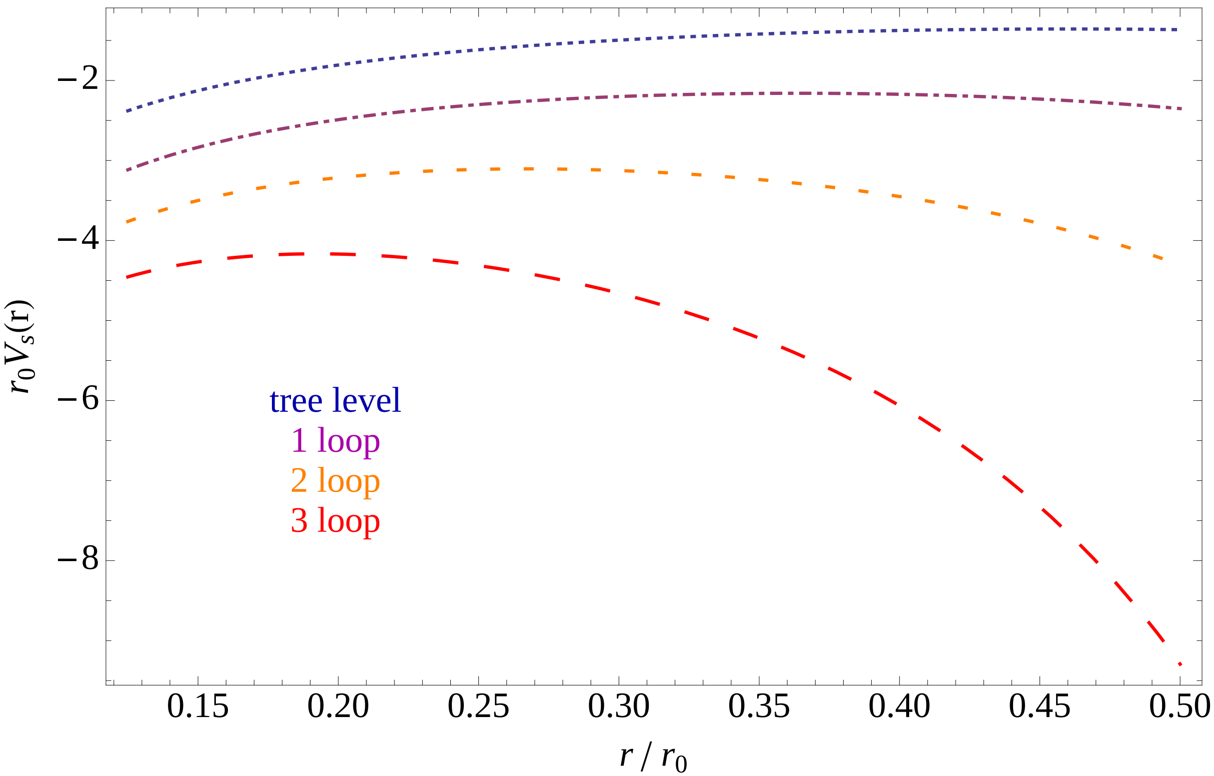
N$^{3(2)}$LL accuracy: $\alpha_s^{1+[3(2)+n]} \ln^n \alpha_s$ with $n \geq 0$

Renormalon effects in the static potential

Directly plotting the previous results ($\overline{\mathrm{MS}}$ scheme) the potential does not present a good convergent behavior



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$$R = \sum_{n=0}^{\infty} r_n \alpha^{n+1} \quad \rightarrow \quad B[R](t) = \sum_{n=0}^{\infty} r_n \frac{t^n}{n!}$$

$$R = \int_0^{\infty} dt\, e^{-t/\alpha} B[R](t)$$

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Previous discussion implicitly assumes pole mass. Badly convergent terms in the potential are also present in the pole mass. Do not use expansion parameters that can be defined less precisely (more sensitive to long-distance contributions, power corrections) than the physical observable we want

We use the so-called RS scheme Pineda'01. Add a subtraction term to the potential

$$V_s^{RS} = V_s^{\overline{\rm MS}} + R_s\, \rho \sum_{n=1}^{m} \left(\frac{\beta_0}{2\pi}\right)^n \alpha_{\rm s}(\rho)^{n+1} \sum_{k=0}^{2} d_k \frac{\Gamma(n+1+b-k)}{\Gamma(1+b-k)}$$

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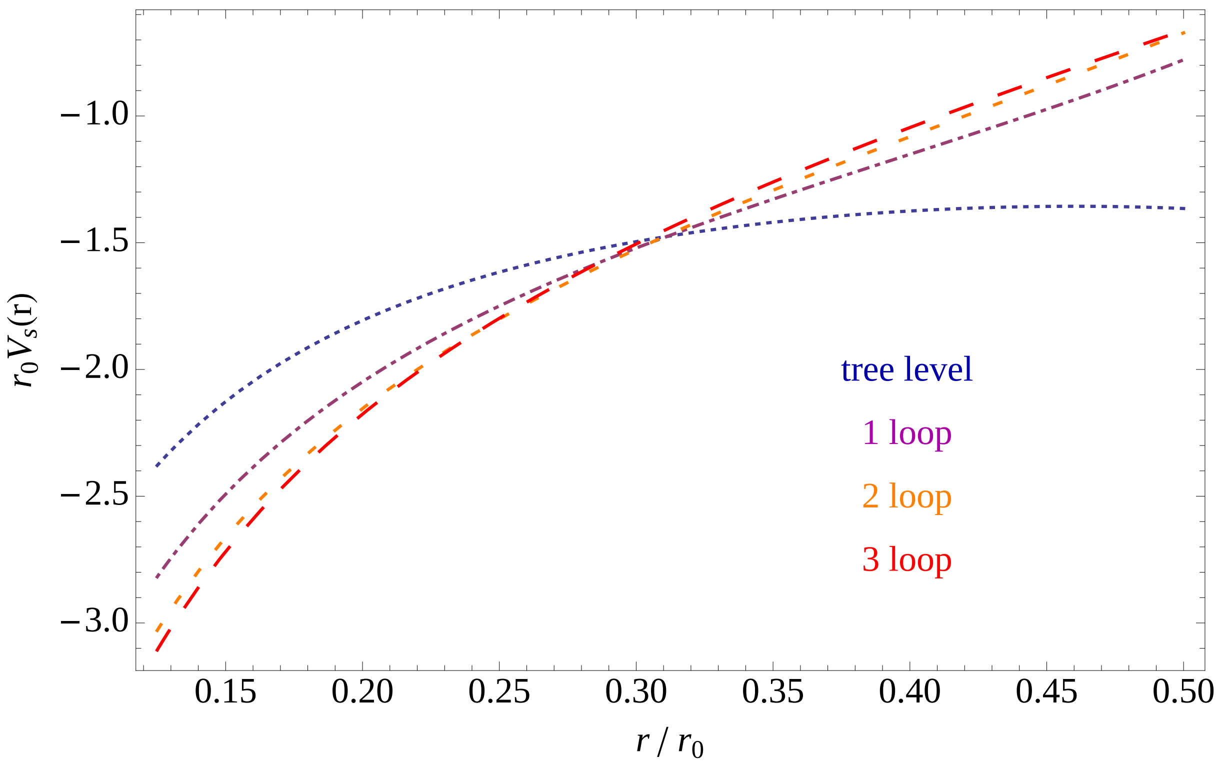
Any scheme of this kind introduces an additional dimensional scale, ϱ. Natural value around the inverse of the center of the r-range, $\varrho \sim 3.25 r_0^{-1} \to (\sim 1.5$ GeV)

Obtain a better convergent behavior.



Comparison. α_s determination

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Agreement with lattice improves when perturbative order is increased

Procedure had been already successfully applied in the quenched case, to extract $r_0\Lambda_{\overline{\rm MS}}$ at N^3LL accuracy from the quenched data of Necco Sommer'01

$$(r_0\Lambda_{\overline{\rm MS}})^{\text{quen.}} = 0.637^{+0.032}_{-0.030}$$

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Previous quenched extractions of $r_0\Lambda_{\overline{\rm MS}}$ from the static energy at lower orders in perturbation theory (e.g. Sumino'05, ...), and analyses with $n_f = 2$ (Jansen Karbstein Nagy Wagner'11, Leder Knechtli'11, Donnellan Knechtli Leder Sommer'10) exist

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In that ϱ range, use reduced χ^2 as weight of fit values $r_0\Lambda_{\overline{\rm MS}}$ and take the average. This gives our central value for $r_0\Lambda_{\overline{\rm MS}}$

Accuracy	$r_0\Lambda_{\overline{\mathrm{MS}}}$
tree level	0.395
1 loop	0.848
2 loop	0.636
N^{2}LL	0.756
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(note that in the quenched case we could use the N^3LL result)

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Cross-check: redo full analysis with energy normalized in units of r_1

$$r^2 \frac{dE_0(r)}{dr}|_{r=r_1} = 1$$

Gives compatible results

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Gives compatible results (comes from the same lattice data set in terms of r/a. But error analysis for the normalization of the energy for each lattice spacing is different in the two cases)

Final result:

$$r_0\Lambda_{\overline{\rm MS}} = 0.70 \pm 0.07,$$

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$$\alpha_s\left(1.5\mathrm{GeV}, n_f = 3\right) = 0.326 \pm 0.019$$

$$\rightarrow \quad \alpha_s\left(M_Z, n_f = 5\right) = 0.1156^{+0.0021}_{-0.0022}$$

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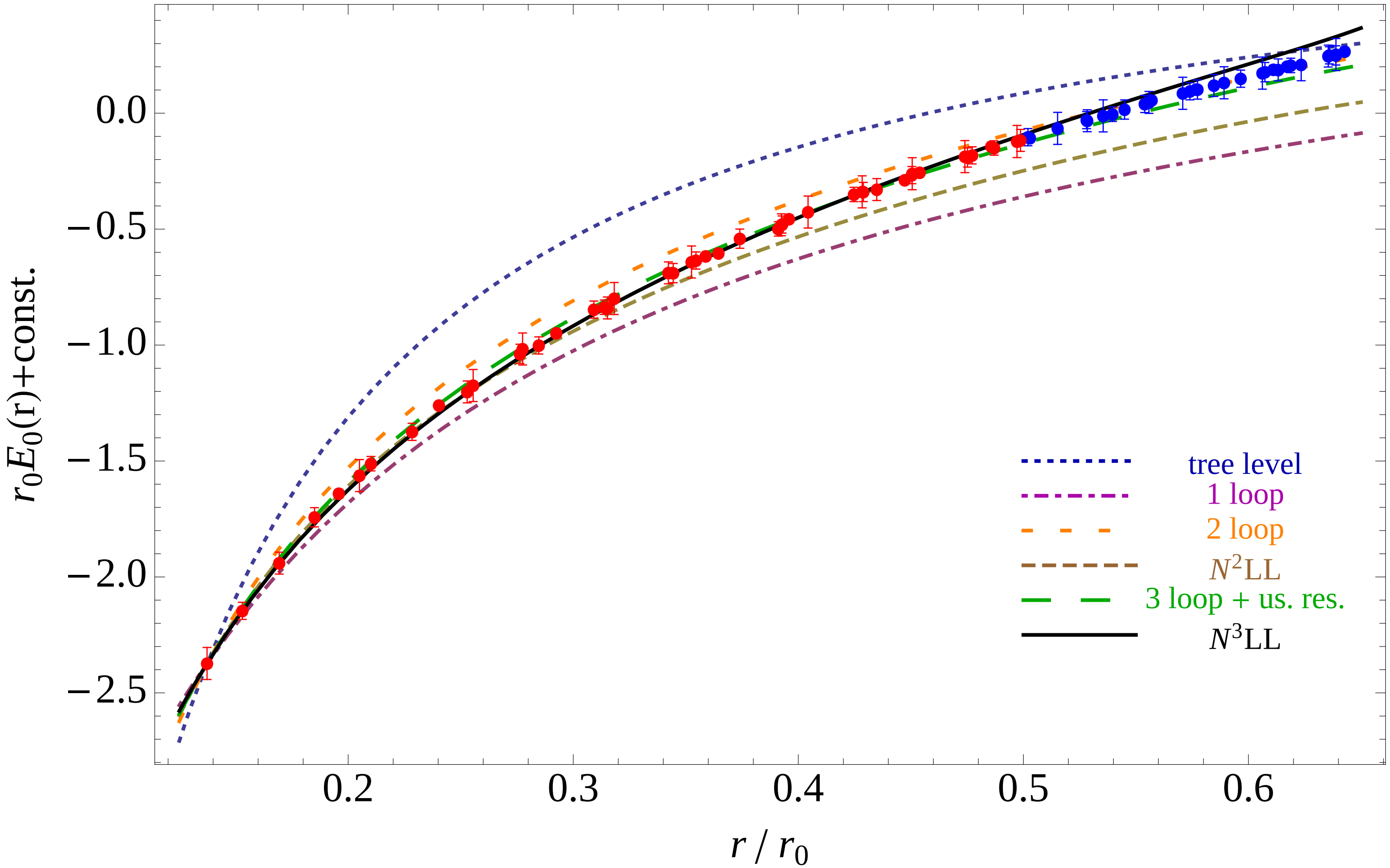
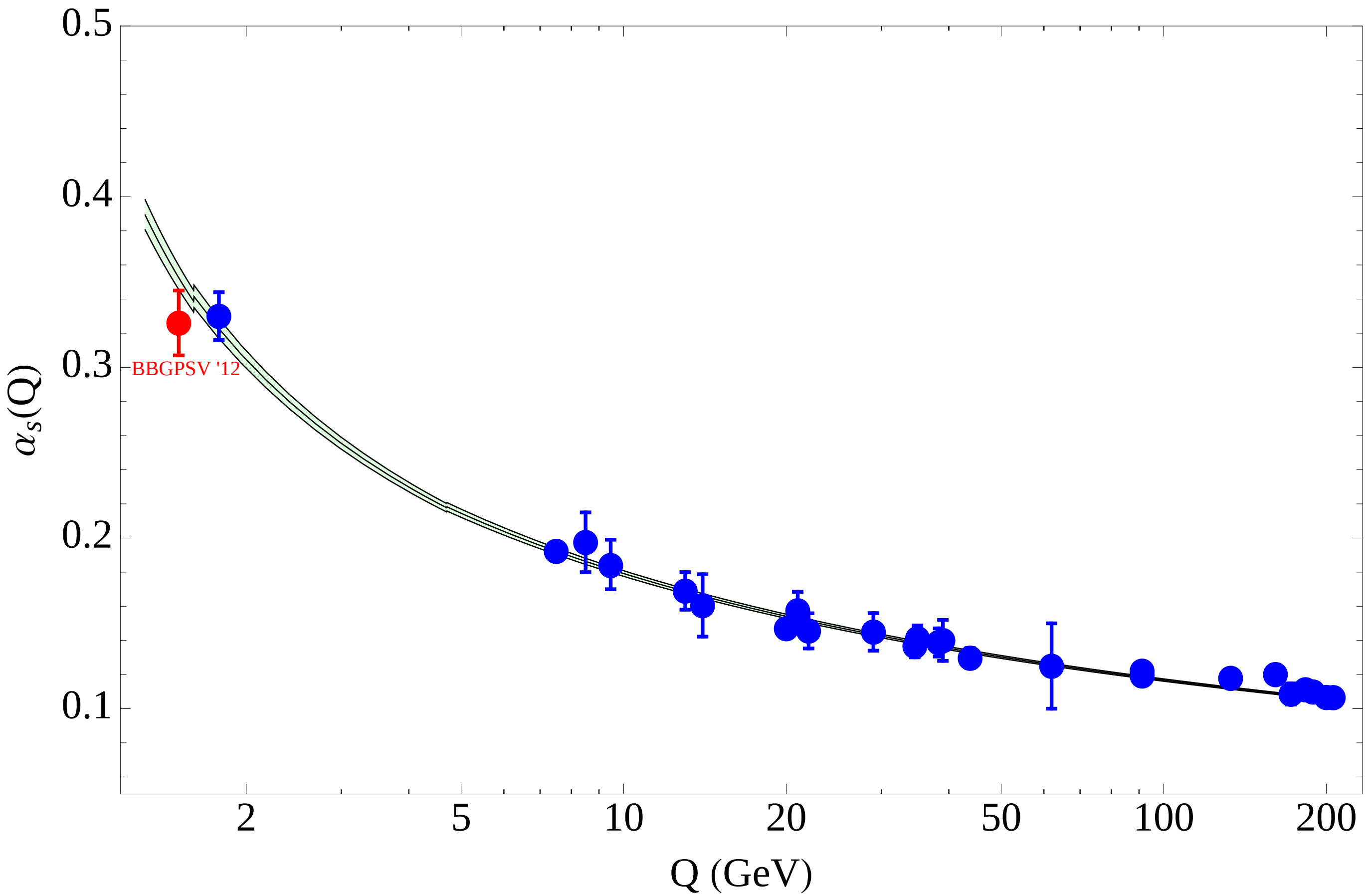
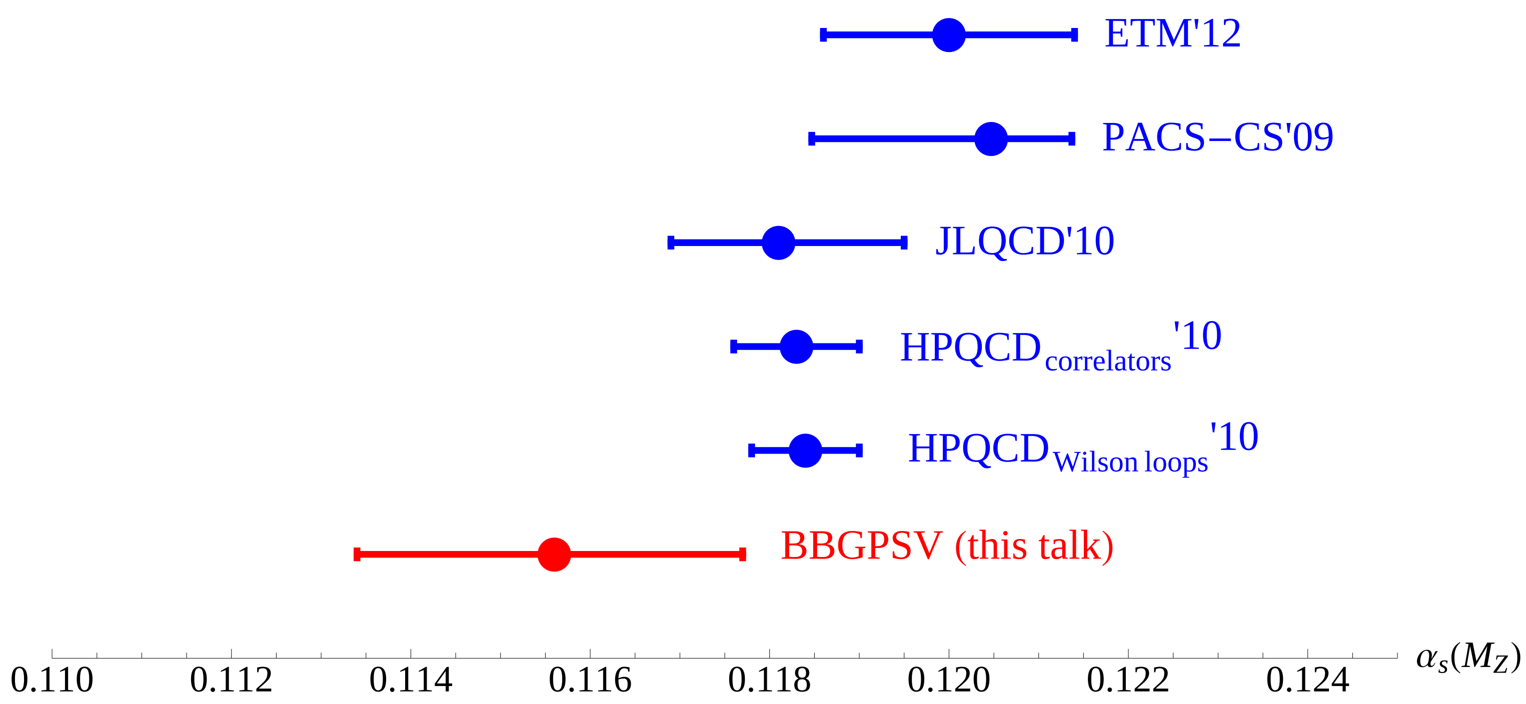


Comparison with other α_s determinations

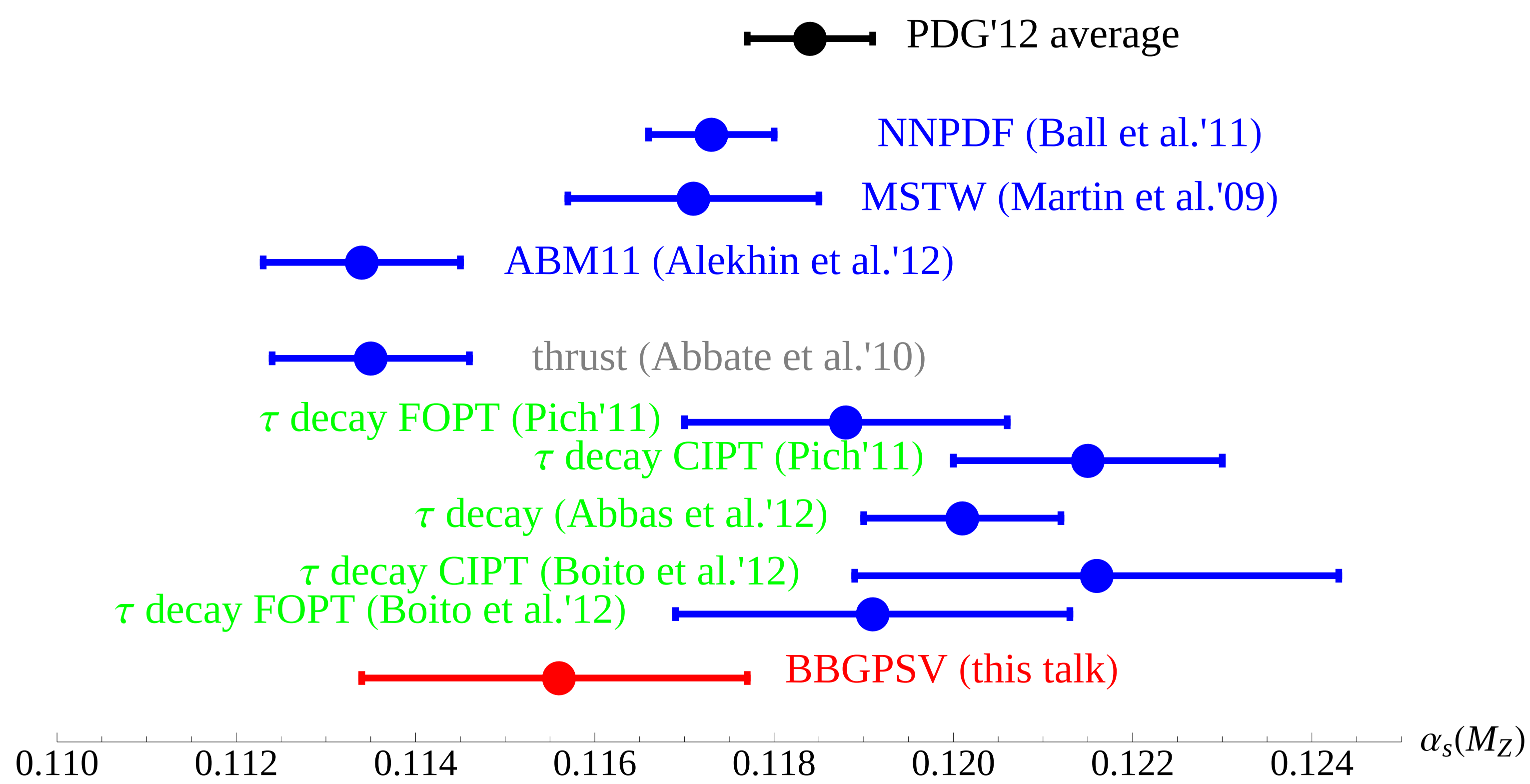


Determinations entering the world average Bethke'09 This result

Comparison with other recent lattice determinations



Comparison with a few recent determinations using other techniques



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- Natural scale of our determination corresponds to the inverse of the typical distance of the lattice data, i.e. around 1.5 GeV
- Result: $\alpha_s\,(1.5\mathrm{GeV}, n_f = 3) = 0.326 \pm 0.019$

$$\longrightarrow \quad \alpha_s\,(M_Z, n_f = 5) = 0.1156^{+0.0021}_{-0.0022}$$

Conclusions

- We have obtained a determination of α_s by comparing lattice data for the short-distance part of the QCD static energy with perturbation theory
- The result is at 3-loop (plus resummation of ultrasoft logs) accuracy
- Natural scale of our determination corresponds to the inverse of the typical distance of the lattice data, i.e. around 1.5 GeV
- Result: $\alpha_s\left(1.5\text{GeV}, n_f = 3\right) = 0.326 \pm 0.019$

$$\longrightarrow \quad \alpha_s\left(M_Z, n_f = 5\right) = 0.1156^{+0.0021}_{-0.0022}$$

- Independent of other determinations in the world average