On the hadronic light-by-light scattering contribution to the muon magnetic anomaly

Kirill Melnikov

Johns Hopkins University

Based on K. Melnikov and A. Vainshtein, PRD 70 (2004), 113006
R. Boughezal and K. Melnikov, PLB 704 (2011), 193

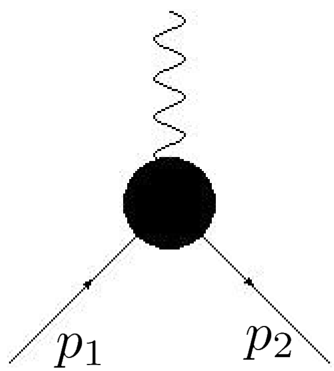
Outline

- *The muon anomalous magnetic moment*
- *Hadronic light-by-light scattering*
- *Short-distance constraints on the light-by-light scattering amplitude*
- *Calculations in the constituent quark model*
- *Conclusions*

The muon anomalous magnetic moment

- *Scattering of a muon on an external electromagnetic field is described by two form-factors*
- *At leading order QED, only Dirac form-factor is present, Pauli form-factor is induced radiatively*
- *At* $q=0$ *, the Dirac form-factor is protected from radiative corrections, but the Pauli form factor is not. The Pauli form-factor changes the muon magnetic moment*

$$q = p_2 - p_1$$



$$\langle u_{p_2} | J^\mu | u_{p_1} \rangle = \bar{u}_{p_2} \left[F_D(q^2)\gamma_\mu + F_P(q^2) \frac{i\sigma_{\mu\nu} q^\nu}{2m} \right] u_{p_1}$$

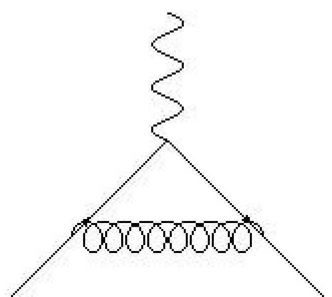
$$H = \frac{(\vec{p} - e\vec{A})^2}{2m} - \vec{\mu}\cdot\vec{B} + e\varphi \qquad \vec{\mu} = 2\mu\vec{s} \qquad \mu = g\frac{e}{2m}$$

$$f = -\frac{m}{2\pi}\chi^+ \left(-\frac{e}{2m}\vec{A}_q \cdot (\vec{p}_1 + \vec{p}_2) + e\varphi_q - i\mu\sigma \left[\vec{q} \times \vec{A}_q \right] \right) \chi_1$$

$$F_D(0) = 1, \quad \mu = \frac{e}{2m}\left(F_D(0) + F_P(0)\right) \qquad a_\mu = \frac{g-2}{2} = F_P(0)$$

Why the muon ?

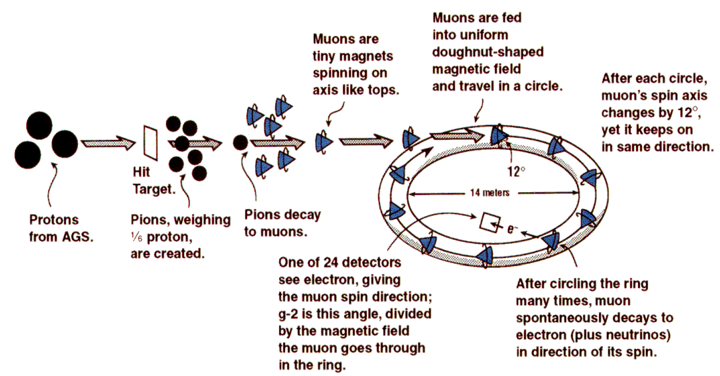
- *Both, electron and muon magnetic moments are modified. In principle, can be measured by studying the spin precession. A much easier task for the electron because it is stable*
- *The muon magnetic anomaly is much more sensitive to physics beyond the Standard Model, because the muon is so heavy !*
- *Muon instability requires special techniques to measure its magnetic moment*



M

$$a_\mu \sim \left(\frac{\alpha}{\pi}\right) \frac{m_\mu^2}{M^2} \sim 230 \left(\frac{100 \text{ GeV}}{M}\right)^2 \times 10^{-11}$$

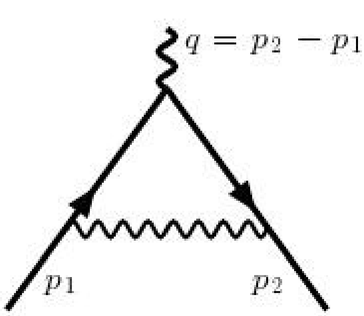
Brookhaven g-2 experiment



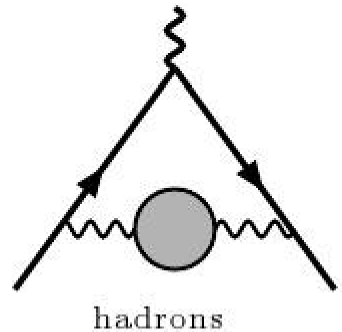
$$a_\mu^{\text{exp}} = 116592080(58) \times 10^{-11}.$$

Theory of muon g-2

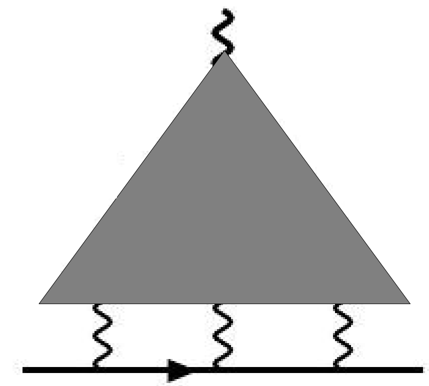
- *Theoretical predictions for muon magnetic anomaly are very advanced; when theoretical and an experimental values are compared, an interesting discrepancy is observed*



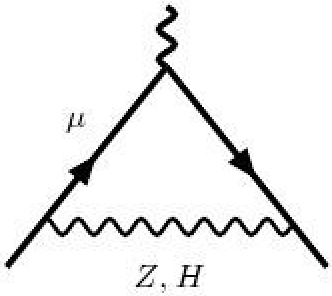
$$a_\mu^{\rm QED} = 116584720(1) \times 10^{-11}$$

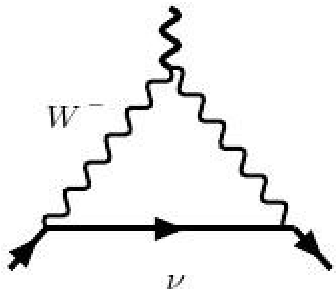


$$a_\mu^{\rm HVP} = 6836(63) \times 10^{-11}$$



$$a_\mu^{\rm Hlbl} = 105(30) \times 10^{-11}$$





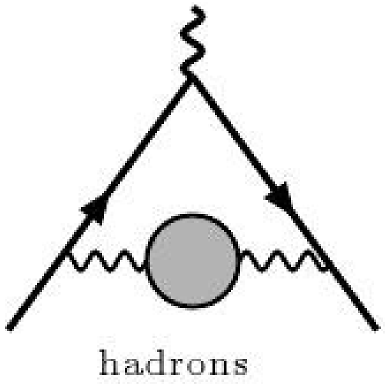
$$a_\mu^{\rm EW} = 154 \times 10^{-11}$$

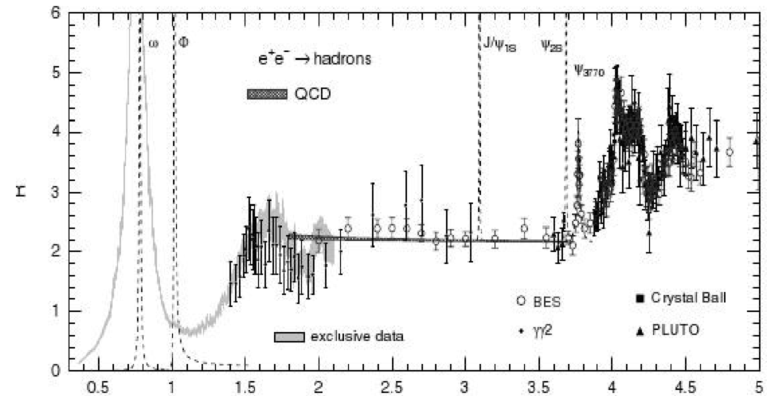
$$a_\mu^{\rm th} = 116591815(80) \times 10^{-11}$$

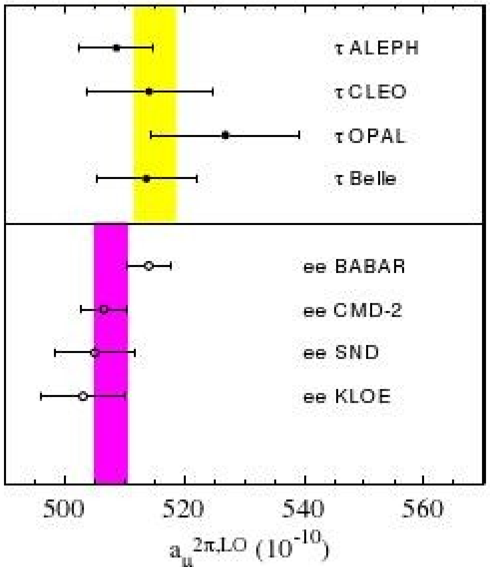
$$a_\mu^{\rm exp} - a_\mu^{\rm th} = 270(80) \times 10^{-11}$$

Hadronic vacuum polarization

- *Hadronic vacuum polarization contribution currently has the largest uncertainty among all Standard Model contributions. It is obtained from a dispersion integral in a relatively clean way. The largest contribution comes from low (less than 1 GeV) energies.*







$$a_\mu^{\rm HVP} = 6836(63) \times 10^{-11}$$

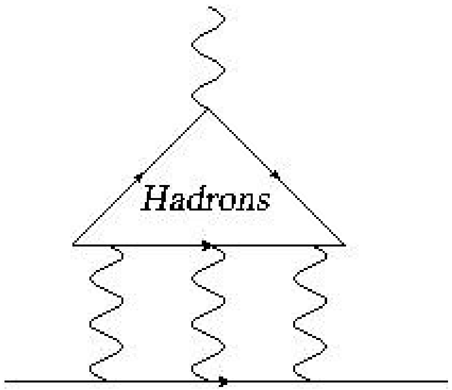
$$a_\mu^{\rm hvp} = \frac{\alpha}{3\pi}\int_{s_0}^{\infty}\frac{{\rm d}s}{s}R^{\rm hadr}(s)a_\mu^{(1)}(s)$$

$$R^{\rm hadr}(s) = \frac{\sigma(e^+e^- \to {\rm hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$a_\mu^{(1)}(s) \approx \frac{\alpha}{2\pi}\begin{cases} \frac{m^2}{3s} & s \gg m^2 \\ \frac{1}{2} - \frac{\pi\sqrt{s}}{2m} & s \ll m^2 \end{cases}$$

Hadronic light-by-light scattering

- *Hadronic light-by-light scattering contribution is a much less clear story because no direct experimental input is available*
- *This contribution is sensitive to relatively low values of the loop momenta; it can not be computed in perturbation theory – some modeling of hadron physics is unavoidable*
- *Need to respect field-theoretic constraints on the correlator of four electromagnetic currents*
  - *long-distance: chiral*
  - *short-distance: perturbative QCD*

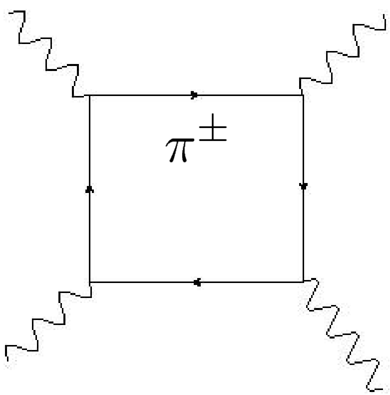


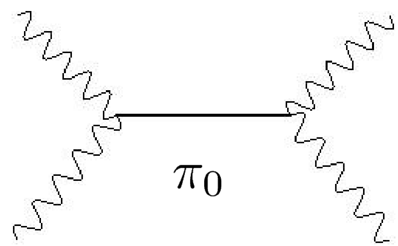
$$a_\mu^{\mathrm{Hlbl}} \sim \int \frac{\mathrm{d}^4 k_1}{(2\pi)^4} \frac{\mathrm{d}^4 k_2}{(2\pi)^4} \frac{\partial}{\partial q_\mu} \left[\Pi_{\lambda\nu\rho\sigma}(q, k_1, k_2, k_3)\right]|_{q=0}$$

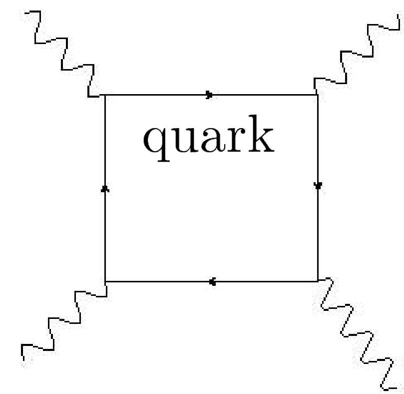
$$\Pi_{\mu\nu\alpha\beta} \sim \langle 0|T j_\mu(x_1) j_\nu(x_2) j_\alpha(x_3) j_\beta(0)|0\rangle$$

Theoretical parameters

- *To make progress, we need a ``small parameter''*
  - *small pion mass – chiral expansion* $m_\mu \sim m_\pi \ll 4\pi f_\pi \sim 1\ \mathrm{GeV}$
  - *large number of colors* $N_c = 3 \gg 1$







Leading chiral contribution

Leading N_c *contributions*

$a_\mu^{\pi^\pm} \sim \mathcal{O}(1-10) \times 10^{-11}$

$a_\mu^{\pi_0} \sim 50 \times 10^{-11}$

$a_\mu^{Q} \sim 50 \times 10^{-11},$

$m_Q = 300\ \mathrm{MeV}$

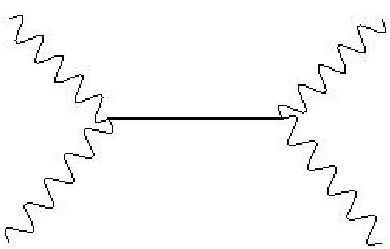
The chiral enhancement is much less important than the N_c *enhancement*

Previous results

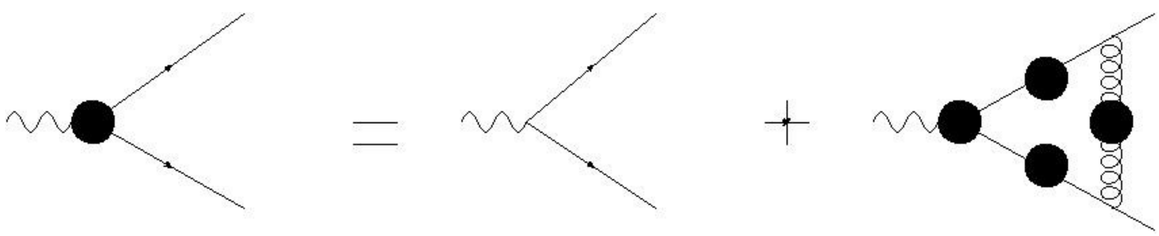
- *All existing results are consistent with this picture of the large-N dominance even if they are not always presented in this way*
- *The majority of calculations of hadronic light-by-light scattering contribution to the muon anomaly employ models of low energy hadron physics where certain mesons interact with photons in certain (form-factor corrected) ways*
- *The current recommended value is* $a_\mu^{\rm hlbl} = 105(26) \times 10^{-11}$ Prades, de Rafael, Vainshtein
- *Recently, the results of a different approach (Dyson-Schwinger) equation were presented; they turned out to be unexpectedly high. The current estimate is*

$$a_\mu^{\rm hlbl}({\rm DS}) = (217 \pm 97) \times 10^{-11}$$

Goecke, Williams, Fisher

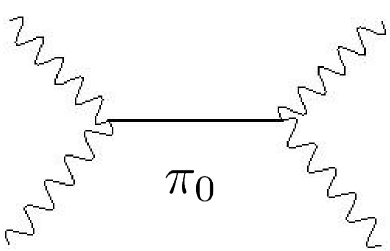


A typical meson contribution

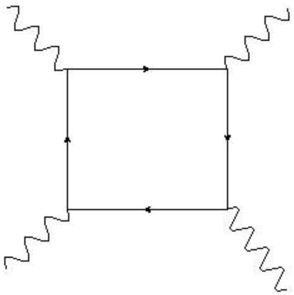


Large number of colors

- *If we work in the approximation of large number of colors, we must consider two approximately equal contributions.*
- *Consistent computation of the two contributions is important but it is not obvious how it can be done. Indeed, the meson contribution is damped in UV by introducing form-factors while the quark loop contribution is damped in IR by the quark masses*
- *Two options:*
  - *extrapolate the meson contributions to high virtualities – no need for the quark loop*
  - *extrapolate the quark loop down to ``small masses'' – no need for the meson contribution*



$$a_\mu^{\pi_0} \sim 50 \times 10^{-11}$$

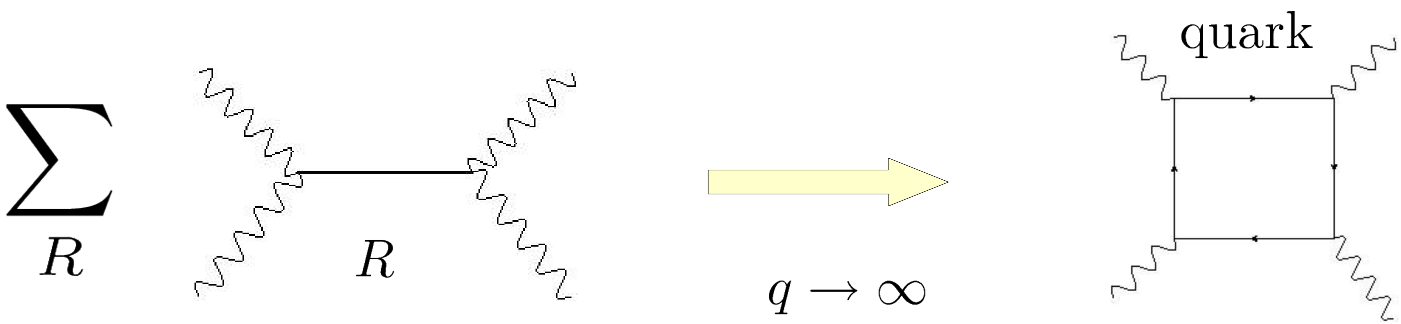


$$a_\mu^{Q} \sim 50 \times 10^{-11},$$

$$m_Q = 300 \text{ MeV}$$

Large-N and short distance contraints

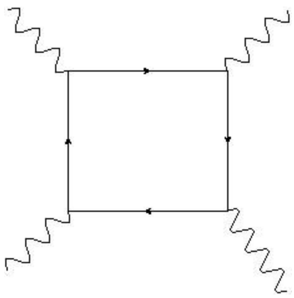
- *In the large-N_c limit, QCD Green's functions are computed as infinite sums of narrow resonances*
- *The correlator of four electromagnetic currents can also be described in this way. But, at short distances, thie sum of resonances for the correlator should match onto the quark-loop contribution since the quark loop is also leading in N_c*
- *We can attempt to match using minimal number of resonances possible, since working with infinite sums is not practical*



High-energy limit of the scattering amplitude

- *The amplitude for off-shell photon scattering is complicated; it is described by nineteen invariant form-factors*

$$
\begin{aligned}
\mathcal{A} = \; & G_1^{(1,2,3)} \{f f_1\}\{f_2 f_3\} + G_1^{(2,3,1)} \{f f_2\}\{f_3 f_1\} + G_1^{(3,1,2)} \{f f_3\}\{f_1 f_2\} \\
+ \; & G_2^{(1,2,3)} \left\{f \tilde{f}_1\right\}\left\{f_2 \tilde{f}_3\right\} + G_2^{(2,3,1)} \left\{f \tilde{f}_2\right\}\left\{f_3 \tilde{f}_1\right\} + G_2^{(3,1,2)} \left\{f \tilde{f}_3\right\}\left\{f_1 \tilde{f}_2\right\} \\
+ \; & G_3^{(1,2,3)} \{\eta_{23} f\, f_1\, \eta_{23}\}\{f_2 f_3\} + G_3^{(2,3,1)} \{\eta_{31} f\, f_2\, \eta_{31}\}\{f_3 f_1\} + G_3^{(3,1,2)} \{\eta_{12} f\, f_3\, \eta_{12}\}\{f_1 f_2\} \\
+ \; & \tilde{G}_3^{(1,2,3)} \{\eta_{23} f\, f_1\, q_1\}\{f_2\, f_3\} + \tilde{G}_3^{(2,3,1)} \{\eta_{31} f\, f_2\, q_2\}\{f_3\, f_1\} + \tilde{G}_3^{(3,1,2)} \{\eta_{12} f\, f_3\, q_3\}\{f_1\, f_2\} \\
+ \; & G_4^{(1,2,3)} \{\eta_{23} f\, f_1\, \eta_{23}\}\{q_2 f_2\, \eta_{31}\}\{q_3 f_3\, \eta_{12}\} + G_4^{(2,3,1)} \{\eta_{31} f\, f_2\, \eta_{31}\}\{q_3 f_3\, \eta_{12}\}\{q_1 f_1\, \eta_{23}\} \\
+ \; & G_4^{(3,1,2)} \{\eta_{12} f\, f_3\, \eta_{12}\}\{q_1 f_1\, \eta_{23}\}\{q_2 f_2\, \eta_{31}\} \\
+ \; & \tilde{G}_4^{(1,2,3)} \{q_1 f\, f_1\, q_1\}\{q_2 f_2\, \eta_{31}\}\{q_3 f_3\, \eta_{12}\} + \tilde{G}_4^{(2,3,1)} \{q_2 f\, f_2\, q_2\}\{q_3 f_3\, \eta_{12}\}\{q_1 f_1\, \eta_{23}\} \\
+ \; & \tilde{G}_4^{(3,1,2)} \{q_3 f\, f_3\, q_3\}\{q_1 f_1\, \eta_{23}\}\{q_2 f_2\, \eta_{31}\} + G_5^{(1,2,3)} \{q_1 f\, q_3\}\{q_1 f_1\, \eta_{23}\}\{q_3 f_3\, \eta_{12}\}\{q_2 f_2\, \eta_{31}\}\,.
\end{aligned}
$$

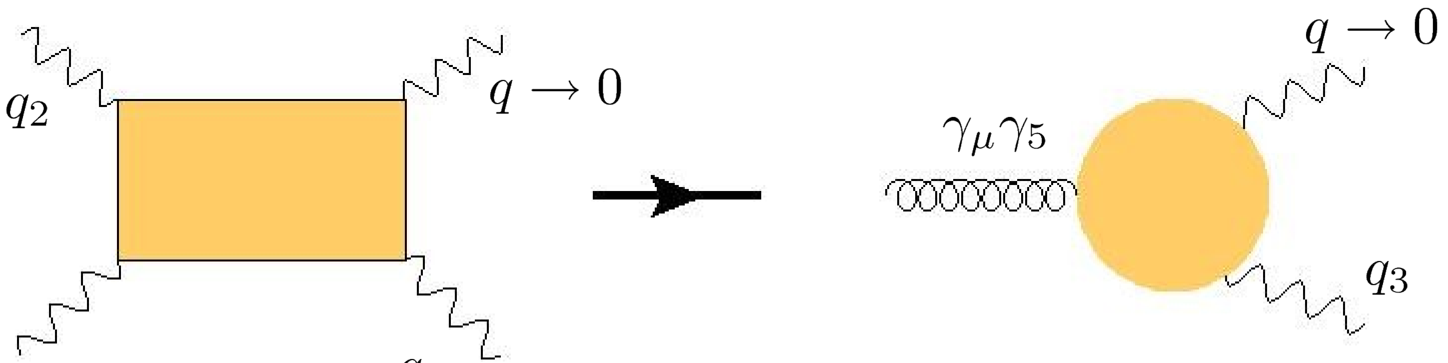


$$
\begin{aligned}
G_1(s_1,s_2,s_3) = & -\left(4s_3s_2^4 - 4s_3^3s_2^2 + 6s_3^2s_1^3 - 4s_3s_1^4 - 22s_1^3s_2^2 + 28s_3s_2^2s_1^2 + 26s_3^2s_2^2s_1 - 11s_2^4s_1 + 32s_1^2s_2^3\right. \\
& \left. -48s_3s_2^3s_1 - 2s_2^5 + s_3^4s_1 + 2s_1^4s_2 - 4s_3s_2s_1^3 - 32s_3^2s_2s_1^2 + 32s_3^3s_2s_1 + 2s_3^4s_2 + s_1^5 - 4s_3^3s_1^2\right) \frac{\ln(s_3)}{D^2 s_1(s_1-s_3-s_2)s_2} \\
& + \left(s_2^5 - 4s_2^4s_1 + 6s_1^2s_2^3 + 6s_3^3s_1^2 + 21s_3s_2^4 - 22s_3^2s_2^3 - 22s_3^3s_2^2 - 26s_3s_2^2s_1^2 + 56s_3^2s_2^2s_1 - 4s_1^3s_2^2 + 21s_3^4s_2\right. \\
& \left. +4s_3s_2s_1^3 - 26s_3^2s_2s_1^2 + s_3^5 + s_1^4s_2 - 4s_3^2s_1^3 - 4s_3^4s_1 + s_3s_1^4\right) \frac{\ln(s_1)}{D^2 s_3(s_1-s_3-s_2)s_2} \\
& - \left(-4s_1^4s_2 - 32s_3s_2^2s_1^2 + 28s_3^2s_2s_1^2 + 32s_3s_2^3s_1 + 26s_3^2s_2^2s_1 - 4s_1^2s_2^3 - 2s_3^5 + 6s_1^3s_2^2 - 4s_3^2s_2^3 + s_2^4s_1 + 2s_3s_2^4\right. \\
& \left. +4s_3^4s_2 - 11s_3^4s_1 + s_1^5 - 4s_3s_2s_1^3 - 48s_3^3s_2s_1 - 22s_3^2s_1^3 + 32s_3^3s_1^2 + 2s_3s_1^4\right) \frac{\ln(s_2)}{D^2 s_1 s_3(s_1-s_3-s_2)} \\
& -2\left(18s_3^2s_2^2 + 7s_2^3s_1 - 7s_1^3s_2 + 18s_3s_2s_1^2 - 19s_3s_2^2s_1 - 4s_3^3s_2 + 3s_1^2s_3^2 - 4s_3s_2^3 - 7s_1^3s_3 + 2s_1^4 + 3s_1^2s_2^2\right. \\
& \left. +7s_1s_3^3 - 19s_3^2s_2s_1 - 5s_3^4 - 5s_2^4\right) \frac{J(s_1,s_2,s_3)}{D^2(s_1-s_3-s_2)} - 4\frac{(-s_2^2 - 3s_1s_3 + s_1^2 - s_3^2 - 3s_2s_1 + 2s_3s_2)}{(s_1-s_3-s_2)s_1 D},
\end{aligned}
$$

Each of the 19 form-factors is a complicated function of external kinematic variables; not obvious how to match on a small number of resonances

Making the matching simpler

- *We simplify the problem by picking up a particular part in the phase-space where expression for the full amplitude simplifies*



$$q_1 \sim q_2 \gg q_3 \gg \Lambda_{\rm QCD}$$

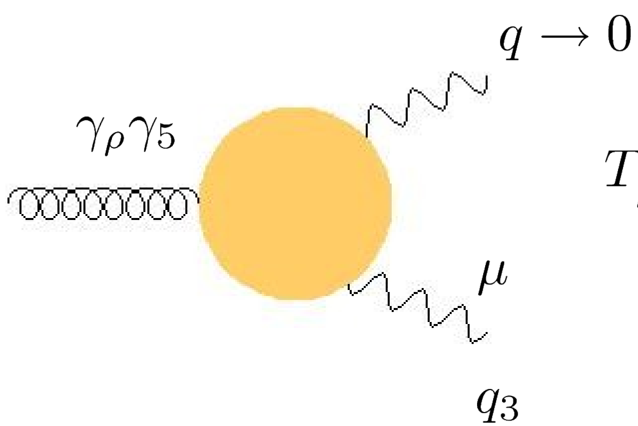
$$i \int {\rm d}^4x {\rm d}^4y e^{-iq_1x-iq_2y} T\{j_{\mu_1}(x), j_{\mu_2}(y)\} = \int {\rm d}^4 z e^{-i(q_1+q_2)z} \frac{2i}{q^2} \epsilon_{\mu_1\mu_2\delta\rho} q_3^\delta j_5^\rho(z) + \ldots$$

$$T_{\mu\rho}^{(a)} = i\langle 0| \int {\rm d}^4 z e^{iq_3 z} T\{j_{5\rho}^{(a)}(z) j_\mu(0)\} |\gamma\rangle, \qquad a = 0, 3, 8.$$

In that limit, the four-point function maps on a the three-point function of an axial current and two vector currents; this is a very special object since it is related to the axial anomaly

The triangle amplitude

- *The triangle amplitude is expressed through two invariant structures, longitudinal and transverse with respect to the momentum of the axial current*



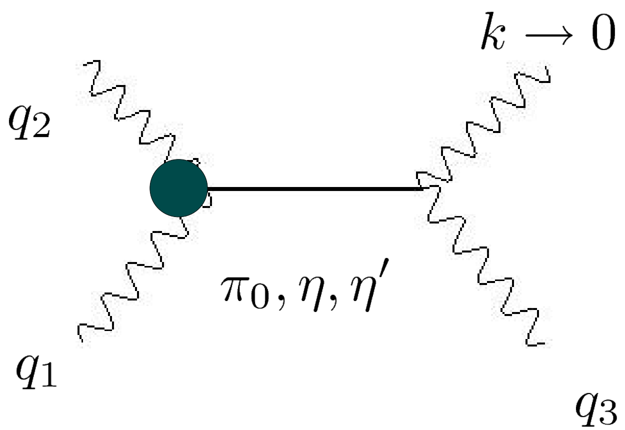
$$T^{(a)}_{\mu\rho} = -\frac{ieN_c \mathrm{Tr}\left[\lambda_a Q^2\right]}{4\pi^2} \Big\{ w_L^{(a)}(q_3^2) q_{3\rho} q_3^\sigma \tilde{f}_{\sigma\mu} + w_T^{(a)}(q_3^2) \Big(-q_3^2 \tilde{f}_{\mu\rho} + q_{3\mu} q_3^\sigma \tilde{f}_{\sigma\rho} - q_{3\rho} q_3^\sigma \tilde{f}_{\sigma\mu} \Big) \Big\}$$

For massless quarks $\quad w_L^{(a)}(q^2) = 2 w_T^{(a)}(q^2) = -\frac{2}{q^2}, \quad q^2 \gg \Lambda^2_{\mathrm{QCD}}$

If no explicit chiral symmetry breaking is present, the longitudinal form factor can be continued to $q^2 \sim \Lambda^2_{\mathrm{QCD}}$ *without modification and can be explicitly matched to the contribution of neutral pseudoscalar mesons to the Green's function*

The model

- *To illustrate the model, we consider the contribution of pseudoscalars*



$$A_{\rm PS} = \sum_{a=3,8,0} W^a \phi^{(a)}(q_1^2, q_2^2) w_L^{(a)}(q_3^2)\{f_2 \tilde{f}_1\}\{\tilde{f} f_3\}$$

$$w_L^{(3)} = \frac{2}{q_3^2 + m_\pi^2}, \qquad \phi_L^{(3)}(0,0) = \frac{N_c}{4\pi^2 F_\pi^2}$$

$$A_{\pi_0} = -\frac{N_c W^{(3)}}{2\pi^2 F_\pi^2} \frac{F_{\pi\gamma^*\gamma^*}(q_1^2, q_2^2)}{q_3^2 + m_\pi^2}\{f_2 \tilde{f}_1\}\{\tilde{f} f_3\} + \ldots$$

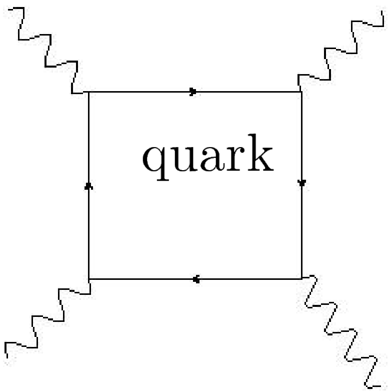
Consistency with the OPE constraint requires that only the form-factor appears in one of the vertices; it should be absent in the vertex with the magnetic field. Numerically, this increases the pseudoscalar contribution by about 30 percent and removes the need for the quark box contribution

With this approach, one obtains 136×10^{-11} *for the large-N contribution*

Perturbative hadronic light-by-light

- *We have seen that the extrapolation of meson contributions to light-by-light scattering amplitude to short distances can be done in a QCD-consistent way thereby avoiding the need to add the (massive) quark box explicitly.*
- *We have also seen that the two contributions are, approximately, equal if both are restricted to their ``natural'' domains*
- *Can we now get rid of the meson contribution entirely and obtain a reasonable description of hadronic light-by-light scattering using the constituent quark diagram?*

Pivovarov; Erler, Sanches

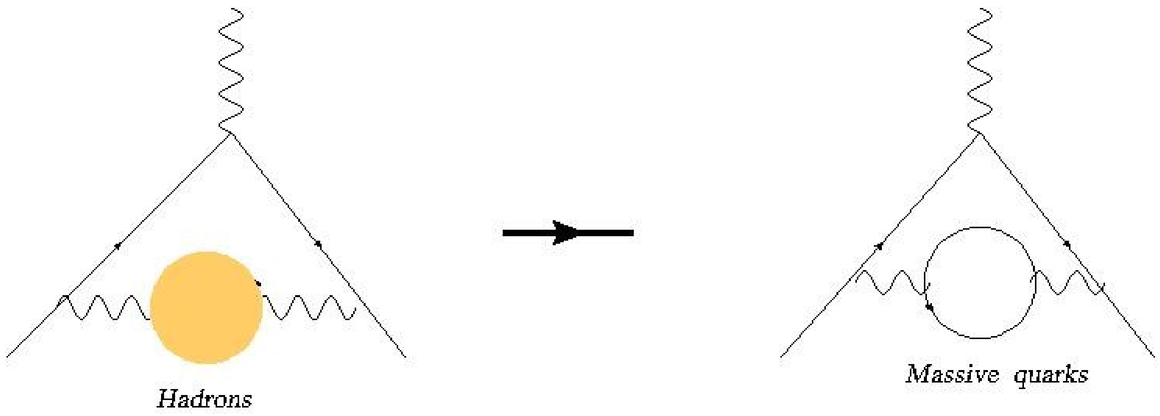


$$a_\mu = \left(\frac{\alpha}{\pi}\right)^3 N_c Q_q^4 \left[\frac{3}{2}\zeta_3 - \frac{19}{16}\right] \frac{m_\mu^2}{m_q^2}. \qquad m_q \gg m_\mu$$

Strong sensitivity to the quark mass – need to fix its value

Fixing the quark mass

- *To fix the quark mass, we compute hadronic vacuum polarization in the constituent quark model; the quark mass is then fixed because the hadronic result is known*
- *Consider the SU(3) symmetric case for simplicity*



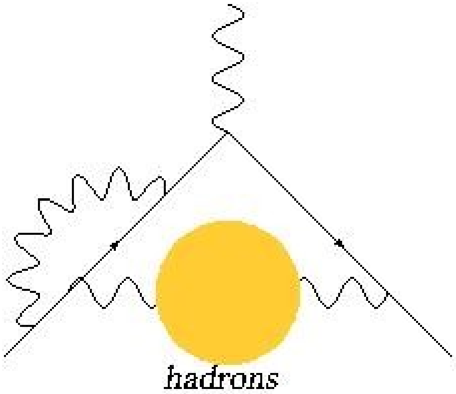
$$a_\mu^{\rm hvp} \approx 6900 \times 10^{-11} \quad \Longleftrightarrow \quad a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \frac{N_c m_\mu^2 \langle Q_q^2 \rangle}{45 M_q^2}$$

$$M_q \sim 200 \text{ MeV} \qquad M_q \sim \frac{M_\rho}{g_{\rho\gamma}}, \quad g_{\rho\gamma} \sim 5$$

Relatively small value of quark mass allows to neglect contributions of pion loops that in pirnciple should be included for a consistent description of the chiral limit

Check of the constituent quark model

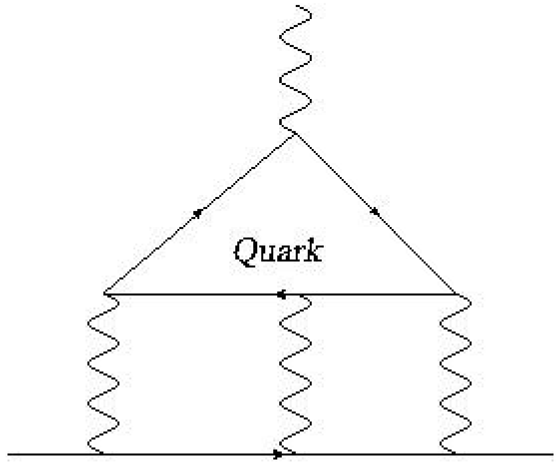
- *We can use the derived value of the quark mass to compute other contributions that can also be obtained using experimental data*
- *The result based on the dispersion relation computation is* -200×10^{-11} *; it is fully consistent with what we get from the constituent quark model computation (in fact, accounting for more mass suppress terms makes the agreement even better)*



$$a_\mu^{\rm th} = -\frac{2N_c\langle Q_q^2\rangle}{3}\left(\frac{m_\mu}{M_q}\right)^2\left(\frac{\alpha}{\pi}\right)^3\left(-\frac{2689}{5400}+\frac{\pi^2}{15}+\frac{23}{90}\ln\frac{M_q}{m_\mu}\right) \approx -150\times 10^{-11}$$

Hadronic light-by-light in the constituent quark model

- *We can now compute the hadronic light-by-light scattering contribution using the constituent quark model. In that model, the hadronic vacuum polarization and the hadronic light-by-light contributions are tightly connected*



$$a_\mu^{\rm hlbl} = a_\mu^{\rm hvp} \frac{\alpha}{\pi} \left[\frac{3}{2}\zeta_3 - \frac{19}{16} \right] \frac{45\langle Q_q^4 \rangle}{\langle Q_q^2 \rangle}$$

$$a_\mu^{\rm hvp} = 6900 \times 10^{-11} \Rightarrow a_\mu^{\rm hlbl} = 148 \times 10^{-11}$$

The result is very well consistent with more sophisticated computation based on the extrpolation from the long- to short-distances

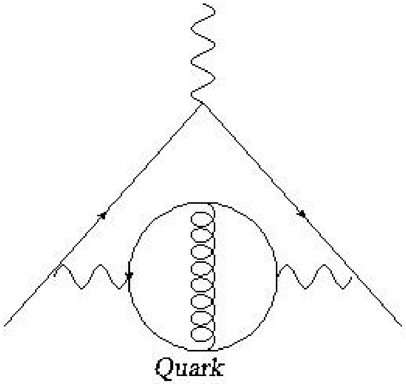
The constituent quark model

- *The physical meaning of the constituent quark model is that (for certain observables) most important non-perturbative effects can be described by ``effective'' quark masses*
- *A possible logical extension of this is that the constituent quark model can also include ``effective gluon fields'' that couple to constituent quarks in a canonical way, with some effective coupling. The coupling does not need to be small.*
- *Also this set up is definitely simplistic, it allows us to ask an important question: does such a deformation of the constituent quark model destroy the relationship between hadronic vacuum polarization contribution and hadronic light-by-light scattering contribution that we just found?*
- *To answer this question, we require radiative corrections to the hadronic vacuum polarization and to the hadronic light-by-light; we need to see that they are consistent*
- *Also: within the Dyson-Schwinger approach, the reason for large discrepancy with other results seems to be a strong modification of the photon-quark vertex. Should probably be detectable in the constituent quark model*

$$\mathcal{L} = \sum_n \bar{\psi}_n \left(i\gamma_\mu D^\mu - M_q \right) \psi_n - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu,a}, \quad D_\mu = \partial_\mu - i g_s A^a_\mu T^a$$

Radiative corrections in the constituent quark model

- *Radiative corrections to the hadronic vacuum polarization contribution can be taken from the literature*
- *We don't quite know what the ``strong coupling constant'' is; as I emphasized earlier, it is some effective coupling*
- *If we take* $\alpha_s = 0.35$ *, we find the radiative corrections to be large, close to sixty percent*
- *On the other hand, they can be absorbed into a re-definition of the quark mass parameter and so they are not very meaningful per se. However, they do become fully meaningful if the hadronic light-by-light scattering contribution is available at the same order in the ``strong coupling constant''*

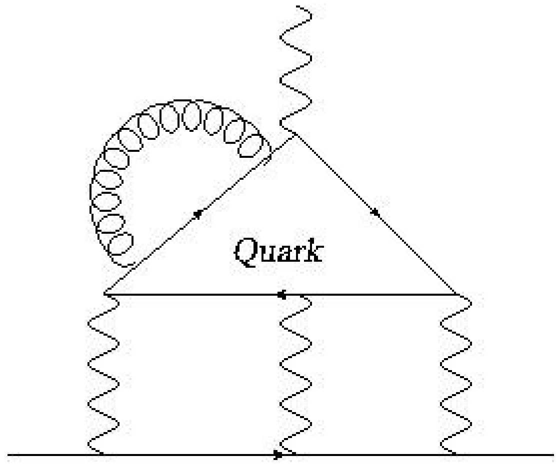


$$a_\mu^{\mathrm{hvp,NLO}} = a_\mu^{\mathrm{hvp}} \left(1 + \frac{205}{54} C_F \frac{\alpha_s}{\pi}\right).$$

Radiative corrections in the constituent quark model

- *Radiative corrections to hadronic light-by-light scattering contribution can be worked out assuming that the heavy quark mass is much larger than the muon mass (this is the same approximation that we employ everywhere)*
- *The problem can be reduced to the evaluation of four-loop vacuum bubble integrals with non-trivial tensor structure*

$$a_\mu^{\mathrm{hlbl,NLO}} = a_\mu^{\mathrm{hlbl,LO}} \left(1 + \frac{\alpha_s}{\pi} C_F \frac{\Delta_1}{\Delta_0}\right) = a_\mu^{\mathrm{hlbl,LO}} \left(1 + C_F \frac{\alpha_s}{\pi}\, 3.851\right)$$



$$\Delta_0 = -\frac{19}{16} + \frac{3}{2}\zeta_3$$

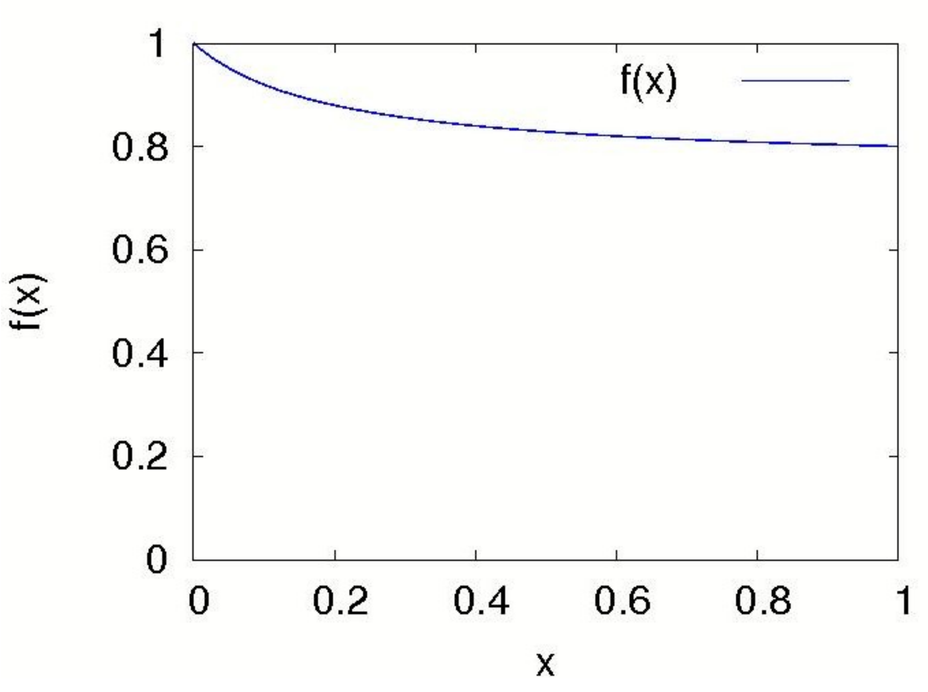
$$\Delta_1 = -\frac{473}{1080}\ln^2 2 + \frac{52\pi^2}{405}\ln^3 2 - \frac{42853\pi^4}{259200} + \frac{5771\pi^4}{32400}\ln 2 + \frac{473}{1080}\ln^4 2 - \frac{52}{675}\ln^5 2 - \frac{8477}{2700} + \frac{473 a_4}{45} + \frac{416 a_5}{45} + \frac{34727\zeta_3}{2400} - \frac{23567\zeta_5}{1440}$$

$$a_{4,5} = \mathrm{Li}_{4,5}(1/2)$$

Four-loop vacuum bubbles computed by Schroder, Vuorinen

Constituent quark model at NLO

- *Using these results we can write down a formula that connects hadronic vacuum polarization and hadronic light-by-light scattering contributions in the constituent quark model, including first order radiative corrections*
- *We find that a tight relation between hadronic vacuum polarization and hadronic light-by-light persists even at next-to-leading order in the ``effective strong coupling constant''. The leading order result is modified by at most twenty percent*



$$a_\mu^{\rm hlbl} = R^{\rm NLO} a_\mu^{\rm hvp}$$

$$R^{\rm NLO} = f\left(\frac{\alpha_s}{\pi}\right)\frac{\alpha}{\pi}\left(\frac{3}{2}\zeta_3 - \frac{19}{16}\right)\frac{45\langle Q_q^4\rangle}{\langle Q_q^2\rangle}$$

$$f(x) = \frac{1+3.851x}{1+5.061x}$$

$$a_\mu^{\rm hlbl} = (118 - -150)\times 10^{-11}$$

Conclusions

- *Hadronic light-by-light scattering contribution to the muon anomalous magnetic moment is a complicated quantity sensitive to non-perturbative details of hadron physics*
- *New experiment to measure muon g-2 with a higher precision is planned; necessitates better understanding of the hadronic light-by-light scattering contribution*
- *Existing results for hadronic light-by-light scattering contribution satisfy a number of field-theoretic constraints including long- and short-distance ones; seem to be quite solid*
- *Recent unexpected result based on the Dyson-Schwinger equation → it differs from more conventional approaches by a factor two*
- *The constituent quark model is capable of describing hadronic light-by-light scattering contributions even if the ``radiative corrections'' are included – the relation between the hadronic vacuum polarization and hadronic light-by-light contributions to the muon anomaly is quit robust*
- *Significant reduction in the theoretical uncertainty in hadronic light-by-light scattering contribution will require a much better control over hadronic contributions that are sub-leading in the number of colors*