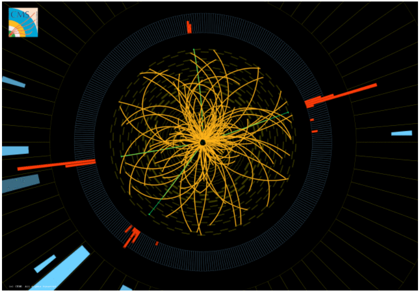
Automated One-Loop Calculations with GoSam

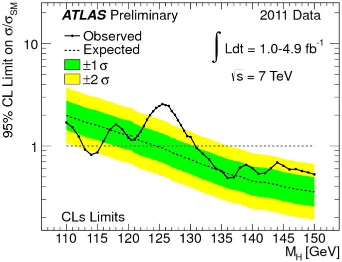
Gavin Cullen, DESY, Zeuthen

12th January 2012

2012: What does it have in store for us?

While the Higgs has been "glimpsed" ...
"... a Christmas toast to the predictability of physics."





2012: What does it have in store for us?

a few questions remain:

- Higgs mass is sensitive to quadratic corrections from heavy particles. Why is it so light?
- Is the minimal Higgs sector behind Electroweak Symmetry Breaking?
- Can we offer an explanation for Dark Matter?

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...the search for Supersymmetry (and other Beyond the Standard Model theories) continues...

- To find them: we need precise theory predictions for theory and background

Outline of Seminar

- NLO calculations outline
- GoSam: Introduction
- GoSam: inside the box
- GoSam: examples
- .end

NLO Calculation Set-Up

Two principles of QCD

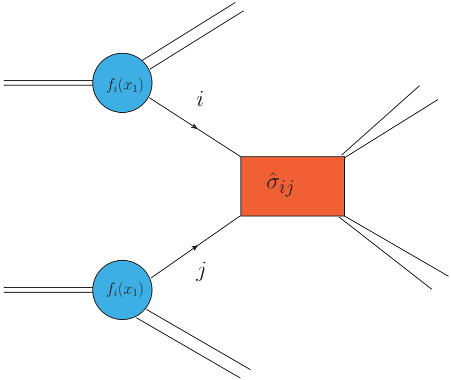
1. Asymptotic freedom $\alpha_s(Q^2) \to 0$ $Q^2 \to \infty$
The hard process can be expanded as a perturbation series in α_s:

$$\hat{\sigma}_{ab} = \hat{\sigma}_0 + \alpha_s(\mu_R)\hat{\sigma}_1 + ...$$

NLO Calculation Set-Up

Two principles of QCD

2. Factorisation $\leftrightarrow$ subdividing our calculation based on energy

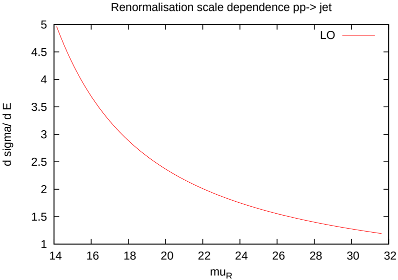


$\sigma_{AB} = \int dx_a dx_b f_{a/A}(x_a, \mu_F^2) f_{b/B}(x_b, \mu_F^2) \left[\hat{\sigma}_{ab}(\mu_F^2, x_a, x_b)\right]$

Both procedures leave an uncertainty in our calculation that is manifest in the choice of our scales

Next to Leading Order for the LHC

- Truncating series introduces scale dependence: calculation at N^{th} order
  $\frac{d\hat{O}bs}{d\log(\mu_R^2)} = \mathcal{O}(\alpha_s^{N+1})$



- At leading order huge scale variation
- At NLO scale dependence reduced

- $\hat{O}_{bs} = \sigma_0\alpha_s^2(\mu_R) \quad +\alpha_s^3(\mu_R)(\sigma_1 + 2b_0\log(\mu_R)\,\sigma_0)$
- For precise predictions we need to go to (at least NLO)

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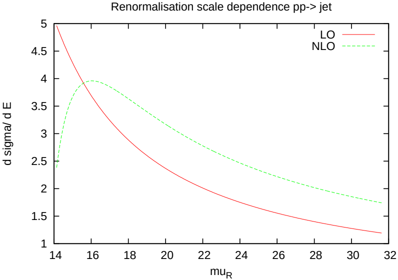

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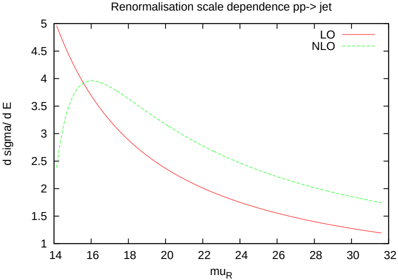

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The problems with LO

- Does not correspond to reality e.g. jets poorly modelled
- LO does not always correctly predict the shape of the distributions
- Does not take into account loop effects from New Physics
- and more

The NLO challenge

- Explosion of diagrams for multi-leg processes
  e.g. for $gg \to t\bar{t}b\bar{b}$ we must calculate $\mathcal{O}(1000)$ diagrams
- One has to consistently deal with the UV and IR poles
- Lots of scope for mistakes

$\to$ automization

What goes into an NLO calculation?

$$\sigma_{NLO} = \int_n d\sigma^{LO} + \int_n \left(d\sigma^V + \int_1 d\sigma^A \right) + \int_{n+1} \left(d\sigma^R - d\sigma^A \right)$$

- Tree level
- Virtual corrections
- Real emissions
- Subtraction terms for soft and collinear singularities

- Exploit modular structure of the calculation
- We focus on the virtual part

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NLO status

- $pp \to W^+W^-b\bar{b}$ [Denner, Dittmaier, Kallweit, Pozzorini '10; Bevilacqua, Czakon, van Hameren, Papadopoulos, Worek '11]
- $pp \to WZ + 4$ jets [BlackHat collaboration '10/'11]
- $pp \to WZ\gamma + 3$ jets [BlackHat collaboration '09/'10]
- $pp \to t\bar{t} + 2$ jets [Bevilacqua, Czakon, Papadopoul., Worek '10]
- $pp \to t\bar{t}b\bar{b}$ [Bredenstein, Denner, Dittmaier, Pozzorini '09; Bevilacqua, Czakon, Papadopoulos, Worek '09]
- $pp \to W\gamma\gamma j$ [Campanario, Englert, Rauch, Zeppenfeld '11]
- $pp \to W^+W^+jj$ [Melia, Melnikov, Rontsch, Zanderighi '10]
- $pp \to W^+W^-jj$ [Melia, Melnikov, Rontsch, Zanderighi '11]
- $pp \to 4b$ [Binoth et al '09; Greiner, Guffanti, Reiter, Reuter '11]
- NGluon ($N < 14$) [Badger, Biedermann, Uwer '11 (public)]
- $e^+e^- \to 5$ jets [Frederix, Frixione, Melnikov, Zanderighi '10]

Automated NLO tools

- Dedicated programs involve high level of automation [Denner, Dittmaier, Pozzorini et al, VBFNLO coll., MCFM, Blackhat, Rocket, ...]

Automated NLO tools

We are interested in the shift towards general tools that can calculate any process

Automation of subtraction terms for IR divergent real radiation

- MadDipole [Frederix, Greiner, Gehrmann]
- Dipole subtraction in Sherpa [Gleisberg, Krauss]
- TevJet [Seymour, Tevlin]
- AutoDipole [Hasegawa, Moch, Uwer]
- Helac-Phegas [Czakon, Papadopoulos, Worek]
- MadFKS [Frederix, Frixione, Maltoni, Stelzer]

Automated NLO tools

And at one-loop

- FeynArts/FormCalc/LoopTools (public) [T. Hahn et al]
- Helac-NLO (public) [Bevilacqua, Czakon, van Hameren, Papadopoulos, Pittau, Worek]
- MadLoop [Hirschi,Frederix,Frixione,Garzelli,Maltoni,Pittau] uses CutTools (public) [Ossola, Papadopoulos, Pittau] and MadFKS
- NGluon (public) [Badger, Biedermann, Uwer]
- GoSam

The challenge today is:

- One-loop component
- Interfacing the seperate components: finding a common language for the various tools to talk to each other

GoSam

[GC, Greiner, Heinrich, Luisoni, Mastrolia, Ossola, Reiter, Tramontano]

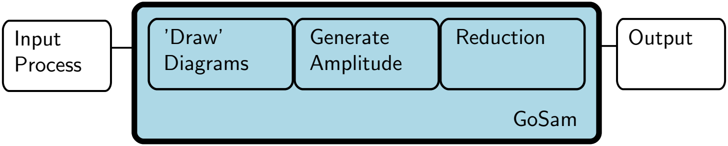
GoSam is a joining of Golem and Samurai:

- Golem: General One Loop Evaluator of Matrix Elements
- Samurai : Scattering Amplitudes from Unitarity based Reduction At Integrand level

Aim: to have a general tool that can compute the **one-loop amplitude** for any process in and beyond the SM.

- Public and open source: download at http://projects.hepforge.org/gosam/ [arXiv: 1111.6534 [hep-ph]]

GoSam: An Overview



- Diagrams drawn by QGRAF [Nogueira] using model files from FeynRules [Duhr et al]
- **Algebraic generation** of D-dimensional integrands based on Feynman diagrams using the Form [Vermaseren] library Spinney [GC et al] and optimized code generation by Haggies [Reiter]
- **Options for reduction:**
  - OPP-type reduction [Ossola, Papadopoulos, Pittau; Ellis, Giele, Kunszt, Melnikov]
  - "traditional" tensor reduction [golem95 library]
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- **Output** is a fortran source code that can fit together with the other components of the calculation

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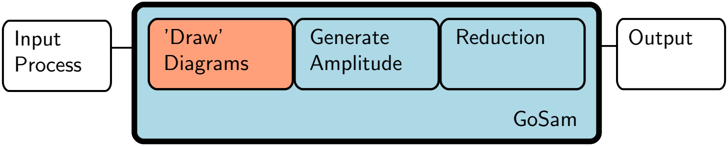

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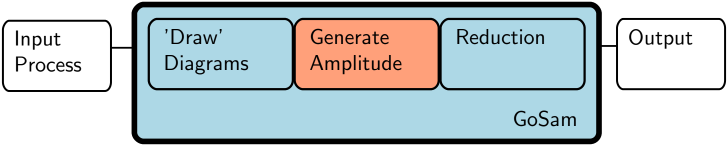

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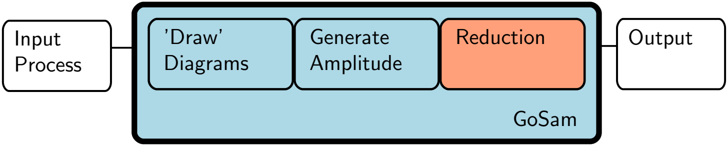

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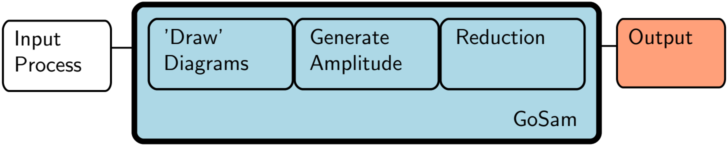

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GoSam: Quick Tutorial

Process: $u\overline{d} \to W^+W^+\overline{c}s \to e^+\nu_e\mu^+\nu_\mu\overline{c}s$

- Prepare input card

```
in=u,d~
out=c~,s,e+,ne,mu+,nmu
model=smdiag
order=QCD,2,4
zero=mU,mD,mC,mS,mB,me,mmu,wB
one=gs,e
helicities=-++-+-+-
extensions=dred,samurai
```

- and run..

```
gosam.py process.in
```

GoSam: Quick Tutorial

Process: $u\overline{d} \to W^+W^+\overline{c}s \to e^+\nu_e\mu^+\nu_\mu\overline{c}s$

- Draw diagrams
  make doc
- Write source files
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- Compile source files
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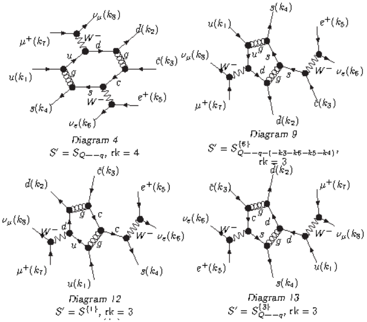

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```
Form is processing loop diagram 1 @ Helicity 0
  1.36 sec out of 1.36 sec
Haggies is processing abbreviations for loop diagram 1
Form is processing loop diagram 2 @ Helicity 0
  1.54 sec out of 1.55 sec
Haggies is processing abbreviations for loop diagram 2
Form is processing loop diagram 3 @ Helicity 0
  0.84 sec out of 0.85 sec
Haggies is processing abbreviations for loop diagram 3
Form is processing loop diagram 4 @ Helicity 0
  0.92 sec out of 0.93 sec
Haggies is processing abbreviations for loop diagram 4
Form is processing loop diagram 5 @ Helicity 0
  0.98 sec out of 0.99 sec
```

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- Compile source files
  make compile

GoSam: Quick Tutorial

Process: $u\overline{d} \to W^+W^+\overline{c}s \to e^+\nu_e\mu^+\nu_\mu\overline{c}s$

- We compare to MMRZ
  [Melia, Melnikov, Rontsch, Zanderighi (1104.2327)]

```
+---------------------------------------------------------------------+
|   __    __    ___   __    __  __                GoSam               |
|  / _)  /  \  / __) (  )  (  \/  )      An Automated One-Loop        |
| ( (/\( () )\__ \  /__\   )    (      Matrix Element Generator       |
|  \__/ \__/ (___/(_)(_)(_/\/\_)              Version 1.0             |
|                                                                     |
|                                  (c) The GoSam Collaboration 2011   |
+---------------------------------------------------------------------+

 LO:          1.143226406875312E-017
 NLO/LO, finite part:   23.3596454824712
 NLO/LO, single pole:   13.6255429253600
 NLO/LO  double pole:  -5.33333333333331

 cpu time (secs) :  5.299200000000000E-002
```

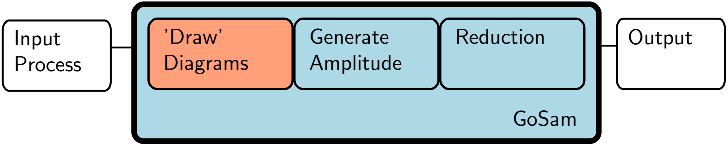
NLO/LO	GoSam	MMRZ
$1/\epsilon^2$	-5.33333333	-5.33333
$1/\epsilon$	13.62554293	13.62554
finite	23.35964548	23.35965

GoSam tests

Processes tested:

- $u\bar{d} \to W^+ s\bar{s} \to e^+ \nu_e s\bar{s}$
- $u\bar{d} \to W^+ gg \to e^+ \nu_e gg$
- $d\bar{d} \to Zgg \to e^+ e^- gg$
- $u\bar{d} \to W^+ gg \to e^+ \nu_e b\bar{b}$ (massive b)
- $u\bar{d} \to W^+ g \to e^+ \nu_e g$ (EW)
- $e^+ e^- \to Z \to d\bar{d}g$
- $\gamma\gamma \to \gamma\gamma\gamma\gamma$
- $q\bar{q} \to b\bar{b}b\bar{b}$
- $gg \to b\bar{b}b\bar{b}$
- $u\bar{d} \to W^+ W^+ s\bar{c} \to e^+ \nu_e \mu^+ \nu_\mu s\bar{c}$
- $u\bar{u} \to W^+ W^+ c\bar{c} \to e^- \bar{\nu}_e \mu^+ \nu_\mu c\bar{c}$
- $u\bar{d} \to W^+ W^- s\bar{c} \to e^- \bar{\nu}_e \mu^+ \nu_\mu \bar{s}c$
- Plus many $2 \to 2$ processes

GoSam: Under the bonnet



Manipulation of QGRAF output overseen by GoSam:

- Give the user greater flexibility in which diagrams to select for calculation (not limited to QGRAF filter)
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Amplitude generated using the Form library Spinney [GC, M. Koch-Janusz, T. Reiter]

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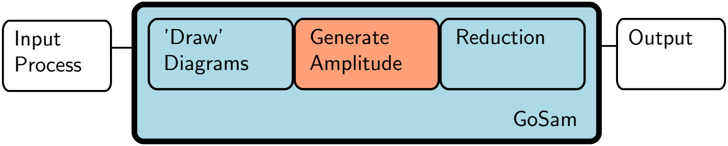

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Spinney- A Form Library for Helicity Spinors

[GC, M. Koch-Janusz, T. Reiter]

Numerator Algebra

Form [Vermaseren] is a symbolic manipulation program

- Form can handle large intermediate expressions
- Form's language = tensors, Lorentz indices, Dirac algebra, traces

Problems:

- Expressions too big $\leftrightarrow$ use helicity projections to break down amplitude into smaller gauge invariant pieces
- Form does not directly support helicity spinors and their manipulation

Spinney- A Form Library for Helicity Spinors

Spinney: available to download at
http://sourceforge.net/projects/spinney-form/

- implementation of helicity spinors and manipulations
- Light cone decomposition for massive spinors
- includes rules for dealing with Majorana fermions $\rightarrow$ inclusion of BSM theories including SUSY
- functions and procedures named to allow easy migration to S@M [D. Maitre, P. Mastrolia, 0710.5559]
- implements t'Hooft-Veltman regularisation scheme to allow extension to $D = 4 - 2\epsilon$ dimensions

Spinney: An Example

Helicity amplitude for $u\overline{u} \to d\overline{d}$

```
Vectors k1,k2,k3,k4;
Indices mu, nu;
#include spinney.hh
Local Amp = UbarSpb(k2) * Sm(mu) * USpa(k1) *
UbarSpb(k4) * Sm(mu) * USpa(k3)*d(mu,nu);
#call tHooftAlgebra
#call SpCollect
#call SpContractMetrics
#call SpContract
#call SpOpen
print;
.end
```

Spinney: An Example

Helicity amplitude for $u\bar{u} \to d\bar{d}$

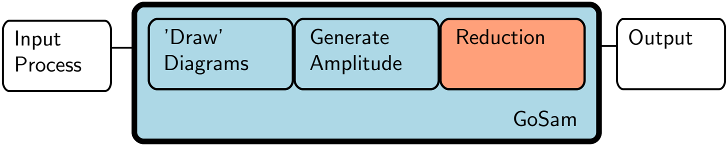
Output:

```
Amp =- 2*Spa2(k1,k3)*Spb2(k2,k4)
```

$$\mathrm{Amp} = -2\langle 13\rangle[24]$$

- Output in terms of spinor products $\rightarrow$ ideal for numerical evaluation
- Can extend this simple example to real-world processes

GoSam: Under the bonnet



GoSam: emphasis on flexibility in the way the amplitude is reduced:

- “Best” choice : ambiguous, strongly process dependent
- *Open laboratory* for testing new methods

Here we discuss the available options.

Traditional Tensor Reduction [Passarino Veltman]

Example: let's compute the tensor triangle

$$I^{\mu} = \int \frac{d^n k}{i\pi^{\frac{n}{2}}} \frac{k^{\mu}}{(k+r_1^2)(k+r_2^2)k^2} = A_1 r_1^{\mu} + A_2 r_2^{\nu}$$

Project out A_1 and A_2 using $2k \cdot r_1 = (k+r_1)^2 - k^2 - r_1^2$ etc. $\rightarrow$

$$\begin{aligned}
r_1^2 A_1 + r_1 \cdot r_2 A_2 &= \frac{1}{2} \int \frac{d^n k}{i\pi^{\frac{n}{2}}} \left[\frac{(k+r_1)^2 - k^2 - r_1^2}{(k+r_1)^2 (k+r_2)^2 k^2} \right] \\
&= \frac{1}{2} \int \frac{d^n k}{i\pi^{\frac{n}{2}}} \left[\frac{1}{k^2 (k+r_1)^2} - \frac{1}{(k+r_1)^2 (k+r_2)^2} - \frac{r_1^2}{(k+r_1)^2 (k+r_2)^2 k^2} \right] \\
&= \frac{1}{2} \left[I_2^n(r_1^2; 0, 0) - I_2^n((r_2 - r_1)^2; 0, 0) - r_1^2 I_3^n(r_1^2, (r_2 - r_1)^2, r_2^2; 0, 0, 0) \right] \\
&\equiv f
\end{aligned}$$

Traditional Tensor Reduction [Passarino Veltman]

Need to solve for A_1, A_2:

$$\begin{pmatrix} r_1^2 & r_1 \cdot r_2 \\ r_1 \cdot r_2 & r_2^2 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \begin{pmatrix} f \\ g \end{pmatrix}$$

$$G\vec{A} = \vec{f}$$

where f,g are functions of scalar integrals (bubbles and triangles):

$$\vec{A} = G^{-1}\vec{f}$$

but

$$G^{-1} \sim 1/(detG)$$

Traditional Tensor Reduction [Passarino Veltman]

Appearance of inverse Gram determinant:

- In some regions of phase space one can approach kinematical points where det G gets very small
- These singularities are a relic of our choice of reduction procedure (namely, our choice of a basis of *scalar* integrals as our end-point)
- They can ruin an automated numerical approach (need cancellations of large contributions from different sections of our result in problematic areas of phase space)
- Look for an alternative

Avoiding the Inverse Gram

- For most of the phase space there are no problems
- For the points where we run into problems we can choose a different basis to express our amplitude (Golem95 Basis) in [Golem95: T. Binoth, GC, J.Ph. Guillet, G. Heinrich, T. Kleinschmidt, E. Pilon, T. Reiter, M. Rodgers]

Define "problematic": ask is relative size of $detG < \Lambda$?
If not:

- Proceed as usual to basis of scalar integrals

If yes:

- Stop the reduction at basis with Feynman parameters in numerator
- Perform these integrals numerically

Golem95

[T. Binoth, GC, J.Ph. Guillet, G. Heinrich, T. Kleinschmidt, E. Pilon, T. Reiter, M. Rodgers]

One-loop amplitudes $\Rightarrow$

- Dimensionally regulated one-loop integrals

$$I_N^{d,\mu_1\cdots\mu_r}(S) = \int \frac{d^d k}{i\pi^{d/2}} \frac{k^{\mu_1}\cdots k^{\mu_r}}{\prod_{j=1}^N \left[(k+r_j)^2 - m_j^2 + i\delta\right]}$$

with $S_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2$.

- Strip away Lorentz structure $\rightarrow$ Form Factor rep.

$$\begin{aligned} I_N^{d,\mu_1\ldots\mu_r}(S) = &\sum_{j_1,\ldots,j_r} [r_{j_1}^{\cdot}\ldots r_{j_r}^{\cdot}]^{\mu_1\ldots\mu_r} A_N^r(j_1,\cdots,j_r;S) \\ &+ \sum_{j_1,\ldots,j_{r-2}} [r_{j_1}^{\cdot}\ldots r_{j_{r-2}}^{\cdot} g^{\cdot\cdot}]^{\mu_1\ldots\mu_r} B_N^r(j_1,\ldots,j_{r-2};S) \\ &+ \sum_{j_1,\ldots,j_{r-4}} [r_{j_1}^{\cdot}\ldots r_{j_{r-4}}^{\cdot} g^{\cdot\cdot} g^{\cdot\cdot}]^{\mu_1\ldots\mu_r} C_N^r(j_1,\ldots,j_{r-4};S) \end{aligned}$$

Golem95

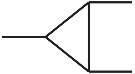
Form Factors are linear combinations of

$$I_N^d(l_1, \ldots, l_p, S) =$$
$$(-1)^N \Gamma\left(N - \frac{d}{2}\right) \int d^N z \frac{\delta(1 - \sum z_j) z_{l_1} \ldots z_{l_p}}{\left[-\frac{1}{2} z^T S z - i\delta\right]^{N-d/2}}$$

- Reduce to scalar integrals
- can introduce dangerous inverse gram determinants for N=3,4
- if det G small Golem95 avoids this step, instead completes numerical one-dimensional integration

Golem95: An Example

3-point, rank 2



$$I_3^{\mu\nu}(S) = \int d\bar{k} \frac{k^\mu k^\nu}{[(k+r_1)^2 - m_1^2][(k+r_2)^2 - m_2^2][k^2 - m_3^2]}$$
$$= r_1^\mu r_1^\nu A_{1,1}^{3,2}(S) + r_1^\mu r_2^\nu A_{1,2}^{3,2}(S) + r_2^\mu r_1^\nu A_{2,1}^{3,2}(S) + r_2^\mu r_2^\nu A_{2,2}^{3,2}(S)$$
$$+ g^{\mu\nu} B^{3,2}(S)$$

and

$$A_{i,j}^{3,2}(S) = I_3^n(i,j,S) \sim \frac{1}{(detG)^2} I_3^n(S) \quad B^{3,2}(S) = -\frac{1}{2} I_3^{n+2}(S)$$

Golem95: An Example

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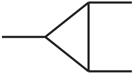

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Explicitly, for N=3,4:

- (N=3) Infra-red divergent $\rightarrow$ explicit expressions
- det G small $\rightarrow$ one-dimensional numerical integration
- otherwise: reduce to scalar integrals

Golem95C

Dedicated Fortran 95 library for the reduction and evaluation of tensor integrals
Latest version 1.2.0 available online
http://projects.hepforge.org/golem including:

- Inclusion of internal masses (Internal call to OneLOop [A. van Hameren] for finite massive scalar box/triangle)
- Scale μ has been added
- Contains all tensor coefficients up to rank six, six point integrals for massive and massless integrals (IR/ UV divergent and finite)
- Can also be used as a library for all types of scalar integrals
- Complex masses now included

Alternative approaches to Tensor Reduction

Reduction to Golem95 basis although numerically robust can result in large final expressions

For complex final states the final code can be too large to compile (upper limit $2 \to 3$)

We can reduce the size of code needed by:

- Building amplitude using its symmetries but...
- We lose using these symmetries as a vital check of the amplitude

Look for a more modern approach to the problem

Unitarity based Techniques

As opposed to *reducing* the tensorial structure we can use the analytic properties of the amplitude to *construct* the coefficients of our final results

OPP method in 4 dimensions:

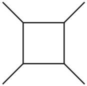
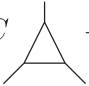
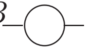
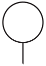
- We can write down the functional form for the amplitude in a universal (process independent way)
- Our problem is mapped to evaluating the coefficients of the scalar integrals

OPP method

Write N-point one-loop amplitude as:

$$\mathcal{A} = \int \frac{d^d k}{i\pi^{d/2}} \frac{\mathcal{N}(k)}{\prod_{j=1}^{N} \left[(k+r_j)^2 - m_j^2 + i\delta \right]} = \int \frac{d^d k}{i\pi^{d/2}} \prod_{j=1}^{N} \frac{\mathcal{N}(k)}{D_j}$$

We *start* from the expansion in terms of basis integrals (4,3,2,1-point functions)

$\mathcal{A} = D$  $+C$  $+B$  $+A$  $+\mathcal{R}$

and construct the loop amplitudes using cuts

- Can construct the coefficients A,B,C,D in d=4
- Solve the system of equations sequentially [OPP method[Ossola, Papadopoulos, Pittau]]
- For the rational terms R we need $d \neq 4$

OPP method

Process-independent functional form for the numerator

$$\mathcal{N}(k) = \sum_{i_0<i_1<i_2<i_3} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(k; i_0 i_1 i_2 i_3)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i$$
$$+ \sum_{i_0<i_1<i_2} \left[c(i_0 i_1 i_2) + \tilde{c}(k; i_0 i_1 i_2)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i$$
$$+ \sum_{i_0<i_1} \left[b(i_0) + \tilde{b}(k; i_0)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i + \sum_{i_0} \left[a(i_0) + \tilde{a}(k; i_0)\right] \prod_{i\neq i_0}^{m-1} D_i$$

$\tilde{a}(k), \tilde{b}(k), \tilde{c}(k), \tilde{d}(k)$ vanish upon integration

OPP method: example

Consider the a three point amplitude with numerator $\mathcal{N}(k)$:
$D_i = (k + r_i)^2 - m_i^2$

$$\int \frac{d^d k}{i\pi^{d/2}} \frac{\mathcal{N}(k)}{D_1(k)D_2(k)D_3(k)} = C_{123} \int \frac{d^d k}{i\pi^{d/2}} \frac{1}{D_1 D_2 D_3}$$
$$+ B_{12} \int \frac{d^d k}{i\pi^{d/2}} \frac{1}{D_1 D_2} + B_{13} \int \frac{d^d k}{i\pi^{d/2}} \frac{1}{D_1 D_3} + B_{23} \int \frac{d^d k}{i\pi^{d/2}} \frac{1}{D_2 D_3}$$
$$+ A_1 \int \frac{d^d k}{i\pi^{d/2}} \frac{1}{D_1} + A_2 \int \frac{d^d k}{i\pi^{d/2}} \frac{1}{D_2} + A_3 \int \frac{d^d k}{i\pi^{d/2}} \frac{1}{D_3}$$

Multiply by $D_1 D_2 D_3 \implies$

$$\begin{aligned} \mathcal{N}(k) = {} & C_{123} \\ & + B_{12} D_3 + B_{13} D_2 + B_{23} D_1 \\ & + A_1 D_2 D_3 + A_2 D_1 D_3 + A_3 D_1 D_2 \end{aligned}$$

OPP method: example

Let $\mathcal{N}_i = \mathcal{N}(k_i)$, and evaluate it seven times:
We need to invert the equation:

$$\begin{pmatrix} \mathcal{N}_1 \\ \mathcal{N}_2 \\ \mathcal{N}_3 \\ \mathcal{N}_4 \\ \mathcal{N}_5 \\ \mathcal{N}_6 \\ \mathcal{N}_7 \end{pmatrix} = \begin{pmatrix} 1 & D_3 & D_2 & D_1 & D_2 D_3 & D_1 D_3 & D_1 D_2 \\ 1 & D_3 & D_2 & D_1 & D_2 D_3 & D_1 D_3 & D_1 D_2 \\ 1 & D_3 & D_2 & D_1 & D_2 D_3 & D_1 D_3 & D_1 D_2 \\ 1 & D_3 & D_2 & D_1 & D_2 D_3 & D_1 D_3 & D_1 D_2 \\ 1 & D_3 & D_2 & D_1 & D_2 D_3 & D_1 D_3 & D_1 D_2 \\ 1 & D_3 & D_2 & D_1 & D_2 D_3 & D_1 D_3 & D_1 D_2 \\ 1 & D_3 & D_2 & D_1 & D_2 D_3 & D_1 D_3 & D_1 D_2 \end{pmatrix} \begin{pmatrix} C_{123} \\ B_{12} \\ B_{13} \\ B_{23} \\ A_1 \\ A_2 \\ A_3 \end{pmatrix}$$

OPP method: example

Choosing k_i such that certain propagators are zero this becomes:

$$\begin{pmatrix} \mathcal{N}_1 \\ \mathcal{N}_2 \\ \mathcal{N}_3 \\ \mathcal{N}_4 \\ \mathcal{N}_5 \\ \mathcal{N}_6 \\ \mathcal{N}_7 \end{pmatrix} = \left(\begin{array}{c|ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 1 & D_3 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & D_2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & D_1 & 0 & 0 & 0 \\ \hline 1 & D_3 & D_2 & 0 & D_2 D_3 & 0 & 0 \\ 1 & D_3 & 0 & D_1 & 0 & D_1 D_3 & 0 \\ 1 & 0 & D_2 & D_1 & 0 & 0 & D_1 D_2 \end{array}\right) \begin{pmatrix} C_{123} \\ B_{12} \\ B_{13} \\ B_{23} \\ A_1 \\ A_2 \\ A_3 \end{pmatrix}$$

Can solve sequentially and algebraically.

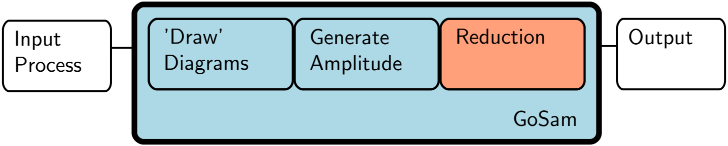
Samurai reduction method [Mastrolia, Ossalo, Reiter, Tramontano]

Our simple example was illustrative of the OPP method in 4-dimensions.
Samurai is a tool based on this:

- OPP Reduction Algorithm [Ossalo, Papadopoulos, Pittau]
- Extended to d-dimensions [Ellis, Giele, Kunszt, Melnikov]
- Coefficients of Polynomials via Discrete Fourier Transform [Mastrolia et al.]

More details and download at
http://projects.hepforge.org/samurai/

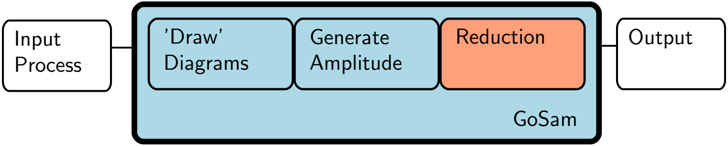
GoSam: Under the bonnet



Reduction options (in brief):

- Samurai: sampling of groups of diagrams
- Samurai: sampling of individual diagrams
- tensor reduction with Golem95
- tensorial reconstruction[Heinrich, Ossola, Reiter, Tramontano] + Samurai
- plus permutations
- i.e. if Samurai "fails" we proceed with Golem95

GoSam: Under the bonnet



Different choices of integral libraries:

- Golem95C (which includes a link to LoopTools [T.Hahn])
- QCDLoop [Ellis, Zanderighi]
- OneLOop [A. van Hameren]
- PJFry [Yundin, Riemann,Fleischer]

Rational terms $\mathcal{R}$ can be produced:

- analytically/ independently of numerator
- as part of the numerator

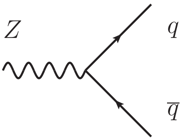
GoSam applications: Neutralino Pair Production

[GC, Heinrich, Greiner]

- We are interested in extending GoSam Beyond the Standard Model, in particular, calculations in SUSY
- We looked at the full NLO QCD corrections to $pp \to \chi_1^0 \chi_1^0$

MSSM in a nutshell

- Supersymmetry: symmetry relating different representations of the Lorentz group i.e. in its minimal formulation (MSSM) : *for every boson (fermion) there exists a fermion (boson)*.
- Realistic versions require some kind of soft SUSY breaking (e.g. must have $m_g = 0 < m_{\tilde{g}}$)
- Schematically: R parity $\Longrightarrow$ we can generate SUSY Feynman rules starting form SM ones if we perform SUSY transformations pairwise:



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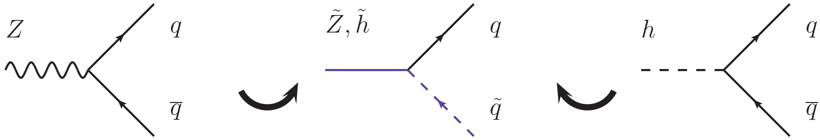


$\tilde{Z}, \tilde{\gamma}, \tilde{h}, \tilde{H}$ 'mix' to form the physical neutralino states χ_i^0.

The Problem with SUSY @ 1-loop

If we apply dimensional regularisation (DREG) we break the supersymmetry of our exactly supersymmetric Lagrangian
How? In t'Hooft-Veltman

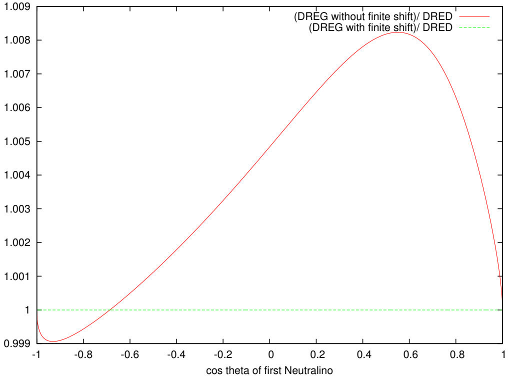
- massless gauge fields extended to n dimensions $\implies$ gluon has $n-2$ degrees of freedom
- Weyl fermion kept in 4 dimensions $\implies$ has 2 degrees of freedom

Supersymmetry is broken as $n_{\tilde{g}} \neq n_g$
The extra degrees of freedom in the loop can give finite differences between schemes.
Options:

- choose a scheme that respects SUSY (dimensional reduction: $n_g = 2$)
- Implement finite SUSY-restoring counter terms in dimensional regularisation

In practice: do both and cross-check!

DRED vs. DREG

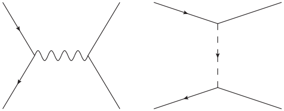


Plot of ratio of renormalised virtual amplitude without and with counterterms to the DRED result.

Neutralino Pair Production

Examples of diagrams

Partonic subprocesses LO: $qq \to \chi_1^0 \chi_1^0$

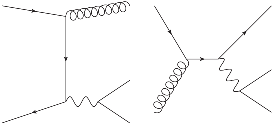


Neutralino Pair Production

Examples of diagrams

real emission: new "LO" type channels open up at NLO. We have

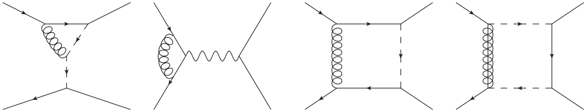
- $qq \to \chi_1^0 \chi_1^0 g$
- $qg \to \chi_1^0 \chi_1^0 q$
- $\overline{q}g \to \chi_1^0 \chi_1^0 \overline{q}$



Neutralino Pair Production

Examples of diagrams

virtual contribution:



Neutralino Pair Production

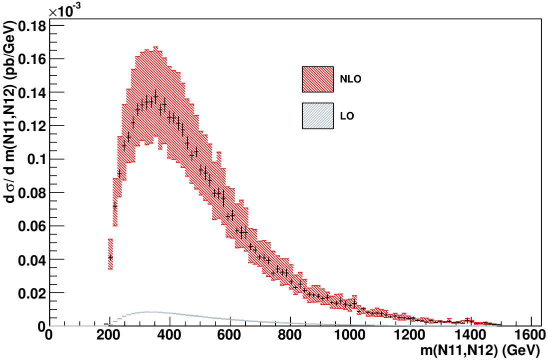
The details

- 5 massless quark scheme (q= u,d,s,c,b)
- Pdf set MSTW08
- SUSY point SPS1amod $\Longrightarrow$ low sparticle masses $\sim$ upper bound on the result
- On-shell renormalisation scheme implemented
- SUSY restoring counter-terms included
- Real emission and IR subtraction terms handled by MadGraph/MadDipole

Neutralino pair production: checks

- Leading Order
  - Two independent calculations: MadGraph and GoSam
- Virtual calculation
  - Cross check of two independent implementations:
    - FeynRules model file, Samurai reduction
    - Home-made model file, Golem95 reduction
- Cancellation of poles in $1/(d-4)$ between the IR subtractions and the virtual part

Neutralino pair production: results



- We vary $\mu_F = \mu_R \in [\frac{m_Z}{2}, 2m_Z]$
- We apply a jet-veto $p_T > 20\text{GeV}$ to suppress "LO" type contributions from new channels opening up at NLO

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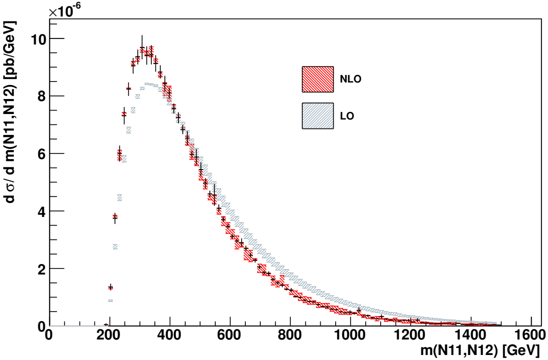

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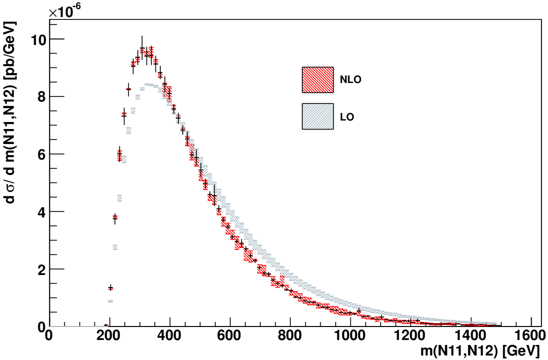


Our intention is to examine only the the radiative corrections to our original process $pp \to \chi_1^0 \chi_1^0 \implies$

- reduced scale uncertainty
- *NLO not a simple scaling of LO result*

Neutralino pair production: Future directions

Now we have the interface between virtual and real contributions we are ready to do some more calculations

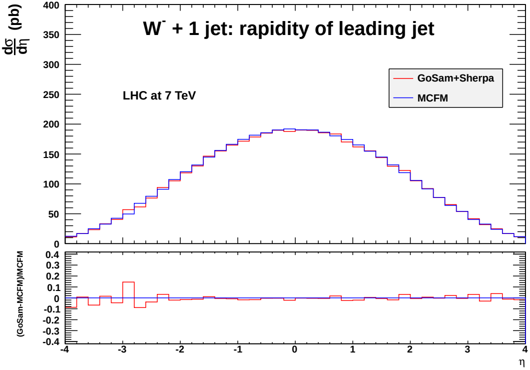
- Add a jet to the final state: $pp \to \chi_1^0 \chi_1^0 j$ ($2 \to 3$)
- More detailed exploration of the SUSY parameter space
- Explore production of heavier neutralinos and their cascade decays (distinctive signatures for LHC)

For GoSam:

- Implementation of SUSY counter terms and renormalization in GoSam $\leftrightarrow$ full MSSM support

GoSam applications: Interface

- GoSam includes a standard interface to real radiation programs using the Binoth Les Houches accord [arxiv 1001.1307]
- tested with Sherpa and PowHeg
- example $pp \to W + jet$ [figures by G. Luisoni, J. Archibald]



- good agreement with MCFM

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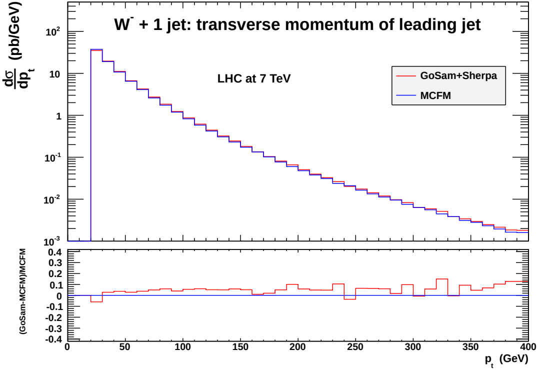

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Summary and Outlook

- Automated tools necessary to have NLO predictions for the LHC, and to be ready for new results from the LHC
- Presented GoSam: a program for the automated calculation of multi-leg and multi-scale one-loop amplitudes in and beyond the Standard Model
- The tool is numerically robust, open source and public: download at http://projects.hepforge.org/gosam
- I encourage you to download and play with it; support at hand in room 3L16
- Presented some applications of GoSam inside full NLO calculations for a SM and BSM process