NNLO DIFFERENTIAL CROSS-SECTIONS FOR HIGGS PRODUCTION DESY Zeuthen, January 2012

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of production rates & on fine details in kinematic distributions.

How is the number of Higgs events estimated ?

Overall normalization from very precise inclusive cross section rates.

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Pythia, Herwig, MC@NLO, POWHEG, Alpgen, Sherpa

Kinematic distributions from parton shower MC (with LO, LL or NLO accuracy).

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Overall normalization from very precise inclusive cross section rates.

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Kinematic distributions from parton shower MC (with LO, LL or NLO accuracy).

Differential distributions from more precise calculations to control MC or to compare directly with binned data.

How well do we understand the kinematic distributions

of the Higgs boson

of its decay products

of associated radiation



Pretty well in general, but there is room for improvement.

Even for the simplest of distributions:

The invariant mass distribution of the Higgs boson

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If the Higgs is light

then it's also thin: an uneventful spike well thinner than the experimental resolution



But if the Higgs is heavy (>500GeV)

which is not excluded experimentally

It could be part of a sensible but more complicated Higgs sector (2HDM, Susy, etc.)

then it's also wide!

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Amplitudes to produce a final state from a not-so-narrow Higgs boson require decay widths at the virtuality, Q, not the Higgs mass.

$$\mathcal{A} \sim \sqrt{\Gamma_{gg \to H}(Q)} \frac{iZ(Q)}{Q^2 - M_{phys}^2 + iZ(Q)\Gamma(Q^2)} \sqrt{\Gamma_{H \to VV}(Q)} + \mathcal{A}_{\text{rest}}$$

$$\mathcal{A} \sim \sqrt{\Gamma_{gg \to H}(Q)} \frac{iZ(Q)}{Q^2 - M_{phys}^2 + iZ(Q)\Gamma(Q^2)} \sqrt{\Gamma_{H \to VV}(Q)} + \mathcal{A}_{rest}$$

Theory predictions can be very sensitive to taking the limit Q~Mh

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iHixs is the only fixed order cross-section calculation which allows for the width and branching ratios to vary with the Higgs virtuality.



There are significant differences in the estimate of the total cross section from the approximation used in experimental studies.



ATLAS and CMS start excluding very wide Higgs bosons.

Is there an effect on exclusion limits?

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The role of the higher order corrections: realistic exclusion limits require the number of hypothetical signal events.



The role of the higher order corrections: realistic exclusion limits require the number of hypothetical signal events.

A very heavy Higgs might be considered unviable for theoretical reasons, but care is needed before we conclude that LHC data disfavors or excludes such a possibility.

More complicated distributions

HQT

resumed transverse momentum distribution with the possibility to match with NNLO (Bozzi, Catani, de Florian, Grazzini 2003&2006, de Florian, Ferrera, Grazzini, Tommasini, 2011). HNNLO

fully differential (Catani & Grazzini 2007, Grazzini 2008)

FeHiPro fully differential but never officially released

WH production fully differential (Ferrera, Grazzini, Tramontano 2011) H→bb production fully differential (Anastasiou, Herzog, AL 2011) fehip fully differential ggF (Anastasiou, Melnikov, Petriello 2005)





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Fehipro Fully differential NNLO, including exact mass dependence, EW effects, ZZ decays etc.

Further improvements (integration of HPro, python interface, ZZ decay): CA, Stoeckli, Lazopoulos

HPro (2009) (NLO with exact mass dependence): public (CA, Kunszt, Bucherer)

Studies and improvements (ANN, WW decay) (2007):CA, Dissertori, Stoeckli

FEHiP (2005): public (CA, Melnikov, Petriello)



Fehipro is based on sector decomposition for the RR

We would prefer:

No sectors

Simpler integrals

Universal treatment of singularities

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Methods



 NNLO needed novel methods for phase-space integrations with arbitrary cuts and experimental observables.



- subtraction, antennae, ktsubtraction, sector decomposition, slicing, physical sectors, ...
- Many conceptual problems remain. Room and need for fresh ideas!

Basic mathematical problem at NNLO



OVERLAPPING SINGULARITIES

- Divergent loop and phase-space (multi-dimensional) integrals
- Evaluated as an expansion in the dimension regulator (epsilon)
- Fixed integration boundaries for loops and inclusive phase-space integrations.
- Infrared safe but otherwise arbitrary boundaries of phase-space for acceptance cuts and differential distributions.

Overlapping singularities

• FIXED BOUNDARIES

Mellin-Barnes, differential equations, successive Feynman_ parameter integrations,...

- ARBITRARY BOUNDARIES (I) Subtraction method based on infrared safety and QCD factorization to divide the integration into a singularity free numerical integral and integrals with fixed boundaries.
- ARBITRARY BOUNDARIES (II)
 Sector decomposition
 NEW: Non-linear mappings

A toy example with sector decomposition Binoth,Heinrich; Denner,Roth; Hepp

Slice phase-space $dxdy = dxdy [\Theta(x \ge y) + \Theta(y \ge x)]$



 $I = \int_0^1 dx dy \frac{x^{\epsilon}}{x(ax+y)}$





Restore boundaries

$$I = \int_0^1 dx dt \frac{(x)^{\epsilon}}{x(a+t)} + \int_0^1 dt dy \frac{(yt)^{\epsilon}}{yt(at+1)}$$
$$y = tx$$
$$x = ty$$

Singularities are factorized! Cost: ntegral proliferation

Non-linear mappings

- Factorizes overlapping singularities
- trivializes extraction of poles
- local ...like sector decomposition.

...but

- Easier to implement
- Does not proliferate integrations
- Transparent and more physical factorization of singularities

A toy example with our new method

$$I = \int_0^1 dx dy \frac{x^{\epsilon}}{x(ax+y)}$$

 $x \to xy$

factorizes the singularity

spoils integration boundaries

 $\mapsto \int_0^1 dy \int_0^{\frac{1}{y}} dx \frac{(xy)^{\epsilon}}{xy(ax+1)}$

 $x \mapsto \frac{x(y/a)}{1 - x + (y/a)} \qquad \mapsto \int_0^{1} dx dy \frac{(xy)^{\epsilon}}{xy} \left(a(1 - x) + y\right)^{-\epsilon}$

factorizes the singularity preserves integration boundaries

A systematic method of nonlinear mappings at NNLO

- Most divergent (massless) two-loop integrals arXiv:1011.4867
 Double real-radiation integrals which emerge in hadron collider processes (Higgs, top-pair,...) arXiv:1011.4867
- Real-virtual. arXiv:1110.2368
- Double real-radiation for decays.arXiv:1110.2368

Ready to do physics...



Butterworth, Davison, Rubin, Salam



- The decay of a Higgs boson to bottom-quarks is dominant for a light Higgs boson.
- A viable discovery decay channel in associated Higgs production.
- Gluon radiation off the bottom-pair system is important for fat-jet analyses.
- Nice proof of principle of our method.

Feynman diagram anatomy



- Easily done analytically... Highly complicated application of non-linear mappings.
- Non-trivial! Overlapping loop and phase-space singularities.
- **Difficult**, but perfect problem for our method.

Double Virtual

$$\int_{p_1}^{p_2} = 4^{2+2\epsilon} \int_0^1 dx_1 dx_2 dz dy dx \frac{zy^{1+\epsilon}(1-y)^{-1-\epsilon}(1-z)^{-1-\epsilon}}{\left[x(1-x)+yz(x-x_1)(x-x_2)\right]^{2+2\epsilon}}$$



- Two-loop integrations numerically from its Feynman parameterization, partial fractioning and non-linear mappings.
- Also analytically with reducing (Laporta algorithm, AIR) to master integrals (known since 1987).
- We use the analytic result in our Monte-Carlo program for the decay width.
- Our method can be useful for two-loop amplitudes which are not yet known analytically (more masses, offshell legs, ...)

Real-Virtual



- Used Laporta algorithm (AIR) to reduce the one-loop amplitude to master integrals (box and bubble)
- Need to integrate the one-loop box over singular phase-space (non-smooth off-shell to on-shell leg limit)

$$\int d\mathrm{PS}_3 \frac{{}_2F_1(1,1-\epsilon,-\epsilon,-\frac{u}{t})}{ut}$$

• Use Euler representation of hypergeometric function

$${}_{2}F_{1}\left(1,1-\epsilon,-\epsilon,-\frac{u}{t}\right) = -\epsilon t \int_{0}^{1} dx_{3} \frac{x_{3}^{-1-\epsilon}}{t+ux_{3}}$$

• Apply non-linear mapping

$$x_3 \mapsto \frac{x_3 t/u}{1 - x_3 + t/u}$$

Real-Virtual (II)



• Our mapping simply "re"-derives a known identity

$${}_{2}F_{1}(a,b,c;z) = (1-z)^{-b} {}_{2}F_{1}\left(c-a,b,c;\frac{z}{z-1}\right)$$

Full regulator dependence must be kept and combined with phase-space measure

carefully expanded in epsilon and subtracted in soft/collinear limits

• Implemented both analytic and semi-analytic (non-linear mapping) methods. Surprisingly, no difference in evaluation time

Double Real

• Overlapping singularities "thrive" in Feynman diagrams with double real emissions

1000000

• We have factorized ALL overlapping singularities with partial fractioning and just three mapping at most!

$$egin{array}{lll} \lambda_2 & \mapsto & lpha(\lambda_2,\lambda_3) \ \lambda_4 & \mapsto & lpha(\lambda_4,\lambda_2ar\lambda_3) \ \lambda_2 & \mapsto & lpha(\lambda_2,ar\lambda_1) \end{array} & egin{array}{lll} lpha(x,A) := \displaystyle rac{xA}{xA+ar x} \ rac{xA}{xA+ar x} \end{array}$$

The inclusive check

• Numerically

$$\Gamma_{H\to b\bar{b}}^{NNLO} = \Gamma_{H\to b\bar{b}}^{LO} \left[1 + \left(\frac{\alpha_s}{\pi}\right) 5.6666(4) + \left(\frac{\alpha_s}{\pi}\right)^2 29.12(4) + \mathcal{O}(\alpha_s^3) \right]$$

• Analytically

$$\Gamma_{H\to b\bar{b}}^{NNLO} = \Gamma_{H\to b\bar{b}}^{LO} \left[1 + \left(\frac{\alpha_s}{\pi}\right) 5.66666666... + \left(\frac{\alpha_s}{\pi}\right)^2 29.146714... + \mathcal{O}(\alpha_s^3) \right]^2 \right]$$

Initial state double real (RR)

We catalogue all possible singular kinematic configurations based on denominators of (physical) Feynman diagrams.



Initial state double real (RR)²

1

Using non-linear mappings we can factorize all singularities for any singular structure in initial-initial and finalfinal radiation.

1. Topology
$$C_1 \otimes C_1$$
:

$$\int \frac{d\Phi_3 N(\{s_{ij}\})}{(s_{13}s_{24})^2}, \int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{13}s_{23}s_{14}s_{24}}$$
2. Topology $C_2 \otimes C_2$:

$$\int \frac{d\Phi_3 N(\{s_{ij}\})}{(s_{34}s_{134})^2}, \int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{34}s_{234}s_{234}}, \int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{13}s_{24}s_{134}s_{234}}$$
3. Topology $C_3 \otimes C_3$:

$$\int \frac{d\Phi_3 N(\{s_{ij}\})}{(s_{13}s_{134})^2}, \int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{13}s_{23}s_{134}s_{234}}, \int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{13}s_{24}s_{134}s_{234}}$$
4. Topology $C_1 \otimes C_2$:

$$\int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{134}s_{13}s_{23}s_{14}}, \int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{134}s_{13}s_{14}}$$
5. Topology $C_1 \otimes C_3$:

$$\int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{34}s_{13}^2s_{13}s_{14}}, \int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{34s}s_{134}s_{234}s_{234}}$$
7. Topology $C_2 \otimes C_3$:

$$\int \frac{d\Phi_3 N(\{s_{ij}\})}{t_{i3}^2t_{j4}^2}, \int \frac{d\Phi_3 N(\{s_{ij}\})}{t_{i3}t_{j4}s_{13}s_{14}}$$
8. Topology $C_4 \otimes C_4$:

$$\int \frac{d\Phi_3 N(\{s_{ij}\})}{t_{i3}t_{j4}s_{13}s_{14}}$$
9. Topology $C_4 \otimes C_2$:

$$\int \frac{d\Phi_3 N(\{s_{ij}\})}{t_{i3}t_{j4}s_{34}s_{134}}$$
10. Topology $C_4 \otimes C_3$:

$$\int \frac{d\Phi_3 N(\{s_{ij}\})}{t_{i3}t_{j4}s_{34}s_{134}}$$

Initial state double real (RR)

The singularity structure in each topology is determined by the kinematic invariants that appear in denominators.

We simultaneously *s*₁₃ factorize them using partial fractioning and three, at most, non-linear mappings.

No differences with double real radiation from the final state.

$$\begin{cases} \mathbf{f} \\ \mathbf{f}$$

$$= \bar{\lambda}_1 \left[\lambda_4 \lambda_3 + \lambda_2 \bar{\lambda}_3 \bar{\lambda}_4 + 2 \cos(\lambda_5 \pi) \sqrt{\lambda_2 \lambda_3 \bar{\lambda}_3 \lambda_4 \bar{\lambda}_4} \right]$$
$$= \bar{\lambda}_1 \left[\lambda_3 \bar{\lambda}_4 + \lambda_2 \bar{\lambda}_3 \lambda_4 - 2 \cos(\lambda_5 \pi) \sqrt{\lambda_2 \lambda_3 \bar{\lambda}_3 \lambda_4 \bar{\lambda}_4} \right]$$

$$\lambda_{2} \mapsto \alpha(\lambda_{2}, \lambda_{3})$$
$$\lambda_{4} \mapsto \alpha(\lambda_{4}, \lambda_{2}\bar{\lambda}_{3})$$
$$\lambda_{2} \mapsto \alpha(\lambda_{2}, \bar{\lambda}_{1})$$

 $\alpha(x,A) := \frac{xA}{xA + \bar{x}}$

On the double real (RR)

Note that the process-specific numerator 4. can be kept arbitrary.

2. Topology
$$C_2 \otimes C_2$$
:

$$\int \frac{d\Phi_3 N(\{s_{ij}\})}{(s_{34}s_{124})^2}, \int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{34}^2 s_{134}s_{234}}$$
3. Topology $C_3 \otimes C_3$:

$$\int \frac{d\Phi_3 N(\{s_{ij}\})}{(s_{13}s_{134})^2}, \int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{13}s_{23}s_{134}s_{234}}, \int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{13}s_{24}s_{134}s_{234}}$$
Or
4. Topology $C_1 \otimes C_2$:

$$\int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{34}s_{234}s_{13}s_{24}}$$
5. Topology $C_1 \otimes C_3$:

$$\int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{134}s_{13}s_{23}s_{14}}, \int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{134}s_{13}^2s_{14}}$$
6. Topology $C_2 \otimes C_3$:

$$\int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{134}s_{13}s_{23}s_{14}}, \int \frac{d\Phi_3 N(\{s_{ij}\})}{s_{134}s_{13}^2s_{14}}$$

 $s_{34}s_{134}^2s_{13}$ (J $s_{34}s_{134}s_{234}s_{23}$

To extend the calculation to a new process we just need to project the new RR matrix elements on the topology basis!

Initial state double real (RR)

The fully soft limit is special: it exposes universal threshold contributions. We parametrize double soft singularities by a singe variable (Q/E) which is never re-mapped.

$$\sigma^{RR} = \widetilde{\sigma}_{ij}^{RR}(1) \int dz (1-z)^{-1-4\epsilon} \mathcal{L}_{ij}(z) + \int dz \mathcal{L}_{ij}(z) (1-z)^{-4\epsilon} \left[\frac{\widetilde{\sigma}_{ij}^{RR}(z) - \widetilde{\sigma}_{ij}^{RR}(1)}{1-z} \right]$$

Threshold contributions: all remaining phase-space variables are integrated once and for all.

Singular in at most three PSP variables. Contains initial state collinear singularities are cancelled numerically against convolutions with splitting functions.

Promise for threshold log resummation and matched to a fully differential NNLO code.

On the Real-Virtual (RV)

Complication: Singular limit from phase space integration of a virtual amplitude. (Non-smooth off-shell to on-shell

limits of master integrals).

$$\int d\mathbf{PS}_3 \frac{{}_2F_1(1,1-\epsilon,-\epsilon,-\frac{u}{t})}{ut}$$

The loop amplitude must be cast in a form that exposes the limit smoothly.

Non-linear mappings is a method to do so.

$${}_{2}F_{1}\left(1,1-\epsilon,-\epsilon,-\frac{u}{t}\right) = -\epsilon t \int_{0}^{1} dx_{3} \frac{x_{3}^{-1-\epsilon}}{t+ux_{3}}$$

$$x_3 \mapsto \frac{x_3 t/u}{1 - x_3 + t/u}$$

On the collinear subtraction

 $(\mathcal{L}_{kl}, g_1, g_2, \mu_{f}) \Delta_{kl} (\mathcal{L}_{l}) \Delta_{ll} (\mathcal{L}_{2})$

 $\tilde{\sigma}_i$

Collinear subtraction terms are non-trivial at NNLO. Usually treated analytically to supply cancelation terms to the partonic cross sections.

$$\sigma = \int_{0}^{1} dx_{1} dx_{2} f_{i}(x_{1}) f_{j}(x_{2}) \sigma_{ij}(x_{1}, x_{2})$$

$$f_{i}(x) = \left(\begin{array}{c} \mathbf{DIV} \\ \Delta_{ij} \otimes \tilde{f}_{j} \end{array} \right) (x)$$

$$\sigma = \int_{0}^{1} dx_{1} dx_{2} \tilde{f}_{k}(x_{1}, \mu_{f}) \tilde{f}_{l}(x_{2}, \mu_{f}) \tilde{\sigma}_{kl}(x_{1}, x_{2})$$

$$\mathbf{FIN} \quad \mathbf{FIN} \quad \mathbf{FIN}$$

$$f_{i}(x_{2}, \mu_{f}) \tilde{\sigma}_{kl}(x_{1}, x_{2})$$

$$f_{i}(x_{2}, \mu_{f}) \tilde{\sigma}_{kl}(x_{1}, x_{2})$$

$$\mathbf{FIN} \quad \mathbf{FIN} \quad \mathbf{FIN}$$

$$\mathbf{FIN} \quad \mathbf{FIN}$$

$$\mathbf{FIN}$$

$$\mathbf{FIN} \quad \mathbf{FIN}$$

$$\mathbf{FIN}$$

$$\mathbf{FIN} \quad \mathbf{FIN}$$

$$\mathbf{FIN}$$

On the collinear subtraction

But if we use the bare PDF's, expanded in strong coupling and the dimensional regulator, we have a universal treatment.

Numerical implementation of bare PDFs in a grid, like the renormalized ones.

$$\sigma = \int_0^1 dx_1 dx_2 f_i(x_1) f_j(x_2) \sigma_{ij}(x_1, x_2)$$
$$f_i(x) = \left(\Delta_{ij} \otimes \tilde{f}_j \right) (x)$$
$$\underbrace{\mathsf{Div}}_{\mathsf{Div}}$$

$$\begin{aligned} \Delta_{ij}^{(0)}(z) &= \delta_{ij}\delta(1-z), \\ \Delta_{ij}^{(1)}(z) &= \frac{P_{ij}^{0}}{\epsilon} \\ \Delta_{ij}^{(2)}(z) &= \frac{P_{ij}^{1}(z)}{2\epsilon} + \frac{1}{2\epsilon^{2}} \left[\left(P_{ik}^{0} \otimes P_{kj}^{0} \right)(z) - \beta_{0} P_{ij}^{0}(z) \right] \end{aligned}$$

$$f_i(z) = f_i^{(0)}(z) + \left(\frac{\alpha_s(\mu)}{\pi}\right) f_i^{(1)} + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 f_i^{(2)} + \dots$$

bb→H differentially @ NNLO

Calculation in progress:

✓ Full LO and NLO

Double virtual

✓ Virtual square

Double real implemented

√ggbbH sub-channel completed.

 \star Real-Virtual in implementation.

- Two independent numerical implementations of the double real subtraction process.
- $gg \rightarrow H$ also in progress: no extra effort.

bb→H differentially @ NNLO

- Very preliminary result: the Higgs rapidity distribution in the gg→bbH channel subchannel.
- Applying cuts on the b-quarks, the total rate is checked against MCFM.



Conclusions

- Years of work by the theory community have resulted to very accurate predictions for the Higgs signal event rates, inclusively and differentially.
- There is still room for improvement, especially in the high mass region, where the Higgs line-shape affects significantly the exclusion/discovery interpretation. iHixs is a flexible tool that can incorporate any
- A lot remains to be done for fully differential calculations that will be even more important when the (some) Higgs is discovered.
- We see a way to systematize the treatment of the double real emission at NNLO. We apply it to gluon fusion and bbH.
- We are building a framework that is fully generic, and is ready to engage processes with colorful and/or massive final states.