JET BROADENING IN EFFECTIVE FIELD THEORY: WHEN DIMENSIONAL REGULARIZATION FAILS

[GUIDO BELL]

based on: T. Becher, GB, M. Neubert, Phys. Lett. B 704 (2011) 276
T. Becher, GB, Phys. Lett. B 713 (2012) 41
T. Becher, GB, work in progress

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OUTLINE

EVENT SHAPE VARIABLES

FACTORIZATION

- BRIEF REVIEW OF THRUST ANALYSIS
- FACTORIZATION BREAKDOWN FOR BROADENING
- ANALYTIC REGULARIZATION IN SCET

RESUMMATION

- COLLINEAR ANOMALY
- EXTENSION TO NNLL

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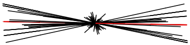
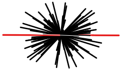
Canonical event shape

Thrust:

$$T = \frac{1}{Q} \max_{\vec{n}} \left(\sum_i |\vec{p}_i \cdot \vec{n}_T| \right)$$

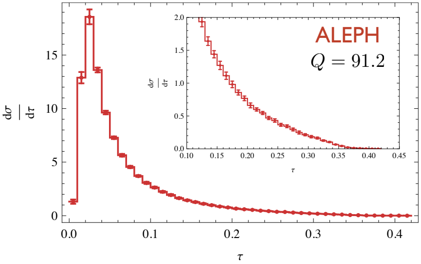


two-jet like: $T \simeq 1$



spherical: $T \simeq 1/2$

Thrust distribution precisely measured at LEP $(\tau = 1 - T)$



in the two-jet region $\tau \simeq 0$

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} \simeq \frac{\alpha_s C_F}{2\pi}\left[-\frac{4\ln\tau+3}{\tau} + \dots\right]$$

$\Rightarrow$ Sudakov logs require resummation

Motivation

Why should we study e^+e^- event shapes in 2012?

- perturbation theory $+$ resummation $+$ non-perturbative effects

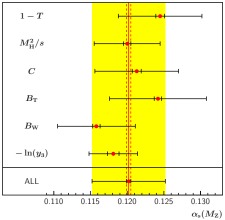
  $\Rightarrow$ clean environment to test our understanding of QCD

- precision determination of α_s

  final result from LEP QCD working group:

$$\boxed{\alpha_s(M_Z) = 0.1202 \pm 0.0003 \text{ (stat)} \pm 0.0049 \text{ (syst)}}$$

  completely dominated by theoretical uncertainty (± 0.0047)



Renewed interest due to theory advances

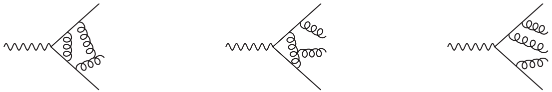
- fixed-order calculation extended to NNLO [Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 07; Weinzierl 08]

- resummations beyond NLL using SCET [Becher, Schwartz 08; Chien, Schwartz 10; Becher, GB in preparation]

The NNLO calculation

[Gehrmann-De Ridder, Gehrmann, Glover, Heinrich 07; Weinzierl 08]

First NNLO calculation for 3-jet observables



- $\mathcal{O}(100)$ diagrams with complicated loop integrals
- individual terms are IR-divergent $\Rightarrow$ highly non-trivial IR subtractions

$\Rightarrow$ two Monte-Carlo implementations for generic 3-jet observables

Used in various α_s determinations

[Dissertori et al 07,09]

- NNLO: $\alpha_s(M_Z) = 0.1240 \pm 0.0008\ \text{(stat)} \pm 0.0010\ \text{(exp)} \pm 0.0011\ \text{(had)} \pm 0.0029\ \text{(theo)}$
- NNLO + NLL: $\alpha_s(M_Z) = 0.1224 \pm 0.0009\ \text{(stat)} \pm 0.0009\ \text{(exp)} \pm 0.0012\ \text{(had)} \pm 0.0035\ \text{(theo)}$

$\Rightarrow$ further improvements require to go beyond NLL resummation!

Beyond NLL?

Traditional resummations are based on the coherent branching algorithm

[Catani, Trentadue, Turnock, Webber 93]

- sums probabilities for independent gluon emissions
- apparently hard to extend beyond NLL

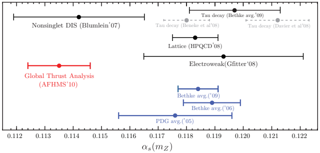
In SCET resummations are formulated in an operator language on the amplitude level

- extension to higher orders requires standard EFT techniques
- thrust analysis extended by two orders to N^3LL accuracy [Becher, Schwartz 08]
- field theoretical treatment of power corrections [Abbate, Fickinger, Hoang, Mateu, Stewart 10]

  two-dimensional fit to world thrust data

  $\alpha_s(M_Z) = 0.1135 \pm 0.0002\ (\text{exp}) \pm 0.0005\ (\text{had}) \pm 0.0009\ (\text{pert})$

  very precise but 3.7σ lower than world average?



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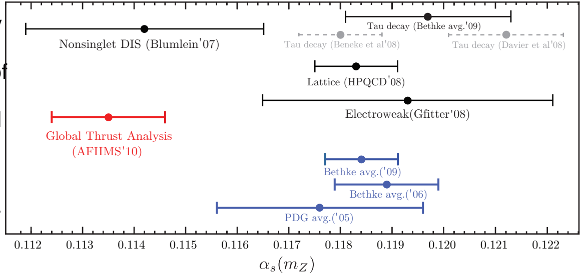

Event shape studies in SCET

Heavy jet mass: [Chien, Schwartz 10]

$$\boxed{\rho = \frac{1}{Q^2} \max\left(M_L^2, M_R^2\right)} \qquad \text{hemisphere jet masses} \quad M_{L/R}^2 = \Big(\sum_{i \in L/R} p_i\Big)^2$$

- similar to thrust $\Rightarrow$ again N^{3}LL resummation
- field theoretical treatment of power corrections more involved [talk by Mateu at SCET 2011]

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Total and wide jet broadening: [Chiu, Jain, Neill, Rothstein 11; Becher, GB, Neubert 11]

$$\boxed{\begin{aligned} b_T &= b_L + b_R \\ b_W &= \max\left(b_L, b_R\right) \end{aligned}}$$

hemisphere jet broadenings $b_{L/R} = \frac{1}{2} \sum_{i \in L/R} |\vec{p}_i \times \vec{n}_T|$

- orthogonal to thrust (measure transverse momentum distribution)
- different type of factorization formula $\Rightarrow$ aim at NNLL resummation

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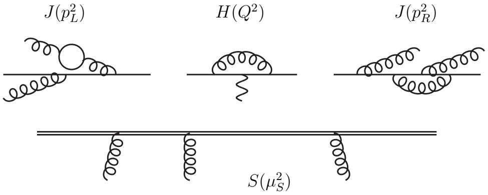
Thrust in SCET

In the two-jet limit $\tau \to 0$ the thrust distribution factorizes as [Fleming, Hoang, Mantry, Stewart 07; Schwartz 07]

$$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau} = H(Q^2,\mu) \int dp_L^2 \int dp_R^2 \; J(p_L^2,\mu) \; J(p_R^2,\mu) \; S\Big(\tau Q - \frac{p_L^2+p_R^2}{Q},\mu\Big)$$

multi-scale problem: $Q^2 \gg p_L^2 \sim p_R^2 \sim \tau Q^2 \gg \tau^2 Q^2$

hard — collinear — soft



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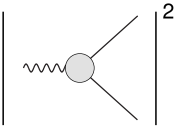
Hard function:

- on-shell vector form factor of a massless quark

$H(Q^2) =$ 

- known to three-loop accuracy [Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser 09; Gehrmann, Glover, Huber, Ikizlerli, Studerus 10]
- also enters Drell-Yan and DIS in the endpoint region

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hard — collinear — soft

Jet function:

- imaginary part of quark propagator in light-cone gauge

$$J(p^2) \sim \text{Im}\left[\text{F.T.}\left\langle 0\left|\frac{\not{n}\bar{\not{n}}}{4}W^\dagger(0)\psi(0)\;\bar\psi(x)W(x)\frac{\bar{\not{n}}\not{n}}{4}\right|0\right\rangle\right] \qquad W(x) = \mathbf{P}\,\exp\left(ig_s\int_{-\infty}^{0} ds\;\bar n\cdot A(x+s\bar n)\right)$$

- known to two-loop accuracy (anomalous dimension to three-loop) [Becher, Neubert 06]
- also enters inclusive B decays and DIS in the endpoint region

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multi-scale problem: $Q^2 \gg p_L^2 \sim p_R^2 \sim \tau Q^2 \gg \tau^2 Q^2$

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Soft function:

- Wilson lines along the directions of energetic quarks

$$S(\omega) = \sum_X \Big|\Big\langle X\Big|S_n^\dagger(0)\,S_{\bar n}(0)\Big|0\Big\rangle\Big|^2 \;\delta(\omega - n\cdot p_{X_n} - \bar n\cdot p_{X_{\bar n}}) \qquad S_n(x) = \mathbf{P}\,\exp\Big(ig_s\int_{-\infty}^0 ds\; n\cdot A_s(x+sn)\Big)$$

- determined to two-loop from matching to fixed order calculation [Becher, Schwartz 08; Hoang, Kluth 08]
- confirmed by several direct calculations [Kelley, Schwartz, Schabinger, Zhu 11; Monni, Gehrmann, Luisoni 11; Hornig, Lee, Stewart, Walsh Zuberi 11]
- anomalous dimension known to three-loop

How does resummation work (roughly)?

Let us have a closer look at the one-loop expressions

$$H(Q^2,\mu) = 1 + \frac{\alpha_s C_F}{4\pi}\left[-2\ln^2\frac{Q^2}{\mu^2} + 6\ln\frac{Q^2}{\mu^2} - 16 + \frac{7\pi^2}{3}\right]$$

$$J(p^2,\mu) = \delta(p^2) + \frac{\alpha_s C_F}{4\pi}\left[\left(\frac{4\ln(p^2/\mu^2)-3}{p^2}\right)_*^{[\mu^2]} + (7-\pi^2)\,\delta(p^2)\right]$$

$$S(\omega,\mu) = \delta(\omega) + \frac{\alpha_s C_F}{4\pi}\left[\left(\frac{-16\ln(\omega/\mu)}{\omega}\right)_*^{[\mu]} + \frac{\pi^2}{3}\,\delta(\omega)\right]$$

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General structure:

- logarithms $\Leftrightarrow$ divergences

  anomalous dimensions of EFT operators $\Rightarrow$ resum logs via RG techniques

$$\frac{d}{d\ln\mu} H(Q^2,\mu) = \left[2\Gamma_{\rm cusp}(\alpha_s)\,\ln\frac{Q^2}{\mu^2} + 4\gamma^q(\alpha_s)\right] H(Q^2,\mu)$$

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Notice: there is no large log when each function is evaluated at its natural scale!

Angularities

Interesting class of event shape variables [Berger, Kucs, Sterman 03]

$$\tau_a = \frac{1}{Q} \sum_i E_i \, (\sin\theta_i)^a \, (1 - |\cos\theta_i|)^{1-a}$$

- interpolates between thrust ($a = 0$) and broadening ($a = 1$)
- infrared safe for $a < 2$, but standard factorization only for $a < 1$

SCET analysis [Hornig, Lee, Ovanesyan 09]

- relevant scales: $\mu_H^2 \sim Q^2 \quad \gg \quad \mu_J^2 \sim Q^2 \tau_a^{\frac{2}{2-a}} \quad \gg \quad \mu_S^2 \sim Q^2 \tau_a^2$

  thrust: $\mu_H^2 \sim Q^2 \quad \gg \quad \mu_J^2 \sim Q^2 \tau \quad \gg \quad \mu_S^2 \sim Q^2 \tau^2$ $\quad(\mathrm{SCET_I})$

  broadening: $\mu_H^2 \sim Q^2 \quad \gg \quad \mu_J^2 \sim Q^2 B^2 \quad \sim \quad \mu_S^2 \sim Q^2 B^2$ $\quad(\mathrm{SCET_{II}})$

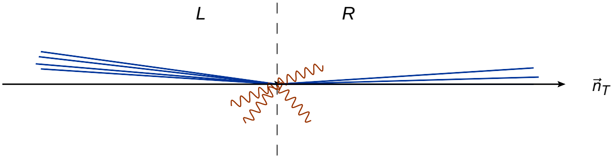
$\Rightarrow$ factorization formula for broadening will be different (and more complicated)

Jet broadening

In the two-jet limit $b_L \sim b_R \to 0$ expect that the broadening distribution factorizes as

$$\frac{1}{\sigma_0}\frac{d^2\sigma}{db_L\,db_R} = H(Q^2,\mu)\int db_L^s \int db_R^s \int d^{d-2}p_L^\perp \int d^{d-2}p_R^\perp \; \mathcal{J}_L(b_L - b_L^s, p_L^\perp,\mu)\; \mathcal{J}_R(b_R - b_R^s, p_R^\perp,\mu)\; \mathcal{S}(b_L^s, b_R^s, -p_L^\perp, -p_R^\perp,\mu)$$

two-scale problem: $Q^2 \quad \gg \quad b_L \sim b_R$



- relevant modes have $p_{\text{coll}}^\perp \sim p_{\text{soft}}^\perp \sim b_{L,R} \quad \Rightarrow \quad$ factorization in SCET_{II}
- jet recoils against soft radiation

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Hard function:

- precisely the same object as for thrust
- recall the RG equation

$$\frac{d}{d\ln\mu} H(Q^2,\mu) = \left[2\Gamma_{\text{cusp}}(\alpha_s)\,\ln\frac{Q^2}{\mu^2} + 4\gamma^q(\alpha_s)\right] H(Q^2,\mu)$$

$\Rightarrow$ there is a hidden Q-dependence in the second line!

thrust $\frac{\mu_J^2}{\mu_S} = \frac{\tau Q^2}{\tau Q} = Q$ $\Leftrightarrow$ broadening $\frac{\mu_J^2}{\mu_S} = \frac{b^2}{b} = b$

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Some manipulations:

- Laplace transform $b_{L,R} \to \tau_{L,R}$
- Fourier transform $p_{L,R}^\perp \to x_{L,R}^\perp$
- define dimensionless variable $z_{L,R} = \frac{2|x_{L,R}^\perp|}{\tau_{L,R}}$

$\Rightarrow$ the naive factorization theorem takes the form

$$\frac{1}{\sigma_0}\,\frac{d^2\sigma}{d\tau_L\,d\tau_R} = H(Q^2,\mu)\int_0^\infty dz_L \int_0^\infty dz_R\;\overline{\mathcal{J}}_L(\tau_L, z_L,\mu)\;\overline{\mathcal{J}}_R(\tau_R, z_R,\mu)\;\overline{\mathcal{S}}(\tau_L,\tau_R,z_L,z_R,\mu)$$

Jet function

The quark jet function for broadening reads

$$\mathcal{J}(b,p^\perp) \sim \sum_X \delta(\bar{n}\cdot p_X - Q)\ \delta^{d-2}(p_X^\perp - p^\perp)\ \delta\Big(b - \tfrac{1}{2}\sum_{i\in X}|p_i^\perp|\Big)\ \Big|\Big\langle X\Big|\bar{\psi}(0)W(0)\tfrac{\bar{\not{n}}\not{n}}{4}\Big|0\Big\rangle\Big|^2$$

- delta-functions ensure that jet has given energy, $p^\perp$ and b
- tree level: $\mathcal{J}(b,p^\perp) = \delta\Big(b - \frac{1}{2}|p^\perp|\Big) \quad \Rightarrow \quad \overline{\mathcal{J}}(\tau,z) = \frac{z}{\big(1+z^2\big)^{3/2}} + \mathcal{O}(\epsilon)$

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At one-loop the calculation involves

- Wilson-line diagrams are not well-defined in dimensional regularization!

  $\displaystyle\int_0^Q \frac{dk_-}{k_-}$ diverges in the soft limit (DR regularizes $d^{d-2}k_\perp$)

- this does not happen for thrust or any SCET_I problem

$\Rightarrow$ one has to introduce an additional regulator

Regularization in SCET$_{II}$

The regularization of individual diagrams is largely arbitrary, one could try e.g.

$$\frac{1}{p^2 + i\varepsilon} \quad \to \quad \frac{1}{p^2 - \Delta + i\varepsilon}, \quad \frac{(\nu^2)^\alpha}{(p^2 + i\varepsilon)^{1+\alpha}}, \quad \dots$$

- trivial for QCD, but regularizes ill-defined EFT diagrams
- spoils gauge-invariance and eikonal structure of Wilson line emissions

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In a massless theory it is sufficient to regularize phase space integrals [Becher, GB 11]

$$\int d^d k \;\delta(k^2)\;\theta(k^0) \quad \Rightarrow \quad \int d^d k \left(\frac{\nu_+}{k_+}\right)^\alpha \delta(k^2)\;\theta(k^0)$$

- does not modify SCET at all $\Rightarrow$ keeps gauge-invariance and eikonal structure
- analytic, minimal and adopted to the problem (LC propagators)

Why does it work?

Our new prescription amounts to

$$\int d^d k \;\delta(k^2)\;\theta(k^0) \quad\Rightarrow\quad \int d^d k \left(\frac{\nu_+}{k_+}\right)^{\alpha} \delta(k^2)\;\theta(k^0)$$

- virtual corrections do not need regularization

  matrix elements of Wilson lines in QCD $\Rightarrow$ the same for thrust and broadening

  technical reason: $\int d^{d-2} k_\perp \;\; f(k_\perp, k_+) \;\sim\; k_+^{-\epsilon}$

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- virtual corrections do not need regularization

  matrix elements of Wilson lines in QCD $\Rightarrow$ the same for thrust and broadening

  technical reason: $\int d^{d-2}k_\perp \; f(k_\perp, k_+) \sim k_+^{-\epsilon}$

- required for observables sensitive to transverse momenta

  $f(k_\perp, k_+) \sim \delta^{d-2}(k_\perp - p_\perp)$ $\Rightarrow$ factor $k_+^{-\epsilon}$ absent $\Rightarrow$ reinstalled as $k_+^{-\alpha}$

  can show that the prescription regularizes all LC singularities in SCET

- not sufficient for cases where virtual corrections are ill-defined

  examples: electroweak Sudakov corrections, Regge limits

Jet function revisited

With the additional regulator in place, the jet functions can be evaluated

$$\mathcal{J}_L(b, p^\perp = 0) = \delta(b) + \frac{C_F\alpha_s}{2\pi}\,\frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\,\frac{1}{b}\left(\frac{\mu}{b}\right)^{2\epsilon}\left[1-\epsilon+\frac{4\,\Gamma(2+\alpha)\,\Gamma(\alpha)}{\Gamma(2+2\alpha)}\left(\frac{Q\nu_+}{b^2}\right)^{\alpha}\right]$$

$$\mathcal{J}_R(b, p^\perp = 0) = \delta(b) + \frac{C_F\alpha_s}{2\pi}\,\frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\,\frac{1}{b}\left(\frac{\mu}{b}\right)^{2\epsilon}\left[1-\epsilon+\frac{4\,\Gamma(-\alpha)}{\Gamma(2-\alpha)}\left(\frac{\nu_+}{Q}\right)^{\alpha}\right]$$

- ordered limit $\alpha \to 0,\ \varepsilon \to 0$ generates a pole in the analytic regulator
- note the characteristic scaling $\left(\frac{\nu_+}{k_+}\right)^{\alpha}$ in each region

For $p^\perp \neq 0$ the computation is considerably more involved $(\to$ later)

$$\overline{\mathcal{J}}_L(\tau, z) = \overline{\mathcal{J}}_L^{(0)}(\tau, z)\left[1 - \frac{C_F\alpha_s}{\pi}\,\frac{1}{\alpha}\left(\frac{1}{\epsilon} + \ln\left(\mu^2\bar{\tau}^2\right) + 2\ln\frac{\sqrt{1+z^2}+1}{4}\right)\left(Q\nu_+\bar{\tau}^2\right)^{\alpha} + \dots\right]$$

- divergent term has non-trivial z-dependence

Soft function

The soft function for broadening reads

$$\mathcal{S}(b_L, b_R, p_L^\perp, p_R^\perp) \sim \sum_{X_L, X_R} \delta^{d-2}(p_{X_L}^\perp - p_L^\perp)\ \delta^{d-2}(p_{X_R}^\perp - p_R^\perp)$$
$$\delta\Big(b_L - \tfrac{1}{2}\sum_{i\in X_L} |p_{L,i}^\perp|\Big)\ \delta\Big(b_R - \tfrac{1}{2}\sum_{j\in X_R} |p_{R,j}^\perp|\Big)\ \Big|\Big\langle X_L X_R \Big| S_n^\dagger(0)\, S_{\bar n}(0) \Big| 0 \Big\rangle\Big|^2$$

- split final state into left and right-moving particles
- tree level: $\mathcal{S}(b_L, b_R, p_L^\perp, p_R^\perp) = \delta(b_L)\,\delta(b_R)\,\delta^{d-2}(p_L^\perp)\,\delta^{d-2}(p_R^\perp) \quad \Rightarrow \quad \overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R) = 1$

At one-loop the calculation involves

$$\Rightarrow\ \overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R) = 1 + \frac{C_F \alpha_s}{\pi} \left\{ \frac{1}{\alpha} \left(\frac{1}{\epsilon} + \ln(\mu^2 \bar\tau_L^2) + 2 \ln \frac{\sqrt{1+z_L^2}+1}{4} \right) \left(\nu_+ \bar\tau_L\right)^\alpha - (L \leftrightarrow R) + \dots \right\}$$

Anomalous Q dependence

Let us now put the jet and soft functions together

$$\overline{\mathcal{J}}_L(\tau_L, z_L)\ \overline{\mathcal{J}}_R(\tau_R, z_R)\ \overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R) \;=\; \overline{\mathcal{J}}_L^{(0)}(\tau_L, z_L)\ \overline{\mathcal{J}}_R^{(0)}(\tau_R, z_R)$$

$$\Bigg\{1 + \frac{C_F\alpha_s}{\pi}\Bigg[\bigg(-\frac{1}{\alpha} - \ln\left(Q\nu_+\bar{\tau}_L^2\right) + \frac{1}{\alpha} + \ln\left(\nu_+\bar{\tau}_L\right)\bigg)\bigg(\frac{1}{\epsilon} + \ln\left(\mu^2\bar{\tau}_L^2\right) + 2\ln\frac{\sqrt{1+z_L^2}+1}{4}\bigg)$$

$$+\bigg(+\frac{1}{\alpha} + \ln\left(\frac{\nu_+}{Q}\right) - \frac{1}{\alpha} - \ln\left(\nu_+\bar{\tau}_R\right)\bigg)\bigg(\frac{1}{\epsilon} + \ln\left(\mu^2\bar{\tau}_R^2\right) + 2\ln\frac{\sqrt{1+z_R^2}+1}{4}\bigg) + \ldots\Bigg]\Bigg\}$$

- well-defined without additional regulators

Anomalous Q dependence

Let us now put the jet and soft functions together

$$\overline{\mathcal{J}}_L(\tau_L, z_L)\ \overline{\mathcal{J}}_R(\tau_R, z_R)\ \overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R)\ =\ \overline{\mathcal{J}}_L^{(0)}(\tau_L, z_L)\ \overline{\mathcal{J}}_R^{(0)}(\tau_R, z_R)$$

$$\Bigg\{1 + \frac{C_F\alpha_s}{\pi}\Bigg[\Bigg(\quad -\ln\left(Q\nu_+\bar{\tau}_L^2\right) \quad + \ln\left(\nu_+\bar{\tau}_L\right)\Bigg)\left(\frac{1}{\epsilon} + \ln\left(\mu^2\bar{\tau}_L^2\right) + 2\ln\frac{\sqrt{1+z_L^2}+1}{4}\right)$$

$$+\Bigg(\quad +\ln\left(\frac{\nu_+}{Q}\right) \quad - \ln\left(\nu_+\bar{\tau}_R\right)\Bigg)\left(\frac{1}{\epsilon} + \ln\left(\mu^2\bar{\tau}_R^2\right) + 2\ln\frac{\sqrt{1+z_R^2}+1}{4}\right) + \dots\Bigg]\Bigg\}$$

- well-defined without additional regulators
- similarly the artificial scale ν_+ drops out

Anomalous Q dependence

Let us now put the jet and soft functions together

$$\overline{\mathcal{J}}_L(\tau_L, z_L)\ \overline{\mathcal{J}}_R(\tau_R, z_R)\ \overline{\mathcal{S}}(\tau_L, \tau_R, z_L, z_R) \;=\; \overline{\mathcal{J}}_L^{(0)}(\tau_L, z_L)\ \overline{\mathcal{J}}_R^{(0)}(\tau_R, z_R)$$

$$\Bigg\{ 1 + \frac{C_F \alpha_s}{\pi} \Bigg[\Bigg(\qquad - \ln\left(Q\bar{\tau}_L\right) \qquad \Bigg) \left(\frac{1}{\epsilon} + \ln\left(\mu^2 \bar{\tau}_L^2\right) + 2 \ln \frac{\sqrt{1+z_L^2}+1}{4} \right)$$

$$+ \Bigg(\qquad - \ln\left(Q\bar{\tau}_R\right) \qquad \Bigg) \left(\frac{1}{\epsilon} + \ln\left(\mu^2 \bar{\tau}_R^2\right) + 2 \ln \frac{\sqrt{1+z_R^2}+1}{4} \right) + \ldots \Bigg] \Bigg\}$$

- well-defined without additional regulators
- similarly the artificial scale ν_+ drops out
- the hidden Q dependence shows up!

$\Rightarrow$ the naive factorization formula does not achieve a proper scale separation

How to resum a logarithm that appears in a matching calculation?

OUTLINE

EVENT SHAPE VARIABLES

FACTORIZATION

BRIEF REVIEW OF THRUST ANALYSIS

FACTORIZATION BREAKDOWN FOR BROADENING

ANALYTIC REGULARIZATION IN SCET

RESUMMATION

COLLINEAR ANOMALY

EXTENSION TO NNLL

Collinear anomaly

Will show that the Q dependence exponentiates using and extending arguments from

- electroweak Sudakov resummation [Chiu, Golf, Kelley, Manohar 07]
- p_T resummation in Drell-Yan production [Becher, Neubert 10]

Start from the logarithm of the product of jet and soft functions

$$\ln P = \ln \overline{\mathcal{J}}_L\Big(\ln\,(Q\nu_+\bar{\tau}_L^2);\ \tau_L, z_L\Big) + \ln \overline{\mathcal{J}}_R\Big(\ln\,(\tfrac{\nu_+}{Q});\ \tau_R, z_R\Big) + \ln \overline{\mathcal{S}}\Big(\ln\,(\nu_+\bar{\tau}_L);\ \tau_L, \tau_R, z_L, z_R\Big)$$

collinear: $k_+ \sim \frac{b^2}{Q}$ anticollinear: $k_+ \sim Q$ soft: $k_+ \sim b$

- use that product does not depend on ν_+ and that it is LR symmetric

$$\Rightarrow\ \ln P = \frac{k_2(\mu)}{4}\ln^2(Q^2\,\bar{\tau}_L\bar{\tau}_R) - F_B(\tau_L, z_L, \mu)\ \ln\,(Q^2\bar{\tau}_L^2) - F_B(\tau_R, z_R, \mu)\ \ln\,(Q^2\bar{\tau}_R^2) + \ln W(\tau_L, \tau_R, z_L, z_R, \mu)$$

- RG invariance implies $k_2(\mu) = 0$ to all orders

$$\Rightarrow\ \boxed{P(Q^2, \tau_L, \tau_R, z_L, z_R, \mu) = (Q^2\bar{\tau}_L^2)^{-F_B(\tau_L, z_L, \mu)}\ (Q^2\bar{\tau}_R^2)^{-F_B(\tau_R, z_R, \mu)}\ W(\tau_L, \tau_R, z_L, z_R, \mu)}$$

Final factorization formula

The corrected all-order generalization of the naive factorization formula becomes

[Becher, GB, Neubert 11]

$$\frac{1}{\sigma_0}\,\frac{d^2\sigma}{d\tau_L\,d\tau_R} = H(Q^2,\mu)\int_0^\infty dz_L \int_0^\infty dz_R\;\left(Q^2\bar{\tau}_L^2\right)^{-F_B(\tau_L,z_L,\mu)}\;\left(Q^2\bar{\tau}_R^2\right)^{-F_B(\tau_R,z_R,\mu)}\;W(\tau_L,\tau_R,z_L,z_R,\mu)$$

Our explicit calculation determines the one-loop anomaly coefficient

$$F_B(\tau,z,\mu) = \frac{C_F\alpha_s}{\pi}\left[\ln(\mu\bar{\tau}) + \ln\frac{\sqrt{1+z^2}+1}{4}\right]$$

To NLL one further needs

$$W(\tau_L,\tau_R,z_L,z_R,\mu) = \frac{z_L\,z_R}{\left(1+z_L^2\right)^{3/2}\left(1+z_R^2\right)^{3/2}}$$

$$H(Q^2,\mu) = \exp\left\{\frac{4C_F}{\beta_0^2}\left[\frac{4\pi}{\alpha_s(Q)}\left(1-\frac{1}{r}-\ln r\right)+\left(K-\frac{\beta_1}{\beta_0}\right)(1-r+\ln r)+\frac{\beta_1}{2\beta_0}\,\ln^2 r+\frac{3\beta_0}{2}\,\ln r\right]\right\}$$

Total and wide jet broadening

To NLL the Mellin inversion can be performed analytically

$$\frac{1}{\sigma_0}\,\frac{d\sigma}{db_T} \;=\; H(Q^2,\mu)\,\frac{e^{-2\gamma_E\eta}}{\Gamma(2\eta)}\,\frac{1}{b_T}\left(\frac{b_T}{\mu}\right)^{2\eta} I^2(\eta)$$

$$\frac{1}{\sigma_0}\,\frac{d\sigma}{db_W} \;=\; H(Q^2,\mu)\,\frac{2\eta\, e^{-2\gamma_E\eta}}{\Gamma^2(1+\eta)}\,\frac{1}{b_W}\left(\frac{b_W}{\mu}\right)^{2\eta} I^2(\eta)$$

where $$\eta \;=\; \frac{C_F\alpha_s(\mu)}{\pi}\,\ln\frac{Q^2}{\mu^2} \;=\; \mathcal{O}(1)$$

The non-trivial z-dependence of the anomaly coefficient is encoded in

$$I(\eta) \;=\; \int_0^\infty dz\,\frac{z}{(1+z^2)^{3/2}}\left(\frac{\sqrt{1+z^2}+1}{4}\right)^{-\eta} \;=\; \frac{4^\eta}{1+\eta}\;{}_2F_1(\eta,1+\eta,2+\eta,-1)$$

Comparison with literature

Traditional resummation

- pioneering work missed quark recoil effects $\Rightarrow$ valid to LL [Catani, Turnock, Webber 92]
- first NLL resummation by Dokshitzer et al [Dokshitzer, Lucenti, Marchesini, Salam 98]

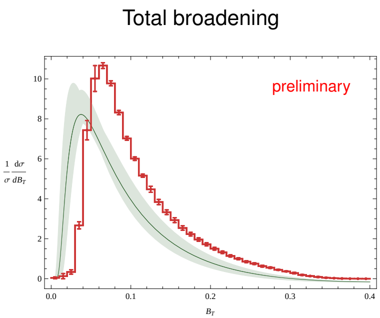
  we find complete analytical agreement with this work

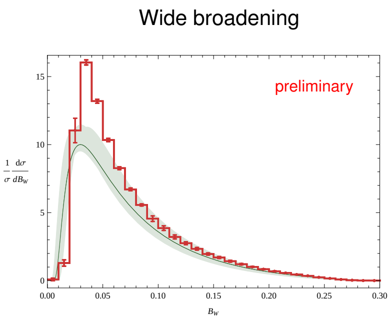
Resummation within SCET [Chiu, Jain, Neill, Rothstein 11,12]

- starts from same naive factorization formula
- uses different regularization prescription (not manifestly gauge-invariant)
- treats additional divergences in a ”rapidity renormalization group”
- 2011 paper missed quark recoil effects $\Rightarrow$ valid to LL

  2012 paper in agreement with Dokshitzer result

A glimpse at the data





- NLL with perturbative uncertainty only
- without matching to fixed order calculation
- does not include any treatment of non-perturbative corrections

⇒ a precision determination of α_s requires NNLL matched to NNLO!

Beyond NLL

The extension to NNLL requires three ingredients

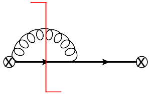
- one-loop soft function
- one-loop jet function
- two-loop anomaly coefficient

The calculation of the one-loop soft function is straight-forward [Becher, GB, Neubert 11]

$$\Rightarrow \quad \overline{\mathcal{S}}(\tau_L,\tau_R,z_L,z_R) = 1 + \frac{\alpha_s C_F}{4\pi}\Bigg\{ (\mu^2\bar{\tau}_L^2)^{\varepsilon}\ (\nu_+\bar{\tau}_L)^{\alpha}\ \Bigg[\frac{4}{\alpha}\left(\frac{1}{\varepsilon} + 2\ln\left(\frac{1+\sqrt{1+z_L^2}}{4}\right)\right) - \frac{2}{\varepsilon^2} + 8\,\mathrm{Li}_2\left(-\frac{\sqrt{1+z_L^2}-1}{\sqrt{1+z_L^2}+1}\right) + 4\ln^2\left(\frac{1+\sqrt{1+z_L^2}}{4}\right) + \frac{5\pi^2}{6}\Bigg] - (L\leftrightarrow R)\Bigg\}$$

One-loop jet function

The calculation of the one-loop jet function is surprisingly complicated



$$\sim \int d^d q\ \delta(q^2)\ \theta(q^0)\ \int d^d k\ \left(\frac{\nu_+}{k_+}\right)^{\alpha} \delta(k^2)\ \theta(k^0)\ \frac{\bar{n}q\ (\bar{n}k + \bar{n}q)}{\bar{n}k\ (q+k)^2}$$

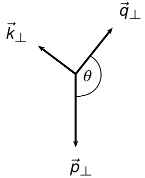
$$\times\ \delta\Big(Q - \bar{n}q - \bar{n}k\Big)\ \delta^{d-2}\Big(p_\perp - q_\perp - k_\perp\Big)\ \delta\Big(b - \tfrac{1}{2}|q_\perp| - \tfrac{1}{2}|k_\perp|\Big)$$

$$\sim \int_0^1 d\eta\, \eta\, (1-\eta)^{-1+\alpha} \int_{1-y}^{1+y} d\xi\ \frac{\xi(2-\xi)^{1-2\alpha}(\xi(2-\xi) - 1 + y^2)^{-\frac{1}{2}-\varepsilon}}{(\xi - 2y\eta)^2 + 4\eta(1-y)(1+y-\xi)}$$

- non-trivial angle complicates calculation
- expansion in α and ϵ is subtle

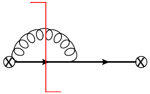
  $\Rightarrow$ have to keep $(2b-p)^{-1-\epsilon}, (2b-p)^{-1-2\epsilon}, \ldots$ to all orders
- computed the integrals in closed form without expanding in ϵ

  $\Rightarrow$ hypergeometric functions of half-integer parameters
- perform Laplace + Fourier transformations analytically



One-loop jet function

The calculation of the one-loop jet function is surprisingly complicated



$$\sim \int d^dq\ \delta(q^2)\ \theta(q^0)\ \int d^dk \left(\frac{\nu_+}{k_+}\right)^{\alpha} \delta(k^2)\ \theta(k^0)\ \frac{\bar{n}q\ (\bar{n}k+\bar{n}q)}{\bar{n}k\ (q+k)^2}$$

$$\times\ \delta\Big(Q-\bar{n}q-\bar{n}k\Big)\ \delta^{d-2}\Big(p_\perp - q_\perp - k_\perp\Big)\ \delta\Big(b-\tfrac{1}{2}|q_\perp| - \tfrac{1}{2}|k_\perp|\Big)$$

$$\sim \int_0^1 d\eta\, \eta\, (1-\eta)^{-1+\alpha} \int_{1-y}^{1+y} d\xi\ \frac{\xi(2-\xi)^{1-2\alpha}\,(\xi(2-\xi)-1+y^2)^{-\frac{1}{2}-\varepsilon}}{(\xi-2y\eta)^2+4\eta(1-y)(1+y-\xi)}$$

The Wilson line diagram yields

$$\overline{\mathcal{J}}_L^{(1b)}(\tau,z) = \overline{\mathcal{J}}_L^{(0)}(\tau,z)\ \frac{\alpha_s C_F}{4\pi}\ (\mu^2\bar{\tau}^2)^{\varepsilon}\ (\nu_+ Q\bar{\tau}^2)^{\alpha}$$

$$\times \Big\{ -\frac{2}{\alpha}\Big[\frac{1}{\varepsilon} + 2\ln\Big(\frac{1+\sqrt{1+z^2}}{4}\Big)\Big] + \frac{2}{\varepsilon^2} + \frac{2}{\varepsilon} - 8\mathrm{Li}_2\Big(-\frac{\sqrt{1+z^2}-1}{\sqrt{1+z^2}+1}\Big)$$

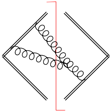
$$+\ 8\mathrm{Li}_2(-\sqrt{1+z^2}) - 4\ln^2\Big(\frac{1+\sqrt{1+z^2}}{4}\Big) + \ln^2(1+z^2) + 2z^2\ln(1+z^2)$$

$$+\ 4(1-z^2)\ln(1+\sqrt{1+z^2}) + 4\sqrt{1+z^2} - 8\ln 2 - \frac{\pi^2}{6}\Big\}$$

Two-loop anomaly coefficient

Most easily extracted from the two-loop soft function

- again two particles in final state $\Rightarrow$ similar integrals as one-loop jet function
- but requires to go one order higher in ϵ-expansion
- encounter Nielsen polylogs and elliptic integrals



$$= \left(\frac{\alpha_s}{4\pi}\right)^2 \frac{C_A C_F}{2} (\mu^2 \bar{\tau}_L^2)^{2\varepsilon} \frac{(\nu_+ \bar{\tau}_L)^\alpha}{\alpha}$$

$$\left\{ \frac{1}{\varepsilon^3} + \frac{4\ln z_L^+ + 2}{\varepsilon^2} + \left[12\text{Li}_2\left(-\frac{z_L^-}{z_L^+}\right) + 8\ln^2 z_L^+ + 8\ln z_L^+ + 4 + \frac{7\pi^2}{6}\right]\frac{1}{\varepsilon} \right.$$

$$+8\pi\, E(-z_L^2) - 16\mathcal{E}(-z_L^2) + 4\text{Li}_3\left(-\frac{z_L^-}{z_L^+}\right) - 8\text{S}_{1,2}\left(-\frac{z_L^-}{z_L^+}\right) - 40\text{Li}_3(1-4z_L^+)$$

$$+8\text{S}_{1,2}(1-4z_L^+) + 8\text{Li}_3(-4z_L^-) - 40\text{S}_{1,2}(-4z_L^-) - 8\text{Li}_3(-2z_L^-) + 40\text{S}_{1,2}(-2z_L^-)$$

$$+8\ln\left(\frac{z_L^+}{64}\right)\text{Li}_2\left(-\frac{z_L^-}{z_L^+}\right) + \left(20\ln(1+z_L^2) + 8\ln(4z_L^+)\right)\text{Li}_2(1-4z_L^+) + \frac{8}{3}\ln^3 z_L^+$$

$$\left. +\text{a few more lines} \right\}$$

Two-loop anomaly coefficient

Most easily extracted from the two-loop soft function

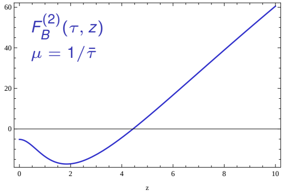
- again two particles in final state $\Rightarrow$ similar integrals as one-loop jet function
- but requires to go one order higher in ϵ-expansion
- encounter Nielsen polylogs and elliptic integrals

We recently finished the two-loop calculation

- can check $\frac{1}{\alpha^2}\left\{\frac{1}{\varepsilon^2}, \frac{1}{\varepsilon}, 1\right\}$ and $\frac{1}{\alpha}\left\{\frac{1}{\varepsilon^3}, \frac{1}{\varepsilon^2}, \frac{1}{\varepsilon}\right\}$ structures ✓
- C_F^2 color structure given by non-abelian exponentiation ✓

$\Rightarrow$ the analytic result is rather lengthy. . .

$\Rightarrow$ we are currently working on the NNLL implementation for b_T and b_W



Conclusions

Resummation beyond standard RG techniques via collinear anomaly

- we proposed an analytic phase space regularization for SCET_{II} problems

$$\int d^d k \;\; \delta(k^2)\, \theta(k^0) \quad \Rightarrow \quad \int d^d k \, \left(\frac{\nu_+}{k_+}\right)^{\alpha} \delta(k^2)\, \theta(k^0)$$

- we have explicitly shown that this prescription works at two-loop order

We have determined all ingredients to perform NNLL resummation for jet broadening

- allows for precision determinations of α_s from b_T and b_W distributions

The formalism is relevant for many interesting LHC observables

- p_T resummation, jet veto, jet substructure, ...

Backup slides

Analytic regularization

[Smirnov 93]

Raise QCD propagators along the fermion line to fractional power

$$\text{quark:} \quad \frac{1}{p^2+i\varepsilon} \to \frac{(\nu_1^2)^\alpha}{(p^2+i\varepsilon)^{1+\alpha}} \qquad \text{antiquark:} \quad \frac{1}{p^2+i\varepsilon} \to \frac{(\nu_2^2)^\beta}{(p^2+i\varepsilon)^{1+\beta}}$$

- trivial for QCD, but regularizes ill-defined SCET diagrams
- modifies Wilson line in the opposite sector

$$\frac{\bar{n}^\mu}{\bar{n}\cdot k} \to \frac{\bar{n}^\mu \, (\nu_2^2)^\beta \; n\cdot p_R}{(\bar{n}\cdot k\, n\cdot p_R)^{1+\beta}} \qquad \frac{n^\mu}{n\cdot k} \to \frac{n^\mu \, (\nu_1^2)^\alpha \; \bar{n}\cdot p_L}{(n\cdot k\, \bar{n}\cdot p_L)^{1+\alpha}}$$

- introduces foreign momentum components $n\cdot p_R = \bar{n}\cdot p_L = Q$ into jet functions!

Analytic regularization

[Smirnov 93]

Raise QCD propagators along the fermion line to fractional power

quark: $\frac{1}{p^2+i\varepsilon} \to \frac{(\nu_1^2)^\alpha}{(p^2+i\varepsilon)^{1+\alpha}}$ antiquark: $\frac{1}{p^2+i\varepsilon} \to \frac{(\nu_2^2)^\beta}{(p^2+i\varepsilon)^{1+\beta}}$

- trivial for QCD, but regularizes ill-defined SCET diagrams
- modifies Wilson line in the opposite sector

$$\frac{\bar{n}^\mu}{\bar{n}\cdot k} \to \frac{\bar{n}^\mu\,(\nu_2^2)^\beta\;n\cdot p_R}{(\bar{n}\cdot k\;n\cdot p_R)^{1+\beta}} \qquad \frac{n^\mu}{n\cdot k} \to \frac{n^\mu\,(\nu_1^2)^\alpha\;\bar{n}\cdot p_L}{(n\cdot k\;\bar{n}\cdot p_L)^{1+\alpha}}$$

- introduces foreign momentum components $n\cdot p_R = \bar{n}\cdot p_L = Q$ into jet functions!

Collinear anomaly:

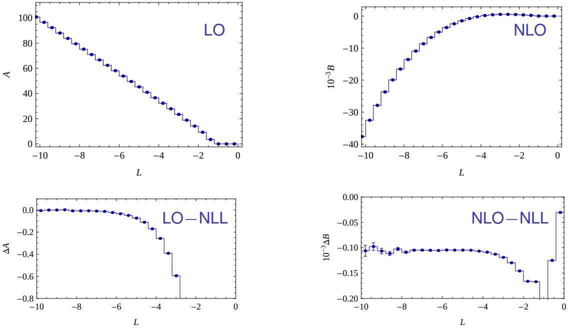
[Becher, Neubert 10]

- classically $\mathcal{J}_L(b_L, p_L^\perp)$ is invariant under rescaling of $p_R \to \lambda p_R$
- in the quantum theory $\mathcal{J}_L(b_L, p_L^\perp)$ requires regularization
- symmetry is not recovered when the regulator is removed

Compare to fixed-order calculation

Confront with output from EVENT2 generator [Catani, Seymour 96]

$$\frac{b_T}{\sigma_0}\frac{d\sigma}{db_T} = \frac{\alpha_s(Q)}{2\pi} A(b_T) + \left(\frac{\alpha_s(Q)}{2\pi}\right)^2 B(b_T)$$



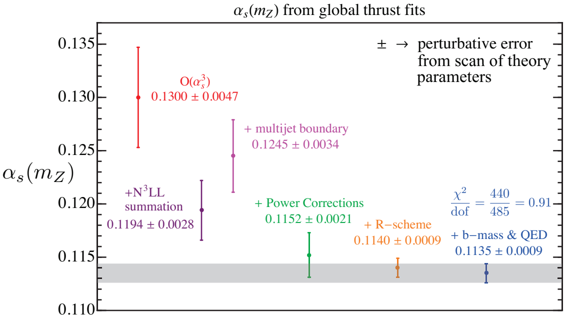
In the two-jet limit $L = \ln(b_T/Q) \to -\infty$ find $\Delta A \to 0$ ✓

$\Delta B \to$ const ✓

Details of thrust analysis

Two-dimensional fit to world thrust data: [Abbate, Fickinger, Hoang, Mateu, Stewart 10]

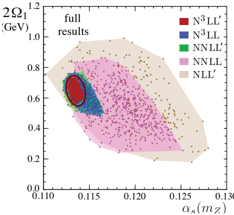
- NNLO + (approx.) N^3LL resummation
- different treatments of peak, tail and multi-jet regions (with profile functions) $+4.3\%$
- field theoretical treatment of power corrections (+ renormalon subtraction) -8.4%
- bottom mass + QED corrections -0.4%

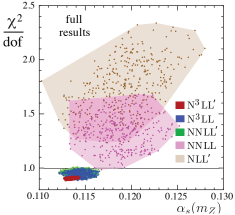


Details of thrust analysis (cont'd)

Pros:

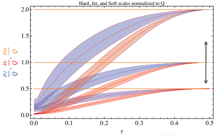
- sophisticated analysis
- uses all available data
- good convergence and fit quality

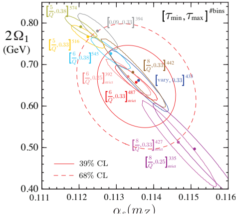




Delicate aspects:

- uncertainty from profile functions
- dependence on fit range
- remnant hadroniz. effects $\sim 0.4\%$?





$\Rightarrow$ thrust: $\alpha_s(M_Z) = 0.1135 \pm 0.0002\ (\text{exp}) \pm 0.0005\ (\text{had}) \pm 0.0009\ (\text{pert})$

moment: $\alpha_s(M_Z) = 0.1140 \pm 0.0004\ (\text{exp}) \pm 0.0013\ (\text{had}) \pm 0.0007\ (\text{pert})$