Computation of the real radiation antenna function

for $S \to Q\bar{Q}q\bar{q}$ at NNLO QCD

Christian Bogner, HU Berlin

Zeuthen, 02.02.2012

joint work with W. Bernreuther and O. Dekkers: JHEP 1106:032,2011,

arXiv:1105.0530

Contents:

1. The **antenna subtraction formalism** and the function B_4^0
2. **Analytic integration** of B_4^0
3. **Checks** and outlook

1. Subtraction terms and the Antenna Formalism

We consider the production of m jets:

$$e^+e^- \to m \text{ jets}$$

(Recently, antenna subtraction also extends to hadronic initial states. (Daleo, Gehrmann, Maitre 2007, Daleo, Gehrmann-De Ridder, Gehrmann, Luisoni 2010, Glover, Pires 2010, Boughezal, Gehrmann-De Ridder, Ritzmann 2011, ...))

Consider the tree-level n-parton contribution to the m-jet cross section

$$d\sigma^{\rm R} = \mathcal{N} \sum_{n-\text{parton configurations}} d\phi_n \frac{1}{S_n} |\mathcal{M}_n|^2 J_m^{(n)}$$

$\mathcal{M}_n$: tree-level n-parton matrix element

$J_m^{(n)}$: jet-function, combining n partons to $m \leq n$ jets

Leading order contribution to the m-jet cross section:

$$d\sigma_{\rm LO} = \int_{\phi_m} d\sigma^{\rm Born}$$

where ϕ_m : m-parton phase-space region and $d\sigma^{\rm Born} = d\sigma^{\rm R}\big|_{n=m}$

NLO contribution to the m-jet cross section:

$$d\sigma_{\rm NLO} = \underbrace{\int_{\phi_{m+1}} \left(d\sigma^{\rm R}_{\rm NLO} - d\sigma^{\rm S}_{\rm NLO} \right)}_{I_1 \text{ finite}} + \underbrace{\underbrace{\int_{d\phi_{m+1}} d\sigma^{\rm S}_{\rm NLO}}_{I_2} + \int_{d\phi_m} d\sigma^{\rm V}_{\rm NLO}}_{\text{explicit cancellation of IR-singularities}}$$

$d\sigma^{\rm R}_{\rm NLO} : d\sigma^{\rm R}\big|_{n=m+1}$, including $J^{(m)}_{m+1}$, one unresolved parton
$d\sigma^{\rm V}_{\rm NLO}$: one-loop virtual correction to $d\sigma^{\rm Born}$
$d\sigma^{\rm S}_{\rm NLO}$: **subtraction term**

Conditions to the subtraction term:

a) I_1 is **finite**, i. e. $d\sigma^{\rm S}_{\rm NLO}$ and $d\sigma^{\rm R}_{\rm NLO}$ coincide in all singular regions;

b) I_2 **can be computed** analytically in all singular regions.

Very common at NLO are the **dipole** (Catani, Seymour 1997) and the **antenna formalism** (Kosower 1998, 2005, Campbell, Cullen, Glover 1999).
At NNLO they extend to a diversity of approaches.

NLO **antenna** subtraction term:

$$d\sigma^{\rm R}_{\rm NLO} - d\sigma^{\rm S}_{\rm NLO} = \mathcal{N} \sum_{m+1} d\phi_{m+1} \frac{1}{S_{m+1}} \Big(|\mathcal{M}_{m+1}(p_1, ..., p_{m+1})|^2 \, J_m^{(m+1)}(p_1, ..., p_{m+1})$$

$$- \sum_j \underbrace{X^0_{ijk}}_{\text{antenna functions}} \quad |\mathcal{M}_m(p_1, ..., \tilde{p}_I, \tilde{p}_K, ..., p_{m+1})|^2 \, J_m^{(m)}(..., \tilde{p}_I, \tilde{p}_K, ...,) \Bigg)$$

The "antenna functions" X^0_{ijk} cover all configurations where parton j is unresolved.

Main idea: The matrix elements are colour-ordered and admit **factorization** in the IR-singular limits.

Example: parton j being a gluon

$$|\mathcal{M}_{m+1}(..., p_i, p_j, p_k, ...)|^2 \stackrel{\text{gluon } j \text{ becomes soft}}{\longrightarrow} \frac{2 s_{ik}}{s_{ij} s_{jk}} \left| \mathcal{M}_m \left(..., \underbrace{\tilde{p}_I, \tilde{p}_K}_{\text{"antenna"}}, ... \right) \right|^2$$

In this limit $\tilde{p}_I = p_i$, $\tilde{p}_K = p_k$ form a colour-connected, hard "antenna", emitting the soft gluon.

NNLO contribution to the m-jet cross section:

$$\begin{aligned} d\sigma_{\mathrm{NNLO}} &= \int_{\phi_{m+2}} \left(d\sigma^{\mathrm{R}}_{\mathrm{NNLO}} - d\sigma^{\mathrm{S}}_{\mathrm{NNLO}} \right) + \int_{\phi_{m+2}} d\sigma^{\mathrm{S}}_{\mathrm{NNLO}} \\ &+ \int_{\phi_{m+1}} \left(d\sigma^{\mathrm{V},1}_{\mathrm{NNLO}} - d\sigma^{\mathrm{VS},1}_{\mathrm{NNLO}} \right) + \int_{\phi_{m+1}} d\sigma^{\mathrm{VS},1}_{\mathrm{NNLO}} \\ &+ \int_{\phi_m} d\sigma^{\mathrm{V},2,}_{\mathrm{NNLO}} \end{aligned}$$

- $d\sigma^{\mathrm{R}}_{\mathrm{NNLO}}$: tree level $(m+2)$-parton cross section
- $d\sigma^{\mathrm{S}}_{\mathrm{NNLO}}$: subtraction term, coinciding with $d\sigma^{\mathrm{R}}_{\mathrm{NNLO}}$ in all singular limits
- $d\sigma^{\mathrm{V},1}_{\mathrm{NNLO}}$: one-loop $(m+1)$-parton cross section
- $d\sigma^{\mathrm{VS},1}_{\mathrm{NNLO}}$: subtraction term, coinciding with $d\sigma^{\mathrm{V},1}_{\mathrm{NNLO}}$ in all singular limits
- $d\sigma^{\mathrm{V},2,}_{\mathrm{NNLO}}$: two-loop correction

The subtraction terms are obtained from colour-ordered squared matrix elements. (See Gehrmann-De Ridder, Gehrmann, Glover 2005)

We compute one of the antenna functions for $d\sigma^{\mathrm{S}}_{\mathrm{NNLO}}$

and integrate it as in $\int_{\phi_{m+2}} d\sigma^{\mathrm{S}}_{\mathrm{NNLO}}$.

Our antenna function is one of several building-blocks of $d\sigma^{\mathrm{S}}_{\mathrm{NNLO}}$.
It is relevant for heavy quark pair production via an uncoloured state S,

$$S \to Q\bar{Q} + X \quad (\text{e.g. } e^{+}e^{-} \to Z \to Q\bar{Q} + X, \;\; H \to Q\bar{Q} + X)$$

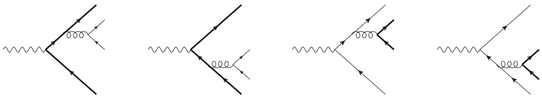
X being $Q\bar{Q}$ (no subtraction needed), gg or $q\bar{q}$.

We focus on the matrix element of $\gamma^{\star} \to Q\bar{Q}q\bar{q}$ in all unresolved limits.

The contribution to the 2-jet cross section is

$$d\sigma^{\mathrm{R},Q\bar{Q}q\bar{q}}_{\mathrm{NNLO}} = 4\pi\alpha(4\pi\alpha_S)^2(N_c^2-1)^2 d\phi_4 J_2^{(4)}$$

$$\times\left(e_Q^2\left|\mathcal{M}^0_{Qq\bar{q}\bar{Q}}\right|^2 + e_q^2\left|\mathcal{M}^0_{qQ\bar{Q}\bar{q}}\right|^2 + 2e_Q e_q Re\left(\mathcal{M}^0_{Qq\bar{q}\bar{Q}}\mathcal{M}^{0,\dagger}_{qQ\bar{Q}\bar{q}}\right)\right)$$



Subtraction of IR-singularities due to $\left|\mathcal{M}^0_{Qq\bar{q}\bar{Q}}\right|^2$ requires the antenna function

$$B^0_4(1_Q,\, 3_q,\, 4_{\bar{q}},\, 2_{\bar{Q}}) := \frac{\left|\mathcal{M}^0_{Qq\bar{q}\bar{Q}}\right|^2}{\left|\mathcal{M}^0_{Q\bar{Q}}\right|^2} = \frac{\left|\mathcal{M}^0_{Qq\bar{q}\bar{Q}}\right|^2}{4\left((1-\epsilon)q^2 + 2m^2\right)}$$

and its integration

$$\mathcal{B}^0_{Qq\bar{q}\bar{Q}}(q^2,\, m,\, \mu,\, \epsilon) = \left(8\pi^2(4\pi)^{-\epsilon}e^{\epsilon\gamma_E}\right)^2 \int d\phi_{X_{Qq\bar{q}\bar{Q}}}\, B^0_4(1_Q,\, 3_q,\, 4_{\bar{q}},\, 2_{\bar{Q}}).$$

$d\phi_{X_{Qq\bar{q}\bar{Q}}}$ ist the corresponding "antenna phase space", related to the two- and four parton phase space by the factorization:

$$d\phi_4 = P_2(q^2,\, m)\, d\phi_{X_{Qq\bar{q}\bar{Q}}}$$

$$P_2(q^2,\, m) = 2^{-3+2\epsilon}\pi^{-1+\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(2-2\epsilon)}\left(\frac{\mu^2}{q^2}\right)^{\epsilon}\left(1-\frac{4m^2}{q^2}\right)^{\frac{1}{2}-\epsilon}$$

The subtraction term $d\sigma^{\mathrm{S}}_{\mathrm{NNLO}}$ can be written as $d\sigma^{\mathrm{S,a}}_{\mathrm{NNLO}} + d\sigma^{\mathrm{S,b}}_{\mathrm{NNLO}}$ (Gehrmann-De Ridder, Gehrmann, Glover 2005), covering:

- Configurations with **one unresolved parton**:

$$d\sigma^{\mathrm{S,a}}_{\mathrm{NNLO}} = 4\pi\alpha(4\pi\alpha_S)^2 e_Q^2 (N_c^2-1)^2 d\phi_4(p_1, p_2, p_3, p_4; q) \left|\mathcal{M}^0_{Q\bar{Q}}\right|^2$$
$$\times \left(E_3^0(1_Q, 3_q, 4_{\bar{q}}) A_3^0\left((\widetilde{13})_Q, (\widetilde{43})_g, 2_{\bar{Q}}\right) J_2^{(3)}(\widetilde{p_{13}}, \widetilde{p_{43}}, p_2) \right.$$
$$\left. + E_3^0(2_Q, 3_q, 4_{\bar{q}}) A_3^0\left(1_Q, (\widetilde{34})_g, (\widetilde{24})_{\bar{Q}}\right) J_2^{(3)}(p_1, \widetilde{p_{34}}, \widetilde{p_{24}}) \right)$$

- Configurations with **two unresolved partons**:

$$d\sigma^{\mathrm{S,b}}_{\mathrm{NNLO}} = 4\pi\alpha(4\pi\alpha_S)^2 e_Q^2 (N_c^2-1)^2 d\phi_4(p_1, p_2, p_3, p_4; q) \left|\mathcal{M}^0_{Q\bar{Q}}\right|^2$$
$$\times \left(B_4^0(1_Q, 3_q, 4_{\bar{q}}, 2_{\bar{Q}}) - E_3^0(1_Q, 3_q, 4_{\bar{q}}) A_3^0\left((\widetilde{13})_Q, (\widetilde{43})_g, 2_{\bar{Q}}\right) \right.$$
$$\left. - E_3^0(2_Q, 3_q, 4_{\bar{q}}) A_3^0\left(1_Q, (\widetilde{34})_g, (\widetilde{24})_{\bar{Q}}\right) \right) J_2^{(2)}(\widetilde{p_{134}}, \widetilde{p_{234}})$$

The antenna functions $A_3^0(i_Q, k_g, j_{\bar{Q}})$ and $E_3^0(i_Q, j_q, k_{\bar{q}})$ were computed (and integrated) in Gehrmann-De Ridder, Ritzmann 2009.

2. The integration of B_4^0

We have to compute the phase-space integral

$$\begin{aligned}\mathcal{B}^0_{Qq\bar{q}\bar{Q}}(q^2,\, m,\, \mu,\, \epsilon) &= \left(8\pi^2(4\pi)^{-\epsilon}e^{\epsilon\gamma_E}\right)^2 \int d\phi_{X_{Qq\bar{q}\bar{Q}}}\, B_4^0(1_Q,\, 3_q,\, 4_{\bar{q}},\, 2_{\bar{Q}}) \\ &= \frac{1}{\epsilon^2}\mathcal{B}^{0\,(-2)}_{Qq\bar{q}\bar{Q}} + \frac{1}{\epsilon}\mathcal{B}^{0\,(-1)}_{Qq\bar{q}\bar{Q}} + \mathcal{B}^{0\,(0)}_{Qq\bar{q}\bar{Q}} + \mathcal{O}(\epsilon).\end{aligned}$$

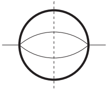
- we express all integrands by an appropriate set of propagators and scalar products D_5, D_6,
- we introduce four auxiliary (Cutkosky) cut-propagators D_1, ..., D_4:

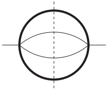
$$\begin{aligned}2\pi i\delta(p_j^2 - m^2) &= \frac{1}{p_j^2 - m^2 + i0} - \frac{1}{p_j^2 - m^2 - i0} =: D_j \text{ with } j = 1,\, 2 \\ 2\pi i\delta(p_j^2) &= \frac{1}{p_j^2 + i0} - \frac{1}{p_j^2 - i0} =: D_j \text{ with } j = 3,\, 4.\end{aligned}$$

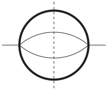
$$\Rightarrow\ d\phi_4\,(p_1,\, p_2,\, p_3,\, p_4;\, q) = \frac{\mu^{12-3d}}{i^4(2\pi)^{3d}}\delta^{(d)}\left(q - \sum_{i=1}^{4} p_i\right)\prod_{i=1}^{4}\frac{d^d p_i}{D_i}$$

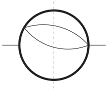
Anastasiou, Melnikov 2004: **IBP-reduction** can be applied to such cut-integrals.

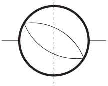
Using AIR (Anastasiou, Lazopoulos 2004) and FIRE (Smirnov 2008) (also possible: REDUZE (Studerus 2008, v.Manteuffel , Studerus 2012))
we express $\mathcal{B}^0_{Qq\bar{q}\bar{Q}}(q^2, m, \mu, \epsilon)$ in terms of **five master-integrals**:

$T_1\left(q^2, m^2, \epsilon\right) =$  $= \int d\phi_4\left(p_1, p_2, p_3, p_4; q\right)$

$T_2\left(q^2, m^2, \epsilon\right) =$  s_{13} $= \int d\phi_4\left(p_1, p_2, p_3, p_4; q\right) s_{13}$

$T_3\left(q^2, m^2, \epsilon\right) =$  s_{134} $= \int d\phi_4\left(p_1, p_2, p_3, p_4; q\right) s_{134}$

$T_4\left(q^2, m^2, \epsilon\right) =$  $= \int d\phi_4\left(p_1, p_2, p_3, p_4; q\right) \frac{1}{s_{134}}$

$T_5\left(q^2, m^2, \epsilon\right) =$  $= \int d\phi_4\left(p_1, p_2, p_3, p_4; q\right) \frac{1}{s_{134} s_{234}}$

Notation: $s_{ij} = 2p_i p_j$, $s_{ijk} = s_{ij} + s_{ik} + s_{jk}$

Computation of $T_1,\ T_2,\ T_3$:
We rewrite the phase-space integration:

$$d\phi_4\left(p_1,\ p_2,\ p_3,\ p_4;\ q\right) = \frac{1}{2\pi}\int_{4m^2}^{q^2} dM^2\, d\phi_2\left(p_4,\ k;\ q\right) d\phi_3\left(p_1,\ p_2,\ p_3;\ k\right);\ k^2 = M^2$$

$\Rightarrow$ We find compact expressions in terms of **hypergeometric functions** ${}_3F_2$.
Example:

$$T_1\left(q^2,\ m^2,\ \epsilon\right) =$$

$$\left(q^2\right)^2\left(\frac{\mu^2}{q^2}\right)^{3\epsilon}\left\{C_1(\epsilon)\,{}_3F_2\left(-\frac{1}{2}+\epsilon,-2+3\epsilon,-3+4\epsilon;\epsilon,-1+2\epsilon;z\right)\right.$$

$$+C_2(\epsilon)\,z^{1-\epsilon}\,{}_3F_2\left(\frac{1}{2},-1+2\epsilon,-2+3\epsilon;2-\epsilon,\epsilon;z\right)$$

$$\left.+C_3(\epsilon)\,z^{2-2\epsilon}\,{}_3F_2\left(\frac{3}{2}-\epsilon,\epsilon,-1+2\epsilon;3-2\epsilon,2-\epsilon;z\right)\right\};\ \text{with } z=\frac{4m^2}{q^2}$$

Similar expressions for $T_2,\ T_3$.

$\Rightarrow$ We have to **expand** functions like
$${}_3F_2\left(-\tfrac{1}{2}+\epsilon, -2+3\epsilon, -3+4\epsilon; \epsilon, -1+2\epsilon; z\right) \text{ in } \epsilon.$$

Such expansions are far from trivial.

Important progress for **integer** arguments was made:

- algorithms of Moch, Uwer, Weinzierl (2001),
- implementations: `Nestedsums` (Weinzierl 2002), `XSummer` (Moch, Uwer 2005),
- alternative, mixed approach: `HypExp` (Huber, Maitre 2005).

Expansion around **half-integers** requires additional work:

- algorithm by Weinzierl (2004),
- **Our cases** can be expanded using `HypExp2` (Huber, Maitre 2007).

$\Rightarrow$ We obtain

$$T_i\left(q^2, m^2, \epsilon\right) =$$
$$T_i^{(0)}\left(q^2, m^2\right) + \epsilon T_i^{(1)}\left(q^2, m^2\right) + \epsilon^2 T_i^{(2)}\left(q^2, m^2\right) + \mathcal{O}(\epsilon^3),\ i = 1, 2, 3 \text{ in terms}$$
of **harmonic polylogarithms**.

Reminder on **harmonic polylogarithms** (HPL) (Remiddi, Vermaseren 1999)

An HPL $H(a_1, a_2, ..., a_w; x)$ of weight w with all $a_i \in \{-1, 0, 1\}$ is **defined as**:

- $H(a_1, a_2, ..., a_w; x) = \int_0^x dx' f(a_1; x)\, H(a_2, ..., a_w; x')$ if some $a_i \neq 0$,
- $H(0, 0, ..., 0; x) = \frac{1}{w!} \ln^w(x)$ if all $a_i = 0$.

Here $f(0; x) = \frac{1}{x}$, $f(1; x) = \frac{1}{1-x}$, $f(-1; x) = \frac{1}{1+x}$
and $H(0; x) = \ln(x)$, $H(1; x) = -\ln(1-x)$, $H(-1; x) = \ln(1+x)$.

Examples: $H(0, 0; x) = \frac{1}{2!} \ln^2(x)$, $H(0, 1; x) = \mathrm{Li}_2(x)$,
$H(-1, 0; x) = \ln(x)\ln(1+x) + \mathrm{Li}_2(-x)$

HPLs are particularly useful when solving Feynman (or cut-) integrals by **differential equations**.

Computation of T_4 and T_5: The differential equations approach (Remiddi 1997, Gehrmann, Remiddi 1999)

We differentiate T_4 and T_5 with respect to the scaleless variable $y := \frac{1-\sqrt{1-z}}{1+\sqrt{1-z}}$ $\left(\text{where } z = \frac{4m^2}{q^2}\right)$.

$$\frac{d}{dy}T_4 = \sum a_i A_i, \; \frac{d}{dy}T_5 = \sum b_i B_i$$

$\Rightarrow$ Apply **IBP-reduction** to the cut-integrals A_i and B_i.
$\Rightarrow$ Linear first order differential equations for T_4 and T_5, involving the other master-integrals in the inhomogeneous part:

$$\frac{d}{dy}T_4 + aT_4 = \sum_{i=1}^{3} c_i T_i, \; \frac{d}{dy}T_5 + bT_5 = \sum_{i=1}^{4} d_i T_i.$$

- For each T_i we formally **expand** $T_i = T_i^{(0)} + \epsilon T_i^{(1)} + \epsilon^2 T_i^{(2)} + \ldots$
- Separating powers of ϵ we obtain a **system of equations** each for $T_4^{(0)}$, $T_4^{(1)}$ and $T_5^{(0)}$, $T_5^{(1)}$. These **can be solved**.

Build up the solutions:

- solve the homogeneous equations (easy)
- integrate the inhomogeneous parts (a bit less easy)

Inhomogeneous parts are known in terms of HPLs.
$\Rightarrow$ Iterate **partial integration** and **partial fraction decomposition** until all terms are of the form:

$$c \cdot (f(y)) \cdot H(\ldots;\, y) \;\; \text{with} \; f(y) \in \left\{ \frac{1}{y}, \frac{1}{1-y}, \frac{1}{1+y} \right\}.$$

Then the integration is immediate from the **definition of HPLs as iterated integrals**.
We use the Mathematica package `HPL` by Maitre, 2006.

A last technical issue: **Fixing the integration constants**

Use known results of T_4 and T_5 at some fixed value for y.

- For T_5 we use the massless limit ($y = 0$), known from Gehrmann, Gehrmann-De Ridder, Heinrich 2004.
- For T_4 the massless limit ($y = 0$) and threshold ($y = 1$) are **inappropriate**, as the differential equations are undefined there (due to a term $\frac{1}{y(1-y)}$).
  "Trick": Choose auxiliary integral, vanishing at $y = 1$, and relate to this limit via IBP-relations.

Final Result:

We obtain the integrated antenna function $\mathcal{B}^0_{Qq\bar{q}\bar{Q}}(q^2, m, \mu, \epsilon)$ to all relevant orders in terms of HPLs.

3. Checks with the literature

Consider the ratio

$$R = \frac{\sigma\left(e^+e^- \to \gamma^\star \to Q\bar{Q} + X\right)}{\sigma_{\rm pt}}, \quad \text{with the massless Born c.s. } \sigma_{\rm pt} = \frac{e^4}{12\pi q^2}$$

at α_s^2 and at lowest order in α.
The $\alpha_s^2 e_Q^2 N_f$-coefficient $R_{\alpha_s^2 e_Q^2 N_f}$ is gauge invariant and **finite**.
We can decompose it as

$$R_{\alpha_s^2 e_Q^2 N_f} = 6\pi e_Q^2 (1-\epsilon) \sum_X \Pi^{Q\bar{Q}X}_{\alpha_s^2 N_f}$$

The contribution $X = q\bar{q}$ is directly related to the integrated antenna function:

$$\Pi^{Q\bar{Q}q\bar{q}}_{\alpha_s^2 N_f} = \frac{(4\pi\alpha_s)^2 4 C_F N_c T_R N_f}{q^2(3-2\epsilon)} \frac{P_2(q^2,\, m)\left|\mathcal{M}_{Q\bar{Q}}\right|^2}{\left(8\pi^2 (4\pi)^{-\epsilon} e^{\gamma_E}\right)^2} \mathcal{B}^0_{Qq\bar{q}\bar{Q}}(q^2,\, m,\, \mu,\, \epsilon)$$

First check: singularities

Finiteness of $R_{\alpha_s^2 e_Q^2 N_f}$ $\Rightarrow$ singular parts of $\Pi^{Q\bar{Q}}_{\alpha_s^2 N_f} + \Pi^{Q\bar{Q}g}_{\alpha_s^2 N_f} + \Pi^{Q\bar{Q}q\bar{q}}_{\alpha_s^2 N_f}$ must **cancel**.

- $\Pi^{Q\bar{Q}q\bar{q}}_{\alpha_s^2 N_f}$: obtained from our result $\mathcal{B}^0_{Qq\bar{q}\bar{Q}}(q^2, m, \mu, \epsilon)$,
- $\Pi^{Q\bar{Q}}_{\alpha_s^2 N_f} = \frac{N_c P_2(q^2, m)}{3-2\epsilon}\Big(4(1+\frac{2y}{(1+y)^2}-\epsilon)\,|F_1|^2_{\alpha_s^2 N_f} + \frac{(1+y^2+y(10-8\epsilon))}{2y}\,|F_2|^2_{\alpha_s^2 N_f} + 4(3-2\epsilon)\mathrm{Re}\,(F_1 F_2^\star)_{\alpha_s^2 N_f}\Big)$
  The heavy quark form factors F_1, F_2 are obtained from Bernreuther et al. 2005, also see Gluza, Moch, Riemann 2009.
- $\Pi^{Q\bar{Q}g}_{\alpha_s^2 N_f} = \frac{(4\pi\alpha_s)\mu^{-2\epsilon} 2 C_F N_c}{q^2(3-2\epsilon)} P_2(q^2, m)\,|\mathcal{M}_{Q\bar{Q}}|^2 \Big(2\delta Z_{1F,\alpha_s N_f}\mathcal{A}^0_{Qg\bar{Q}}\Big)$
  The integrated antenna function $\mathcal{A}^0_{Qg\bar{Q}}$ is obtained from Gehrmann-De Ridder, Ritzmann 2009.

$\Rightarrow$ The exact cancellation provides a strong check for our singular terms.

Second check: finite part

For $d = 4$ the coefficient $R_{\alpha_s^2 e_Q^2 N_f}$ can be obtained from Hoang, Kühn, Teubner 1995.

Technical remark:
Their result is expressed using certain combination of the integrals

$$T_2(\eta,\,\xi) = \int_0^1 dx \frac{\arctan(\xi x)}{x^2+\eta^2},\;\; T_2^\star(\eta,\,\xi) = \int_0^1 dx \frac{\ln(x^2+\xi^2)}{x^2+\eta^2}$$

$$T_3(\eta,\,\xi,\,\chi) = \int_0^1 dx \frac{\ln(x^2+\xi^2)\arctan(\chi x)}{x^2+\eta^2}.$$

We had to solve this expression in terms of polylogarithms for our comparison.

$\Rightarrow$ Strong analytical check for our finite part.

Outlook (work in progress with Bernreuther and Dekkers)

Next steps: Compute (integrated) antenna functions for

- two unresolved gluons,
- real/virtual contribution with one unresolved gluon.

Nice feature of the approach:

The now known master-integrals T_1, ..., T_5 **appear again**. We don't have to start from zero.

Possible issues of the approach:

- There always may be systems of differential equations which can not be decoupled / solved.
- There may be generalized hypergeom. functions which can not be expanded to the desired order.

Up to now we were **lucky**, but limitations of the techniques seem not very far away.

Summary:

- We computed the **real radiation antenna function** $B_4^0(1_Q, 3_q, 4_{\bar{q}}, 2_{\bar{Q}})$ contributing to the antenna subtraction term $d\sigma^S_{\mathrm{NNLO}}$ for $S \to Q\bar{Q} + X$.
- We computed the **integrated antenna function** $\mathcal{B}^0_{Qq\bar{q}\bar{Q}}(q^2, m, \mu, \epsilon)$ analytically in terms of HPLs.
- Our computation proceeded along:
  indoducing cut-propagators $\to$ IBP-reduction $\to$ expanding hypergeom. functions / differential equations
- We compared our result analytically with the literature.