asymptotically safe gravitation

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1102.4624 [hep-th]

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gravitation

physics of classical gravity

Einstein's theory $G_N = 6.7 \times 10^{-11} \frac{m^3}{\mathrm{kg}\, s^2}$

classical action

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_N} \int \sqrt{\det g}\,(-R(g_{\mu\nu}) + 2\Lambda)$$

long distances

gravity not tested beyond $10^{28}\mathrm{cm}$

short distances

gravity not tested below $10^{-2}\mathrm{cm}$

gravitation

physics of classical gravity

Einstein's theory $G_N = 6.7 \times 10^{-11} \frac{m^3}{\mathrm{kg}\, s^2}$

physics of quantum gravity

Planck length $\ell_{\mathrm{Pl}} = \left(\frac{\hbar G_N}{c^3}\right)^{1/2} \approx 10^{-33}\,\mathrm{cm}$

Planck mass $M_{\mathrm{Pl}} \approx 10^{19}\mathrm{GeV}$

Planck time $t_{\mathrm{Pl}} \approx 10^{-44}\,\mathrm{s}$

Planck temperature $T_{\mathrm{Pl}} \approx 10^{32}\,\mathrm{K}$

expect **quantum modifications** at energy scales M_{Pl}

perturbation theory

- **structure of UV divergences**

  gravity: $[g_{\mu\nu}] = 0\,, \quad [\mathrm{Ricci}] = 2\,, \quad [G_N] = 2 - d$

  effective expansion parameter: $g_{\mathrm{eff}} \equiv G_N\, E^2 \sim \frac{E^2}{M_{\mathrm{Pl}}^2}$

  N-loop Feynman diagram $\sim \int dp\, p^{A - [G] N}$

  $[G] > 0 :$ superrenormalisable

  $[G] = 0 :$ renormalisable

  $[G] < 0 :$ dangerous interactions

- **perturbative non-renormalisability**

  gravity with matter interactions

  pure gravity (Goroff-Sagnotti term)

perturbation theory

- **effective theory for gravity** (Donoghue '94)

  quantum corrections computable for energies $E^2/M_{\rm Pl}^2 \ll 1$

  knowledge of UV completion not required

- **higher derivative gravity I** (Stelle '77)

  R^2 gravity perturbatively renormalisable

  unitarity issues at high energies

- **higher derivative gravity II** (Gomis, Weinberg '96)

  all higher derivative operators

  gravity 'weakly' perturbatively renormalisable

  no unitarity issues at high energies

quantum field theory

running couplings

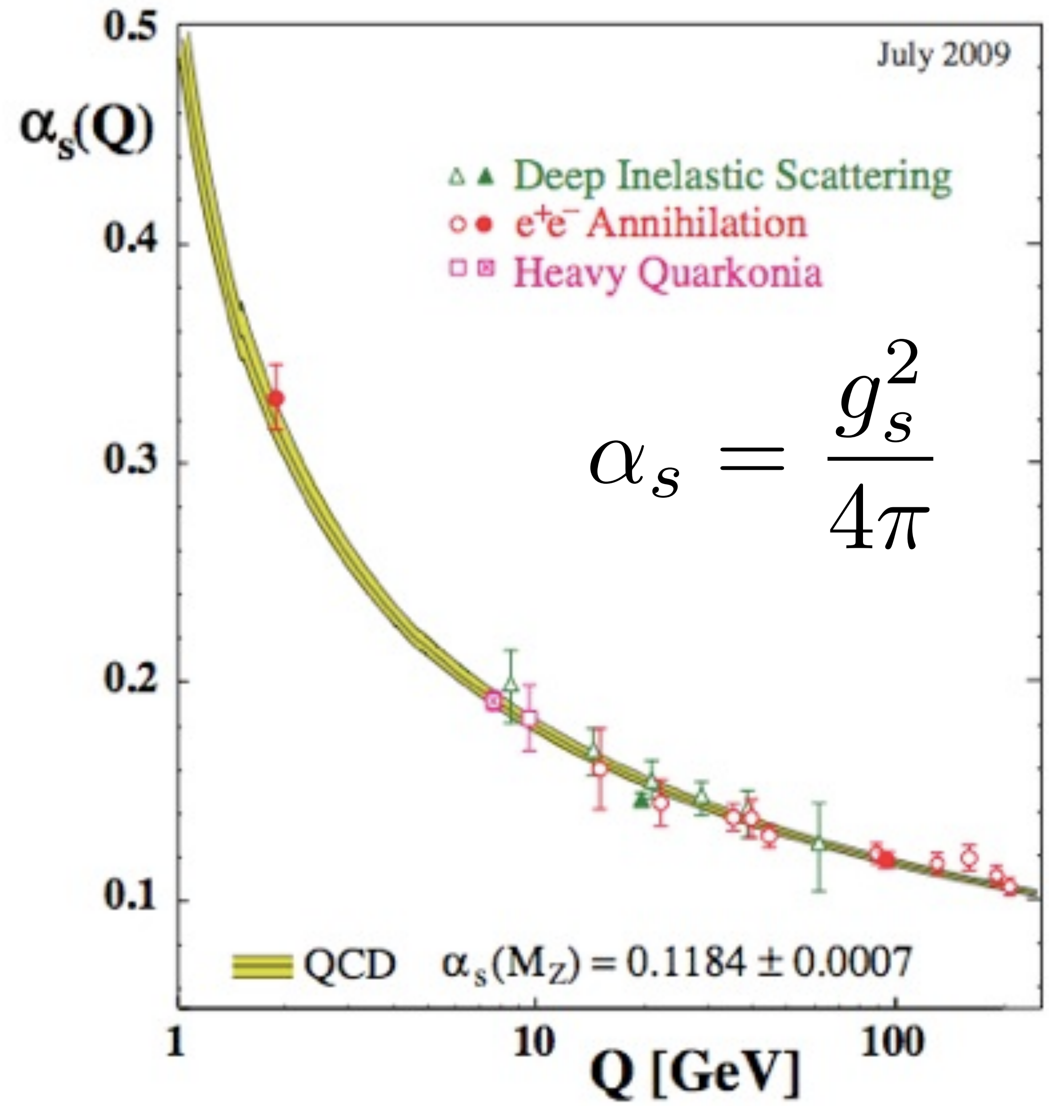
quantum fluctuations modify interactions

couplings depend on eg. energy or distance

asymptotic freedom of the strong force

(taken from PDG)

$$S_{\rm YM} = \frac{1}{4g_s^2}\int F^2$$



$$\alpha_s = \frac{g_s^2}{4\pi}$$

quantum field theory

running couplings

quantum fluctuations modify interactions

couplings depend on eg. energy or distance

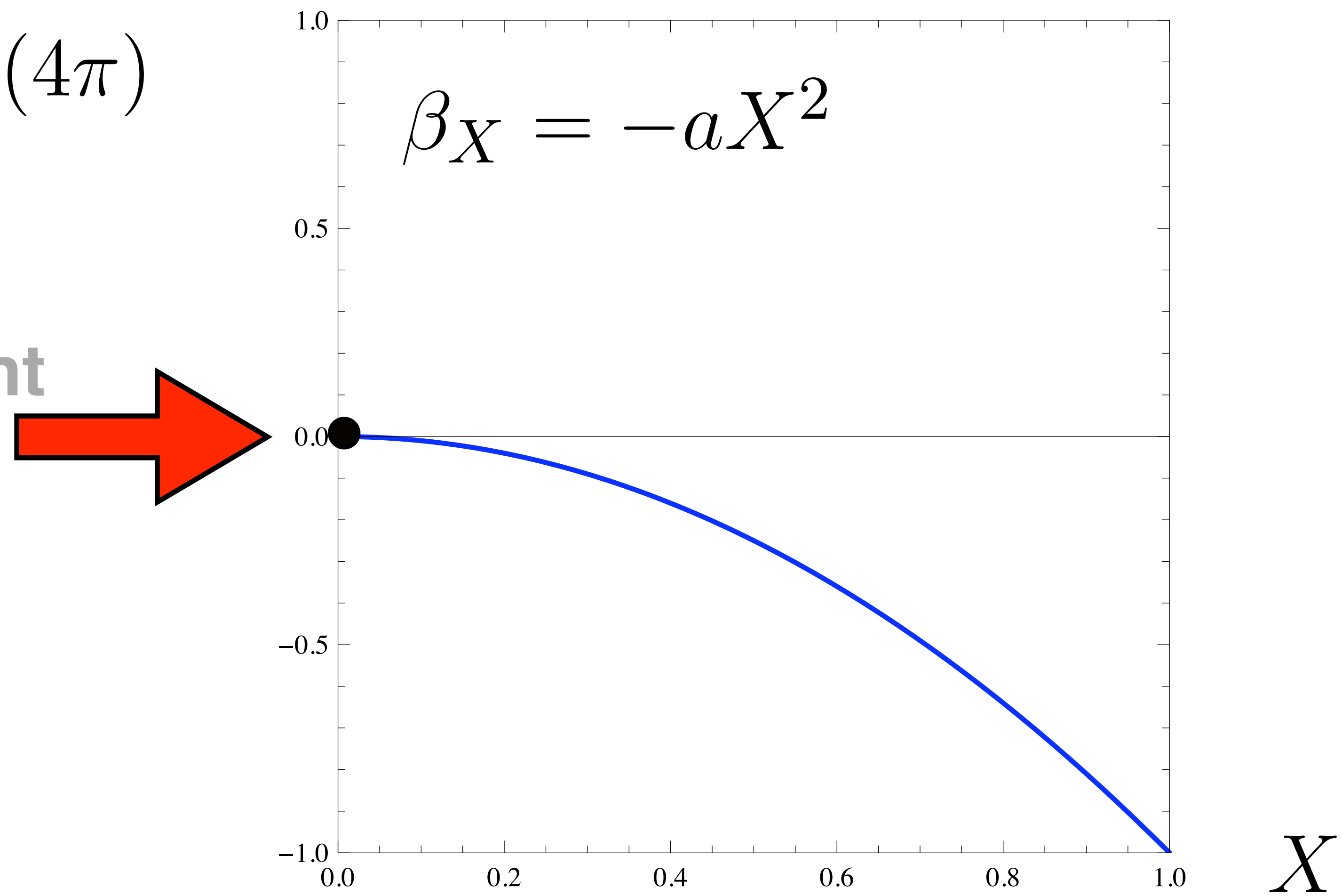
asymptotic freedom of the strong force

coupling $X = g_s^2/(4\pi)$

$$\beta_X \equiv \frac{dX}{d\ln\mu}$$

trivial UV fixed point

$$X_* = 0$$



quantum field theory

running couplings

quantum fluctuations modify interactions

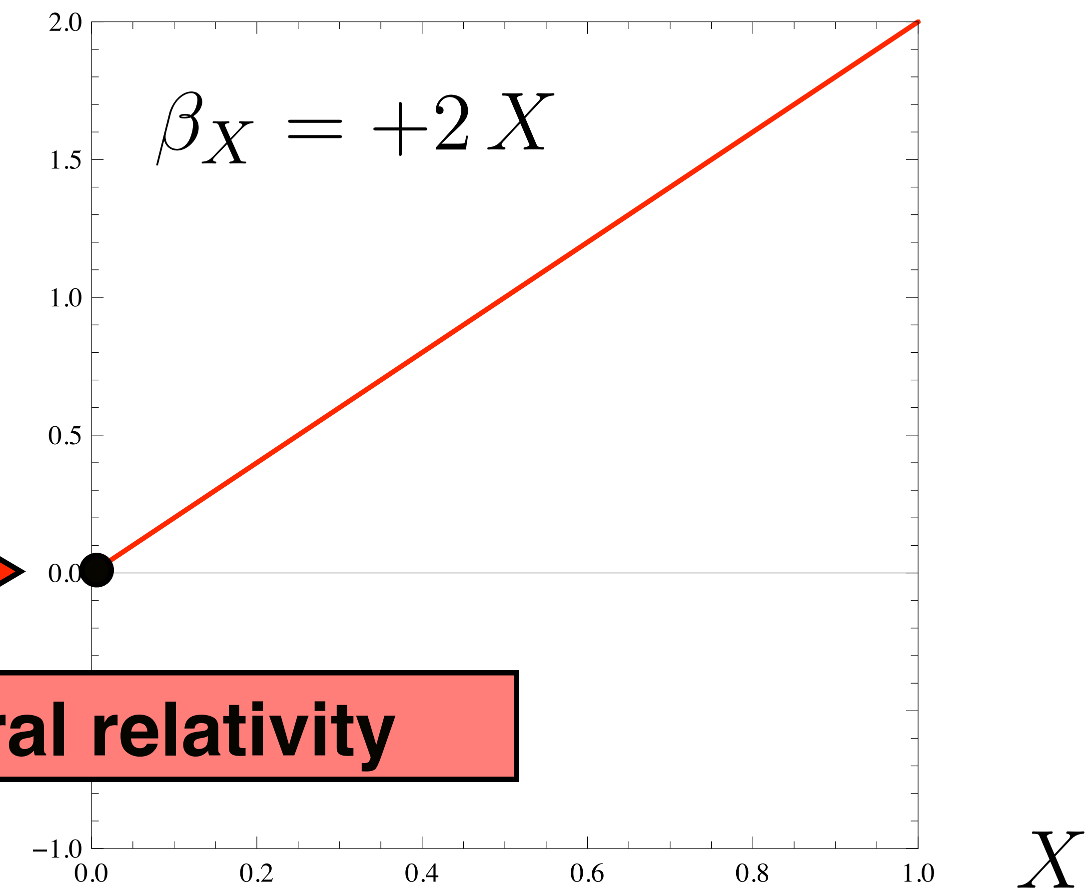
couplings depend on eg. energy or distance

gravitation

coupling $X = G_N \, \mu^2$

$$\beta_X \equiv \frac{dX}{d \ln \mu}$$

trivial IR fixed point



classical general relativity

quantum field theory

running couplings

quantum fluctuations modify interactions

couplings depend on eg. energy or distance

gravitation

replace

$$G_N \to G(\mu)$$

$$g_{\text{eff}} = G_N\,\mu^2 \to g(\mu) \equiv G(\mu)\,\mu^2$$

quantum field theory

running couplings

quantum fluctuations modify interactions

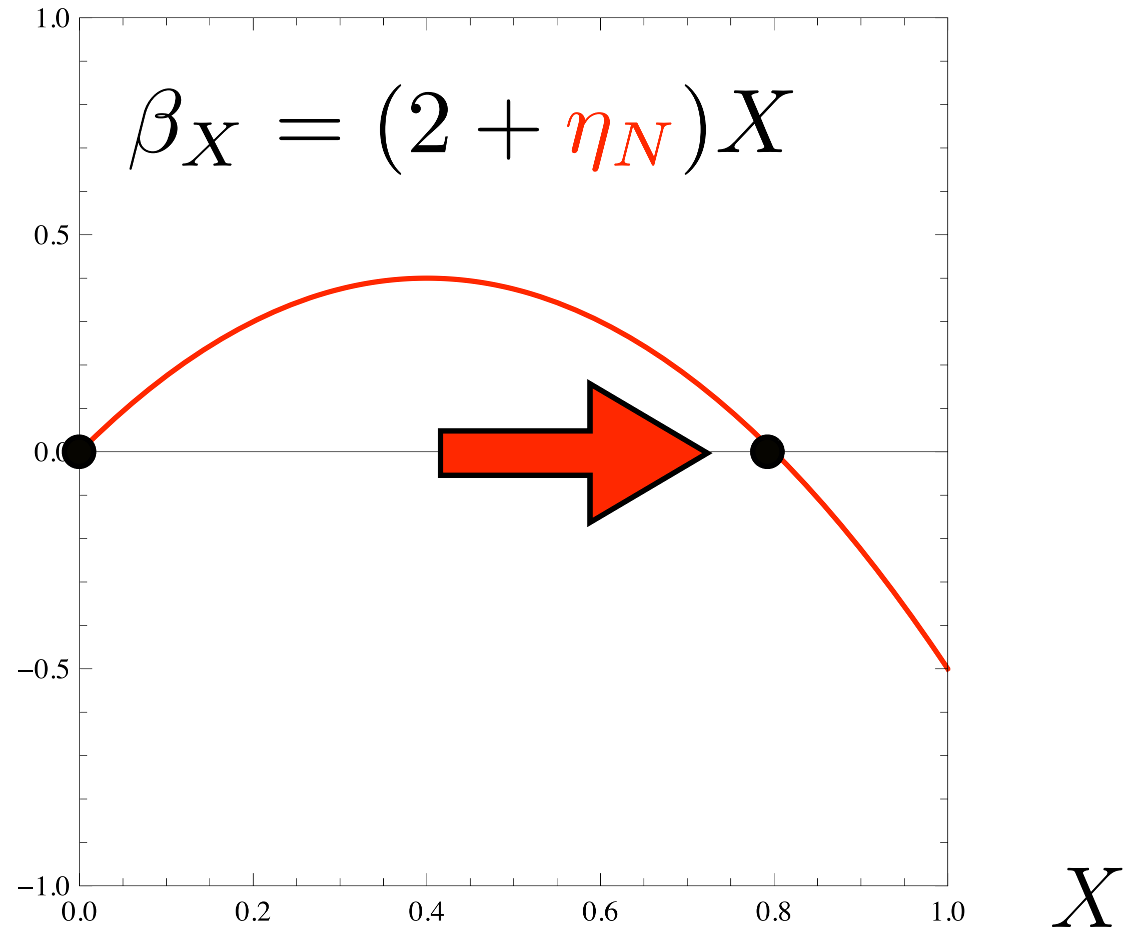
couplings depend on eg. energy or distance

gravitation

coupling $X = G(\mu)\,\mu^2$

$$\beta_X \equiv \frac{dX}{d\ln\mu}$$

non-trivial UV fixed point



quantum field theory

running couplings

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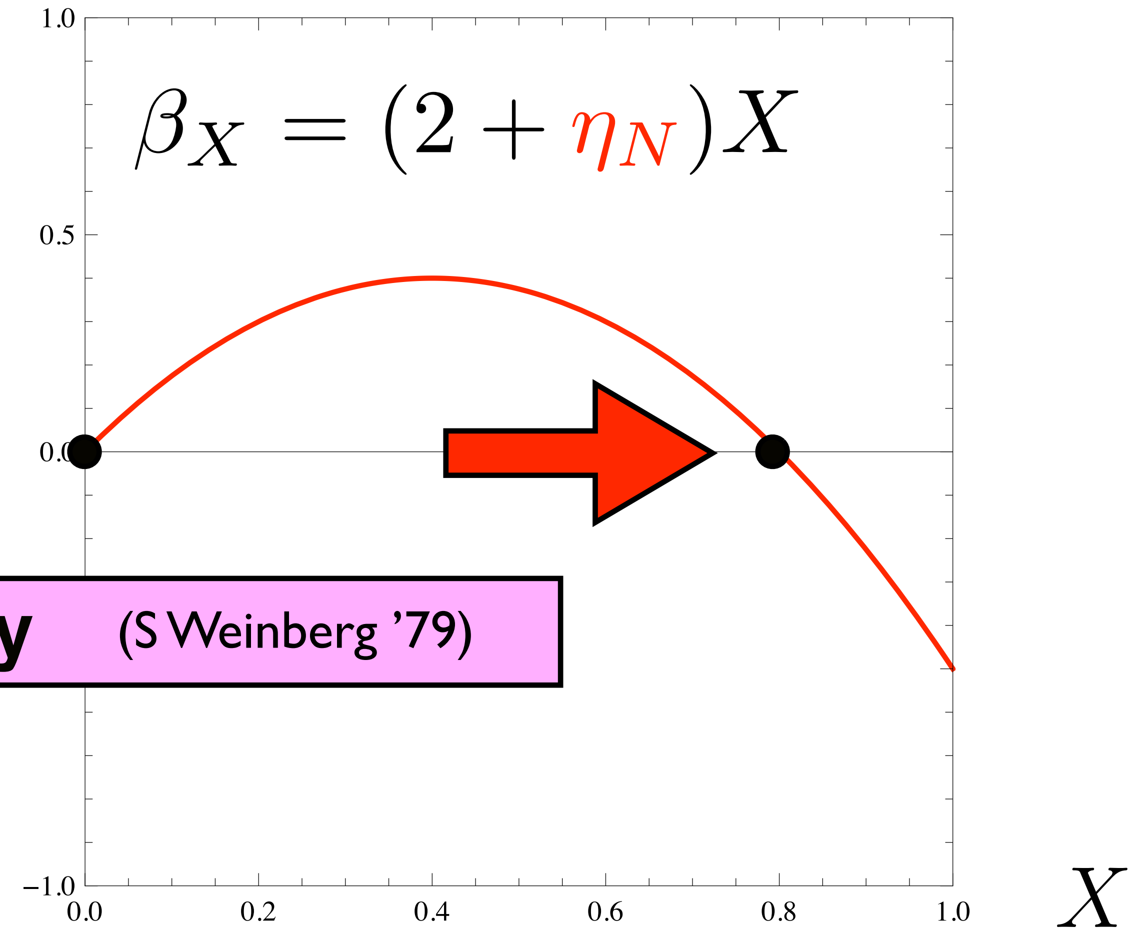

asymptotic safety (S Weinberg '79)

quantum field theory

running couplings

quantum fluctuations modify interactions

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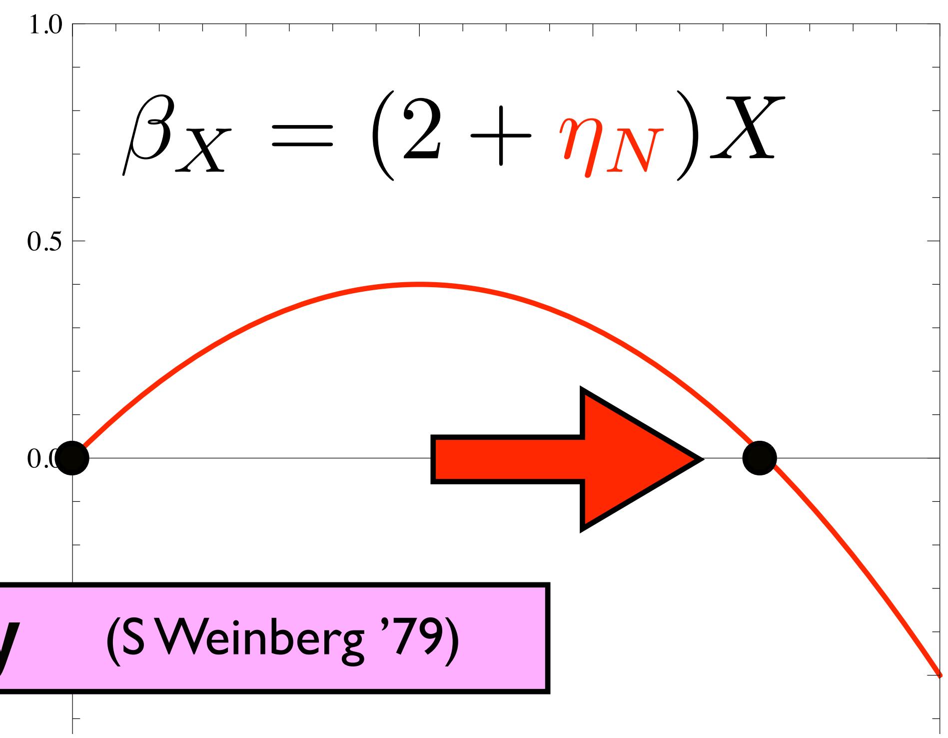


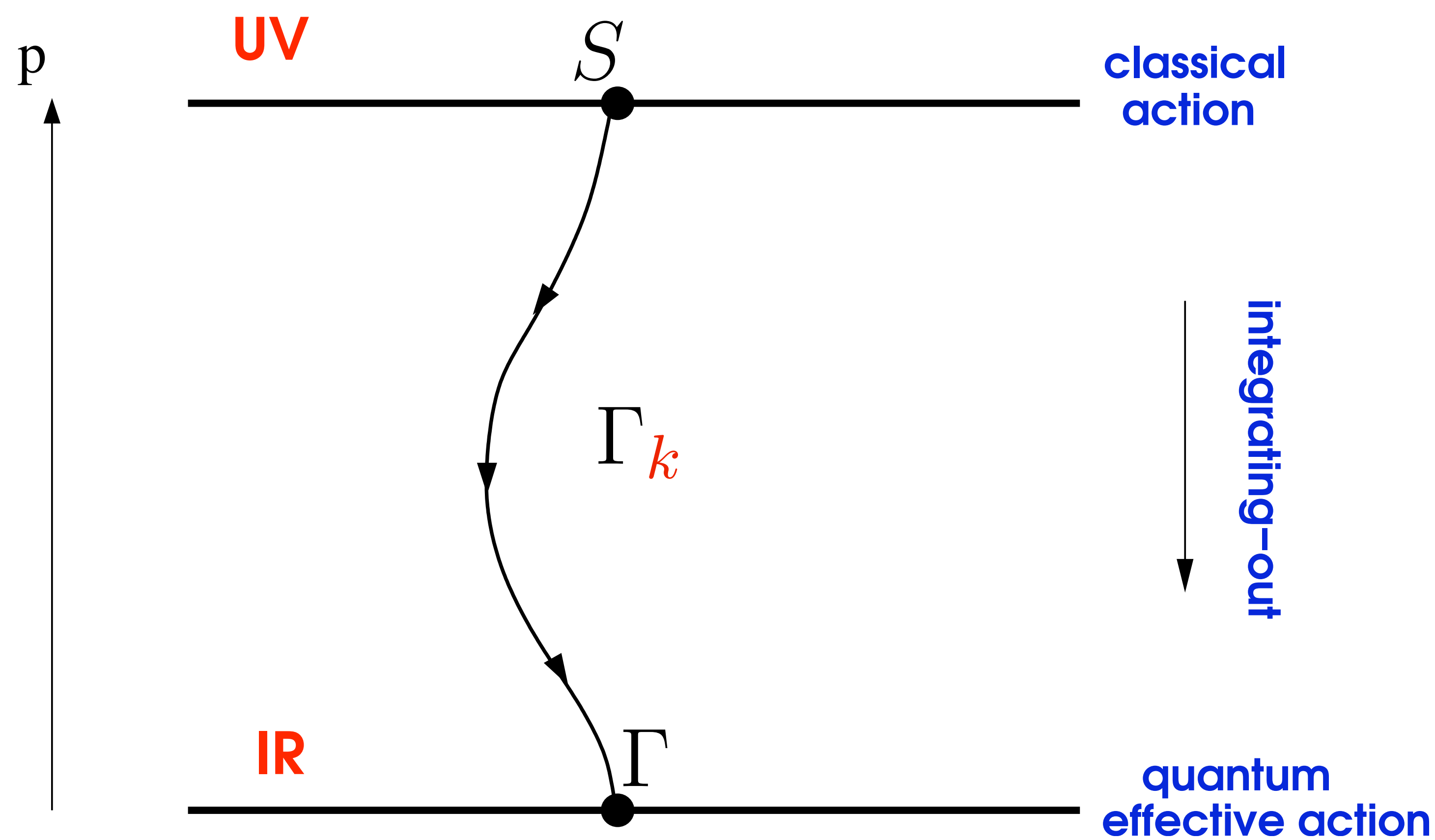
asymptotic safety (S Weinberg '79)

UV fixed point implies **weakly coupled gravity** at **high energies**

$$\mu \to \infty : \qquad G(\mu) \to g_* \mu^{2-D} \ll G_N$$

renormalisation group

integrating-out momentum degrees of freedom: "top-down" (Wilson '71)

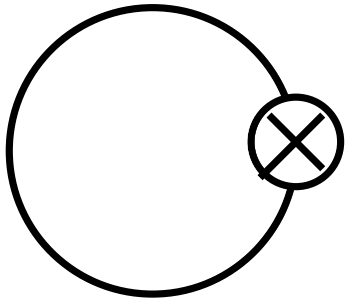


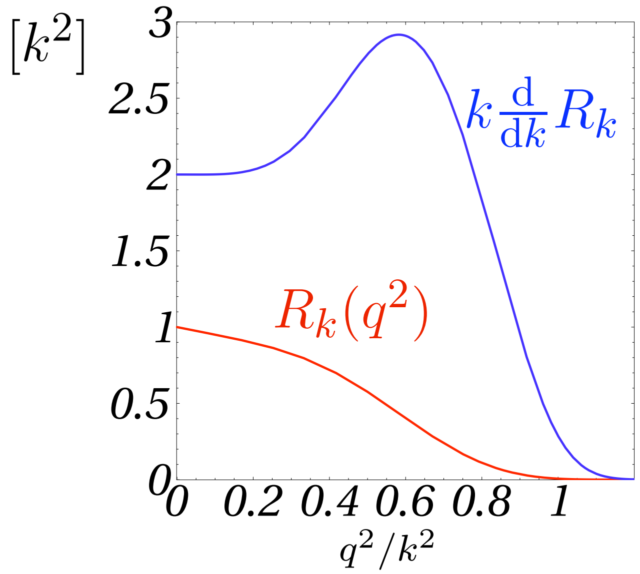
`coarse-graining' of quantum fields

renormalisation group

functional RG (Wetterich '93)

$$k\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2}\,\mathrm{Tr}\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right] \quad = \frac{1}{2}$$





\`all-in-one'
\`exact'
finite
systematic

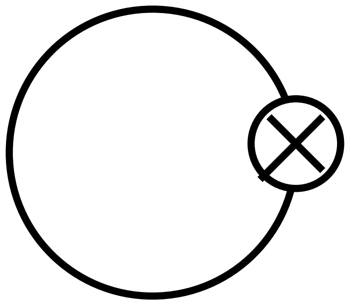
\`optimised' choices of R_k

→ stability, analyticity, convergence (DL '01,'02)

renormalisation group

functional RG (Wetterich '93)

$$k\frac{\mathrm{d}\Gamma_k}{\mathrm{d}k} = \frac{1}{2}\,\mathrm{Tr}\,\left[\left(\frac{\delta^2\Gamma_k[\phi]}{\delta\phi\,\delta\phi} + R_k\right)^{-1} k\frac{\mathrm{d}R_k}{\mathrm{d}k}\right] = \frac{1}{2}$$



QCD

signatures of confinement (Pawlowski, DL, Nedelko, Smekal '03)

Ising-type universality

phase transitions, high precision exponents (DL '02, Bervillier, Juettner, DL '07)

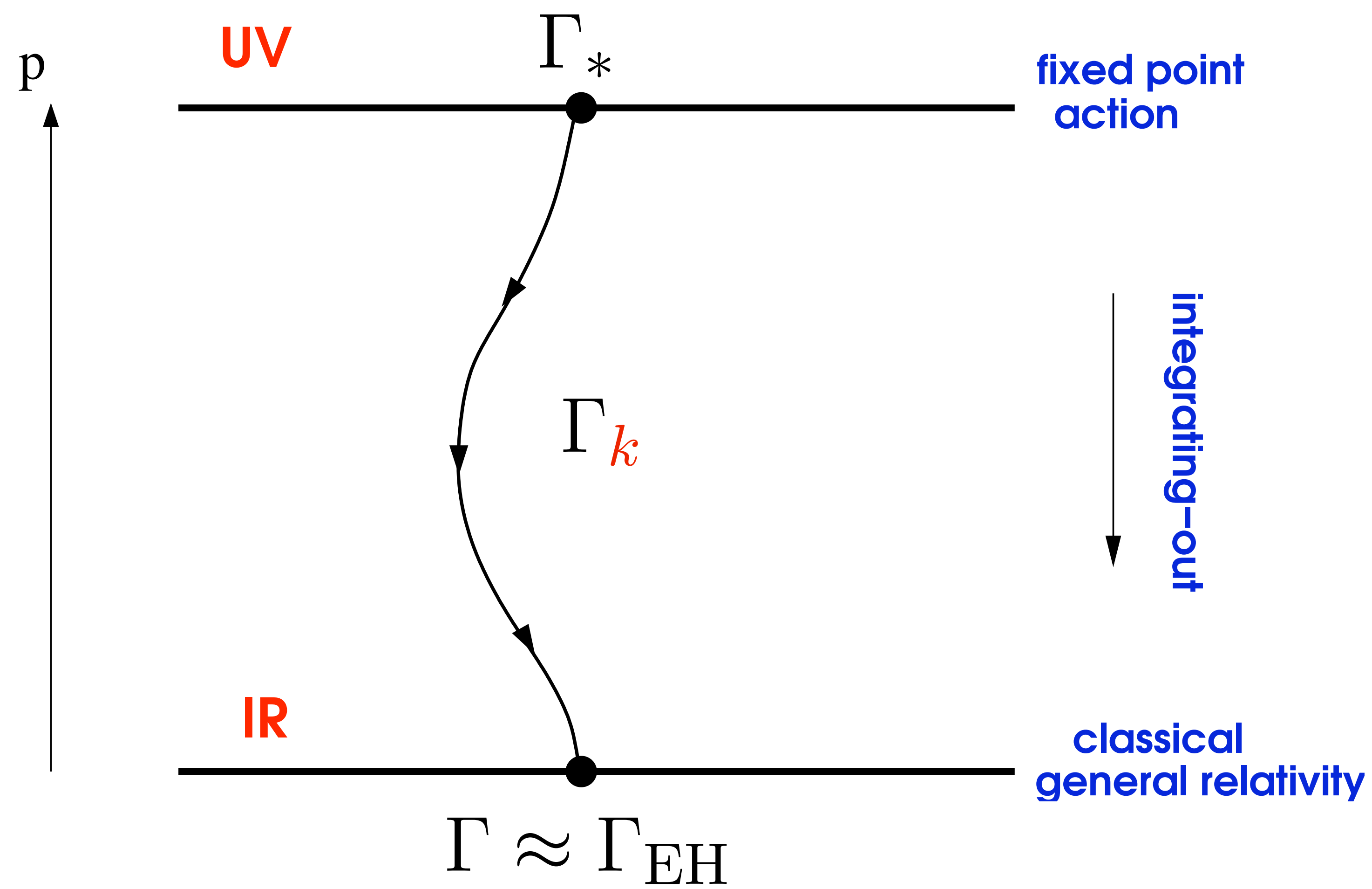
quality control, systematic uncertainties

	η	ν	ω
resummed PT	**0.0335(25)**	**0.6304(13)**	**0.799(11)**
ϵ-expansion	**0.0360(50)**	**0.6290(25)**	**0.814(18)**
world average	**0.0364(5)**	**0.6301(4)**	**0.84(4)**
Monte Carlo	**0.03627(10)**	**0.63002(10)**	**0.832(6)**
functional RGs	**0.034(5)**	**0.630(5)**	**0.82(4)**

(DL, Zappala '10)

renormalisation group

for quantum gravity: "bottom-up" (Reuter '96)



`coarse-graining' of quantum fields

quantum gravity

effective action

$$\Gamma_k = \int \sqrt{g} \left(\frac{-R + 2\Lambda_k}{16\pi G_k} + \cdots \right) + S_{\mathrm{matter},k} + S_{\mathrm{gf},k} + S_{\mathrm{ghosts},k}$$

quantum gravity

effective action

$$\Gamma_k = \int \sqrt{g} \left(\frac{-R + 2\Lambda_k}{16\pi G_k} + \cdots \right) + S_{\mathrm{matter},k} + S_{\mathrm{gf},k} + S_{\mathrm{ghosts},k}$$

Einstein-Hilbert theory

$$\beta_g = (D - 2 + \eta)\, g \qquad g_k = G_k\, k^{D-2} \qquad \eta = \frac{g\, b_1(\lambda)}{1 + g\, b_2(\lambda)}$$

$$\beta_\lambda = (-2 + \eta)\lambda + g(a_1 - \eta\, a_2) \qquad \lambda_k = \Lambda_k / k^2$$

$$\begin{aligned}
a_1 &= \frac{D(D-1)(D+2)}{2(1-2\lambda)} + \frac{D(D+2)}{1-2\alpha\lambda} - 2D(D+2) \\
a_2 &= \frac{D(D-1)}{2(1-2\lambda)} + \frac{D}{1-2\alpha\lambda} \\
b_1 &= -\frac{1}{3}(1+\frac{2}{D})(D^3+6D+12) \\
&\quad - \frac{(D+2)(D^3-4D^2+7D-8)}{(D-1)(1-2\lambda)^2} + \frac{D(D+2)(D^3-2D^2-11D-12)}{12(D-1)(1-2\lambda)} \\
&\quad - \frac{2(D+2)(\alpha D^2-2\alpha D-D-1)}{D(1-2\alpha\lambda)^2} + \frac{(D+2)(D^2-6)}{6(1-2\alpha\lambda)} \\
b_2 &= -\frac{D^3-4D^2+7D-8}{(D-1)(1-2\lambda)^2} + \frac{(D+2)(D^3-2D^2-11D-12)}{12(D-1)(1-2\lambda)} \\
&\quad - \frac{2(\alpha D^2-2\alpha D-D-1)}{D(1-2\alpha\lambda)^2} + \frac{(D+2)(D^2-6)}{6D(1-2\alpha\lambda)}
\end{aligned}$$

(DL′03)

quantum gravity

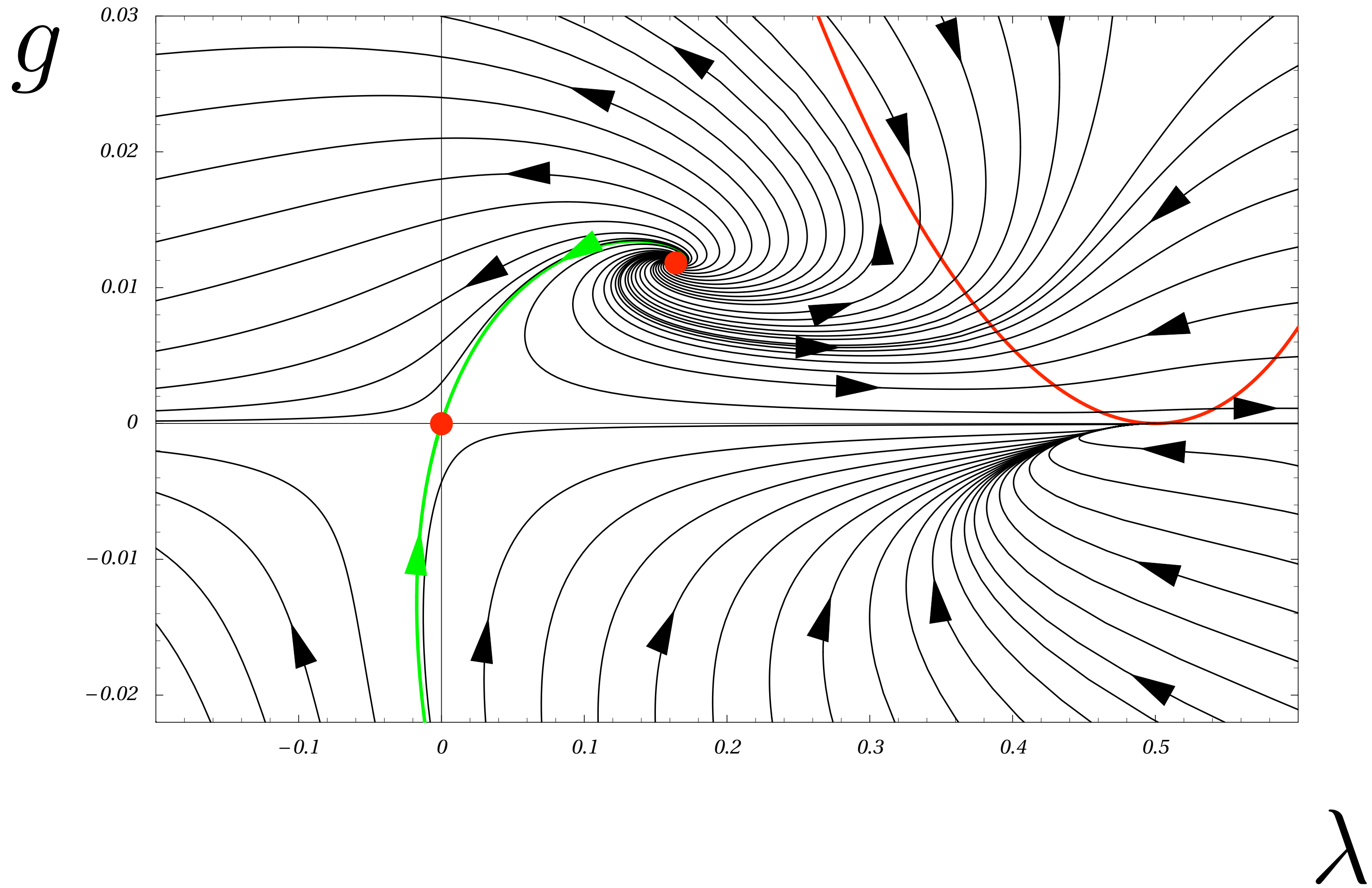
effective action

$$\Gamma_k = \int \sqrt{g}\left(\frac{-R + 2\Lambda_k}{16\pi G_k} + \cdots\right) + S_{\text{matter},k} + S_{\text{gf},k} + S_{\text{ghosts},k}$$

Einstein-Hilbert theory

$$\Lambda_k = \lambda\, k^2$$

$$G_k = g/k^2$$



(DL '03)

quantum gravity

effective action

$$\Gamma_k = \int \sqrt{g}\left(\frac{-R + 2\Lambda_k}{16\pi G_k} + \cdots\right) + S_{\text{matter},k} + S_{\text{gf},k} + S_{\text{ghosts},k}$$

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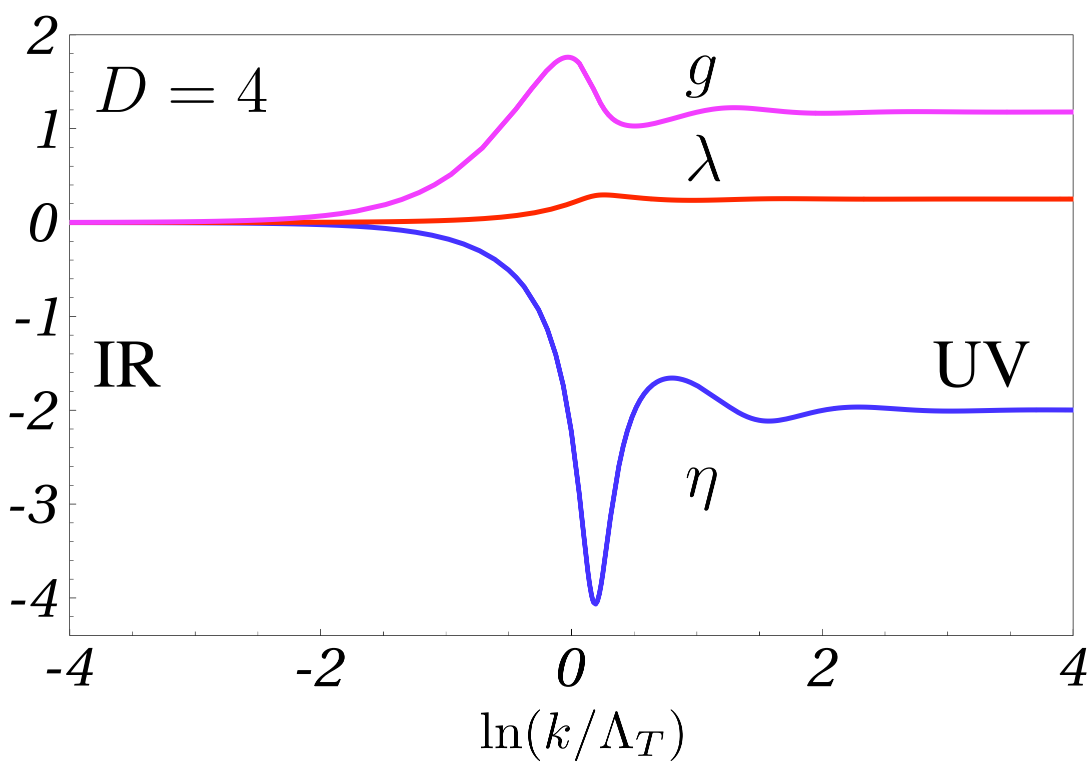
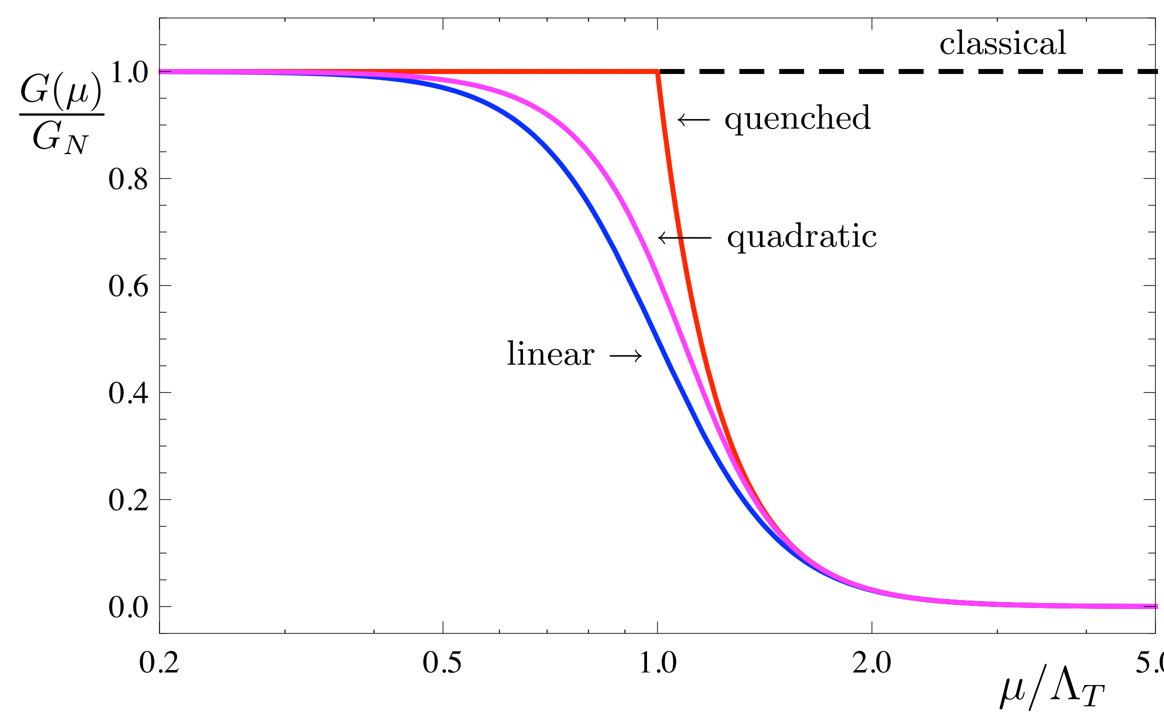


(DL '03)



(Gerwick,DL, Plehn '11)

quantum gravity

effective action

$$\Gamma_k = \int \sqrt{g} \left(\frac{-R + 2\Lambda_k}{16\pi G_k} + \cdots \right) + S_{\text{matter},k} + S_{\text{gf},k} + S_{\text{ghosts},k}$$

Einstein-Hilbert theory

$$\Lambda_k = \lambda\, k^2$$

$$G_k = g/k^2$$

higher dimensions

Einstein-Hilbert, extensions (DL ’03, Fischer, DL ’05)

$$\lambda_* = \frac{D^2 - D - 4 - \sqrt{2D(D^2 - D - 4)}}{2(D-4)(D+1)}$$

$$g_* = \Gamma(\tfrac{D}{2} + 2)(4\pi)^{D/2-1} \frac{(\sqrt{D^2 - D - 4} - \sqrt{2D})^2}{2(D-4)^2(D+1)^2}$$

RG connected with perturbative infrared regime

quantum gravity

effective action

$$\Gamma_k = \int \sqrt{g} \left(\frac{-R + 2\Lambda_k}{16\pi G_k} + \cdots \right) + S_{\text{matter},k} + S_{\text{gf},k} + S_{\text{ghosts},k}$$

Einstein-Hilbert theory

$$\Lambda_k = \lambda\, k^2$$

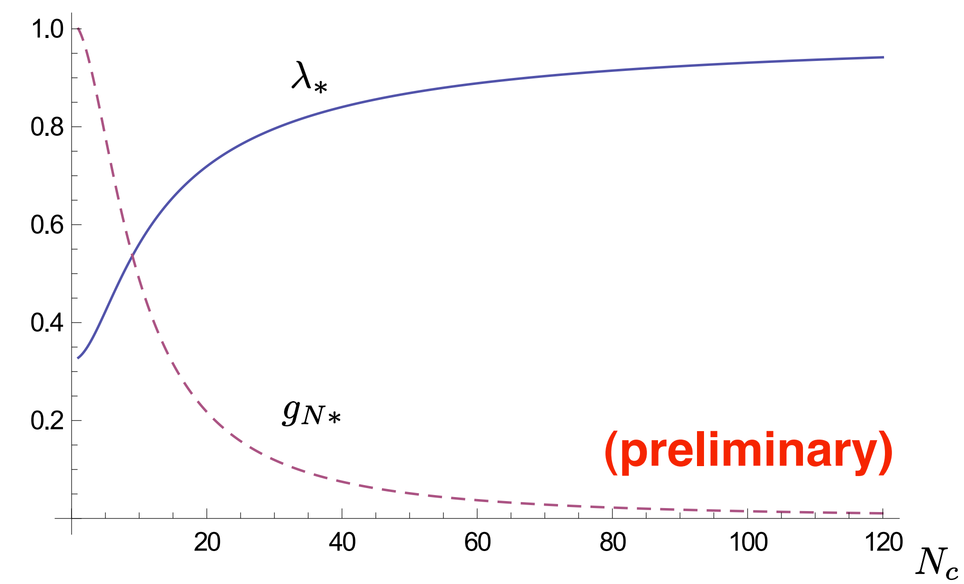
$$G_k = g/k^2$$

higher dimensions

Yang-Mills + gravity (Folkerts, DL, Pawlowski '11)

1-loop and quantum gravity: asymptotic freedom persists

fully coupled system: asymptotic freedom and safety persist



conformal symmetry

higher derivative gravity

1-loop — Codello, Percacci ('05) Niedermaier ('09)

1-loop and beyond — Benedetti, Machado, Saueressig ('09)

conformally invariant operator

Weyl coupling

$$\frac{1}{\sigma}\int d^4x\sqrt{g}\,C_{\mu\nu\rho\tau}\,C^{\mu\nu\rho\tau}$$

asymptotically `free' fixed point $\sigma_* = 0$ — DL, Rahmede (to appear)

entails $g_* > 0 \quad \lambda_* \neq 0$

QCD versus quantum gravitation: ``as close as it gets''

consistency with lattice

- **simplicial gravity formulation**

  ultraviolet fixed point in 3d and 4d (Hamber '00, Hamber, Williams '05)

  4d scaling exponents

  lattice: $\nu \approx 3$ **RG study:** $\nu = 8/3$ (DL '03)

  large-d scaling exponents

  lattice: $1/\nu \approx d - 1$ **RG study:** $1/\nu = 2d$ (DL '03)

- **causal dynamical triangulation**

  dimensional crossover from 4d Monte Carlo study (Ambjorn et. al. '05)

  large distances

  lattice: $D_{\text{eff}} \approx 4$ **RG studies:** $\eta \approx 0$

  short distances

  lattice: $D_{\text{eff}} \approx 2$ **RG studies:** $\eta \approx -2$ (Reuter et. al. '01)

quantum gravity at colliders

Standard Model

what if the fundamental Planck scale is as low as

$$M_* \approx \mathcal{O}(M_{\mathrm{EW}}) \ll M_{\mathrm{Pl}} \,?$$

circumnavigates the SM hierarchy problem

scenario with extra dimensions (Arkani-Hamed, Dimopoulos, Dvali '98)

$D = 4 + n$ compact extra dimensions of size L,

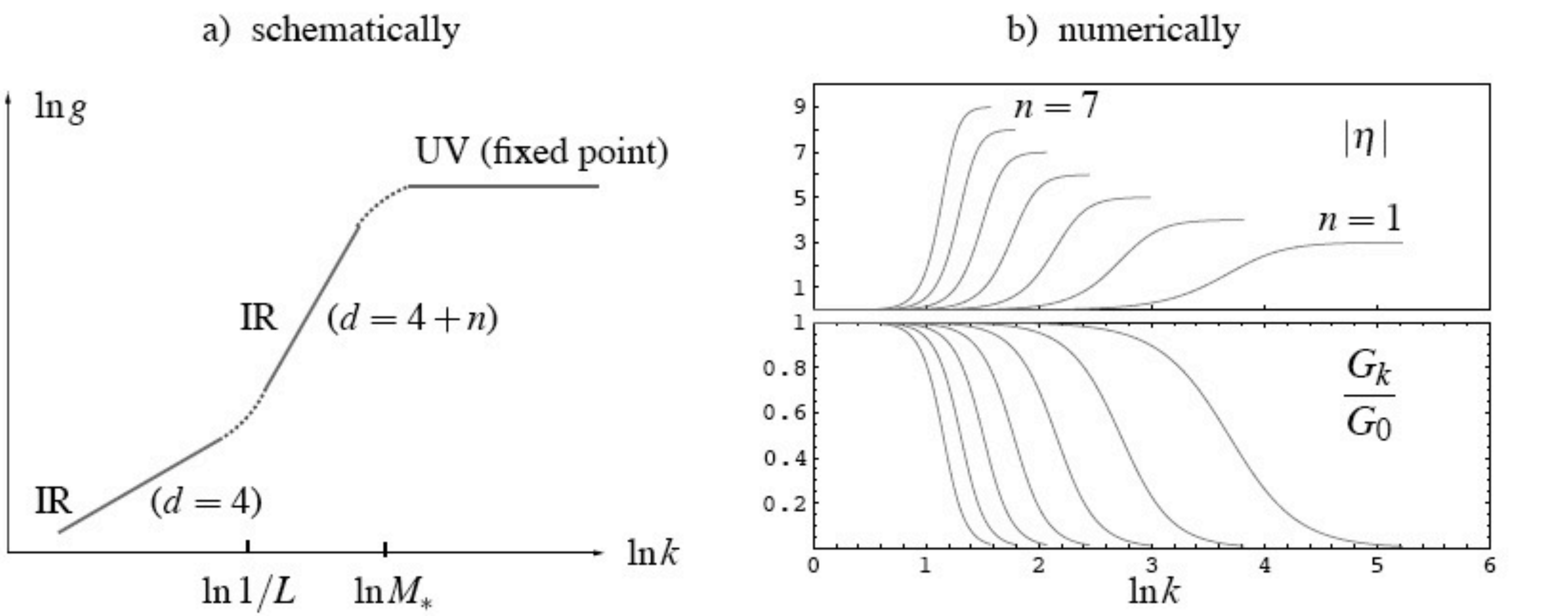
$$M_{\mathrm{Pl}}^2 \sim M_*^2 (M_* \, L)^n$$

scale separation $1/L \ll M_* \ll M_{\mathrm{Pl}}$

→ high-energetic particle colliders can **test quantisation of gravity**

running gravitational coupling

DL ('03), Fischer, DL ('05)



$$g(\mu) \equiv G(\mu)\,\mu^2$$

$$G(\mu) = G_N \cdot Z_N(\mu)^{-1}$$

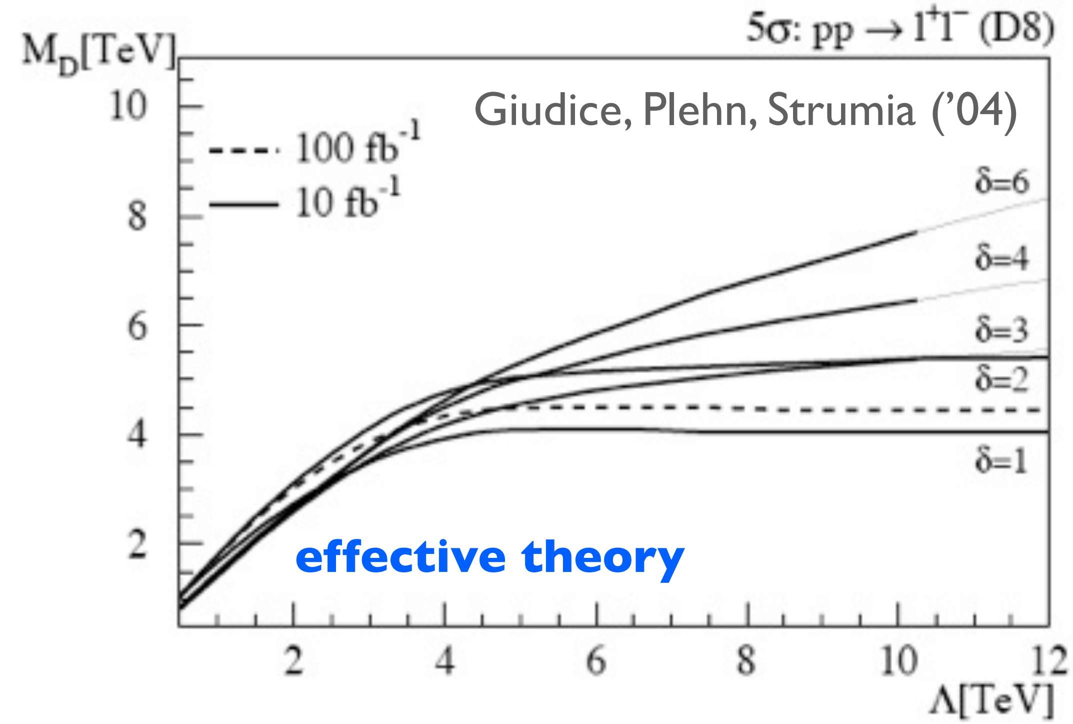
Drell Yan production

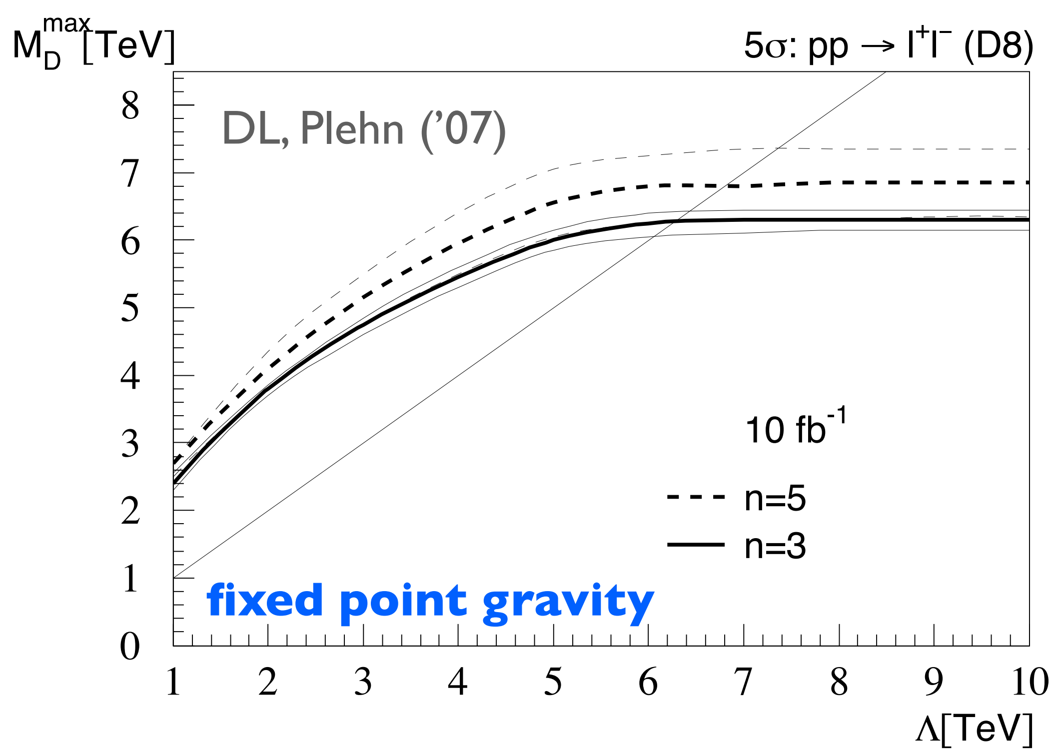
$$q\bar{q}, gg \xrightarrow{G} \mu^+\mu^-$$

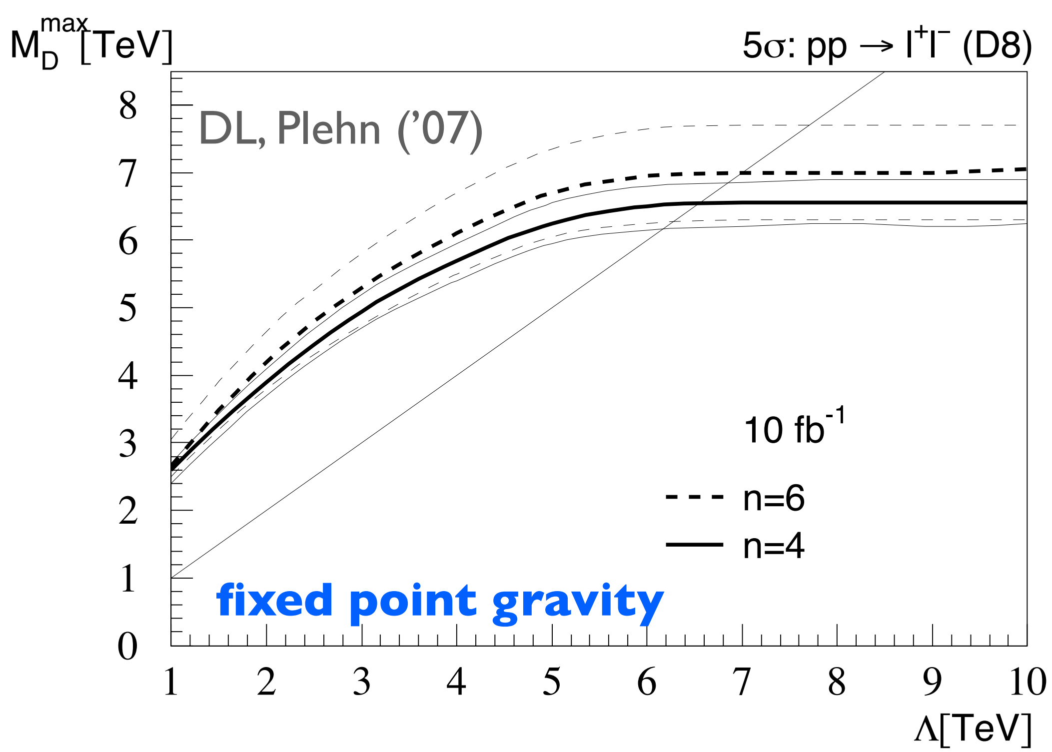
discovery reach

effective theory vs fixed point gravity

$$\sim \frac{1}{M_*^{n+2}} \int_0^\infty dm \frac{m^{n-1}}{s-m^2} \frac{G(k)}{G_D}$$ (DL, Plehn '07)







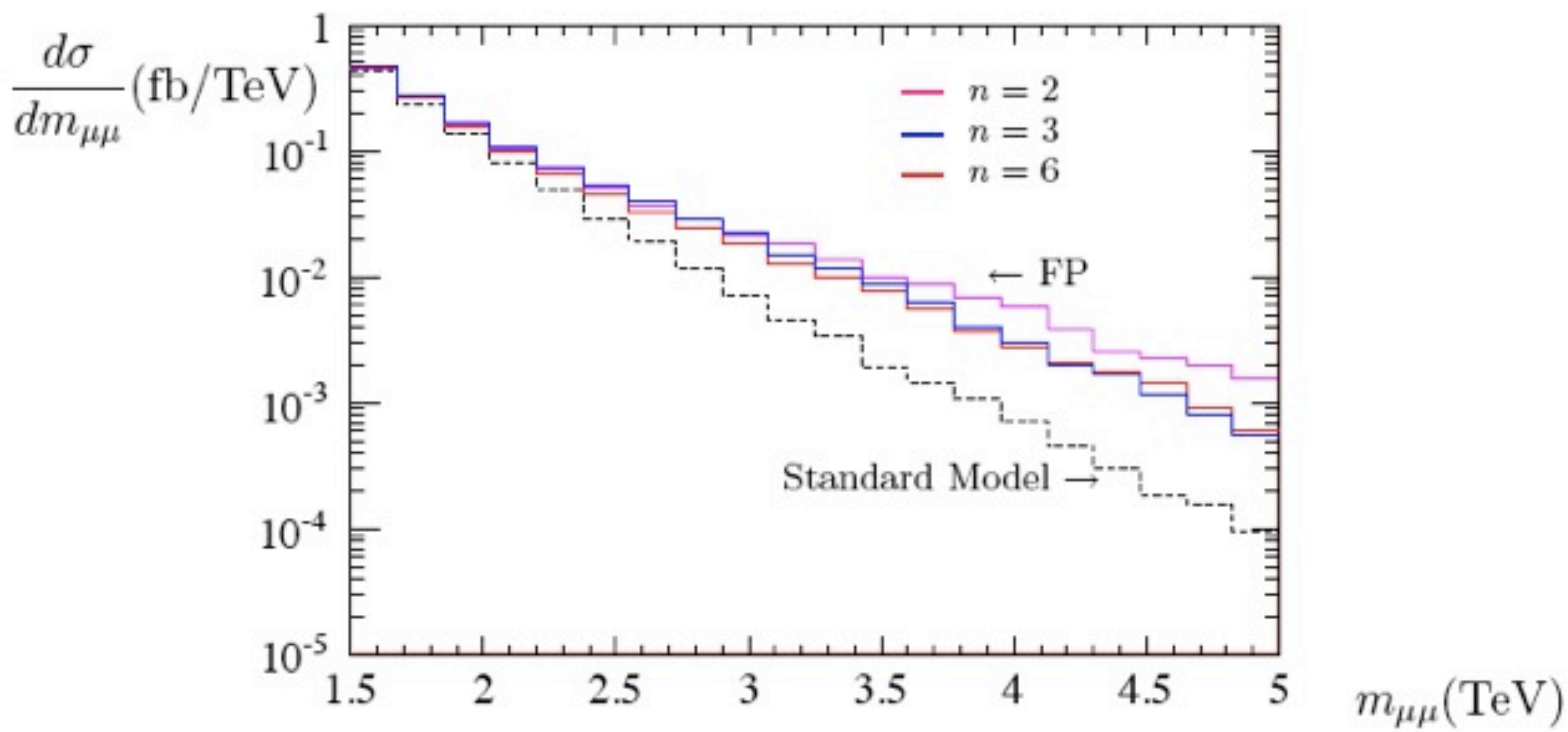
quantum gravity at colliders

gravitational Drell-Yan production

$$q\bar{q}, gg \xrightarrow{G} \mu^+\mu^-$$

RG dressing of propagator
finite amplitudes

renormalisation group + Monte Carlo simulations



$$D = 4 + n$$

(Gerwick, DL, Plehn '11)

quantum gravity and black holes

(Falls, DL, Raghuraman '10)

Schwarzschild black holes

horizon radius

prediction: smallest black hole

$$r_s = G_N\, M \to r_s = G\left(k = \frac{1}{r_s}\right) M$$

black hole thermodynamics

temperature from surface gravity

prediction: mini black holes are cold

$$T = \frac{d-3}{4\pi r_s}\left(1 + \frac{\eta(r_s)}{d-3}\right)$$

black hole production at the LHC

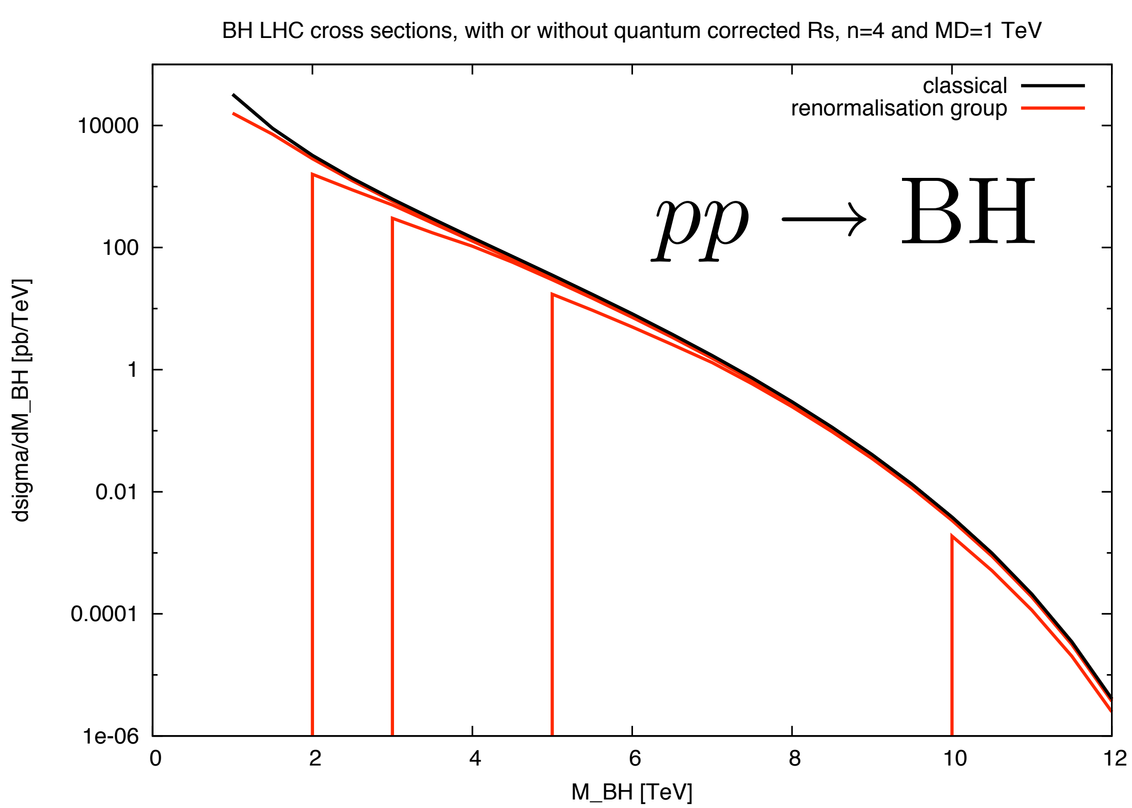
prediction:

semi-classical production

cross section suppressed

CTEQ61 parton distribution function with $Q^2 = M^2_{\rm BH}$

(Falls, Hiller, DL to appear)



conclusions and outlook

quantum gravity

increasing evidence for asymptotic safety
quantitative and structural insights, black holes

particle physics

towards a Standard Model including quantum gravity
low-scale quantum gravity: signatures at colliders

cosmology

late-time acceleration, IR fixed points
very early universe, inflation (Weinberg '09)
asymptotically safe cosmology (Hindmarsh, DL, Rahmede '11)

challenges

lattice $\leftrightarrow$ **RG** $\leftrightarrow$ loops $\longleftrightarrow$ strings $\leftrightarrow$ other

conclusions and outlook

quantum gravity

increasing evidence for asymptotic safety
metric field remains fundamental carrier of gravitational force
challenges
- more curvature
- structural insights
- black holes - entropy, information, thermodynamics

particle physics

towards a Standard Model including quantum gravity
fully coupled system
low-scale quantum gravity: signatures at colliders

outlook: cosmology

late-time acceleration, IR fixed points
very early universe, inflation
asymptotically safe cosmology

thank you!

Extra material

quantum gravity

effective action

$$\Gamma_k = \int \sqrt{g} \left(\frac{-R + 2\Lambda_k}{16\pi G_k} + \cdots \right) + S_{\text{matter},k} + S_{\text{gf},k} + S_{\text{ghosts},k}$$

- **extensions including** $\sqrt{g}$ **and** $\sqrt{g}(R)^i$, i $= 1, \cdots, n$.

n	θ'	θ''	θ_2	
1	$1.1 - 2.3$	$2.5 - 7.0$	$-$	(Lauscher, Reuter '01)
1	$1.4 - 2.0$	$2.4 - 4.3$	$-$	(DL '03)
1	$1.5 - 1.7$	$3.0 - 3.2$	$-$	(Fischer, DL '06)
1	2.4	2.2	$-$	(Codello, Percacci, Rahmede '07)
2	$2.1 - 3.4$	$3.1 - 4.3$	$8.4 - 28.8$	(Lauscher, Reuter '02)
2	1.4	2.8	25.6	(DL '07)
2	1.7	3.1	3.5	(DL '07)
2	1.4	2.3	26.9	(Codello, Percacci, Rahmede '07)

quantum gravity

effective action

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- **extensions including** $\sqrt{g}$ **and** $\sqrt{g}(R)^i$, i $= 1, \cdots, n$.

n	θ'	θ''	θ_2	θ_3	θ'_4	θ''_4	θ_6	θ_7	θ
1	2.38	2.17							
2	1.38	2.32	26.9						
3	2.71	2.27	2.07	-4.23					
4	2.86	2.45	1.55	-3.91	-5.22				
5	2.53	2.69	1.78	-4.36	-3.76	-4.88			
6	2.41	2.42	1.50	-4.11	-4.42	-5.98	-8.58		
7	2.51	2.44	1.24	-3.97	-4.57	-4.93	-7.57	-11.1	
8	2.41	2.54	1.40	-4.17	-3.52	-5.15	-7.46	-10.2	-12.

(Codello, Percacci, Rahmede '07, Machado, Saueressig '07)

asymptotic safety

overviews: DL **0810.3675** and **1102.4624**

Niedermaier **gr-qc/0610018** Reuter, Saueressig **0708.1317**

gravitation

Einstein-Hilbert (Souma '99, Reuter, Lauscher '01, DL '03)

f(R), polynomials in R (Codello, Percacci, Rahmede '08, Machado, Saueressig '09)

higher-derivative gravity (Codello, Percacci '05)
(Benedetti, Saueressig, Machado '09, Niedermaier '09)
(DL, Rahmede, '11)

higher dimensions, dimensional reduction (DL '03, Fischer, DL '05)

conformally reduced gravity (Reuter, Weyer '09, Machado, Percacci '10)

Holst action + Immirzi parameter (Daum, Reuter '10, Benedetti, Speciale '11)

signature effects (Manrique, Rechenberger, Saueressig '11)

gravitation + matter

matter (Percacci '05, Perini, Percacci '05)
(Narain, Percacci '09, Narain, Rahmede '09, Codello '11)

Yang-Mills gravity

1-loop: (Robinson, Wilczek '05, Pietrokowski, '06, Toms '07, Ebett, Plefka, Rodigast '08)

beyond: (Manrique, Reuter, Saueressig '09, Folkerts, DL, Pawlowski, 11, Harst, Reuter '11)

asymptotic safety

overviews: DL **0810.3675** and **1102.4624**

black hole physics

RG improvement in 4d (Bonanno, Reuter '00, '01, Reuter, Tiuran '10)

RG improvement in higher dimensions (Falls, DL '10, DL, Nikolakopoulos '10)

particle physics

gravitational Drell Yan at the LHC (DL, Plehn '07, Gerwick, DL, Plehn '11)

scattering and unitarity bounds (Brinckmann, Hiller, DL, Schroeder '11)

RG improved BH production (Falls, Hiller, DL at PASCOS '09)

cosmology

late-time acceleration, IR fixed points

very early universe, inflation (Reuter et. al '01, Weinberg '09)

asymptotically safe cosmology (Hindmarsh, DL, Rahmede '11)

gravitation + lattice

consistency with

- simplicial gravity / Regge calculus (Hamber '00, Hamber, Williams '04)
- causal DT (Amjorn, Jurkiewicz, Loll '04)
- euclidean DT (Laiho, Coumbe '11)

higher-dimensional black holes

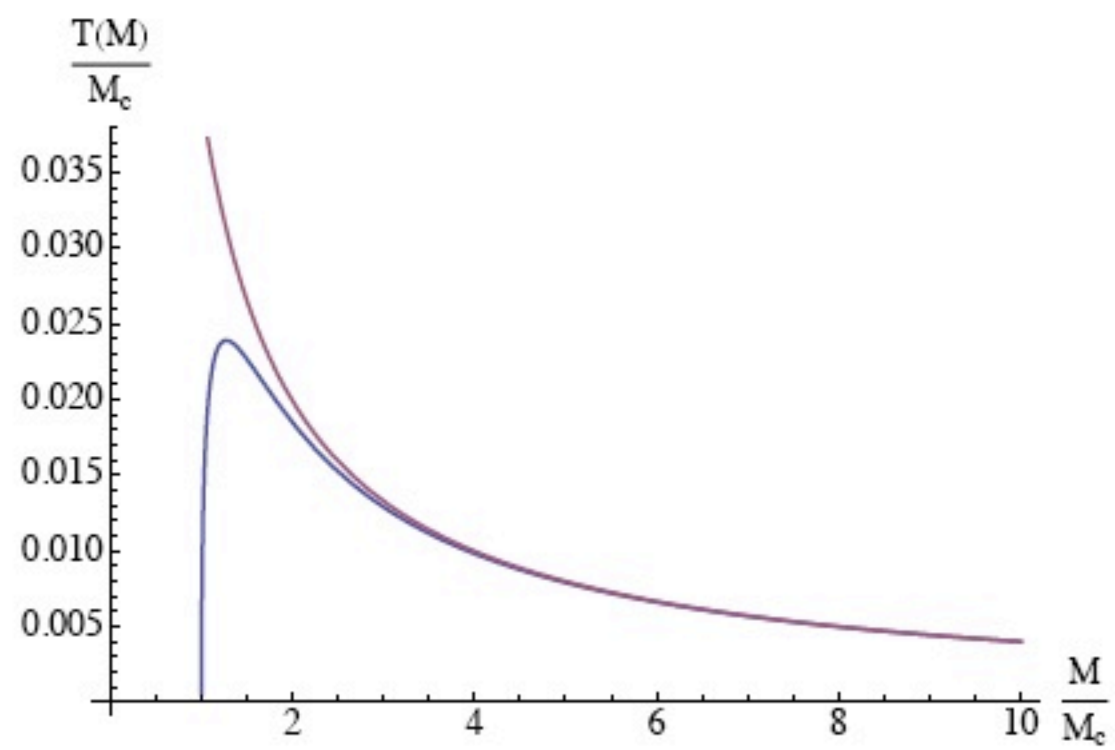
thermodynamics

Falls, DL, Raghuraman ('10)

temperature from surface gravity

$$T = \frac{\kappa}{2\pi} = \frac{d-3}{4\pi\, r_s}\left(1 + \frac{\eta(r_s)}{d-3}\right)$$

vanishes at $M = M_c$.



gravitation

physics of classical gravity

Einstein's theory $G_N = 6.7 \times 10^{-11} \frac{m^3}{\mathrm{kg}\, s^2}$

physics of quantum gravity

effective expansion parameter: $g_{\rm eff} \equiv G_N\, \mu^2 \approx \frac{\mu^2}{M_{\rm Pl}^2}$

$[G] > 0:$ superrenormalisable

$[G] = 0:$ renormalisable

$[G] < 0:$ **dangerous** interactions $[G_N] = 2 - d$

perturbative non-renormalisability

gravity with matter interactions

pure gravity (Goroff-Sagnotti term)

quantum gravity

effective action

$$\Gamma_k = \int \sqrt{g} \left(\frac{-R + 2\Lambda_k}{16\pi G_k} + \cdots \right) + S_{\mathrm{matter},k} + S_{\mathrm{gf},k} + S_{\mathrm{ghosts},k}$$

Einstein-Hilbert theory

$$\Lambda_k = \lambda\, k^2$$

$$G_k = g/k^2$$

higher dimensions

Yang-Mills + gravity

higher-derivative gravity + conformal symmetry (DL, Rahmede, in prep. '11)

Weyl tensor coupling $\frac{1}{\sigma_W} \int \sqrt{g}\, C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$

weakly coupled fixed point $\sigma_W = 0$ entails $g_* \neq 0 \quad \lambda_* \neq 0$